Optimal predictive maintenance policy for multi-component systems

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Maintenance optimization problem

Equipments

- with several components,
- required for missions,
- subject to random degradation and failures.

Find a maintenance policy ..

- what action : mission / workshop (repair or change) ?
- when ?

.. in order to optimize some criterion

- **minimize** maintenance costs,
- **maximize** availability.

non-trivial compromise

Industrial context (1)

Missions

- System required for fixed frequencies and durations missions,
- Over a finite time horizon,
- When the system is not functioning, it can not degrade or fail.

Equipments with 3 components

- Dynamics of the components 1, 2 and 3 :

$$\text{stable} \xrightarrow{\text{Weibull}} \text{degraded} \xrightarrow{\text{Exponential}} \text{failed}.$$

Global equipment state

- stable if all its components are in a stable state,
- failed if at least one of its component is in failed state,
- and degraded otherwise.

Industrial context (2)

Possible maintenance operations

- do nothing : in stable, degraded and failed states,
- change : in stable, degraded and failed states,
- repair : in stable and degraded states.

Workshop

- Immobilize the entire system,
- As good as new (stable state, functioning times reset to 0).

Costs

- Maintenances : repair, change,
- Penalties in failed state : failed missions, unavailability,
- repair $<$ change $<$ unavailability $<$ **failure**.

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Our approach

- Propose a *mathematical model* for the evolution of the multi-component system by using the formalism of a Markov Decision Processes (MDP).
  1. System degradation modeling,
  2. Explicit the cost function.

- *Simulate* the process under different reference *maintenance policies* (corrective or preventive) and compare their *costs*.

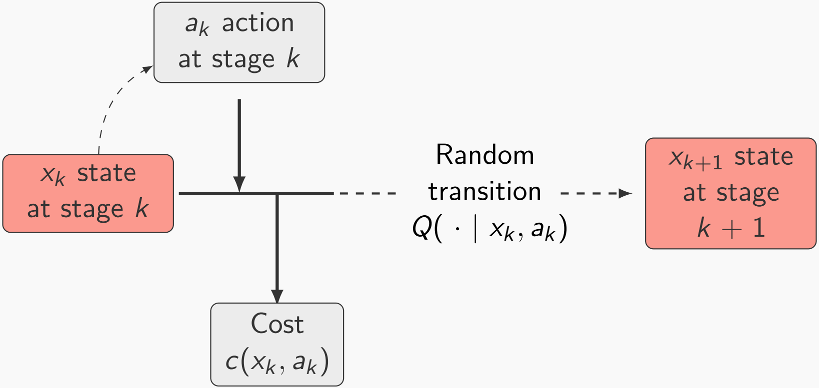
Markov Decision Processes (MDP)

A MDP is defined by the following parameters :

$$(\mathbb{X};\ \mathbb{A};\ \{\mathbb{A}(x) \mid x \in \mathbb{X}\};\ Q;\ c)$$

- A state space $\mathbb{X}$
  $\mathbb{X} = \{x = (e_i, r_i)_{i \in \{1,2,3\}},\ e_i \in \{\text{stable}, \text{degraded}, \text{failed}\},\ r_i \in \mathbb{R}^+\}$.
- An action space $\mathbb{A}$,
  $\mathbb{A} = \{a = (a_1, a_2, a_3),\ a_i \in \{\text{nothing},\ \text{repair},\ \text{change}\}\}$.
- A set $\mathbb{A}(x)$ of admissible actions when the system is in state x ; is such that a failed component cannot be repaired.
- A transition kernel $Q(\cdot \mid x, a)$ which provides the distribution of the next state of the system, when the current state is $x \in X$ and the action $a \in \mathbb{A}(x)$.
- A cost function $c : \mathbb{X} \times \mathbb{A}(x) \to \mathbb{R}$ depending on state-action.

Construction of controlled trajectories



Optimization problem

The total **cost** until the finite horizon N, with initial state $x \in \mathbb{X}$ and under the *policy* π :

$$V_N(\pi, x) = \mathbb{E}_x^\pi \Big[\sum_{n=0}^{N} c(x_n, a_n) \Big].$$

The optimal control problem associated to a MDP is to *minimize*, over all *admissible policies* Π, the function $\pi \to V_N(\pi, x)$.

The optimum is called the *value function* and is given by

$$V(x) = \inf_{\pi \in \Pi} V_N(\pi; x).$$

A strategy $\pi^* \in \Pi$ is called *optimal* if it satisfies

$$V_N(\pi^*, x) = V(x).$$

Reference policies

π_1 - Policy without any intervention

Do nothing (no change, no repair) during the studied period.

π_2 - Corrective maintenance policy

Send back the equipment to the workshop, 1 day after the failure,

- repair each degraded component,
- change each failed one.

π_3 - Preventive maintenance policy

After 1 day spent in a degraded or failed state, send back the equipment to the workshop,

- repair each degraded components,
- change each failed one.

Policy Comparisons

We compare the performances of these reference policies.
Their *cost* was evaluated through 10^5 Monte Carlo simulations.

Policy	cost	95% CI
π_1	22892	$[22884, 22900]$
π_2	18134	$[18121, 18147]$
π_3	15435	$[15423, 15447]$

Table – *Costs of the reference policies*

As expected, a preventive maintenance policy π_3 effectively *reduces maintenance costs* by intervening on the system before the failure.

This yields a *relative gain* with respect to the uncontrolled policy π_1 of 33% and 15% with respect to the corrective policy π_2.

Non standard optimization problem

State space

- Discrete variables and *continuous variables* (functioning times of the components) : the state space is *not finite*.

Transition kernel $Q(dy|x, a)$

- Not analytically explicit, it can be simulated.

$\rightsquigarrow$ Standard optimization technique for MDPs *do not apply*.

$\rightsquigarrow$ The *next step* toward solving the global optimization problem will be to *discretize the state space*.

Discretization of the state space

State space
$\mathbb{X} = \{(e_i, r_i); i \in \{1, 2, 3\}, e_i \in \{\text{stable}, \text{degraded}, \text{failed}\}, r_i \in \mathbb{R}^+\}$

Discretize the state space, as a trade-off between :

$$\left.\begin{array}{l}\blacktriangleright \text{ precision of the approximation} \\ \blacktriangleright \text{ numerical complexity}\end{array}\right\} \textbf{Non-trivial compromise}$$

Reference policy costs will be used to assess the impact of discretization on costs.

Problems :

- No "universal method",
- No theoritical result.

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Conclusions

- Propose a *mathematical model* for the evolution of the system by using the formalism of a Markov Decision Processes (MDP).
  - System degradation modeling,
  - Explicit the cost functions.
- *Simulate* the process under different reference *maintenance policies* (corrective or preventive) and compare their *costs*.

Work in progress

- Discretize the state space, as a compromise between numerical complexity and precision of the approximation.
- Use *simulation-based* optimisation algorithm to compute an approximation of the optimal *cost* and *policy*, over the whole space Π of admissible policies.

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Thank you for your attention !

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