5. Prove that for any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3.

Pre-proof note:

$n+4$ is equivalent to $n+1+3$. This is clearly divisible by 3 if, and only if, $n+1$ is divisible by three. We will therefore consider $n+1$ instead of $n+4$ with no loss of generality.

Theorem: For all integers n at least one of the following expressions is divisible by 3

1. n
2. $n+1$
3. $n+2$

Proof. For any integer n, n is either divisible by 3 (case 1.) or there is some remainder r (where $r=1$ or $r=2$). If $r=2$, we can add 1 to n to get a number divisible by 3 (case 2.). If $r=1$, we can add 2 to n to get a number divisible by 3 (case 3.).

This proves that the theorem is true, for any integer n one of the three cases stated above is divisible by 3.

$\square$