Given m data points $\{X^{(i)}\}_{i=1}^{m}$ in $[-1,1]^d$ and function values $f_i = f(X^{(i)})$, choose n well-separated points $\{X^{(i_j)}\}_{j=1}^{n}$ (e.g., approximately solve the problem of finding the subset of n points with maximum minimum distance between any pair of points). Build a hypercube centered at $X^{(i_j)}$ by building your box as before, and then taking the smallest hypercube centered at $X^{(i_j)}$ containing that box. The basis functions $B_j(x)$ are multilinear (total degree d) box splines defined on these (covering) hypercubes. The approximation to the data is then the best least squares fit of the form

$$\sum_{j=1}^{m} B_j(x).$$

[Code to find these well-separated n points is essentially in the QNSTOP subroutine QNSTOPS, at Step 2, where instead of picking N points out of $5N$ points, you pick n out of the given m points. Step 2 is doing a lot more (random sampling within an ellipsoid, etc.) that you can ignore—all you need is the part that picks out well-separated points.] [pawnee:/f/ltw/mathsoft/QNSTOP.zip]

This is now an algorithm (for $d < 20$) that is comparable to LSHEP (local LS fitting) and MARS (LS fitting), and differs from MARS primarily in the choice of basis functions (box splines vs. truncated power functions).