AIRCRAFT DESIGN

Wing and Power Loading – Constraint Diagram

Performance Requirements

- Normally there few performance requirements that the aircraft needs to achieve:
  - Stall Speed
  - Take-off and Landing distance
  - Cruise Speed (or maximum speed)
  - Rate of climb
  - Time to climb
  - Maneuverability

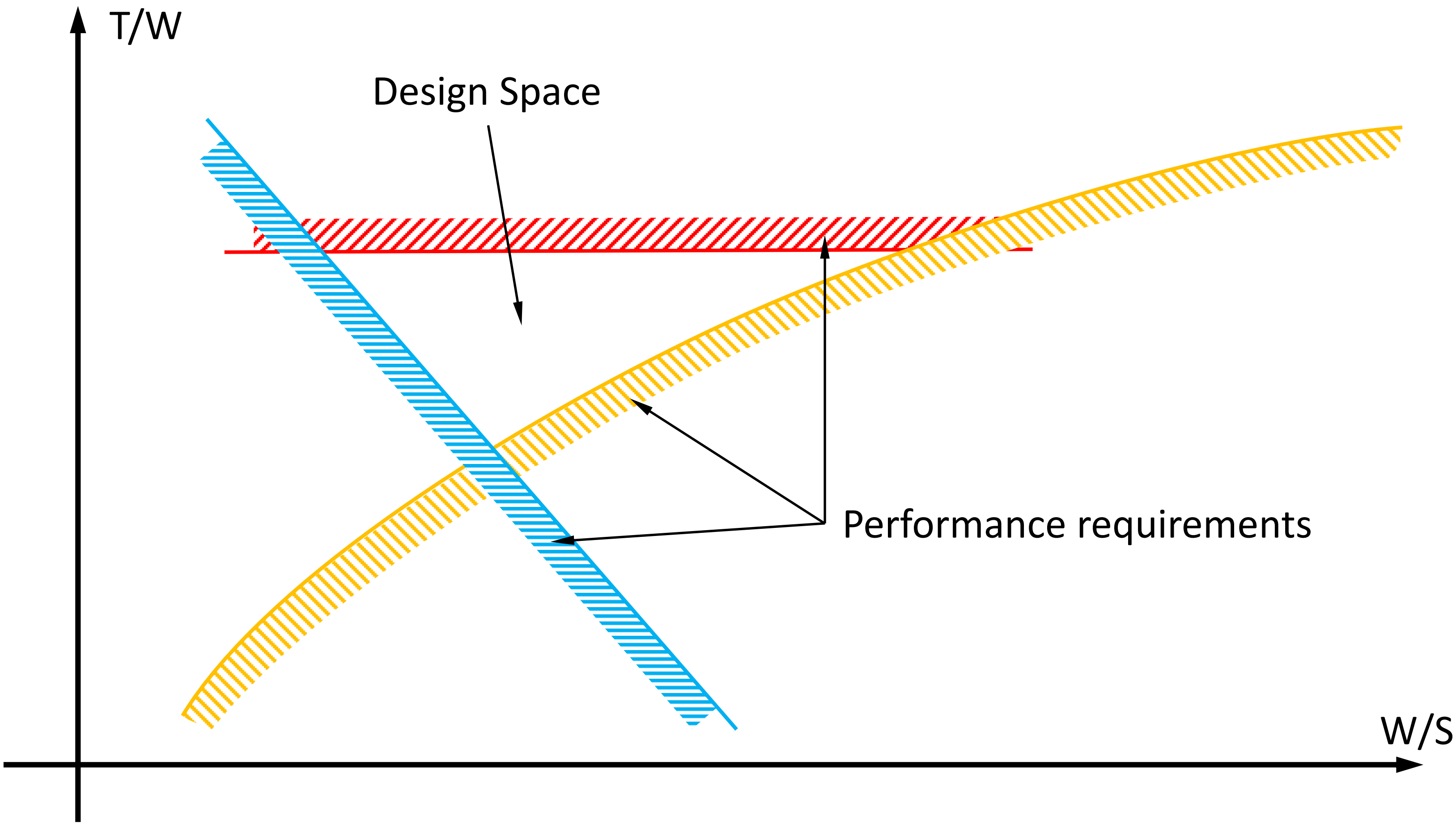
Design Parameters

- Few design parameters are directly related with these performance requirements:
  - Wing Loading
  - Thrust or Power
  - Maximum Lift Coefficient (for all the configurations possible)

Algorithm

- The constraint diagram is a method to graphically determine a range of design parameters (wing loading, thrust/weight and maximum lift coefficient) that guarantee that the aircraft will comply with the performance requirements.

Algorithm



Stall Speed

- In general, the requirements states:

$$V_S \leq\sim V_{S\lim ite}$$

Requirement	CS-VLA	Part 23	Part 25	LSA
$V_{Slimite}$	45kts (flaped)	61kts	NA	45kts(no flap)

Stall Speed

- Assuming equilibrium of vertical forces :

$$V_S = \sqrt{\frac{2W/S}{\rho C_{L\max}}}$$

- Knowing the required stall speed, there is a clear relation between wing loading, for a given maximum lift coefficient that need to be satisfied.

Stall Speed

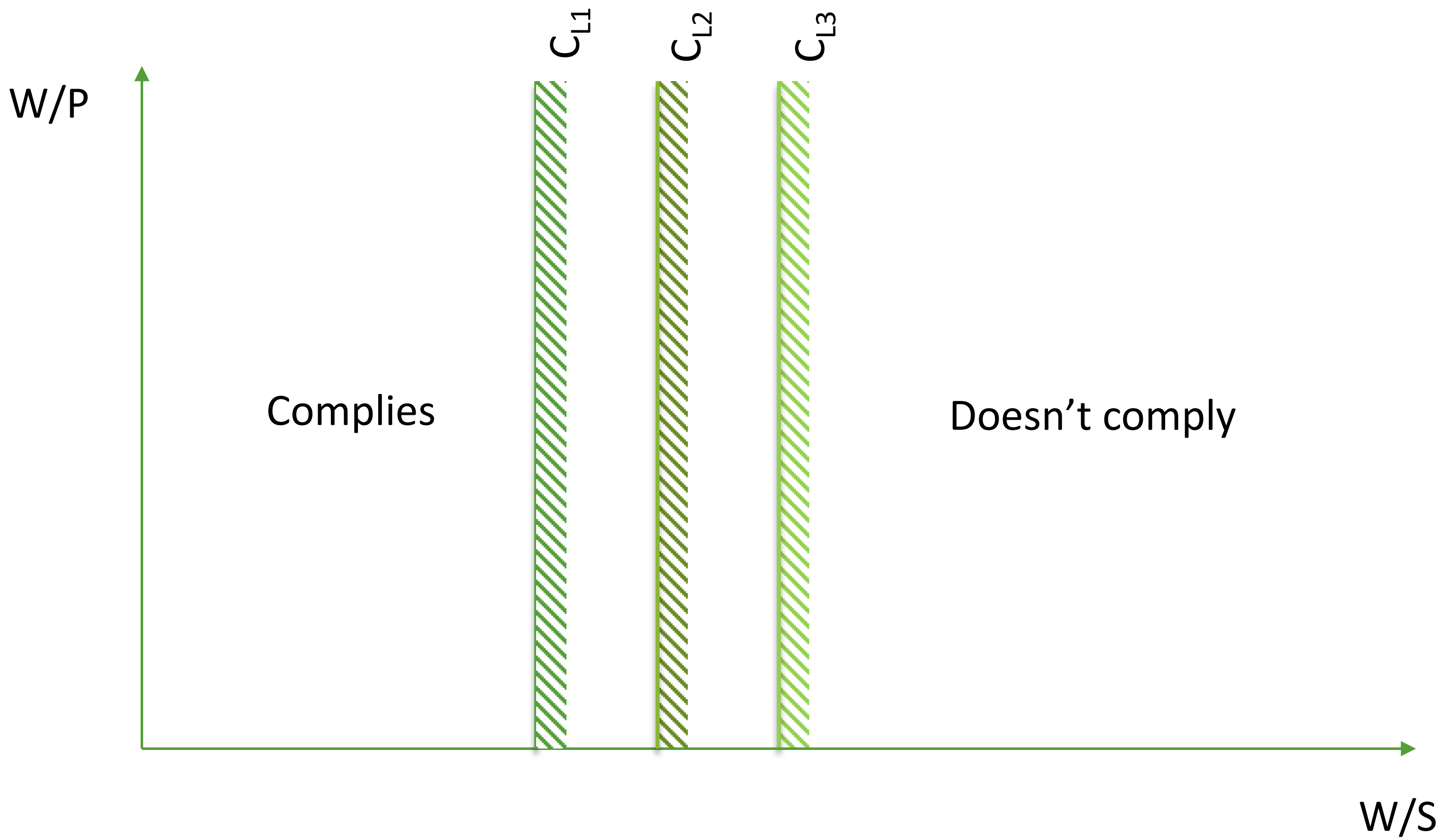
$$\frac{V_{S\,\lim ite}^2 \rho}{2} C_{L\max} \geq W/S$$

- The maximum lift coefficient changes with:
  - Wing airfoil type
  - Type and position of the high lifting devices

Stall Speed

Aircraft type	C_{Lmax}	C_{LmaxTO}	C_{LmaxLD}
HomeBuilt	1.2-1.8	1.2-1.8	1.2-2.0
Single engine	1.3-1.9	1.3-1.9	1.6-2.3
Twin Engine	1.2-1.8	1.4-2.0	1.6-2.5
Crop duster	1.3-1.9	1.3-1.9	1.3-1.9
Corporate Jet	1.4-1.8	1.6-2.2	1.6-2.6
TP Regional	1.5-1.9	1.7-2.1	1.9-3.3
Transport Jet	1.2-1.8	1.6-2.2	1.8-2.8
Military trainner	1.2-1.8	1.4-2.0	1.6-2.2
Fighter	1.2-1.8	1.4-2.0	1.6-2.6
Bomb./Patr.	1.2-1.8	1.6-2.2	1.8-3.0
Anphybian	1.2-1.8	1.6-2.2	1.8-3.4
Supersonic Transp.	1.2-1.8	1.6-2.0	1.8-2.2

Stall Speed



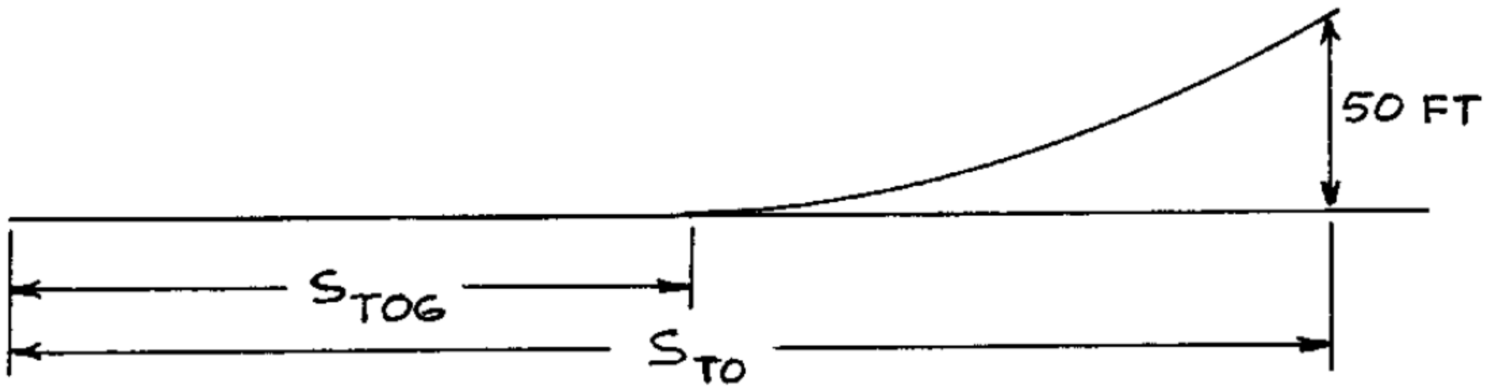
$$C_{L1} < C_{L2} < C_{L3}$$

Take off distance

- Take off distance is a parameter that depends of:
  - Takeoff weight
  - Takeoff speed
  - Weight-Power ration
  - Aerodynamic drag
  - Rolling resistance
  - Flight technique
  - Type of runway

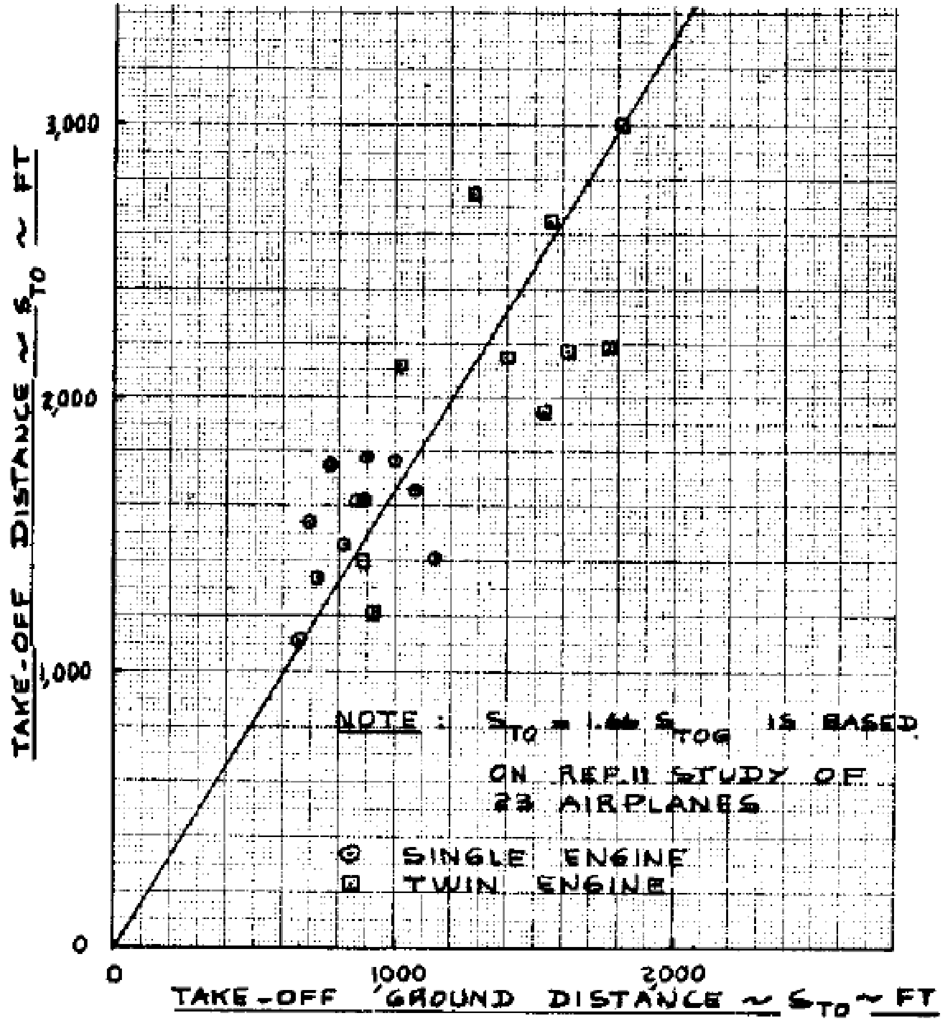
- For our cases we will assume hard runways.

Take off distance

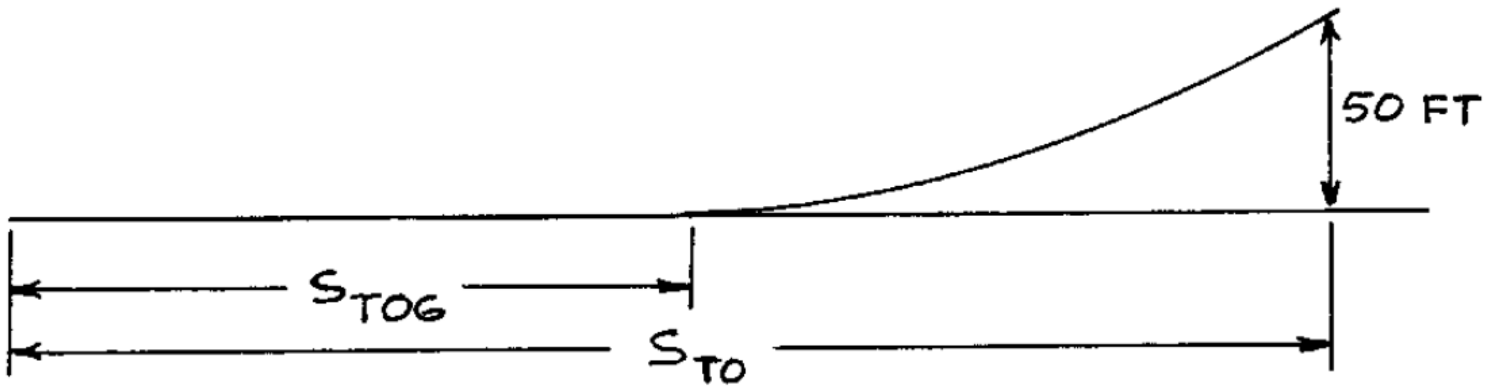


- Roskan (1997) presents an approximated relation between S_{TO} and S_{TOG} for Part 23 airplanes.

Take off distance



Take off distance



- For PART 23 (propeller) airplanes, the following relation can be assumed:

$$S_{TOG} \propto \frac{\left(W/S\right)_{TO}\left(W/P\right)_{TO}}{\sigma C_{L\max TO}} = TOP_{23}$$

Take off distance

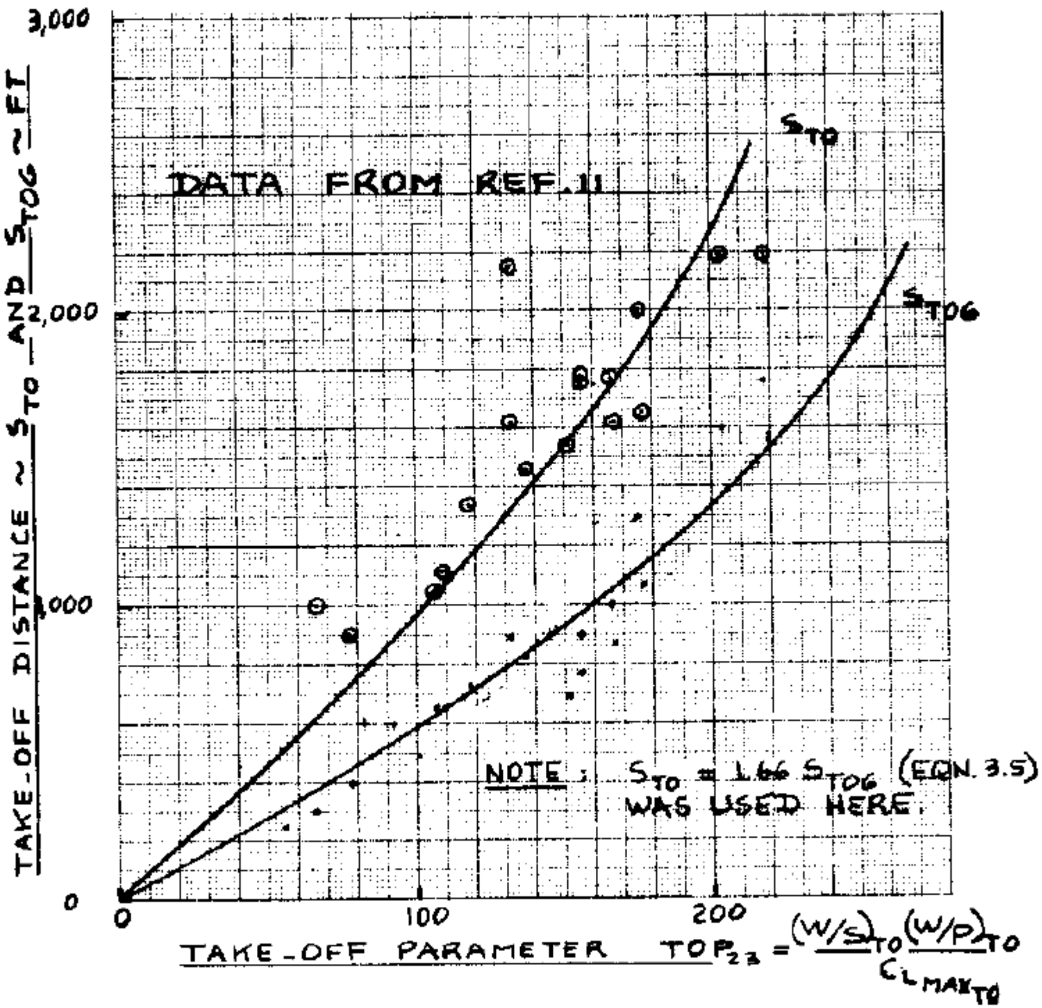
- TOP_{23} units are: lb^2/ft^2hp

- In general, it is correct to assume:

$$\underset{LIFT-OFF}{C_{L_{TO}}} = \frac{C_{L\max TO}}{1.21}$$

- Roskan (1997) presents a relation between S_{TOG} and TOP_{23}

Take off distance



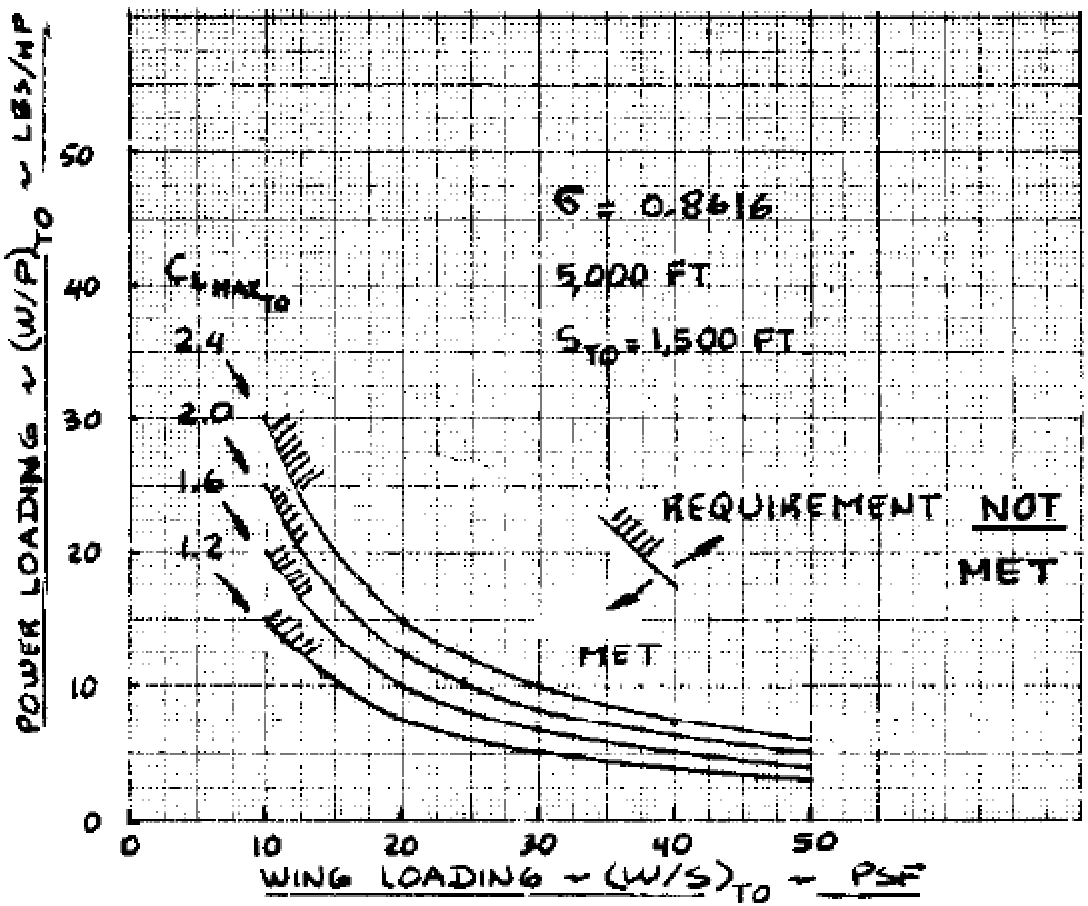
Take off distance

- Therefore, it is possible to establish a relation between wing loading and power loading, for given values of maximum lift coefficient, such way that the take off distance stays smaller than the specified:

$$TOP_{23} \;\; S_{TOG} \geq \frac{\left(W/S\right)_{TO} \left(W/P\right)_{TO}}{\sigma C_{L\max TO}}$$

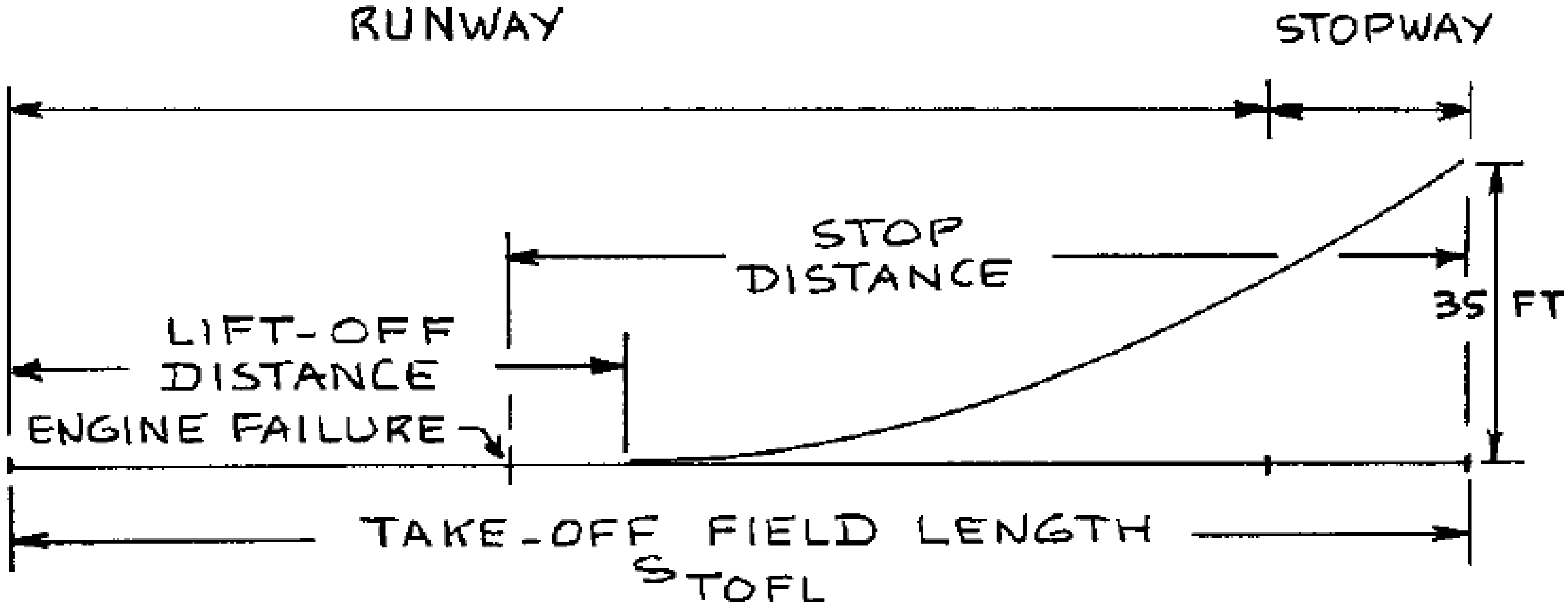
$$\left(W/P\right)_{TO} \leq \frac{TOP_{23} \;\; S_{TOG} \;\; \sigma C_{L\max TO}}{\left(W/S\right)_{TO}}$$

Take off distance



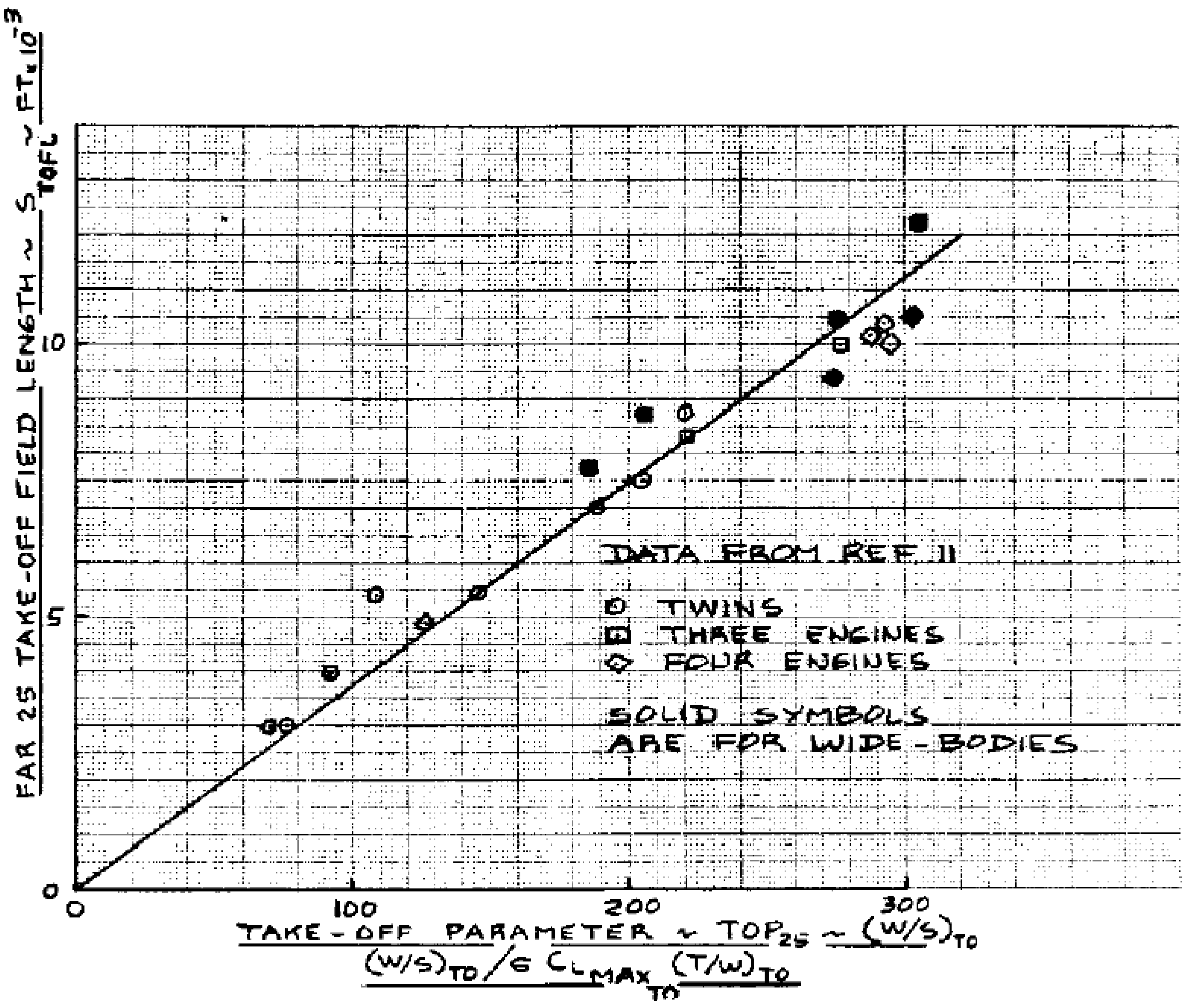
Take off distance

- For Part 25 airplanes:



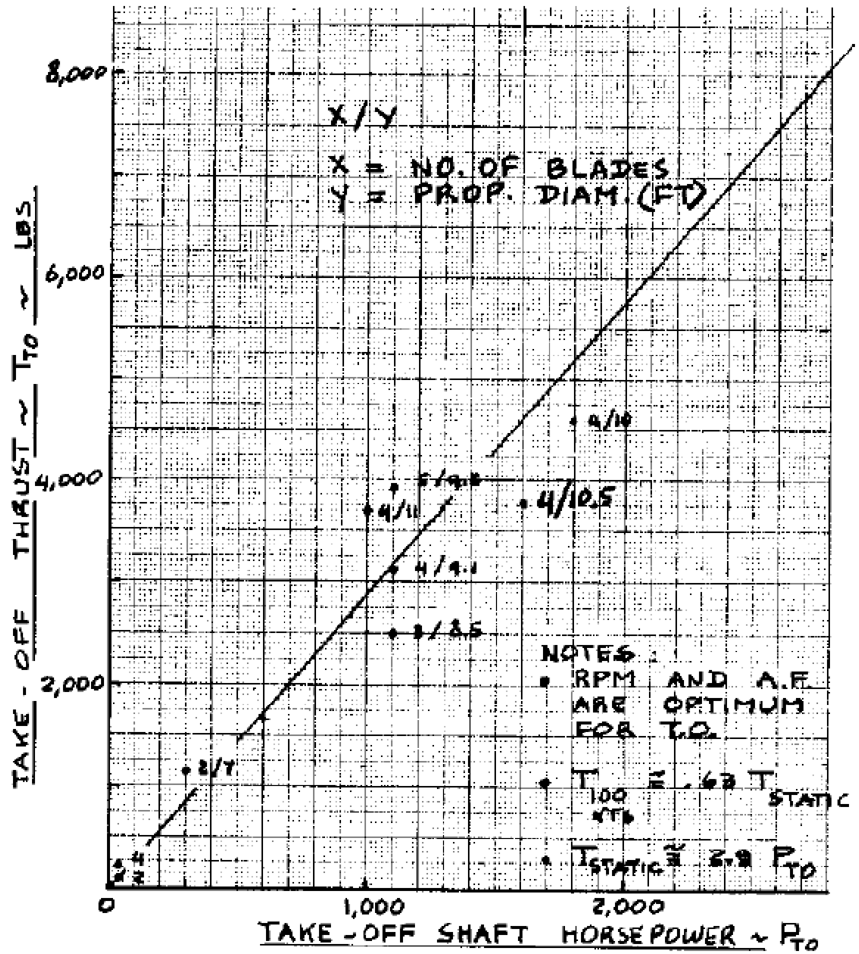
$$S_{TOFL} \propto \frac{\left(W/S\right)_{TO}}{\sigma C_{L\max TO}\left(T/W\right)_{TO}} = TOP25$$

Take off distance



Take off distance

- Since on PART25 it is possible to have propeller driven airplanes, (turbo props) it is necessary to have a relation between thrust and power during take off.



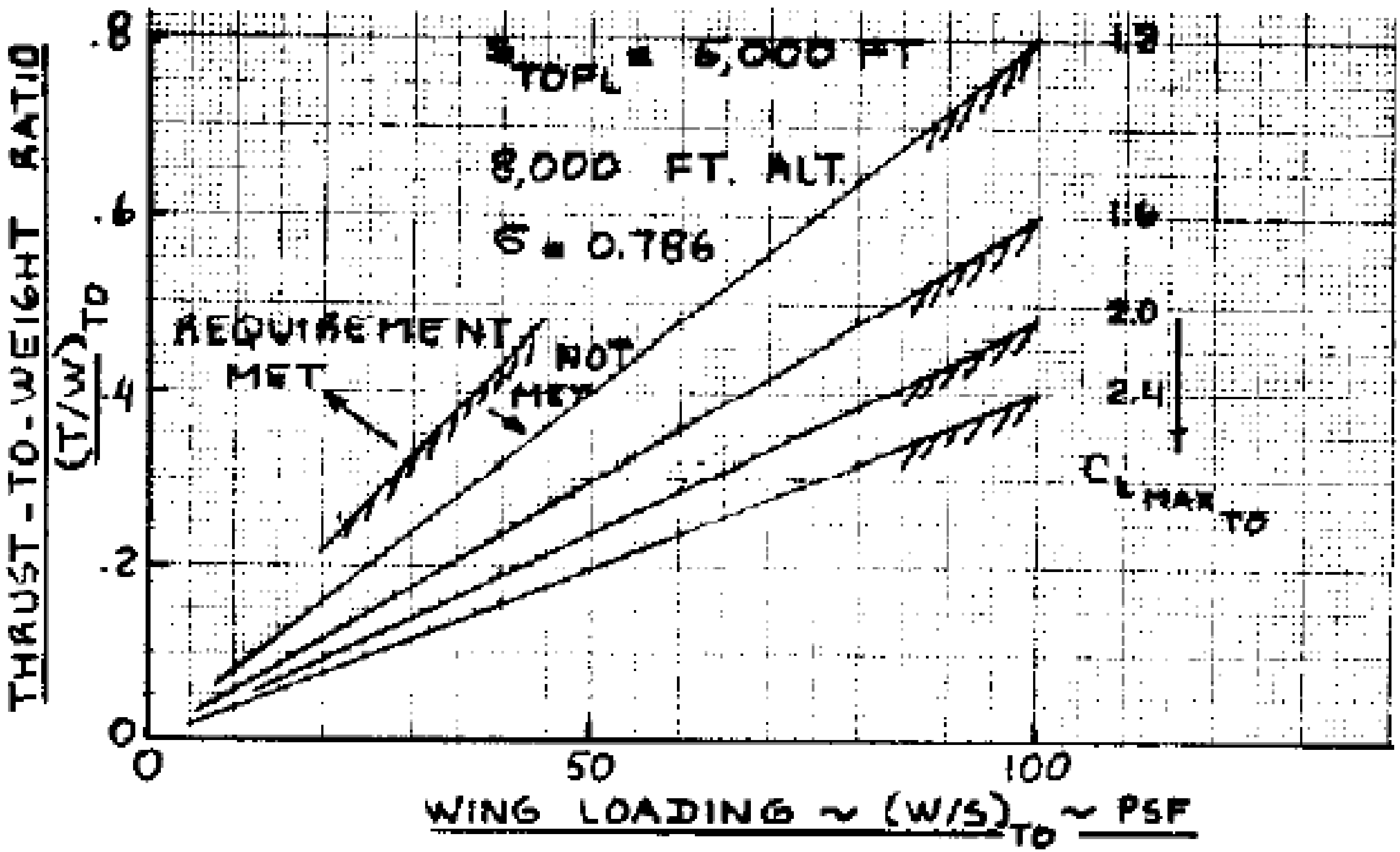
Take off distance

- Therefore:

$$TOP25\ \ S_{TOFL}\ \geq \frac{W/S_{\ TO}}{\sigma C_{L\max TO}\ \ T/W_{\ \ TO}}$$

$$T/W_{\ \ TO} \geq \frac{W/S_{\ TO}}{\sigma C_{L\max TO} TOP25\ \ S_{TOFL}}$$

Take off distance



Landing distance

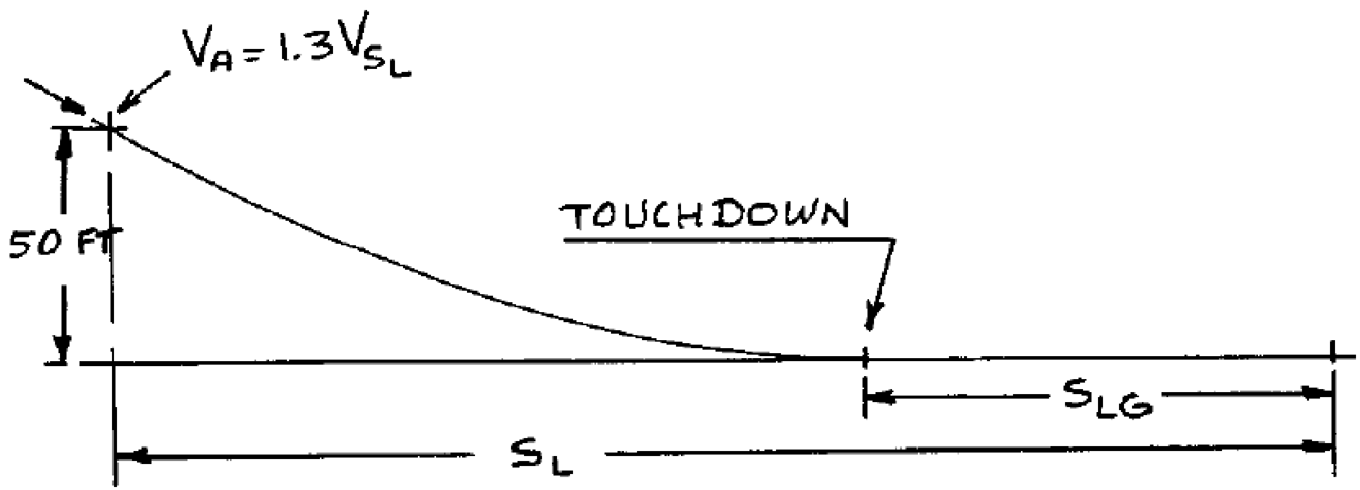
- It is more complicated that the take off distance, since it depends of:
  - Weight
  - Approach speed
  - Deceleration devices (brakes, thrust reverser, drogue chute, tail hook, barriers)
  - Airplane handling
  - Flight technique

Relation W_L/W_{TO}

Aircraft type	Minimum	Average	Maximum
HomeBuilt	0.96	1.0	1.0
Single engine	0.95	0.997	1.0
Twin engine	0.88	0.99	1.0
Crop duster	0.7	0.94	1.0
Corporate Jet	0.69	0.88	0.96
Regional TP	0.92	0.98	1.0
Transport Jet	0.65	0.84	1.0
Military trainner	0.87	0.99	1.0
Fighter	0.78(jet)/0.57(TP)		1.0
Bomb./Patr.	0.68(jet)/0.77(TP)	0.76(jet)/0.84(TP)	0.83(jet)/1.0(TP)
Amphybian	0.79(jet)/0.98(TP)		0.95(jet)/1.0(TP)
Supersonic	0.63	0.75	0.88

Landing distance

- Part 23:

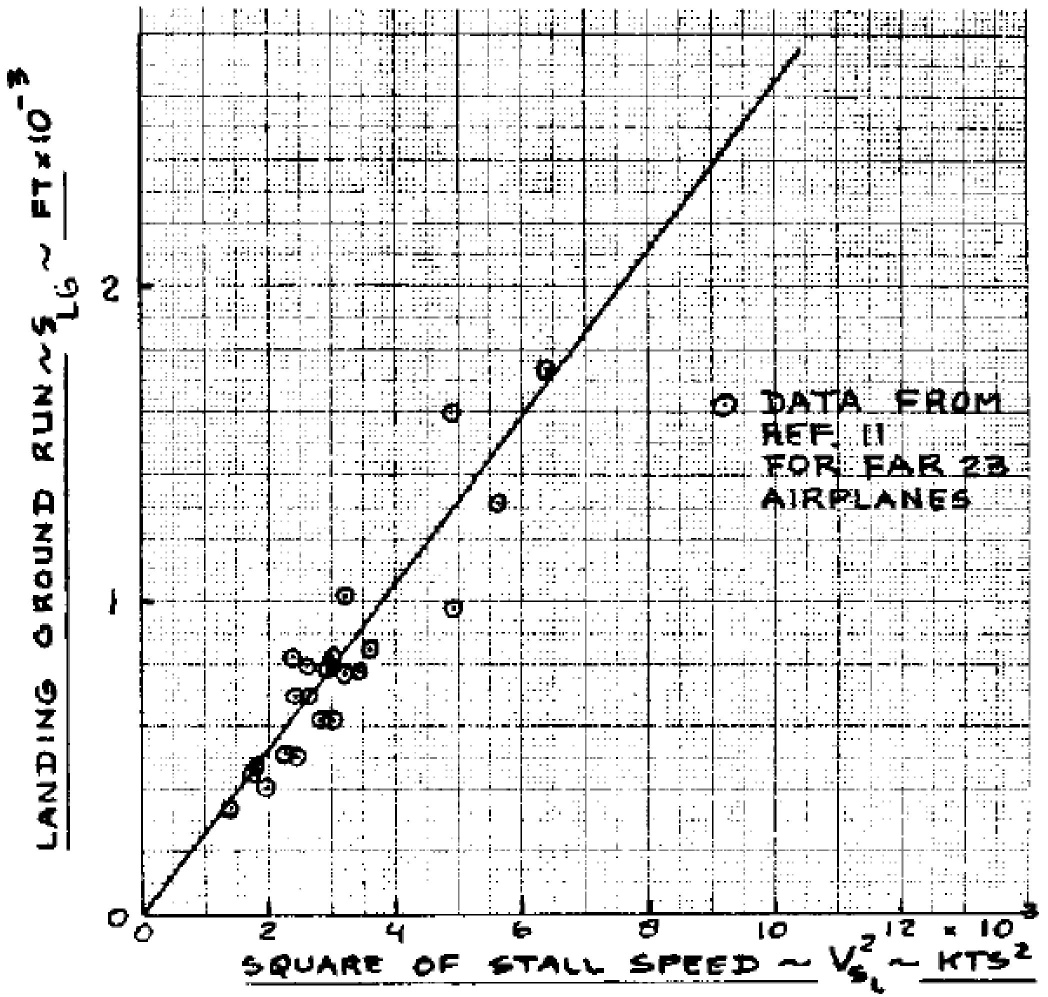


$$S_{LG}[ft] = 0.265 V_{SL}^2 [kt]$$

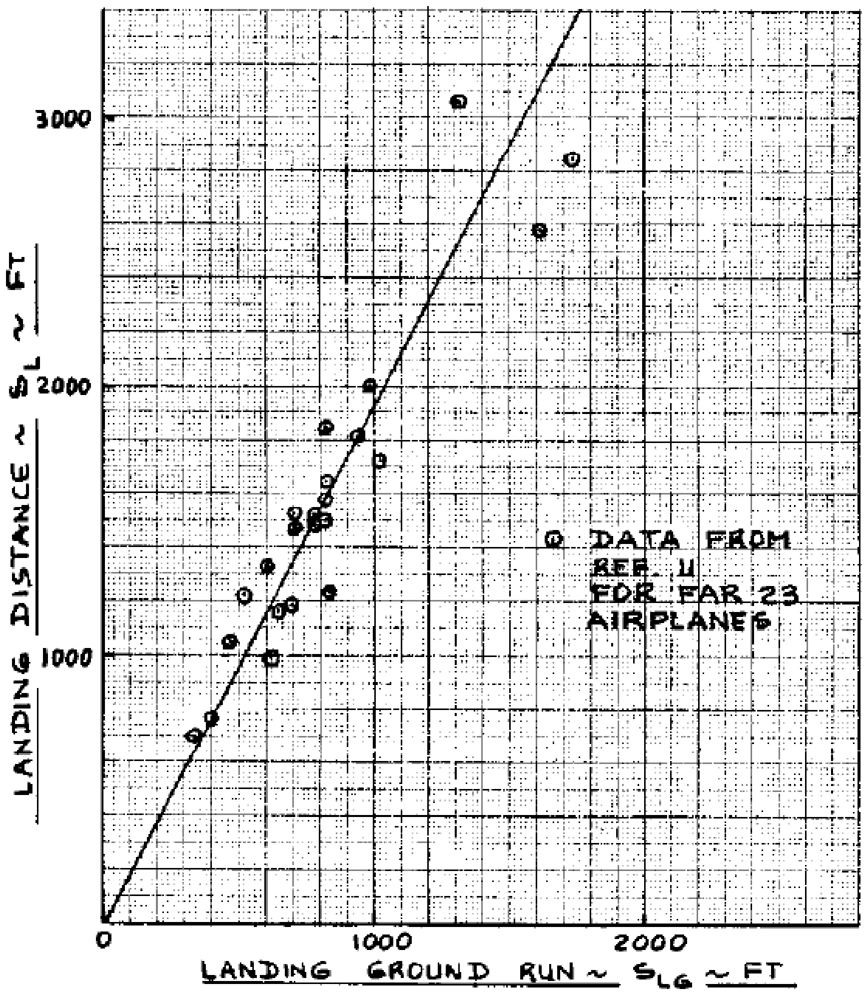
$$S_L = 1.938 S_{LG}$$

$$\therefore S_L = 0.5136 V_{SL}^2$$

Landing distance



Landing distance



Landing distance

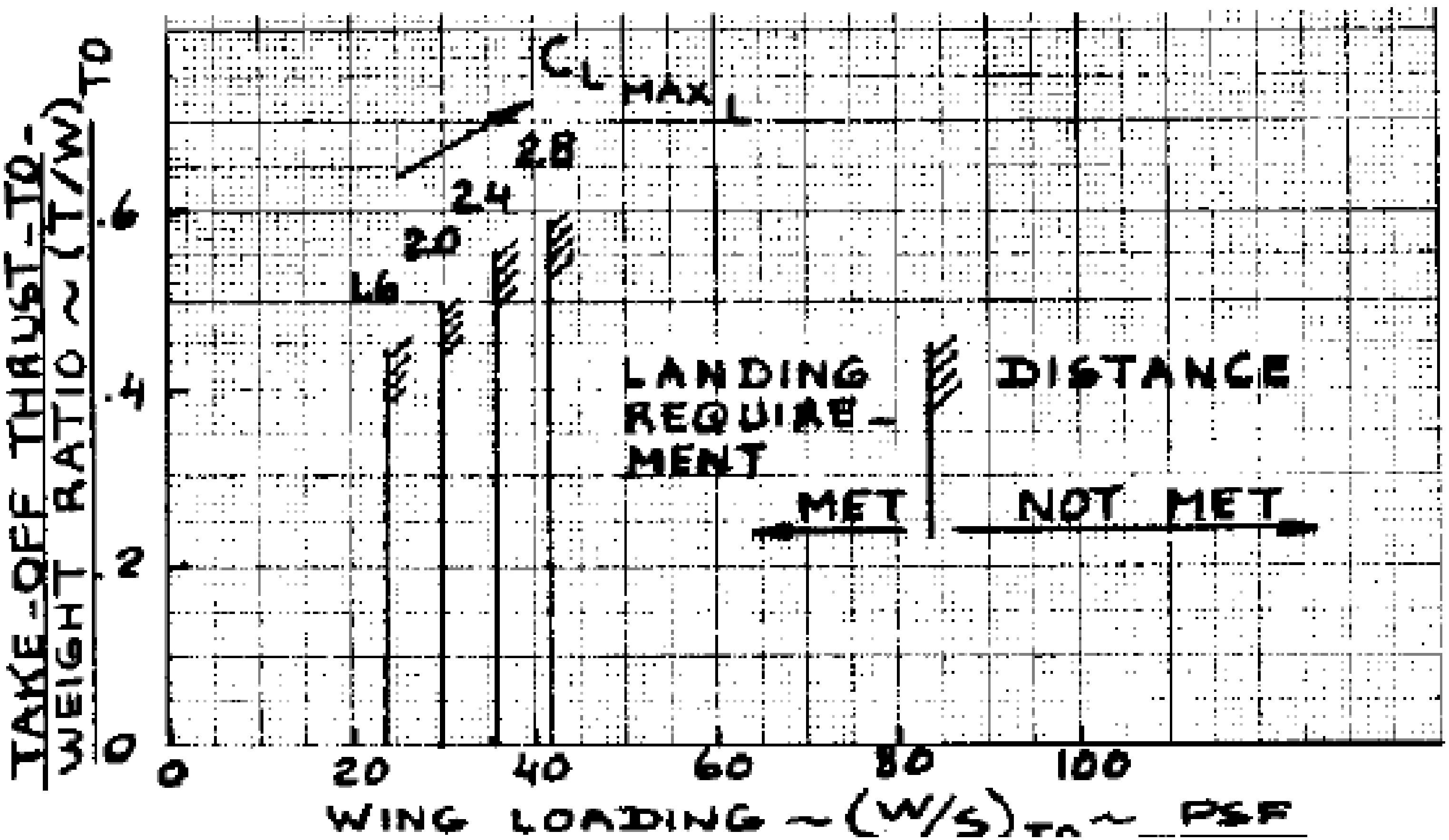
- However, we can write:

$$V_{SL}^2 = \frac{2W_L}{\rho S C_{L\max L}}$$

- Therefore:

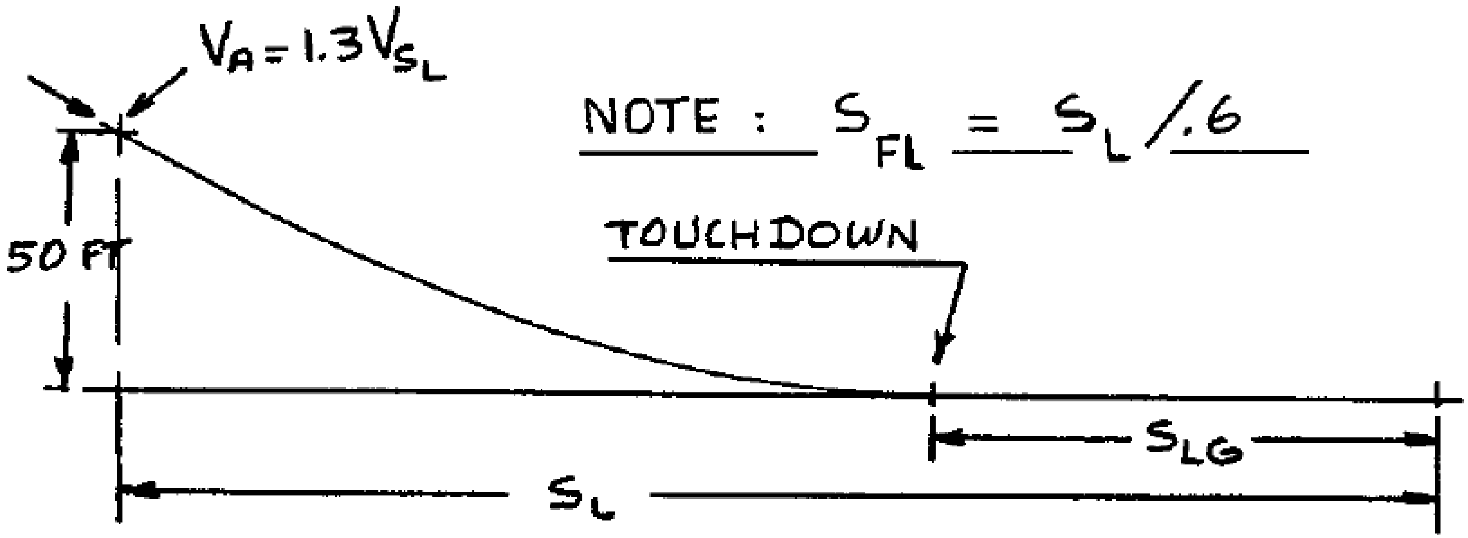
$$\left(\frac{W_L}{S}\right) \leq \frac{S_L \rho C_{L\max L}}{2 \cdot 0.5136}$$

Landing distance



Landing distance

- Part 25:



$$S_{FL}[ft] = 0.3 V_A^2 [kt]$$

Landing distance

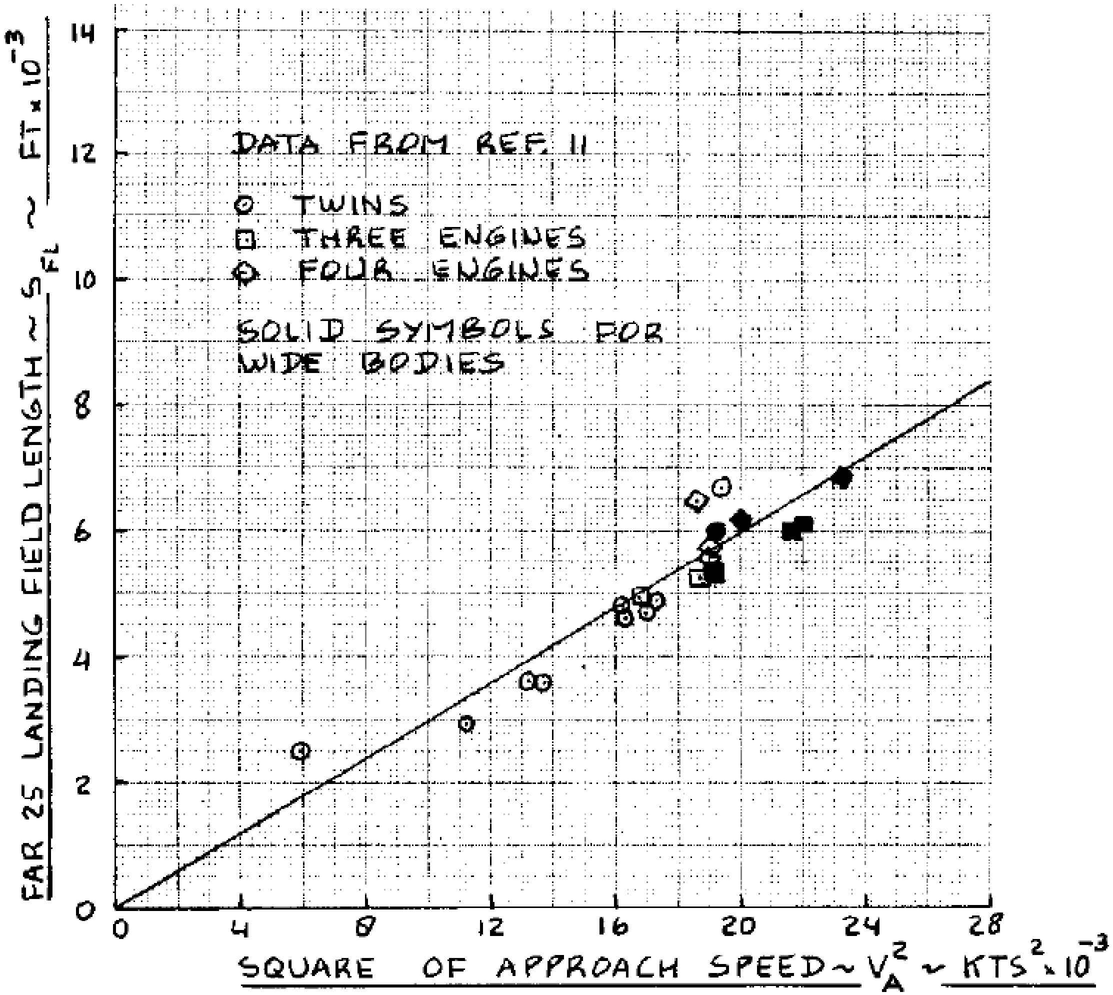
- Same way that was done on Part 23 case:

$$V_A^2 = \frac{2W_L}{\rho S C_{L\max L}}$$

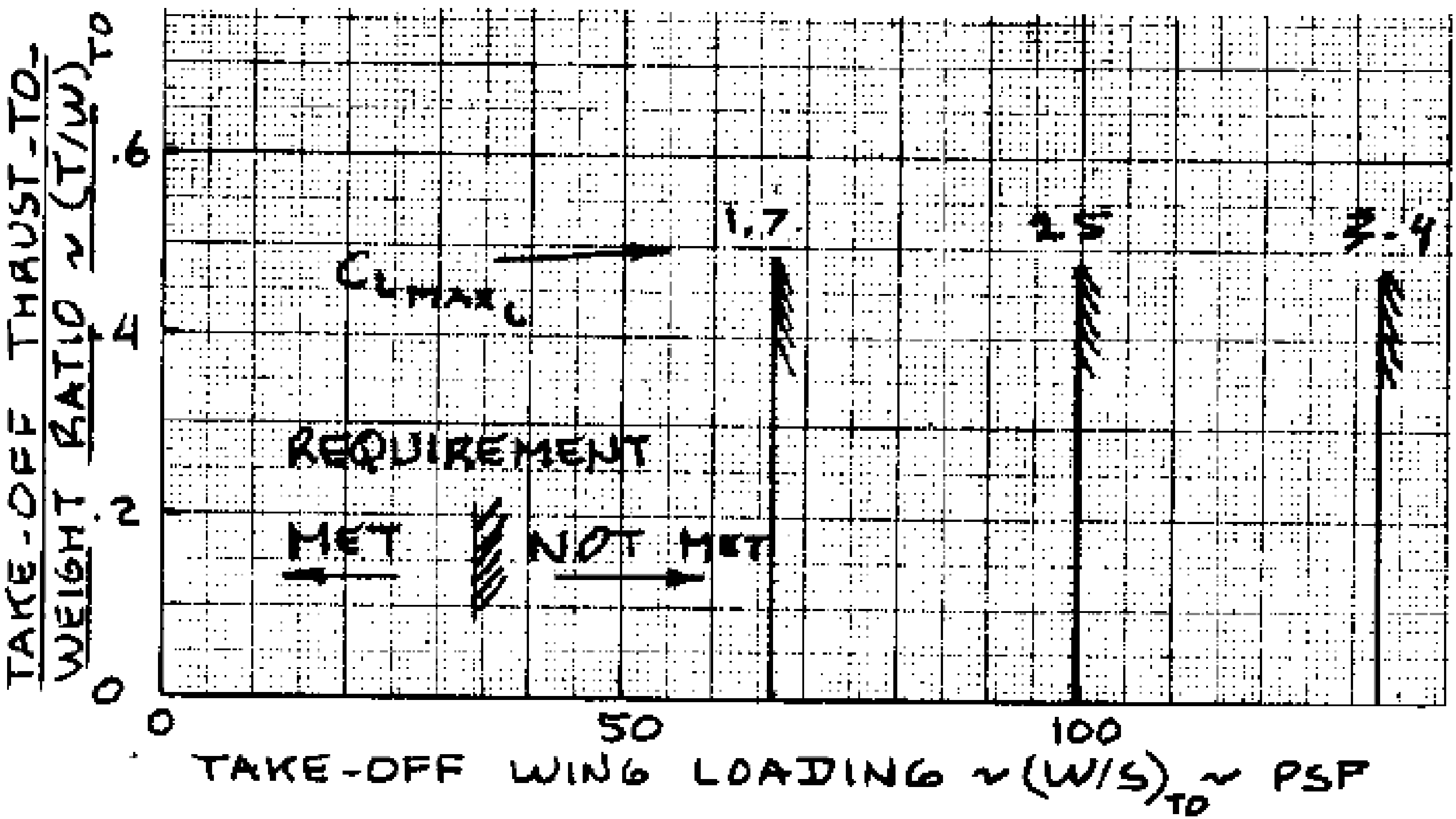
- Therefore:

$$\left(\frac{W_L}{S}\right) \leq \frac{S_{FL}\rho C_{L\max L}}{2 \cdot 0.3}$$

Landing distance



Landing distance



Rate of climb

- Conditions for PART 23 regulations
  - Power and engine conditions (§23.45)
  - AOE (All Engine Operating - (§23.65)
    - Minimum rate of climb 300ft/min
    - Min. climb gradient 1:12 (land)/1:15(amph.)
    - Retracted landing gear
    - TO flaps
    - Cowl flaps on adequate position

Rate of Climb

- OEI (One Engine Inoperative - §23.67)
  - Multi engine $W_{TO} < 6000 lbs$
  - Min. rate of climb

    $$0.027 V_{SO}^2 [kts] ft/min \ @ \ 5000ft$$

  - Critical Engine not operating
  - Non operating prop on minimum drag position
  - Landing gear retracted
  - Flaps on most favorable position
  - Cowl-flaps adequate
  - Verify conditions for $W_{TO} > 6000 lbs$ and jet planes.

Rate of Climb

- Go-around - §23.77
  - Minimum gradient 1:30
  - Take off weight
  - Extended landing gear
  - Landing flaps, unless they can't be retracted in less than 2 seconds.

Rate of Climb

- Dimensioning by rate of climb

$$RC = \frac{dh}{dt} = 33000 RCP [ft / \min]$$

$$RCP = \frac{\eta_p}{W/P} - \left(\frac{\left(\frac{W}{S}\right)^{\frac{1}{2}}}{19 \; C_L^{3/2} / C_D \;\; \sigma^{\frac{1}{2}}} \right)$$

Rate of Climb

- For propeller driven airplanes the maximum rate of climb will be with maximum:

$$C_L^{3/2} / C_D$$

- Therefore, it must happen:

$$C_{L RC\,\max} = 3C_{D0}\pi A e^{\frac{1}{2}}$$

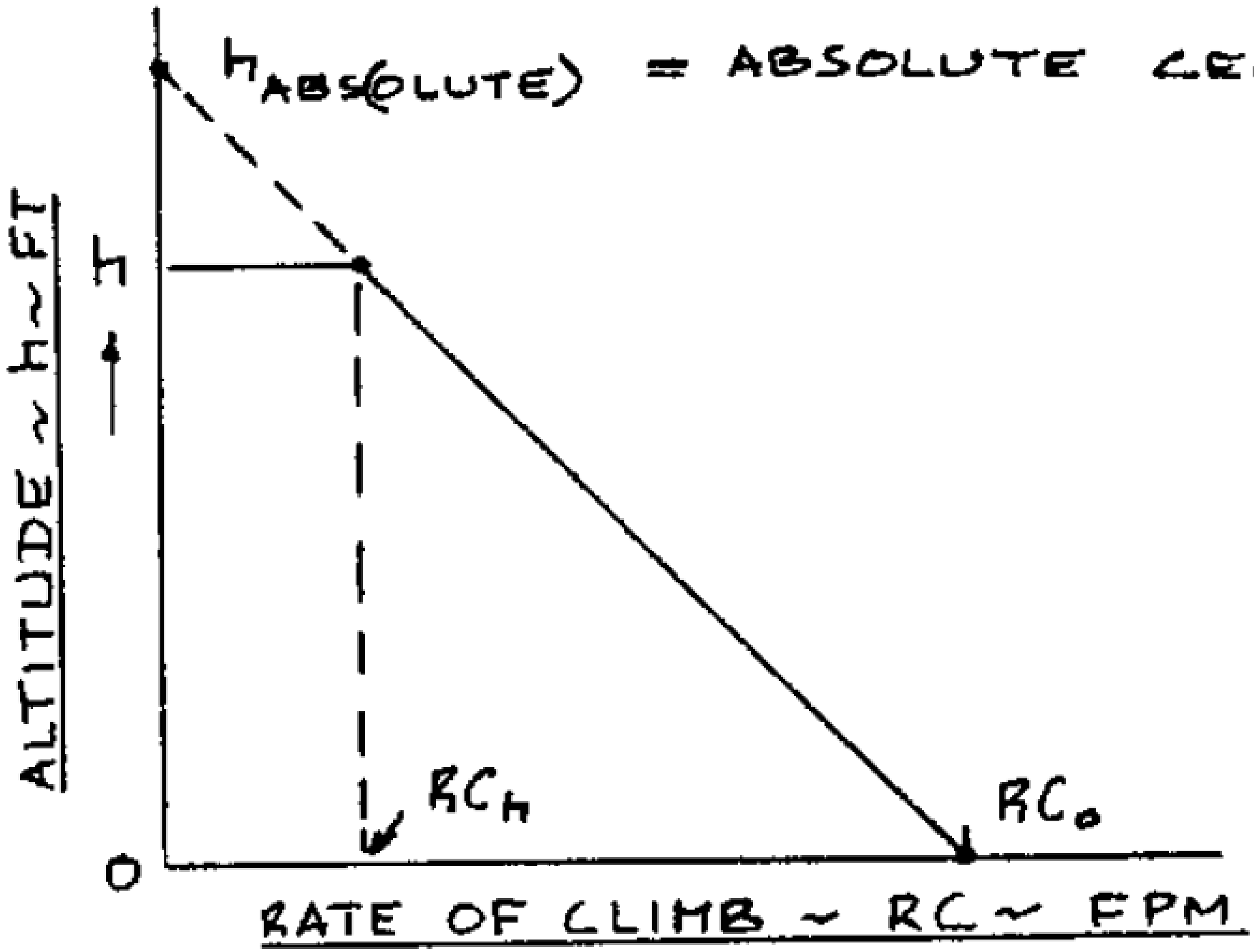
$$C_L^{3/2} / C_D \;{}_{RC\,\max} = \frac{1.345\ A e^{\,3/4}}{C_{C0}^{1/4}}$$

Rate of Climb

- In general the conditions for best gradient happens at really low airspeed, really close to the stall. Therefore, it is necessary to define a margin from minimum speed, for instance:

$$C_{LCGR} = C_{L\max} - 0.2$$

Time to Climb



Time to Climb

- Assuming that the relation between rate of climb and altitude is linear, we can write:

$$R_C = \left(1 - \frac{h}{h_{abs}}\right) R_{C0}$$

Time to Climb

Table 3.7 Typical Values for the Absolute Ceiling, h_{abs}

Airplane Type	h_{abs} (ft)x10^{-3}
Airplanes with piston-propeller combinations:	
normally aspirated	12-18
supercharged	15-25
Airplanes with turbojet or turbofan engines:	
Commercial	40-50
Military	40-55
Fighters	55-75
Military Trainers	35-45
Airplanes with turbopropeller or propfan engines:	
Commercial	30-45
Military	30-50
Supersonic Cruise Airplanes (jets)	55-80

Tempo de Subida

- If a time to climb is specified and the absolute ceiling is estimated, this problem becomes to determine the rate of climb at sea level.

$$R_{C0} = \frac{h_{abs}}{t_{cl}} \ln\left(1 - \frac{h}{h_{abs}}\right)^{-1}$$

Ceiling

- The ceiling specification is generally a specification of rate of climb at altitude. Therefore, we should procedure normally as a rate of climb especification.

Ceiling

Ceiling Type	Minimum Required Climb Rate
Absolute ceiling	0 fpm
Service ceiling	
Commercial/Piston-propeller	100 fpm
Commercial/jet	500 fpm
Military at maximum power	100 fpm
Combat ceiling	
Military/Subsonic/maximum power	500 fpm at $M<1$
Military/Supersonic/maximum power	1,000 fpm at $M>1$
Cruise ceiling	
Military/Subsonic/max.cont. power	300 fpm at $M<1$
Military/Supersonic/max.cont. power	1,000 fpm at $M>1$

Power Excess

- The definition of power excess is:

$$P_S = \frac{dh_e}{dt} = \frac{T - D \;\; V}{W} = \left[\left(\frac{T}{W}\right) - \left(\frac{D}{W}\right)\right]V$$

where (specific energy):

$$h_e = \frac{V^2}{2g} + h$$

Maneuverability

- The specification of maneuverability, in general, is done with the concept of sustained turn.
- Which is the maximum load factor that the airplane can fly, at an certain altitude (sometimes with airspeed constraints as well).
- It is strongly dependent of maximum lift coefficient and installed power (or thrust).

Maneuverability

- From the leveled flight, we can write:

$$nW = qSC_L \Rightarrow n = qC_L \left(\frac{W}{S}\right)^{-1}$$

- This load factor can only be sustained if there is thrust available.

$$T = qSC_{D0} + qS\left(\frac{C_L^2}{\pi A e}\right)$$

$$\frac{T}{W} = qC_{D0}\left(\frac{W}{S}\right)^{-1} + \left(\frac{W}{S}\right)n^2\left(\frac{1}{\pi A e q}\right)$$

Maneuverability

- In terms of power:

$$P = VqSC_{D0} + VqS\left(\frac{C_L^2}{\pi A e}\right)$$

$$\frac{W}{P} = \left[\frac{1}{2}\rho V^3 C_{D0}\left(\frac{W}{S}\right)^{-1} + \left(\frac{W}{S}\right)n^2\left(\frac{1}{\pi A e \frac{1}{2}\rho V}\right)\right]^{-1}$$

Maneuverability

- It is possible to derive a relation between the load factor and the turn rate.

$$\dot{\psi} = \frac{g}{V}\; {n^2 - 1}^{\frac{1}{2}}$$

$$n = \left[\left(\frac{V\dot{\psi}}{g}\right)^2 + 1\right]^{\frac{1}{2}}$$

Speed

- It must have power or thrust equilibrium:

$$P = TV = VqSC_D$$

$$550SHP\eta_p = \frac{1}{2}\rho V^3 SC_D$$

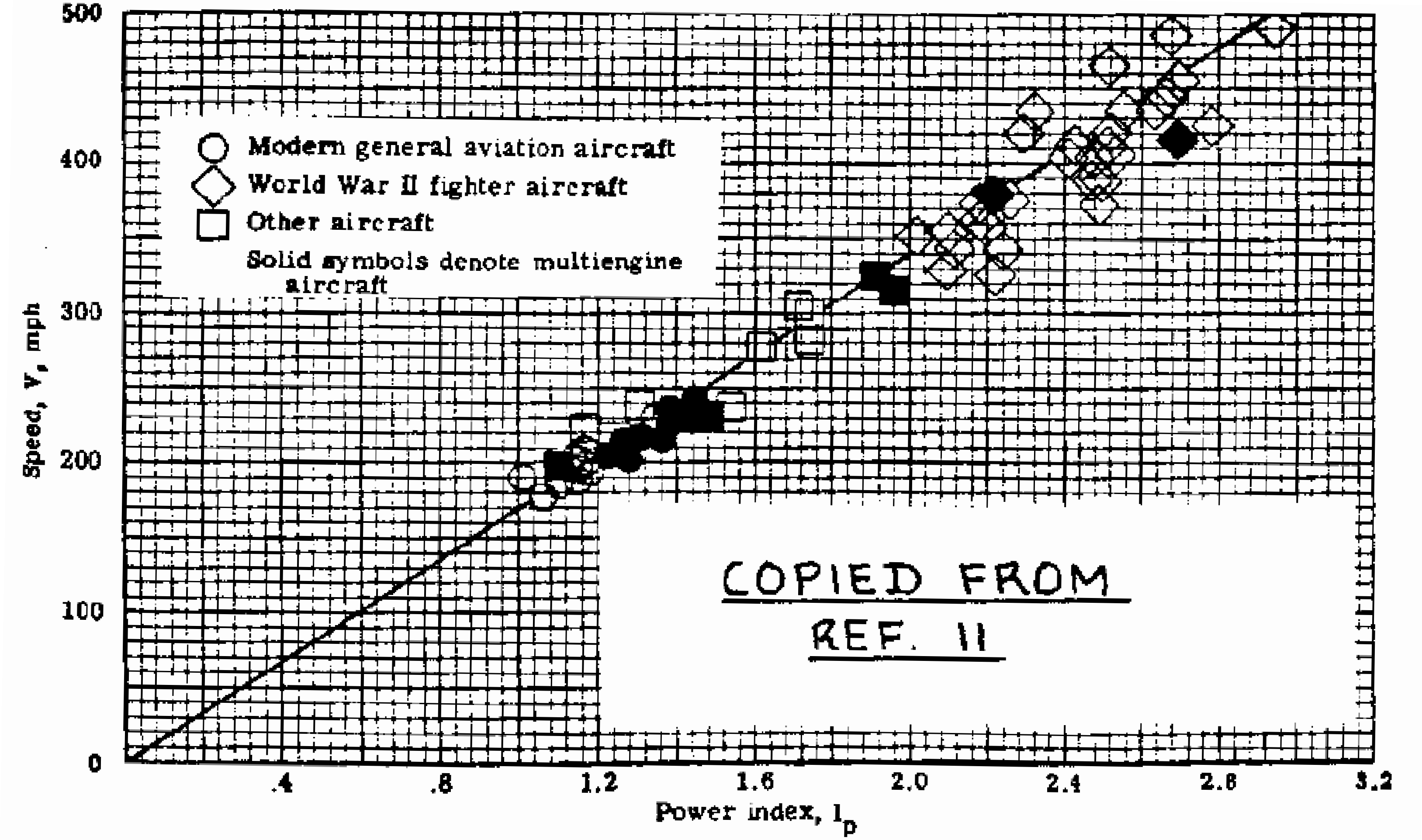
Speed

- Assuming small induced drag for maximum or cruise speed.

$$V \propto \left[\left(\frac{W}{S}\right)\left(\frac{P}{W}\right)\left(\frac{\sigma C_{D0}}{\eta_p}\right)\right]^{\frac{1}{3}} \Rightarrow V \propto I_P$$

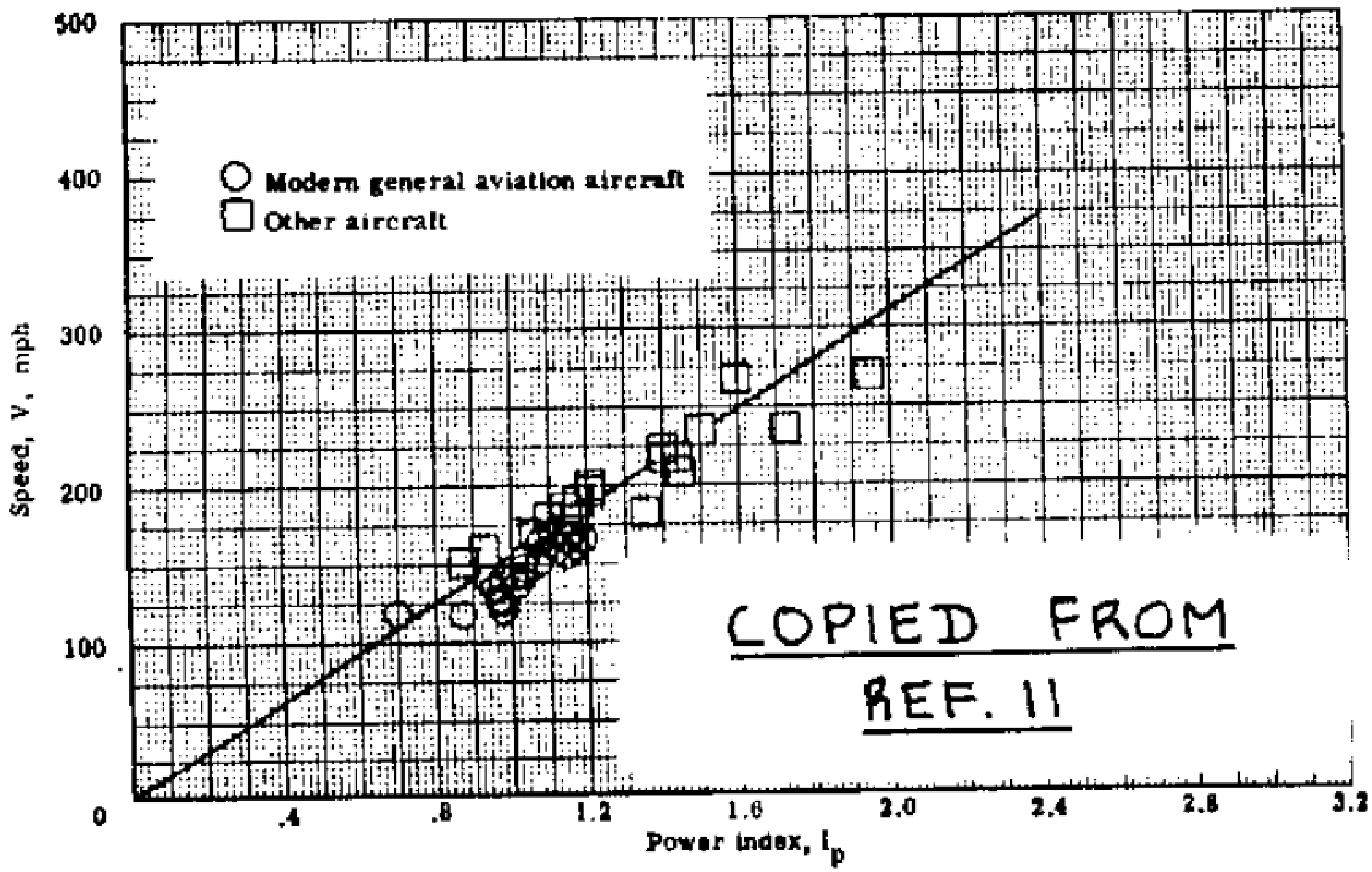
$$I_P = \left[\left(\frac{W}{S}\right)\left(\frac{P}{W}\right)\frac{1}{\sigma}\right]^{\frac{1}{3}}$$

Speed



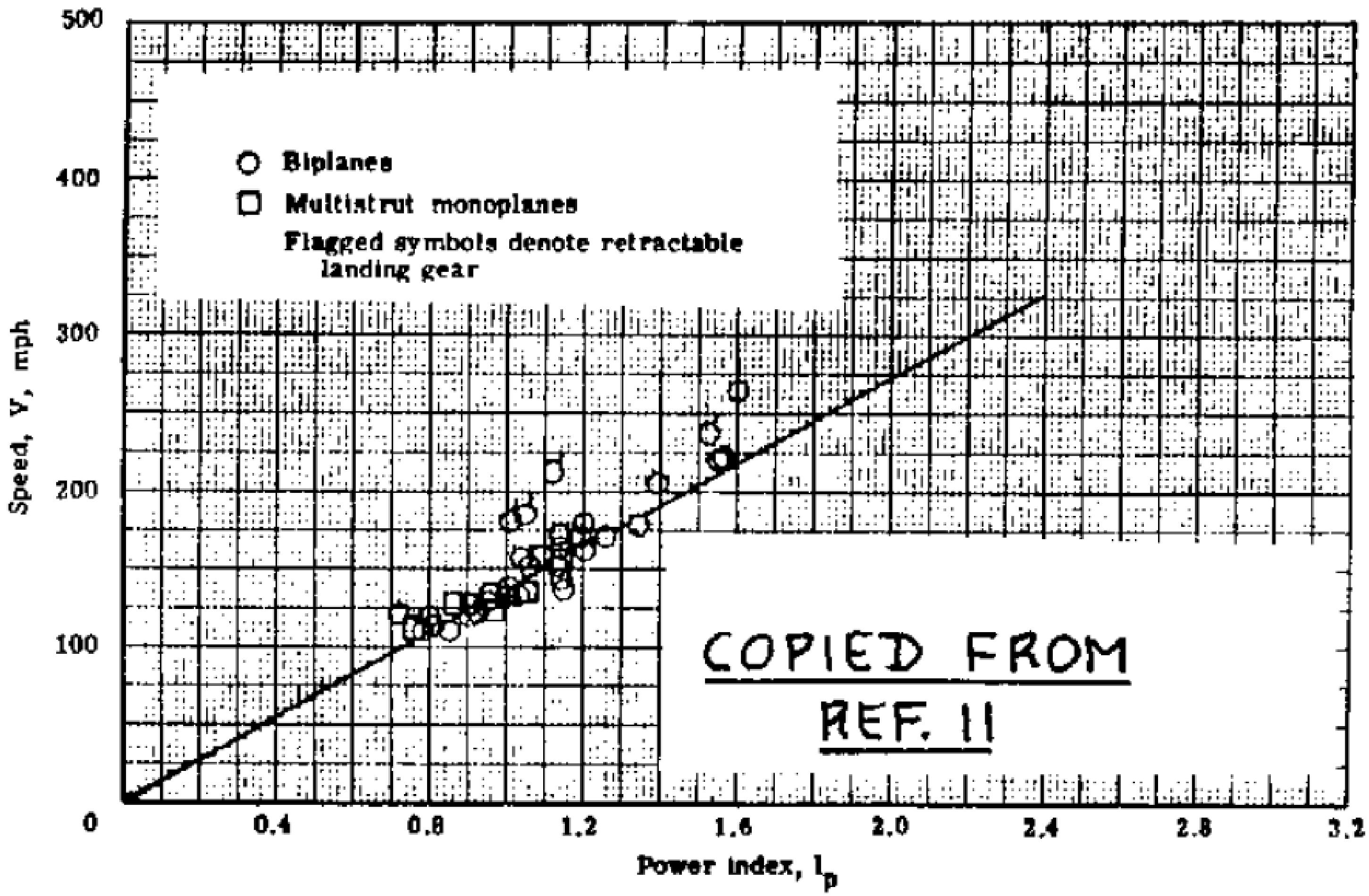
500
400
300
200
100
0
Speed, V, mph
Modern general aviation aircraft
World War II fighter aircraft
Other aircraft
Solid symbols denote multiengine aircraft
COPIED FROM REF. 11
.4
.8
1.2
1.6
2.0
2.4
2.8
3.2
Power index, I_p

Speed



Modern general aviation aircraft
Other aircraft
Speed, V, mph
0
100
200
300
400
500
.4
.8
1.2
1.6
2.0
2.4
2.8
3.2
Power index, I_p
COPIED FROM
REF. 11

Speed



500
400
300
200
100
0
Speed, V, mph
Biplanes
Multistrut monoplanes
Flagged symbols denote retractable landing gear
COPIED FROM REF. 11
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
Power index, I_p

Speed

- For jet planes:

$$\frac{T}{W} = C_{D0} q \left(\frac{W}{S} \right)^{-1} + \frac{W}{S} \frac{1}{q \pi A e}$$