Lemma 1 Let $f : \mathbb{R} \to \mathbb{R}$ be a function with a γ-Lipschitz continuous first derivative and $x_0 < x_1 \in \mathbb{R}$. Without loss of generality, let $f(x_0) = f(x_1) = 0$ and let $z \in \mathbb{R}$, $x_0 < z < x_1$ such that $f'(z) = 0$. Then

$$\left|f'(x_0) - f'(z)\right| \leq \frac{\gamma(x_1 - x_0)}{2}.$$

Proof. By definition of Lipschitz continuity, $\left|f'(x_0) - f'(z)\right| \leq \gamma(z - x_0)$. It must also be true that

$$\int_{x_0}^{z} f'(t)\, dt = -\int_{z}^{x_1} f'(t)\, dt.$$

Now, these two conditions occur simultaneously only when $z = (x_0 + x_1)/2$.

$$\implies |f'(x_0) - f'(z)| \leq \frac{\gamma(x_1 - x_0)}{2}$$

This is normal text. $\square$