Unit-3: Joint Distributions

1. For the following bivariate probability distribution find (i) Marginal distributions of X and Y (ii) $p(X \le 1, Y = 2)$ (iii) $p(X \le 2, Y < 2)$

X\Y	0	1	2	3
0	0.840	0.030	0.020	0.010
1	0.060	0.010	0.008	0.002
2	0.010	0.005	0.004	0.001

2. For the following bivariate probability distribution find,

(i) $E(X)$ and $E(Y)$

(ii) V(X) and V(Y)

(iii) correlation coefficient between X and Y

X\Y	0	1	2	3
0	0.840	0.030	0.020	0.010
1	0.060	0.010	0.008	0.002
2	0.010	0.005	0.004	0.001

3. For the following bivariate probability distribution find,

(i) $E(X)$ and $E(Y)$

(ii) V(X) and V(Y)

(i) Conditional densities of X given Y=1

(ii) Conditional densities of Y given X=2

X\Y	1	2	3	4
1	4/36	3/36	2/36	1/36
2	1/36	3/36	3/36	2/36
3	5/36	1/36	1/36	1/36
4	1/36	2/36	1/36	5/36

4. Let X denote the number of "do loops" in fortran program and Y the number of runs needed for a novice to debug the program. Assume that the joint density for (X, Y) is given table.

x/y	1	2	3	4
0	0.059	0.100	0.050	0.001
1	0.093	0.120	0.082	0.003
2	0.065	0.102	0.100	0.010
3	0.050	0.075	0.070	0.020

(a) Find the probability that a randomly selected program contains at most one do loop and requires at least two runs to debug the program.
(b) Find E[XY].
(c) Find the marginal densities for X and Y. Use these to find the mean and variance for both X and Y.
(d) Find the probability that a randomly selected program requires at least two runs to debug given that it contains exactly one do loop.
(e) Find $Cov(X,Y)$. Find the correlation between X and Y. Based on the observed value of ρ, can you claim that X and Y are not independent? Explain.

5. For the following bivariate probability distribution obtain,
(i) marginal distributions of X and Y
(ii) the conditional distribution of X given Y=0
(iii) the conditional distribution of Y given X=1

X\Y	0	1	2	3
0	0.02	0.08	0.10	0.04
1	0.05	0.02	0.25	0.06
2	0.03	0.12	0.15	0.08

6. Given the following bivariate probability distribution, obtain (i) Marginal distributions of X and Y (ii) the conditional distribution of X given Y=2 (iii) E[XY]

X\Y	0	1	2
-1	1/15	3/15	2/15
0	2/15	2/15	1/15
1	1/15	1/15	2/15

7. Let X denote the temperature $(°C)$ and let Y denote the time in minutes that it takes for the diesel engine on an automobile to get ready to start. Assume that the joint density for (X,Y) is given by $f(x,y)=c(4x+2y+1); \quad 0\le x\le 40, 0\le y\le 2$

(i) Find the value of c that makes this a density

(ii) Find the marginal densities for X and Y.

(iii) Find the probability that on a randomly selected day it will take at least one minute for the car to be ready to start.

(iv) Find the probability that on a randomly selected day the air temperature will exceed $20°C$

(v) Are X and Y independent?

8. The joint density for (X,Y) is given by $f(x,y)=\dfrac{x^3y^3}{16} \quad 0\le x\le 2, 0\le y\le 2$

(i) Find the marginal densities for X and Y.

(ii) Are X and Y independent?

(iii) Find $p(X\le 1)$

(iv) Find $p(X\le 1, Y\le 1)$

9. The joint density for (X,Y) is given by $f(x,y)=xye^{-x}e^{-y} \quad x>0, y>0$

(i) Find the marginal densities for X and Y.

(ii) Cov(X,Y)

(iii) Are X and Y independent?

(iv) Find $p(X\le 1)$

10. The joint density for (X,Y) is given by f(x) = 2 ; 0 < x < 1 , 0 < y < x

= 0, elese where

(i) Find the marginal densities for X and Y.

(ii) Find the conditional density of Y given X

(iii) Find the conditional density of X given Y

(iv) Are X and Y independent?

11. The joint density for (X,Y) is given by $f(x,y)=\dfrac{x^3y^3}{16} \quad 0\le x\le 2, 0\le y\le 2$

obtain the correlation coefficient between X and Y.

12. The joint density for (X,Y) is given by f(x,y) = 8xy ; 0 < x < y < 1

= 0, elese where

(i) Verify f(x,y) satisfies the conditions necessary to be density

(ii) Find the marginal densities for X and Y.

(iii) Are X and Y independent?

(iv) Find V(X)

(v) Find the conditional density of Y given X=x

(vi) Find E[XY]

13. The joint density for (X,Y) is given by f(x,y) = $\frac{1}{x}$ for 0<y<x
    (v) Find the marginal densities for X and Y .
    (vi) Find E[X] and E[Y]
    (vii) Find E[XY]
    (viii) Find Cov[XY]

14. Let X be a random variable with density $f_x(x)=\frac{1}{4}xe^{-\frac{x}{2}}$, $x \geq 0$ and let $y=-\frac{1}{2}x+2$. Find the density for y.
15. Let X be a random variable with density $f_x(x)=e^{-x}$, x>0 and let Y= e^x . Find the density for Y.
16. Let C denote the temperature in degrees Celsius to which a computer will be subjected in the field. Assume that C is normally distributed over the interval (15,21). Let F denote the field temperature in degrees Fahrenheit so that F=(9/5)C+32. Find the density of F.
17. Let X be a random variable with density $f_x(x)=2x$, $0<x<1$. If (x)=Y=3x+6 then find $f_y(y)$
18. Assume that X and Y are independent uniformly distributed random variables over (0,2) and (0,3) respectively. If U= X-Y and V= X+Y find the density function of (U,V)

19. Consider the linear transformation defined by T: $\begin{cases} U = 2x + y \\ V = x + 3y \end{cases}$

    (a) Is this transformation invertible? If so, find the defining equations for T^{-1}
    (b) Find the Jacobian for T^{-1}

20. If X and Y are independent standard normal variables over and If U= X+Y and V= Y find the density function of (U,V)