Name________________________________

Math 125 **Quiz 4 — 40 Minutes** **12:40-1:20, Tuesday, Nov. 7, 2017**

(40 points total, no notes or calculator permitted)

Please show your work clearly. In 1(b) use the triangle method, and show your triangle as part of your work.

1. Evaluate

(a) (12 points) $\int \sin^4(x/4)dx$ (b) (12 points) $\int \frac{dx}{(8x-x^2)^{3/2}}$

2. (16 points) Using the method of partial fractions, evaluate

$$\int \frac{3x^3+13x^2+13x-15}{x^2(x^2+4x+5)}dx.$$

Answers

1. (a) Write $\sin^4(x/4) = (\sin^2(x/4))^2 = (1-\cos(x/2))^2/4 = \frac{1}{4} - \frac{1}{2}\cos(x/2) + \frac{1}{4}\cos^2(x/2)$. The last term is equal to $\frac{1}{8}(1+\cos(x))$, and so the integral becomes

$$\int \left(\frac{3}{8} - \frac{1}{2}\cos(x/2) + \frac{1}{8}\cos(x) \right) dx = \frac{3}{8}x - \sin(x/2) + \frac{1}{8}\sin(x) + C.$$

(b) First complete the square, much like an example done in class: $8x - x^2 = -(x^2 - 8x) = -(x^2 - 8x + 16 - 16) = -((x-4)^2 - 4^2) = 4^2 - (x-4)^2$. Next, make a triangle with 4 on the hypotenuse and $x-4$ on the leg opposite θ. Then $\sqrt{8x - x^2}$ is the other leg, and so $\sqrt{8x-x^2} = 4\cos(\theta)$. Also, $x = 4 + 4\sin(\theta)$, and $dx = 4\cos(\theta)d\theta$. The integral now becomes $\int \frac{4\cos(\theta)d\theta}{4^3\cos^3(\theta)} = \frac{1}{16}\int \sec^2(\theta)d\theta = \frac{1}{16}\tan(\theta) + C = \frac{x-4}{16\sqrt{8x-x^2}} + C$.

2. Setting the function in the integral equal to $\frac{a}{x} + \frac{b}{x^2} + \frac{cx+d}{x^2+4x+5}$ and clearing denominators, we get

$$3x^3 + 13x^2 + 13x - 15 = ax(x^2+4x+5) + b(x^2+4x+5) + (cx+d)x^2.$$

Gathering together constant terms on both sides, we get $-15 = 5b$ (because the only constant term on the right is the term next to b), and so $b = -3$. Doing the same with the x-terms, we get $13 = 5a + 4b = 5a - 12$, and so $a = 5$. Equating the x^2-terms gives $13 = 4a + b + d = 17 + d$, and so $d = -4$. Finally, equating the x^3-terms gives $3 = a + c = 5 + c$, and so $c = -2$. So our integral becomes

$$\int \left(\frac{5}{x} - \frac{3}{x^2} - \frac{2x+4}{x^2+4x+5} \right) dx = 5\ln|x| + \frac{3}{x} - \ln(x^2+4x+5) + C,$$

where the last term can be integrated by the u-substitution $u = x^2+4x+5$, $du = (2x+4)dx$.