Methods of Estimating Derivatives (cont.)

In the context of quintic spline interpolation

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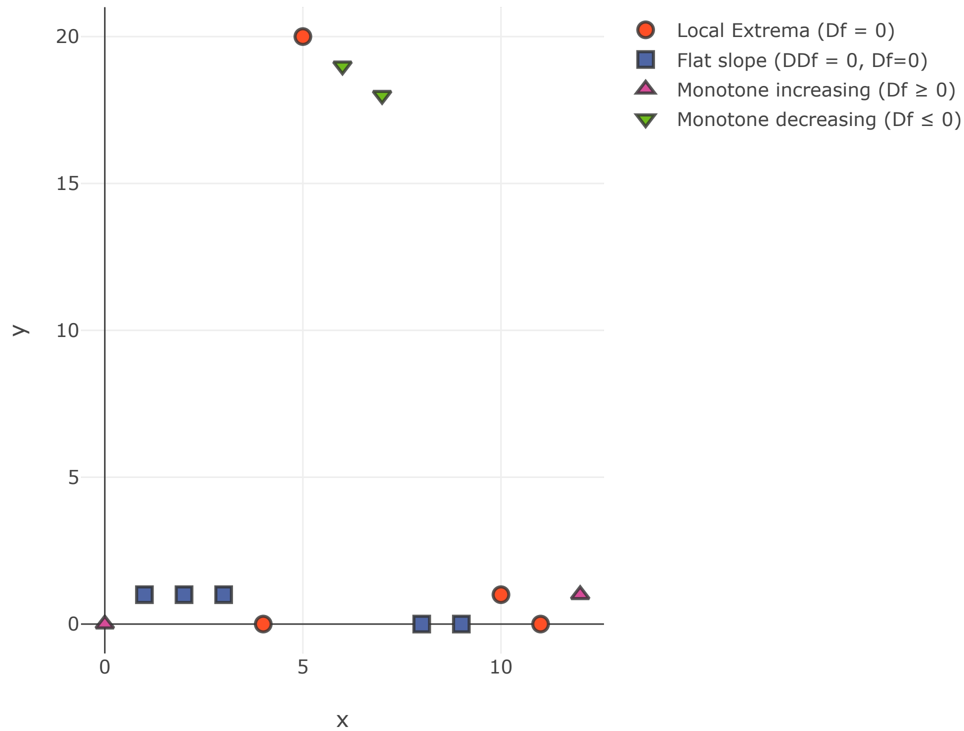
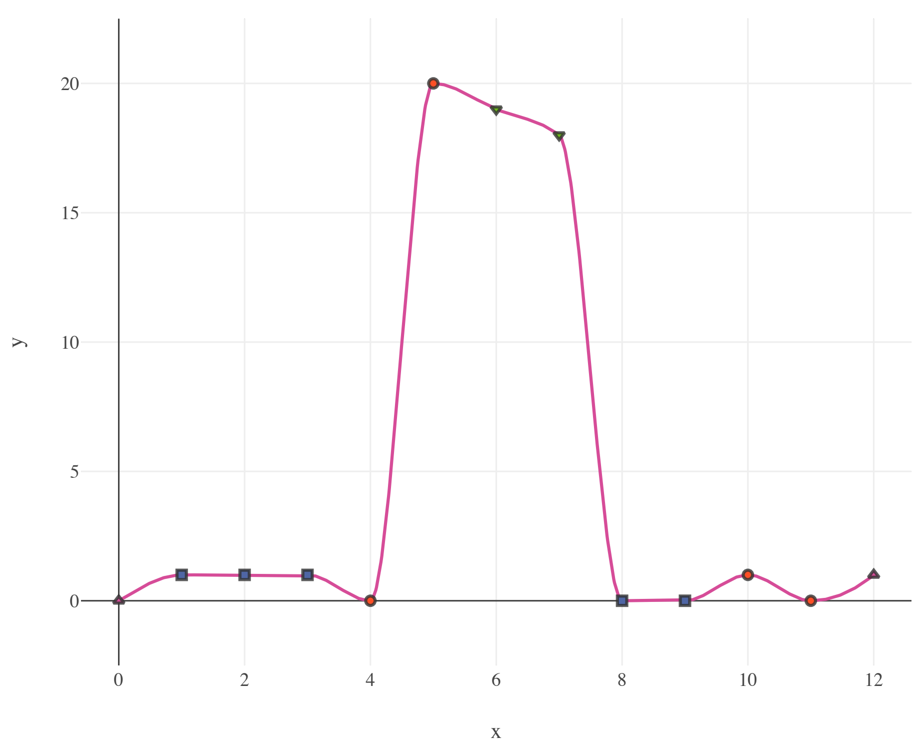
The setup for this report is the same as for the previous report. However, in this report the following two methods are considered.

METHOD		Description
facet	*3 pt. facet model*	Constructs local linear regression models over $\{x_{j-2}, x_{j-1}, x_j\}$, $\{x_{j-1}, x_j, x_{j+1}\}$, and $\{x_j, x_{j+1}, x_{j+2}\}$. The assigned slope at x_j is that of the regression model with the lowest sum of squared error (over its domain) of the three regression models. When the first derivative is given at x_j it is directly assigned. The second derivative is always assigned to zero.
quadratic	*Quadratic interpolant*	Interpolates at x_{j-1}, x_j, and x_{j+1}. When a first derivative is given at x_j, that constraint is included in the interpolant and the function value at the nearer of x_{j-1} or x_{j+1} is interpolated.

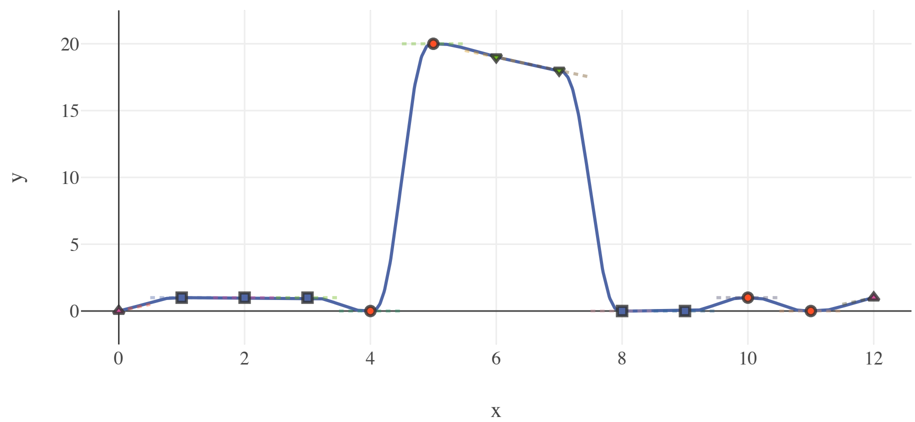
What follows are examples of the resulting piecewise monotone quintic spline interpolants constructed after each method provides initial derivative estimates. The spline will be a solid blue line, the provided points are styled according to constraints (see legend below), and the shape of the local fit for each point is a dotted line interpolating that point (if you can't see it, it's because it overlaps the blue line). The first test problem considered is as follows.



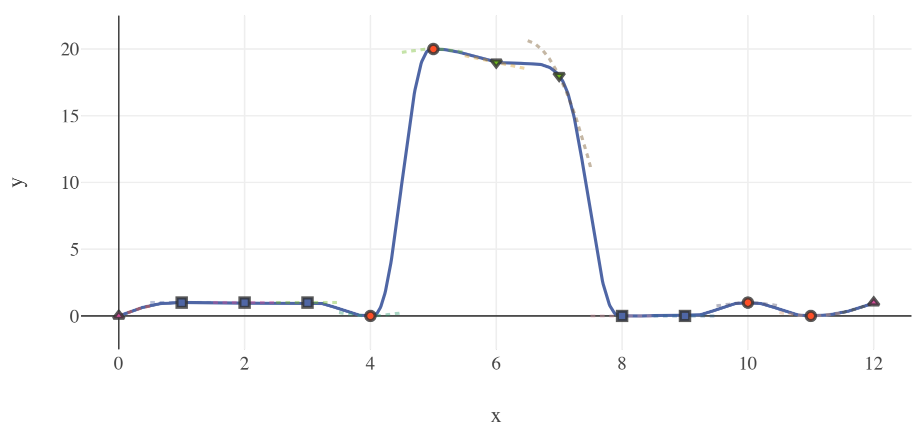
For context, here is the PCHIP (Piecewise **Cubic** Hermite Interpolating Polynomial) interpolant produced over these points and values.



Method 1: Facet model

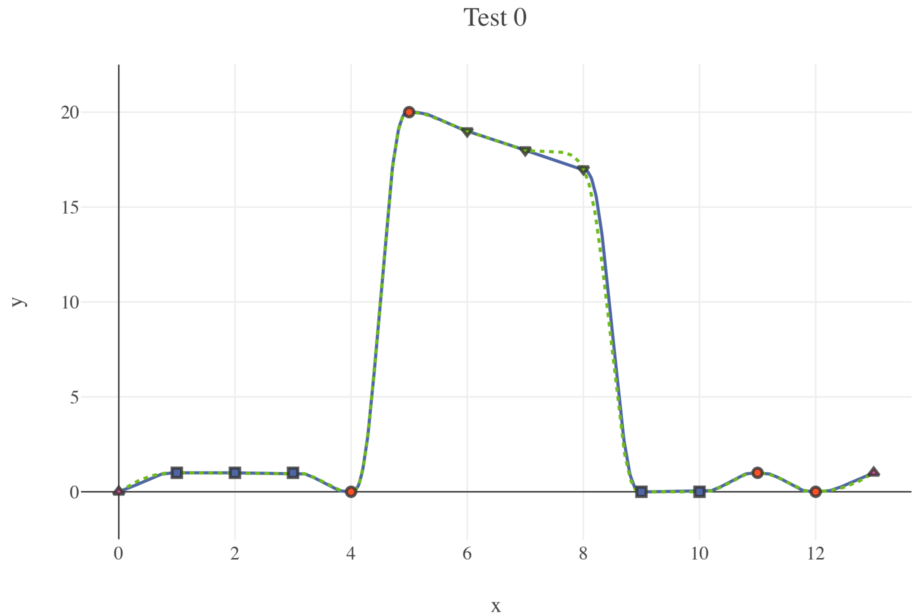


Method 2: Quadratic interpolant

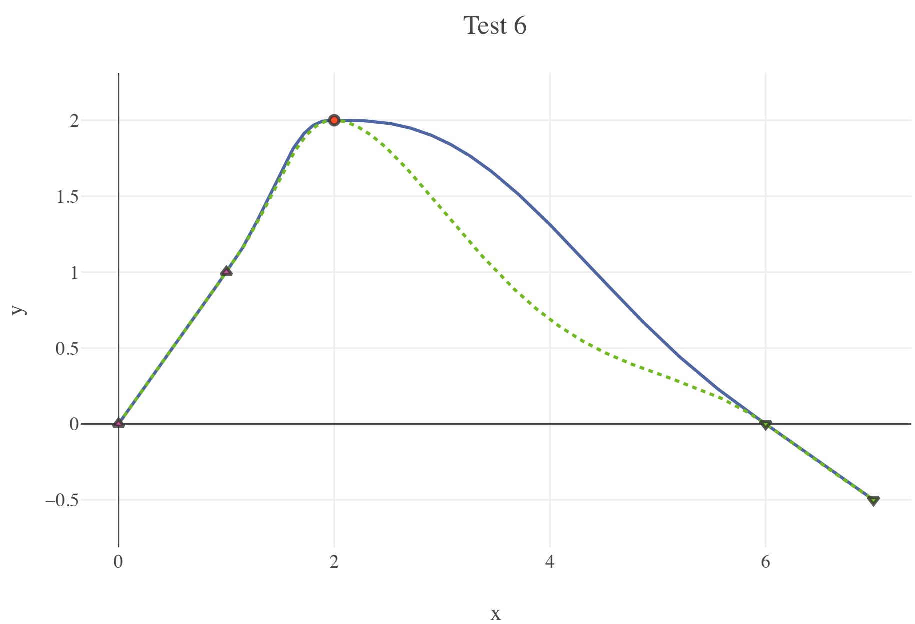


Extended Test Problems

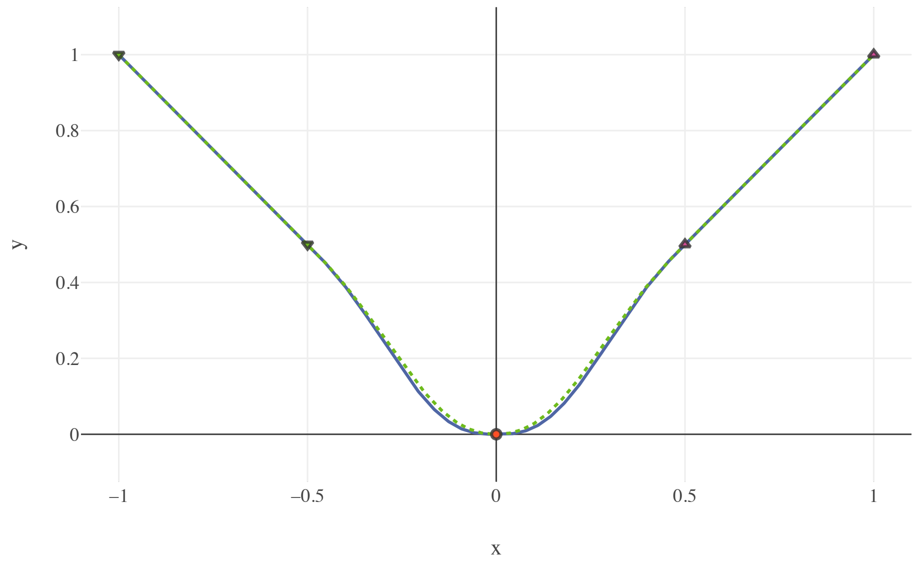
In addition to that one problem, the following examples may also be of interest. The resulting piecewise monotone quintic spline interpolant from the facet model is a blue solid line, from the local quadratic interpolant is the dotted green line.



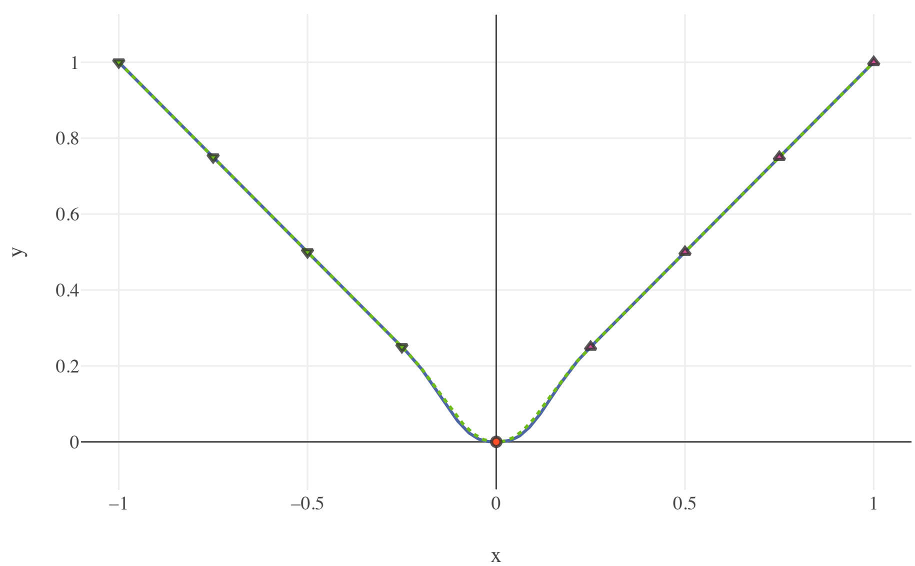
Test 0 above is similar to the point set from before, but with one more point in the upper linear segment.

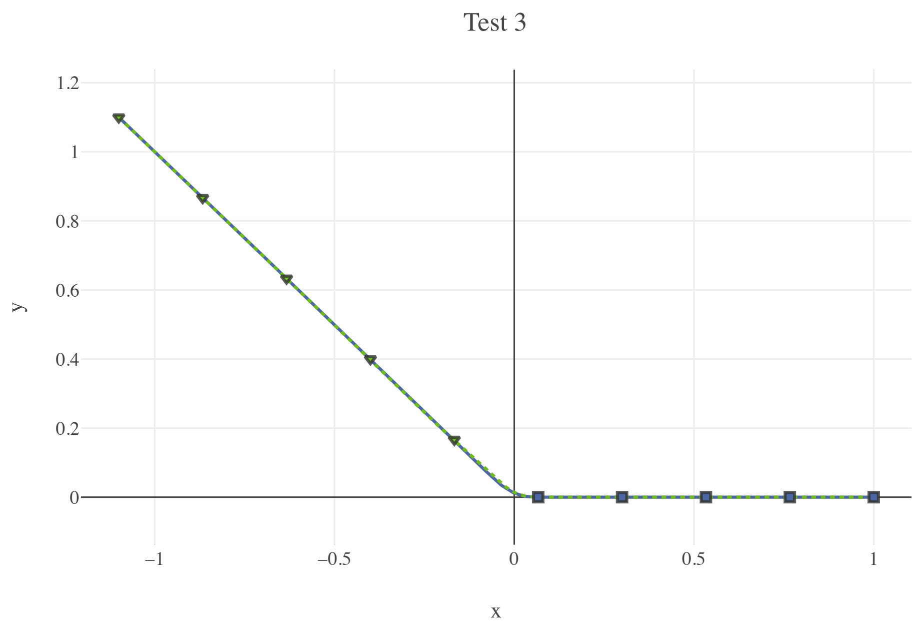


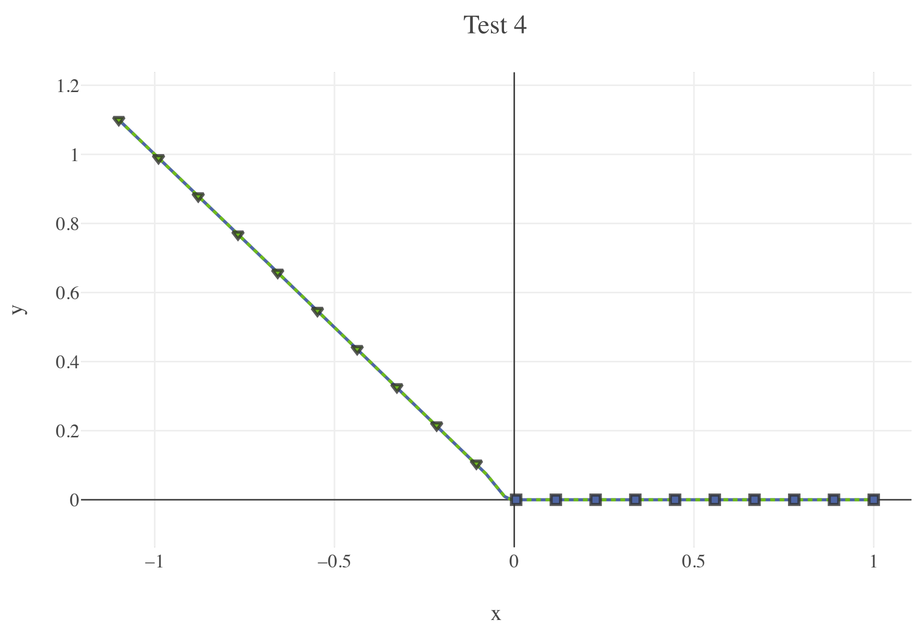
Test 1

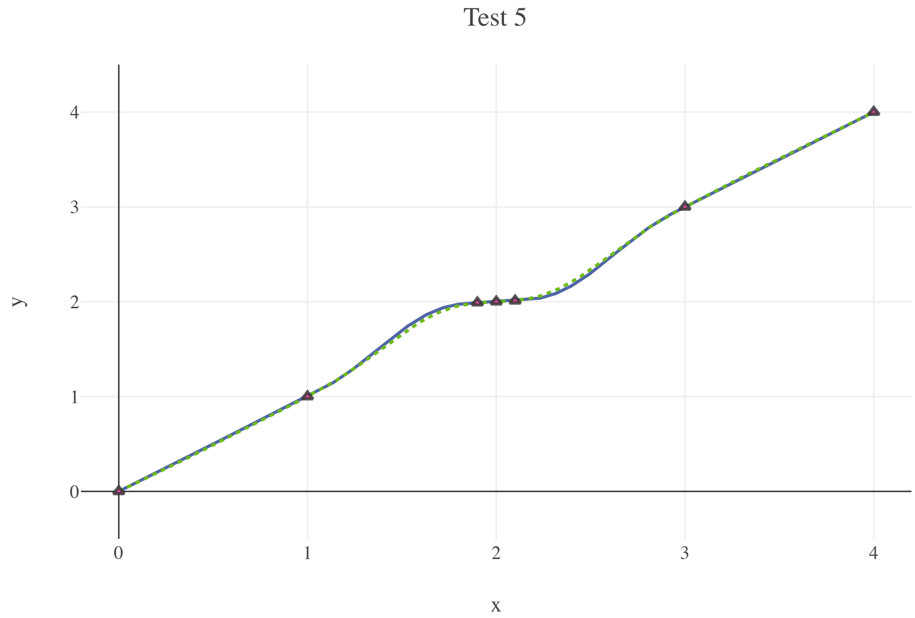


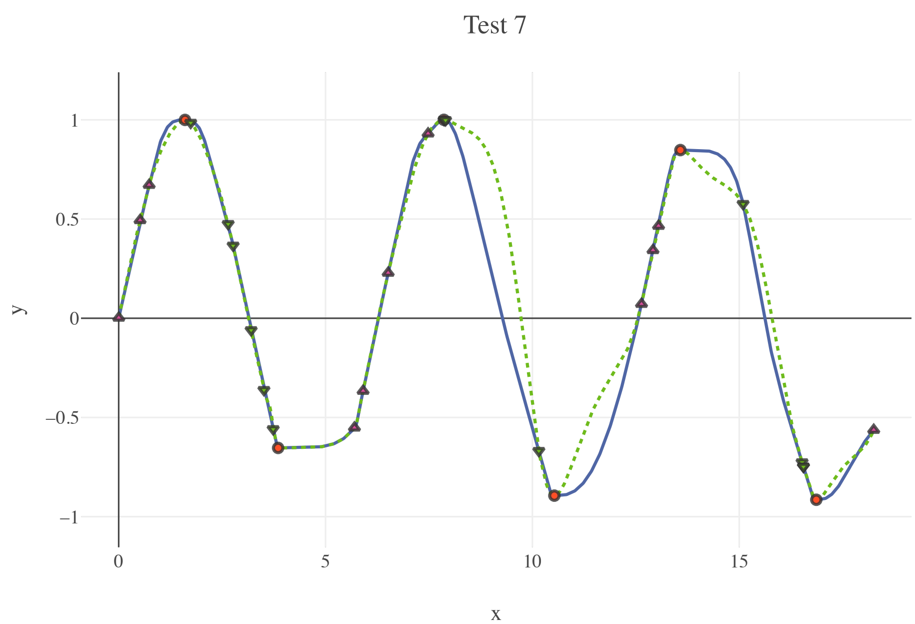
Test 2




Test 3
1.2
1
0.8
0.6
0.4
0.2
0
y
−1
−0.5
0
0.5
1
x


Test 4
1.2
1
0.8
0.6
0.4
0.2
0
y
−1
−0.5
0
0.5
1
x





Discussion of Results

The facet model is producing much more favorable results in general. There are a few instances where the local quadratic estimate to the second derivative is better. It seems like the facet model is the best going forward, especially if a better estimate of second derivative can be calculated.

On possible methods for estimating the second derivatives:

1. A local quintic interpolant could interpolate the first derivative at x_{j-1}, x_j, and x_{j+1}. The second derivative of this interpolant could be used.
2. The local quadratic interpolant over x_{j-1}, x_j, x_{j+1} could also be used to estimated the second derivative.