1. Say whether the following is true or false and support your answer by a proof.

$$(\exists m, n \in \mathbb{N})(3m + 5n = 12)$$

This statement is false.

Proof. Let $n = 1$. Solve the equation for incremental values of m starting with the smallest natural number until the solution is greater than 12

$$\begin{aligned} 3m + 5n &= x \\ (3 \times 1) + 5 &= 8 \\ (3 \times 2) + 5 &= 11 \\ (3 \times 3) + 5 &= 14 \end{aligned}$$

And similarly for $m = 1$, solve the equation for incremental values of n

$$\begin{aligned} 3m + 5n &= x \\ 3 + (5 \times 1) &= 8 \\ 3 + (5 \times 2) &= 13 \end{aligned}$$

Clearly any further increment of either m or n results in a solution that is greater than 12. The theorem is therefore false. □