

On the Convergence of Conjugate Gradient Variants in Finite Precision Arithmetic

Tyler Chen

This is a companion piece to the publication:

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A preprint will be on ArXiv in the near future.

Why should I care?

Need new algorithms to deal with modern HPC architecture. But can't sacrifice accuracy.

Take advantage of lower precision for ML type applications.

Introduction

If you are not familiar with the Conjugate Gradient method, it may be worth reading this page first.

The Conjugate Gradient algorithm is a widely used method for solving $Ax = b$ when A is positive definite (all eigenvalues are positive).

Conjugate Gradient In Finite Precision

- The CG algorithm can be derived by minimizing the A -norm of the error at each step.

Numerical Problems

- introduction to floating points failing to be associative
- even worse in low precision
- give an example of error analysis for HSCG
- full results in paper

Avoiding Communication

- give example with inner product vs sparse matrix product
- derive CGCG and GVCG by replacing recurrences

Concusion