

Name_____

Math 125

Quiz 5 — 30 Minutes

12:50-1:20, Tuesday, Nov. 28, 2017

(40 points total, no notes or calculator permitted)

Please show your work clearly.

1. (20 points) Find the x - and y -coordinates of the center of mass of a uniform flat plate contained between the two curves $y = \sqrt{x}$ and $y = x^3$, $0 \leq x \leq 1$. Please simplify your answer (writing the coordinates as fractions).
2. (20 points) Set up a definite integral and write an approximation for the arclength of the curve $y = \ln(x)$ between $x = 4$ and $x = 16$. You may use either the trapezoid rule with $n = 6$ or Simpson's rule with $n = 6$ (but please state which of them you are using). You cannot give the final decimal value because that would require a calculator.

Answers

1. To find \bar{x} divide $\int_0^1 x(x^{1/2} - x^3)dx = \frac{2}{5} - \frac{1}{5} = \frac{1}{5}$ by $\int_0^1 (x^{\frac{1}{2}} - x^3)dx = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$. The result is $\bar{x} = 12/25$. To find \bar{y} divide $\frac{1}{2} \int_0^1 (x - x^6)dx = \frac{1}{2}(\frac{1}{2} - \frac{1}{7}) = \frac{5}{28}$ by $5/12$, getting $\bar{y} = 12/28 = 3/7$. Alternatively, the numerator of \bar{y} is $\int_0^1 y(y^{1/3} - y^2)dy = \frac{3}{7} - \frac{1}{4} = 5/28$.

$$2. \int_4^{16} \sqrt{1+x^{-2}} dx \approx \\ \approx \frac{2}{3}(\sqrt{1+1/16} + 4\sqrt{1+1/36} + 2\sqrt{1+1/64} + 4\sqrt{1+1/100} + \\ + 2\sqrt{1+1/144} + 4\sqrt{1+1/196} + \sqrt{1+1/256}) \text{ (Simpson's rule),}$$

or else

$$\approx 2(\frac{1}{2}\sqrt{1+1/16} + \sqrt{1+1/36} + \sqrt{1+1/64} + \sqrt{1+1/100} + \sqrt{1+1/144} + \\ + \sqrt{1+1/196} + \frac{1}{2}\sqrt{1+1/256}) \text{ (trapezoid rule).}$$