hw1

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Contents

Set a seed to ensure reproducibility of results.

```
library(ISLR)
library(glmnet)
library(caret)
library(tidymodels)
library(corrplot)
library(ggplot2)
library(plotmo)
library(ggrepel)
library(pls)
```

```
training <- read.csv("housing_training.csv")
testing <- read.csv("housing_test.csv")
```

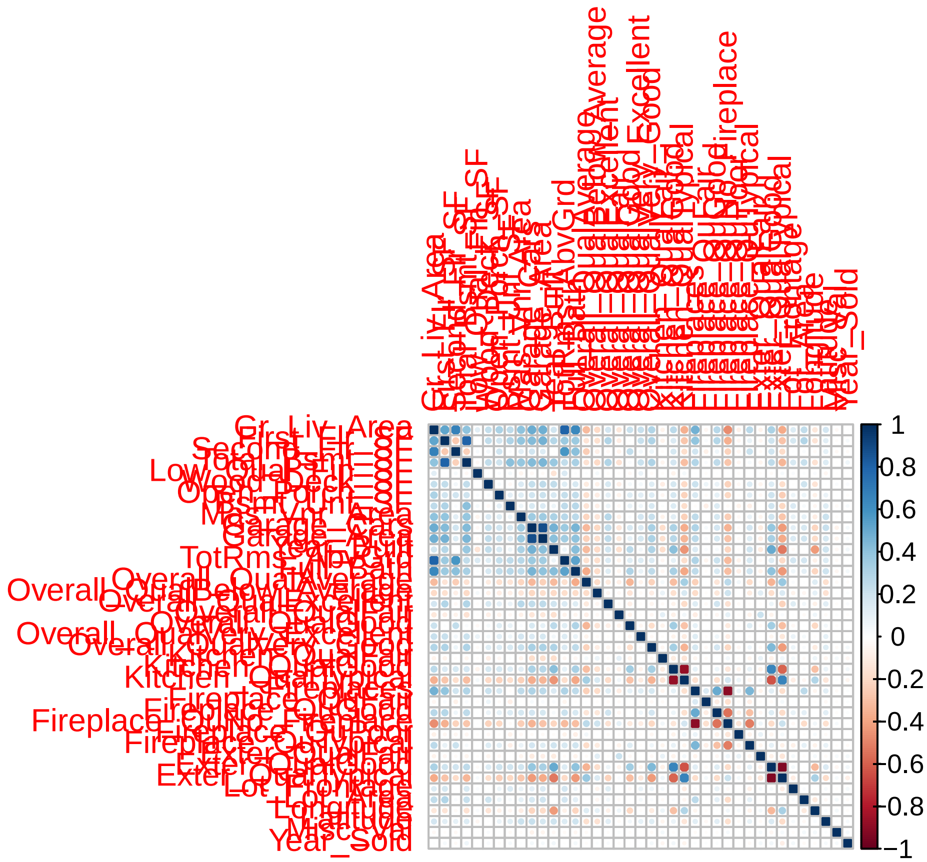
Question a

We will be using the `caret` metaengine to conduct a 10-fold cross-validation of a lasso regression on the training dataset. We will also need to configure the predictors as a matrix, the response variable as a vector, and the testing data as a matrix without the response variable.

```
predictors <- model.matrix(Sale_Price ~ ., training)[, -1]
response <- training[, "Sale_Price"]
test <- model.matrix(Sale_Price ~ ., testing)[, -1]
```

Before fitting a model, we should also check for correlations between predictors, which may cause problems with lasso regression.

```
corrplot::corrplot(
  cor(predictors),
  method = "circle",
  type = "full"
)
```



There are some predictors which are correlated with each other, such as `Total_Bsmt_SF` and `First_Flr_SF`, `Second_Flr_SF` and `Gr_Liv_Area`, `Kitchen_QualTypical` and `Kitchen_QualGood`, `Fireplaces` and `Fireplace_QuNo_FirePlace`.

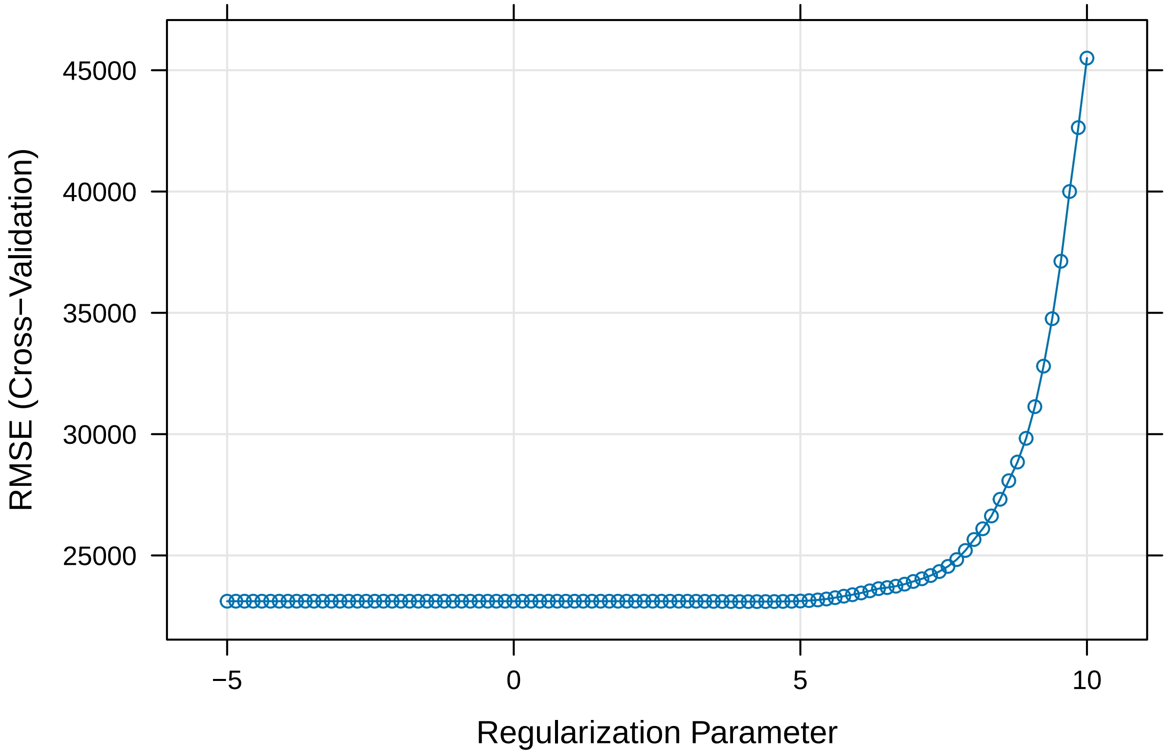
For 10-fold cross validation, we need to use `trainControl()` to specify control parameters, i.e. specifying the resampling method we are using and the number of folds - in this case, cross-validation and 10, respectively. The formula results are stored in `ctrl1`.

We then pass this to the `train()` function, along with the desired model statement as a formula (`Sale_Price ~ .`), the training dataset, `alpha = 1` for lasso regression, and a corresponding lambda grid hopefully wide enough to capture the optimal lambda value.

```
ctrl1 <- trainControl(method = "cv", number = 10)

lasso_caret <-
  train(
    Sale_Price ~.,
    data = training,
    method = "glmnet",
    tuneGrid = expand.grid(
      alpha = 1,
      lambda = exp(seq(10, -5, length = 100))
    ),
    trControl = ctrl1
  )
```

```r
plot(lasso_caret, xTrans = log)
```




```r
lasso_caret$bestTune
```

```
##    alpha   lambda
## 61     1 59.79423
```

```r
coef(lasso_caret$finalModel, lasso_caret$bestTune$lambda)
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                s1
## (Intercept)         -4.840779e+06
## Gr_Liv_Area          6.546330e+01
## First_Flr_SF         7.987905e-01
## Second_Flr_SF        .
## Total_Bsmt_SF        3.540118e+01
## Low_Qual_Fin_SF     -4.102390e+01
## Wood_Deck_SF         1.166675e+01
## Open_Porch_SF        1.549835e+01
## Bsmt_Unf_SF         -2.088901e+01
## Mas_Vnr_Area         1.086359e+01
## Garage_Cars          4.095060e+03
## Garage_Area          8.138121e+00
## Year_Built           3.234957e+02
```

```
## TotRms_AbvGrd               -3.635125e+03
## Full_Bath                   -3.880963e+03
## Overall_QualAverage         -4.868338e+03
## Overall_QualBelow_Average  -1.248694e+04
## Overall_QualExcellent       7.529368e+04
## Overall_QualFair           -1.078634e+04
## Overall_QualGood            1.213551e+04
## Overall_QualVery_Excellent  1.353248e+05
## Overall_QualVery_Good       3.790245e+04
## Kitchen_QualFair           -2.498642e+04
## Kitchen_QualGood           -1.733299e+04
## Kitchen_QualTypical        -2.543213e+04
## Fireplaces                  1.061166e+04
## Fireplace_QuFair           -7.682725e+03
## Fireplace_QuGood            .
## Fireplace_QuNo_Fireplace    1.548227e+03
## Fireplace_QuPoor           -5.656288e+03
## Fireplace_QuTypical        -7.013339e+03
## Exter_QualFair             -3.368377e+04
## Exter_QualGood             -1.541581e+04
## Exter_QualTypical          -1.984825e+04
## Lot_Frontage                9.987240e+01
## Lot_Area                    6.043141e-01
## Longitude                  -3.308279e+04
## Latitude                    5.539552e+04
## Misc_Val                    8.359457e-01
## Year_Sold                  -5.671644e+02
```

Based on the CV RMSE, the optimal lambda that minimizes the CV RMSE is 59.7942254, which corresponds to 4.0909091 on the graph.

Excluding the intercept, there are 37 predictors included in this final model.

```r
lasso_caret_pred <- predict(lasso_caret, newdata = testing)

lasso_caret_testerror <- mean((lasso_caret_pred - testing[, "Sale_Price"])^2)
```

The test error for the selected lasso regression model is 4.4052006×10^8.

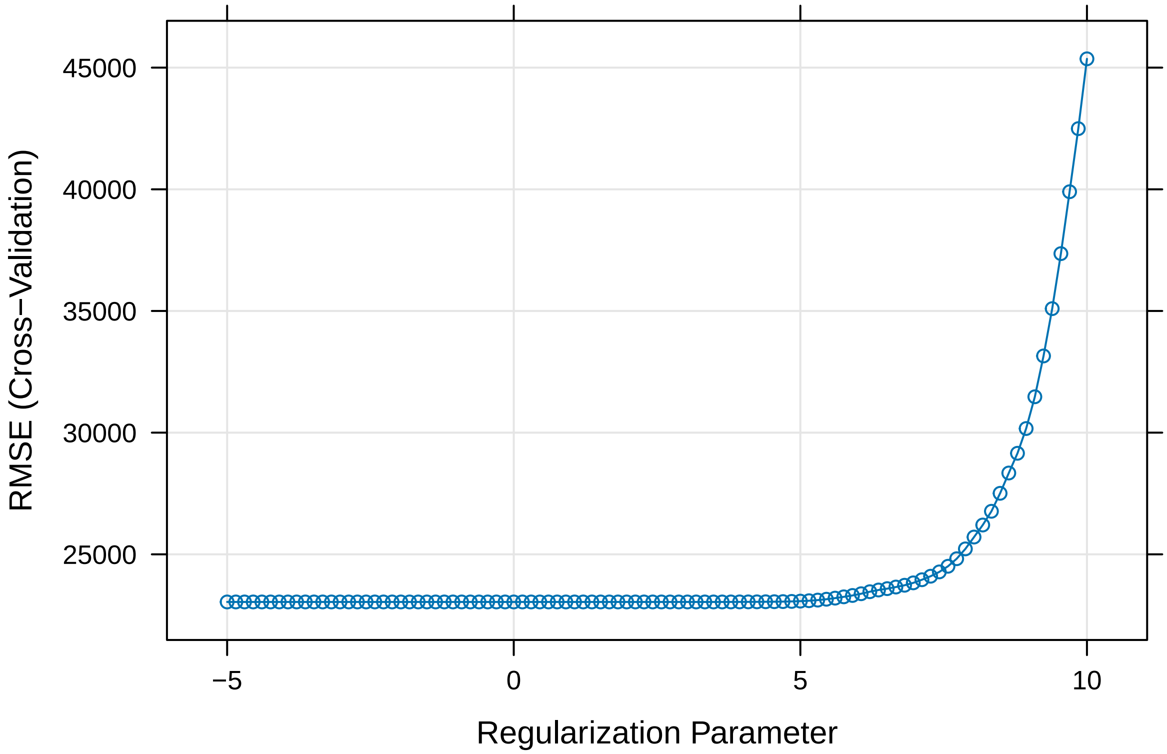
For `caret`, to get the lambda within the 1 SE rule, a different trainControl object needs to be initialized, with `oneSE` specified for the `selectionFunction` argument.

```r
ctrl2 <- trainControl(method = "cv", number = 10, selectionFunction = "oneSE")

lasso_caret_1se <-
  train(
    Sale_Price ~.,
    data = training,
    method = "glmnet",
    tuneGrid = expand.grid(
      alpha = 1,
      lambda = exp(seq(10, -5, length = 100))
    ),
    trControl = ctrl2
```

```r
  )

plot(lasso_caret_1se, xTrans = log)
```




```r
lasso_caret_1se$bestTune
```

```
##    alpha   lambda
## 79     1 914.3211
```

```r
coef(lasso_caret_1se$finalModel, lasso_caret_1se$bestTune$lambda)
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                  s1
## (Intercept)                 -1.833645e+06
## Gr_Liv_Area                  5.590387e+01
## First_Flr_SF                 1.159582e+00
## Second_Flr_SF                .
## Total_Bsmt_SF                3.675634e+01
## Low_Qual_Fin_SF             -2.442870e+01
## Wood_Deck_SF                 8.139740e+00
## Open_Porch_SF                7.319679e+00
## Bsmt_Unf_SF                 -1.911105e+01
## Mas_Vnr_Area                 1.434249e+01
## Garage_Cars                  3.083568e+03
```

```
## Garage_Area                  1.153597e+01
## Year_Built                   3.161492e+02
## TotRms_AbvGrd               -9.559177e+02
## Full_Bath                    .
## Overall_QualAverage         -2.939069e+03
## Overall_QualBelow_Average   -8.768967e+03
## Overall_QualExcellent        8.955958e+04
## Overall_QualFair            -5.666292e+03
## Overall_QualGood             9.534028e+03
## Overall_QualVery_Excellent   1.590825e+05
## Overall_QualVery_Good        3.571584e+04
## Kitchen_QualFair            -4.705230e+03
## Kitchen_QualGood             .
## Kitchen_QualTypical         -9.745366e+03
## Fireplaces                   6.594120e+03
## Fireplace_QuFair             .
## Fireplace_QuGood             4.550964e+03
## Fireplace_QuNo_Fireplace     .
## Fireplace_QuPoor             .
## Fireplace_QuTypical          .
## Exter_QualFair              -1.400666e+04
## Exter_QualGood               .
## Exter_QualTypical           -5.226267e+03
## Lot_Frontage                 6.640048e+01
## Lot_Area                     5.492446e-01
## Longitude                   -7.796931e+03
## Latitude                     1.250104e+04
## Misc_Val                     .
## Year_Sold                    .
```

This generates a lambda of 914.3210959. In this model, there are 29 predictors included, excluding the intercept.

Question b

With elastic net, the **alpha** argument should be supplied a sequence of values from 0 to 1, being in between the extremes of 0 and 1, representing ridge and lasso regression.

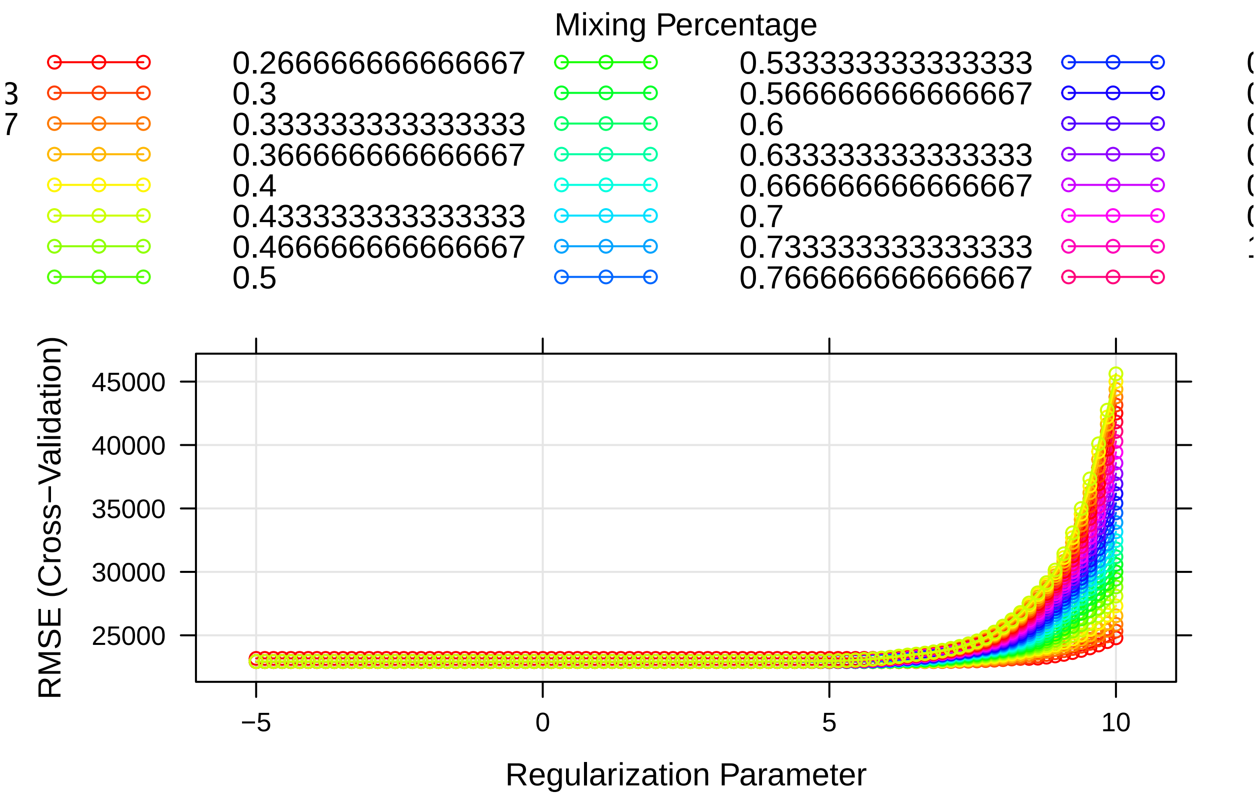
```
elastic_caret <-
  train(
    Sale_Price ~.,
    data = training,
    method = "glmnet",
    tuneGrid = expand.grid(
      alpha = seq(0, 1, length = 31),
      lambda = exp(seq(10, -5, length = 100))
    ),
    trControl = ctrl1
  )

elastic_caret$bestTune
```

```
##          alpha   lambda
## 176 0.03333333 580.3529
```

```r
myCol <- rainbow(25)
myPar <- list(superpose.symbol = list(col = myCol),
              superpose.line = list(col = myCol))

plot(elastic_caret, par.settings = myPar, xTrans = log)
```




```r
coef(elastic_caret$finalModel, elastic_caret$bestTune$lambda)
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                   s1
## (Intercept)            -5.144924e+06
## Gr_Liv_Area             3.808779e+01
## First_Flr_SF            2.746150e+01
## Second_Flr_SF           2.624387e+01
## Total_Bsmt_SF           3.491390e+01
## Low_Qual_Fin_SF        -1.524123e+01
## Wood_Deck_SF            1.238235e+01
## Open_Porch_SF           1.698967e+01
## Bsmt_Unf_SF            -2.072841e+01
## Mas_Vnr_Area            1.161738e+01
## Garage_Cars             4.060711e+03
## Garage_Area             8.855567e+00
```

```
## Year_Built                  3.193289e+02
## TotRms_AbvGrd              -3.467301e+03
## Full_Bath                  -3.750131e+03
## Overall_QualAverage        -5.139922e+03
## Overall_QualBelow_Average  -1.275582e+04
## Overall_QualExcellent       7.555666e+04
## Overall_QualFair           -1.153906e+04
## Overall_QualGood            1.201138e+04
## Overall_QualVery_Excellent  1.359035e+05
## Overall_QualVery_Good       3.769049e+04
## Kitchen_QualFair           -2.394460e+04
## Kitchen_QualGood           -1.632514e+04
## Kitchen_QualTypical        -2.436249e+04
## Fireplaces                  1.095271e+04
## Fireplace_QuFair           -7.791909e+03
## Fireplace_QuGood            2.577608e+02
## Fireplace_QuNo_Fireplace    2.102623e+03
## Fireplace_QuPoor           -5.739219e+03
## Fireplace_QuTypical        -6.870557e+03
## Exter_QualFair             -3.342600e+04
## Exter_QualGood             -1.499088e+04
## Exter_QualTypical          -1.957084e+04
## Lot_Frontage                1.005810e+02
## Lot_Area                    6.034191e-01
## Longitude                  -3.549894e+04
## Latitude                    5.828185e+04
## Misc_Val                    8.828488e-01
## Year_Sold                  -5.856503e+02
```

The lambda which minimizes the cross validation MSE is 580.3528982. This model has all 39 predictors (excluding the intercept).

```r
elastic_caret_pred <- predict(elastic_caret, newdata = testing)

elastic_caret_testerror <- mean((elastic_caret_pred - testing[, "Sale_Price"])^2)
```

The test error from this optimal elastic net model is 4.3937318×10^8.

Unlike lasso, the 1SE rule is not applicable to elastic net regression. This is because elastic net has two different regularization parameters, alpha (the mixing of the two penalties), and lambda itself. The premise of the 1SE rule then becomes arbitrary when there are more than 1 regularization parameters that the 1SE can refer to.

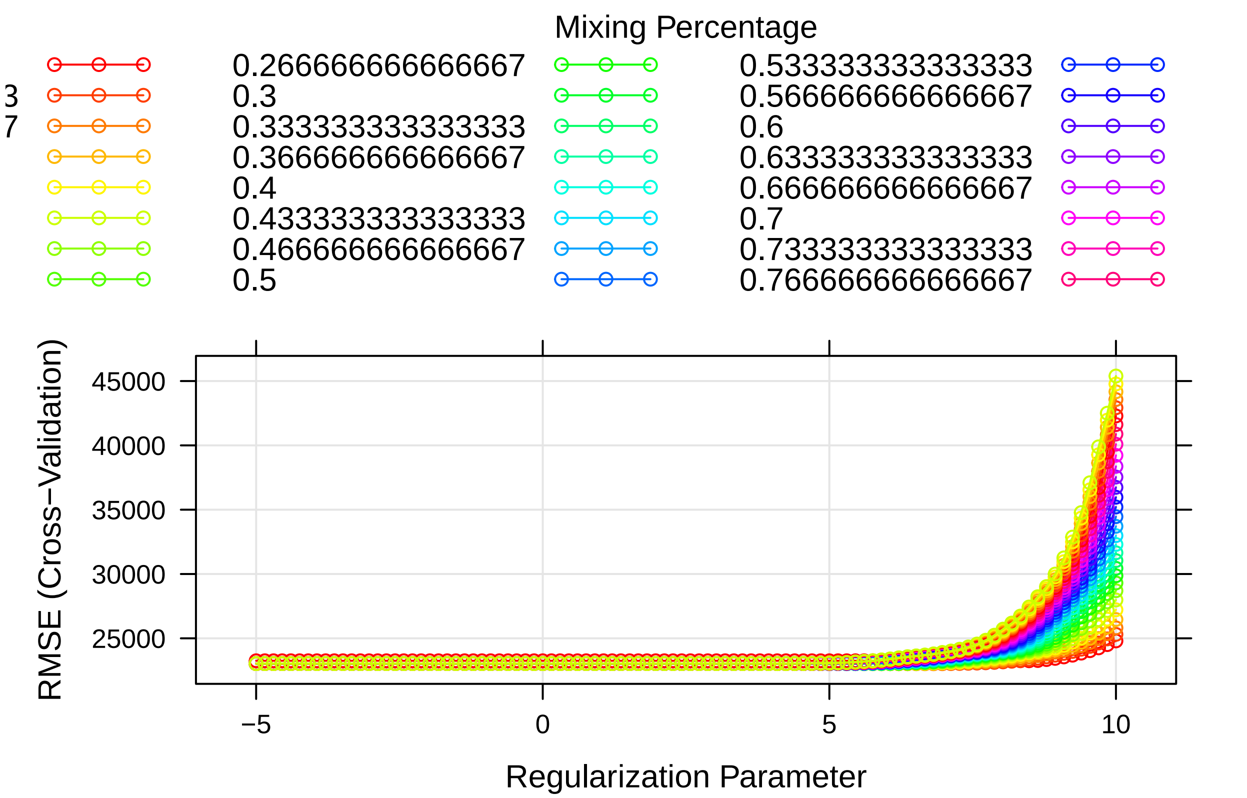
```r
elastic_caret_1se <-
  train(
    Sale_Price ~.,
    data = training,
    method = "glmnet",
    tuneGrid = expand.grid(
      alpha = seq(0, 1, length = 31),
      lambda = exp(seq(10, -5, length = 100))
    ),
    trControl = ctrl2
  )
```

```r
elastic_caret_1se$bestTune
```

```
##    alpha   lambda
## 93     0 7626.573
```

```r
myCol <- rainbow(25)
myPar <- list(superpose.symbol = list(col = myCol),
              superpose.line = list(col = myCol))

plot(elastic_caret_1se, par.settings = myPar, xTrans = log)
```




```r
#coef(elastic_caret_1se$finalModel, elastic_caret_1se$bestTune$lambda)

elastic_caret_1se_pred <- predict(elastic_caret, newdata = testing)

elastic_caret_1se_testerror <- mean((elastic_caret_pred - testing[, "Sale_Price"])^2)
```

The test error when the 1SE rule is applied is 4.3937318×10^8.

Question c

```r
pls_caret <-
  train(
    predictors, response,
    method = "pls",
    tuneGrid = data.frame(ncomp = 1:19),
    trControl = ctrl1,
    preProcess = c("center", "scale")
  )

pls_caret_pred <- predict(pls_caret, newdata = test)

pls_caret_testerror <- mean((pls_caret_pred - testing[, "Sale_Price"])^2)
```

The test error for the optimal partial least squares model is 4.4962272×10^8.

```r
coef(pls_caret$finalModel, pls_caret$bestTune$ncomp)
```

```
## , , 12 comps
##
##                                .outcome
## Gr_Liv_Area                 18756.88537
## First_Flr_SF                11050.66463
## Second_Flr_SF               11959.73073
## Total_Bsmt_SF               14259.35013
## Low_Qual_Fin_SF              -615.81992
## Wood_Deck_SF                 1654.78686
## Open_Porch_SF                1147.52758
## Bsmt_Unf_SF                 -8636.78765
## Mas_Vnr_Area                 1718.35603
## Garage_Cars                  3539.98870
## Garage_Area                  1118.94737
## Year_Built                   9634.87380
## TotRms_AbvGrd               -6198.15321
## Full_Bath                   -2442.38900
## Overall_QualAverage         -2498.87545
## Overall_QualBelow_Average   -3475.02136
## Overall_QualExcellent       12344.65239
## Overall_QualFair            -1460.56548
## Overall_QualGood             4817.44603
## Overall_QualVery_Excellent  12624.88913
## Overall_QualVery_Good       11487.67494
## Kitchen_QualFair            -3416.40799
## Kitchen_QualGood            -9420.46352
## Kitchen_QualTypical        -13528.39030
## Fireplaces                   7648.59007
## Fireplace_QuFair            -1436.27750
## Fireplace_QuGood              -70.35994
## Fireplace_QuNo_Fireplace     1601.09601
## Fireplace_QuPoor             -806.93544
## Fireplace_QuTypical         -3043.58467
## Exter_QualFair              -3315.91744
## Exter_QualGood              -7309.97106
```

```
## Exter_QualTypical           -9510.00921
## Lot_Frontage                 3320.00348
## Lot_Area                     4977.45033
## Longitude                    -982.24036
## Latitude                     1139.65155
## Misc_Val                      523.60220
## Year_Sold                    -725.60818
```

This optimal PLR model also has 39 predictors, excluding the intercept.

Question d

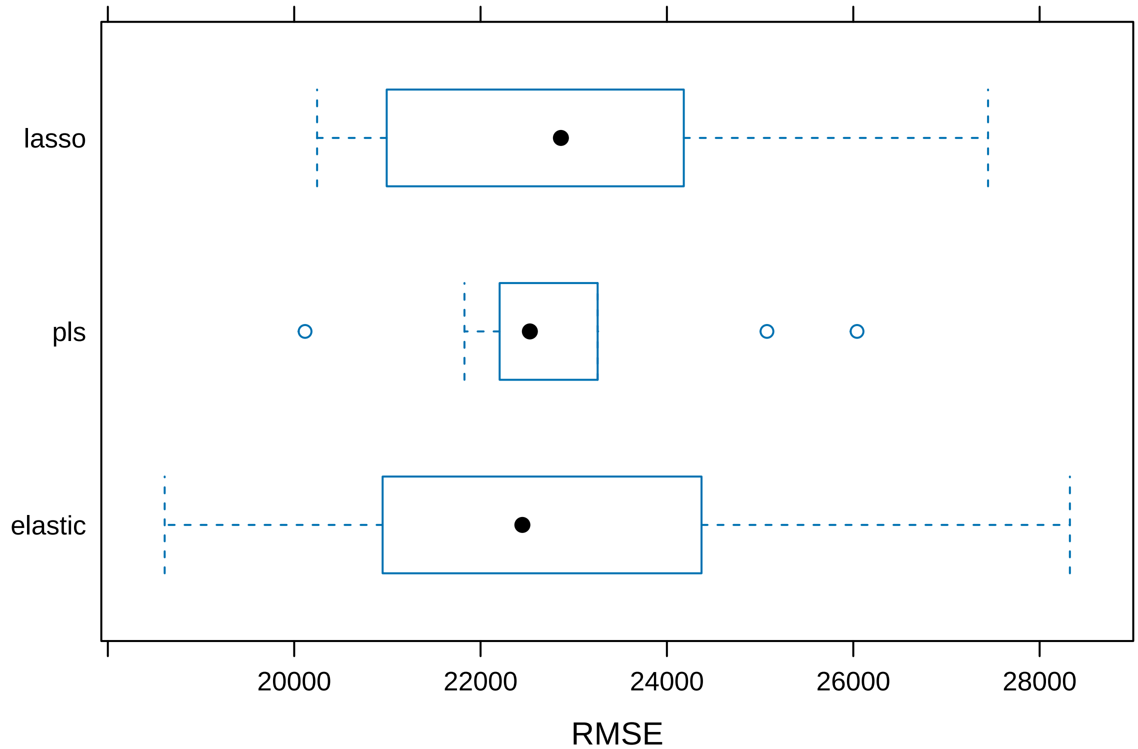
The best model to predict the response is the model with the lowest training error - which is lasso regression. Applied to the training data, the lasso model has a mean RMSE of 22709.16, while partial least squares has a mean RMSE of 22909.59, and elastic has a mean RMSE of 23001.77. Therefore, the best model is the lasso regression model.

```
resamp <- resamples(
  list(lasso = lasso_caret, elastic = elastic_caret, pls = pls_caret)
)

summary(resamp)
```

```
##
## Call:
## summary.resamples(object = resamp)
##
## Models: lasso, elastic, pls
## Number of resamples: 10
##
## MAE
##             Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## lasso   15033.80 15597.48 16602.06 16703.33 17525.28 18919.79    0
## elastic 13900.98 16057.78 16439.93 16669.51 17507.72 19969.15    0
## pls     14996.23 16376.26 16694.34 16679.54 17102.06 18829.17    0
##
## RMSE
##             Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## lasso   20245.70 21099.59 22862.38 23098.46 23993.08 27446.44    0
## elastic 18609.90 21213.61 22448.86 22921.75 24324.40 28324.74    0
## pls     20116.59 22214.73 22529.29 22903.31 23244.87 26040.55    0
##
## Rsquared
##              Min.   1st Qu.    Median      Mean   3rd Qu.      Max. NA's
## lasso   0.8611835 0.8902872 0.9016153 0.9016068 0.9178027 0.9266175    0
## elastic 0.8537776 0.8958124 0.9063268 0.9038439 0.9193077 0.9339173    0
## pls     0.8791265 0.8933876 0.9050800 0.9020743 0.9128534 0.9187453    0
```

```
bwplot(resamp, metric = "RMSE")
```



Question e

We will be using `glmnet` to fit a lasso model on the training data.

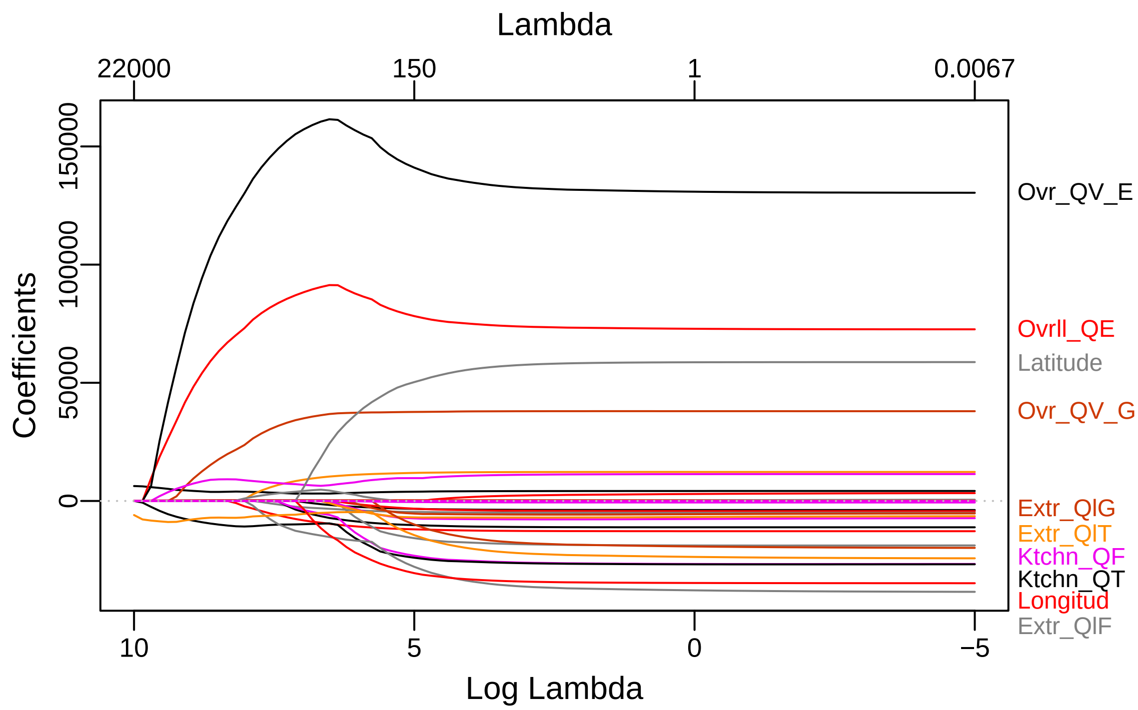
Fitting the lasso model with `glmnet` requires `alpha` to be 1 for lasso regression, and passing a sequence of `lambda` values to hopefully capture the optimal lambda. The lambda values must be in descending order, hence the initial sequence value of 10 and the terminal sequence value of -20.

```
lasso_glmnet <-
  glmnet(
    predictors,
    response,
    alpha = 1,
    lambda = exp(seq(10, -5, length = 100))
)

mat.coef <- coef(lasso_glmnet)
dim(mat.coef)
```

```
## [1]  40 100
```

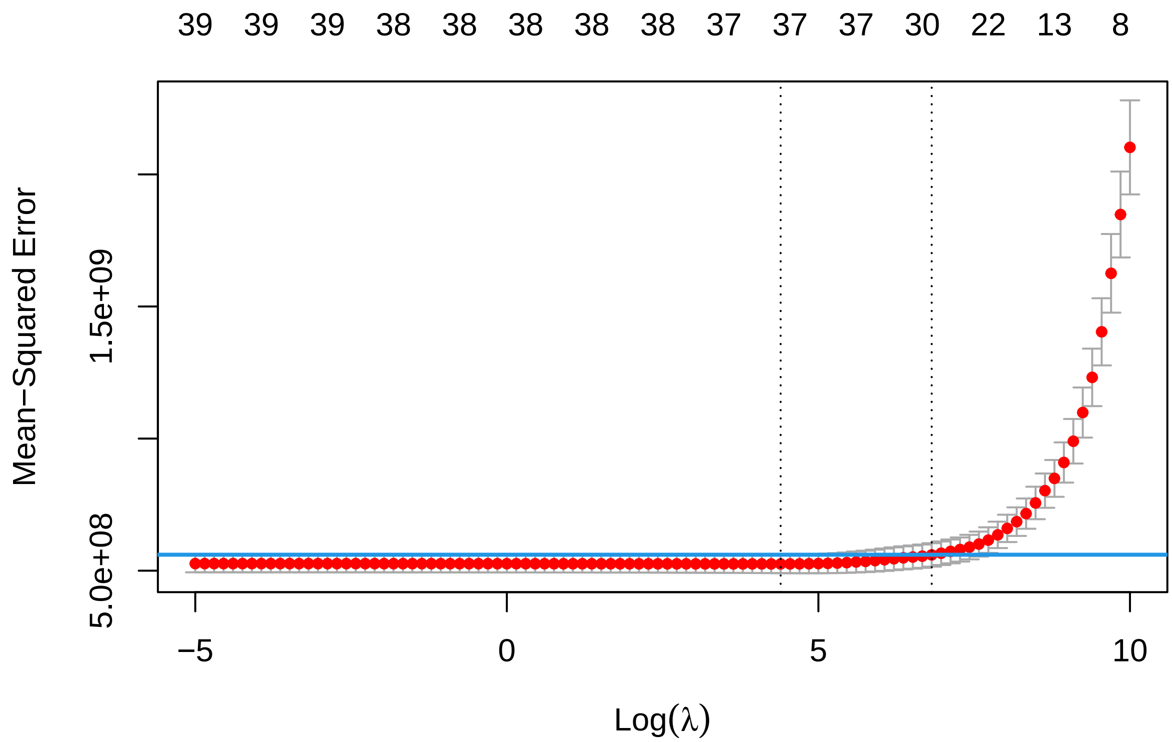
```
plot_glmnet(lasso_glmnet)
```



Use 10-fold cross validation to determine optimal lambda and regression parameters.

```
lasso_glmnet_cv <-
  cv.glmnet(
    predictors,
    response,
    alpha = 1,
    lambda = exp(seq(10, -5, length = 100))
  )

plot(lasso_glmnet_cv)
abline(h = (lasso_glmnet_cv$cvm + lasso_glmnet_cv$cvsd)[which.min(lasso_glmnet_cv$cvm)], col = 4, lwd =
```




```
lasso_glmnet_cv$lambda.1se
```

```
## [1] 914.3211
```

```
lasso_glmnet_cv$cvm[which.min(lasso_glmnet_cv$cvm)]
```

```
## [1] 525595138
```

The lambda value, our tuning parameter, that minimizes MSE is given by 80.9587199, and the smallest lambda value within 1 SE of the MSE is given by 914.3210959. With the lambda that minimizes MSE, the test error is 5.2559514×10^8.

To get the parameters of the model which minimizes MSE, we need to pass the corresponding lambda, `lasso_glmnet_cv$lambda_min`, to the `s` argument of `predict()`.

```
predict(lasso_glmnet_cv, s = lasso_glmnet_cv$lambda.min, type = "coefficients")
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                     s1
## (Intercept)                -4.776647e+06
## Gr_Liv_Area                 6.523458e+01
## First_Flr_SF                8.099936e-01
## Second_Flr_SF               .
## Total_Bsmt_SF               3.546165e+01
```

```
## Low_Qual_Fin_SF            -4.077091e+01
## Wood_Deck_SF                1.153538e+01
## Open_Porch_SF               1.525018e+01
## Bsmt_Unf_SF                -2.086593e+01
## Mas_Vnr_Area                1.096835e+01
## Garage_Cars                 4.062059e+03
## Garage_Area                 8.239439e+00
## Year_Built                  3.234063e+02
## TotRms_AbvGrd              -3.571120e+03
## Full_Bath                  -3.756191e+03
## Overall_QualAverage        -4.805719e+03
## Overall_QualBelow_Average  -1.235997e+04
## Overall_QualExcellent       7.572879e+04
## Overall_QualFair           -1.061939e+04
## Overall_QualGood            1.205545e+04
## Overall_QualVery_Excellent  1.363974e+05
## Overall_QualVery_Good       3.779305e+04
## Kitchen_QualFair           -2.489531e+04
## Kitchen_QualGood           -1.727310e+04
## Kitchen_QualTypical        -2.542113e+04
## Fireplaces                  1.034447e+04
## Fireplace_QuFair           -7.615630e+03
## Fireplace_QuGood            .
## Fireplace_QuNo_Fireplace    1.122172e+03
## Fireplace_QuPoor           -5.601195e+03
## Fireplace_QuTypical        -7.002598e+03
## Exter_QualFair             -3.223191e+04
## Exter_QualGood             -1.405814e+04
## Exter_QualTypical          -1.850266e+04
## Lot_Frontage                9.899308e+01
## Lot_Area                    6.038976e-01
## Longitude                  -3.245461e+04
## Latitude                    5.402129e+04
## Misc_Val                    8.088360e-01
## Year_Sold                  -5.415878e+02
```

```
dim(predict(lasso_glmnet_cv, s = lasso_glmnet_cv$lambda.min, type = "coefficients"))
```

```
## [1] 40  1
```

```
predict(lasso_glmnet_cv, s = lasso_glmnet_cv$lambda.1se, type = "coefficients")
```

```
## 40 x 1 sparse Matrix of class "dgCMatrix"
##                                       s1
## (Intercept)                -1.844218e+06
## Gr_Liv_Area                 5.594300e+01
## First_Flr_SF                1.185739e+00
## Second_Flr_SF               .
## Total_Bsmt_SF               3.677880e+01
## Low_Qual_Fin_SF            -2.446653e+01
## Wood_Deck_SF                8.166610e+00
## Open_Porch_SF               7.369809e+00
## Bsmt_Unf_SF                -1.912732e+01
```

```
## Mas_Vnr_Area                 1.432827e+01
## Garage_Cars                  3.126503e+03
## Garage_Area                  1.139961e+01
## Year_Built                   3.158255e+02
## TotRms_AbvGrd               -9.650770e+02
## Full_Bath                    .
## Overall_QualAverage         -2.941056e+03
## Overall_QualBelow_Average   -8.770246e+03
## Overall_QualExcellent        8.952784e+04
## Overall_QualFair            -5.663642e+03
## Overall_QualGood             9.527042e+03
## Overall_QualVery_Excellent   1.590395e+05
## Overall_QualVery_Good        3.569317e+04
## Kitchen_QualFair            -4.657608e+03
## Kitchen_QualGood             .
## Kitchen_QualTypical         -9.707437e+03
## Fireplaces                   6.576694e+03
## Fireplace_QuFair             .
## Fireplace_QuGood             4.549987e+03
## Fireplace_QuNo_Fireplace     .
## Fireplace_QuPoor             .
## Fireplace_QuTypical          .
## Exter_QualFair              -1.404558e+04
## Exter_QualGood               .
## Exter_QualTypical           -5.250572e+03
## Lot_Frontage                 6.641687e+01
## Lot_Area                     5.488127e-01
## Longitude                   -7.897670e+03
## Latitude                     1.254227e+04
## Misc_Val                     .
## Year_Sold                    .
```

```r
dim(predict(lasso_glmnet_cv, s = lasso_glmnet_cv$lambda.1se, type = "coefficients"))
```

```
## [1] 40  1
```

Both models, with and without the 1SE rule, have 40 parameters included in the model.

```r
predy2_lasso <- predict(lasso_glmnet_cv, s = lasso_glmnet_cv$lambda.min, newx = test, type = "response")

test_error_lasso <- mean((testing$Sale_Price - predy2_lasso)^2)
```

After fitting the lasso regression model to the testing dataset, the test error is 4.3879387×10^8.