RBF-FD MESHLESS OPTIMIZATION USING DIRECT SEARCH (GLODS)

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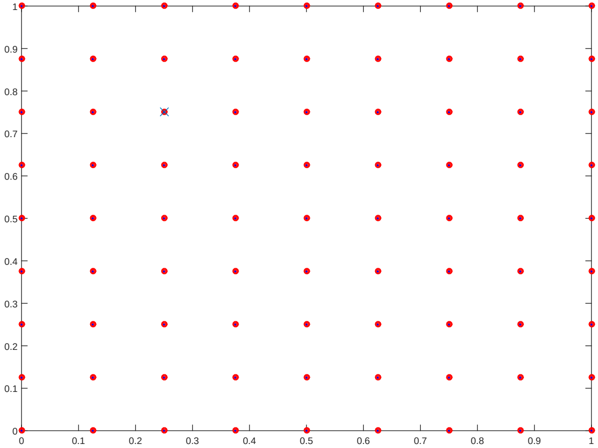
CEM 2016

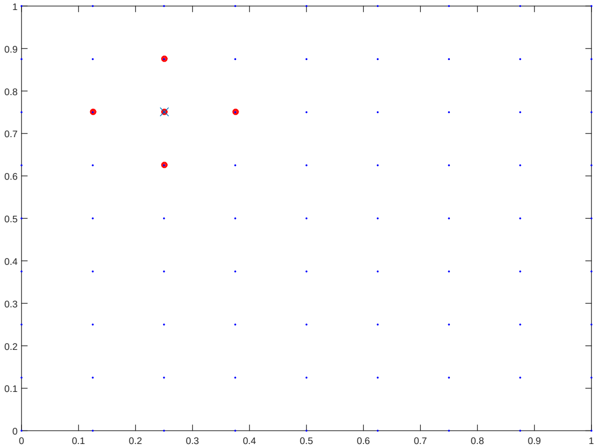
Content

Motivation



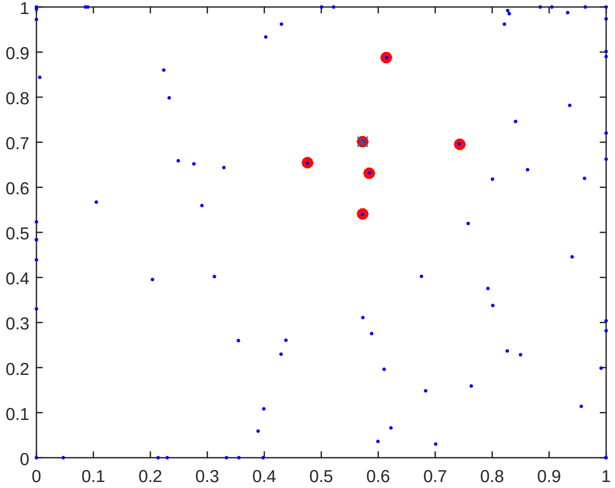
Grid for RBF collocation method

Motivation



Grid for RBF-FD and FD method

Motivation



Grid for RBF-FD method

Motivation

Advantages:

easy to apply to any geometry
just needs scattered nodes
no connectivity between nodes, (no triangles,... or mappings)

Radial basis functions

- depend on a distance to a central point and may depend on a shape parameter, ϵ and are of the form $g(\|\mathbf{x} - \mathbf{x}^{(j)}\|, \epsilon)$

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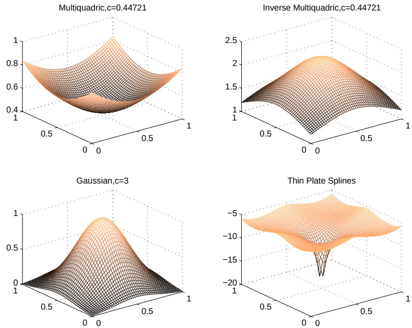
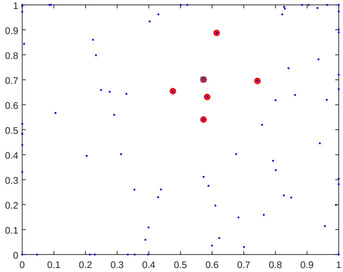
- Example: multiquadric RBF

$$g = (c^2 + r^2)^{1/2}; \, r - \text{euclidian distance}$$





Objectives

- use radial basis function - finite difference (RBF-FD) to study the bending of plates with minimum intervention by the user.

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- use Global and Local Optimization using Direct Search (GLODS) to find good relation shape parameter- stencil size.

Content

RBF (global collocation)

$$Lu(x) = f(x),\ \ x \in \Omega \subset \mathbb{R}^n$$
$$Bu(x)|_{\partial\Omega} = q(x),\ \ x \in\ \partial\Omega$$
L, B differential operators

$$s(\mathbf{x}) = \sum_{k=1}^{n} \lambda_k \phi(||\mathbf{x} - \mathbf{x}_k||) + \sum_{k=1}^{l} \mu_k p_k(\mathbf{x})$$

Radial basis functions method-System of equations to be solved

$$\begin{bmatrix} L_i g(\|\mathbf{x}-\mathbf{x}_j\|,\epsilon) \\ B_b g(\|\mathbf{x}-\mathbf{x}_j\|,\epsilon) \end{bmatrix} [\mathbf{a}] = \begin{bmatrix} f_i \\ q_b \end{bmatrix}; \qquad \text{or } [\mathcal{L}]\ [\mathbf{a}] = [\lambda]$$

i, b - domain and boundary nodes
f_i, q_b - external conditions in domain and boundary (in plates in bending these can be external forces)

- The function g represents the multiquadric function, defined as:

$$g(r,c) = (c^2 + r^2)^{1/2};$$

RBF-FD method

Radial Basis Function - Finite Difference mode (RBF-FD)

Use RBF methods to generate weights in scattered node FD formulas.

$$Lu(x) = \sum_{k=1}^{n} w_k u_k$$

n: stencil size

w_k: differentiation weights

$$s(\mathbf{x}) = \sum_{k=1}^{n} \lambda_k \phi(||\mathbf{x} - \mathbf{x}_k||) + \sum_{k=1}^{l} \mu_k p_k(\mathbf{x})$$

RBF-FD method

matrix form

$$\begin{bmatrix} \phi(x_1 - x_1) & \cdots & \phi(x_1 - x_n) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \phi(x_n - x_1) & \cdots & \phi(x_n - x_n) & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_2 \\ w_{n+1} \end{bmatrix} = \begin{bmatrix} L(x_c - x_1) \\ \vdots \\ L(x_c - x_n) \\ L(1) \end{bmatrix}$$

assemble w_i to form a global matrix L so that the approximate RBF-FD solution can be obtained by solving the linear system

$$Lu = f$$

Content

Displacement field proposed by Reddy

Displacement field:

$$u(x, y, z) = u_0(x, y) + z\phi_x(x, y) - c_1 z^3 \left(\phi_x(x, y) + \frac{\partial w_0(x, y)}{\partial x} \right)$$

$$v(x, y, z) = v_0(x, y) + z\phi_y(x, y) - c_1 z^3 \left(\phi_y(x, y) + \frac{\partial w_0(x, y)}{\partial y} \right)$$

$$w(x, y, z) = w_0(x, y)$$

Equations for the plate theory are derived by using the principle of virtual work...

Equilibrium equations

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0$$

$$\frac{\partial \bar{Q}_x}{\partial x} + \frac{\partial \bar{Q}_y}{\partial y} + c_1 \left(\frac{\partial^2 P_{xx}}{\partial x^2} + 2\frac{\partial^2 P_{xy}}{\partial x \partial y} + \frac{\partial^2 P_{yy}}{\partial y^2} \right) + q = 0$$

$$\frac{\partial \bar{M}_{xx}}{\partial x} + \frac{\partial \bar{M}_{xy}}{\partial y} - \bar{Q}_x = 0$$

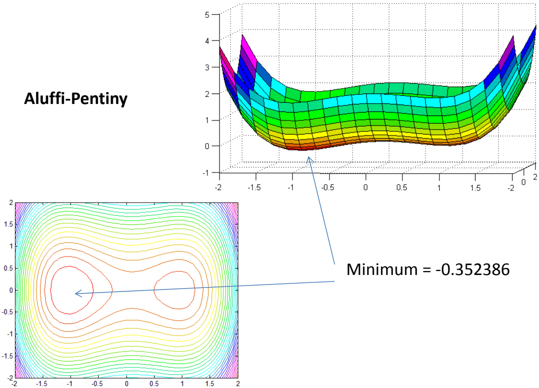
$$\frac{\partial \bar{M}_{xy}}{\partial x} + \frac{\partial \bar{M}_{yy}}{\partial y} - \bar{Q}_y = 0$$

$$
\begin{aligned}
&-\left(A_{55}-\tfrac{4}{h^2}D_{55}\right)\left(\phi_x+\tfrac{\partial w}{\partial x}\right)+\tfrac{4}{h^2}\left(D_{55}-\tfrac{4}{h^2}F_{55}\right)\left(\phi_x+\tfrac{\partial w}{\partial x}\right)+\left(D_{11}-\tfrac{4}{3h^2}F_{11}\right)\frac{\partial^2\phi_x}{\partial x^2}\\
&+\left(D_{12}-\tfrac{4}{3h^2}F_{12}\right)\frac{\partial^2\phi_y}{\partial y\partial x}-\tfrac{4}{3h^2}\left(F_{11}-\tfrac{4}{3h^2}H_{11}\right)\left(\frac{\partial^2\phi_x}{\partial x^2}+\frac{\partial^3 w}{\partial x^3}\right)-\\
&-\tfrac{4}{3h^2}\left(F_{12}-\tfrac{4}{3h^2}H_{12}\right)\left(\frac{\partial^2\phi_y}{\partial y\partial x}+\frac{\partial^3 w}{\partial y^2\partial x}\right)+\left(D_{33}-\tfrac{4}{3h^2}F_{33}\right)\left(\frac{\partial^2\phi_x}{\partial y^2}+\frac{\partial^2\phi_y}{\partial x\partial y}\right)-\\
&-\tfrac{4}{3h^2}\left(F_{33}-\tfrac{4}{3h^2}H_{33}\right)\left(\frac{\partial^2\phi_x}{\partial y^2}+\frac{\partial^2\phi_y}{\partial y\partial x}+2\frac{\partial^3 w}{\partial y^2\partial x}\right)=0
\end{aligned}
$$

Content

Global and Local Optimization using Direct Search (GLODS)

Aluffi-Pentiny



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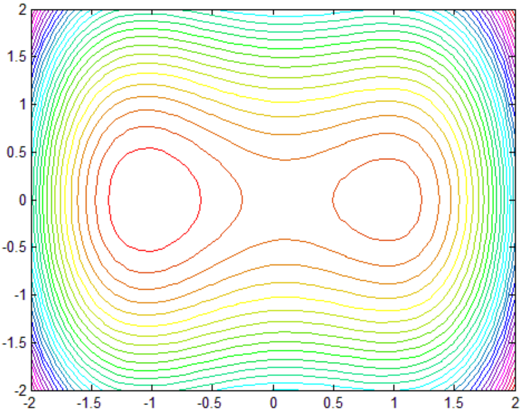

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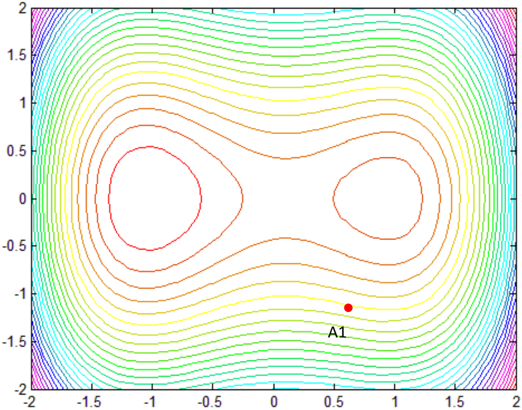

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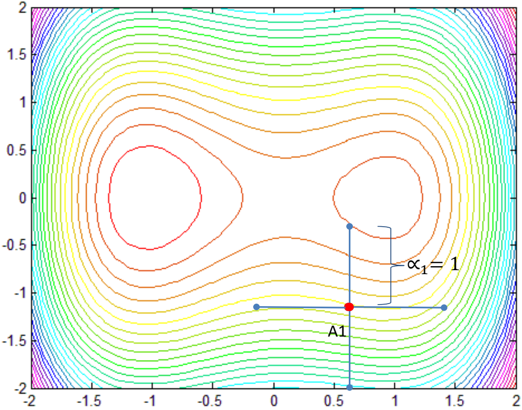

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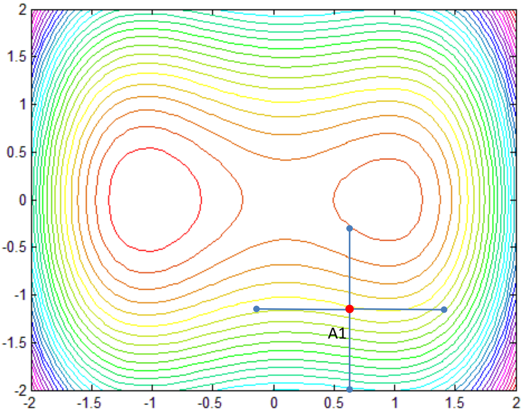

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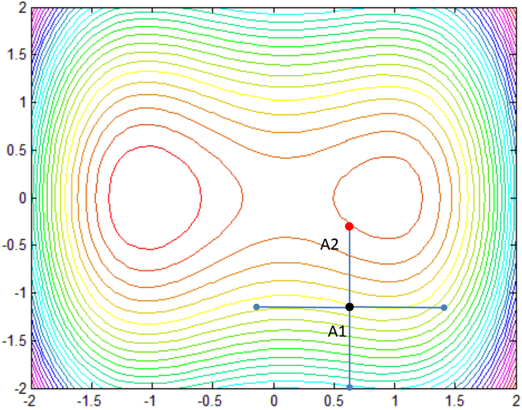

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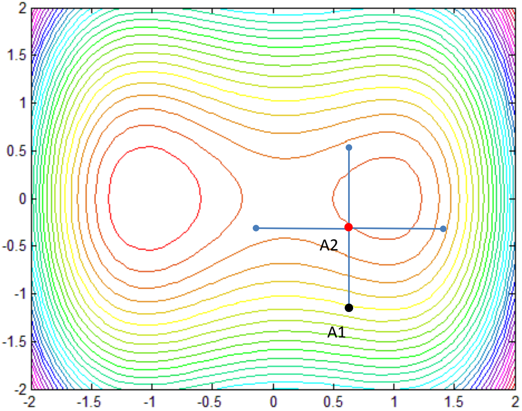

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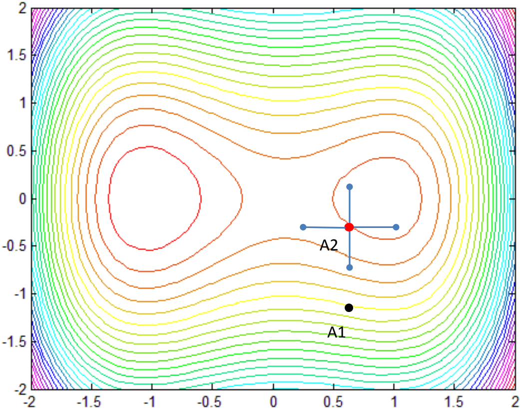

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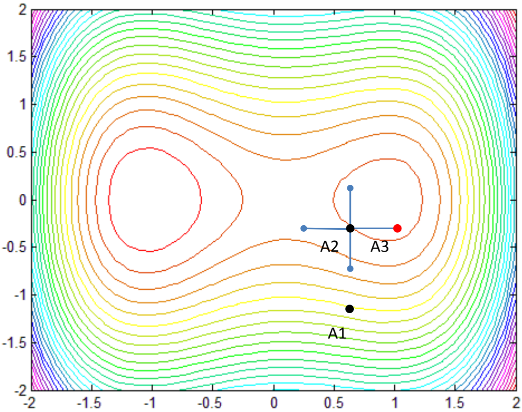

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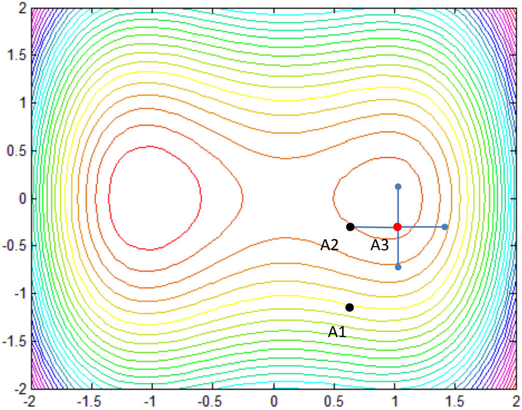

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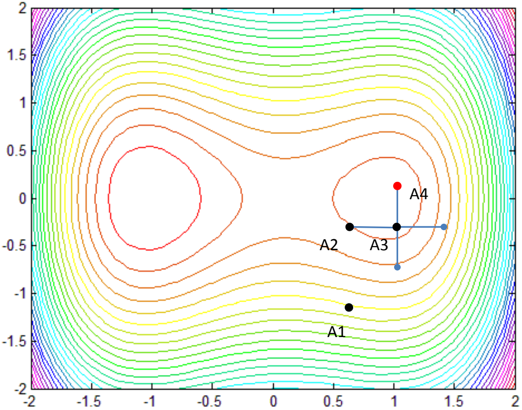

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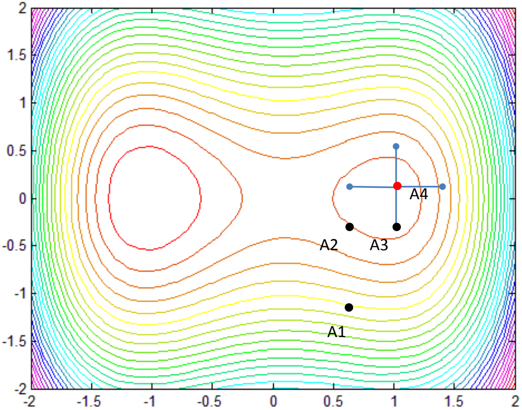

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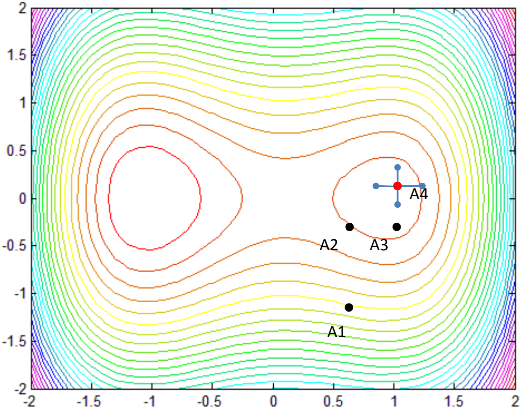

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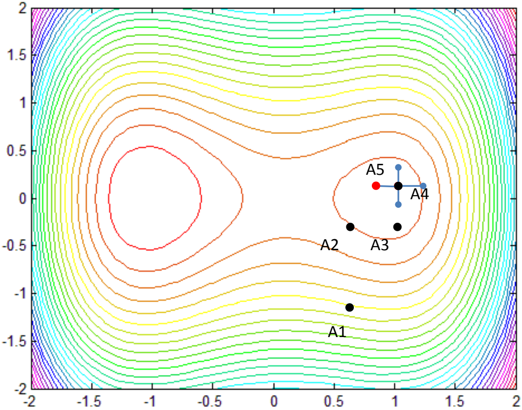

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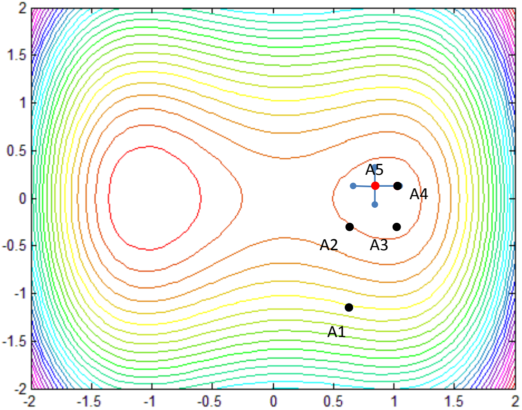

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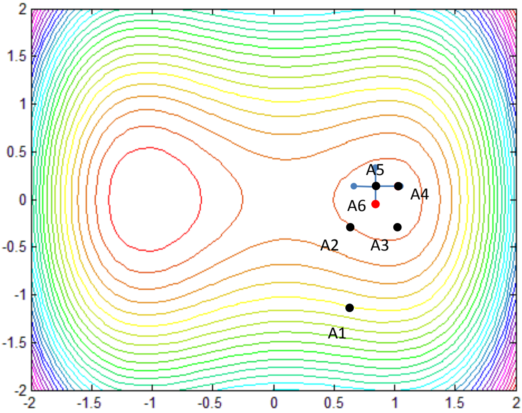

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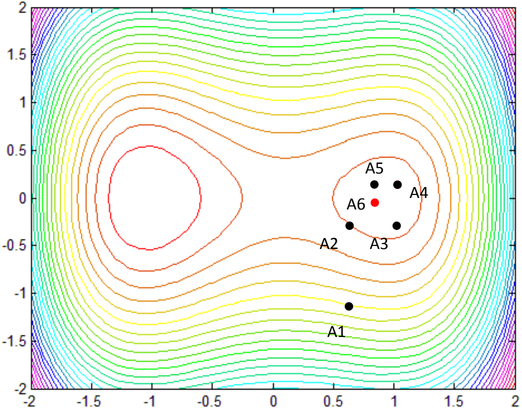

Each generated point will have associated a comparison radius

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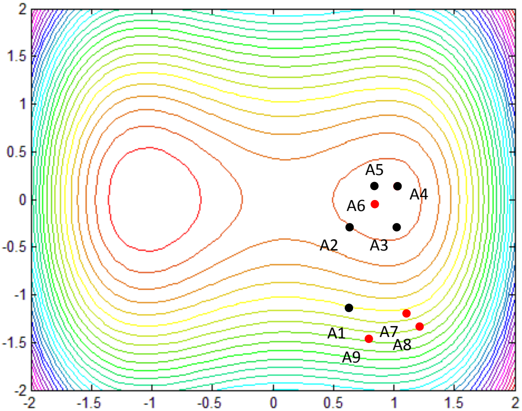

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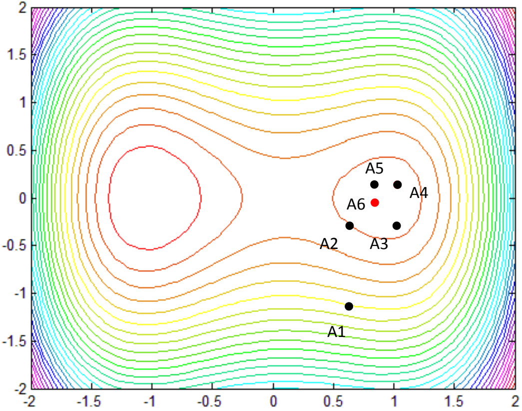

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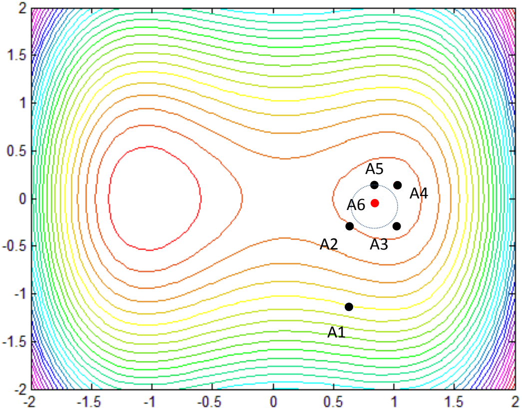

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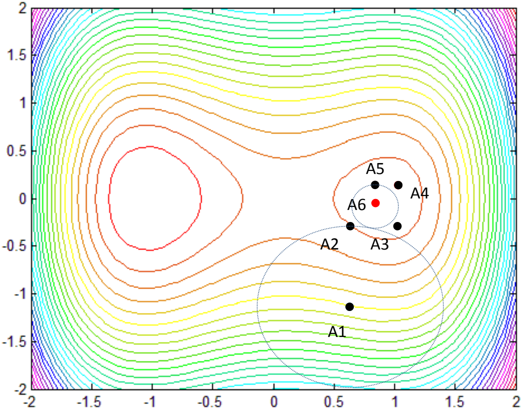

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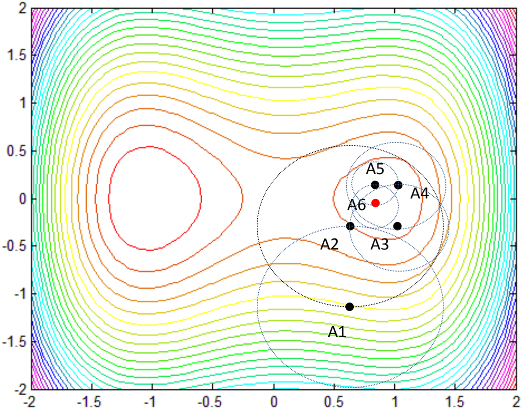

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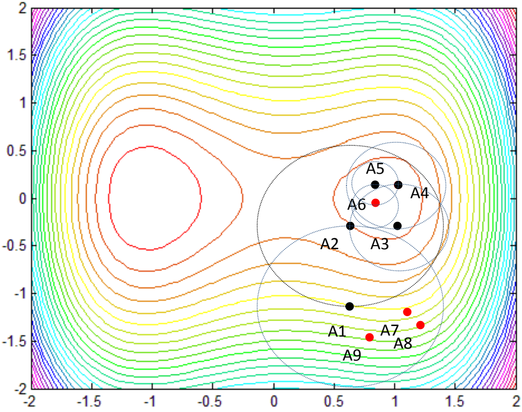

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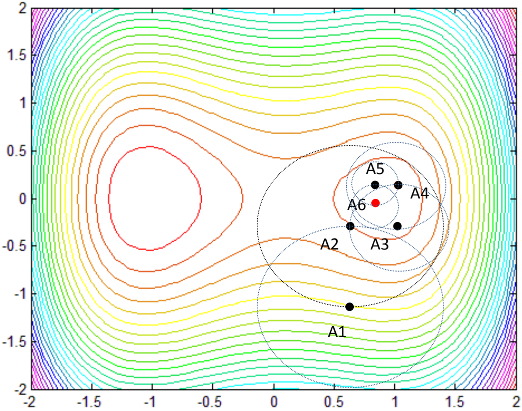

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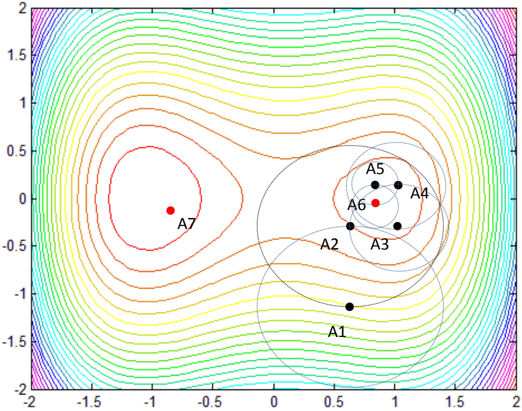

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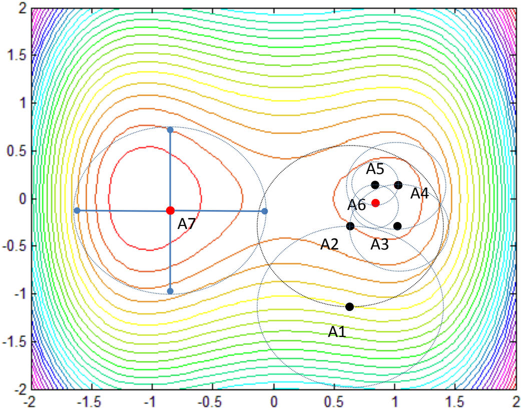

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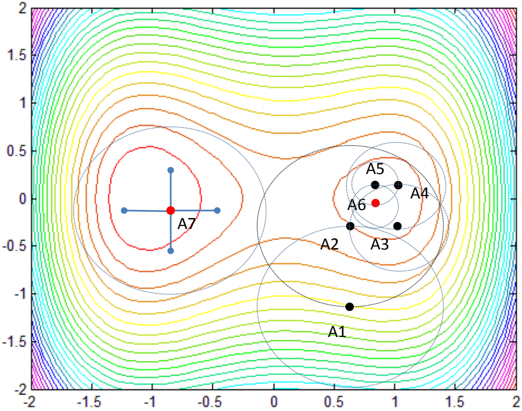

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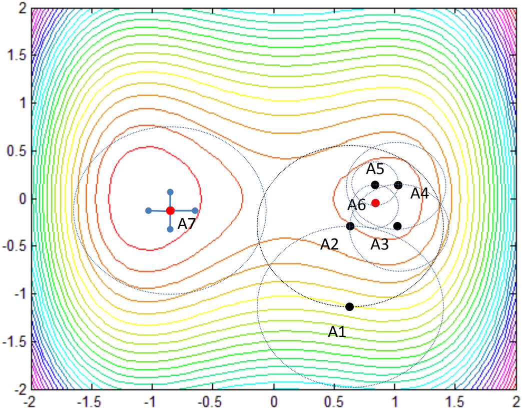

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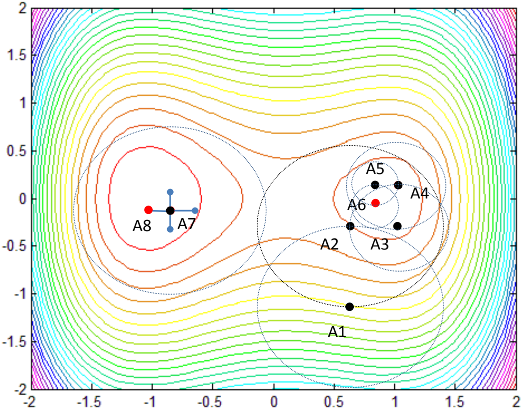

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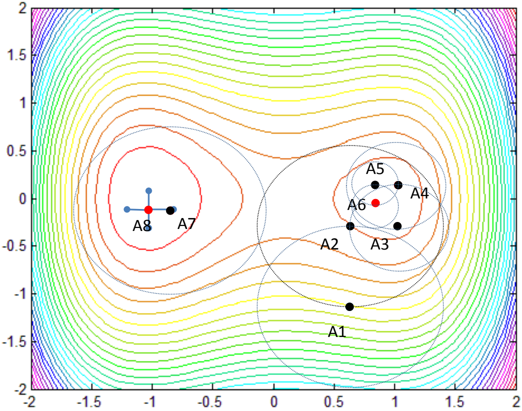

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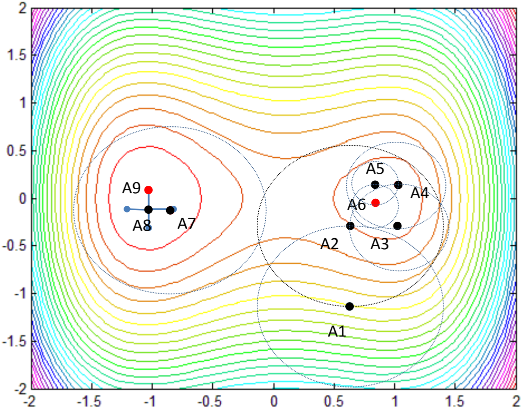

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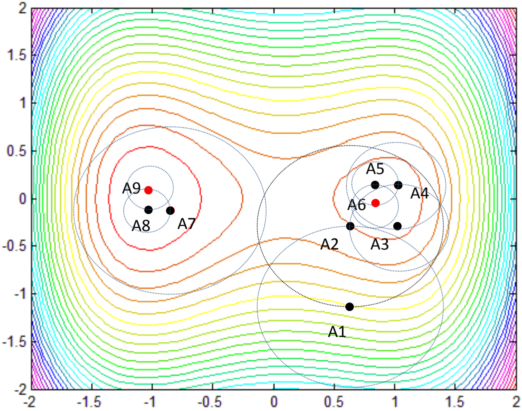

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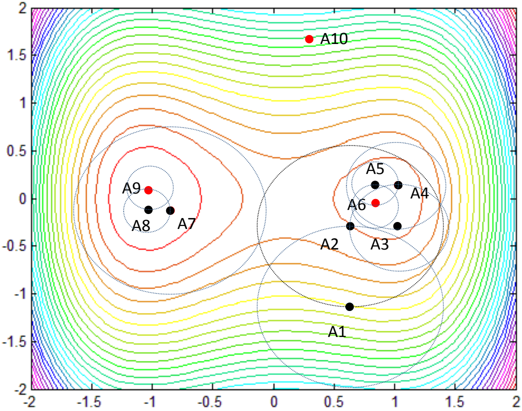

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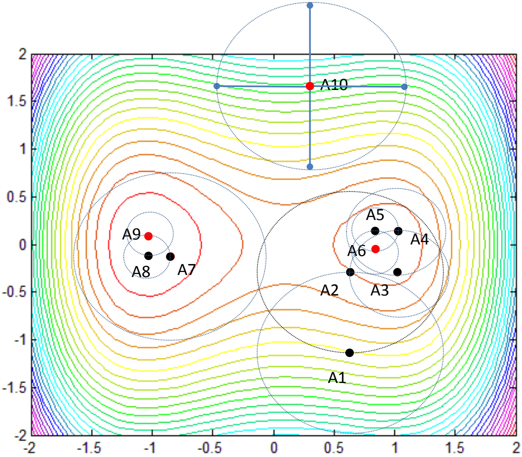

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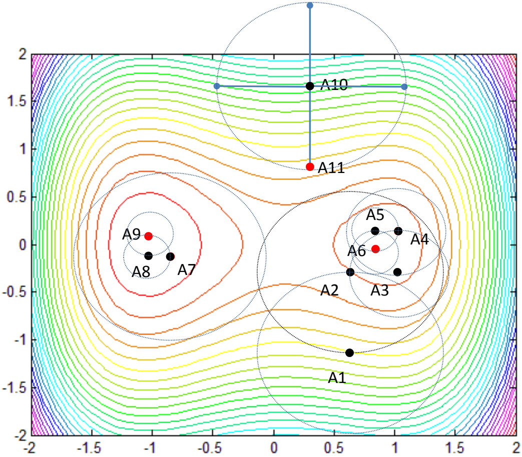

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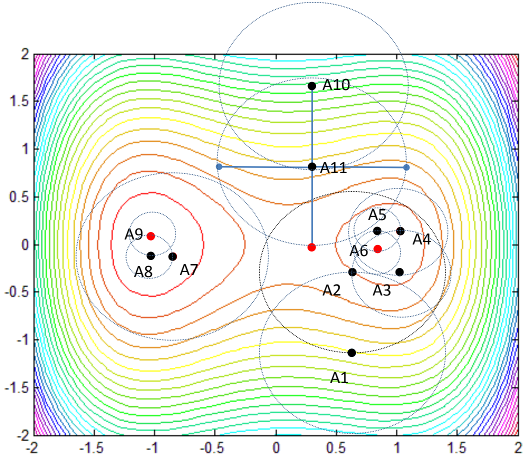

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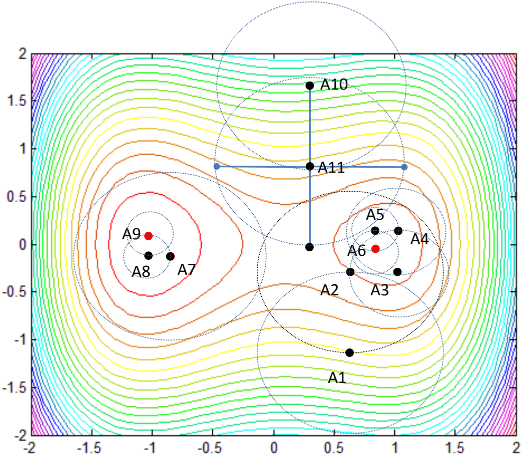


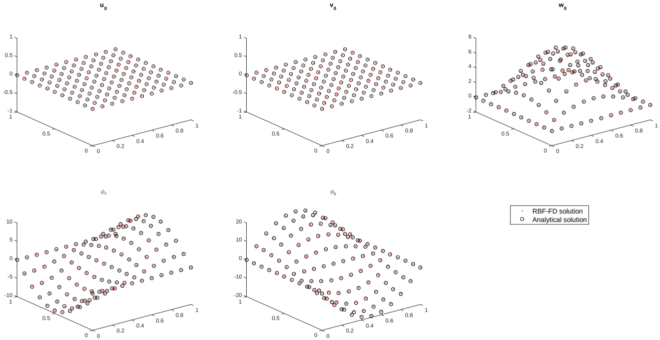
Content

Simply supported plate, under sinusoidal load.
Cross-ply laminate $[0, \pi/2, \pi/2, 0]$

Problem 1
$min(E)$
$r = 0.2$
$0<c<3$
11×11 regular grid

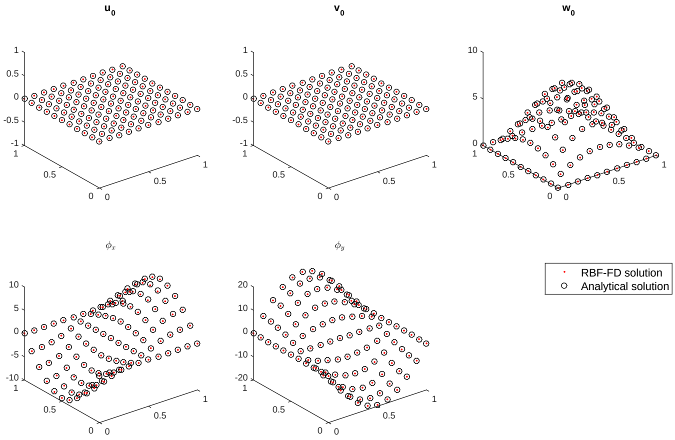
Problem 2
$min(E)$
$c = \frac{2}{\sqrt{n}}$
$0.1414 < r < 1.4142$
11×11 regular grid

Problem 1



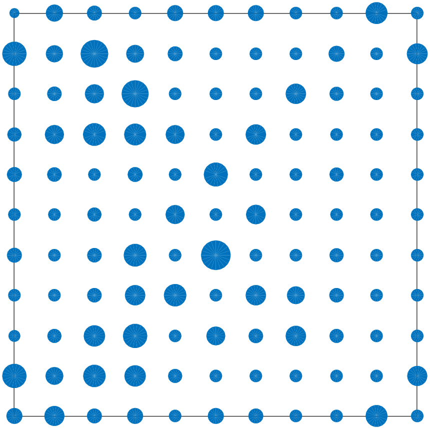
Best solution: constant shape parameter, $c = 2.99$

Optimization results, Problem 2



Best solution: variable distance with center

Optimization results, Problem 2



$r_{min} = 0.14$; $r_{max} = 1.22$

Content

Comments

- GLODS found excellent solutions with minimum intervention by the user.
- the RBF-FD method seems to benefit from variable stencil size.
- Future work: work with irregular grids.

Acknowledgments

This work was supported by FCT, under LAETA, project UID/EMS/50022/2013. The support of LAETA to project "Aplicação de optimização global ao método numérico sem malha RBF–FD para estudo de materiais compósitos" is also gratefully acknowledged. The support of Ministerio da Ciencia Tecnologia e do Ensino Superior and Fundo Social Europeu (MCTES and FSE) under programs POPH-QREN and Investigador FCT is also acknowledged.

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