4. Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

Proof. The sequence produced by $4n + 1$ for non-negative integers n is:

$$
\begin{aligned}
&= (4.0 + 1), (4.1 + 1), (4.2 + 1), (4.3 + 1), (4.4 + 1), ...(4n + 1) \\
&= 1, 5, 9, 13, 17, ...
\end{aligned}
$$

The sequence produced by $4n + 3$ for non-negative integers n is:

$$
\begin{aligned}
&= (4.0 + 3), (4.1 + 3), (4.2 + 3), (4.3 + 3), (4.4 + 3), ...(4n + 3) \\
&= 3, 7, 11, 15, ...
\end{aligned}
$$

It is trivial to see that the combination of these two sequences represents the sequence of all natural numbers. Therefore every natural odd number can be represented by one of the forms as stated in the theorem.

□