Wing Design

Subsonic

Paulo Iscold

Goals:

- Review the basic concepts that lead to:

    - Lifting Line Theory
    - Understanding of Drag

        - Induced Drag
        - Parasite Drag

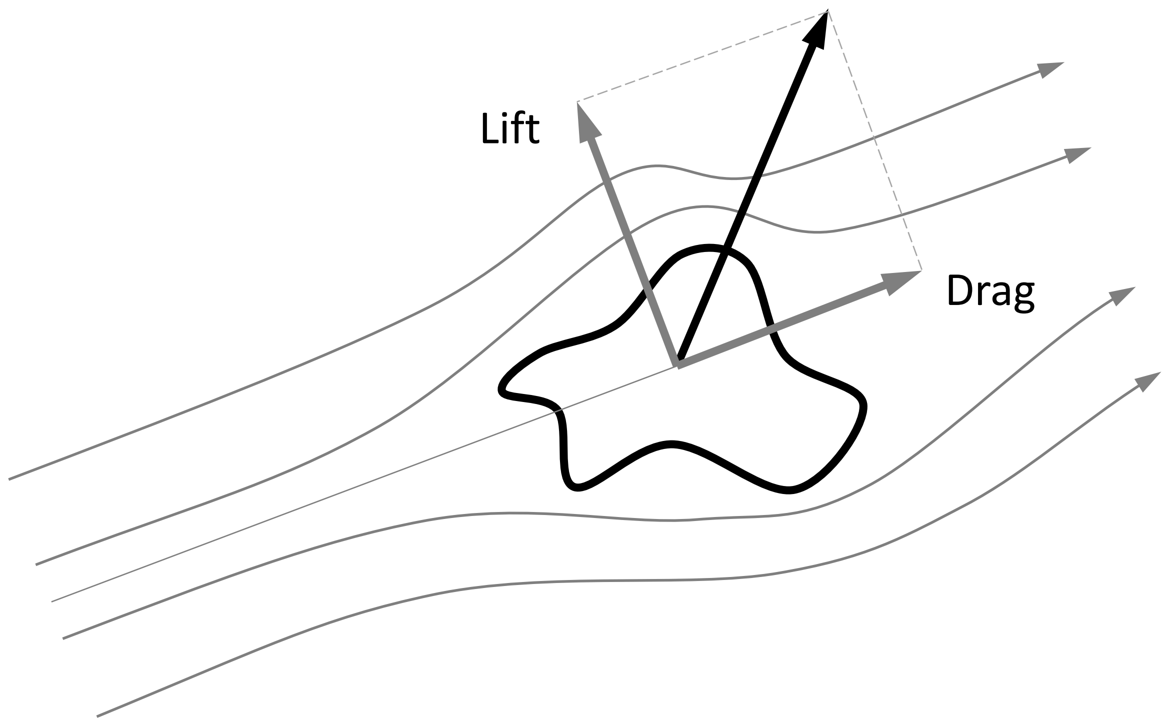
- Introduce the wing design problem and strategies.

- Introduce free software that can be used to assist the wing design process

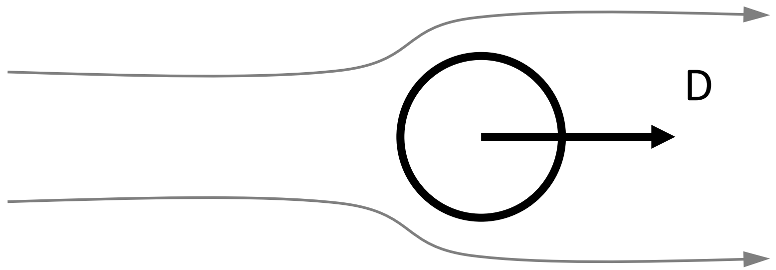
Lift

A brief review

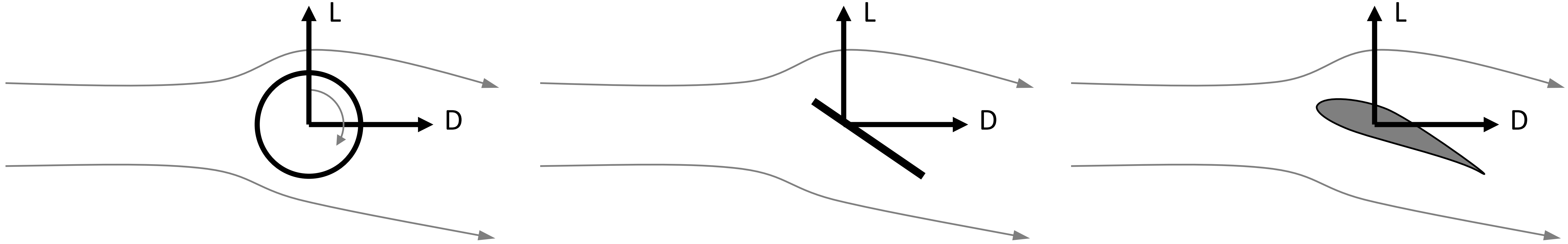
Any body surrounded by a moving fluid will experience a reaction force. According to Newton's third law, this force has equal magnitude to the force exerted, by the body, on the fluid.



By definition, the component of this force that is perpendicular to the flow direction upstream from the body is called LIFT and the component parallel to the flow direction is called DRAG.



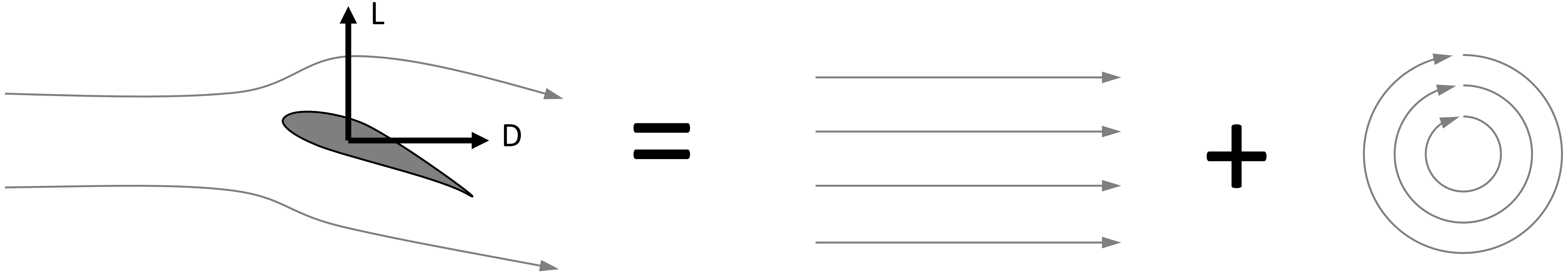
Not all bodies will experience a reaction force with a component in the lift direction. In order to have the lift component it is necessary that the body be able to **change the momentum direction of the flow**, producing a downward component on the flow, normally called **downwash**.



When performing a mathematical modeling of a flow field it is convenient to use a quantity called Circulation (line integral around a closed curve of the velocity field) to determine/model the change in the direction of the flow momentum.

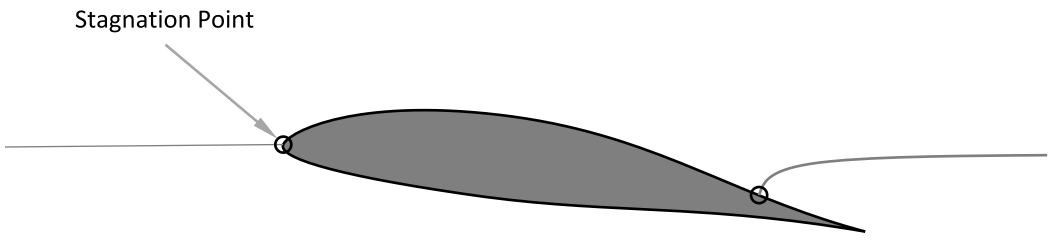
The **Kutta-Joukowski theorem** states that the lift of a body in two-dimensional flow is proportional to the circulation of the flow. The proportionality constant is the product of fluid specific mass and speed.

When modeling inviscid potential flows (Laplace equation) it is convenient to model circulation using vortex elements, since it has direct correlation to the flow circulation.

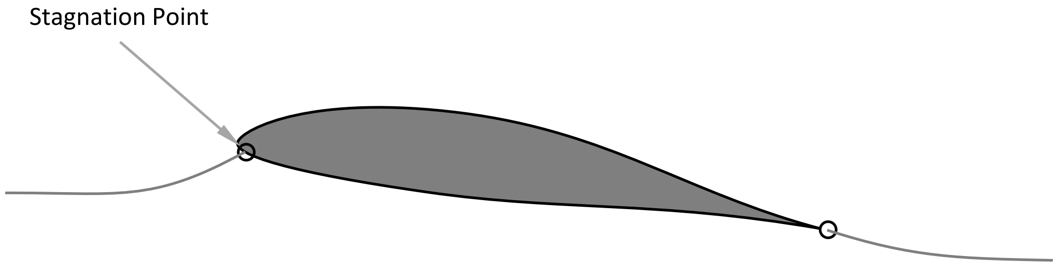


When solving an inviscid potential flow around airfoils (with sharp trailing edge), any value of circulation (including zero) will be a valid solution for the Euler equation.

Therefore, it is necessary to add an additional condition to solve the problem. This condition is called **Kutta Condition**. It states that on an inviscid potential steady flow, the circulation created by a body with a sharp trailing edge is such that the <u>rear stagnation point will be at the trailing edge</u>.



Valid Solution for Euler Equation
Without Kutta Condition



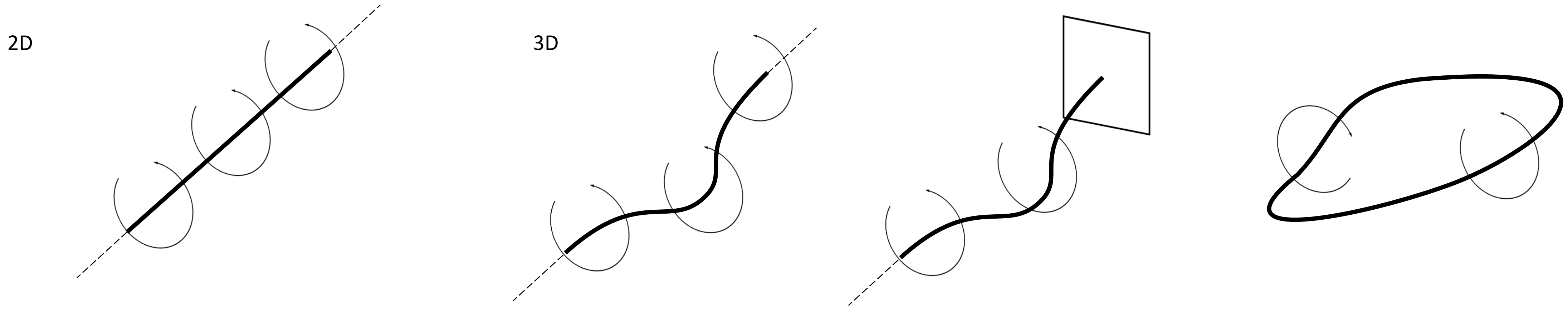
Valid Solution for Euler Equation
With Kutta Condition

The concept of circulation and vortex modeling in two dimensional flow can be extended to three dimensional flow, where the vortex flow can be modeled around any arbitrary shape – vortex filament.

However, it is important to know and understand some principles about vortex in three dimensional flow.

First and Second **Helmholtz Vortex Theorems** state that:

1. *The strength of the vortex is constant along its length*
2. *The vortex can't end inside the fluid. It must either:*
    a) *Extend to infinite in both directions, or*
    b) *End on a solid boundary, or*
    c) *Form a close loop*

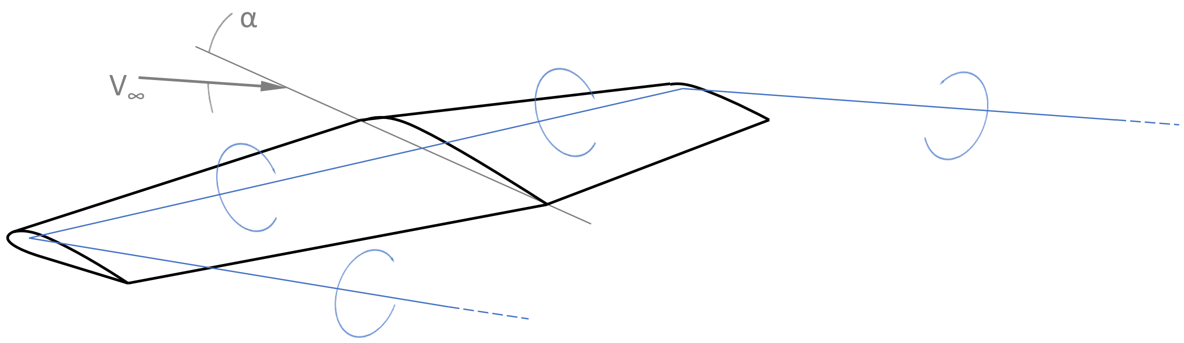


Finite Wings

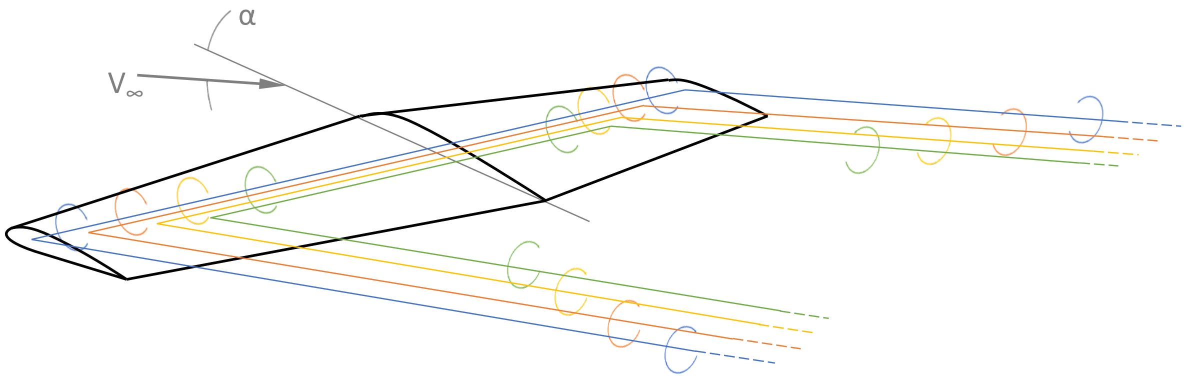
Lifting Line Theory

A finite wing is a body that generates lift; therefore, it can be modeled using vortices filaments. The vortex filament is naturally placed span wise bonded to the wing (bounded vortex). However, to obey the Second Helmholtz Vortex Theorem, the filament can't end on the tip of the wing. Instead, a good strategy (that matches flow observations) is to extend these filaments stream wise up to the infinite* (trailing vortices). This setup is often called horse-shoe vortex.

Since the intensity of the vortex is constant over the filament (First Helmholtz Vortex Theorem) a wing modeled with a single horse-shoe vortex will present constant circulation (lift) distribution. **Ludwig Prandtl** proposed a model consisting of multiple horse-shoe vortices which can be solved, using some approximations, as a system of linear equations.



Single Filament
Constant circulation/lift



Multiple Filament
Variable circulation/lift

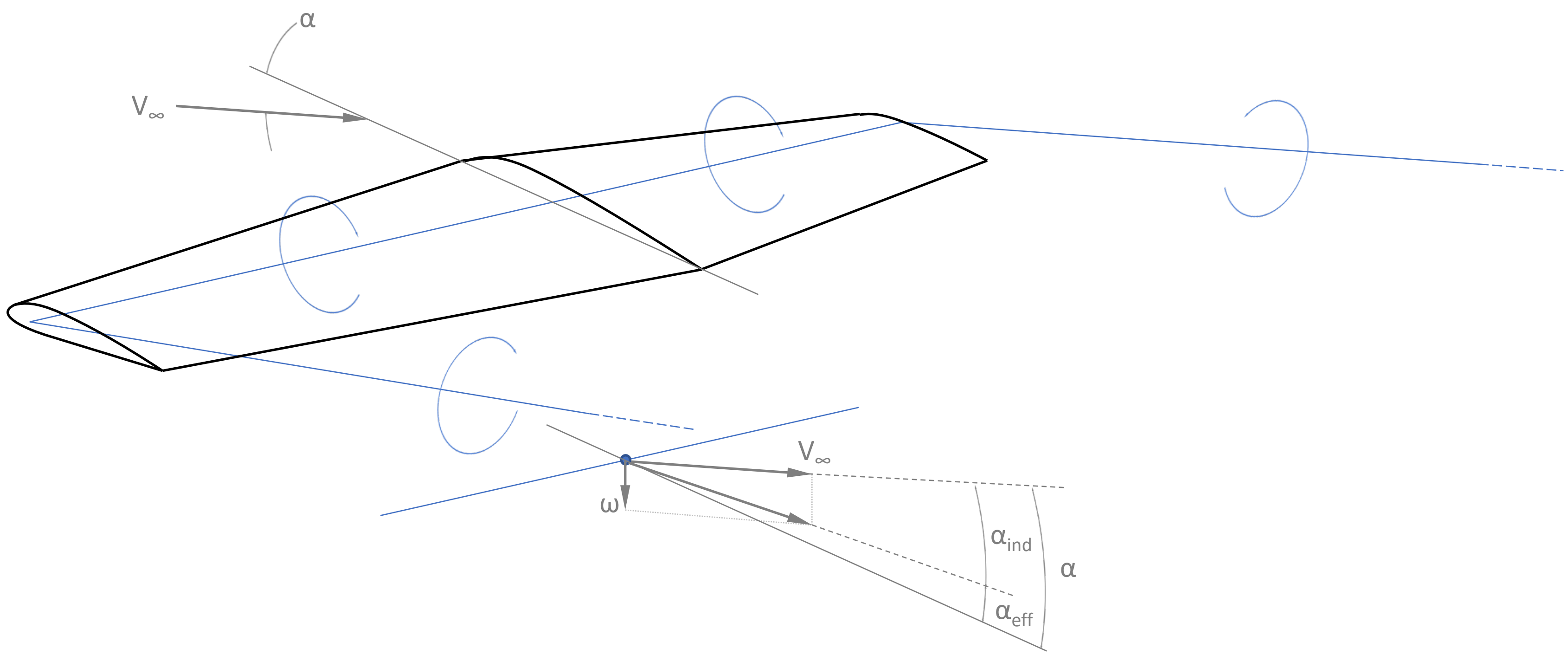
* - In fact, to fulfill the third Helmholtz Vortex Theorem or the Kevin Theorem the trailing vortex are connected to each other on the origin of the wing movement, but the influence of this segment can be neglected for the purpose of wing analysis.

The horse-shoe vortex filament will induce vertical velocity (ω) on flow at the wing plane (downward inside the horse-shoe and upward outside the horse-shoe). Therefore, the local flow on any position can be calculated as a composition of the incoming flow and this induced velocity.

The local attack angle on any position will also be affected and two new quantities can be defined:

- Induced attack angle: a function of the circulation distribution of the horse-shoe vortices arrangement.
- Effective attack angle: the attack angle that a portion of the wing is experiencing.

For multiple filament model these quantities are not constant on the span wise direction.



On the **Lifting Line Theory**, Prandtl uses:

1) The **Bio-Savart Law** to calculate the induced (by all the horse-shoe vortex filaments) attack angle along the span

2) Two dimensional airfoil data (zero lift angle and lift line slope = 2π) to correlate the effective attack angle and the circulation along the span

The geometric attack angle (incoming flow) is the sum of the induced and effective attack angle and can be used to create a equation that needs to be satisfied by a correct distribution of circulation. This equation is the fundamental equation of **Prandtl's Lifting Line Theory**.

Three important characteristics of a finite wing can be directly calculated by the Lifting Line Theory

1) Lift distribution over the span

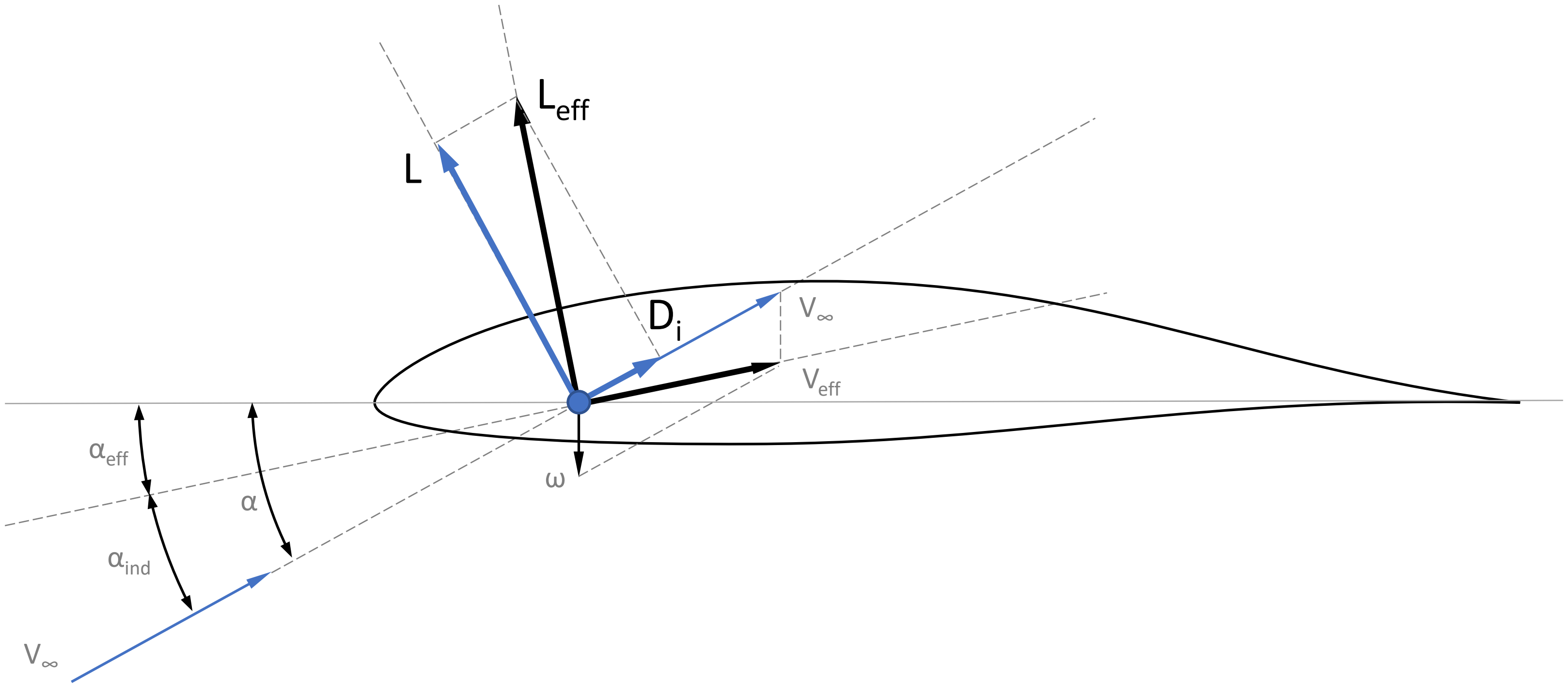
2) Lift slope of the wing

3) Induced drag of the wing

Assuming one specific section along the span, it is possible to notice that the **local lift force** (effective lift force) is perpendicular to the flow on that position (**effective attack angle**).

However, the lift of the wing is measured perpendicular to the incoming (upstream) flow (**geometric attack angle**). Therefore, the effective lift force will have a component perpendicular to the geometric attack angle (lift) and one component parallel to the geometric attack angle, called **Induced Drag**.

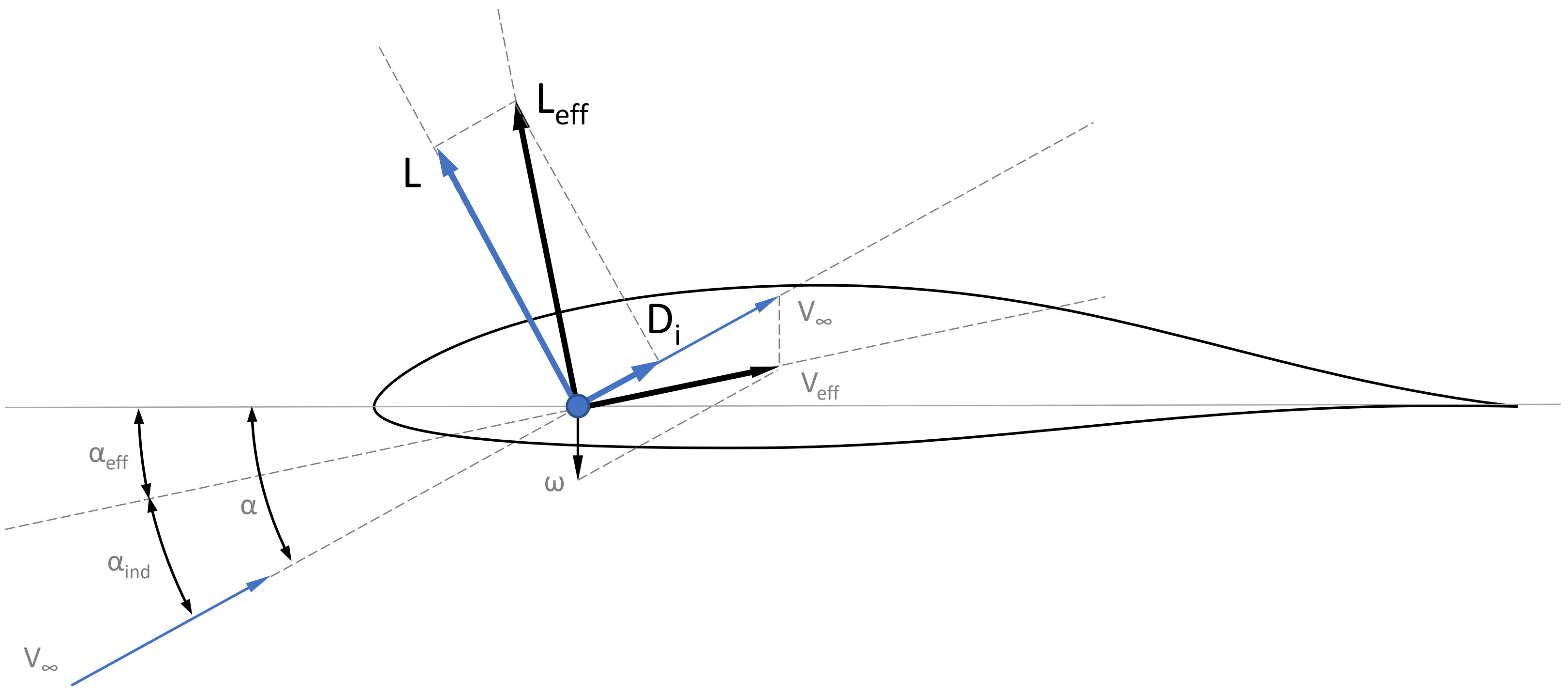
So, on the three dimensional case, the inviscid potential flow doesn't follow the **d'Alambert paradox**.



Also, notice:

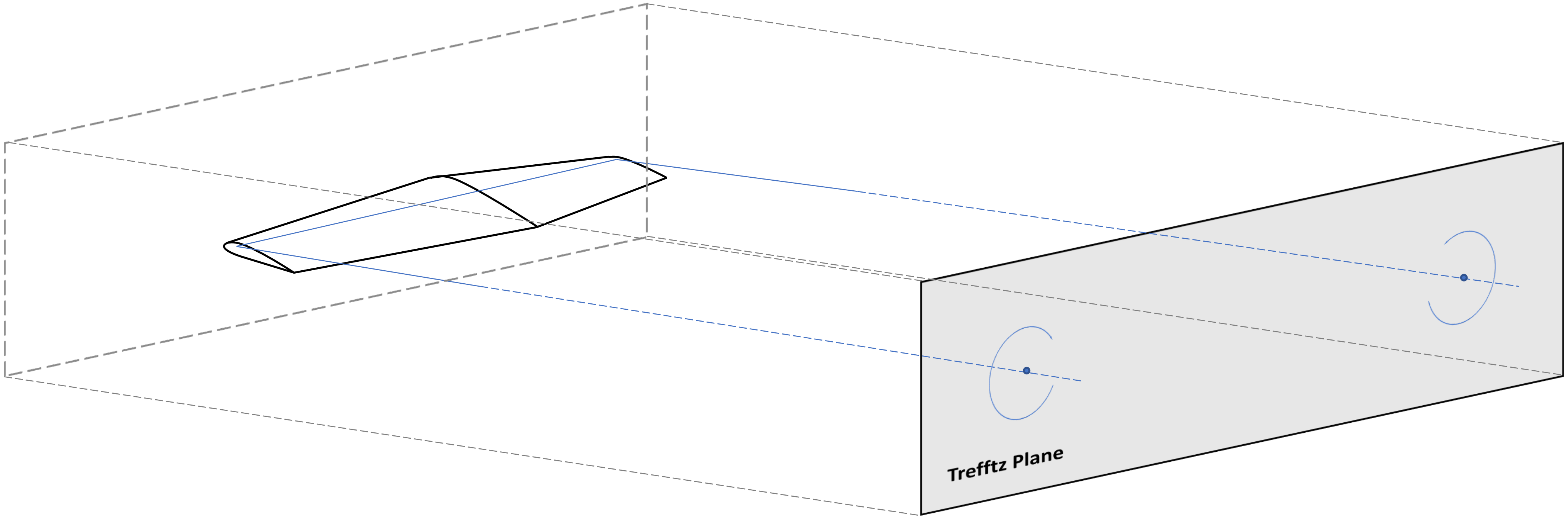
1) The effective attack angle is smaller than the geometric attack angle,
2) The lift component (L) is smaller than the effective lift.

For a specific geometric attack angle, the lift produced by a finite wing is always smaller than the lift produced by a two dimensional airfoil. Therefore, the **lift slope** of a finite wing is **smaller** than the lift slope of a **two dimensional airfoil**.



It is convenient to perform the analysis of forces acting on the body using the integral form of the **Momentum Equation** on a large control volume. On the limit as the box goes to infinity, it is possible to prove that the forces will depend on the velocities induced by the wing and its wake on a far field plane down stream the wing – **Trefftz Plane.**

This approach is more convenient for analysis of non planar wings and free wakes conditions.



Max **Munk**, one of Prandtl students, published a study defining a series of theorems that are important for the minimization of induced drag (**Munk's Stagger Theorems**), they can be summarized as:

1) The necessary and sufficient condition for the minimum induced drag of a planar lifting element is that the downwash velocity produced by the stream wise vortices is constant along the span of the lifting element.

2) The induced drag of a three dimensional system of parallel lifting elements (biplanes, wing and tail, swept wings, etc) does not depend on the longitudinal position of these lifting elements.

3) For non-planar wing elements the condition for minimum induced drag is that the velocity induced normal to the element, produced by the stream wise vortices, is proportional, at all lifting elements, to the cosine of the angle of lateral inclination (dihedral) of the lifting element.

Limitations

- Doesn’t take into consideration span wise flow effects, therefore, it is not suitable for swept wings, extreme side slip angle or very low aspect ratio.
- Doesn’t handle compressive flow.
- Doesn’t handle unsteady flow.
- Doesn’t take into consideration viscous effects.

Extensions

- Non planar configurations – **Weissinger Method**
- Unsteady flow using **Wagner functions**
- Viscous drag can be computed using effective attack angle and two dimensional airfoil data

Drag

Another very brief review

To understand the physics involved in the drag force, it is convenient to divide the contributions of this force into:

Total Drag

=

Components

- Parasitic Drag = Pressure Drag + Friction Drag
- \+ Induced Drag

\+

Vehicle

- Interference Drag
- \+ Trim Drag

\+

Supersonic

Wave Drag = Wave Drag due to lift + Wave Drag due to volume

Induced drag was explained in the previous session, so we will now focus on the **Parasitic Drag**

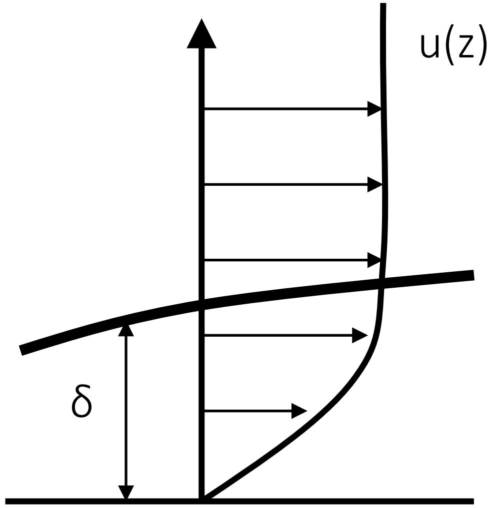
To understand parasitic drag it is important to understand the concept of boundary layer and its three possible conditions.

The concept of **Boundary Layer** was introduced by Prandtl and Blasius and refers to the layer of fluid in the immediate vicinity of a surface, where the effects of **viscosity** are important.

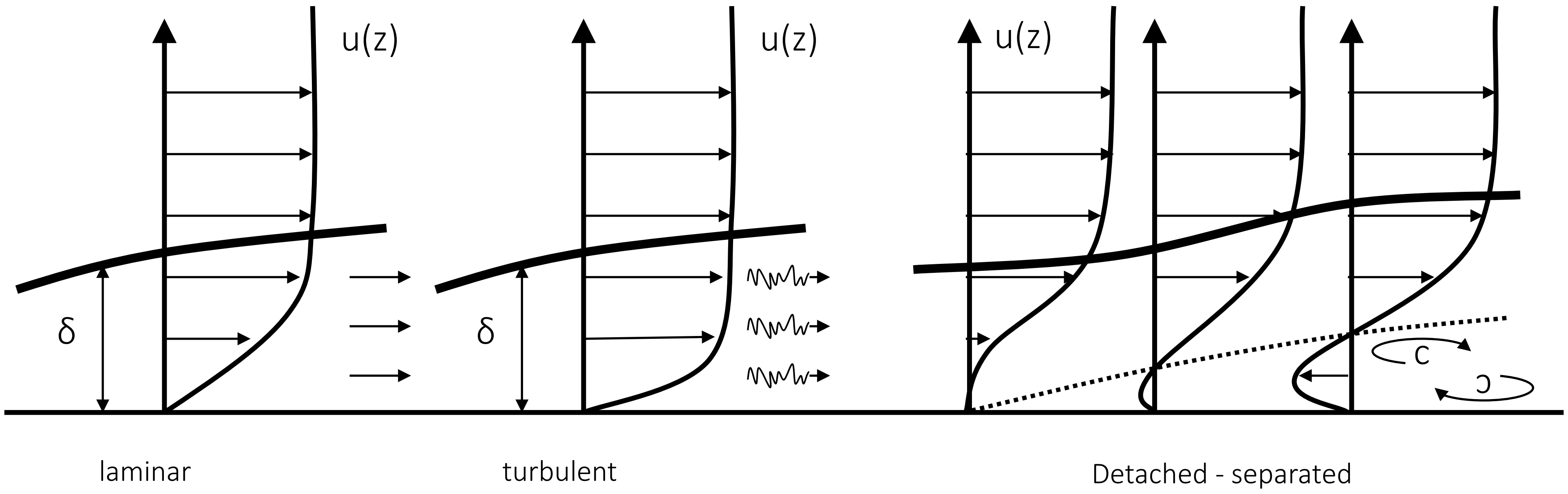
At the surface, the **non-slip condition** assumes that the fluid will have zero velocity.

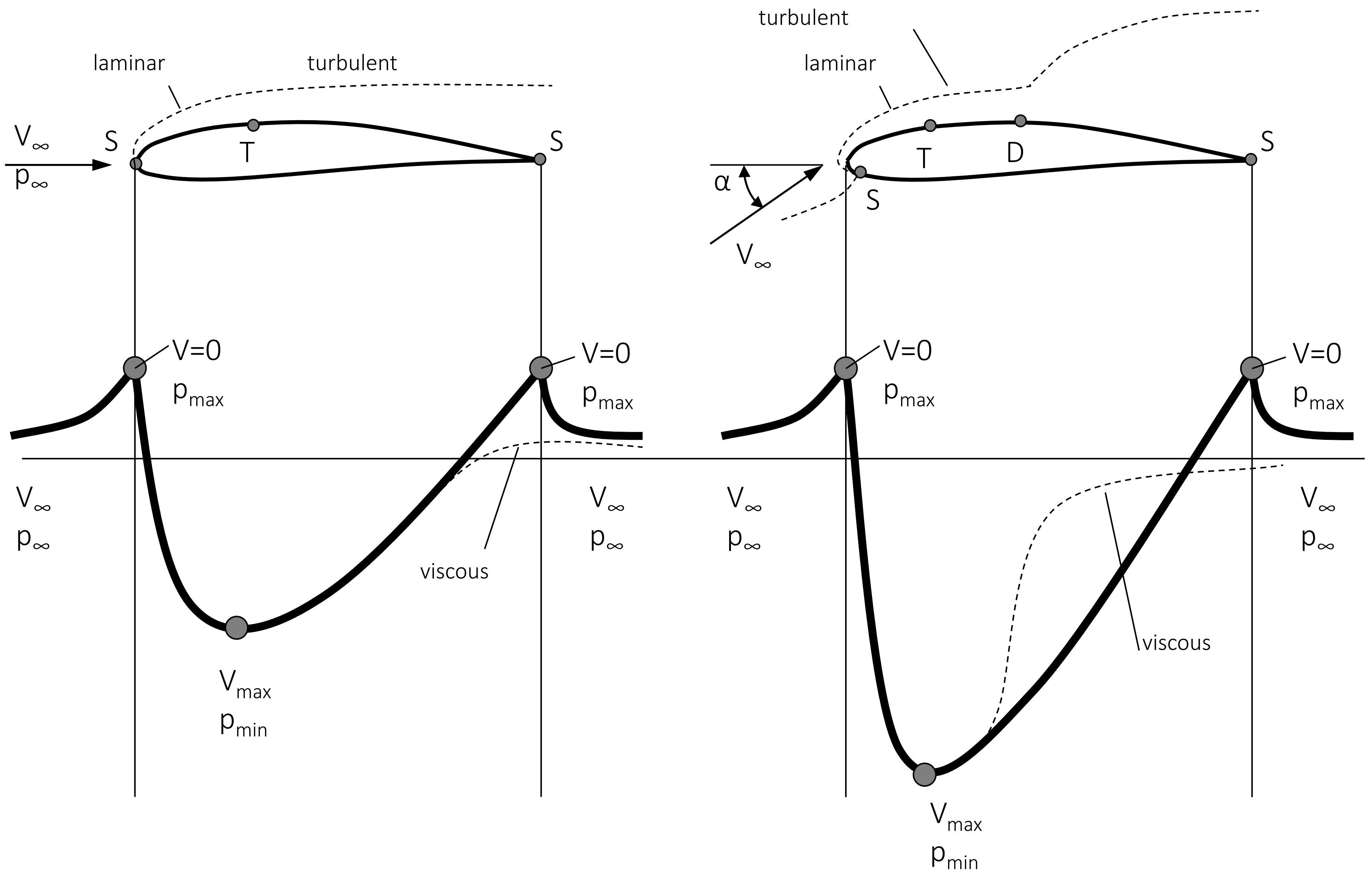
Away from the wall the fluid increases velocity until it reaches a velocity that its not affected by the viscosity.

To produce this variation of momentum it is necessary to impose a force on the fluid (**shear**) and the reaction of this force on the body is called **Friction Drag**



A boundary layer can assume three distinct conditions:





laminar
turbulent
V_∞
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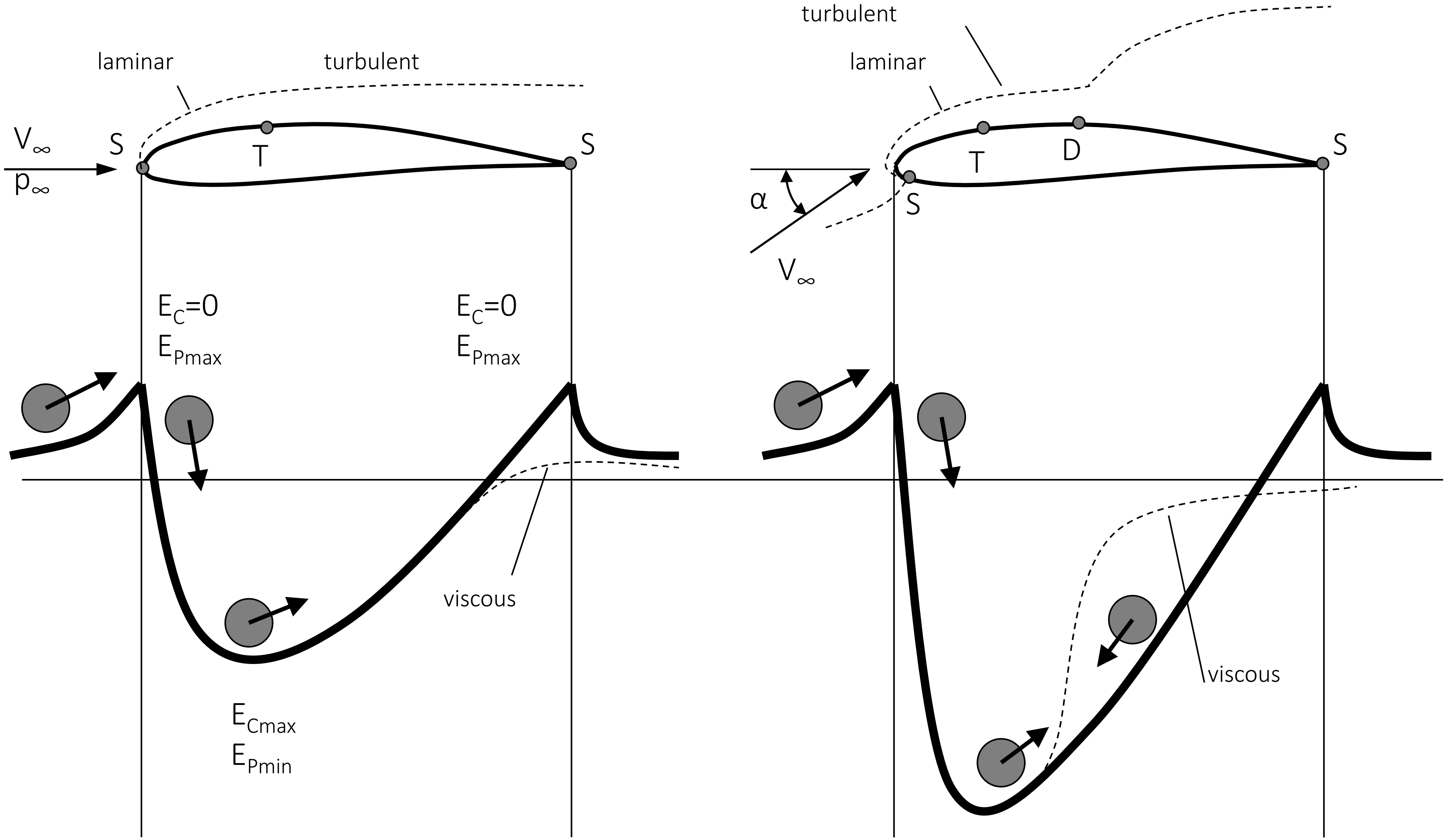
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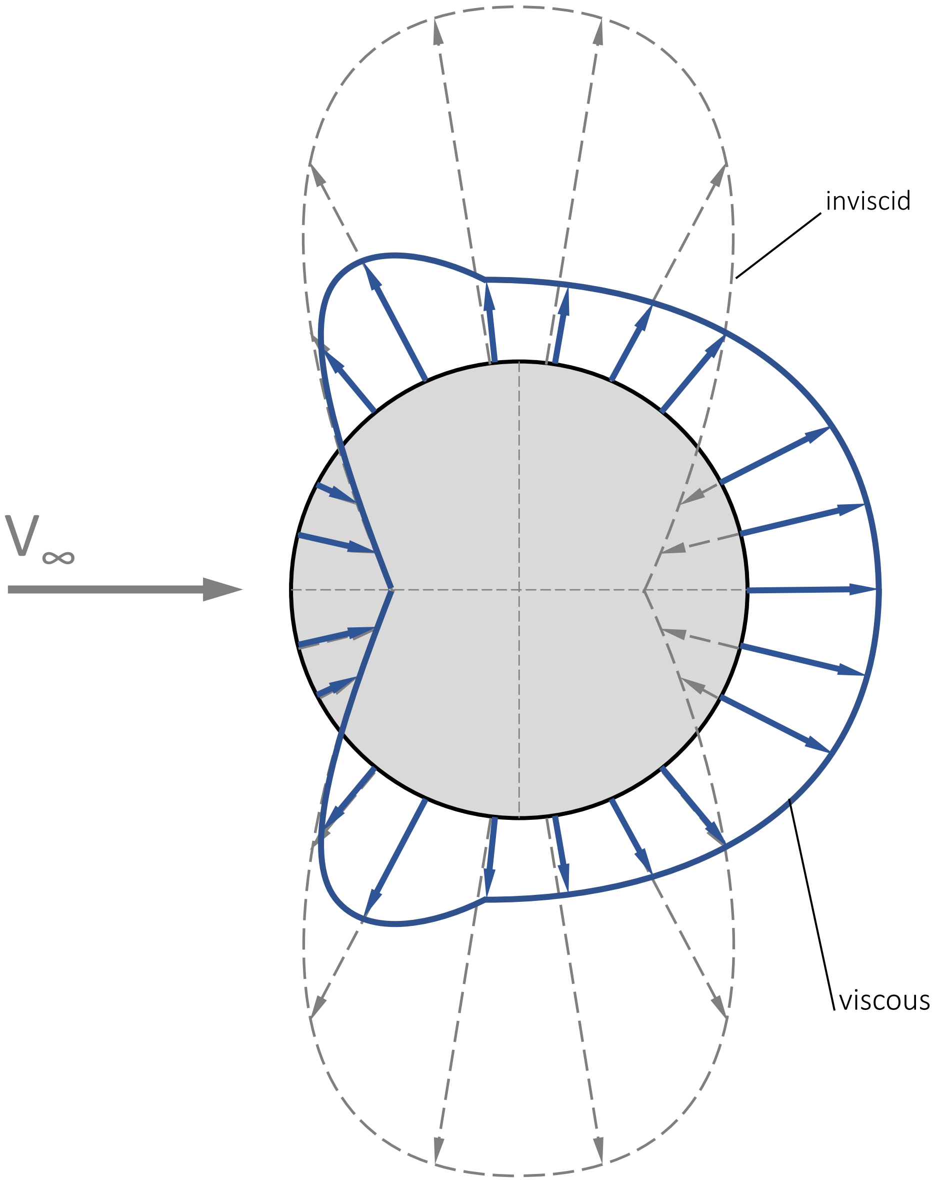
viscous

Due to the effects of the boundary layer (viscosity), the pressure distribution on a body (especially the after part – separation) deviates from what the inviscid potential solution predicts (**d'Alambert paradox**).

On the viscous flow, the integral of pressure over the surface will result on a force tangential to the inflow direction. This force is called **Pressure Drag**.

On a cylinder (or sphere) it is easy to notice the difference between the inviscid and viscous pressure distribution.

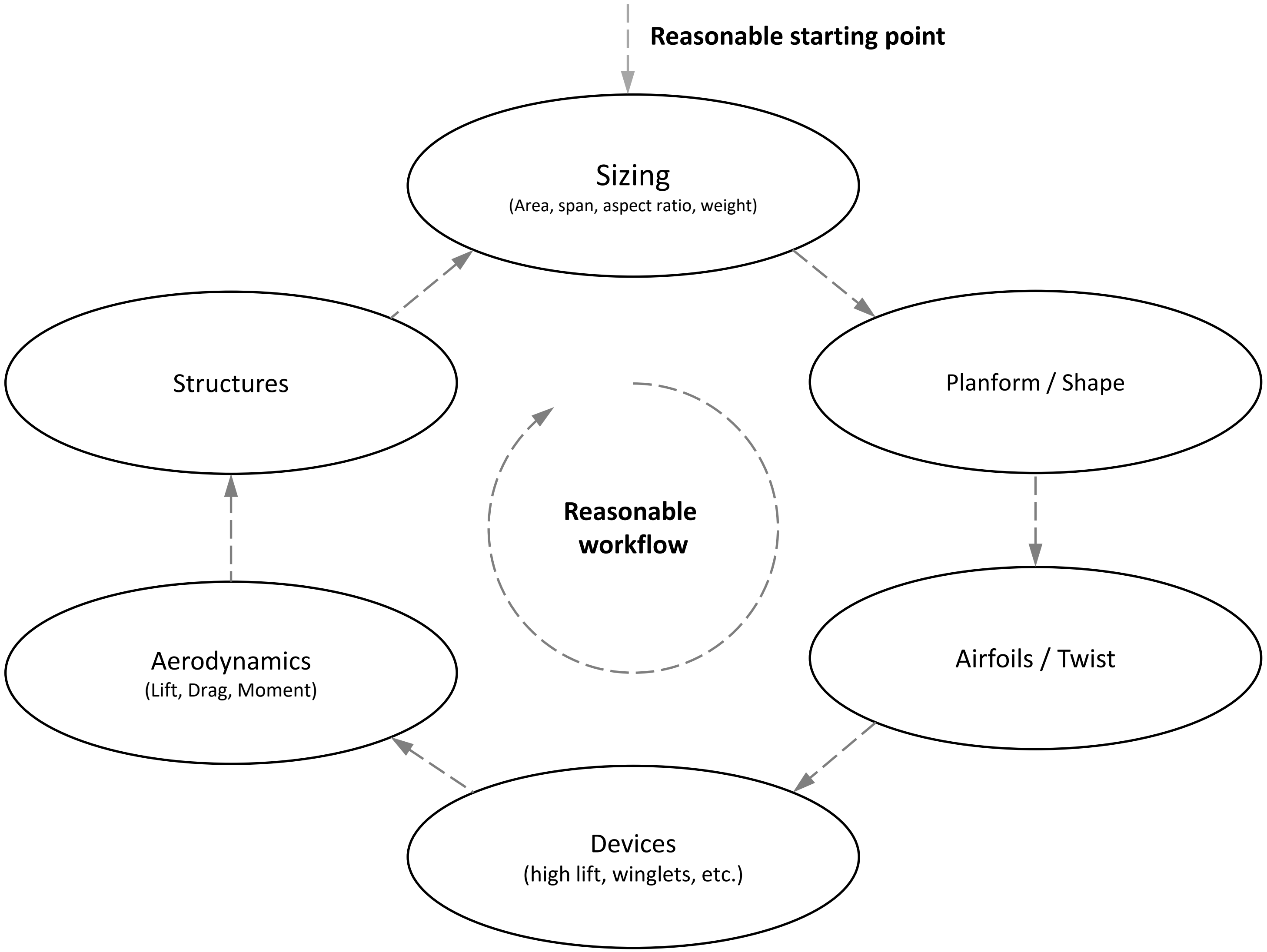
On airfoils without separated flow this difference is not very noticeable (therefore the pressure drag is not big).



Preliminary Wing Design

How to apply the theory

The preliminary wing design is an iterative process, without a clear beginning or end point.



Planform

Some preliminary aspects

Aspects that might influence on the **Planform** selection decision:

1. Induced drag – **Munk's theorem** can be used but there are preliminary results that lead to good results.

2. Fabrication techniques – some geometries are really difficult to fabricate.

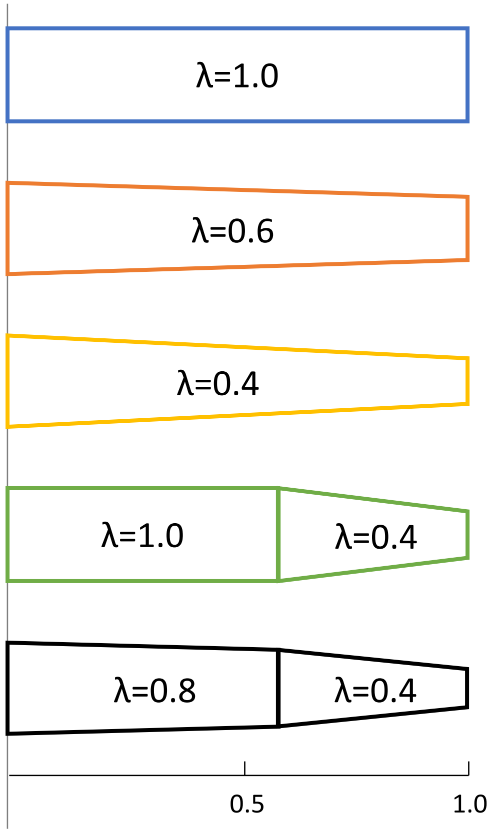
3. Handling – the development of stall over the span is strongly influenced by the planform and has direct impact on the airplane handling.

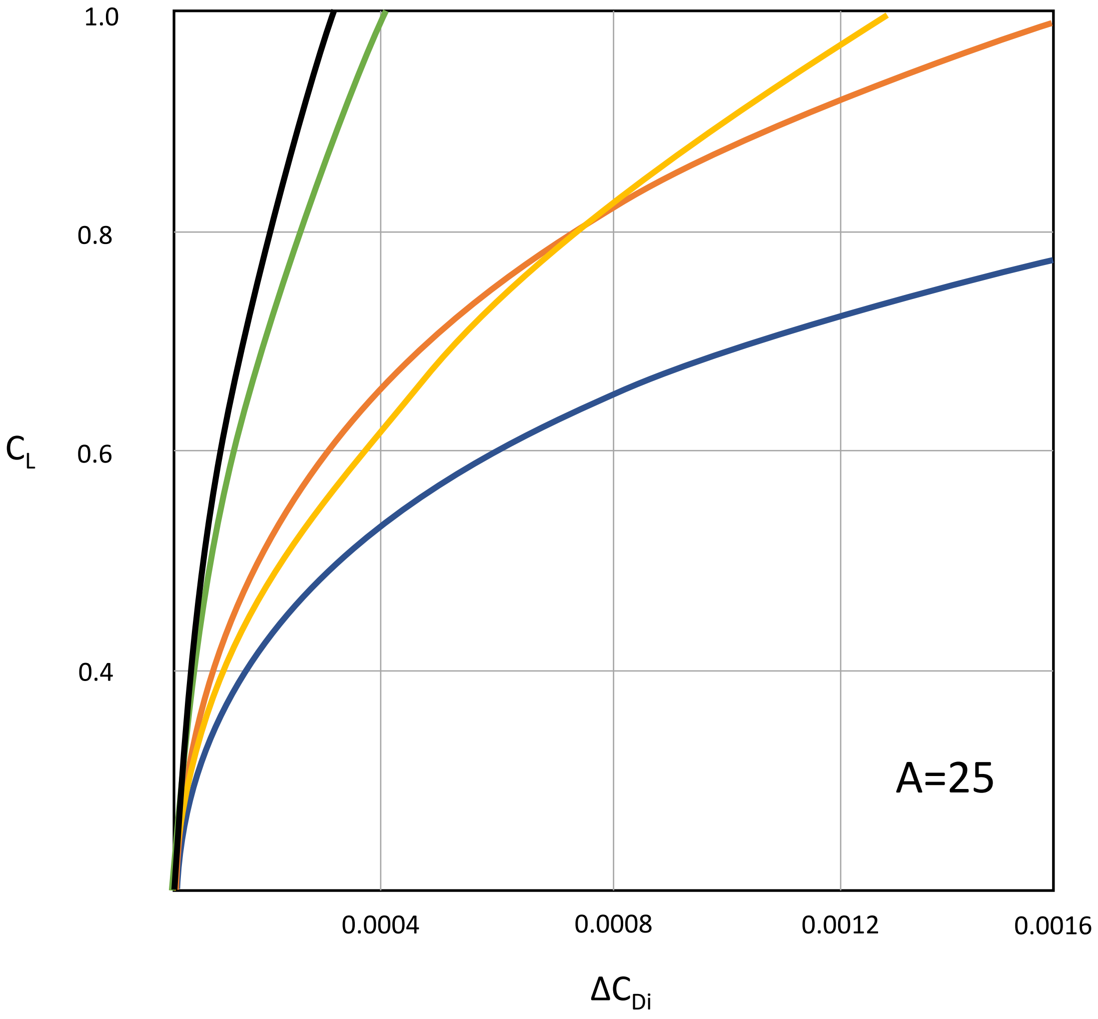
4. Structures – It is important to check how the wing structure will be accommodated inside the wing and how it will be fixed to the other structures.

5. Center of Gravity – adjusts on the planform (sweep angle) can be used to perform small adjustments of the airplane CG position.

6. Systems – the space inside the wing is important to carry some systems (landing gear, fuel tanks, engines, etc.) and the planform have impact on this aspect.

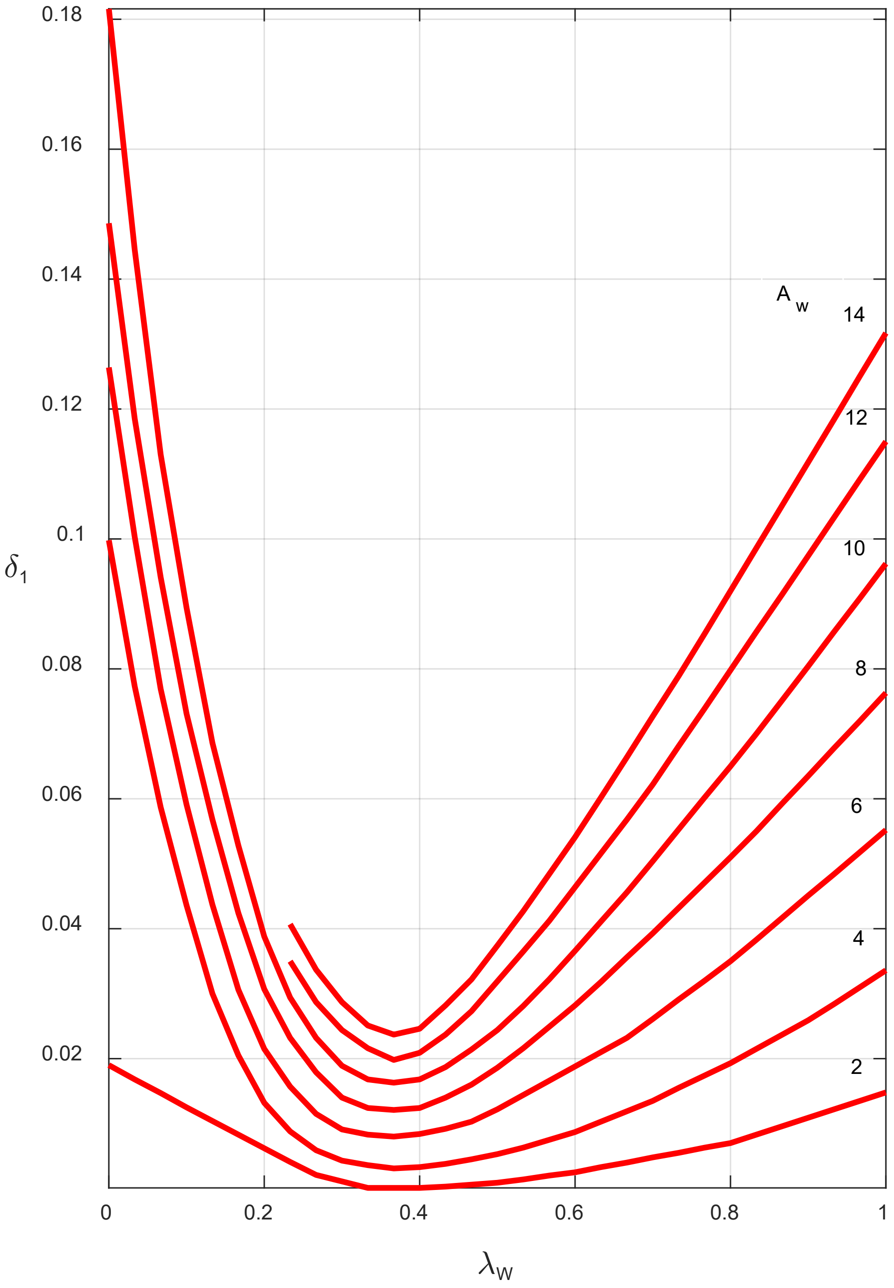
This figure is an example of results obtained in the literature regarding the efficiency of different planforms in relation to its induced drag

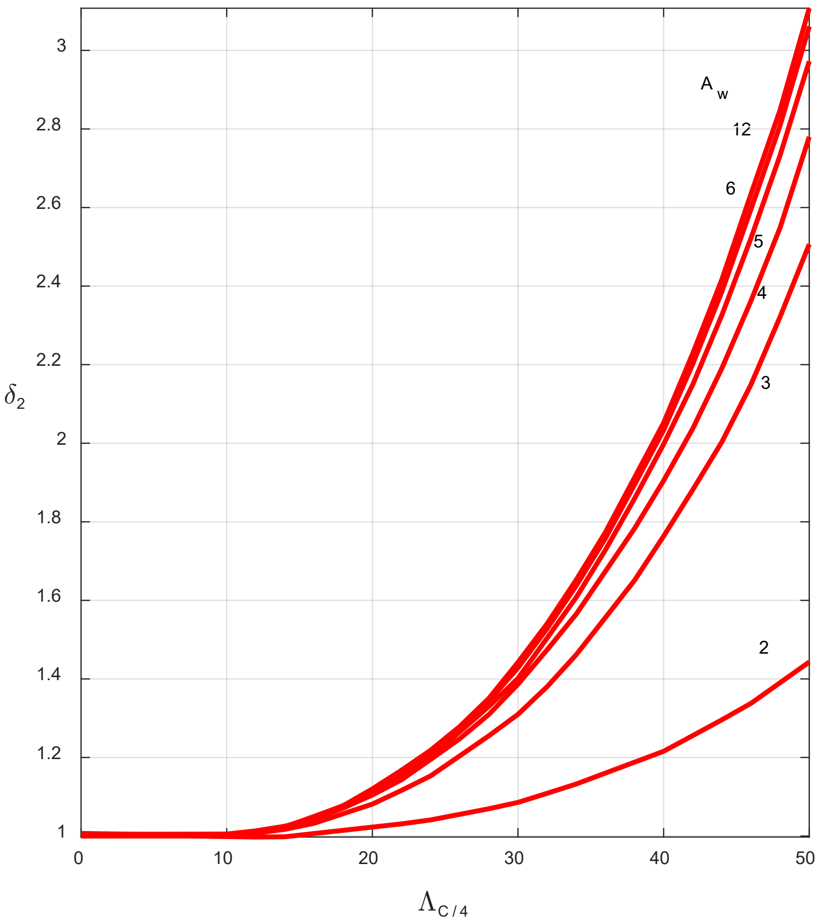




An easy way to estimate the induced drag of tapered wings with sweep angle is using this procedure:

$$(C_{Di})_w = \frac{(C_L)_w^2}{\pi A_w}(1 + \delta_1 \delta_2)$$





The optimization of the induced drag can be done using:

- Planform – chord distribution
- Twist – zero lift angle distribution
- Airfoil – lift slope distribution

However, a good strategy to design subsonic wings with minimum drag is:

1. Optimize the chord distribution assuming zero twist (constant zero lift angle) and constant airfoil (constant lift slope) over the span.
2. Design the airfoil variation over the span to ensure constant zero lift angle and constant lift slope – notice that depending on the chord distribution the Reynolds number of the airfoil will change dramatically over the span.
3. If the wing will be equipped with camber change device – flaps actuated during normal flight – it is also desirable to design these surface such that the variation of lift with surface deflection is constant over the span – the easiest way to do this is change the chord of the flap over the span.
4. Avoid the use of geometric twist since is can only be optimized for one specific flight condition.

On the issue of induced drag reduction, there is vast literature. It is common to have studies dealing with other constraints like:

- Span
- Length
- Bending moment

For planar wings without constraints I strongly recommend:

J. Ashenberg and D. Weihsradius. "Minimum Induced Drag of Wings with Curved Planform", Journal of Aircraft, Vol. 21, No. 1 (1984), pp. 89-91. - https://doi.org/10.2514/3.56733

C. P. Van Dam. "Induced-drag characteristics of crescent-moon-shaped wings", Journal of Aircraft, Vol. 24, No. 2 (1987), pp. 115-119. - https://doi.org/10.2514/3.45427

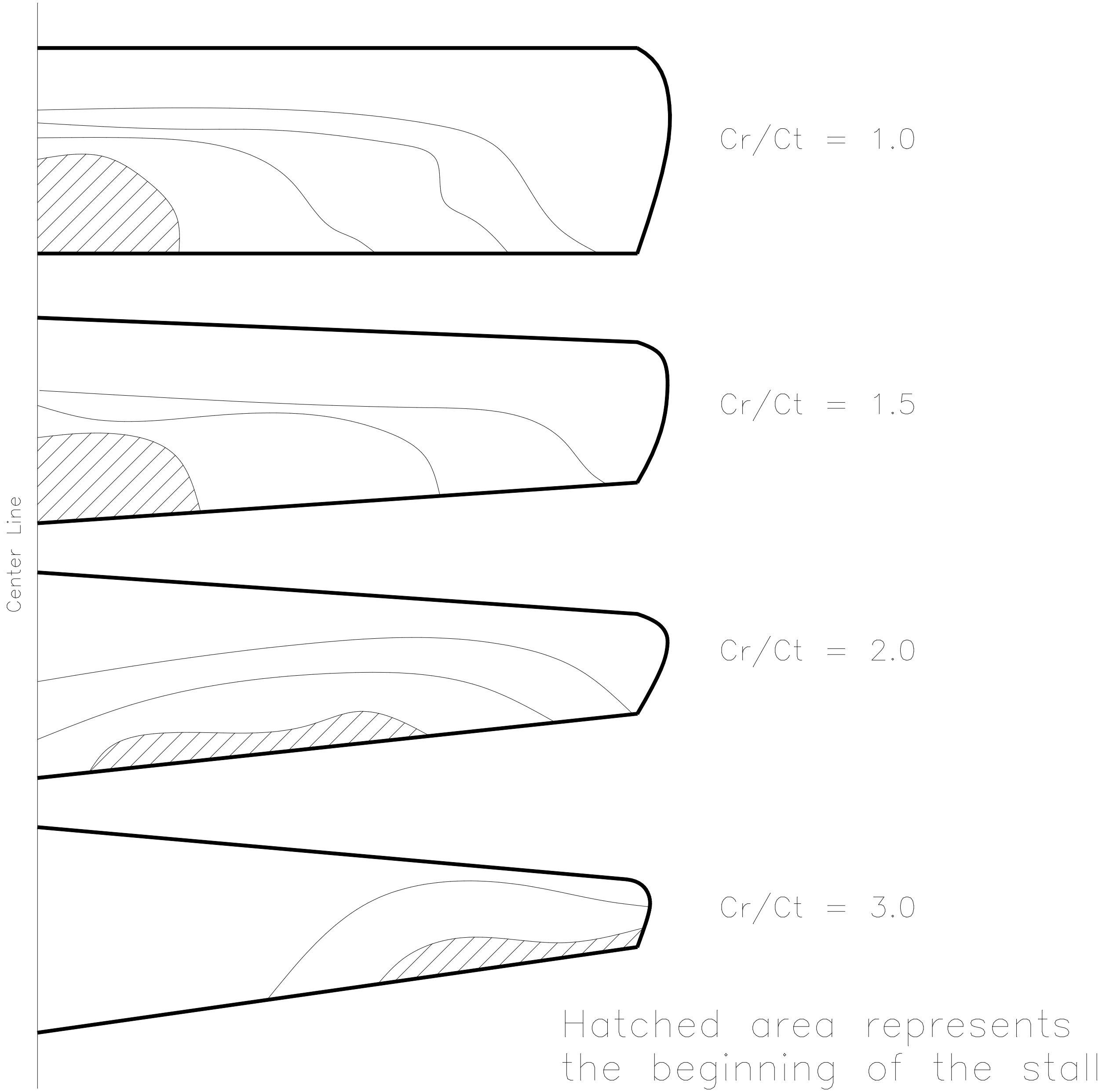
Free software that can be used for induced drag analyses of wings:

- XFLR 5 - http://www.xflr5.com/xflr5.htm
- AVL - http://web.mit.edu/drela/Public/web/avl/

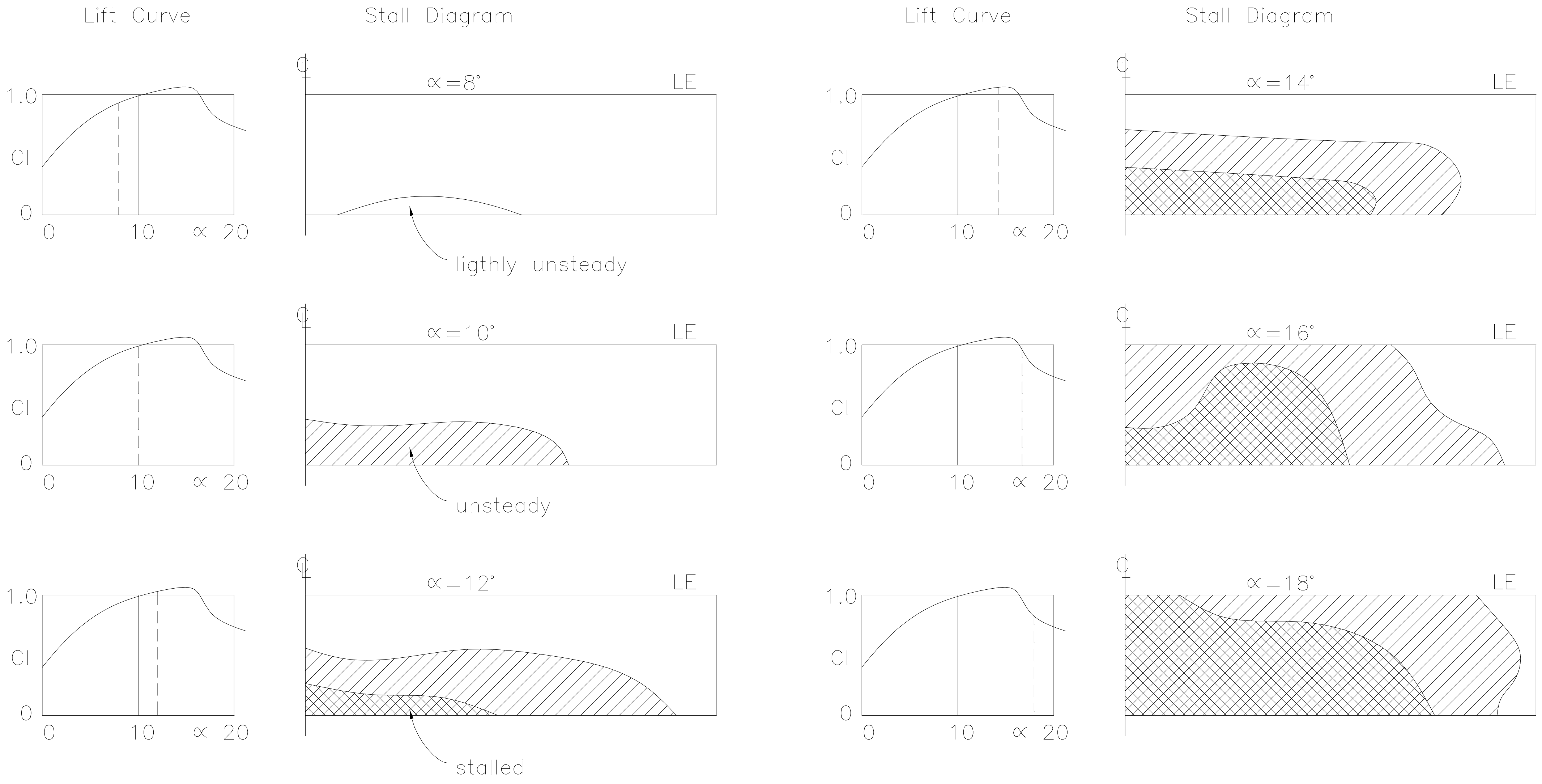
Effect of planform on stall behavior

This figure shows the evolution of the stall on trapezoidal wings with different tapper ratios without sweep angle.

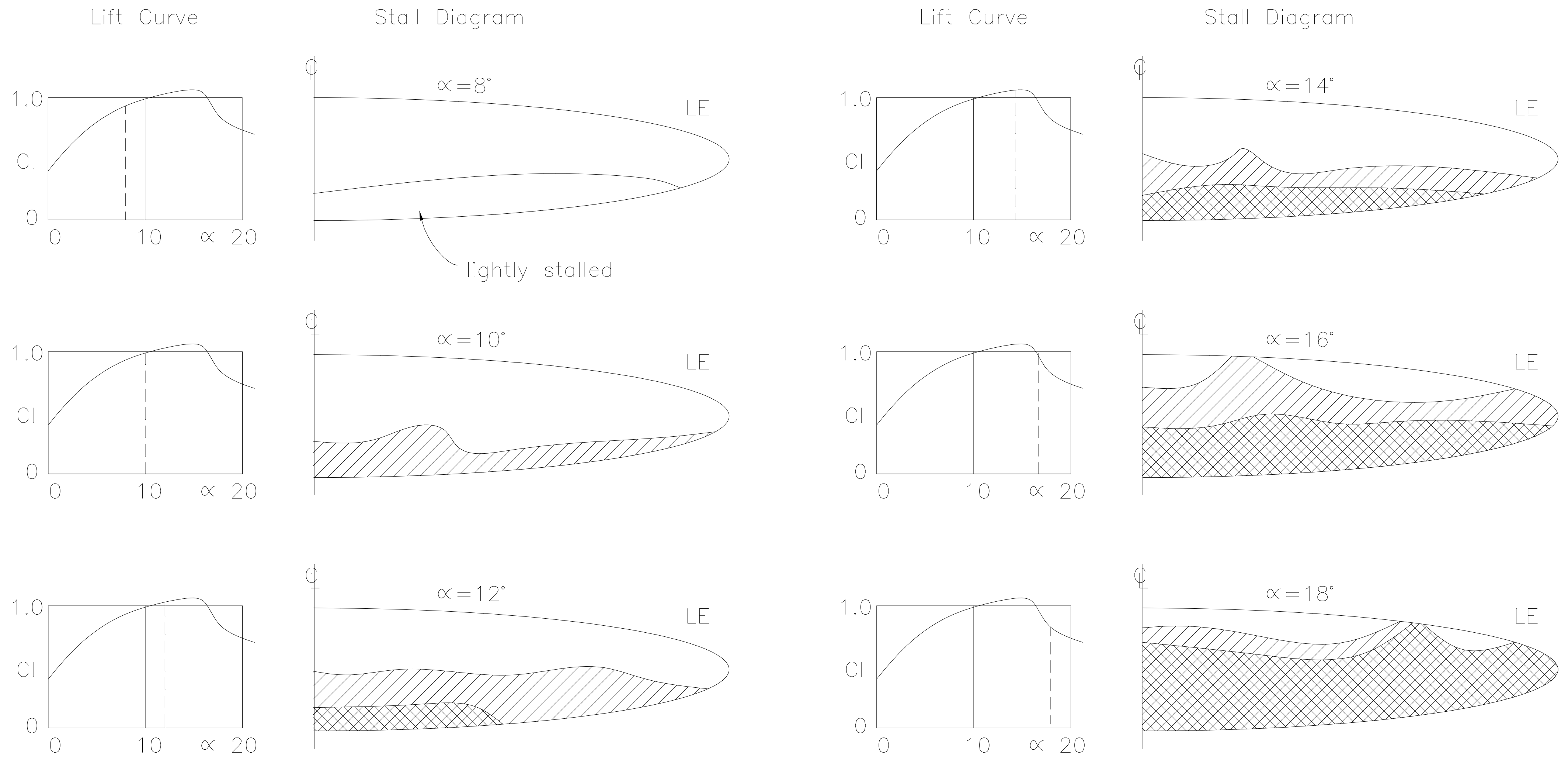
The next slides will show the evolution of the stall on four different wings. Notice the effect of the sweep angle.



Rectangular Wing $\lambda=1.0$ $A=7.2$

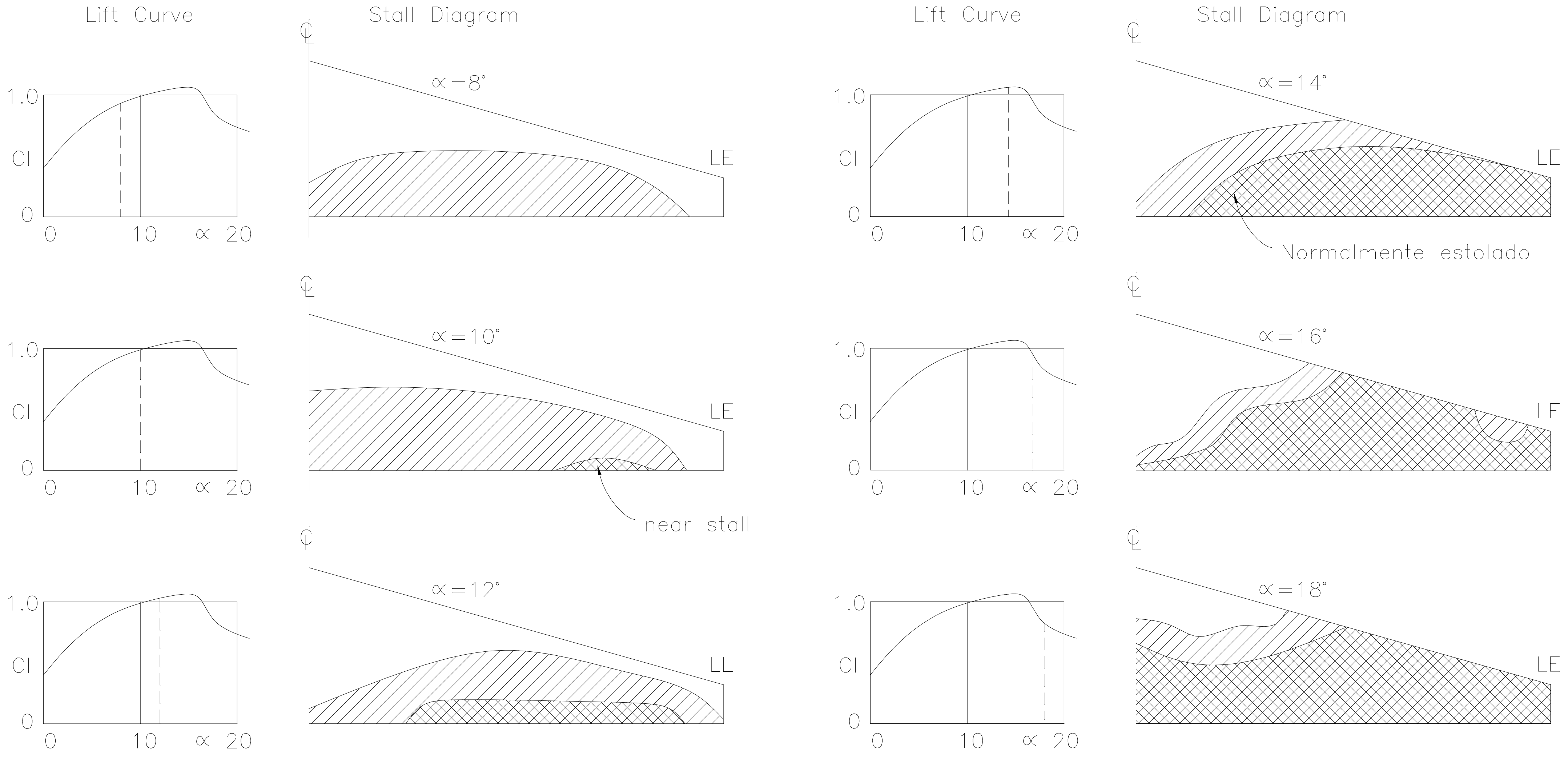


Elliptical Wing   A=7.2



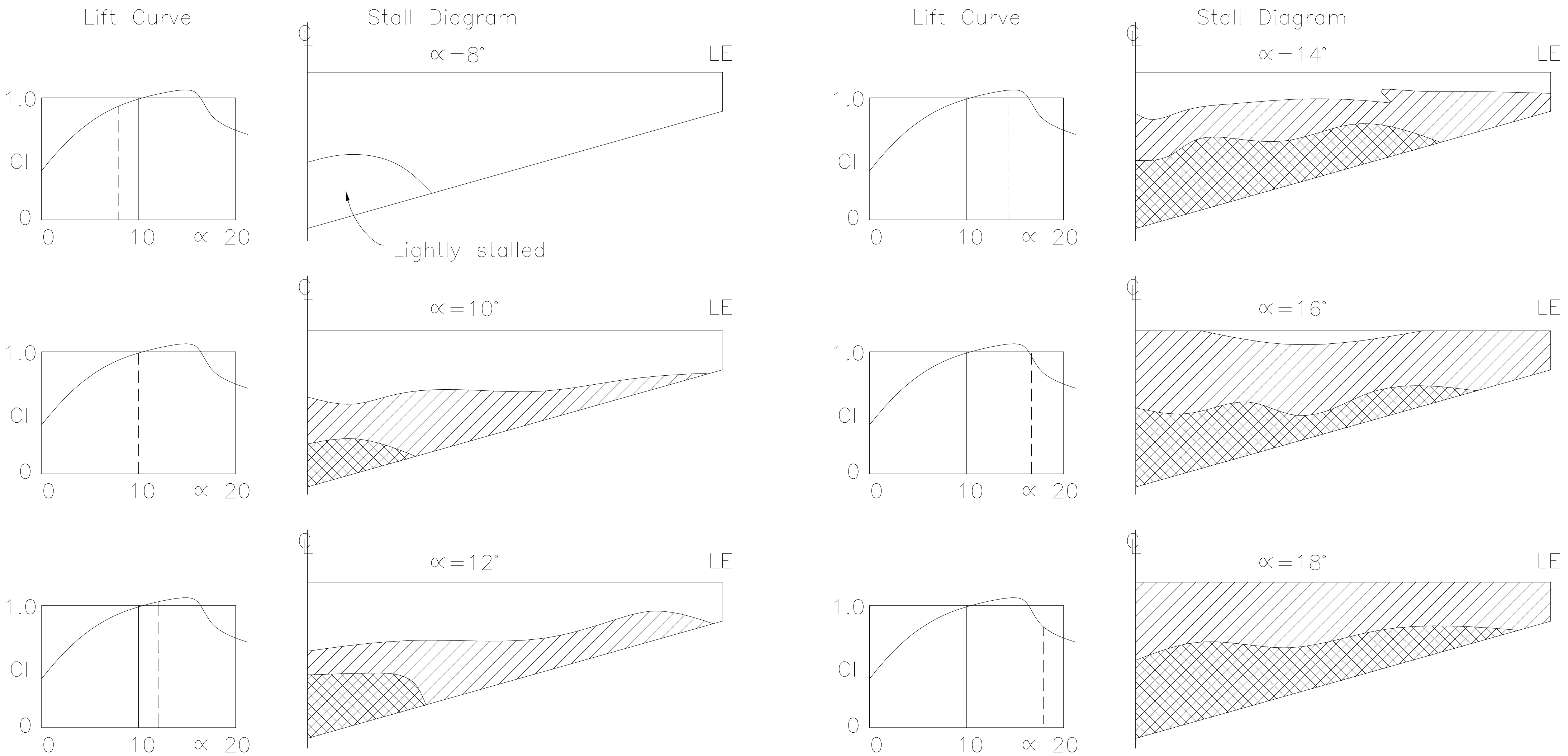
Trapezoidal Wing

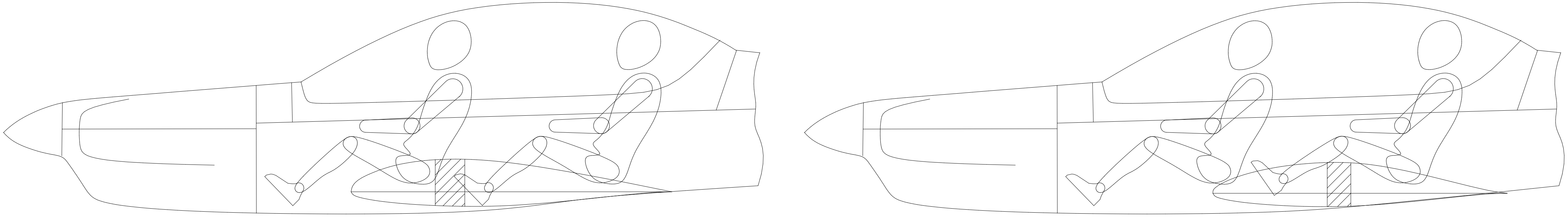
$\lambda=0.25 \quad A=7.2$



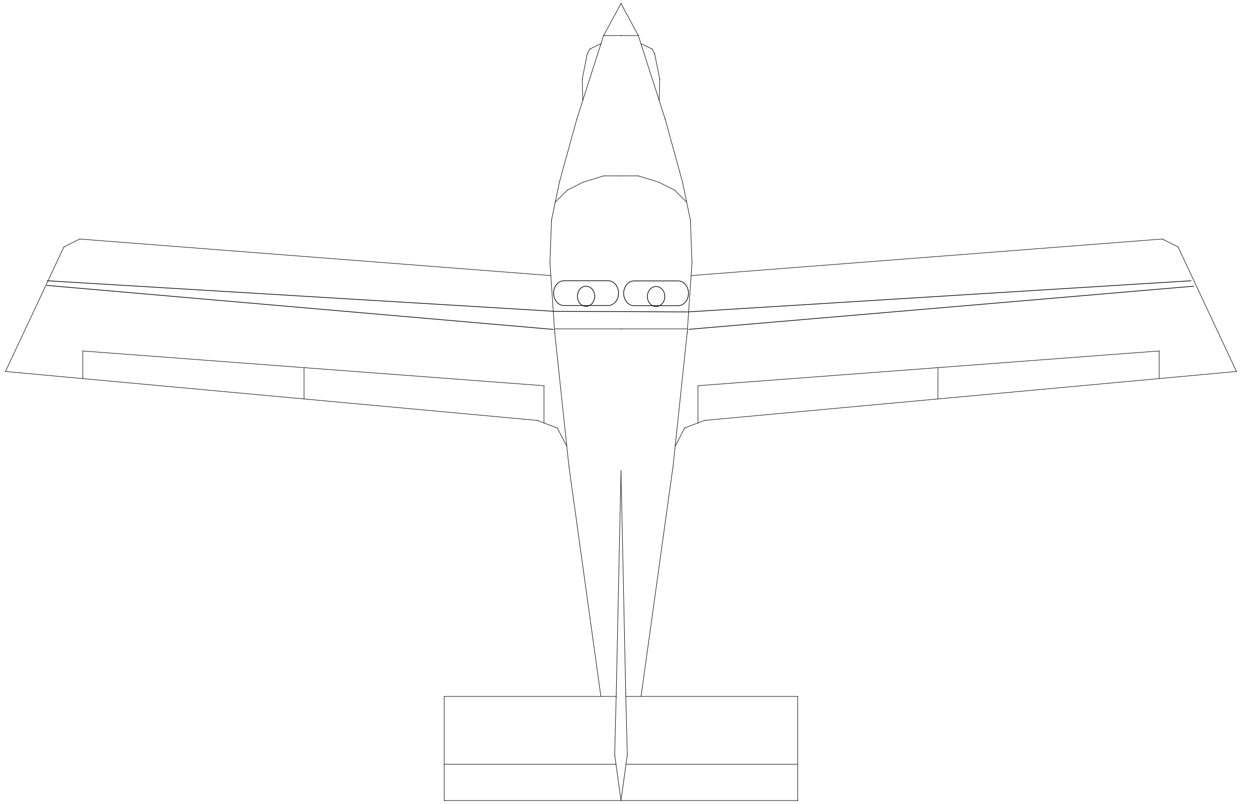
Trapezoidal Wing

$\lambda = 0.25 \quad A = 7.2$



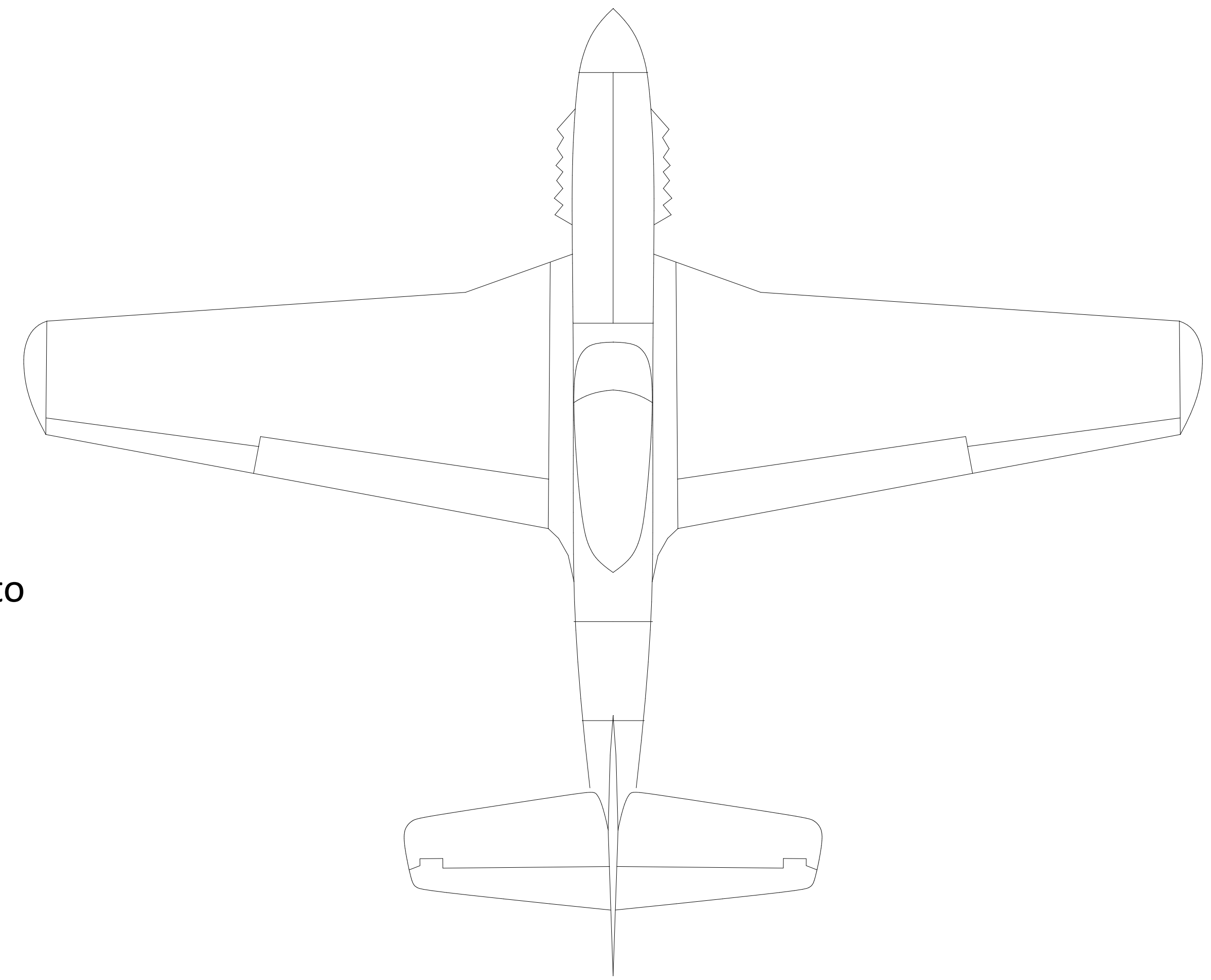


Sometimes the structure of the wing will interfere with systems or ergonomics of the airplane. It is common to see configurations with sweep angle to accommodate these interferences while maintaining the appropriate positioning of the wing to the airplane center of gravity.



P-51D Mustang

Why does it have this leading edge extension close to the fuselage?



Pressures

Total pressure (p_0) – is the sum of static pressure, dynamic pressure and gravitation head ($\rho g z$). It is not a pressure on it strict sense, but a quantity that has dimension of pressure.

$$p_0 = p + q + \rho g z$$

Dynamic pressure (q) – it is the kinetic energy per unit of volume of a fluid particle. it is also not a pressure, but a quantity that has dimension of pressure.

$$q = \frac{1}{2}\rho V^2$$

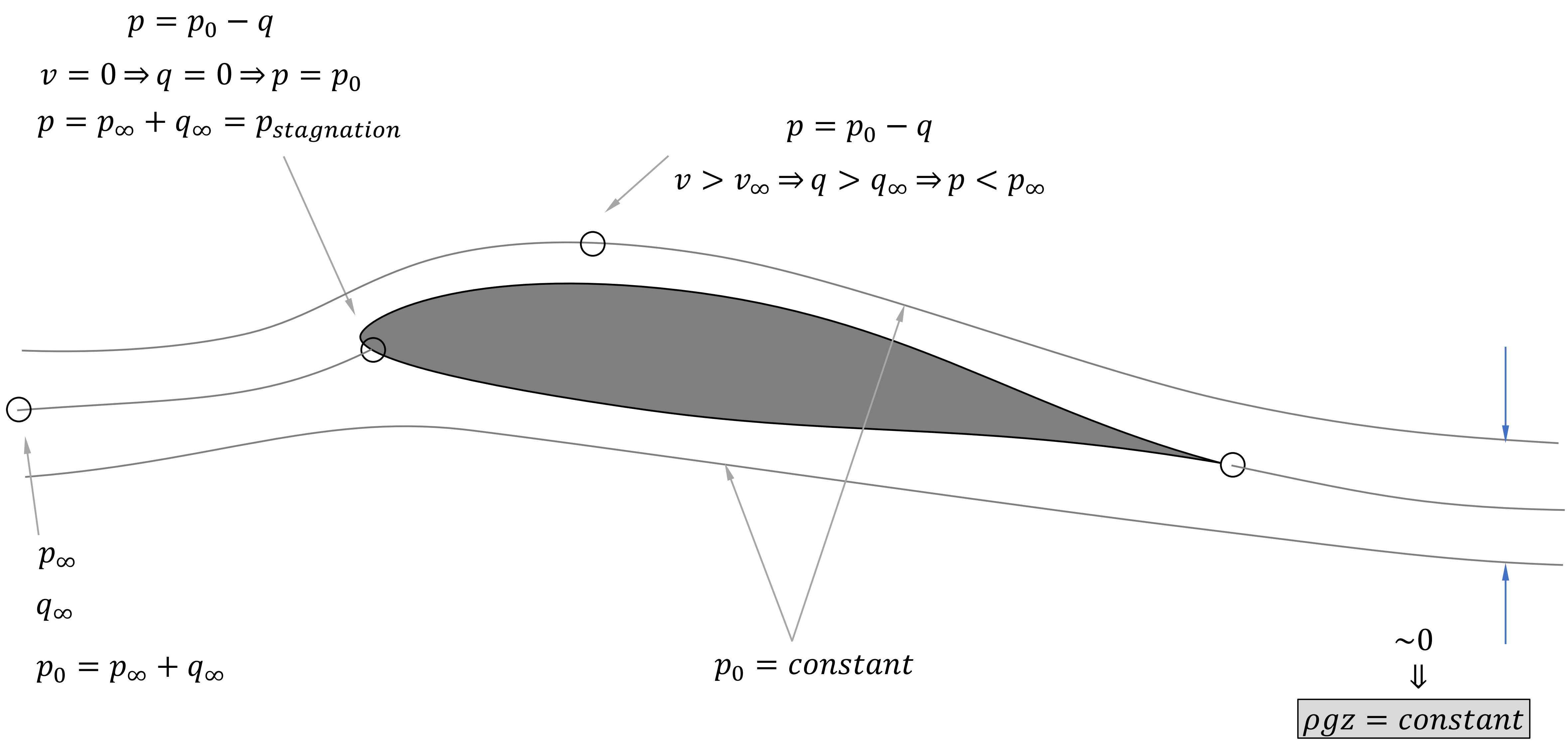
Static Pressure (p) – it is pressure, on its strict sense. The term static is used to distinguish it from total and dynamic pressure, but we can understand static pressure as the pressure that you can measure on a point of the fluid flow using a “manometer”.

Stagnation pressure – is the static pressure at the stagnation pressure of a fluid flow. In the absence of gravitational head, it is equal to total pressure.

$$p = p_{stagnation} = p_\infty + q_\infty$$

Bernoulli Equation states that **total pressure is constant** on **a stream line** of a fluid flow.

$$p_0 = constant$$


$p = p_0 - q$
$v = 0 \Rightarrow q = 0 \Rightarrow p = p_0$
$p = p_\infty + q_\infty = p_{stagnation}$
$p = p_0 - q$
$v > v_\infty \Rightarrow q > q_\infty \Rightarrow p < p_\infty$
p_∞
q_∞
$p_0 = p_\infty + q_\infty$
$p_0 = constant$
~ 0
$\Downarrow$
$\boxed{\rho g z = constant}$

Pressure Coefficient:

$$C_p = \frac{p - p_\infty}{p_0 - p_\infty} = \frac{p - p_\infty}{q_\infty}$$

$$\frac{(p_0 - q) - p_\infty}{q_\infty} = \frac{(p_\infty + q_\infty - q) - p_\infty}{q_\infty} = \frac{q_\infty - q}{q_\infty} = 1 - \left(\frac{V^2}{V_\infty^2}\right)$$

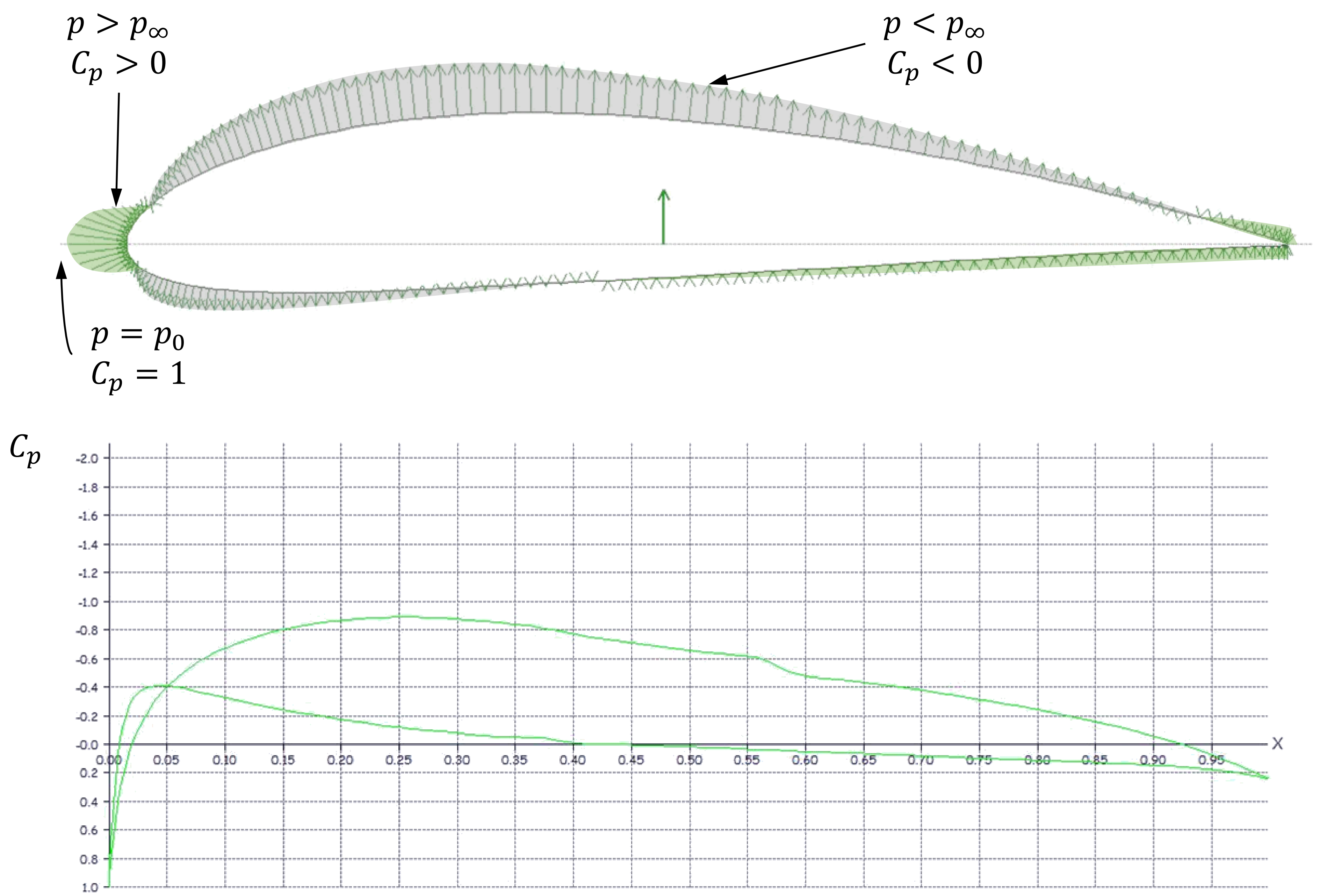
$C_p = 0$, means that $V = V_\infty$

C_p =1, means that $V = 0$, i.e. stagnation point

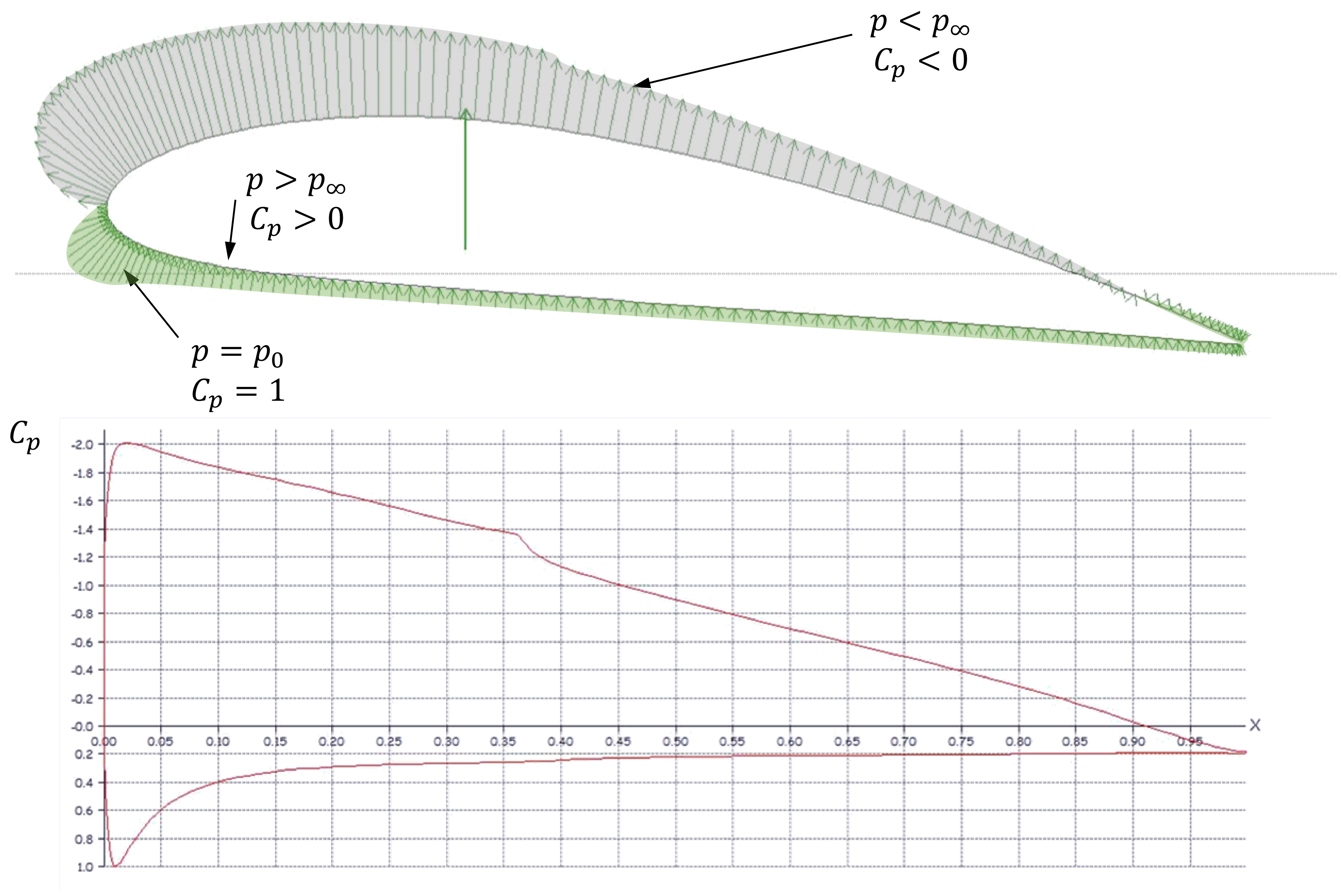
$C_p < 0$, means that $V > V_\infty$, i.e. higher speed

$C_p = -1$, is it an important value? Why?

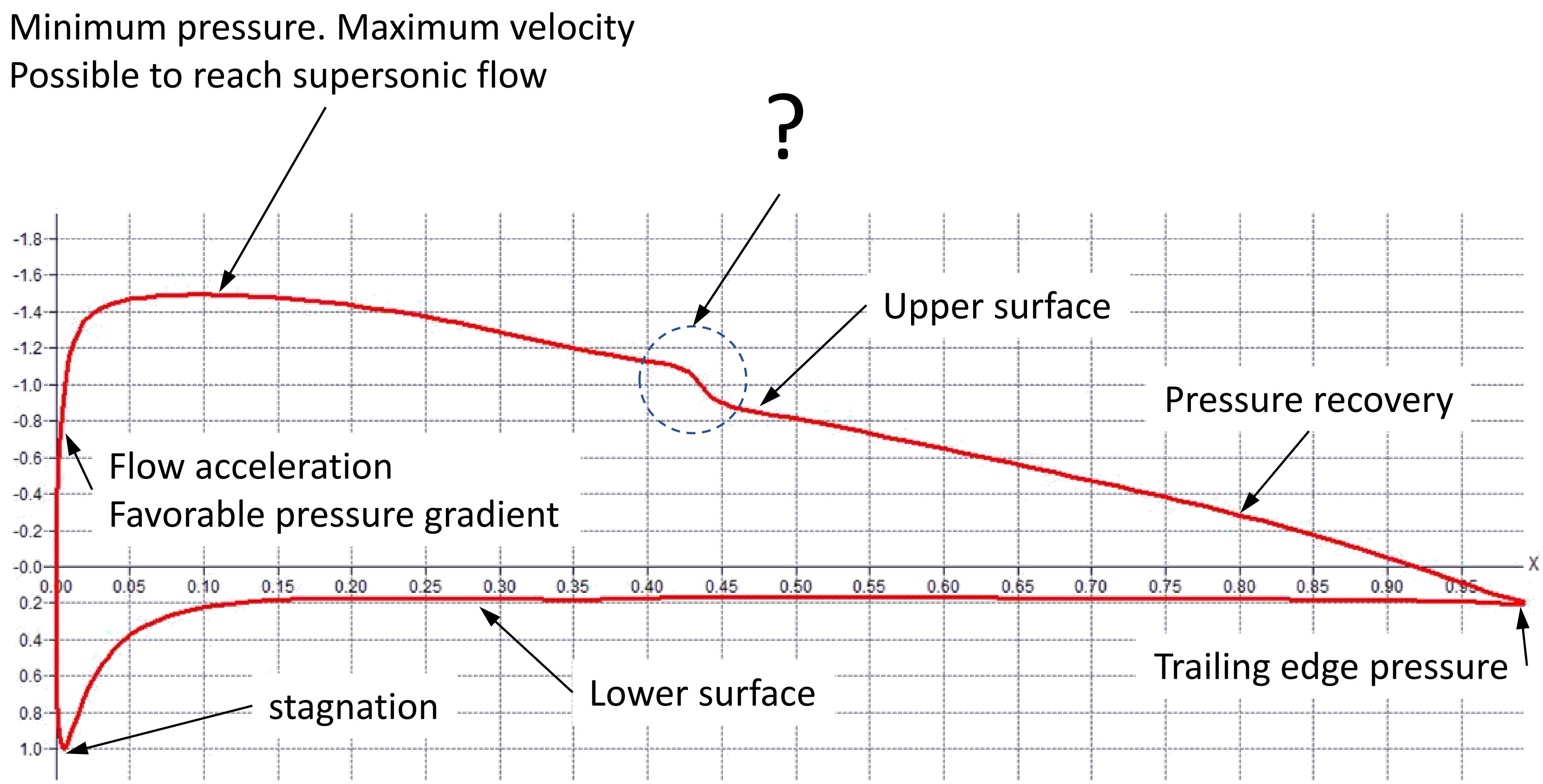
$C_p > 1$, is it possible? When?



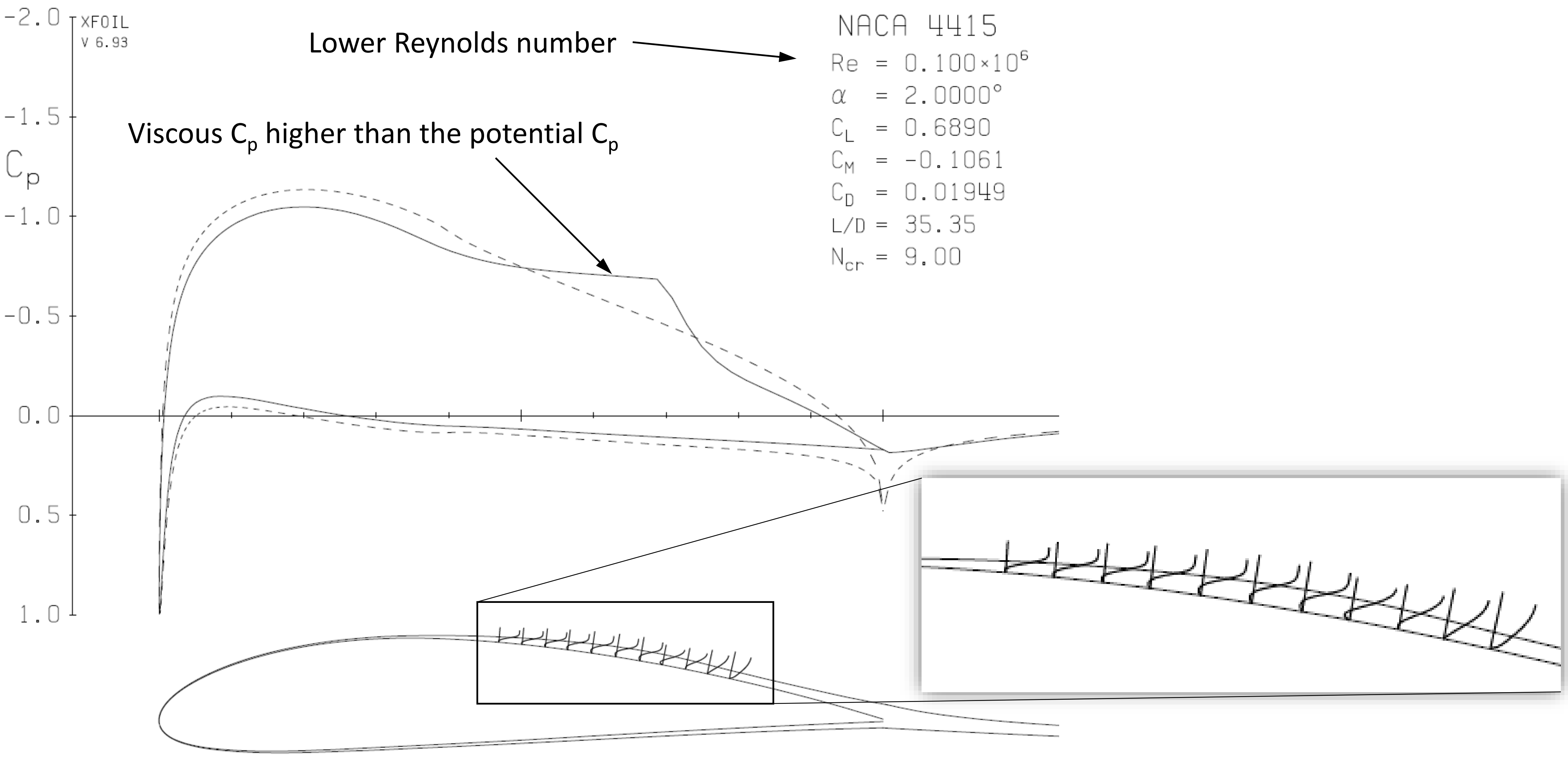
$p > p_\infty$
$C_p > 0$
$p < p_\infty$
$C_p < 0$
$p = p_0$
$C_p = 1$
C_p
-2.0
-1.8
-1.6
-1.4
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
-0.0
0.2
0.4
0.6
0.8
1.0
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95
X



$p < p_\infty$
$C_p < 0$
$p > p_\infty$
$C_p > 0$
$p = p_0$
$C_p = 1$
C_p
X


Minimum pressure. Maximum velocity
Possible to reach supersonic flow
?
Upper surface
Pressure recovery
Flow acceleration
Favorable pressure gradient
stagnation
Lower surface
Trailing edge pressure
-1.8
-1.6
-1.4
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
-0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
X

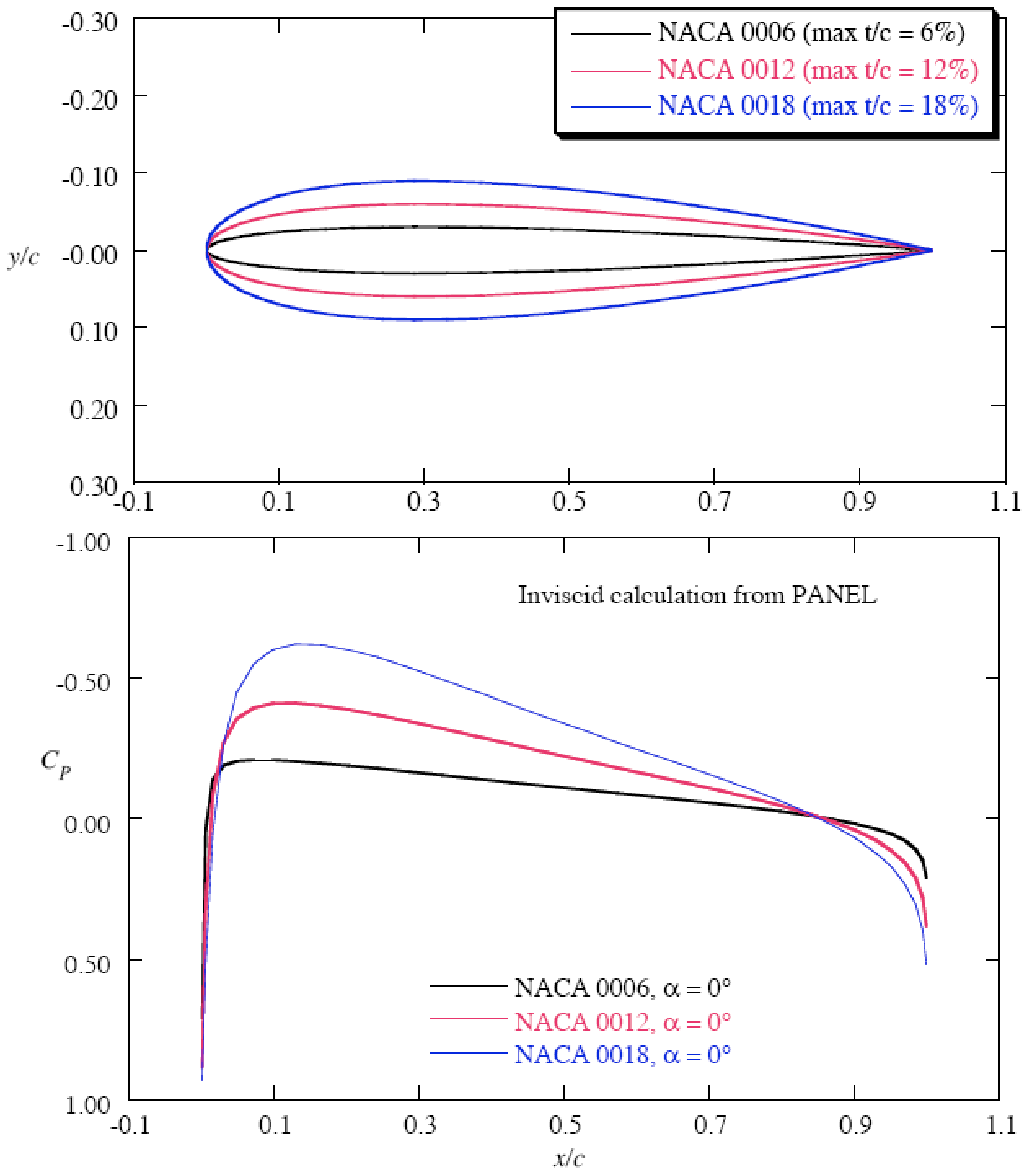
Laminar separation – Laminar bubble



Effect of thickness

Notice that the increase of thickness increase the minimum pressure (more negative). However, the pressure gradient (accelerating) is less aggressive.

Notice that the adverse pressure gradient, after the minimal pressure is more aggressive for higher thickness.

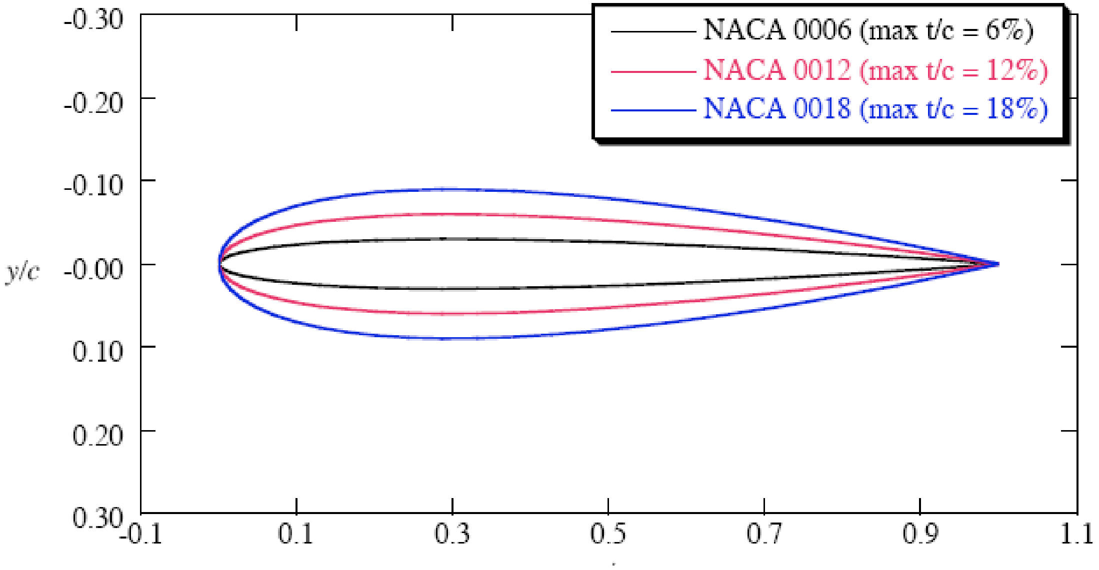


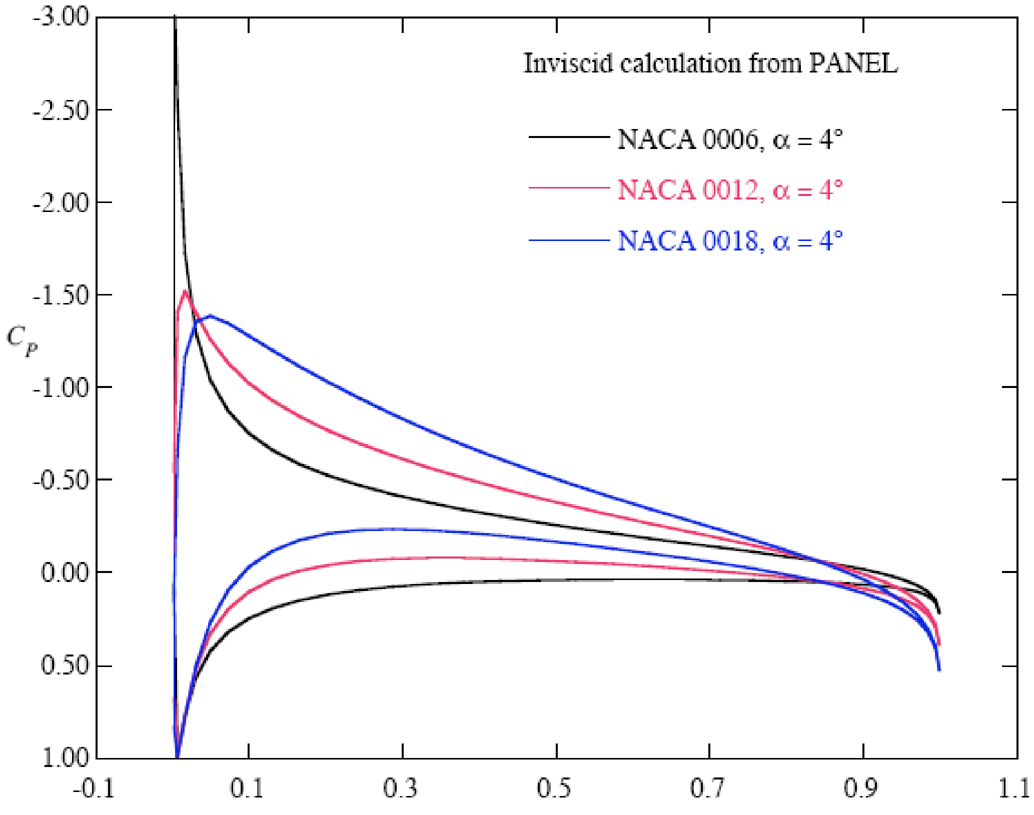
For attack angle different of zero, notice that the thinner airfoil produce a peak of lower pressure close to its leading edge (high acceleration) and therefore, shows a really aggressive adverse gradient to recover this pressure.

This, in most of the cases, causes a really aggressive stall behavior.

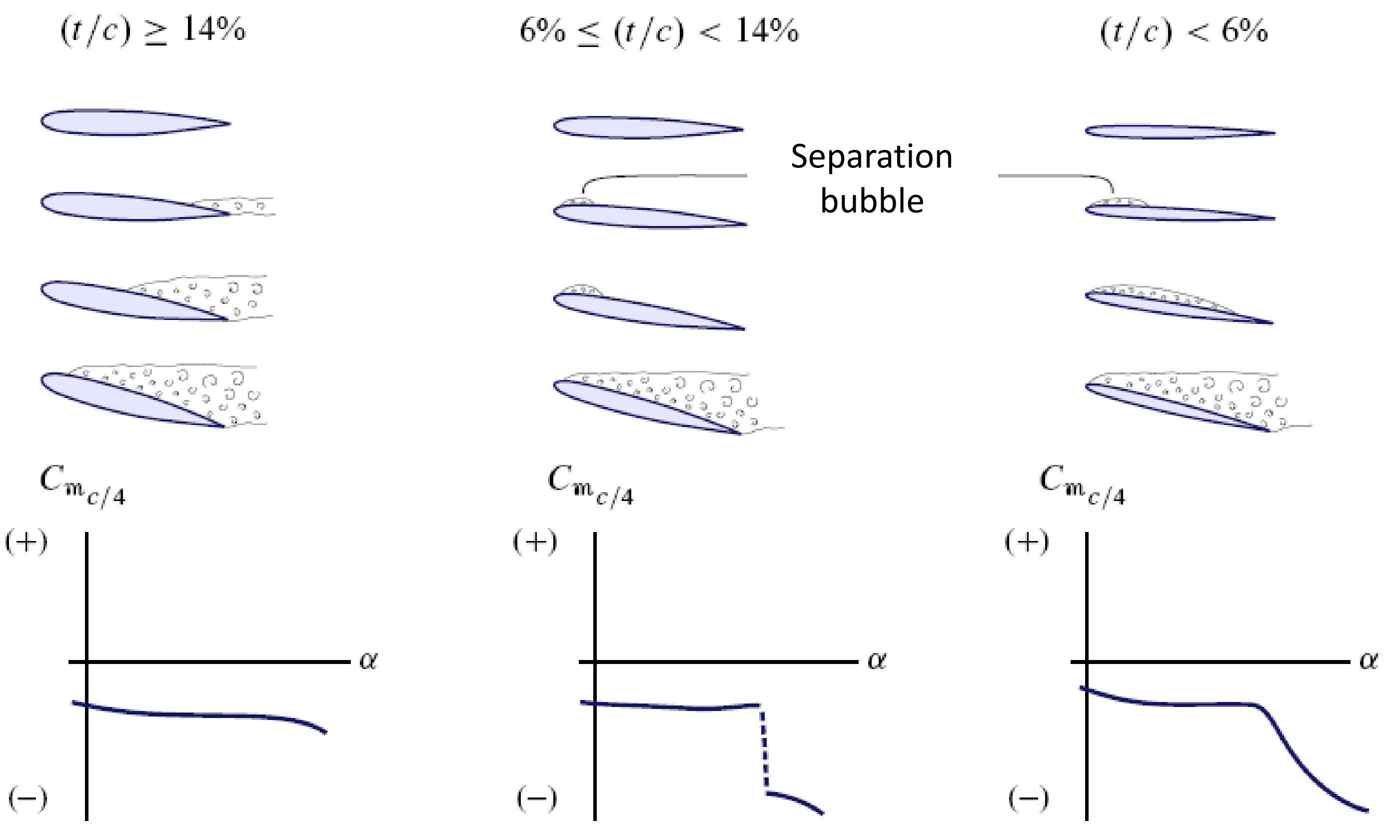
For thick airfoils, the flow behavior is similar to zero attack angle condition.

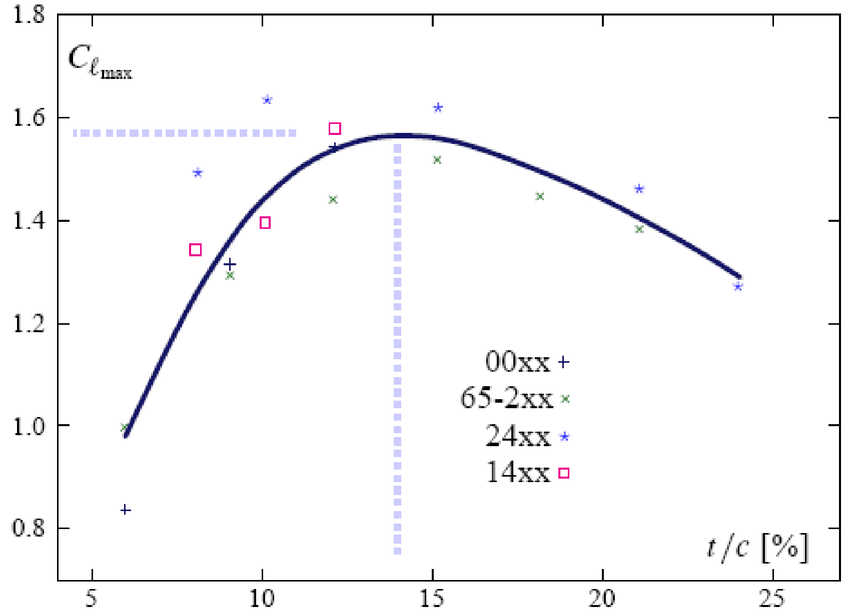
This behavior is similar for cambered airofils.



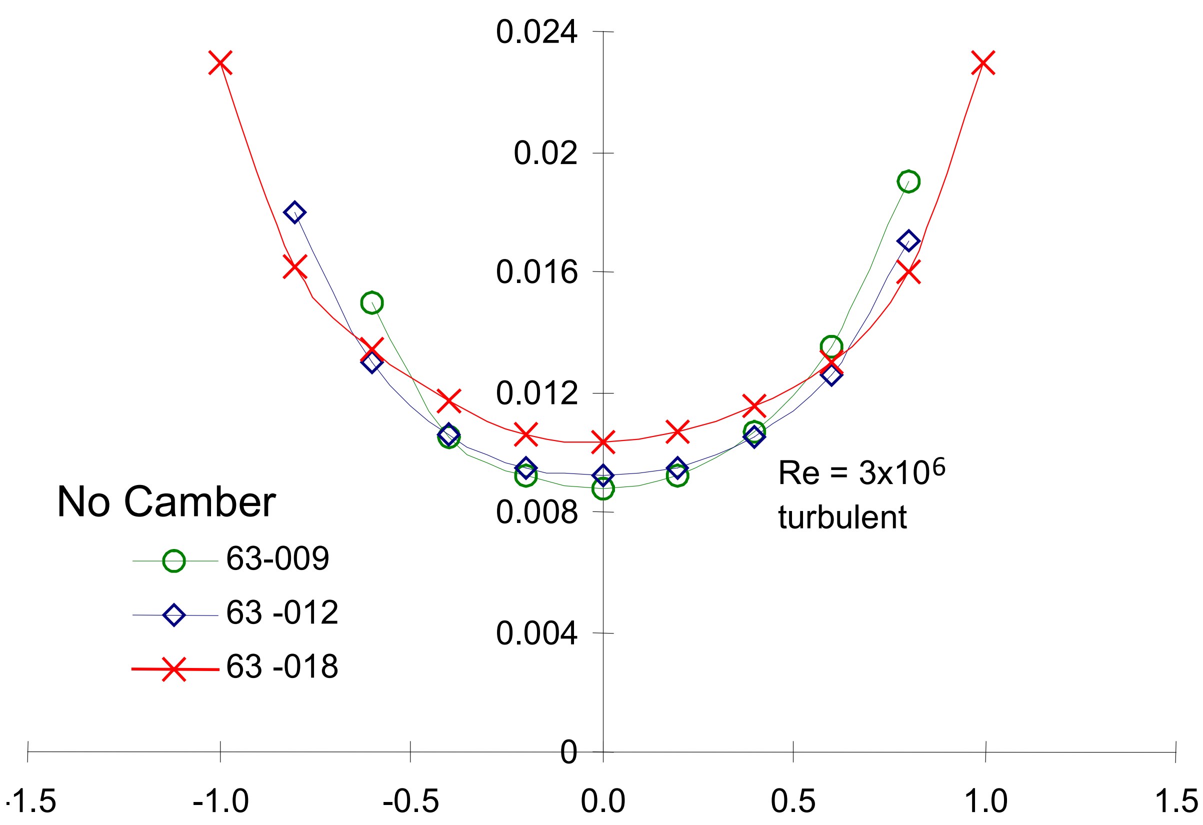


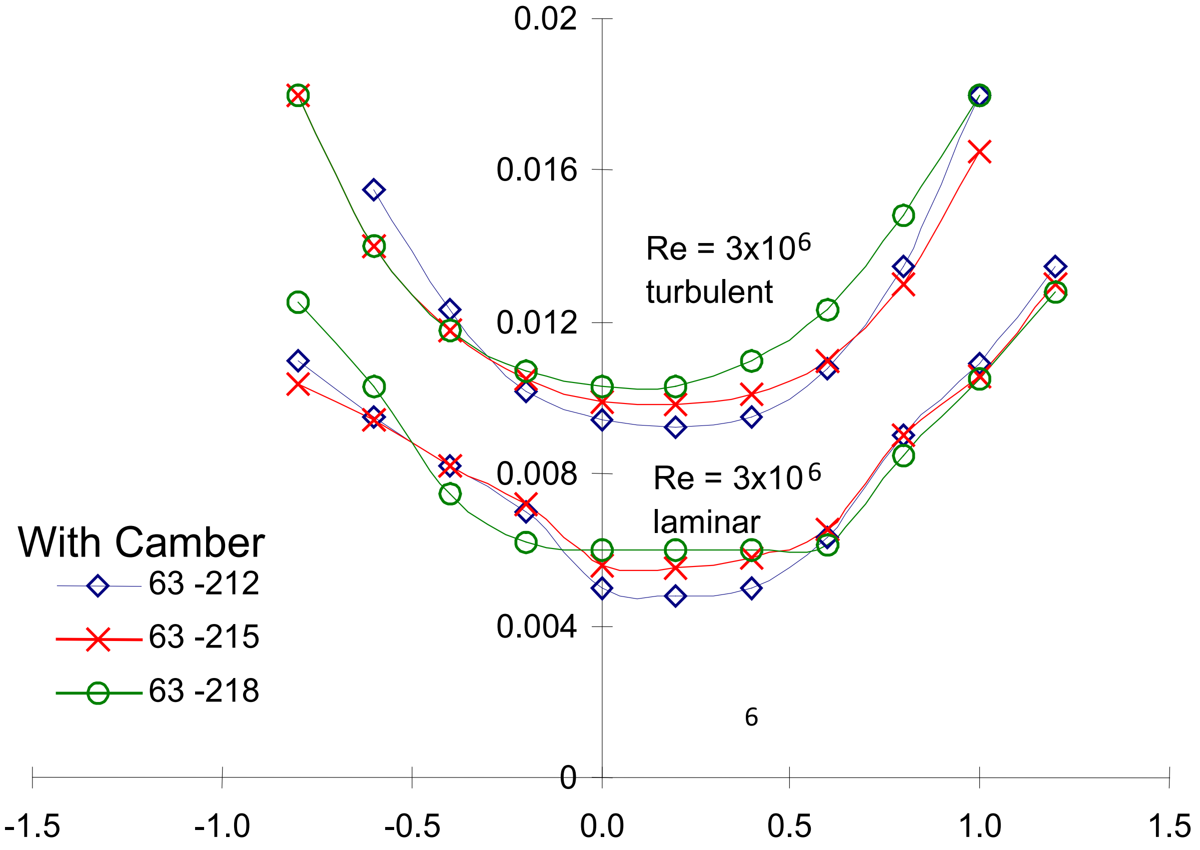
For stall evolution, this is a typical behavior for airfoils, based on its thickness.

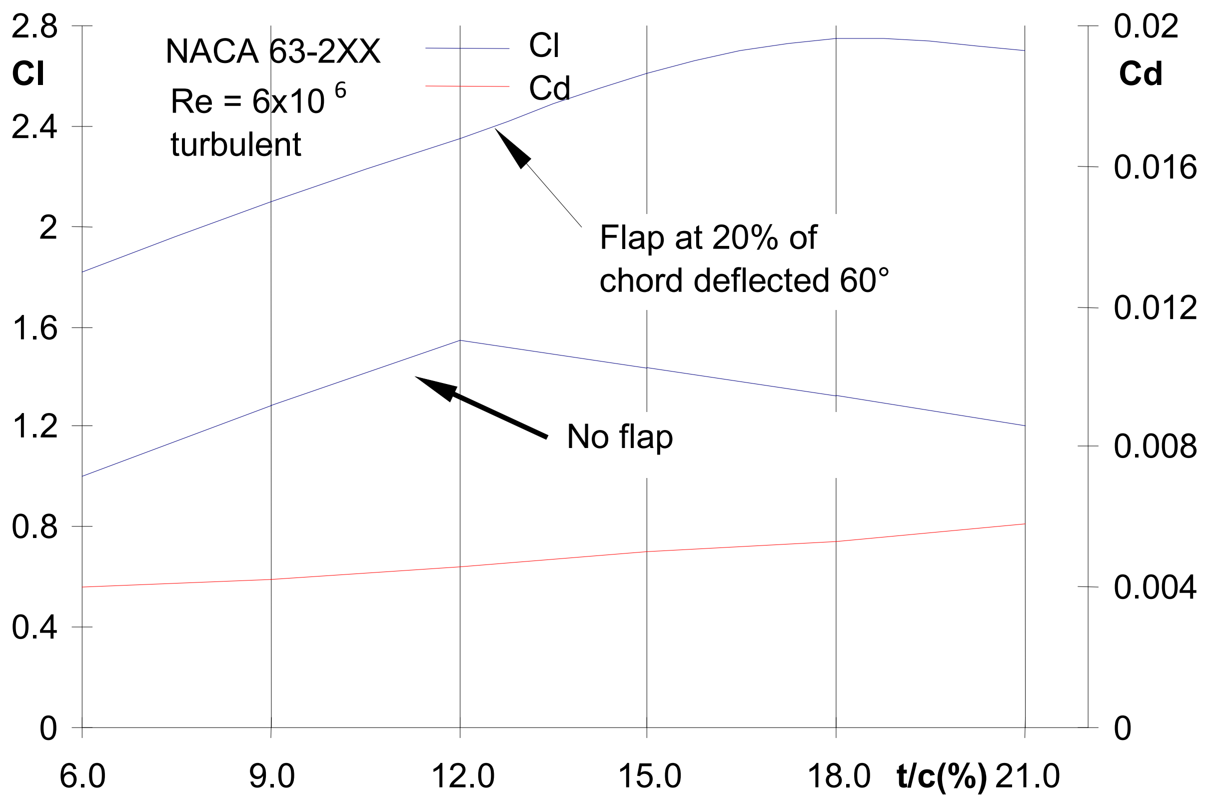




Thickness effect:

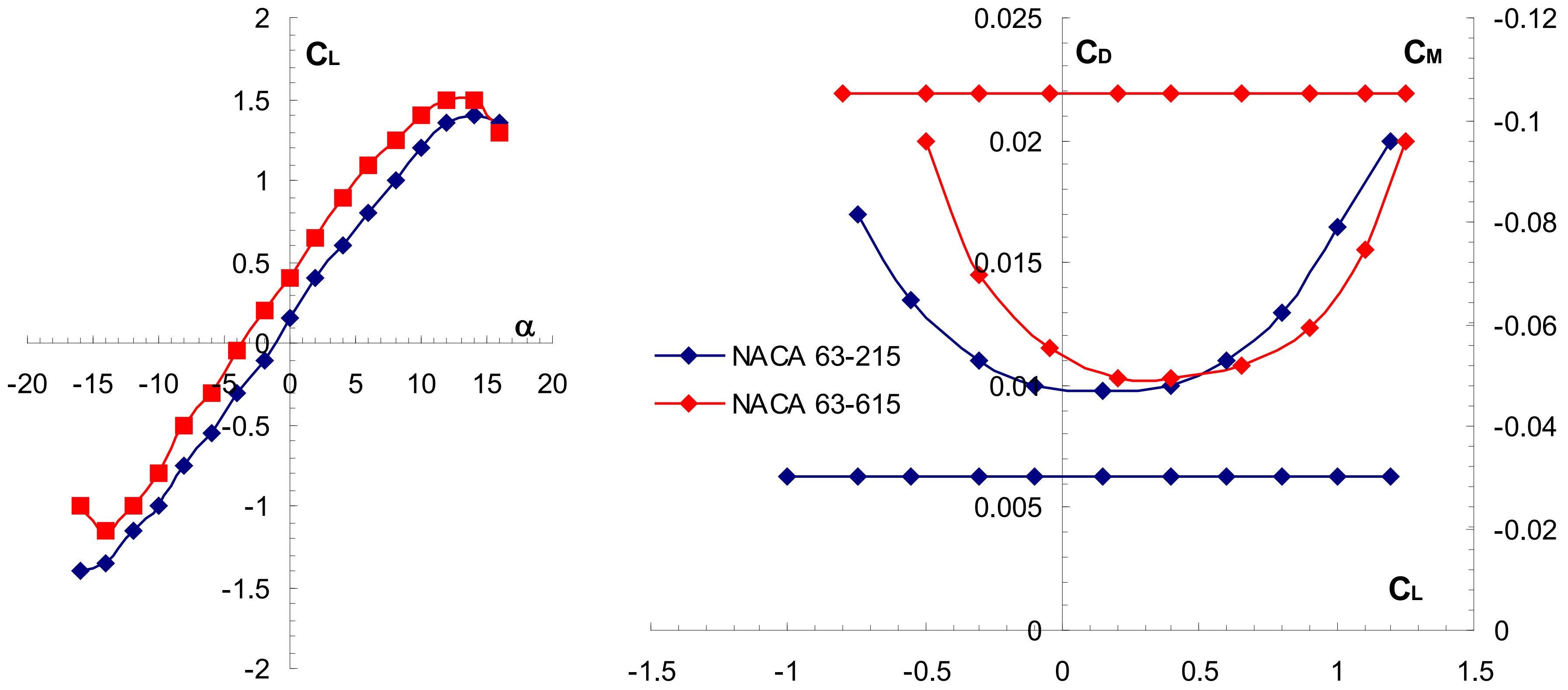




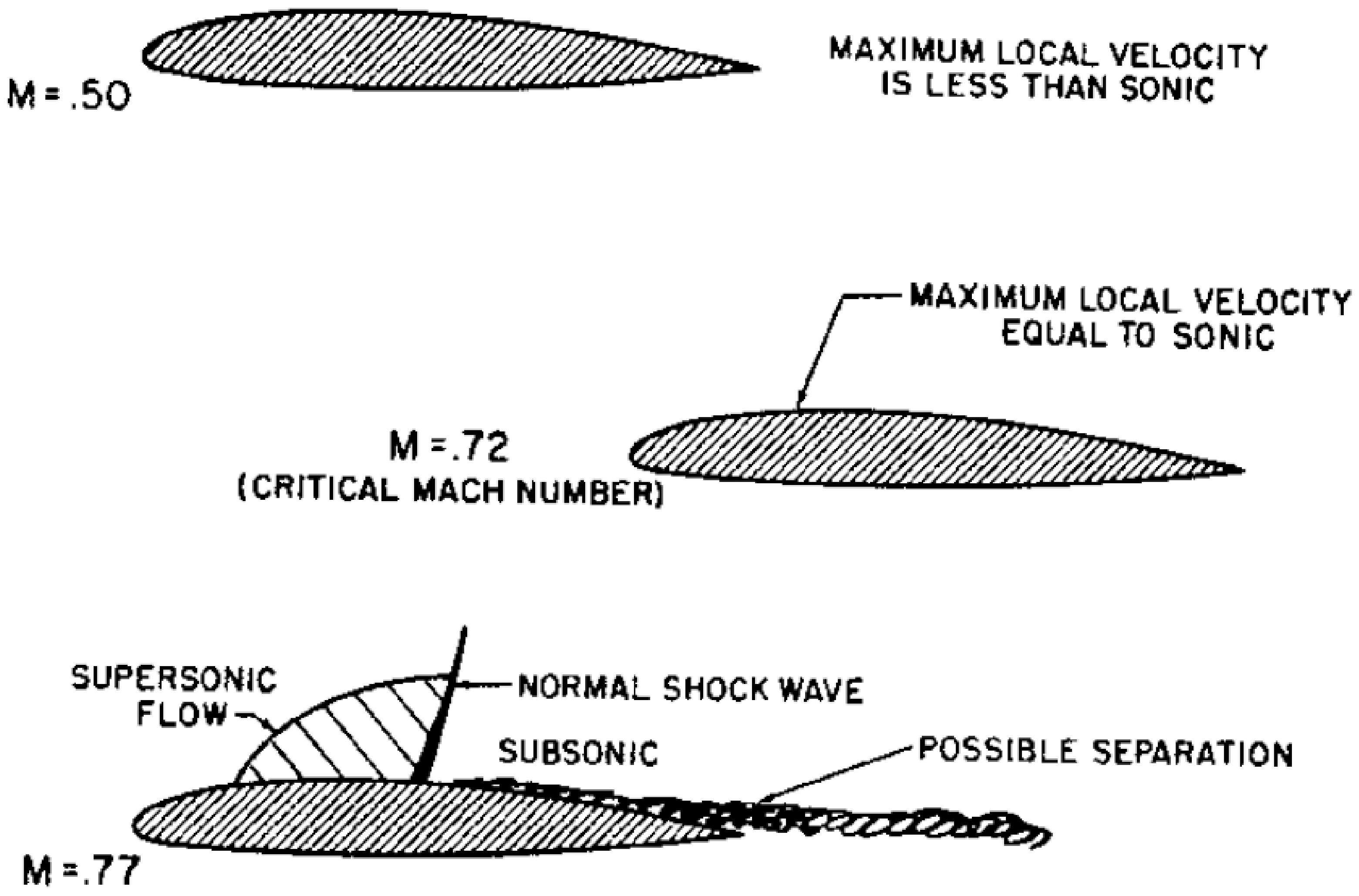


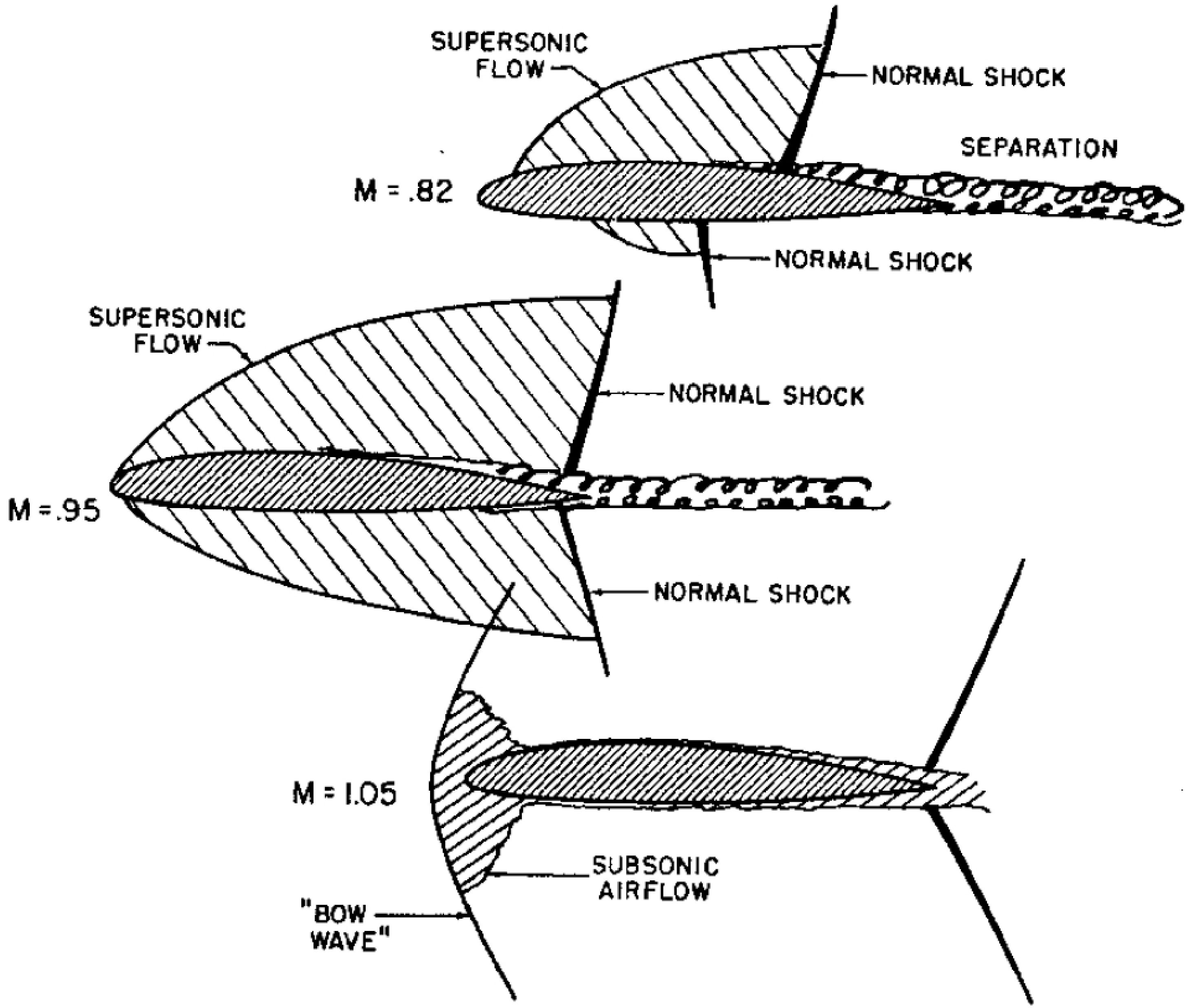
NACA 63-2XX
Re = 6x10 6
turbulent
Cl
Cd
Flap at 20% of
chord deflected 60°
No flap
Cl
Cd
t/c(%)
2.8
2.4
2
1.6
1.2
0.8
0.4
0
0.02
0.016
0.012
0.008
0.004
0
6.0
9.0
12.0
15.0
18.0
21.0

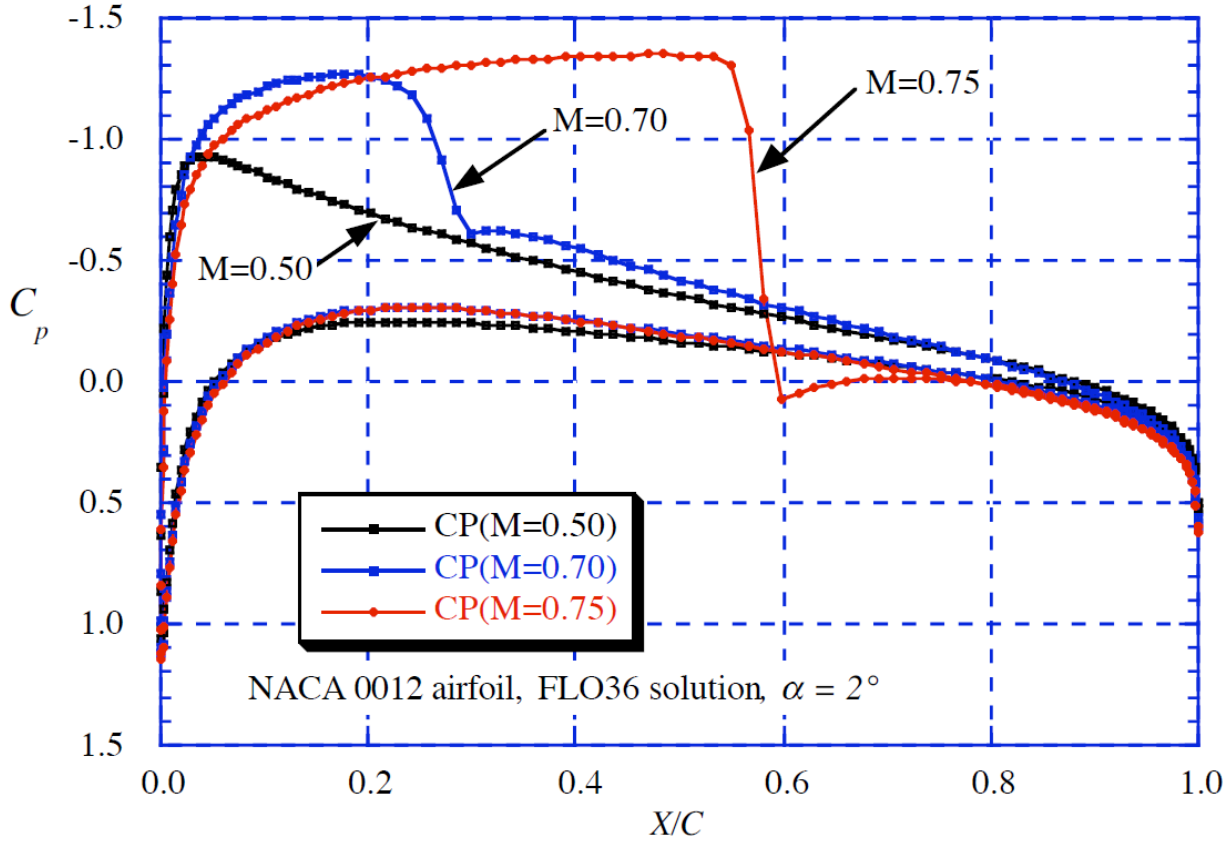
Camber effect:



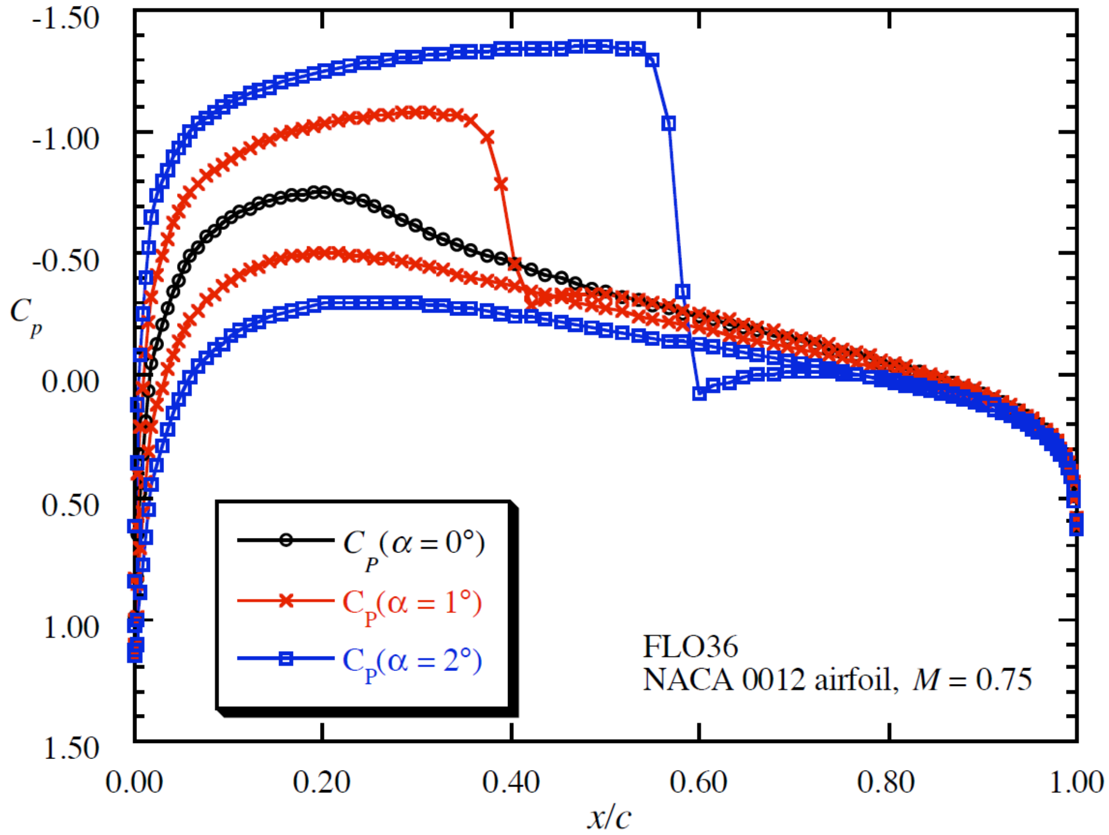
Transonic


M = .50
MAXIMUM LOCAL VELOCITY
IS LESS THAN SONIC
MAXIMUM LOCAL VELOCITY
EQUAL TO SONIC
M = .72
(CRITICAL MACH NUMBER)
SUPERSONIC
FLOW
NORMAL SHOCK WAVE
SUBSONIC
POSSIBLE SEPARATION
M = .77

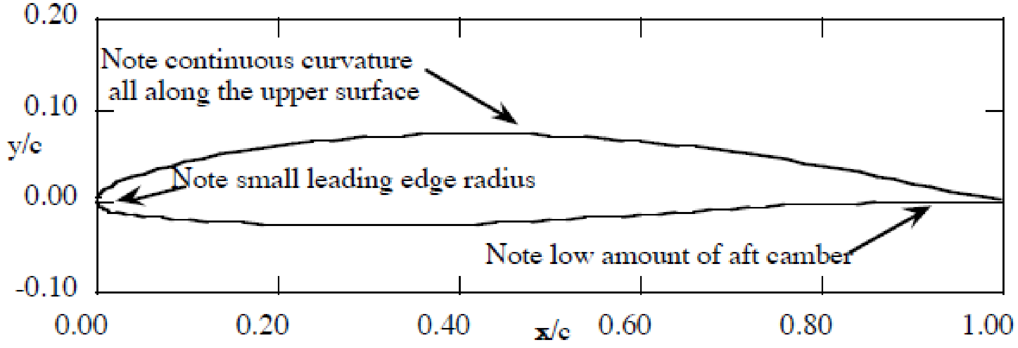

SUPERSONIC
FLOW
NORMAL SHOCK
SEPARATION
M = .82
NORMAL SHOCK
SUPERSONIC
FLOW
NORMAL SHOCK
M = .95
NORMAL SHOCK
M = 1.05
SUBSONIC
AIRFLOW
"BOW
WAVE"



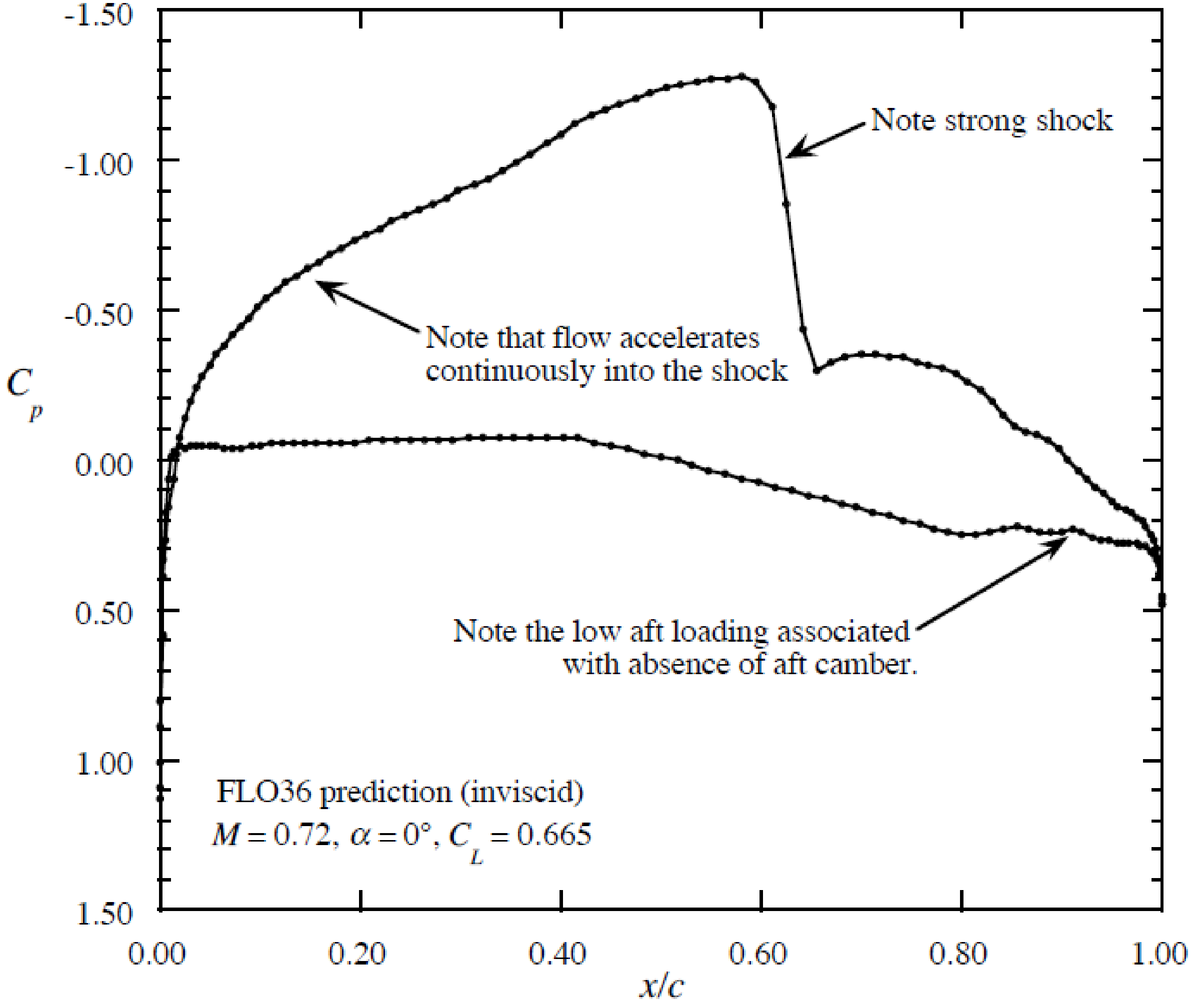
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
C_p
0.0
0.2
0.4
0.6
0.8
1.0
X/C
M=0.70
M=0.75
M=0.50
CP(M=0.50)
CP(M=0.70)
CP(M=0.75)
NACA 0012 airfoil, FLO36 solution, $\alpha = 2°$

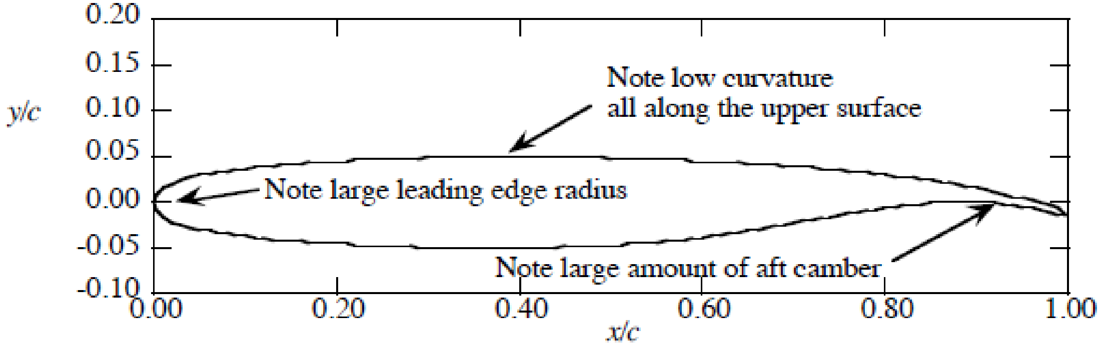


-1.50
-1.00
-0.50
0.00
0.50
1.00
1.50
C_p
0.00
0.20
0.40
0.60
0.80
1.00
x/c
$C_P(\alpha = 0°)$
$C_P(\alpha = 1°)$
$C_P(\alpha = 2°)$
FLO36
NACA 0012 airfoil, $M = 0.75$

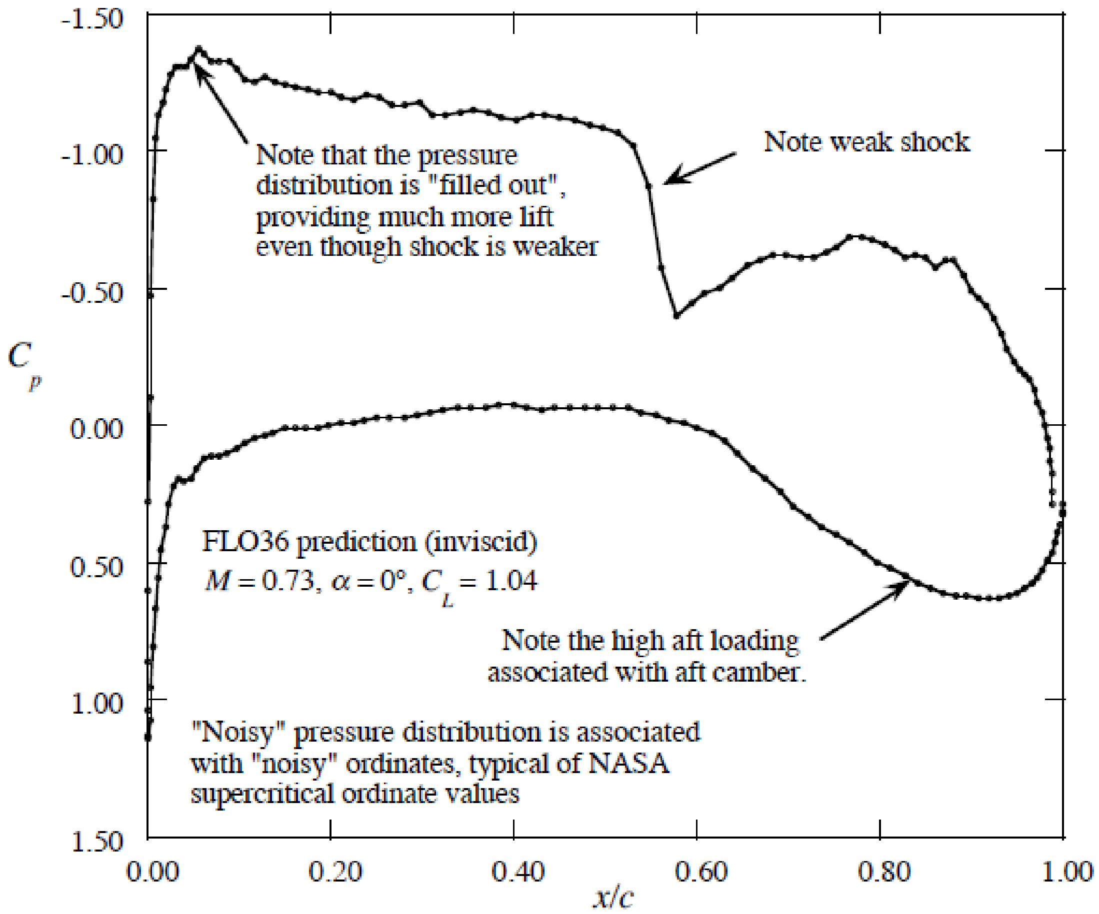


a) NACA 64A410 airfoil





a) FOIL 31



Choice of leading edge radius

Airfoil thickness and high lift geometry (LE device?)

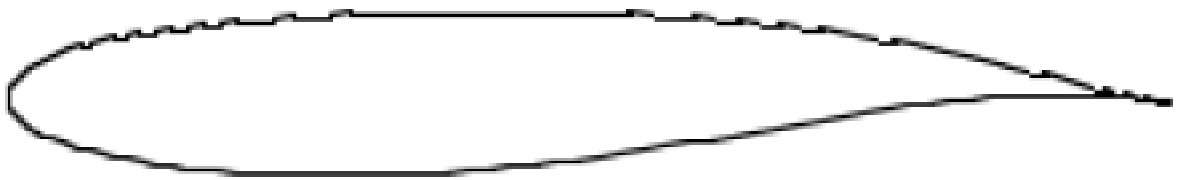
Increase favors
- clean wing low speed CLmax
- Mach divergence at moderate to high lift coefficients

Decrease favors
- elimination of drag creep
- drag divergence Mach numbers at low lift coefficients

Choice of aft camber

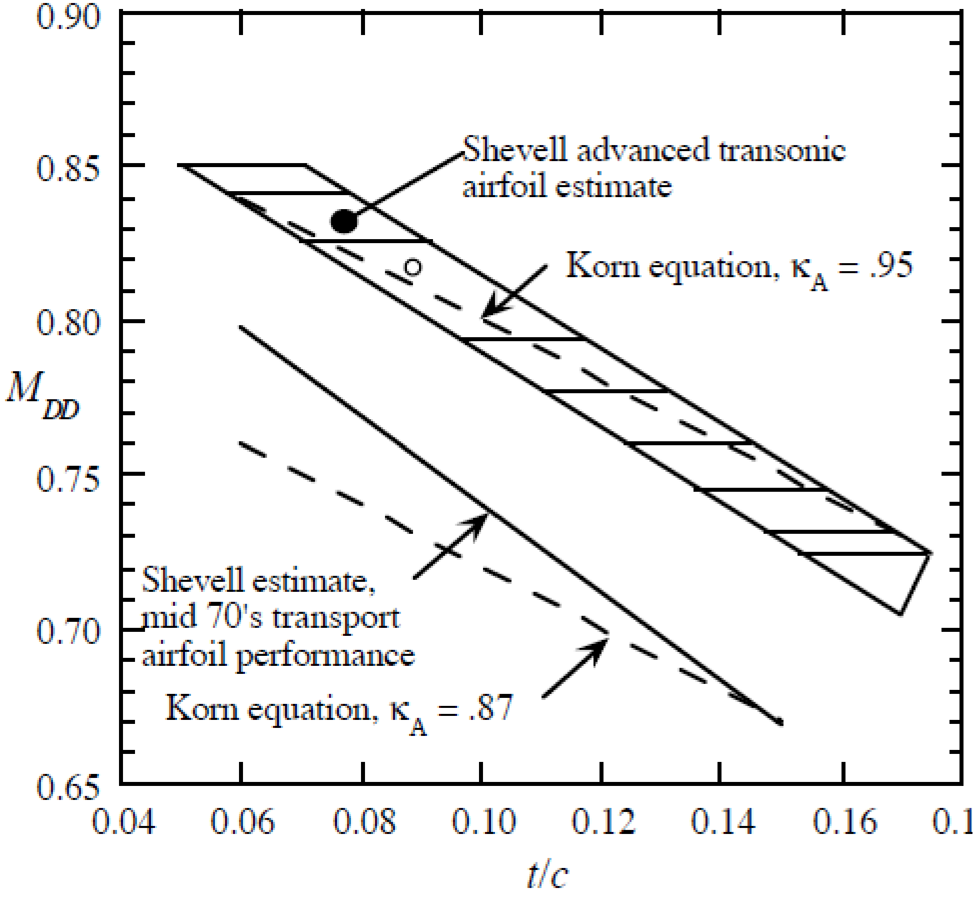
Design lift coefficient and flight Reynolds number

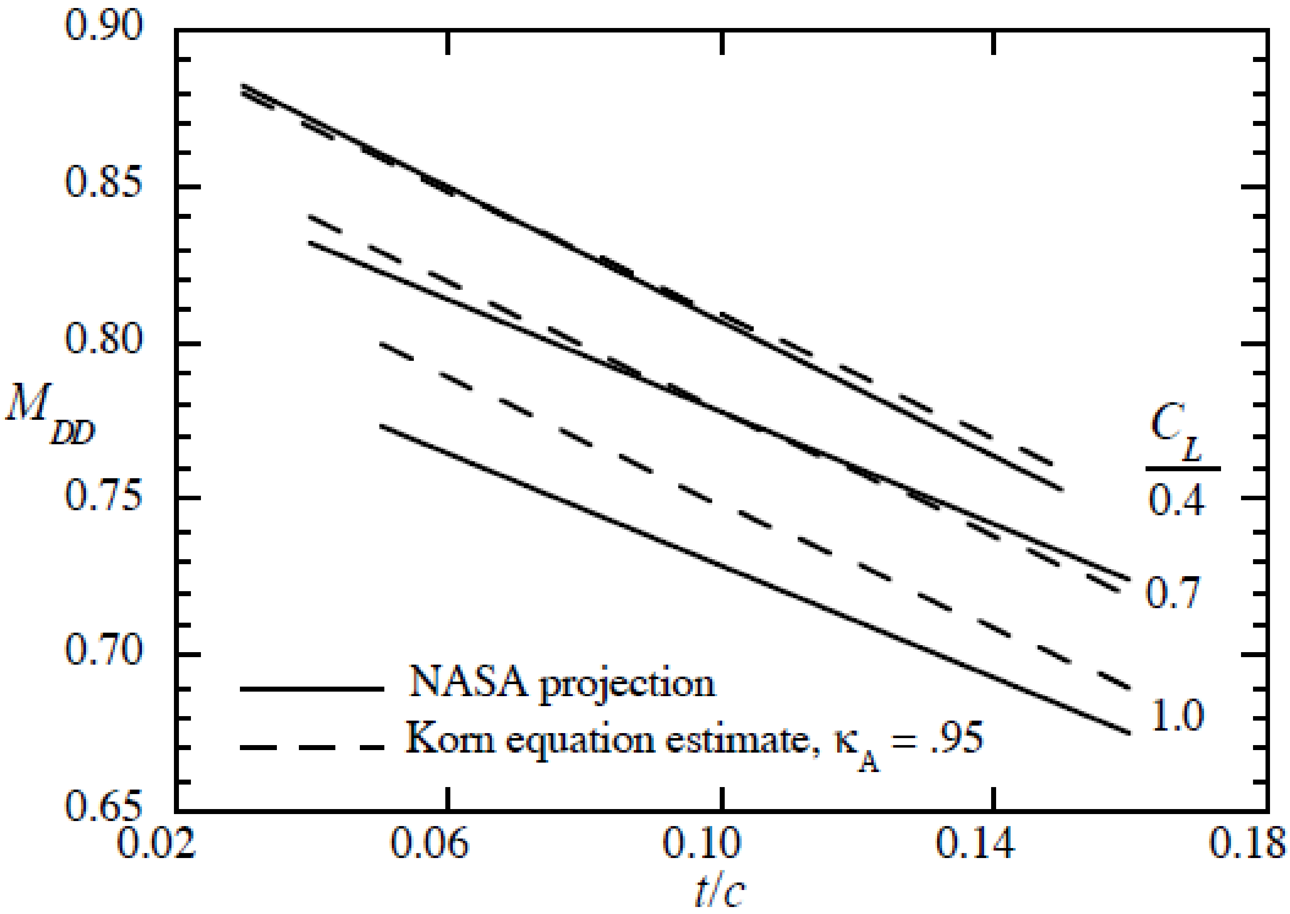
Increase favors
- clean wing low speed CLmax
- drag divergence Mach number at high lift coefficients



Decrease favors
- trim drag by decreasing CM0
- lower surface interference problems (flap hinge line fairings, etc.)
- risk of premature separation at flight conditions
- control surface hinge moments

Other considerations
- spanwise location of airfoils on swept wings (root requires special treatment)
- chordwise distribution of thickness (determined by both aerodynamic and structural considerations





Korn's equation: $$M_{dd} = \frac{\kappa_A}{cos\Lambda} - \frac{(t/c)}{\cos^2\Lambda} - \frac{C_L}{10\cos^3\Lambda}$$

$$M_{dd} = \frac{\kappa_A}{cos\Lambda} - \frac{(t/c)}{\cos^2 \Lambda} - \frac{C_L}{10\cos^3 \Lambda}$$

$$M_{dd} \rightarrow \frac{\partial C_D}{\partial M} = 0.1$$

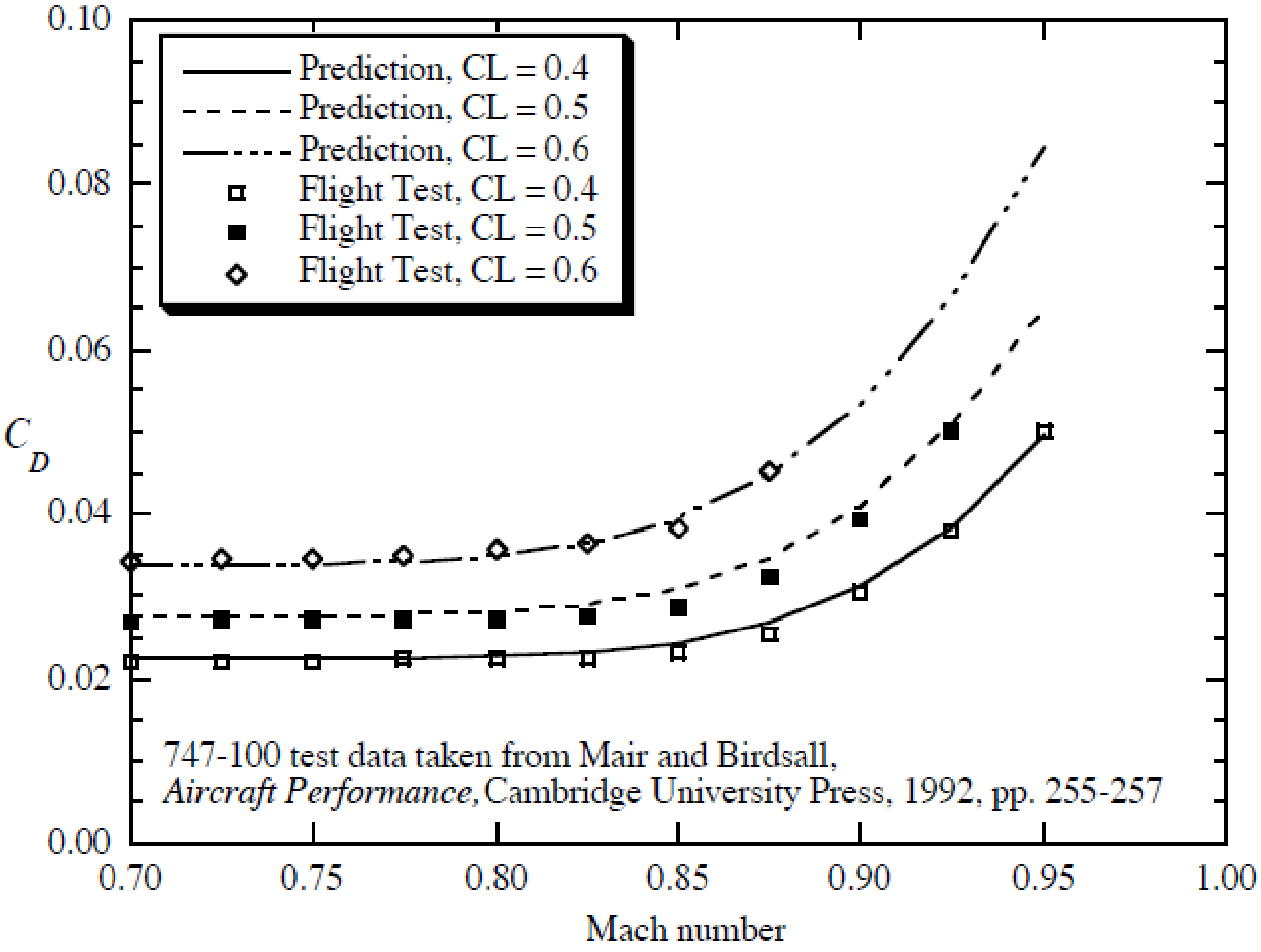
Using Lock's approximation:

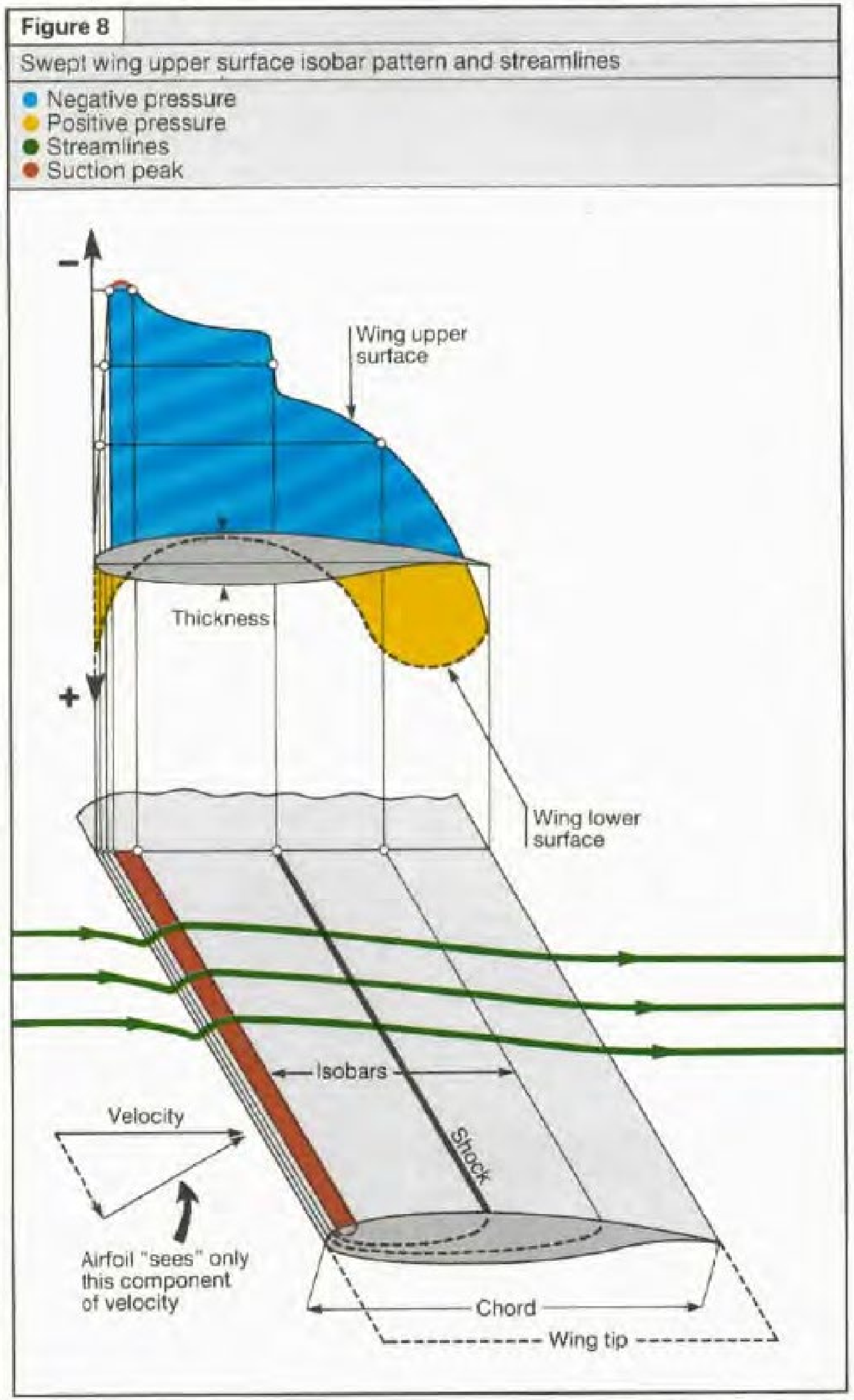
$$C_D = 20(M - M_{crit})^4$$

$$\frac{\partial C_D}{\partial M} = 80(M - M_{crit})^3$$

$$0.1 = 80(M_{dd} - M_{crit})^3$$

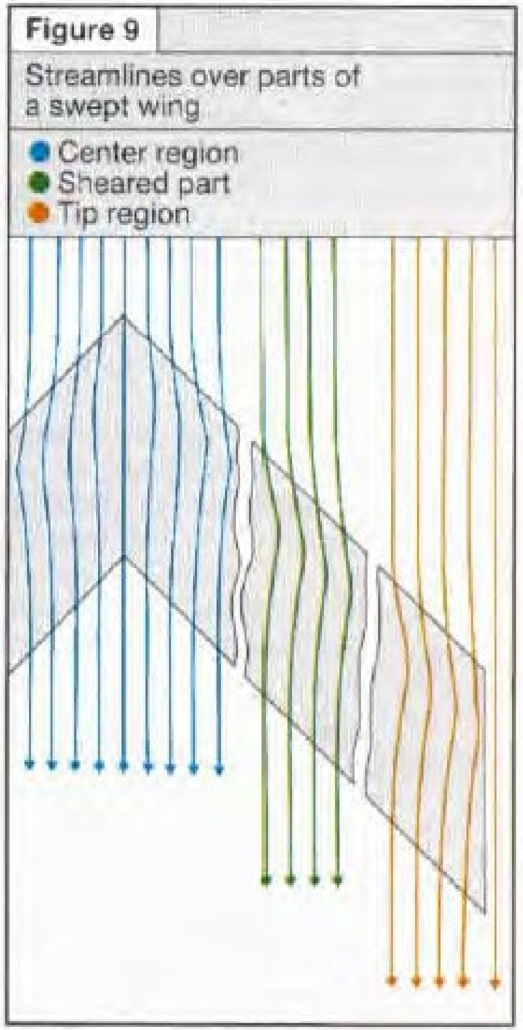
$$M_{crit} = M_{dd} - \left(\frac{0.1}{80}\right)^{\frac{1}{3}}$$

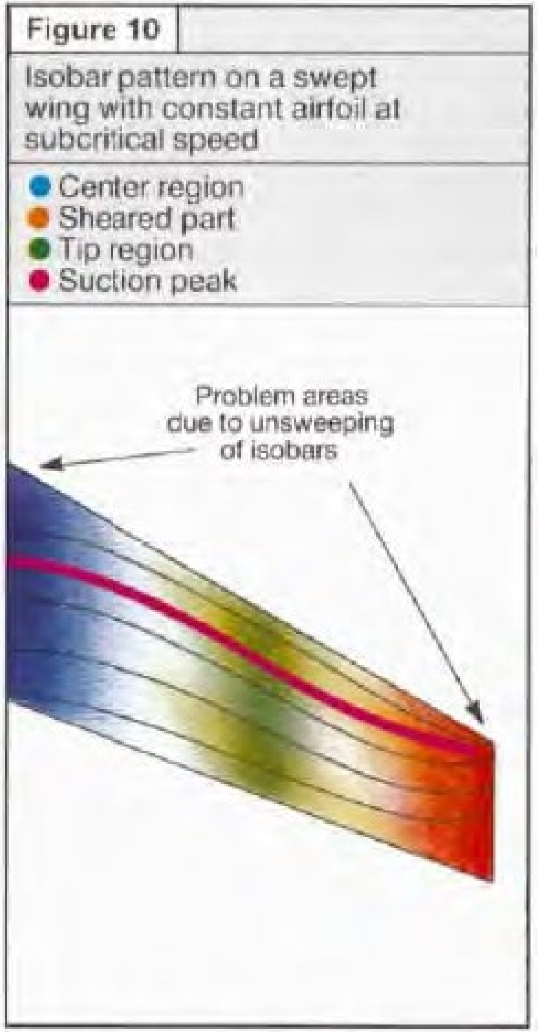




Streamlines pattern on swept wings.

- Influence of staggered sections
- Apex and Tip deviations
- Isobars





COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
MPX5X WING-BODY

REN = 101.00 , CL = 0.610