I have used Mathematica to verify some of the computations I did by hand.

Problem 1

In[78]:=
```
A = {{1, 0}, {0, 1}, {1, 0}};
```

In[85]:=
```
A.Inverse[Aᵀ.A].Aᵀ // MatrixForm
```

Out[85]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

In[86]:=
```
A.Inverse[Aᵀ.A].Aᵀ.{1, 2, 3}
```

Out[86]= $\{2, 2, 2\}$

In[92]:=
```
Q = {{√2 / 2, 0}, {0, 1}, {√2 / 2, 0}};
```

In[95]:=
```
Q.Qᵀ // MatrixForm
```

Out[95]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

In[96]:=
```
DD = DiagonalMatrix[{1, 2, 3}];
q0 = {1/2, 0, 1/2};
q1 = {0, 1/√2, 0};
```

In[99]:=
```
q0.DD.q0
q1.DD.q1
q1.DD.q0
```

Out[99]= 1

Out[100]= 1

Out[101]= 0

In[103]:=
```
w = {3, 2, 1};
```

In[104]:=
```
(w.DD.q0) q0 + (w.DD.q1) q1
```

Out[104]= $\left\{\frac{3}{2}, 2, \frac{3}{2}\right\}$

Problem 3

In[32]:=
```
u[t_] := Exp[t a] u[0] + Integrate[Exp[(t - τ) a] b (u[τ]), {τ, 0, t}]
```

In[42]:=
```
Exp[tn1 a] Integrate[Exp[-a τ], {τ, tn, tn1}] // FullSimplify
```

Out[42]=
$$\frac{-1 + e^{a\,(-tn+tn1)}}{a}$$

Problem 4

In[45]:=
```
U[j_, n_] := Normal[Series[u[x + j h z, t + n k z], {z, 0, 3}]] /. {z → 1}
```

In[106]:=
```
Collect[ExpandAll[ReplaceRepeated[
    U[0, 2] - U[-1, 0] - U[0, 1] + U[-1, 1] - 2 k/h (U[0, 1] - U[-1, 1]), {k → μ h}]], h]
```

Out[106]=
$$h\left(2\,\mu\,u^{(0,1)}[x,t] - 2\,\mu\,u^{(1,0)}[x,t]\right) + h^2\left(2\,\mu^2\,u^{(0,2)}[x,t] - \mu\,u^{(1,1)}[x,t] - 2\,\mu^2\,u^{(1,1)}[x,t] + \mu\,u^{(2,0)}[x,t]\right) + h^3\left(\frac{4}{3}\,\mu^3\,u^{(0,3)}[x,t] - \frac{1}{2}\,\mu^2\,u^{(1,2)}[x,t] - \mu^3\,u^{(1,2)}[x,t] + \frac{1}{2}\,\mu\,u^{(2,1)}[x,t] + \mu^2\,u^{(2,1)}[x,t] - \frac{1}{3}\,\mu\,u^{(3,0)}[x,t]\right)$$