

3. Say whether the following is true or false and support your answer by a proof: For any integer n , the number $n^2 + n + 1$ is odd.

Proof. Proof by contradiction.

Assume: $\exists n \in \mathbb{Z}$ such that n is even.

Since $n^2 + n + 1$ is even then $n^2 + n$ must be odd. Since $n^2 + n$ is odd, either n^2 or n must be odd (since only the sum of an odd and even number gives an odd number). But since n is even, n^2 must be even - this is a contradiction.

□