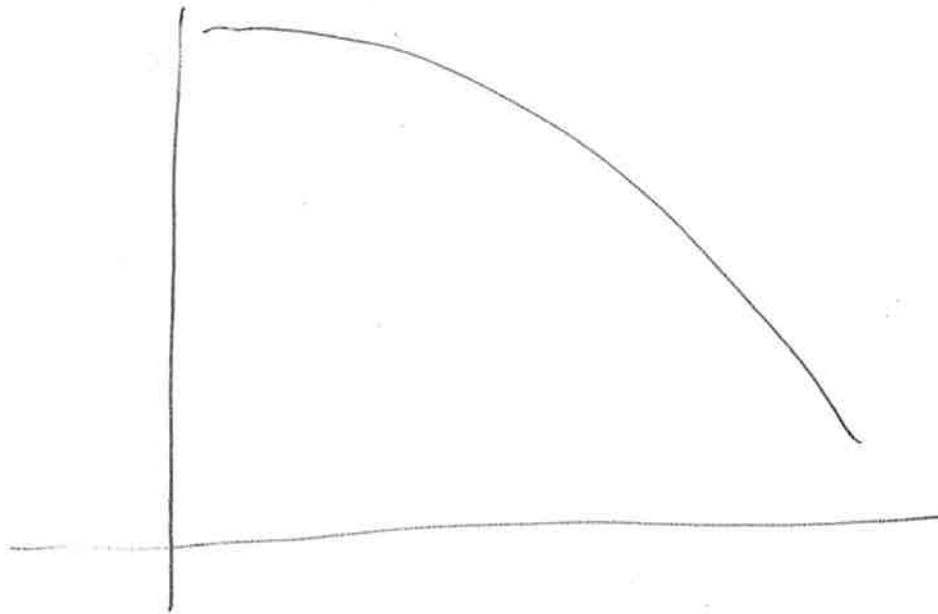


— mean, median, mode plot



for each value of  $E[\tau]$  or  $\lambda$

data file with 10,000 trials

trial data

10	trial 1, trial 2, . . . , trial 10,000
15	trial 10,001, . . . , trial 20,000
20	

$\lambda$

$$F: \mathbb{R}^6 \rightarrow \mathbb{R}^6$$

$$F(V, \bar{I}, I, F, B, A)$$

$$= \begin{pmatrix} \rho I - c V - \mu VA - \beta VT \frac{V}{V_m + V} \\ \vdots \\ \vdots \end{pmatrix}$$

$$\phi(x_0, \cdot)$$



$$\phi(\cdot, \cdot)$$

$$\phi(\bar{x}_{T_2}, \bar{t}_3) + K_3$$

$K_3$

$$\bar{x}_{T_2} = \phi(\bar{x}_{T_1}, \bar{t}_2) + K_2$$

$\bar{x}_{T_1}$

$$= \phi(x_0, \bar{t}_1) + K_1$$

$K_1$

$$\begin{aligned} & t=1 \quad \phi(x_0, 1) \\ & t=2 \quad \phi(x_0, 2) \\ & x_0 \end{aligned}$$

$$\tau_n = T_n - T_{n-1}$$

$$\tau_1 = T_1 - T_0$$

$$= T_1 - 0$$

$$= T_1$$

0

Poisson process

$$N_1 \sim \text{Pois}(5)$$

$$N_{1/2} \sim \text{Pois}(5 \cdot \frac{1}{2})$$

$$N_{10} \sim \text{Pois}(5 \cdot 10)$$

Given  $\lambda > 0$ ,  $(N_t)_{t \geq 0}$  is called

a Poisson process if it's a collection of random variables such that

$$(1) N_0 = 0$$

$$(2) N_t \sim \text{Pois}(\lambda t) \quad \forall t > 0$$



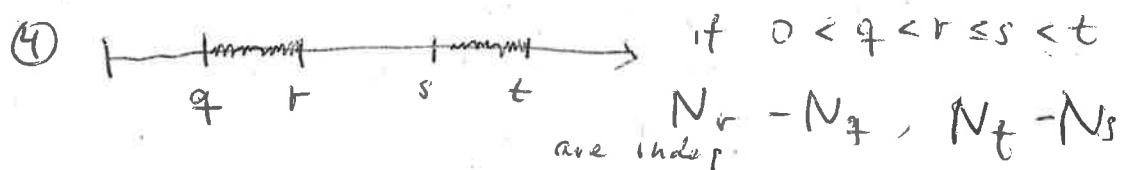
$N_{t+s} - N_t$  is distributed as

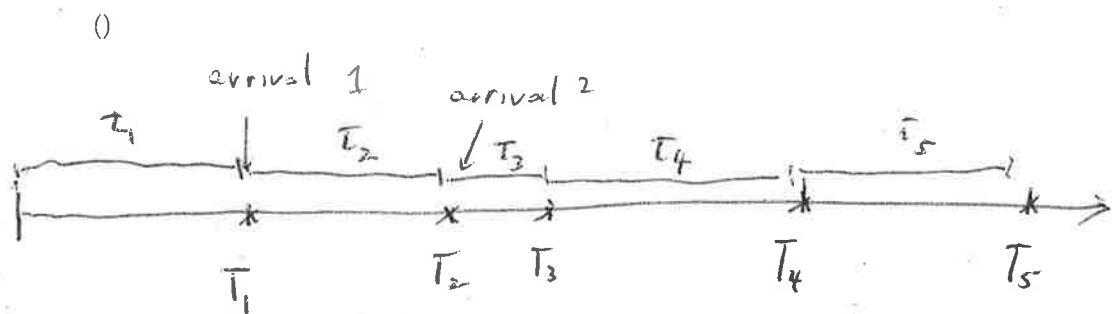
$$\text{Pois}(s\lambda), \text{ i.e.}$$

12

the same as  $N_s$

(stationary increments)





$\tau_1, \tau_2, \tau_3, \dots$  are i.i.d.  $\text{Exp}(\lambda)$

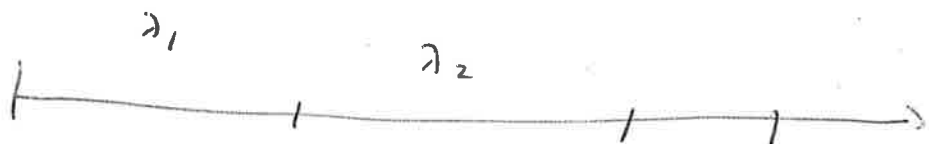
$T_1, T_2, T_3, \dots$  are all Gamma distributed

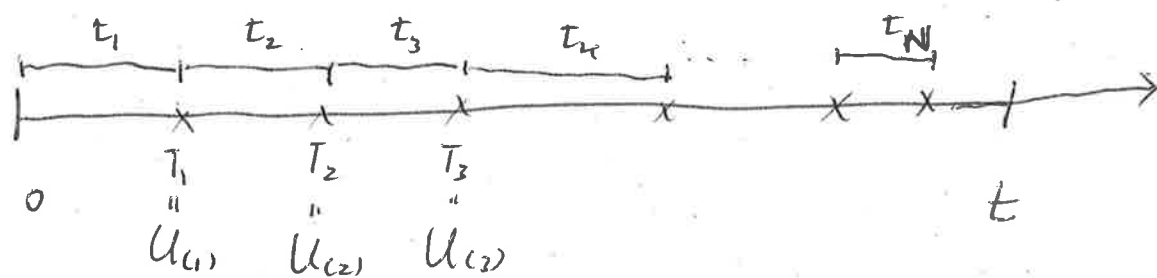
$$T_1 \sim \text{Gamma}(1, \lambda)$$

$$T_2 \sim \text{Gamma}(2, \lambda)$$

$\vdots$

$$T_n \sim \text{Gamma}(n, \lambda)$$





$N_t$  is # of arrivals in  $[0, t]$

Suppose  $N_t = N$  for some constant integer  $N \geq 0$ .

Let  $U_1, U_2, \dots, U_N$  be i.i.d.

~~Uniform r.v.~~  $\text{Unif}[0, t]$

Let them be sorted and call the sorted ones

$U_{(1)} \leq U_{(2)} \leq \dots \leq U_{(N)}$  (called the order statistics of  $U_1, \dots, U_N$ )

Then  $U_{(1)}, U_{(2)} - U_{(1)}, U_{(3)} - U_{(2)}, \dots$

$U_{(N)} - U_{(N-1)}$

have the same distribution as  $T_1, \dots, T_N$ .