# **Numerics** Methods and Useful Facts

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# 1 Linear Algebra

## 1.1 Classification of Matrices

Matrices are assumed to be complex unless specified otherwise.

### 1.1.1 Hermetian

*Definition*: A matrix $A$ is Hermetian if $A^* = A$

*Properties*:

$\Rightarrow$ Real eigenvalues
$\Rightarrow$ Normal
$\Rightarrow$ Unitarily diagonalizable
$\Rightarrow$ $\|A\|_2 = \rho(A)$

### 1.1.2 Skew symmetric

*Definition*: A real matrix $A$ is skew symmetric if $A^T = -A$

*Properties*:

$\Rightarrow$ pure imaginary eigenvalues
$\Rightarrow$ $I + A$ is invertible

### 1.1.3 Normal

*Definition*: A matrix $A$ is normal if $A^*A = AA^*$

*Properties*:

$\Leftrightarrow$ Unitarily diagonalizable

### 1.1.4 Positive definite

*Definition*: A matrix $A$ is positive definite if all eigenvalues are positive.

*Properties*:

$\Leftrightarrow$ $v^*Av > 0$ for all $v$
$\Leftrightarrow$ Has Cholesky factorization

## 1.2 Matrix Decompositions

### 1.2.1 SVD

$$A = U\Sigma V^*$$

- $U$ unitary
- $\Sigma$ diagonal, with real positive entries in non-increasing order
- $V$ unitary

*Existence*: Always

*Uniqueness*: *Note:* double check Unique up to complex sign of columns of $U$ and $V$

*Computing*:

*Why it is useful*:

- Rank revealing
- Numerical stability of algorithms using SVD

### 1.2.2 (P)LU

*Existence*:

*Uniqueness*:

*Computing*: Gaussian Elimination

When is pivoting needed?

### 1.2.3 Cholesky

*Existence*: If $A$ is Hermetian positive definite

*Uniqueness*: Unique up to sign

*Why it is useful*:

- Save storage space

### 1.2.4 QR

### 1.2.5 Eigen

## 1.3 Matrix and Vector Norms

$$\|A\| = \sup_{u \neq 0} \frac{\|Au\|}{\|u\|} = \sup_{\|u\|=1} \|Au\|$$

$$\|A\| = \sup_{u,v\neq 0} \frac{\langle Au, v\rangle}{\|u\|\,\|v\|} = \sup_{\|u\|=\|v\|=1} \langle Au, v\rangle$$

If $A$ is self Hermetian, *Note:* double check this

$$\|A\| = \sup_{u\neq 0} \frac{\langle Au, u\rangle}{\|u\|^2} = \sup_{\|u\|=1} \langle Au, u\rangle$$

## 1.4 Rayleigh Quotients

*Note:* Anne seems to like these

## 1.5 Miscellaneous

### 1.5.1 Gradients of Matrix Vector Forms

$$\nabla x^* A x = 2Ax$$

## 1.6 Iterative Methods

### 1.6.1 Simple Iteration

### 1.6.2 Conjugate Gradient

### 1.6.3 GMRES

# 2 Boundary Value Problems

## 2.1 Laplacian

# 3 Initial Value Problems

*Note:* need better section names

## 3.1 Runge-Kutta Methods

## 3.2 LMMs

## 3.3 Stability

zero stable

region of abs stability

A stable L stable etc

MOL

Von Neumann