Optimal predictive maintenance policy for multi-component systems

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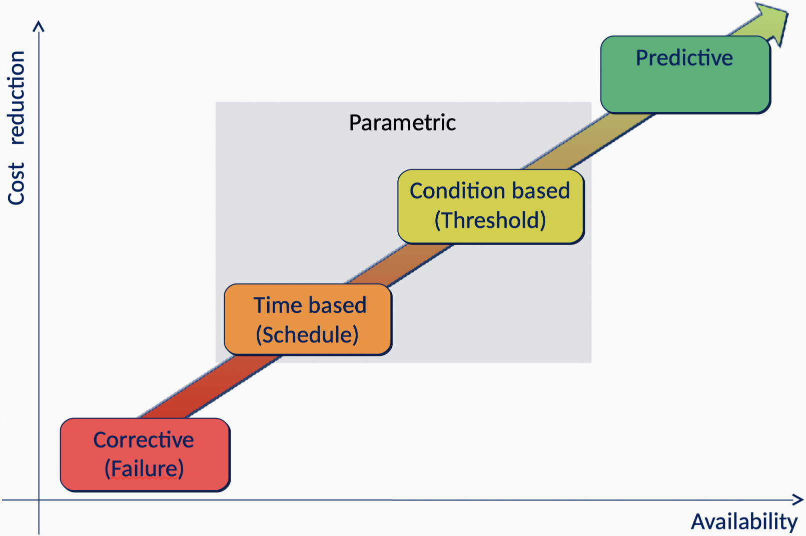
Evolution of maintenance



Maintenance optimization

Equipments

- with several components
- required for missions,
- subject to random failures.

Find a maintenance policy ..

- what action : mission / workshop (repair or change) ?
- when ?

.. in order to optimize some criterion

- **minimize** maintenance costs
- **maximize** availability

Non-trivial compromise

Our approach

1. Define a *simplified* version of the industrial problem.
2. Propose a *mathematical model* for the evolution of the system by using the formalism of a Markov Decision Processes (MDP).
   - System degradation modeling,
   - Explicit the cost functions.
3. *Simulate* the process under different reference *maintenance policies* (corrective or preventive) and compare their *costs*.
4. Compute an approximation of the optimal *cost* and *policy* over the whole space Π of admissible policies :
   - Discretize the state space,
   - Use simulation-based optimisation algorithm to compute the optimal cost and policy.

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Industrial context

Missions

- System required for fixed frequencies and durations missions,
- Over a finite time horizon,
- When the system is not functioning, it can not degrade or fail.

Equipments with several components

- Component i : stable $\xrightarrow{\text{Weib}(\alpha_i, \beta_i)}$ degraded $\xrightarrow{\text{Exp}(\lambda_i)}$ failed .

Global equipment state

- stable stable if all its components are in a stable mode,
- failed if at least one of its component is in failed mode,
- and degraded otherwise.

Maintenance operations

- Nothing : in stable, degraded and failed states,
- repair : in stable and degraded states,
- change : in stable, degraded and failed states.

Workshop

- Immobilize the entire system,
- As good as new (stable state, functioning times reset to 0).

Costs

- Maintenances : repair , change ,
- Penalties in failed state : failed missions, unavailability.

- repair < change < unavailability < **failure**

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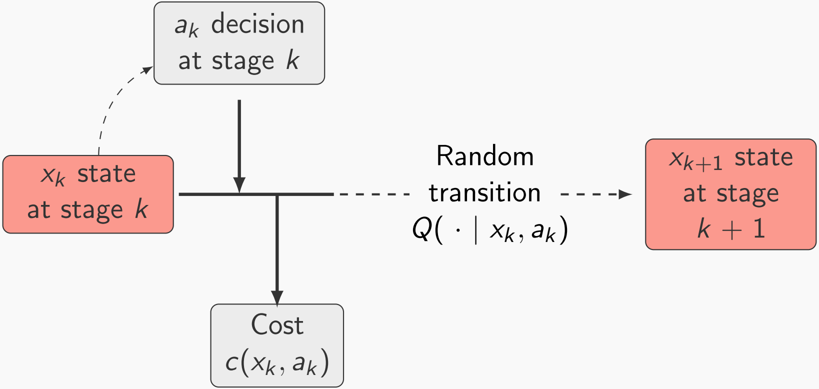
Markov Decision Processes (MDP)

A MDP is defined by the following parameters :

$$(\mathbb{X};\ \mathbb{A};\ \{\mathbb{A}(x) \mid x \in \mathbb{X}\};\ Q;\ c)$$

- A state space $\mathbb{X}$,
  $\mathbb{X} = \{(e_i, r_i); e_i \in \{\text{stable}, \text{degraded}, \text{failed}\}, r_i \in \mathbb{R}^+\}$.
- An action space $\mathbb{A}$,
  $\mathbb{A} = \{a = (a_1, a_2, a_3),\ a_i \in \{\text{nothing},\ \text{repair},\ \text{change}\}\}$.
- $\mathbb{K} = \{(x; a) \mid x \in \mathbb{X}; a \in \mathbb{A}(x)\} \neq \emptyset$, where $\mathbb{A}(x)$ representing the set of admissible actions when the system is in state x ;
- A Markov transition kernel $Q(\cdot \mid x, a)$ which provides the distribution of the next state of the system, when the current state is $x \in X$ and the action $a \in A(x)$;
- A cost function $c : \mathbb{K} \to \mathbb{R}$ depending on the state-action pair

Construction of controlled trajectories



Optimization problem

The total **cost** until the finite horizon N, with initial state $x \in \mathbb{X}$ and under the *policy* π :

$$V_N(\pi, x) = \mathbb{E}_x^{\pi}\Big[\sum_{n=0}^{N} c(x_n, a_n)\Big].$$

The optimal control problem associated to a MDP is to *minimize*, over all *admissible policies* Π, the function $\pi \to V_N(\pi, x)$.

The optimum is called the *value function* and is given by

$$V(x) = \inf_{\pi \in \Pi} V_N(\pi; x).$$

A strategy $\pi^* \in \Pi$ is called *optimal* if it satisfies

$$V_N(\pi^*, x) = V(x).$$

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Reference policies

Policy without any intervention : π_1

There is **no maintenance intervention** (no change, no repair) during the studied period.

Corrective maintenance policy : π_2

After 1 day spent in a failed state, the system is sent back to the workshop,

- change each component in failed state,
- repair each component in degraded state.

Preventive maintenance policy : π_3

After 1 day spent in a degraded or failed state, the system is sent back to the workshop,

- change each component in failed state,
- repair each component in degraded state.

Policy Comparisons

We compare the performances of these reference policies.
Their *cost* was evaluated through 10^5 Monte Carlo simulations.

Policy	cost	95% CI
π_1	22892	$[22884, 22900]$
π_2	18134	$[18121, 18147]$
π_3	15435	$[15423, 15447]$

Table – *Costs of the reference policies*

As expected, a preventive maintenance policy π_3 effectively *reduces maintenance costs* by intervening on the system before the failure.

This yields a *relative gain* with respect to the uncontrolled policy π_1 of 33% and 15% with respect to the corrective policy π_2.

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Dynamic programming

Algorithm 1: Dynamic programming

Input: $\mathbb{X}, \mathbb{A}, Q, costs$

Output: v^*, π^*

begin

  forall $x \in \mathbb{X}$ **do**

    $v_N(x) = C_N(x)$

  for k *de* $N - 1$ *à* 0 **do**

    forall $x \in \mathbb{X}$ **do**

$$v_k(x) = \min_{a \in \mathbb{A}(x)} \left[c(x,a) + \int_{\mathbb{X}} v_{k+1}(y) Q(dy \mid x, a) \right]$$

$$\pi_k^*(x) = \operatorname*{argmin}_{a \in \mathbb{A}(x)} \left[c(x,a) + \int_{\mathbb{X}} v_{k+1}(y) Q(dy \mid x, a) \right]$$

  return v_0, π^*

Non standard optimization problem

State space

- Discrete variables and *continuous variables* (functioning times of the components) : the state space is *not finite*.

Transition kernel $Q(dy|x,a)$

- Not analytically explicit, it can be simulated.

Different time scales

- Physical phenomenon *(continuous time)*,
- Sequential decisions *(discrete time)*, fixed by missions frequency,
- Workshop *(discrete time)*.

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Discretization of the state space

State space
$\mathbb{X} = \{x = (e_1, e_2, r_1, r_2); e_i \in \{\text{stable}, \text{degraded}, \text{failed}\}, r_i \in \mathbb{R}^+\}$

Discretize the state space, as a compromise between :

$\left.\begin{array}{l} \blacktriangleright \text{ precision of the approximation} \\ \blacktriangleright \text{ numerical complexity} \end{array}\right\}$ **Non-trivial compromise**

Reference policy costs will be used to assess the impact of discretization on costing.

Problems :

- No "universal method"
- No theoritical result

Example : Discrete uniform distribution

- Component i : stable $\xrightarrow{\mathrm{U}(a_i, b_i)}$ degraded $\xrightarrow{\mathrm{Geo}(\lambda_i)}$ failed

  With (a_i, b_i) *resp* (λ_i) choosen as expectation of $Weib(\alpha_i, \beta_i)$ *resp* $Exp(\lambda_i)$

Discretization errors :

- Less than 1 %

Problems :

- The state space is not finite
- The kernel is not analytically explicit

$\rightarrow$ **Non standard optimization problem**

Example : Fixed Grid

State space

$\mathbb{X} = \{x = (e_1, e_2, r_1, r_2); e_i \in \{\text{stable}, \text{degraded}, \text{failed}\}, r_i \in \mathbb{R}^+\}$

Propose a fixed grid

- Find δ such as $r_i \in D = \{\delta, 2\delta, \ldots, k\delta\}$

Where k must be a tradeoff between

- precision
- numerical complexity

Quantization

Approximate X by $\hat{X}$ taking finitely many values such that $||X - \hat{X}||_p$ is minimum

- Find a finite grid with K points

Input: nb of points K, nb of runs NR, Séquence (γ_n), Simulator of target law ν, initial Grid

Output: Optimized Grids (Γ^*_n) $0 \leq n \leq N$

begin

  for *$m \leftarrow$ 0 to NR-1* **do**

    simulate x according to law ν

    select y as the closest neighbor of x in Γ^m

    set $y' = y - \gamma_{n+1}(y - x_n)$

    $\Gamma^{m+1} \leftarrow \Gamma^m \cup \{y'\} \backslash \{y\}$

  return *Optimized grid Γ^**

Example : $\mathcal{N}(0; I_2)$:

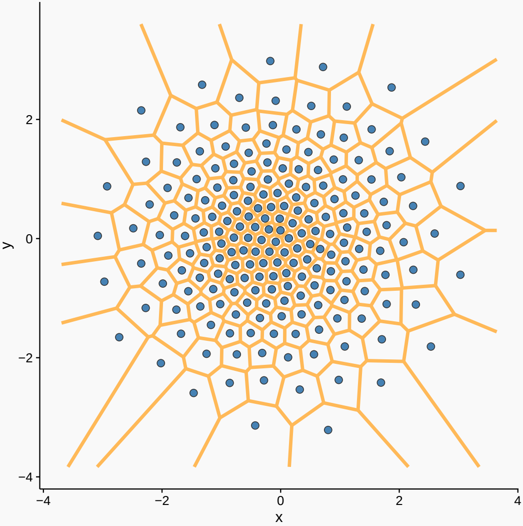

Figure – *Quantization grid (200 points).*

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Conclusions and perspectives

Conclusions

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Perspectives

Compute an approximation of the optimal *cost* and *policy* over the whole space Π of admissible policies :

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References