CS 839: Systems Verification

Fall 2025

Today's agenda

1. Recq demo / review
2. Informal proofs
3. Induction

$P(n) \triangleq 1 + 2 \cdots + n = n(n+1)/2$

$P(1) \quad 1 = 1(1+1)/2$

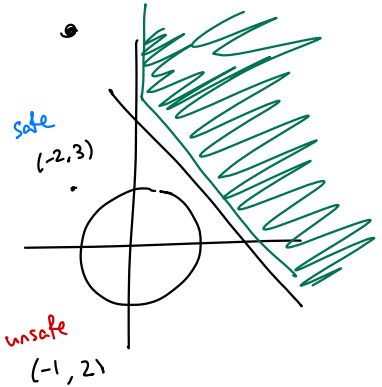
$= 1$

$\forall k, \quad P(k) \rightarrow P(k+1)$

IH: $1 + \cdots + k = k(k+1)/2$

$= \begin{cases} \boxed{1 + \cdots + k} + (k+1) = (k+1)(k+2)/2 \\ k(k+1)/2 + (k+1) = (k+1)(k+2)/2 \end{cases}$

$= \begin{cases} \frac{k(k+1)}{2} + \frac{2(k+1)}{2} \\ \frac{(k+2)(k+1)}{2} = \frac{(k+1)(k+2)}{2} \end{cases}$



$e = \sigma_1, \sigma_2, \sigma_3, \ldots$

$\uparrow$ eg. $(5,3)$

$tr(\sigma, \sigma') = \sigma \searrow \sigma' \vee \sigma \uparrow \sigma' \vee$ noop

$\sigma_1 = (0,5)$

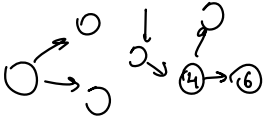
$\forall i,\ tr(e(i), e(i+1))$

$\forall i,\ e(i)$ safe

$\forall \sigma.\ inv(\sigma) \rightarrow P(\sigma)$

$\forall \sigma, \sigma'.\ P(\sigma) \rightarrow tr(\sigma, \sigma') \rightarrow P(\sigma')$

induction for transition systems

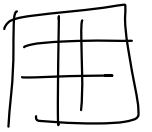


$\forall e,$ valid(e) $\rightarrow$ $\forall i,\ P(e(i))$

$\uparrow$ follows inv and tr

Lecture 4: Abstraction

- Most of today will be a fun (?!) activity
- Read the notes



```
location := {
    row: int [0, 2]
    col: int [0, 2]
}

player := black | white

cell := empty | full (p: player)

state := location → cell

game_over (s: state)

move (s, s') : Prop
```

Contents of box: three 1's, two 2's,
two 3's, two 4's,
one 5 of each color

8 "Clock" tokens (hints)

4 "Black Fuse" tokens (lives/misplays)

assume 3 players, hand size of 5 if you like

Lecture 5: Hoare Logic (part 1)

Learning Outcomes:

1. Explain what pre- and post-conditions mean
2. Formally analyze a "whiteboard" programming language

$\{P\}\ e\ \{Q\}$ — if P holds and we run e, and it finishes, then Q } soundness

- semantics: "run e"?
- logic: set of rules for $\{P\}\ e\ \{Q\}$
  - soundness

$\{P\}\ e\ \{\lambda v.\ Q(v)\}$

$e \rightsquigarrow v'$

$Q(v')$

code

recursive calls mod

proof

inductive

proof of euclid "call" proof of mod

euclid(a, b)

mod(a, b)

$\{\_\}$ mod(a, b) $\{c.\ \_\_\_\}$

$\{\_\}$ euclid(a, b) $\{c.\ gcd(a, b, c)\}$

$3 : \text{nat}$

$\overline{3} : \text{val}$

expr $e ::= x \mid v \mid \lambda x.e \mid e_1\ e_2$

$\mid$ if e then e_1 else e_2

$\mid e_1 + e_2$

$\mid (e_1, e_2) \mid \pi_1\ e \mid \pi_2\ e$

values $v ::= \lambda x.\ e \mid \overline{n} \mid \text{true} \mid \text{false} \mid (v_1, v_2)$

let $x := e_1$ in e_2 $::=$ $(\lambda x.\ e_2)\ e_1$

$(\lambda x.\, e)\ v \longrightarrow e[v/x]$ β-reduction $(\lambda x.\ x + \bar{3})\ \bar{5}$

$\bar{5} + \bar{3}$

if false then e_1 else $e_2 \to e_2$

$\pi_1\ (v_1, v_2) \longrightarrow v_1$

$\pi_2\ (v_1, v_2) \longrightarrow v_2$

$\bar{n}_1 + \bar{n}_2 \longrightarrow \overline{(n_1 + n_2)\ \%\ 2^{64}}$

step

$e_1 \to e_2$

$e_1 \to^* e_2$

we have products (tuples)

add sums (inductives / enums)

$A + B$

Either a b

Result $\langle A, B \rangle$

| Ok (a : A)

| Err (b : B)

e ::= .... |

ok e | err e |

match e with

| ok x ⇒ e_1

| err x ⇒ e_2

case (e, ok_f, err_f)

case (ok e, ok_f err_f) ⟶ ok_f e

case (Err e, ok_f, err_f) ⟶ err_f e

$e ::= x \mid v \mid \lambda x.e \mid e_1 + e_2 \mid \cdots$

$v ::= \bar{n} \mid \text{true} \mid \text{false} \mid \cdots$

$e_1 \longrightarrow e_2$ step relation

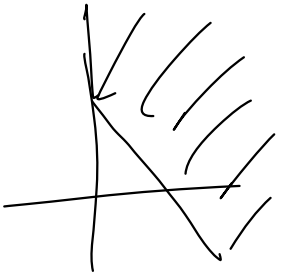
$e_1 \longrightarrow^* e_2$ semantics

$\{P\}\ e\ \{\lambda v.\ Q(v)\}$

if

$(\sigma, pc) \longrightarrow^* (\sigma', pc')$ then

$Q(\sigma')$ $\sigma'(r_6) = 0 + \cdots + 10$



$\forall v;\ P \wedge e \longrightarrow^* v' \Rightarrow$

$Q(v')$

$\{Q(v)\} \; v \; \{v.\, Q(v)\}$

$$\frac{P \vdash Q(v)}{\{P\} \; v \; \{v.\, Q(v)\}}$$

(2 → $P \vdash Q(v)$; 1 → $\{P\}$; 3 → $\{v.\, Q(v)\}$)

$$\frac{\{p\}\; e_1 \;\{v.\, Q(v)\} \qquad \forall v,\; \{Q(v)\}\; e_2[v/x]\; \{R\}}{\{p\}\; \underline{\text{let}}\; x := e_1\; \underline{\text{in}}\; e_2 \;\{R\}}\; \text{hoare-let}$$

Lecture 6: Hoare Logic (part 2)

Learning outcomes:

1. Prove reasoning principles in Hoare Logic
2. Analyze pre- and post-conditions

Hoare logic x2

Separation logic x2

Iris Proof Mode (Rocq)

$\{P\}\ e\ \{v.\ Q(v)\}$

1. $\forall v',\ P \wedge (e \to^* v') \Rightarrow Q(v')$

reasonable?

$\{P\}\ 3\ \{Q\}$

✓

2 ✓ maybe

4 too strong

3 X

5 X

5. $(P \wedge e \to^* 17) \Rightarrow Q(17)$

$P \wedge \forall v'', \quad (\text{let } x := e_1 \text{ in } e_2) \to^* v''$

prove: $R(v'')$

use $\{P\}\ e_1\ \{Q\}$ prove P ✓

$e_1 \to^* v' \Rightarrow$

$Q(v')$ ✓

conclude $R(v'')$

$(\text{let } x := e_1 \text{ in } e_2) \to^*$

$(\text{let } x := v' \text{ in } e_2) \to^*$

$e_2[v'/x] \to^* v''$ ✓

$e_1 \to^* v'$

2. $\{\text{True}\}\ \text{add}\ \bar{n}\ \bar{m}\ \{v.\ v = \overline{n+m}\}$

$\{n < 2^{64} - 1\}$

$3 + \text{true} \not\to .$

$f\ \bar{n}$

$\{\exists p.\ y = \bar{p} \wedge p \leq 1\}$

What did you learn?

What are you confused about?

Lecture 7: Separation Logic (part 1)

- syntax, semantics ← add a heap
- P, Q $\quad \forall x,\ P(x) \quad P \vee Q \quad$ Prop ← heap predicates
- $\{P\}\ e\ \{v.\ Q(v)\}_{SL}$ → soundness
  → rules for constructing proofs

$\text{alloc}\ e : T \quad : \text{ref}\ T \quad \ell : T \quad \ell : \text{loc}$

$!e \quad *e$

$e \leftarrow v \quad *e = v$

$h : \text{loc} \to \text{option val}$

"->" on slides

$(e, h) \rightsquigarrow (e', h')$

!5

Q: val → (heap → Prop)

val → heap → Prop "currying"

Q v h

SL propositions

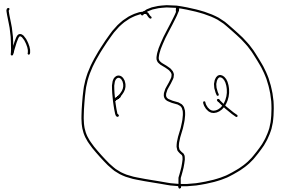
model as heap → Prop

$(\ell \mapsto v)(h) \triangleq h(\ell) = v \wedge dom(h) = \{\ell\}$

$h = \{\ell \mapsto v\}$

$P \vdash Q \triangleq \forall h,\ P(h) \to Q(h)$

$(P * Q)(h) \triangleq \exists h_1, h_2,\ h = h_1 \cup h_2 \wedge \underbrace{h_1 \perp h_2}_{\text{disjoint}} \wedge P(h_1) \wedge Q(h_2)$



$emp(h) \triangleq dom(h) = \emptyset$

$True(h) \equiv True$

$(P * Q)(h)$ $\qquad$ $\forall h_1,\ P(h) \to P'(h)$

---

goal: $(P' * Q)(h)$

$h = h_1 \cup h_2$

$P(h_1)$ $\qquad$ $Q(h_2)$

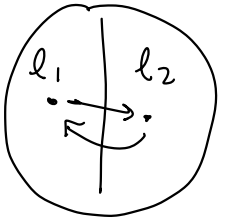
$\downarrow$

$P'(h_1)$

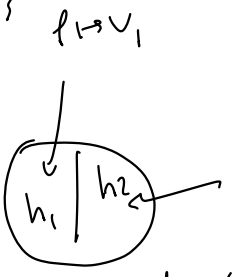
$P * Q \vdash Q * P$ $\qquad$ sep-comm

$\vdash Q * P'$ $\qquad$ sep-monotone-right

$\vdash P' * Q$ $\qquad$ sep-comm



$\{ l_1 \mapsto l_2 ; \quad l_2 \mapsto l_1 \}$



$dom(h_1) = \{l\}$

$dom(h_2) = \{l\}$

$h_1 \perp h_2$

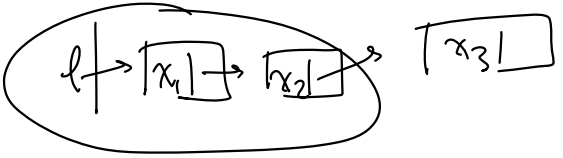
$l \mapsto v_1 * l \mapsto v_2 \vdash \text{False}$

$l_1 \mapsto v_1 * l_2 \mapsto v_2 \vdash l_1 \neq l_2$

$\text{list}(l_1, xs) * \text{list}(l_2, ys)$

↑

list int

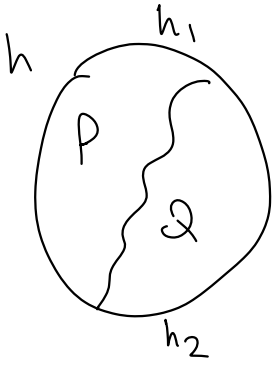


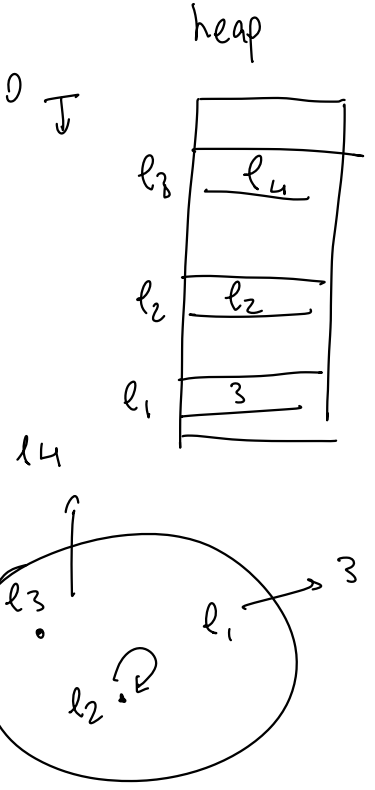
Lecture 8

discriminate missing

caution on using induction tactic

Strategy for even proof is not obvious





$\{P\}\ e\ \{v.\ Q(v)\}$

$\varphi : Prop$

$\lceil \varphi \rceil : hProp$

$w = \#\ell$

$\uparrow$ val

$\nwarrow$ loc

$\lceil \varphi \rceil\ (h) \triangleq \varphi \wedge h = \{\}$

$emp \vdash \lceil True \rceil$

$P, Q : hProp$

$(P \wedge Q)\ (h) \triangleq P(h) \wedge Q(h)$

$\lceil \varphi \rceil * P \vdash P \wedge \lceil \varphi \rceil$

$$\frac{P' \vdash P \quad Q \vdash Q' \quad \{P\}\ e\ \{Q\}}{\{P'\}\ e\ \{Q'\}}$$

$$\frac{\{emp\}\ \text{alloc } 42\ \{y \mapsto 42\}}{\{x \mapsto 0\}\ \text{alloc } 42\ \{x \mapsto 0 * y \mapsto 42\}}$$

RET #();

$\{l_1 \mapsto 0\}\ f(l_1, l_2)\ \{l_1 \mapsto 42\}$

$\{emp\}$ assert #true $\{emp\}$

let $t := !l_1$ in
let $t_2 := !l_2$ in
$l_1 \leftarrow t_2$;
$l_2 \leftarrow t$

$\{emp\}$
let $x :=$ alloc 0 in
$\{x \mapsto 0\}$
let $y :=$ alloc 42 in
$\{x \mapsto 0 * y \mapsto 42\}$
$f(x, y)$
$\{x \mapsto 42 * y \mapsto 42\}$
let $a := !x$ in
$\{\lceil a = 42 \rceil * x \mapsto 42 * y \mapsto 42\}$
let $b := !y$ in
$\{\lceil b = 42 \rceil * x \mapsto 42 * y \mapsto 42\}$
assert #(bool_decide $(a = b)$)
$\{x \mapsto 42 * y \mapsto 42\}$
$\{True\}$

$\{\ell_1 \mapsto a \ast \ell_2 \mapsto b\}$

let $t := !\ell_1$ in

$\{\ulcorner t = a \urcorner \ast \ell_1 \mapsto a \ast \ell_2 \mapsto b\}$

let $t_2 := !\ell_2$ in

$\{\ulcorner t_2 = b \urcorner \ast \ell_2 \mapsto b \ast \ell_1 \mapsto a\}$

$\ell_1 \leftarrow t_2$;

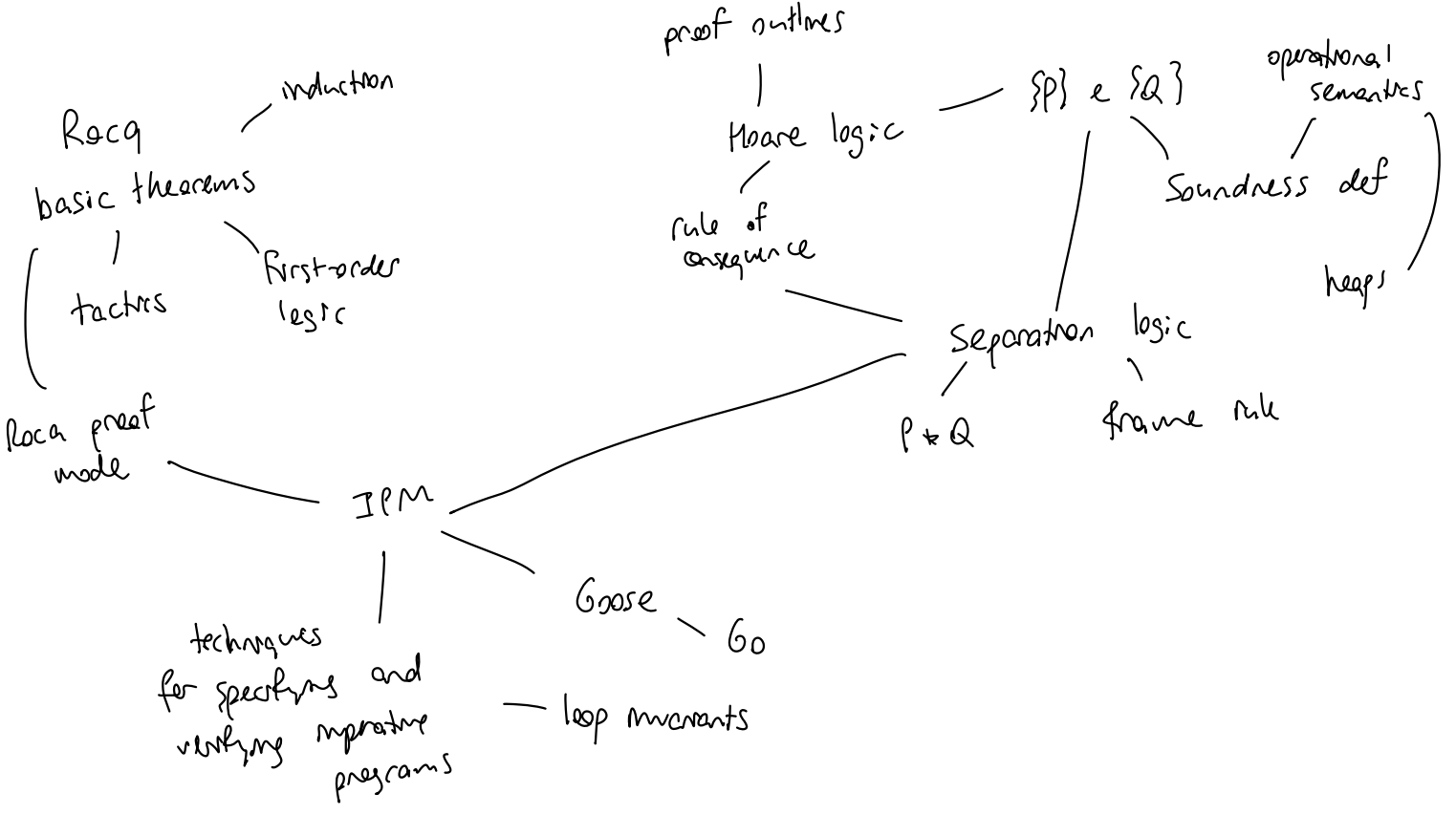
$\{\ell_1 \mapsto t_2 \ast \ell_2 \mapsto b\}$

$\ell_2 \leftarrow t$

$\{\ell_2 \mapsto \cancel{t}^{\,a} \ast \ell_1 \mapsto \cancel{t_2}^{\,b}\}$

$\{\ell_1 \mapsto b \ast \ell_2 \mapsto a\}$

Video Lecture: Recursion


induction
Rocq
basic theorems
tactics
first-order logic
Rocq proof mode
proof outlines
Hoare logic
rule of consequence
{P} e {Q}
operational semantics
Soundness def
heaps
Separation logic
P * Q
frame rule
IPM
Goose
Go
techniques for specifying and verifying imperative programs
loop invariants

$P * Q$ $\quad \exists x.\ P(x)$ $\quad \lceil \varphi \rceil$ $\quad emp$

$\forall x.\ P(x)$

intro

$P -\!\!* Q$ $\quad P \vdash (Q -\!\!* R) \iff P * Q \vdash R$

$P * (P -\!\!* Q) \vdash Q$ $\quad elim$

$P \wedge (P \to Q) \vdash Q$

$$\lceil t = a \rceil * (x \mapsto b * y \mapsto t) \vdash x \mapsto b * y \mapsto a$$

$$P \vdash \lceil \varphi \rceil \qquad \lceil \varphi \rceil \to (P \vdash Q) \to P * \lceil \varphi \rceil \vdash Q$$

$P, Q : \mathrm{iProp}$

$$P * Q \vdash Q * P \qquad P \vdash \mathrm{emp}$$

"H_1" : P

~~"H_2" : Q~~

~~Q~~ $*$ R

iIntros "[H1 H2]" "(H1 & H2 & H3)"

iDestruct "H" as "[H1 H2]"

(x) "H2"

$A \to B \to C$

$A \wedge B \to C$

iExists

iSplitL, iSplitR

iAssumption, iFrame

iApply

(H with "[H]")

with "[$H1 H2]"

Specialization patterns

with "[H1] [H2]" with "[$]"

Learning outcomes

1. Translate program proof goals from IPM to paper
2. Use IPM for entailments / WPs

$e ::= x \mid v \mid \cancel{\lambda x.\, e} \mid e_1\ e_2$

$(\text{rec}\ f\ x.\ e)$

↑ name ↑ name (argument)

$\lambda x.\ e \triangleq (\text{rec}\ \_\ x.\ e)$

Sum ::

$(\text{rec}\ f\ n.\ \text{if}\ n = 0\ \text{then}\ 0\ \text{else}\ n + f\ (n-1))$

$$fib\ n = fib\ (n-2) + fib\ (n-1)$$

$$fib\ 0 = 0$$
$$fib\ 1 = 1$$

$$(rec\ f\ x.\ e)\ v \longrightarrow e\,[(rec\ f\ x.\ e)/f]\,[v/x]$$

$$SumN := (rec\ f\ n.\ \text{if } n=0 \text{ then } 0 \text{ else } n + f\ (n-1))$$

$$SumN\ 3 \longrightarrow \text{if } 3=0 \text{ then } 0 \text{ else } 3 + \overbrace{(rec\ f\ n.\ \text{if } n=0 \text{ then } 0 \text{ else } n + f\ (n-1))}^{=SumN}\ (3-1)$$

$\{P\}\ e\ \{Q\}$

if P, then if

$e \rightsquigarrow v'$,

then $Q(v')$

if,

$e \rightsquigarrow e'$

then either e' is not stuck

or e' is a value v'

and $Q(v')$

$\{P\} \; e \; \{Q\}$

$P * Q \vdash Q * P$

```
P, Q : iProp
--------------------------------------
"H1" : P
"H2" : Q
--------------------------------------∗
Q * P
```

Htmp

```
iDestruct (lem with "[H1 H2]") as "[H3 H4]".
```

```
lem : A -∗ B
```

```
iPoseProof (lem with "[H1 H2]")
  as Htmp.
```

goal 1:

```
H1 : _
H2 : _
--------------------------------------∗
A
```

goal 2:

```
Htmp : B
--------------------------------------
goal
```

```
H3 : B1
H4 : B2
```

$\{P\}\ e\ \{Q\} \iff P \vdash wp(e, Q)$

$wp(e, Q) : iProp$

$\uparrow \quad \uparrow$

$expr \quad val \to iProp$

$\{ wp(e, Q) \}\ e\ \{Q\}$

assert (!x == 0);

x ← 0

$\{ \ell \mapsto 0 \}$

IgnoreOne $\ell\ \ell'$

$\{ \ell \mapsto 42 \}$

$wp(e_1, wp(e_2, \Phi)) \vdash wp((e_1; e_2), \Phi)$

$$\frac{\{P\}\ e_1\ \{Q\} \qquad \{Q\}\ e_2\ \{R\}}{\{P\}\ e_1; e_2\ \{R\}}$$

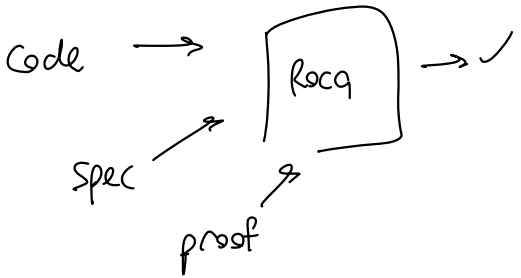
$$\{P\}\ e\ \{Q\} \triangleq P \vdash wp(e, Q)$$

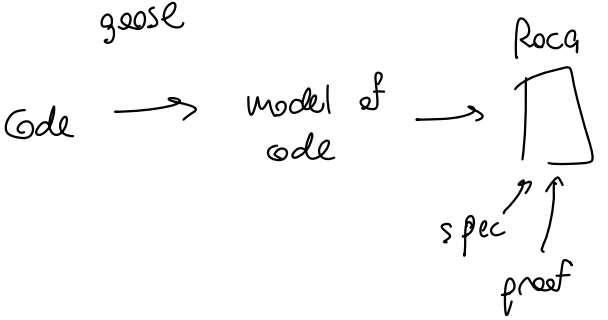
$$\forall \phi,\ P * \underbrace{(\forall v,\ Q(v) \twoheadrightarrow \phi(v))}_{H\phi} \vdash wp(e, \phi)$$

Lecture 11 : Goose

How to model Go code for verification?

Today's focus: control flow





Go

```
for {
    e
}
```

↓

```
for e
```

1. Determine Go evaluation order in $f(x, y)$

2. Determine Goose evaluation order

$f(\text{collatz}(2000), \text{collatz}(3000), \text{collatz}(4000))$

1 (do: e)

2 (return: e)

3 $m_1 ;;; m_2$

(do: e) ≜ ("execute", e)

(return: e) ≜ ("return", e)

do: $e_1 ;;; m_2 \equiv e_1 ;;; m_2$

return: $e_1 ;;; m_2 \equiv$ return: e_1

4 exception_do (return: e) ≡ e

exception_do (do: e) ≡ e

break: #() ≜ ("break", #())

continue: #() ≜ ("continue", #())

exception_do m ≡ Snd m

Lecture 12: Ownership reasoning

1. Articulate the meaning of $\ell \mapsto v$ as ownership
2. Work with fractional permissions
3. Work with struct ownership

$$\ell \overset{q}{\mapsto} v \;\dashv\vdash\; \ell \overset{q_1}{\mapsto} v \;*\; \ell \overset{q_2}{\mapsto} v$$

$$\ell \overset{q}{\mapsto} v \;\dashv\vdash\; \ell \overset{q_1}{\mapsto} v \;*\; \ell \overset{q_2}{\mapsto} v \;*\; \ulcorner q_1 + q_2 \leq 1 \urcorner$$

$$\ell \overset{q_1}{\mapsto} v_1 \;*\; \ell \overset{q_2}{\mapsto} v_2 \;\vdash\; \ulcorner v_1 = v_2 \urcorner$$

$$\{\ell \overset{1/2}{\mapsto} v_0\}\ \ell \leftarrow v\ \{\ell \overset{1/2}{\mapsto} v\}$$

what goes wrong?

$$\forall q,\ \{\ell \overset{q}{\mapsto} v\}\ !\ell\ \{\mathrm{RET}\ v;\ \ell \overset{q}{\mapsto} v\}$$

$$\{\ell \overset{1}{\mapsto} v_0\}\ \ell \leftarrow v\ \{\ell \overset{1}{\mapsto} v\}$$

$$\ell \mapsto \left(\ell \overset{1}{\mapsto} v_0 * \ell \overset{q}{\mapsto} v\right) \vdash \text{False}$$

$$\{\text{True}\}\ \ell := \text{alloc } 2\ \{\ell \mapsto 2\}$$

$$\{\ell \overset{1/2}{\mapsto} 2 * \boxed{\ell \overset{1/2}{\mapsto} 2}\}\ \ell \leftarrow 3$$

$$\{\ell \overset{1/2}{\mapsto} 3 * \ell \overset{1/2}{\mapsto} 2\}\ \{\ulcorner 3 = 2 \urcorner\}\ \{\text{False}\}$$

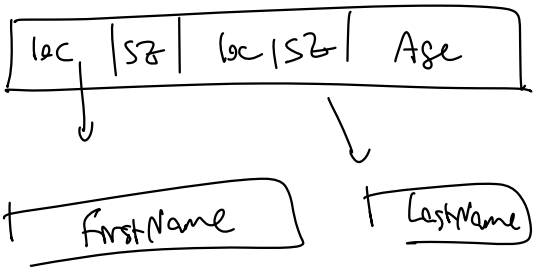
$$\{\ell \overset{1}{\mapsto} v_0 * F\}\ \ell \leftarrow v\ \{\ell \overset{1}{\mapsto} v * F\}$$

$\ell \mapsto p$ $\quad \{\ell := p\}$

RET a_ℓ.

$\{p\_\ell \mapsto p\}$ GetAge p_ℓ $\{\exists a\_\ell.\ a\_\ell \mapsto (p.\text{Age})\}$

$\{\ell := p.\text{FirstName},\ \ell+1 := p.\text{LastName},\ \ell+2 := p.\text{Age}\}$



Lecture 13: Ownership (part 2)

1. Review ownership of structs
2. Use ownership of slices correctly

$\ell \mapsto v$

$\ell \mapsto_S [S :: f]\ v$ Struct field points-to

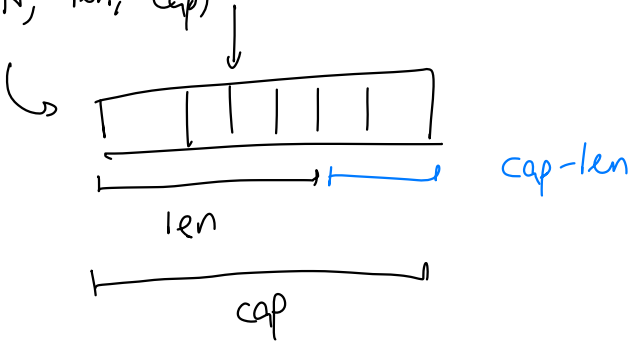
just syntax

s []int

s ↦* vs

(vs : list w64)

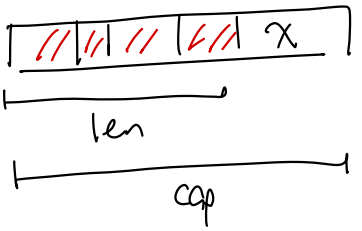
s = (ptr, len, cap)



- get an element reference
- load
- store

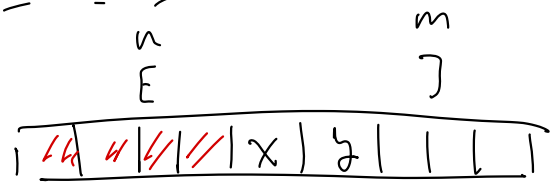
- subslice
- append

append (s, x)



s = (ptr, len, cap)

append (s, x) = (ptr, len+1, cap)



append (append (s, x), y)

(ptr', len+1, cap×2)

$$s[n:m] = (ptr+n,\ m-n,\ cap-n)$$

includes elements from n to end (and original spare capacity in s)

Lecture 14: Loop invariants

1. Recall the rules for a correct loop invariant
2. Struggle to come up with loop invariants

own_slice s xs

s: slice.t

xs: list V

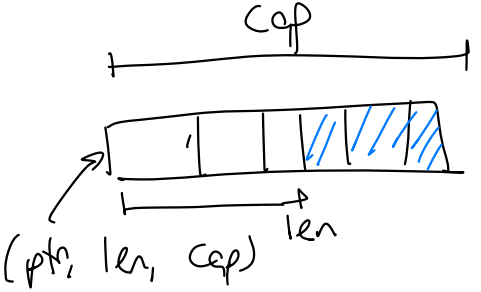
own_slice_cap s

$s \mapsto^* xs$

own_slice_len

$x_i : V$

$\#x : val$



[list_rep l (x :: xs)]

tail l

[∀ v. list_rep v xs]

[list_rep l (x :: xs)]

[list_rep l (x :: xs) ∧ tail_tl list_rep v xs]

[∃ v. l ↦ x * list_rep v xs]

len cap-len

$\{ s \mapsto\!* \; xs \;*\; \text{own\_slice\_cap } s \}$

append(s, x)

$\{ s',\ \text{RET } \#s';\ s' \mapsto\!* \; (xs ++ [x]) \;*\; \text{own\_slice\_cap } s' \}$

len+1 cap-len-1

$s \mapsto\!* xs \;*\; \text{own\_cap}(s) \vdash s[0:n] \mapsto\!* \text{take}(xs, n) \;*\; \text{own\_cap}(s[0:n])$

$s \mapsto\!* xs \vdash s[0:n] \mapsto\!* \text{take}(xs, n) \;*\; s[n:\text{len}(s)] \mapsto\!* \text{drop}(xs, n) \;*\; \text{own\_cap}(s) \;*\; \text{own\_cap}(s[n:\text{len}(s)])$

$s := []int\{1,2,3\}$

$s \mapsto * [1;2;3] * own\_cap(s)$

$s_1 := s[:1]$

$s_2 := s[1:]$

$\{ s_1 \mapsto * [1] * s_2 \mapsto * [2;3] * own\_cap(s_2) \}$

$s2[0]$ ✓

$s_1 = append(s_1, s)$

$\{True\}\ s[:n]\ \{RET\ s[:n];\ True\}$

$s[:1:1]$

$\forall I,$

$$\frac{\{I\}\ \text{body}\ ()\ \{I\}}{\{I\}\ \text{loop body}\ \{I\}}$$

loop body

$\{P\}$ loop body $\{Q\}$

$$P \vdash I$$
$$I \vdash Q$$

$$\sum_{0 \le i \le n} i = \frac{n(n+1)}{2}$$

```
sum := 0
i := 0
if i > n
    sum += i
    i++
```

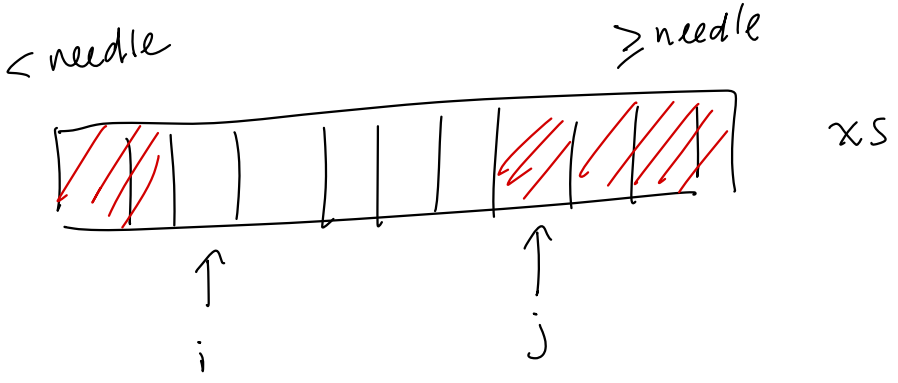
prove I (manually)

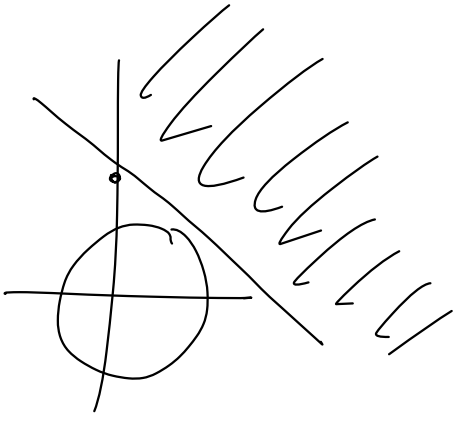
return sum

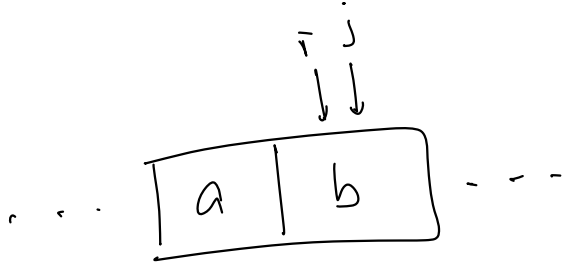
prove postcondition

$\{I\}$

Lecture 15







$a < needle < b$

$\{ l_1 \mapsto l_1' * \text{list\_rep } l_1' \; xs \}$

match !l_1 with

| Inil ⇒ $\{\text{list\_rep Inil } xs\}$

// why does $xs = []$?

list_rep v xs :=

match xs with

| [] ⇒ v = Inil

| $hd :: xs'$ ⇒ ... v = Icons hd tl ...

end

$\{\text{list\_rep } l_2 \; ys\}$

$\{\text{List\_rep } l_2 \; (xs ++ ys)\}$

$\text{list\_rep Inil } xs \vdash \lceil xs = [] \rceil$

Lecture 16: Persistently modality

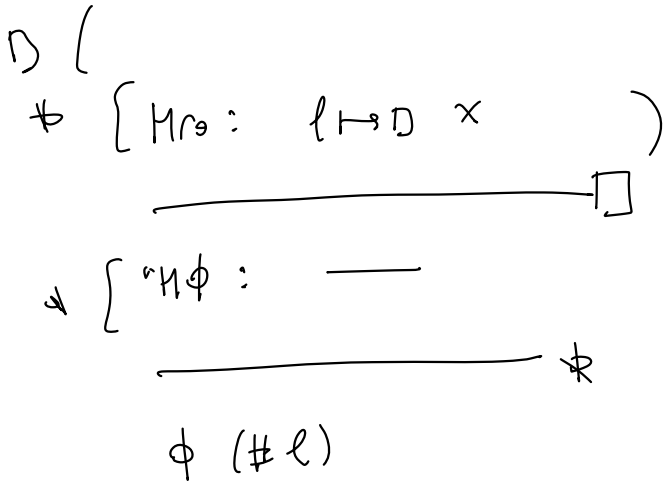
$P : iProp$ $\quad \ell \mapsto v$

Persistent (P) $\quad \square P$ "persistently P"

$\square P : iProp$

$\ell \mapsto_q v$ $\quad q \leq 1$: read-only $\quad 0 < q$

$\ell \mapsto_{q_1+q_2} v \dashv\vdash \ell \mapsto_{q_1} v * \ell \mapsto_{q_2} v$



$\square\, H_{r0} * H\phi \vdash \phi(\#\ell)$

(h_r, h_w) $\quad h_r \perp h_w \quad h = h_r \cup h_w$

$(\ell \mapsto v)\ (h_r, h_w) \triangleq h_w = \{\ell := v\}$ $\quad (\ell \mapsto_\square v)(h_r, h_w) \triangleq h_r = \{\ell := v\}$

$\square P\ (h_r, h_w) \triangleq P(h_r, \emptyset)$

$P * Q\ (h_r, h_w) \triangleq$

$\exists h_1, h_2,\ h_w = h_1 \cup h_2$

$h_1 \perp h_2$

$P(h_r, h_1) \wedge$

$Q(h_r, h_2)$

1. $\square P \vdash P$
2. $P \vdash \square P$
3. $P \vdash Q$ implies $\square P \vdash \square Q$
4. $\square P * Q \vdash \square P \wedge Q$
5. $\square P \wedge Q \vdash \square P * Q$
6. what is $\square(\ell \mapsto v)$?

$\lceil \varphi \rceil * Q \dashv\vdash \lceil \varphi \rceil \wedge Q$

$\square P * Q \dashv\vdash \square P \wedge Q$

~~$A * B \dashv\vdash A \wedge B$~~

Persistent $(P) \quad := \quad P \vdash \Box P$

$P \vdash \Box Q \quad \rightarrow \quad P \vdash \Box Q * P$

$P \overset{?}{\vdash} \Box Q \wedge P \overset{\checkmark}{\vdash} \Box Q * P$

$P \vdash P$ ✓

$P \vdash \Box Q$ ✓

$\square P \vdash P * \square P$

$\{P\}\ e\ \{Q\} : \mathrm{Prop}$

$\forall \Phi,\quad P * (\forall v, Q(v) -\!\!* \Phi(v)) \vdash \mathrm{WP}(e, \Phi)$

$P \vdash \mathrm{WP}(e, Q)$

$\{P\}\ e\ \{Q\} : \mathrm{iProp}$

$(P -\!\!* Q) * P \vdash Q$

$P -\!\!* \mathrm{WP}(e, Q)$

$$\square\left(\forall x,\ \text{true} \twoheadrightarrow \text{wp}\ f\_\text{code}\ x\ \{\lambda y.\ \ulcorner y = f\ x \urcorner\}\right)$$

$$\ell \mapsto_{p} v \triangleq \left(\exists q,\ \ell \mapsto_{q} v\right) \qquad \ell \mapsto_{p} v \vdash \ell \mapsto_{p/2} v * \ell \mapsto_{p/2} v$$

Lecture 17: Concurrency introduction

Learning outcomes:

1. Formalize concurrency semantics with interleavings
2. Appreciate why concurrency is challenging

Lecture 18: Lock invariants

$\{P\}\ e\ \{Q\}$ if $(e, h) \rightsquigarrow (e', h')$

either (a) (e', h') is not stuck

(b) $\exists v'.\ e' = v' \wedge \varphi(v')$

$\{\text{True}\}\ e\ \{v.\ \lceil \varphi(v) \rceil\}$

$\varphi : \text{val} \to \text{Prop}$

$P(h)$

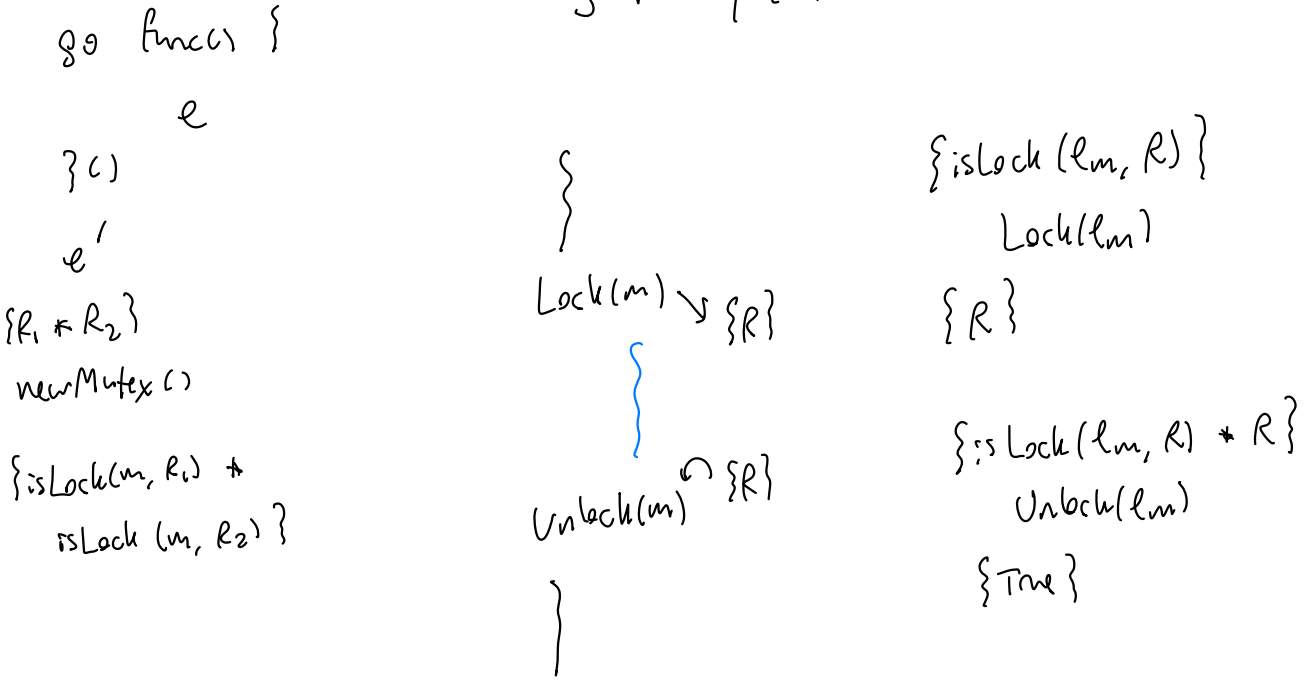
if $([e], h) \rightsquigarrow_{tp} ([e'] ++ T', h')$

either (a) $([e'] ++ T', h)$ is not stuck

(b) $\exists v'.\ e' = v' \wedge \varphi(v')$ ←

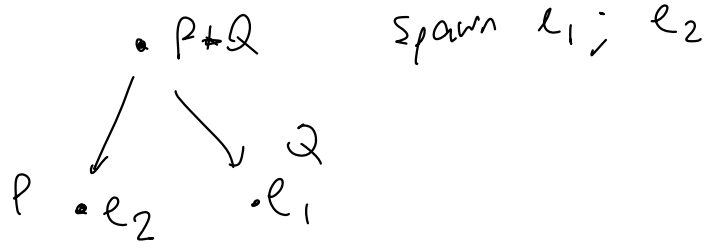
and no thread in $[e'] ++ T'$ is stuck in h

same thread can take a step


S ⊢ wp (e, True) * P
go func() {
e
}()
e'
{R1 * R2}
newMutex()
{isLock(m, R1) *
isLock(m, R2)}
{
Lock(m)
{R}
Unlock(m)
{R}
}
{isLock(lm, R)}
Lock(lm)
{R}
{isLock(lm, R) * R}
Unlock(lm)
{True}

Lecture 19: Ghost state

1. Explain the API for (plain) ghost variables
2. Use ghost variables in simple concurrency proofs.
3. (maybe) Appreciate resource algebras as a foundation for ghost state.



Spawn(...) var x = 0

FAA(&x, 2) || FAA(&x, 2)

{Q1} {Q2}

Q1 * Q2

print(x) // 4

var x = 0

Lock() || Lock()

x += 2 || x += 2

Unlock() || Unlock()

Lock(); !x; Unlock()

x += 2

{ WP f () {P} }

t := Spawn(f)

{ is_JoinHandle(t, P) }

{ is_JoinHandle(t, P) }

t.Join()

{P}

$i = 0$

$x_1 = 0$

$x_2 = 0$

$\{x_1\text{-ptr} \overset{2/3}{\mapsto} 0\}$

Lock()

$i += 2$

$x_1 = 2$

Unlock()

$\{x_1\text{-ptr} \overset{2/3}{\mapsto} 2\}$

is_Mutex($i\text{-ptr} \mapsto i$ * $x_1\text{-ptr} \overset{1/3}{\mapsto} x_1$ * $x_2\text{-ptr} \overset{1/3}{\mapsto} x_2$ * $\lceil i = x_1 + x_2 \rceil$)

$\{x_2\text{-ptr} \overset{2/3}{\mapsto} 0\}$

Lock()

$i += 2$

$x_2 = 2$

Unlock()

$\{x_2\text{-ptr} \overset{2/3}{\mapsto} 2\}$

$x\text{-ptr} \mapsto v \dashv\vdash$

split/combine $x\text{-ptr} \overset{1/2}{\mapsto} v * x\text{-ptr} \overset{1/2}{\mapsto} v$

agree $x\text{-ptr} \overset{q_1}{\mapsto} v_1 * x\text{-ptr} \overset{q_2}{\mapsto} v_2 \vdash \lceil v_1 = v_2 \rceil$

$$\frac{H:\ i = x_1 + x_2}{(i+2) = 2 + x_2}$$

Lock()

$y := i$

Unlock()

y ?

$\gamma \overset{1/2}{\mapsto} v_1 * \gamma \overset{1/2}{\mapsto} v_1 \equiv\!* \gamma \overset{1/2}{\mapsto} v_2 * \gamma \overset{1/2}{\mapsto} v_2$

ghost_var(γ, 1, v_1)

$\text{True} \equiv\!* \exists \gamma,\ \gamma \overset{1/2}{\mapsto} v * \gamma \overset{1/2}{\mapsto} v$

$P \equiv\!* Q \quad \sim \quad \{P\}\ \text{skip}\ \{Q\}$

deep

Lecture 20: Atomic specs

Learning outcomes

1. Appreciate the challenge of specifying atomicity
2. Recall how to specify atomicity using atomic updates

$\exists x,\ \gamma\_ptr \mapsto x * \ldots$

var m sync.Mutex

var x int

$\gamma_1 = 0$

$x_2 = 0$

is_Mutex(_____)

$\exists x_1,\ \gamma_1 \overset{1/2}{\mapsto} x_1 *$

$\exists x_2\ \ \gamma_2 \overset{1/2}{\mapsto} x_2 *$

$x = x_1 + x_2$

Spawn($(\gamma_1 \overset{1/2}{\mapsto} 0)$

m.Lock()

x += 2 ← $x_1 = 2$

m.Unlock()

)

$(\gamma_2 \overset{1/2}{\mapsto} 2)$

Spawn($(\gamma_2 \overset{1/2}{\mapsto} 0)$

m.Lock()

x += 2 ← $x_2 = 2$

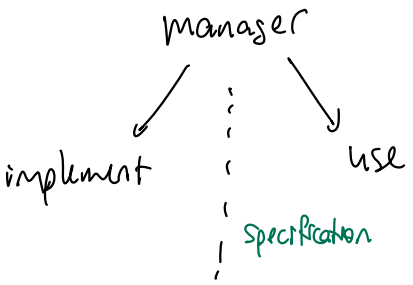
m.Unlock()

$(\gamma_2 \overset{1/2}{\mapsto} 2)$

m.Lock(); y := x; m.Unlock()

Assert(y = 4)

↖ why = 4?



integer

$\ell.\mathrm{Get}() : \mathrm{nt}$

$\ell.\mathrm{Inc}()$

$\ell \mapsto x$

$\{ \mathrm{int\_rep}(\ell, x) \}$

$\ell.\mathrm{Get}()$

$\{ \mathrm{RET}\ \#x;\ \ \mathrm{int\_rep}(\ell, x) \}$

$\{ \mathrm{int\_rep}(\ell, x) \}$

$\ell.\mathrm{Inc}()$

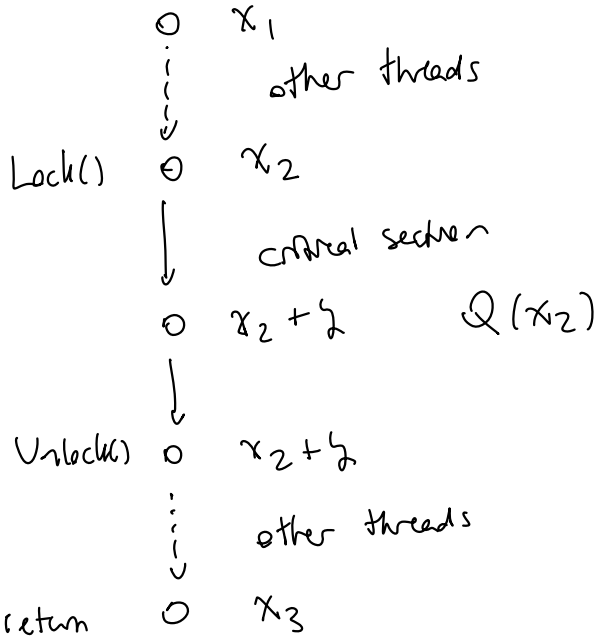
$\{ \mathrm{RET}\ \#();\ \ \mathrm{int\_rep}(\ell, x+1) \}$

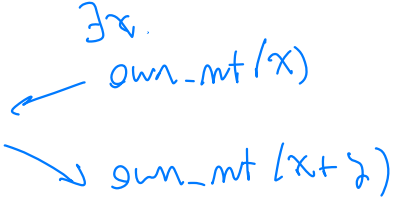
is_atomic_mt(ℓ, γ) : iProp

own_mt(γ, x) : iProp

inv I

$I \twoheadrightarrow \exists x.\ own\_mt(x) * (own\_mt(x+2) \twoheadrightarrow I)$





$\text{True} \overset{\top}{\Rrightarrow}_{\emptyset}$

$\{ \overset{\top}{\Rrightarrow}_{\emptyset} \exists x,\ own\_mt(\gamma, x) * (own\_mt(\gamma, x) \overset{\emptyset}{\Rrightarrow}_{\top} Q(x)) \}$

FAA
- movement by 1
- returns old value

$\ell.\text{Get}()$

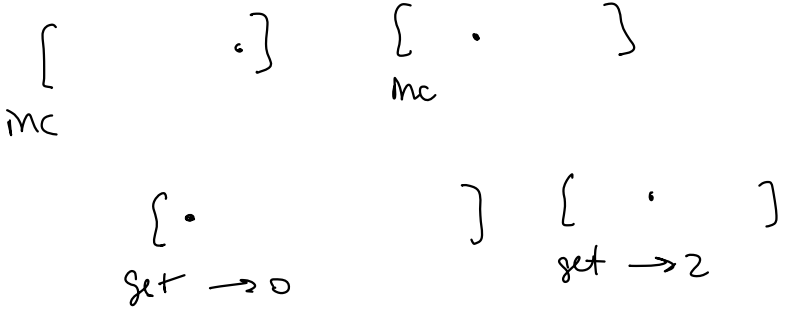
$AU\langle \exists x. own\_mt(\gamma, x) ;\ \text{RET } x.\ own\_mt(\gamma, x) \rangle$

$\{ \exists x.\ \text{RET } \#x,\ Q(x) \}$

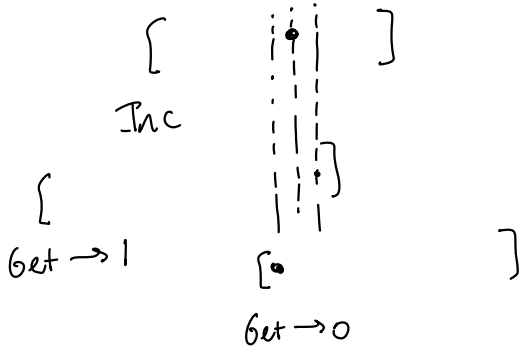
$inv\ I \vdash \text{True} \overset{\top}{\Rrightarrow}_{\emptyset} I$

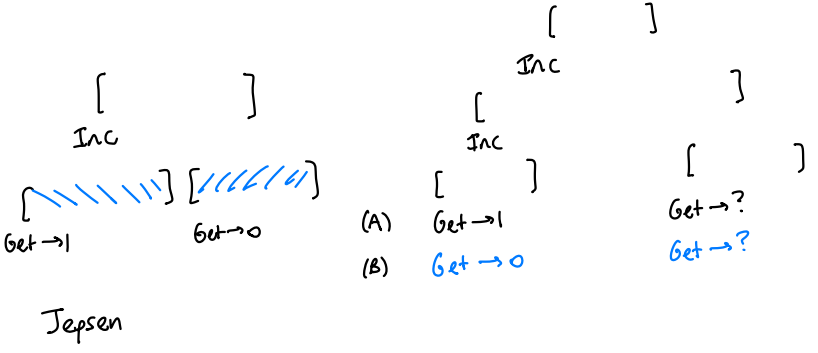
$A \overset{\top}{\Rrightarrow}_{\emptyset} B$

$B \overset{\emptyset}{\Rrightarrow}_{\top} C$


inc
inc
set → 0
set → 2

Lecture 21: Atomic specs (part 2)





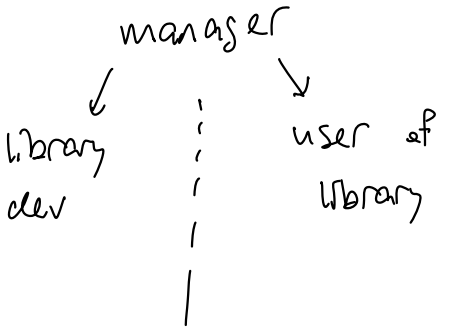
linearizability: $\forall tr$, behavior(code, tr) $\rightarrow$ linearizable(tr)

$\exists$ ordering

$\exists$ ordering(code), $\forall tr$, behavior(code, tr) $\rightarrow$ linearizable(tr)

ordering(code)

1. dynamic lin points
2. helping
3. "future dependent"



$\boxed{I} : iProp$

inv N I

$\forall \phi, \{ \phi \}$ Noop $\{ \phi \}$

Exercise solution (FAA spec)

$\{ \top \Rrightarrow \emptyset \; \exists x.\ own\_mt(\gamma, x) * (own\_mt(\gamma, x+1) \Rrightarrow \emptyset \; \phi(x)) \}$

$\#$

$l.FAA()$

$\{ \exists x.\ RET\ \#x, \phi(x) \}$

ϕ