Developing an Optimal Controller for Energy Minimization of an Electric Car

ME231A/EE220B Fall 2017 Final Project

AUTHORS

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Abstract-Designing energy-efficient optimal controllers for vehicle control is an essential step towards achieving a ubiquitous, environmentally-friendly autonomous transportation model. This study explores the design and implementation of optimal control strategies that minimize the energy used by an electric car traversing a pre-established roadway. The explored control strategies dictate the vehicle's output driving force as a function of its position on the chosen roadway through the minimization of a highly non-linear cost function that is representative of total energy use. The cost function is formulated subject to speed constraints (due to road speed limits), power limitations (due to physical limitations in the motor) and non-linear system dynamics that describe the vehicle's speed as a function of driving force and road position. The study investigates the efficacy of several control strategies based on Dynamic Programming (DP) and Model Predictive Control (MPC) approaches and compares the respective results. This study finally analyzes the feasibility and pragmatism of implementing the above control approaches in various real-world scenarios. A github repo of the code we used can be found here. A summary video has been recorded on youtube

Contents

Purpose of Documentation

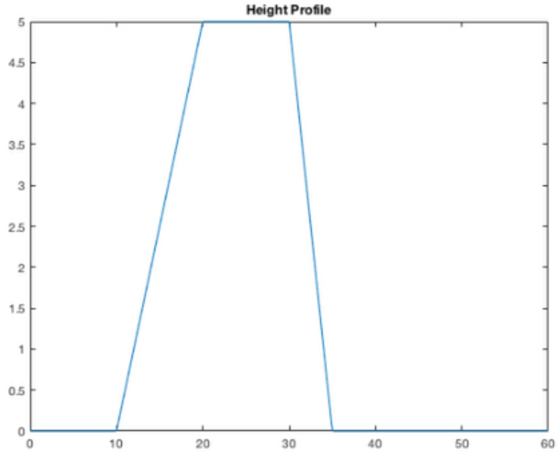
The following documentation is used to highlight the MATLAB methods used for the Final Project Case study. The results and findings of the report are highlighted in a seperate report. Therefore the contents of this report is to purely show the sequence of code used during our project.

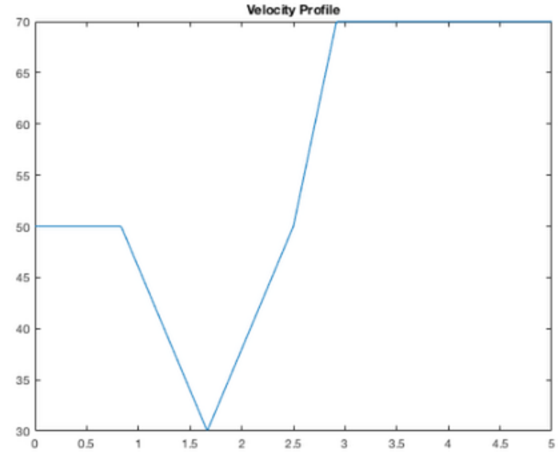
```
clear;clc;close all;
```

Intution For the Problem.

When cars typically go up hills, they tend to slow down, as the force of gravity increasingly becomes a problem. The opposite happes as it rolls down a hill. We started with this intuitive understanding and modeled what our velocity profiles may look like when faced with height variations.

```
dt = 0.01;
T_final = 5;
T0 = 0;
t = T0:dt:T_final;
X = linspace(0,60,numel(t));
Y = zeros(numel(t),1);
theta = size(numel(X),1); % Slope

for i=1:numel(X)
    if 0<=X(i) && X(i)<=10
        Y(i) = 0;
        theta(i) = 0;
        V(i) = 50;
    elseif 10<=X(i) && X(i)<=20
        Y(i) = 0.5*(X(i)-10);
        theta(i) = atan(0.5);
        V(i) = -2*(X(i)-10)+50;
    elseif 20<=X(i) && X(i)<=30
        Y(i) = 5;
        theta(i) = 0;
        V(i) = 2*(X(i)-20)+30;
    elseif 30<=X(i) && X(i)<=35
        Y(i) = -(X(i)-30)+5;
        theta(i) = atan(-1);
        V(i) = 4*(X(i)-30)+50;
    else
        Y(i) = 0;
        theta(i) = 0;
        V(i) = 70;
    end
end

L = 60; % Length of the road
V_mean = L/(T_final-T0);

figure (1)
plot(X,Y)
title('Height Profile')

figure (2)
plot(t,V)
title('Velocity Profile')
```





We developed our initial models based on this intuitive understanding. Using dynamics equations, we proceeded to create the equations which would be used in our study. Much of these models were developed using the midterm examinations found in our class, and credit is soley due to our professor Francesco Borrelli.

The following figure shows a free-body diagram that explains the major forces acting on our vehicle. The vehicle is subject to some dissipative aerodynamic force Fair, rolling resistance Froll, resistance due to the horizontal component of gravity Fg-x and would be propelled forward by the electric motor, providing a force, Fdrive.

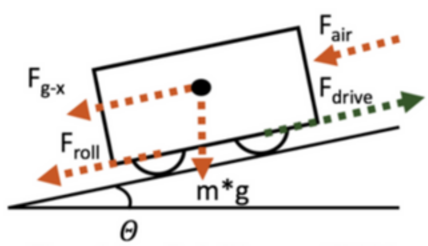


Figure 1: Free-Body Diagram of Vehicle

The system model can be described as follows:

$$M\,\dot{v} = F_{drive} - \left[F_{g-x} + F_{roll} + F_{air}\right]$$

$$\frac{P_{k+1} - P_k}{\Delta t} = v_k$$

$$v_{k+1} - v_k = \Delta t \cdot (\Sigma F / M)$$

Figure 3a

$$J^*(v_0) =$$

$$\min_{F_{\text{drive}}} \sum_{p=0}^{N-1} \left(\frac{F_{\text{drive}} + |F_{\text{drive}}|}{2} + \left(|F_{\text{drive}}| - F_{\text{drive}} \right) \right) \frac{\Delta P}{\eta(F_{\text{drive}}, v_k)}$$

$$\begin{aligned} \text{s.t.} \quad & v_{k+1} = v_k + \frac{\Delta P}{v_k}\left(\frac{\Sigma F}{m}\right) \\ & \Sigma F = A - Bv_k - Cv_k^2 - mg(\sin\theta) + F_{\text{drive}} \\ & \Delta P = p_{k+1} - p_k \\ & v_{k=0} = v_0 \\ & F_{\min} \leq F_{\text{drive}} \leq F_{\max} \\ & v_{\min} \leq v_k \leq v_{\max} \end{aligned}$$

Figure 3b

In order to minimize the amount of energy consumed by our car as it traverses across a roadway, we must minimize the Fdrive of the vehicle. We defined a cost function as follows that reflects this desire to minimize energy:

Creating a Representative Model for Route Planning

The path was created using an online path creator

```
T=gpxread('path.gpx');
states = geoshape(shaperead('usastatehi', 'UseGeoCoords', true));
latlim = [min(T.Latitude) max(T.Latitude)];
lonlim = [min(T.Longitude) max(T.Longitude)];
ocean = [0.7 0.8 1]; land = [0.9 0.9 0.8];
%% Overlay the GPS points on the map.
webmap('World Street Map', 'WrapAround', false)
colors = {'red'};
wmline(T, 'Color', colors)
wmlimits(latlim, lonlim)
%% Overlay the GPS points on the map.
figure
ax = usamap(latlim-[0.01,-.01], lonlim-[.01,-.01]);
setm(ax, 'FFaceColor', ocean)
geoshow(states,'FaceColor',land)
geoshow(T.Latitude,T.Longitude,'DisplayType','Point','Marker','.',...
    'MarkerSize',4,'MarkerEdgeColor',[0 0 1])
title('Trip Data')
xlabel('Bay Area')%%
```

Using this path, the profile for our function was created.

```
function [p,slope,lat,lon,el] = gpxtoPandSlope(filename)
% This function converts the raw path data into usable position and slope.
% It returns p- distance from initial point and slope- sin(theta) at p.
```

```
file=gpxread(filename);          % read the gps file
points=numel(file.Latitude);     % extract number of discrete points by measuring latitude points

lat=file.Latitude;               % read latitudes
lon=file.Longitude;              % read longitudes
el=file.Elevation;               % read elevations
p=zeros(points,1);               % initialize position

noise_slope=zeros(points,1);     % initialize raw slope array
for i=1:points-1
   [a,b]=distance(lat(i+1),lon(i+1),lat(i),lon(i)); % Assigns arclen in degrees and azimuthal to a and b to
   deltaP=a*111120;                                 % Converts arclen into meters
   deltaH=el(i+1)-el(i);                            % Slope
   if deltaP>1                                      % To avoid small deltaP blowing up the slope
        noise_slope(i)= deltaH/sqrt(deltaH^2+deltaP^2); % sin(theta)
   else
        noise_slope(i)=noise_slope(i-1); % Assign slope to the region with small deltaP
   end
        p(i+1)=p(i)+deltaP;             % Increment position
end
slope=medfilt1(noise_slope,3);          % Smoothen slope values
```

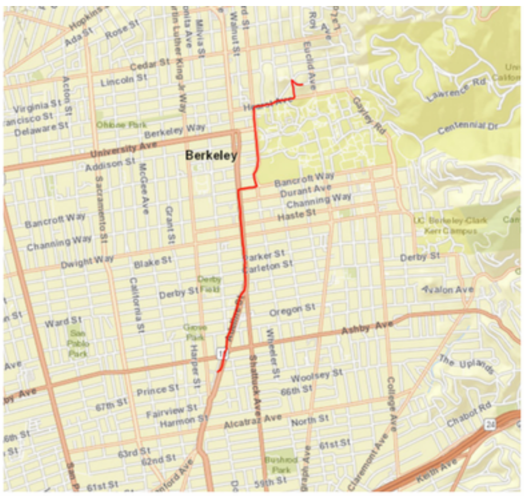


Figure 4a

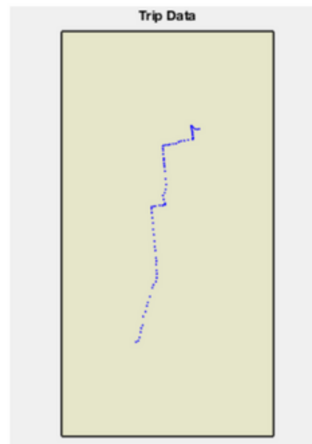


Figure 4b

Developing a DP Controller Model

The DP approach involves discretizing the state (i.e., speed, v), input (i.e., driving force Fdrive), and position spaces to generate a look-up table that correlates the measured speed at a given position to the optimal input. This approach should produce not only the best solution, but rather all the solutions possible.

- **1. Define Parameters**

```
tic
load NewAshbytoBerk
g = 9.8;                          % Gravitational acceleration in m/s^2
M = 2000;                         % Mass of car in kg
A = 200;                          % Rolling resistance coeffcient in N
B = 5.5;                          % Drivetrain losses in Ns/m
C = 0.39;                         % Aerodynamic drag coeffcient in N(s^2)/(m^2)
umax = 880;                       % Max input force in N
umin = -880;                      % Minimum input force (think of this as 'max' breaking, thus negative acceleration and subsequently force) in N;
minspeed = 3;                     % Minimum velocity of car in m/s (~10 km/h);
maxspeed = 30;                    % Maximum velocity of car in m/s (~100 km/h);
R=0.4;                            % Radius of wheel in m
gr=6;                             % Gear ratio
n=4;                              % Number of motors
delta_F= 1*gr/R*n;                % Grid distance force
delta_v= 10*R/gr*2*(pi/60);       % Grid distance velocity
Fgrid= delta_F:delta_F:88*delta_F;
vgrid= delta_v:delta_v:1000*delta_v;
E_grid=load('Motor_data.mat');
E_grid = E_grid.Motor_data;
j = 2;

for i=1:size(E_grid,2)-1
    if i == 1;
    NewEGrid(:,i) = E_grid(:,i);
    end
    if rem(i,10) == 0
    Store = E_grid(:,i);
    NewEGrid(:,j)=Store;
    j = j+1;
    end
end
```

```
E_grid = NewEGrid;
%}
```

% Run the following when including with Efficiency

```
%E_grid = [flipud(E_grid);E_grid(88,:);E_grid];
%Fgrid = [fliplr(Fgrid),0,-Fgrid];
```

- **2. Grid states and inputs**

```
vpoints = 40;
v_sampled = linspace(minspeed,maxspeed,vpoints); % Grid from minspeed to maxspeed
upoints = 30;
u_sampled = linspace(umin,umax,upoints);         % Grid from minimum to max force
%}
```

- **3. Grid position**

```
track_length = profile(end,1);                  % in meters
dp = 10;                                        % Sampling in space
p_sampled = 0:dp:track_length;                  % Sampled train position
N_p = length(p_sampled);
%}
```

- **4. Create Arrays to save cost and inputs**

```
Jsave = cell(N_p,1);
USave = cell(N_p,1);
Jtogo = cell(N_p+1,1);
Uopt = cell(N_p,1);
%}
```

- **5. Define anononymous functions**

```
computeVNext = @(v,u,p) v+dp/(v*M)*(-A-B*v-C*v^2-M*g*slope(p,profile)+u);
efficiency = @(u,v) interpn(Fgrid,vgrid,E_grid,u,v);
Jstage = @(u,v) (((u+abs(u))/2)+(abs(u)-u))*dp
Jtogo{end} = @(x) x*0;
```

Run the following when including with Efficiency

```
% Jstage = @(u,v) (((u+abs(u))/2)+(abs(u)-u))*dp/efficiency(u,v);
% Jstage = @(u,v) abs(u)*dp/efficiency(u,v);
% Jstage = @(u,v) u*dp/efficiency(abs(u),v);
%}
```

- **6. Dynamic Programming**

```
for pIndex = N_p:-1:1
    J = nan(vpoints,1);
    UGR = nan(vpoints,1);
    for i = 1:vpoints
            vt = v_sampled(i) ;
            Jbest = inf;
            Ubest = nan;
        for j=u_sampled
            ut = j;
            vnext = computeVNext(vt,ut,pIndex);
            if vnext<minspeed || vnext>maxspeed
                continue;
            end

            Jt = Jstage(ut,vt)+Jtogo{pIndex+1}(vnext);

            if isnan(Jt)
                continue;
            end
            if Jt < Jbest
                Jbest = Jt;
                Ubest = ut;
            end
        end
        J(i) = Jbest;
        UGR(i) = Ubest;
    end
    Jsave{pIndex} = J;
    USave{pIndex} = UGR;
    Jtogo{pIndex} = @(v) interpn(v_sampled,J,v,'linear');
```

```
    UOpt{pIndex} = @(v) interpn(v_sampled,UGR,v,'linear');
end
save('NegSlope20BrakingSmallMotorAshtoBerk.mat')
%}
```

```
Jstage =

  function_handle with value:

    @(u,v)(((u+abs(u))/2)+(abs(u)-u))*dp
```

- **7. Plotting**

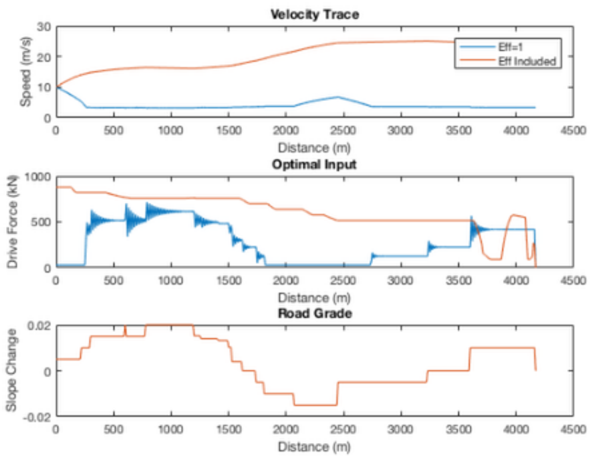
```
load('NegSlope20BrakingSmallMotorAshtoBerk.mat')
V0=10;

VSim = zeros(1,N_p+1);
VSim(:,1) = V0;
USim = zeros(1,N_p);
JSim = zeros(1,N_p);
sl = zeros(1,N_p);

for t = 1:N_p-1
    USim(:,t) = UOpt{t}(VSim(:,t));
    if any(isnan(USim(:,t)))
        disp('infeasible')
    break;
    end
    VSim(:,t+1) = computeVNext(VSim(:,t),USim(:,t),p_sampled(t));
    JSim(t) = Jstage(USim(:,t),VSim(:,t));
    sl(t)=slope(p_sampled(t),profile);
end

figure (5)
subplot(3,1,1)
plot(p_sampled,VSim(1:end-1))
xlabel('Distance (m)')
ylabel('Speed (m/s)')
title('Velocity Trace')
hold on
subplot(3,1,2)
plot(p_sampled,USim);
xlabel('Distance (m)')
ylabel('Drive Force (kN)')
title('Optimal Input')
hold on
subplot(3,1,3)
plot(p_sampled,sl)
xlabel('Distance (m)')
ylabel('Slope Change')
title('Road Grade')
hold on
```

Loading the Efficiency Data

```
load('NegSlope20BrakingSmallMotorAshtoBerkEff.mat')
V0=10;
VSim = zeros(1,N_p+1);
VSim(:,1) = V0;
USim = zeros(1,N_p);
JSim = zeros(1,N_p);
sl = zeros(1,N_p);

for t = 1:N_p-1
    USim(:,t) = UOpt{t}(VSim(:,t));
    if any(isnan(USim(:,t)))
        disp('infeasible')
    break;
    end
    VSim(:,t+1) = computeVNext(VSim(:,t),USim(:,t),p_sampled(t));
    JSim(t) = Jstage(USim(:,t),VSim(:,t));
    sl(t)=slope(p_sampled(t),profile);
end
JSum = sum(JSim);
subplot(3,1,1)
plot(p_sampled,VSim(1:end-1))
xlabel('Distance (m)')
ylabel('Speed (m/s)')
title('Velocity Trace')
legend ('Eff=1','Eff Included')
subplot(3,1,2)
plot(p_sampled,USim);
xlabel('Distance (m)')
ylabel('Drive Force (kN)')
title('Optimal Input')
```

```
subplot(3,1,3)
plot(p_sampled,sl)
xlabel('Distance (m)')
ylabel('Slope Change')
title('Road Grade')
%toc;
fprintf('Elapsed time is 177.555808 seconds.')
hold off
```

```
Elapsed time is 177.555808 seconds.
```



Developing an MPC Controller Model

The MPC approach first solves an optimal control problem over a chosen prediction horizon given a measured speed at a position. The solution to this optimal control problem is a sequence of optimal inputs over the prediction horizon. The first input in this sequence is then applied to the system and the new state of the system becomes the initial state for the next optimal control problem. This pattern is then repeated over the entire length of the chosen roadway

- **1. Define Parameters**

load NewAshbytoBerk.mat profile=[po,slo];

```
track_length=profile(end,1);
dp=10;
p_s=0:dp:track_length;
Mpc=numel(p_s); % MPC Horizon
N=3; % Batch Horizon
v0=10;
vOpt(1)=v0;
objective=0;
g = 9.8; % gravitational acceleration in m/s^2
M = 2000; % mass of car in kg
A = 200; % rolling resistance coeffcient in N
B = 5.5; % drivetrain losses in Ns/m
C = 0.39; % aerodynamic drag coeffcient in N(s^2)/(m^2)
R=0.4; %radius of wheel in m
gr=6;  %gear ratio
n=4;   %number of motors
```

- **2. Efficiency Values**

```
E_grid=load('Motor_data.mat');
E_grid = E_grid.Motor_data;
delta_F= 1*gr/R*n; % grid distance force
delta_v= 1*R/gr*2*(pi/60); % grid distance velocity
Fgrid= delta_F:delta_F:88*delta_F;
vgrid= delta_v:delta_v:10000*delta_v;
```

- **3. Batch Approach Function**

```
function [feas vOpt uOpt]= car_batch(N, v0,p_sampled,dp,vgrid,Fgrid,E_grid,profile)
%clear all;

%% Define parameters
%load TrainDataNew
g = 9.8; % gravitational acceleration in m/s^2
M = 2000; % mass of car in kg
A = 200; % rolling resistance coeffcient in N
B = 5.5; % drivetrain losses in Ns/m
C = 0.39; % aerodynamic drag coeffcient in N(s^2)/(m^2)
R=0.4; %radius of wheel in m
```

```
gr=6;  %gear ratio
n=4;   %number of motors

%% function arguments
vMin= 3; %m
vMax= 30; %m
%%
% Define state matrix(velocity)
v= sdpvar(1,N+1);
% Define decision variables
u = sdpvar(1,N);

% Define objective function and constraints
uMin = -880; % N
uMax = 880; % N

% Define anonymous function
computeVNext = @(v,u,p) v+dp/(v*M)*(-A-B*v-C*v^2-M*g*slope(p,profile)+u);
objective=0;
constraints=[];


efficiency = @(f,v) interp2(Fgrid,vgrid,E_grid,f,v);

%% run optimization
for i=1:N
    %objective= objective + ((u(i)+abs(u(i)))/2+(abs(u(i))-u(i)))*dp/efficiency(u(i),v(i));
   %objective= objective + ((u(i)+abs(u(i)))/2+(abs(u(i))-u(i)))*dp;
    objective= objective + u(i)*dp;

    constraints = [constraints vMin<=v(i)<=vMax uMin<=u(i)<=uMax v(i+1)==computeVNext(v(i),u(i),p_sampled(i))];
    sl(i)=slope(p_sampled(i),profile);
end
constraints = [constraints v(1)==v0];

% Set options for YALMIP and solver
options = sdpsettings('verbose',0,'solver','fmincon','usex0',0,'cachesolvers', 1);

% Solve
sol = optimize(constraints, objective,options);

feas=sol.problem;
vOpt=value(v);
uOpt=value(u);
end
```

- **4. MPC Controller**

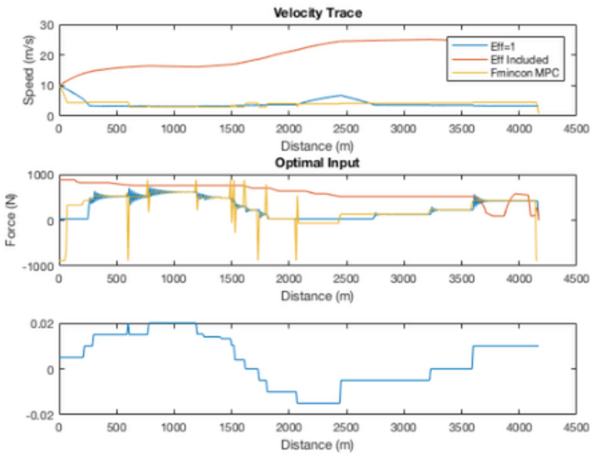
```
for i=1:Mpc
    if i<Mpc-N
        p_sampled= p_s(i:i+N);
        [feas(i), vOpt_ol, uOpt_ol] = car_batch(N,v0,p_sampled,dp,vgrid,Fgrid,E_grid,profile);
        vOpt(i+1)=vOpt_ol(2);
        uOpt(i)=uOpt_ol(1);
        v0=vOpt(i+1);

        if feas > 0
            break;
        end
    end
    if i==Mpc-N
        p_sampled=p_s(i:i+N)
        [feas(i), vOpt_ol, uOpt_ol] = car_batch(N,v0,p_sampled,dp,vgrid,Fgrid,E_grid,profile);
        vOpt(i+1:i+N)=vOpt_ol(2:end);
        uOpt(i:i+N-1)=uOpt_ol;
    end
       sl(i)=slope(p_s(i),profile);
       if i<Mpc
           objective= objective + ((uOpt(i)+abs(uOpt(i)))/2+ (abs(uOpt(i))-uOpt(i)))*dp;
       end

end
```

- **5. Plotting**

```
subplot(3,1,1)
plot(p_s(1:Mpc),vOpt(1:Mpc))
legend ('Eff=1','Eff Included','Fmincon MPC')
ylabel('Speed (m/s)')
subplot(3,1,2)
ylabel('Force (N)')
plot(p_s(1:Mpc-1),uOpt(1:Mpc-1))
subplot(3,1,3)
ylabel('Slope')
```

```
plot(p_s(1:Mpc),sl(1:Mpc))
xlabel('Distance (m)')
```

```
p_sampled =

        4140        4150        4160        4170
```



The optimal input and state trajectories for the three scenarios outlined above are shown in the top two graphs in Figure 4 below. The bottom graph demonstrates the changes in slope as a function of position along the chosen pathway. Scenario #1 (DP control approach that properly accounts for efficiency) yields expected results, as the speed almost inversely scales with slope, and the upper constraints on speed limits are easily met. Scenarios #2-3 on the other hand (i.e., MPC and DP assuming η= 1) demonstrate that incorporating the powertrain efficiency function described earlier significantly changes the optimal speed profile. As seen from Fig. 2, lower input torque and lower motor speeds yield lower efficiency values. As a result, low speeds and low driving force do not necessarily imply minimum cost because efficiency penalizes the cost if the driving force and the speed are low. Physically, this means that going too slow may not be the best way to save energy.

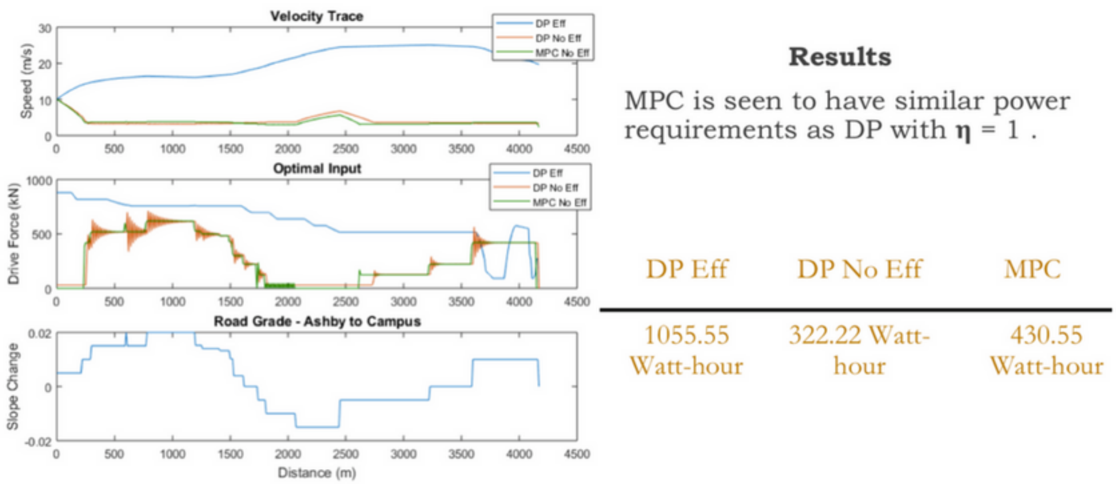


Figure 7 - Using MPC IPOPT Method

Attribution

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```
% | 09/09/17 | Class # 39895
```

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