Thứ .... ngày .... tháng ....

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max = A[0]
i = 1
while i < n do :
    if max < A[i] then
        max = A[i]
    endif
    i = i + 1
endw
```

$$\begin{cases} n-1 , & i < n \\ 1 , & i = n \end{cases}$$ phép so sánh

α_i

Số phép so sánh : $2n-1$ phép so sánh

$$\sum_{i=1}^{n}\left(1+\sum_{i=1}^{n}1\right)+1$$

Số phép gán : $2+\sum_{i=1}^{n-1}[\alpha_i+1]$, $\alpha_i = \begin{cases} 1 & \text{nếu } max < A[i] \\ 0 & \text{nếu } max \geq A[i] \end{cases}$

$\Rightarrow n+1 \leq \text{Gán} \leq 2+2(n-1) = 2n$

$\Rightarrow \text{Gán} = O(n)$

Ta có : $A \rightarrow N$ phần tử

$P_{n,K}$ là xác suất có $\alpha = K$ lần với 1 mảng có n phần tử đôi 1 khác nhau với :

$$0 \leq P_{n,K} \leq 1$$

$$\sum_{K=0}^{\infty} P_{n,K} = 1 , \quad P_{n,K} = 0 \ (K \geq n)$$

$X = \{0, 1, 2, \ldots\}$

$\downarrow \ \downarrow$

$q_0 \ q_1$ $\qquad q_1 = P(X=i)$

$$\Rightarrow \overline{\alpha_n} = \sum_{K=0}^{\infty} P_{n,K}\,K = (F(X))$$

HONG HA

Hàm sinh $G_n(z)$ tương ứng dãy $\{P_{n,k}\}_{k=0}^{\infty}$ là

$$G_n(z) = \sum_{k=0}^{\infty} P_{n,k} \cdot z^k$$

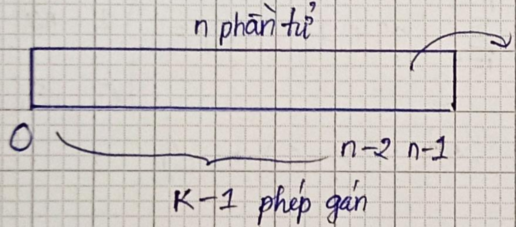
$$\Rightarrow \frac{d}{dz} G_n(z) = \sum_{k=1}^{\infty} P_{n,k}\, k\, z^{k-1}$$

$$\Rightarrow \overline{\alpha_n} = \frac{d}{dz} G_n(z) \Big|_{z=1}$$

$$P_{n,0} = \frac{(n-1)!}{n!} = \frac{1}{n}$$

$P_{n,k} = 0 \,,\ k \geq n$

$1 \leq k \leq n-1$



B_n là biến cố, phần tử cuối cùng của mảng n phần tử là max (A(n))

$$P(B_n) = \frac{1}{n} \quad , \quad P(\overline{B_n}) = 1 - \frac{1}{n} = \frac{n-1}{n}$$

$$P_{n,k} = P(B_n) \cdot P_{n-1,k-1} + P(\overline{B_n}) \cdot P_{n-1,k} \quad , \forall\, 1 \leq k \leq n-1$$

$$\Rightarrow P_{n,k} = \frac{1}{n} P_{n-1,k-1} + \left(\frac{n-1}{n}\right) P_{n-1,k} \quad , \forall\, 1 \leq k \leq n-1$$

$$\{P_{n,k}\}_{k=0}^{\infty} = \begin{cases} P_{n,0} = \frac{1}{n} \,,\ P_{n,k} = 0 \,,\ \forall k \geq n \\ P_{n,k} = \frac{1}{2} \cdot P_{n-1,k-1} + \left(\frac{n-1}{n}\right) P_{n-1,k} \quad (1 \leq k \leq n-1) \end{cases}$$

$$G_n(z) = \sum_{k=0}^{\infty} P_{n,k}\, z^k \quad , \quad \overline{\alpha_n} = \frac{d}{dz} G_n(z) \Big|_{z=1}$$

Thứ ....ngày..........tháng........

$$G_n(z) = P_{n,0} + \sum_{K=1}^{n-1} P_{n,K}\, z^K \quad (\text{vì } P_{n,K} = 0,\ K \geqslant 0)$$

$$= \frac{1}{n} + \sum_{K=1}^{n-1} \left[\frac{1}{n} \cdot P_{n-1,\,n-1} + \left(\frac{n-1}{n}\right) P_{n-1,K} \right] z^K$$

$$= \frac{1}{n} + \frac{1}{n}\left[\sum_{K-1}^{n-1} P_{n-1,K-1}\, z^K \right] + \left(\frac{n-1}{n}\right) \sum_{K=1}^{n-1} P_{n-1,K}\, z^K$$

$$G_{n-1}(z) = P_{n-1} + \sum_{j=1}^{n-2} P_{n-1,j}\, z^j \quad (\text{do } P_{n-1,j} = 0$$

$j \geqslant n-1)$

$$P_{n-1} = \frac{1}{n-1}$$

$$\sum_{K=1}^{n-1} P_{n-1,K}\, z^K = \sum_{n=1}^{n-2} P_{n-1,K}\, z^K + \underbrace{P_{n-1,n-1}\, z^{n-1}}_{=0}$$

$$= G_{n-1}(z) - \frac{1}{n-1}$$

$$\sum_{K=1}^{n-1} P_{n-1,K-1} \cdot z^K = z \cdot \sum_{K=1}^{n-1} P_{n-1,K-1}\, z^{K-1} \quad (\text{đặt } j = K-1)$$

$$= z \cdot \sum_{j=0}^{n-2} P_{n-1,j} \cdot z^j$$

$$= z\left[G_{n-1}(z) \right]$$

$$\Rightarrow G_n(z) = \frac{1}{n} + \frac{z}{n} G_{n-1}(z) + \left(\frac{n-1}{n}\right)\left[G_{n-1}(z) - \frac{1}{n-1} \right]$$

$$= \frac{1}{n} + \left(\frac{z}{n} + \frac{n-1}{n}\right) G_{n-1}(z) - \frac{1}{n}$$

$$= \left(\frac{z}{n} + \frac{n-1}{n}\right) G_{n-1}(z), \quad \forall n \geqslant 2$$

$$G_1(z) = P_{1,0} + P_{1,1} z^1 + \dots$$
$$= \frac{1}{1} = 1$$

$$\Rightarrow \begin{cases} G_n(z) = \left(\frac{z}{n} + \frac{n-1}{n}\right) G_{n-1}(z), & n \geq 2 \\ G_1(z) = 1 \end{cases}$$

* Tìm $G_n(z)$

$$G_n(z) = \left(\frac{z}{n} + \frac{n-1}{n}\right) G_{n-1}(z)$$

$$G_{n-1}(z) = \left(\frac{z}{n-1} + \frac{n-2}{n-1}\right) G_{n-2}(z)$$

$$\vdots$$

$$G_2(z) = \left(\frac{z}{2} + \frac{1}{2}\right) G_1(z) = \left(\frac{z}{2} + \frac{1}{2}\right)$$

$$\Rightarrow G_n(z) = \prod_{i=2}^{n} \left(\frac{z}{i} + \frac{i-1}{i}\right) \quad (\text{Do } G_1(z) = 1)$$

$$f_i(z) = \frac{z}{i} + \frac{i-1}{i}, \quad i = \overline{2,n}, \quad \frac{df_i}{dz} = \frac{1}{i}, \quad \forall i \in \overline{2,n}$$

Lại có: $f = f_2 f_3 \dots f_n$

$$\frac{df}{dz} = \sum_{i=2}^{n} \frac{df_i}{dz} \cdot \prod_{\substack{j \neq i \\ j \in \{\overline{2,n}\}}} f_i$$

$$\Rightarrow \overline{\alpha_n} = \frac{d}{dz} G_n(z) = \sum_{i=2}^{n} \frac{1}{i}$$

Với $n = 3$, $\overline{\alpha_3} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$