

2. Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).

Proof. By mathematical induction.

For any integer x the sum of the first 5 consecutive integers is given by:

$$x + (x + 1) + (x + 2) + (x + 3) + (x + 4) = 5n$$

For $x = 1$, we have:

$$1 + (1 + 1) + (1 + 2) + (1 + 3) + (1 + 4) = 5n$$

$$1 + 2 + 3 + 4 + 5 = 5n$$

$$15 = 5n$$

$$3 = n$$

Substituting $x + 1$ for x we have:

$$= (x + 1) + ((x + 1) + 1) + ((x + 1) + 2) + ((x + 1) + 3) + ((x + 1) + 4)$$

(substitute $x+1$)

$$= 5 + (x + (x + 1) + (x + 2) + (x + 3) + (x + 4))$$

(bring plus ones out the front)

$$= 5 + 5n$$

(from the inductive statement)

$$= 5(1 + n)$$

This proves that the inductive statement is true for $x+1$, therefore proving by the principle of mathematical induction that the theorem is true.

□