

Name_____

Math 125

Quiz 5 — 30 Minutes

11:50-12:20, Tuesday, Nov. 28, 2017

(40 points total, no notes or calculator permitted)

Please show your work clearly.

1. (20 points) Find the x - and y -coordinates of the center of mass of a uniform flat plate contained between the two curves $y = x^3$ and $y = x^2$, $0 \leq x \leq 1$. Please simplify your answer (writing the coordinates as fractions).
2. (20 points) Set up a definite integral and write an approximation for the arclength of the curve $y = \ln(x)$ between $x = 1$ and $x = 4$. You may use either the trapezoid rule with $n = 6$ or Simpson's rule with $n = 6$ (but please state which of them you are using). You cannot give the final decimal value because that would require a calculator.

Answers

1. To find \bar{x} divide $\int_0^1 x(x^2 - x^3)dx = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$ by $\int_0^1 (x^2 - x^3)dx = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$. The result is $\bar{x} = 12/20 = 3/5$. To find \bar{y} divide $\frac{1}{2} \int_0^1 (x^4 - x^6)dx = \frac{1}{2}(\frac{1}{5} - \frac{1}{7}) = \frac{1}{35}$ by $1/12$, getting $12/35$. Alternatively, the numerator of \bar{y} is $\int_0^1 y(y^{1/3} - y^{1/2})dy = \frac{3}{7} - \frac{2}{5} = 1/35$.

2. $\int_1^4 \sqrt{1+x^{-2}} dx \approx \frac{1}{6}(\sqrt{2} + 4\sqrt{1+4/9} + 2\sqrt{1+1/4} + 4\sqrt{1+4/25} + 2\sqrt{1+1/9} + 4\sqrt{1+4/49} + \sqrt{1+1/16})$ (Simpson's rule), or else $\approx \frac{1}{2}(\frac{1}{2}\sqrt{2} + \sqrt{1+4/9} + \sqrt{1+1/4} + \sqrt{1+4/25} + \sqrt{1+1/9} + \sqrt{1+4/49} + \frac{1}{2}\sqrt{1+1/16})$ (trapezoid rule).