# AERO 443 – Initial Sizing - Example

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During the initial sizing of an airplane, we want to define 3 unknowns weights:

- Empty Weight ———————— $W_e$
- Maximum Take-off Weight ———————— $W_{TO}$
- Fuel Weight ———————— $W_f$

Other unknows weights will be considered requirements at this point:

- Payload Weight ———————— $W_{pl}$
- Reserves ———————— $W_{res}$
- Others ———————— $W_{misc}$

So, to solve for three unknows, we need three equations:

1. Maximum Take-off Weight

$$W_{TO} = W_e + W_f + W_{pl} + W_{res} + W_{misc}$$

2. Empty weight as function of Maximum Take-off Weight

$$W_e = f(W_{TO})$$

3. Breguet Range/Endurance equation

$$W_f = f(W_{TO}, V, \frac{L}{D}, SFC, R)$$

## Breguet Range/Endurance equations

**<u>!!! Make sure you are using the right units!!!</u>**

| | Jet | Propeller |
|---|---|---|
| Range | $R = \frac{L}{D} \cdot V \cdot \frac{1}{SFC} \cdot \ln\left(\frac{W_i}{W_o}\right)$ | $R = \frac{L}{D} \cdot \frac{\eta_p}{SFC} \cdot \ln\left(\frac{W_i}{W_o}\right)$ |
| Endurance | $E = \frac{L}{D} \cdot \frac{1}{SFC} \cdot \ln\left(\frac{W_i}{W_o}\right)$ | $E = \frac{L}{D} \cdot \frac{1}{V} \cdot \frac{\eta_p}{SFC} \cdot \ln\left(\frac{W_i}{W_o}\right)$ |

Where:

- Initial Weight ———————— $W_i$
- Final Weight ———————— $W_o$

Notice that range $(R)$ (or endurance), $\frac{L}{D}$, specific fuel consumption $(SFC)$, propeller efficiency $(\eta_p)$ and speed $(V)$ can be treated as:

- Requirements – need to meet a specific value due to design requirements

OR

- Design variables – can vary within a certain range and depending on sizing results will be selected to give the design a specific goal/requirement.

Also, notice that $\frac{L}{D}$ and $V$ will be tight together in order to follow a drag polar.

Let's see how can we start to solve this problem, slowly increasing the level of complexity…

Requirements

- 2-seater propeller driven airplane (200lbs per person)
- Range 1000 nm
- Payload 100 lbs
- 25% fuel reserve

Design Variables

- Aerodynamic efficiency $\left(\frac{L}{D}\right)$
- Speed $(V)$
- Propulsion system efficiency $(SFC, \eta_p)$
- Structural efficiency $\left(\frac{W_e}{W_{TO}}\right)$

## Requirements

- 2-seater propeller airplane (200lbs per person) → $W_{pl} = 500lbs$
- Range 1000 nm → $R = 1000nm$
- Payload 100 lbs
- 25% fuel reserve → $W_{res} = 0.25 \cdot W_f$

## Design Variables

- $\frac{L}{D}$ → $10.0 < \frac{L}{D} < 13.0$
- Speed → $80kts < V < 150kts$
- Propulsion system efficiency → $0.35\frac{lb}{hp \cdot h} < SFC < 0.55\frac{lb}{hp \cdot h}$, $0.7 < \eta_p < 0.85$

Guess! Based on competitive assessment, literature or experience

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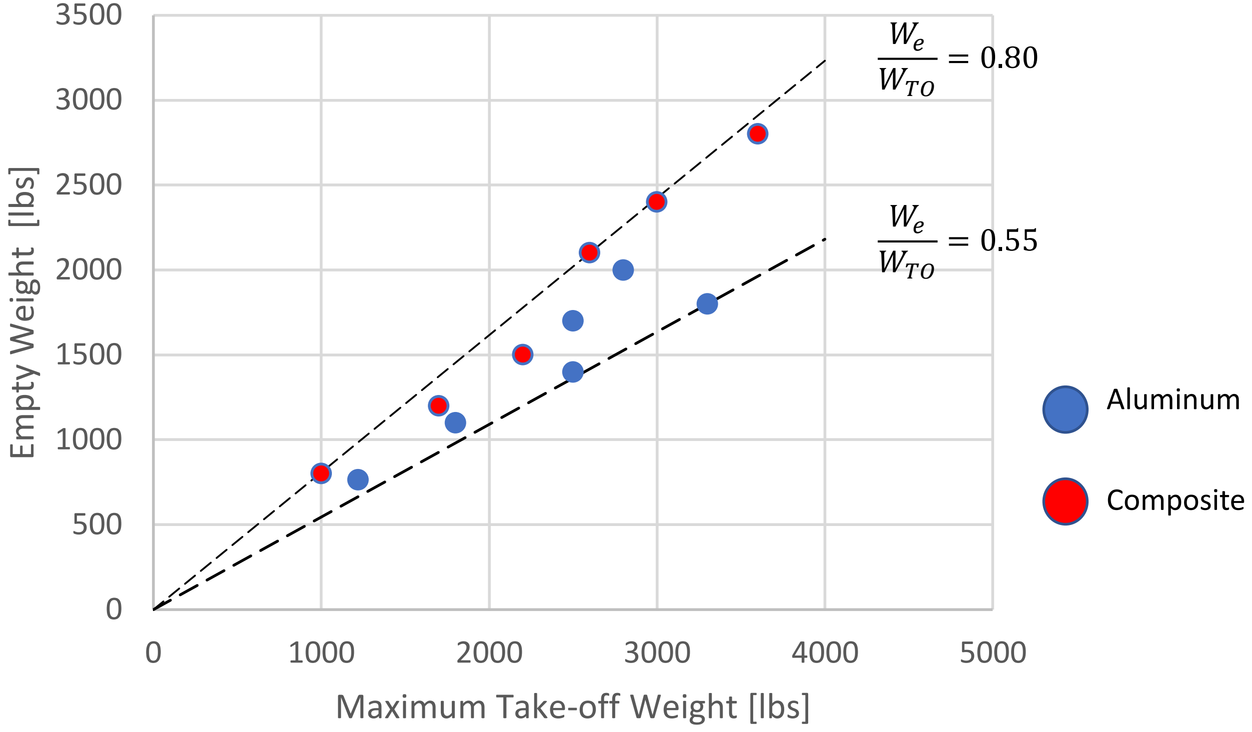
## Assumptions

- Empty Weight can be written as a linear function of the Maximum Take-off Weight – relation is called empty weight fraction – can be estimated based on competitive assessment.




Let's assume for now an empty weight fraction of 0.7 because we are thinking to make a composite airplane.

Depending on results we should investigate the effect of different empty weight fractions and re-consider our fabrication method.

At this point we can write:

$$W_{TO} = W_e + 1.25 \cdot W_f + 500lbs$$

$$W_e = 0.7 \cdot W_{TO}$$

$$1000nm = \frac{L}{D} \cdot \frac{\eta_p}{SFC} \cdot \ln\left(\frac{W_{TO}}{W_{TO} - W_f}\right)$$

For a first approach, let's consider $\boxed{\frac{L}{D} = 10, SFC = 0.5\frac{lb}{hp \cdot h}, \text{ and } \eta_p = 0.8}$

$$0.3 \cdot W_{TO} = 1.25 \cdot W_f + 500lbs$$

$$1000nm = 10 \cdot \frac{0.8}{0.5\frac{lb}{hp \cdot h}} \cdot 325 \cdot \ln\left(\frac{W_{TO}}{W_{TO} - W_f}\right)$$

Notice:

- $1\frac{hp \cdot h}{lbs} = 325nm$

$$0.24 \cdot W_{TO} - 400 = W_f$$

$$0.1923 = \ln\left(\frac{W_{TO}}{0.76W_{TO} - 400}\right)$$

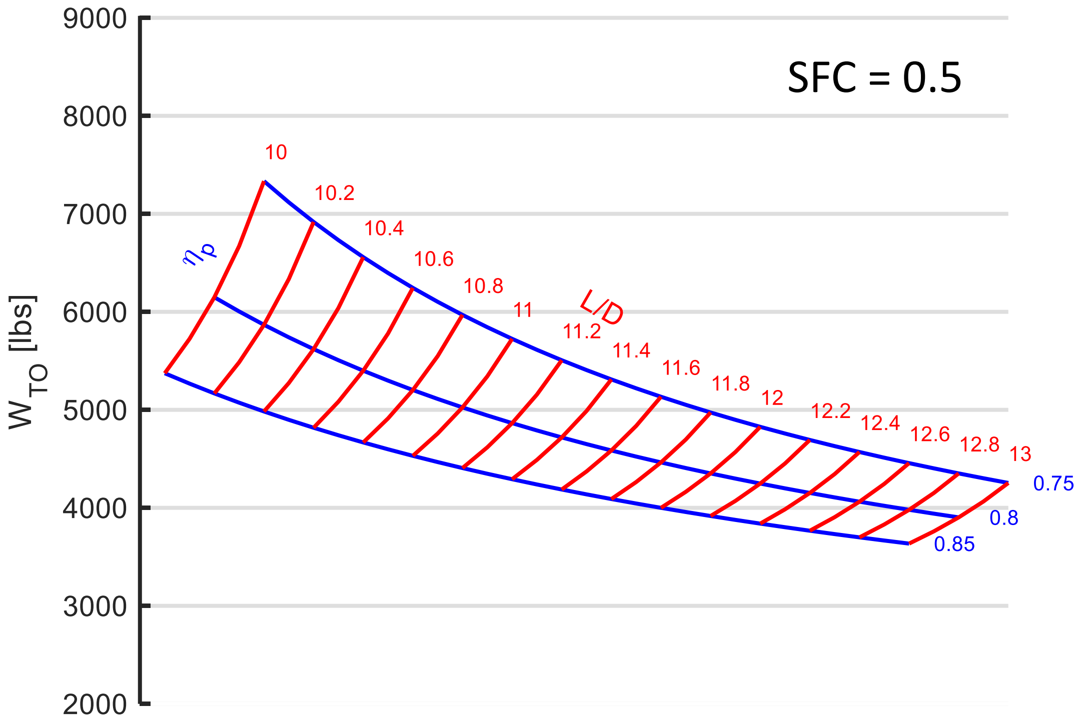
$$1.2120 = \frac{W_{TO}}{0.76W_{TO} - 400}$$

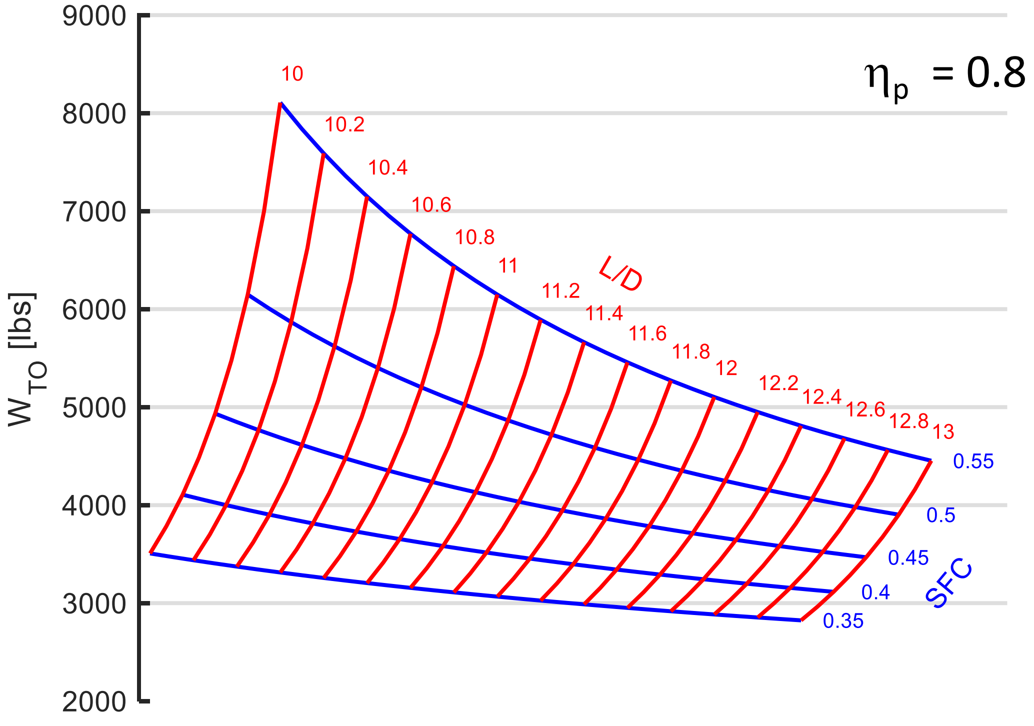
$$0.0789 \cdot W_{TO} = 484.8$$

| | |
|---|---|
| $W_{TO} = 6144\ lbs$ | $W_{res} = 269\ lbs$ |
| $W_E = 4300\ lbs$ | $W_{pl} = 500\ lbs$ |
| $W_f = 1075\ lbs$ | |

But what if the design variables ($\frac{L}{D}$, $SFC$ and $\eta_p$) are not the ones we choose?

Repeating this procedure for multiple conditions, we can create carpet plots to study the impact of design variables in our design.



At this point, we should start to ask, what are the reasonable values for our design variables?

More analysis, competitive assessment and technology will be necessary.

Let's go ahead and try to refine our estimative of SFC and L/D.

The following procedure is just an example – it is not the only way to analyze this problem!

For SFC, let's look at some existing engines (notice, we still don't know how much power we will need…)

| Engine | Max Power [hp] | SFC - cruise[lbs/hp/h] |
|---|---|---|
| Lycoming IO-540 | 300 | 0.45 |
| Rotax 912 | 80 | 0.50 |
| Rotax 915 | 140 | 0.46 |
| Austro Engine AE300 | 170 | 0.35 |

Now, we need to start to make design decisions. In this case, I have the following rationale (just an example):

- The AE300 is the best efficiency, but it is currently only used by Diamond and might be difficult to get proper support from the manufacturer.
- The second-best option in terms of SFC is a Lycoming IO-540, but the engine is old technology and heavy.
- The new Rotax 915 is a very modern engine, turbo charged, and with a minor penalty for the IO-540 in terms of SFC

So, let's assume that we can use the Rotax 915 for now (we still don't know if it has enough power).

For L/D, we know that it depends on the airplane drag polar. Assuming a parabolic drag polar, and using data from the literature, to estimate drag ($C_{D0}$ *and* $e$), aspect ratio ($AR$) and wing area ($S$):

$$C_D = C_{D0} + \frac{C_L^2}{\pi \cdot AR \cdot e}$$

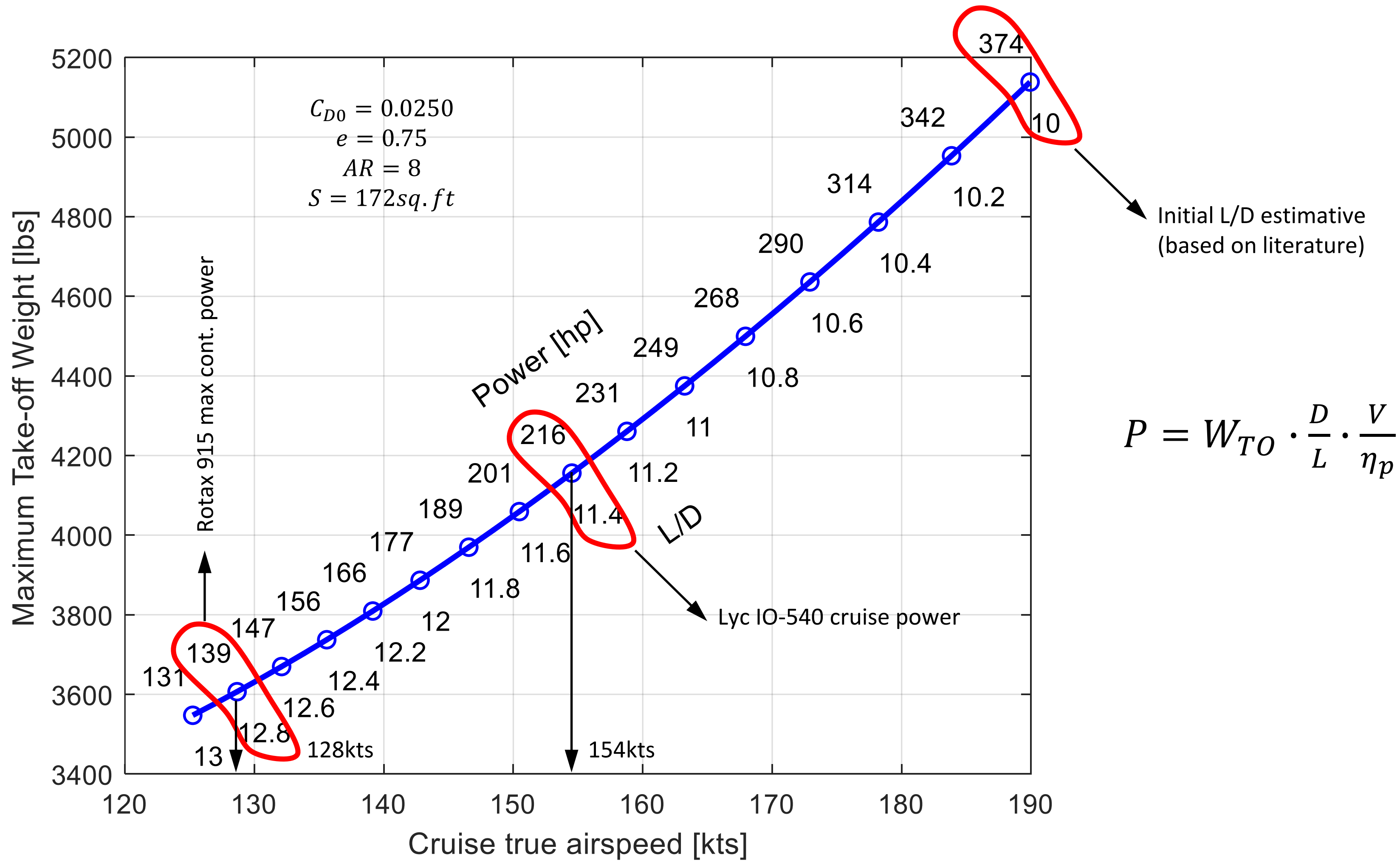
$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D0} + \frac{C_L^2}{\pi \cdot AR \cdot e}} \xrightarrow{solving\ for\ C_L} \frac{L}{D}\frac{1}{\pi \cdot AR \cdot e}C_L^2 - C_L + \frac{L}{D}C_{D0} = 0$$

and, remembering that:

$$L = \frac{1}{2}\rho V^2 \cdot S \cdot C_L = W \rightarrow C_L = \frac{2W}{\rho V^2 S}$$

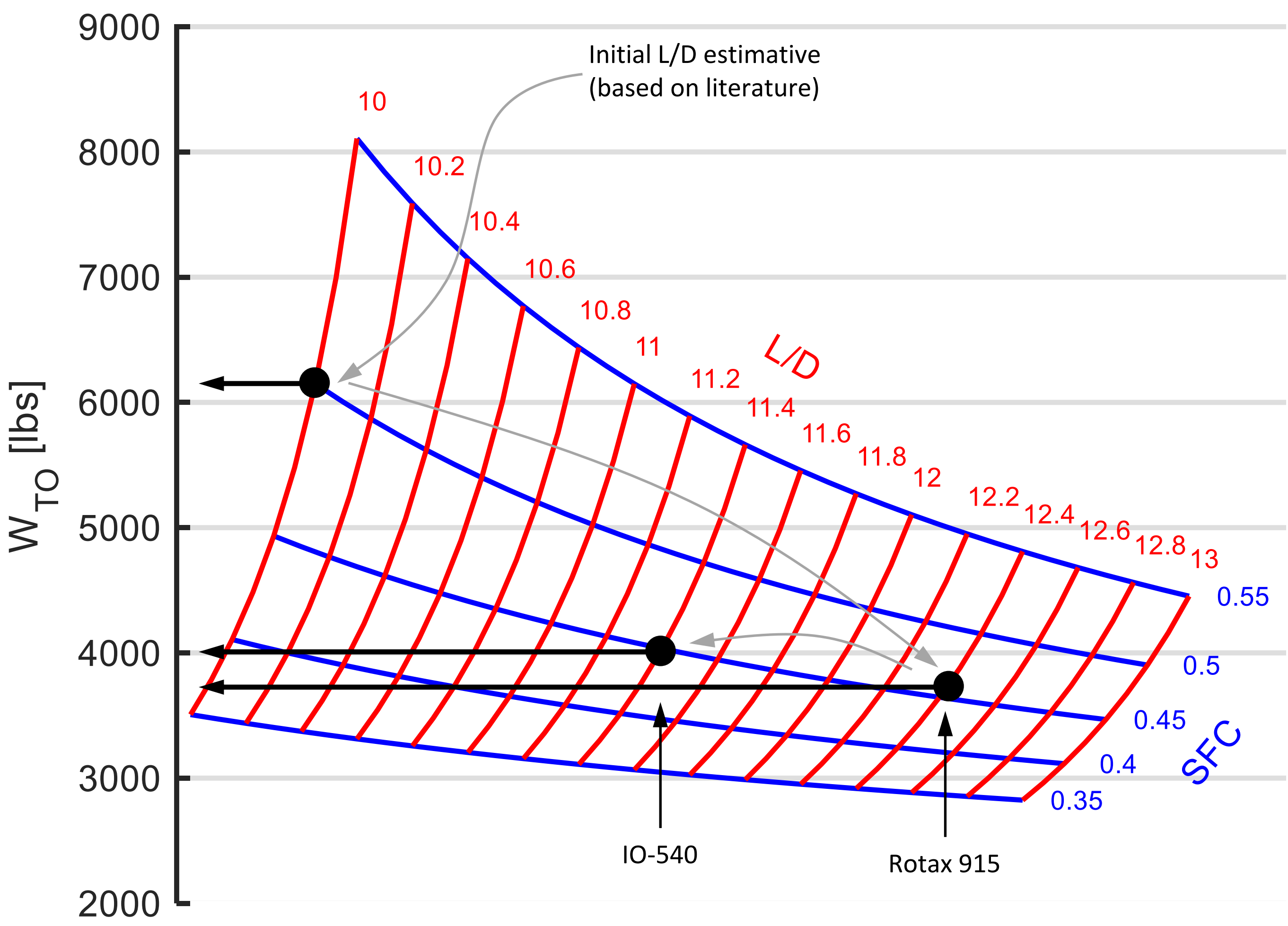
Allow us to calculate which speed the airplane need to fly to reach a specific value of $L/D$, given a grad polar.

So, using $SFC = 0.46$, $\eta_p = 0.8$ and a proposed drag polar, it is possible to calculate:




$$P = W_{TO} \cdot \frac{D}{L} \cdot \frac{V}{\eta_p}$$

Back to the initial carpet plot:



At this point, after 3 iterations, seems that the best estimative of weight for this airplane is:

$$W_{TO} = 4030lbs$$
$$W_E = 2820\ lbs$$
$$W_F = 568lbs$$

To achieve this results, many assumptions were made, including:

Drag polar ($C_{D0} = 0.025$ and Oswald Factor $e = 0.8$)
Wing size (area $S = 172sq.ft$ and aspect ratio $AR = 8.0$)
Engine type and size (Lycoming IO-540 $300hp$)

At this point, it will be important to go back and re-evaluate these assumption and make studies (carper plots, etc) to evaluate their impact on the design and their feasibility.

This design process is heavily iterative, and the solution should converge to a final value as more fidelity is added to these models.

The goal of the designer should be to investigate all possible solutions and justify WHY his choices are feasible and adequate.