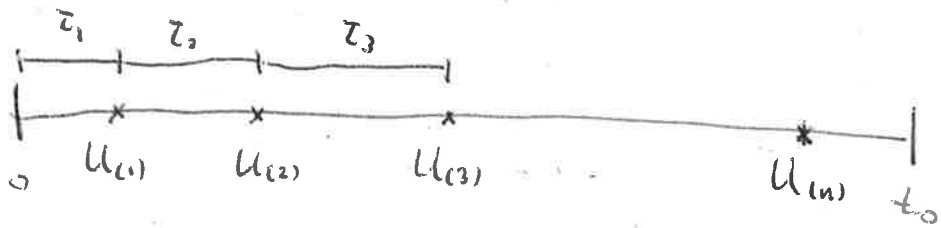


Given time interval $[0, t_0]$

Goal sample Poisson process over $[0, t_0]$.
(constant rate of the process given as λ)

Generate ① how many arrivals in $[0, t_0]$ — average number of arrivals in 1 time unit (e.g. $([0,1])$)

$$N \sim \text{Pois}(\lambda t_0)$$

② Suppose N has been sampled
(i.e. $N = n$)
Then sample $U_1, \ldots, U_n$ i.i.d.
from $\text{Unif}([0, t_0])$

③ Sort $U_1, \ldots, U_n$ into ascending order
and call them

$$U_{(1)} < U_{(2)} < \cdots < U_{(n)}$$

(i.e. $U_{(1)} = \min\{U_1, \ldots, U_n\}$,
$U_{(n)} = \max\{U_1, \ldots, U_n\}$)

$U_{(1)}, \ldots, U_{(n)}$ represent the times of

() arrivals of the process

$\tau_1, \tau_2, \ldots, \tau_n$ are defined as

$$\tau_i = U_{(i)} - U_{(i-1)}, \quad U_{(0)} = 0.$$

These are the flow times (a.k.a. the times between kicks or interkick times)

Facts

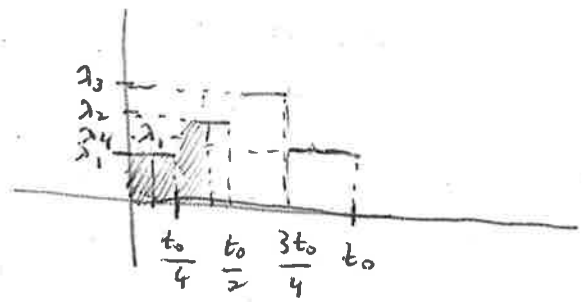
1. $U_{(1)}, U_{(2)}, \ldots, U_{(n)}$ are Gamma distributed $U_{(i)} \sim \text{Gamma}(i, \lambda)$

2. $\tau_1, \ldots, \tau_n$ are i.i.d. $\text{Exp}(\lambda)$ distributed.

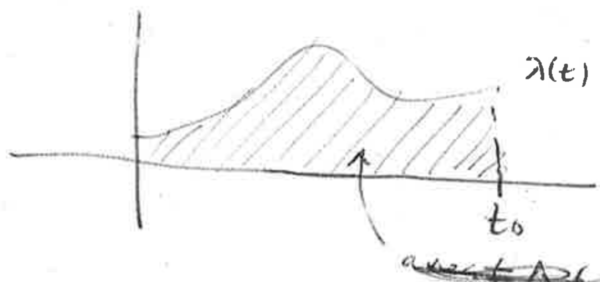
(non-homogeneous)

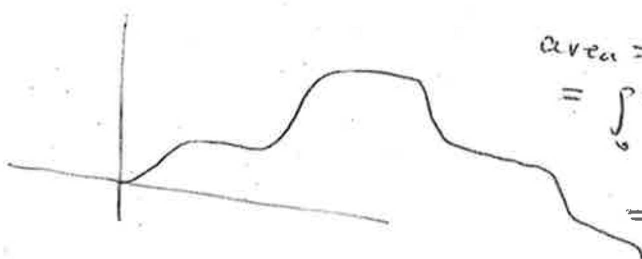
<u>Inhomogeneous case</u> ~~t_0~~ writing code: let t_0 be a variable that gets set early on in code (suggestion: $t_0 = 91 \times \cancel{5}\,4$.)

$\lambda(t)$



$\lambda(t)$





area $= \Lambda(t_0)$

$= \int_0^{t_0} \lambda(t)\,dt$

$= t_0 \cdot \underbrace{\left(\frac{1}{t_0} \int_0^{t_0} \lambda(t)\,dt\right)}_{\text{average value of } \lambda(t)}$

$$\Lambda(t) = \int_0^{t} \lambda(s)\,ds$$

$$= \begin{cases} \lambda_1 t & 0 < t < \frac{t_0}{4} \\ \frac{t_0}{4}\lambda_1 + \left(t - \frac{t_0}{4}\right)\lambda_2 & \frac{t_0}{4} < t < \frac{t_0}{2} \\ \vdots \end{cases}$$

()

$$\Lambda(t_0) = \frac{t_0}{4}\left(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\right)$$

Generate (1) $N \sim \underbrace{\mathrm{Pois}(\Lambda(t_0))}$

$\mathrm{Pois}\left(\frac{t_0}{4}(\lambda_1 + \cdots + \lambda_4)\right)$

$= \mathrm{Pois}(t_0 \cdot$ average of the λ-values $)$

$$\lambda(t) = \cancel{\frac{1}{t^2}}\; e^{-t}$$

(2) Suppose N has been sampled

and $N = n$.

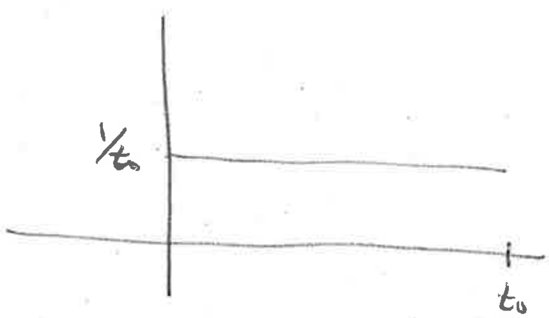
Sample $U_1, \ldots, U_n$ i.i.d.

from dist. with CDF $F(t) = \dfrac{\Lambda(t)}{\Lambda(t_0)}$

$$F(t) = \begin{cases} \left(\dfrac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}\right)\dfrac{4t}{t_0} & 0 < t < \dfrac{t_0}{4} \\ \end{cases}$$

$$f(t) = F'(t) = \begin{cases} \left(\dfrac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}\right)\cdot \dfrac{4}{t_0} & 0 < t < \dfrac{t_0}{4} \\ \end{cases}$$

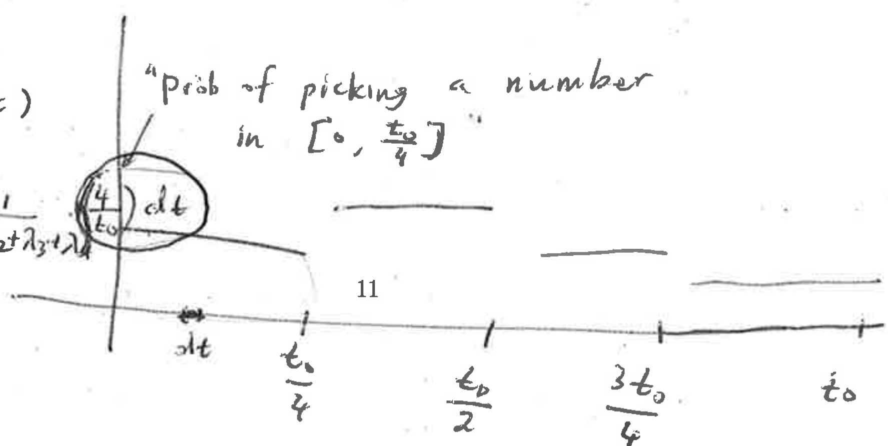
$Unif([0, t_0])$



$f(t)$

prob. of picking first subinterval

$$\left(\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}\right)\left(\frac{4}{t_0}\right) dt$$



$$\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4} + \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}$$