Instructor's Answer Book

To Accompany

Mathematical Methods in the Physical Sciences, 3rd Edition

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Chapter 1

1.1 $(2/3)^{10} = 0.0173$ yd; $6(2/3)^{10} = 0.104$ yd (compared to a total of 5 yd)

1.3   5/9   1.4   9/11   1.5   7/12

1.6   11/18   1.7   5/27   1.8   25/36

1.9   6/7   1.10   15/26   1.11   19/28

1.13   \$1646.99   1.15   Blank area $= 1$

1.16   At $x = 1$: $1/(1+r)$; at $x = 0$: $r/(1+r)$; maximum escape at $x = 0$ is $1/2$.

2.1   1   2.2   1/2   2.3   0

2.4 ∞ 2.5   0   2.6 ∞

2.7 e^2 2.8   0   2.9   1

4.1 $a_n = 1/2^n \to 0$; $S_n = 1 - 1/2^n \to 1$; $R_n = 1/2^n \to 0$

4.2 $a_n = 1/5^{n-1} \to 0$; $S_n = (5/4)(1 - 1/5^n) \to 5/4$; $R_n = 1/(4 \cdot 5^{n-1}) \to 0$

4.3 $a_n = (-1/2)^{n-1} \to 0$; $S_n = (2/3)[1 - (-1/2)^n] \to 2/3$; $R_n = (2/3)(-1/2)^n \to 0$

4.4 $a_n = 1/3^n \to 0$; $S_n = (1/2)(1 - 1/3^n) \to 1/2$; $R_n = 1/(2 \cdot 3^n) \to 0$

4.5 $a_n = (3/4)^{n-1} \to 0$; $S_n = 4[1 - (3/4)^n] \to 4$; $R_n = 4(3/4)^n \to 0$

4.6 $a_n = \dfrac{1}{n(n+1)} \to 0$; $S_n = 1 - \dfrac{1}{n+1} \to 1$; $R_n = \dfrac{1}{n+1} \to 0$

4.7 $a_n = (-1)^{n+1}\left(\dfrac{1}{n} + \dfrac{1}{n+1}\right) \to 0$; $S_n = 1 + \dfrac{(-1)^{n+1}}{n+1} \to 1$; $R_n = \dfrac{(-1)^n}{n+1} \to 0$

5.1   D   5.2   Test further   5.3   Test further

5.4   D   5.5   D   5.6   Test further

5.7   Test further   5.8   Test further

5.9   D   5.10   D

6.5   (a) D   6.5   (b) D

Note: In the following answers, $I = \int^{\infty} a_n\, dn$; $\rho =$ test ratio.

6.7   D, $I = \infty$ 6.8   D, $I = \infty$ 6.9   C, $I = 0$

6.10   C, $I = \pi/6$ 6.11   C, $I = 0$ 6.12   C, $I = 0$

6.13   D, $I = \infty$ 6.14   D, $I = \infty$ 6.18   D, $\rho = 2$

6.19   C, $\rho = 3/4$ 6.20   C, $\rho = 0$ 6.21   D, $\rho = 5/4$

6.22   C, $\rho = 0$ 6.23   D, $\rho = \infty$ 6.24   D, $\rho = 9/8$

6.25   C, $\rho = 0$ 6.26   C, $\rho = (e/3)^3$ 6.27   D, $\rho = 100$

6.28   C, $\rho = 4/27$ 6.29   D, $\rho = 2$ 6.31   D, cf. $\sum n^{-1}$

6.32   D, cf. $\sum n^{-1}$ 6.33   C, cf. $\sum 2^{-n}$ 6.34   C, cf. $\sum n^{-2}$

6.35   C, cf. $\sum n^{-2}$ 6.36   D, cf. $\sum n^{-1/2}$

7.1	C	7.2	D	7.3	C	7.4	C
7.5	C	7.6	D	7.7	C	7.8	C

9.1   D, cf. $\sum n^{-1}$ 9.2   D, $a_n \not\to 0$

9.3   C, I $= 0$ 9.4   D, I $= \infty$, or cf. $\sum n^{-1}$

9.5   C, cf. $\sum n^{-2}$ 9.6   C, $\rho = 1/4$

9.7   D, $\rho = 4/3$ 9.8   C, $\rho = 1/5$

9.9   D, $\rho = e$ 9.10   D, $a_n \not\to 0$

9.11   D, I $= \infty$, or cf. $\sum n^{-1}$ 9.12   C, cf. $\sum n^{-2}$

9.13   C, I $= 0$, or cf. $\sum n^{-2}$ 9.14   C, alt. ser.

9.15   D, $\rho = \infty$, $a_n \not\to 0$ 9.16   C, cf. $\sum n^{-2}$

9.17   C, $\rho = 1/27$ 9.18   C, alt. ser.

9.19   C   9.20   C

9.21   C, $\rho = 1/2$

9.22   (a) C   (b) D   (c) $k > e$

10.1 $|x| < 1$ 10.2 $|x| < 3/2$ 10.3 $|x| \le 1$

10.4 $|x| \le \sqrt{2}$ 10.5   All x 10.6   All x

10.7 $-1 \le x < 1$ 10.8 $-1 < x \le 1$ 10.9 $|x| < 1$

10.10 $|x| \le 1$ 10.11 $-5 \le x < 5$ 10.12 $|x| < 1/2$

10.13 $-1 < x \le 1$ 10.14 $|x| < 3$ 10.15 $-1 < x < 5$

10.16 $-1 < x < 3$ 10.17 $-2 < x \le 0$ 10.18 $-3/4 \le x \le -1/4$

10.19 $|x| < 3$ 10.20   All x 10.21 $0 \le x \le 1$

10.22   No x 10.23 $x > 2$ or $x < -4$ 10.24 $|x| < \sqrt{5}/2$

10.25 $n\pi - \pi/6 < x < n\pi + \pi/6$

13.4 $\dbinom{-1/2}{0} = 1$; $\dbinom{-1/2}{n} = \dfrac{(-1)^n(2n-1)!!}{(2n)!!}$

Answers to part (b), Problems 5 to 19:

13.5 $-\sum_{1}^{\infty} \dfrac{x^{n+2}}{n}$ 13.6 $\sum_{0}^{\infty} \dbinom{1/2}{n} x^{n+1}$ (see Example 2)

13.7 $\sum_{0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n+1)!}$ 13.8 $\sum_{0}^{\infty} \dbinom{-1/2}{n}(-x^2)^n$ (see Problem 13.4)

13.9 $1 + 2\sum_{1}^{\infty} x^n$ 13.10 $\sum_{0}^{\infty} \dfrac{(-1)^n x^{4n+2}}{(2n+1)!}$

13.11 $\sum_{0}^{\infty} \dfrac{(-1)^n x^n}{(2n+1)!}$ 13.12 $\sum_{0}^{\infty} \dfrac{(-1)^n x^{4n+1}}{(2n)!(4n+1)}$

13.13 $\sum_{0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{n!(2n+1)}$ 13.14 $\sum_{0}^{\infty} \dfrac{x^{2n+1}}{2n+1}$

13.15 $\sum_{0}^{\infty} \dbinom{-1/2}{n}(-1)^n \dfrac{x^{2n+1}}{2n+1}$

13.16 $\sum_{0}^{\infty} \dfrac{x^{2n}}{(2n)!}$ 13.17 $2\sum_{\text{odd}\,n}^{\infty} \dfrac{x^n}{n}$

13.18 $\sum_{0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$ 13.19 $\sum_{0}^{\infty} \dbinom{-1/2}{n} \dfrac{x^{2n+1}}{2n+1}$

13.20 $x + x^2 + x^3/3 - x^5/30 - x^6/90 \cdots$

13.21 $x^2 + 2x^4/3 + 17x^6/45 \cdots$

13.22 $1 + 2x + 5x^2/2 + 8x^3/3 + 65x^4/24 \cdots$

13.23 $1 - x + x^3 - x^4 + x^6 \cdots$

13.24 $1 + x^2/2! + 5x^4/4! + 61x^6/6! \cdots$

13.25 $1 - x + x^2/3 - x^4/45 \cdots$

13.26 $1 + x^2/4 + 7x^4/96 + 139x^6/5760 \cdots$

13.27 $1 + x + x^2/2 - x^4/8 - x^5/15 \cdots$

13.28 $x - x^2/2 + x^3/6 - x^5/12 \cdots$

13.29 $1 + x/2 - 3x^2/8 + 17x^3/48 \cdots$

13.30 $1 - x + x^2/2 - x^3/2 + 3x^4/8 - 3x^5/8 \cdots$

13.31 $1 - x^2/2 - x^3/2 - x^4/4 - x^5/24 \cdots$

13.32 $x + x^2/2 - x^3/6 - x^4/12 \cdots$

13.33 $1 + x^3/6 + x^4/6 + 19x^5/120 + 19x^6/120 \cdots$

13.34 $x - x^2 + x^3 - 13x^4/12 + 5x^5/4 \cdots$

13.35 $1 + x^2/3! + 7x^4/(3 \cdot 5!) + 31x^6/(3 \cdot 7!) \cdots$

13.36 $u^2/2 + u^4/12 + u^6/20 \cdots$

13.37 $-(x^2/2 + x^4/12 + x^6/45 \cdots)$

13.38 $e(1 - x^2/2 + x^4/6 \cdots)$

13.39 $1 - (x - \pi/2)^2/2! + (x - \pi/2)^4/4! \cdots$

13.40 $1 - (x - 1) + (x - 1)^2 - (x - 1)^3 \cdots$

13.41 $e^3[1 + (x - 3) + (x - 3)^2/2! + (x - 3)^3/3! \cdots]$

13.42 $-1 + (x - \pi)^2/2! - (x - \pi)^4/4! \cdots$

13.43 $-[(x - \pi/2) + (x - \pi/2)^3/3 + 2(x - \pi/2)^5/15 \cdots]$

13.44 $5 + (x - 25)/10 - (x - 25)^2/10^3 + (x - 25)^3/(5 \cdot 10^4) \cdots$

14.6 Error $< (1/2)(0.1)^2 \div (1 - 0.1) < 0.0056$

14.7 Error $< (3/8)(1/4)^2 \div (1 - \frac{1}{4}) = 1/32$

14.8 For $x < 0$, error $< (1/64)(1/2)^4 < 0.001$
For $x > 0$, error $< 0.001 \div (1 - \frac{1}{2}) = 0.002$

14.9 Term $n + 1$ is $a_{n+1} = \frac{1}{(n+1)(n+2)}$, so $R_n = (n + 2)a_{n+1}$.

14.10 $S_4 = 0.3052$, error < 0.0021 (cf. $S = 1 - \ln 2 = 0.307$)

15.1 $-x^4/24 - x^5/30 \cdots \simeq -3.376 \times 10^{-16}$

15.2 $x^8/3 - 14x^{12}/45 \cdots \simeq 1.433 \times 10^{-16}$

15.3 $x^5/15 - 2x^7/45 \cdots \simeq 6.667 \times 10^{-17}$

15.4 $x^3/3 + 5x^4/6 \cdots \simeq 1.430 \times 10^{-11}$

15.5 0   15.6 12   15.7 10!

15.8 1/2   15.9 $-1/6$ 15.10 -1

15.11 4   15.12 1/3   15.13 -1

15.14 $t - t^3/3$, error $< 10^{-6}$ 15.15 $\frac{2}{3}t^{3/2} - \frac{2}{5}t^{5/2}$, error $< \frac{1}{7}10^{-7}$

15.16 $e^2 - 1$ 15.17 $\cos \frac{\pi}{2} = 0$

15.18 $\ln 2$ 15.19 $\sqrt{2}$

15.20 (a) 1/8   (b) $5e$ (c) 9/4

15.21 (a) 0.397117   (b) 0.937548   (c) 1.291286

15.22 (a) $\pi^4/90$ (b) 1.202057   (c) 2.612375

15.23 (a) 1/2   (b) 1/6   (c) 1/3   (d) $-1/2$

15.24 (a) $-\pi$ (b) 0   (c) -1
(d) 0   (e) 0   (f) 0

15.27 (a) $1 - \frac{v}{c} = 1.3 \times 10^{-5}$, or $v = 0.999987c$
(b) $1 - \frac{v}{c} = 5.2 \times 10^{-7}$
(c) $1 - \frac{v}{c} = 2.1 \times 10^{-10}$
(d) $1 - \frac{v}{c} = 1.3 \times 10^{-11}$

15.28 $mc^2 + \frac{1}{2}mv^2$

15.29 (a) $F/W = \theta + \theta^3/3 \cdots$
(b) $F/W = x/l + x^3/(2l^3) + 3x^5/(8l^5) \cdots$

15.30 (a) $T = F(5/x + x/40 - x^3/16000 \cdots)$
　　　 (b) $T = \frac{1}{2}(F/\theta)(1 + \theta^2/6 + 7\theta^4/360 \cdots)$
15.31 (a) finite　　　　　　　　(b) infinite

16.1　(c) overhang:　　　　　　　2　　　3　　　　　10　　　　　　100
　　　　　books needed:　　　32　　228　　2.7×10^8　　4×10^{86}
16.4　C, $\rho = 0$　　　　　　16.5　D, $a_n \nrightarrow 0$　　　　　16.6　C, cf. $\sum n^{-3/2}$
16.7　D, $I = \infty$　　　　　16.8　D, cf. $\sum n^{-1}$　　　　16.9　$-1 \leq x < 1$
16.10 $|x| < 4$　　　　　　16.11 $|x| \leq 1$　　　　　　16.12 $|x| < 5$
16.13 $-5 < x \leq 1$
16.14 $1 - x^2/2 + x^3/2 - 5x^4/12 \cdots$
16.15 $-x^2/6 - x^4/180 - x^6/2835 \cdots$
16.16 $1 - x/2 + 3x^2/8 - 11x^3/48 + 19x^4/128 \cdots$
16.17 $1 + x^2/2 + x^4/4 + 7x^6/48 \cdots$
16.18 $x - x^3/3 + x^5/5 - x^7/7 \cdots$
16.19 $-(x - \pi) + (x - \pi)^3/3! - (x - \pi)^5/5! \cdots$
16.20 $2 + (x - 8)/12 - (x - 8)^2/(2^5 \cdot 3^2) + 5(x - 8)^3/(2^8 \cdot 3^4) \cdots$
16.21 $e[1 + (x - 1) + (x - 1)^2/2! + (x - 1)^3/3! \cdots]$
16.22 arc tan $1 = \pi/4$　　　　　　　16.23 $1 - (\sin\pi)/\pi = 1$
16.24 $e^{\ln 3} - 1 = 2$　　　　　　　16.25 -2
16.26 $-1/3$　　　　　　　　　　16.27 $2/3$
16.28 1　　　　　　　　　　　　16.29 $6!$
16.30 (b) For $N = 130$, $10.5821 < \zeta(1.1) < 10.5868$
16.31 (a) 10^{430} terms. For $N = 200$, $100.5755 < \zeta(1.01) < 100.5803$
16.31 (b) 2.66×10^{86} terms.　　For $N = 15$, $1.6905 < S < 1.6952$
16.31 (c) $e^{e^{200}} = 10^{3.1382 \times 10^{86}}$ terms.　　For $N = 40$, $38.4048 < S < 38.4088$

Chapter 2

	x	y	r	θ
4.1	1	1	$\sqrt{2}$	$\pi/4$
4.2	-1	1	$\sqrt{2}$	$3\pi/4$
4.3	1	$-\sqrt{3}$	2	$-\pi/3$
4.4	$-\sqrt{3}$	1	2	$5\pi/6$
4.5	0	2	2	$\pi/2$
4.6	0	-4	4	$-\pi/2$
4.7	-1	0	1	π
4.8	3	0	3	0
4.9	-2	2	$2\sqrt{2}$	$3\pi/4$
4.10	2	-2	$2\sqrt{2}$	$-\pi/4$
4.11	$\sqrt{3}$	1	2	$\pi/6$
4.12	-2	$-2\sqrt{3}$	4	$-2\pi/3$
4.13	0	-1	1	$3\pi/2$
4.14	$\sqrt{2}$	$\sqrt{2}$	2	$\pi/4$
4.15	-1	0	1	$-\pi$ or π
4.16	5	0	5	0
4.17	1	-1	$\sqrt{2}$	$-\pi/4$
4.18	0	3	3	$\pi/2$
4.19	4.69	1.71	5	$20° = 0.35$
4.20	-2.39	-6.58	7	$-110° = -1.92$
5.1	$1/2$	$-1/2$	$1/\sqrt{2}$	$-\pi/4$
5.2	$-1/2$	$-1/2$	$1/\sqrt{2}$	$-3\pi/4$ or $5\pi/4$
5.3	1	0	1	0
5.4	0	2	2	$\pi/2$
5.5	2	$2\sqrt{3}$	4	$\pi/3$
5.6	-1	0	1	π
5.7	$7/5$	$-1/5$	$\sqrt{2}$	$-8.13° = -0.14$
5.8	1.6	-2.7	3.14	$-59.3° = -1.04$
5.9	-10.4	22.7	25	$2 = 114.6°$
5.10	$-25/17$	$19/17$	$\sqrt{58/17}$	$142.8° = 2.49$
5.11	17	-12	20.8	$-35.2° = -0.615$
5.12	2.65	1.41	3	$28° = 0.49$
5.13	1.55	4.76	5	$2\pi/5$
5.14	1.27	-2.5	2.8	$-1.1 = -63°$
5.15	$21/29$	$-20/29$	1	$-43.6° = -0.76$
5.16	1.53	-1.29	2	$-40° = -0.698$
5.17	-7.35	-10.9	13.1	$-124° = -2.16$
5.18	-0.94	-0.36	1	$201°$ or $-159°$, 3.51 or -2.77

5.19 $(2+3i)/13$; $(x-yi)/(x^2+y^2)$
5.20 $(-5+12i)/169$; $(x^2-y^2-2ixy)/(x^2+y^2)^2$
5.21 $(1+i)/6$; $(x+1-iy)/[(x+1)^2+y^2]$
5.22 $(1+2i)/10$; $[x-i(y-1)]/[x^2+(y-1)^2]$
5.23 $(-6-3i)/5$; $(1-x^2-y^2+2yi)/[(1-x)^2+y^2]$
5.24 $(-5-12i)/13$; $(x^2-y^2+2ixy)/(x^2+y^2)$

5.26   1   5.27 $\sqrt{13/2}$ 5.28   1
5.29 $5\sqrt{5}$ 5.30 $3/2$ 5.31   1
5.32   169   5.33   5   5.34   1
5.35 $x=-4,\ y=3$ 5.36 $x=-1/2,\ y=3$
5.37 $x=y=0$ 5.38 $x=-7,\ y=2$
5.39 $x=y=$ any real number   5.40 $x=0,\ y=3$
5.41 $x=1,\ y=-1$ 5.42 $x=-1/7,\ y=-10/7$
5.43 $(x,y)=(0,0)$, or $(1,1)$, or $(-1,1)$
5.44 $x=0,\ y=-2$
5.45 $x=0$, any real y; or $y=0$, any real x
5.46 $y=-x$
5.47 $(x,y)=(-1,0),\ (1/2,\pm\sqrt{3}/2)$
5.48 $x=36/13,\ y=2/13$
5.49 $x=1/2,\ y=0$
5.50 $x=0,\ y\geq 0$
5.51   Circle, center at origin, radius $=2$
5.52 y axis
5.53   Circle, center at $(1,0)$, $r=1$
5.54   Disk, center at $(1,0)$, $r=1$
5.55   Line $y=5/2$
5.56   Positive y axis
5.57   Hyperbola, $x^2-y^2=4$
5.58   Half plane, $x>2$
5.59   Circle, center at $(0,-3)$, $r=4$
5.60   Circle, center at $(1,-1)$, $r=2$
5.61   Half plane, $y<0$
5.62   Ellipse, foci at $(1,0)$ and $(-1,0)$, semi-major axis $=4$
5.63   The coordinate axes
5.64   Straight lines, $y=\pm x$
5.67 $v=(4t^2+1)^{-1},\ a=4(4t^2+1)^{-3/2}$
5.68   Motion around circle $r=1$, with $v=2,\ a=4$

6.2 $D,\ \rho=\sqrt{2}$ 6.3 $C,\ \rho=1/\sqrt{2}$ 6.4 $D,\ |a_n|=1\not\to 0$
6.5 D 6.6 C 6.7 $D,\ \rho=\sqrt{2}$
6.8 $D,\ |a_n|=1\not\to 0$ 6.9 C 6.10 $C,\ \rho=\sqrt{2}/2$
6.11 $C,\ \rho=1/5$ 6.12 C 6.13 $C,\ \rho=\sqrt{2/5}$

7.1   All z 7.2 $|z|<1$ 7.3   All z
7.4 $|z|<1$ 7.5 $|z|<2$ 7.6 $|z|<1/3$
7.7   All z 7.8   All z 7.9 $|z|<1$
7.10 $|z|<1$ 7.11 $|z|<27$ 7.12 $|z|<4$
7.13 $|z-i|<1$ 7.14 $|z-2i|<1$ 7.15 $|z-(2-i)|<2$
7.16 $|z+(i-3)|<1/\sqrt{2}$

8.3   See Problem 17.30.

9.1 $(1-i)/\sqrt{2}$ 9.2 i 9.3 $-9i$

9.4 $-e(1+i\sqrt{3})/2$ 9.5 -1 9.6 1

9.7 $3e^2$ 9.8 $-\sqrt{3}+i$ 9.9 $-2i$

9.10 -2 9.11 $-1-i$ 9.12 $-2-2i\sqrt{3}$

9.13 $-4+4i$ 9.14 64 9.15 $2i-4$

9.16 $-2\sqrt{3}-2i$ 9.17 $-(1+i)/4$ 9.18 1

9.19 16 9.20 i 9.21 1

9.22 $-i$ 9.23 $(\sqrt{3}+i)/4$ 9.24 $4i$

9.25 -1 9.26 $(1+i\sqrt{3})/2$ 9.29 1

9.30 $e^{\sqrt{3}}$ 9.31 5 9.32 $3e^2$

9.33 $2e^3$ 9.34 $4/e$ 9.35 21

9.36 4 9.37 1 9.38 $1/\sqrt{2}$

10.1 $1,\ (-1\pm i\sqrt{3})/2$ 10.2 $3,\ 3(-1\pm i\sqrt{3})/2$

10.3 $\pm1,\ \pm i$ 10.4 $\pm2,\ \pm2i$

10.5 $\pm1,\ (\pm1\pm i\sqrt{3})/2$ 10.6 $\pm2,\ \pm1\pm i\sqrt{3}$

10.7 $\pm\sqrt{2},\ \pm i\sqrt{2},\ \pm1\pm i$ 10.8 $\pm1,\ \pm i,\ (\pm1\pm i)/\sqrt{2}$

10.9 $1,\ 0.309\pm0.951i,\ -0.809\pm0.588i$

10.10 $2,\ 0.618\pm1.902i,\ -1.618\pm1.176i$

10.11 $-2,\ 1\pm i\sqrt{3}$ 10.12 $-1,\ \left(1\pm i\sqrt{3}\right)/2$

10.13 $\pm1\pm i$ 10.14 $(\pm1\pm i)/\sqrt{2}$

10.15 $\pm2i,\ \pm\sqrt{3}\pm i$ 10.16 $\pm i,\ (\pm\sqrt{3}\pm i)/2$

10.17 $-1,\ 0.809\pm0.588i,\ -0.309\pm0.951i$

10.18 $\pm(1+i)/\sqrt{2}$ 10.19 $-i,\ (\pm\sqrt{3}+i)/2$

10.20 $2i,\ \pm\sqrt{3}-i$ 10.21 $\pm(\sqrt{3}+i)$

10.22 $r=\sqrt{2},\ \theta=45°+120°n:\ 1+i,\ -1.366+0.366i,\ 0.366-1.366i$

10.23 $r=2,\ \theta=30°+90°n:\ \pm(\sqrt{3}+i),\ \pm\left(1-i\sqrt{3}\right)$

10.24 $r=1,\ \theta=30°+45°n:$
$\pm(\sqrt{3}+i)/2,\ \pm\left(1-i\sqrt{3}\right)/2,\ \pm(0.259+0966i),\ \pm(0.966-0.259i)$

10.25 $r=\sqrt[10]{2},\ \theta=45°+72°n:\ 0.758(1+i),\ -0.487+0.955i,$
$-1.059-0.168i,\ -0.168-1.059i,\ 0.955-0.487i$

10.26 $r=1,\ \theta=18°+72°n:\ i,\ \pm0.951+0.309i,\ \pm0.588-0.809i$

10.28 $\cos3\theta=\cos^3\theta-3\cos\theta\sin^2\theta$
$\sin3\theta=3\cos^2\theta\sin\theta-\sin^3\theta$

11.3 $3(1-i)/\sqrt{2}$ 11.4 -8 11.5 $1+i$ 11.6 $13/5$

11.7 $3i/5$ 11.8 $-41/9$ 11.9 $4i/3$ 11.10 -1

12.20 $\cosh3z=\cosh^3z+3\cosh z\sinh^2z,\ \ \sinh3z=3\cosh^2z\sinh z+\sinh^3z$

12.22 $\sinh x=\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}\ ,\ \ \cosh x=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$

12.23 $\cos x,\ |\cos x|$

12.24 $\cosh x$

12.25 $\sin x\cosh y-i\cos x\sinh y,\ \ \sqrt{\sin^2x+\sinh^2y}$

12.26 $\cosh2\cos3-i\sinh2\sin3=-3.725-0.512i,\ 3.760$

12.27 $\sin4\cosh3+i\cos4\sinh3=-7.62-6.55i,\ 10.05$

12.28 $\tanh1=0.762$ 12.29 1

12.30 $-i$ 12.31 $(3+5i\sqrt{3})/8$

12.32 $-4i/3$ 12.33 $i\tanh1=0.762i$

12.34 $i\sinh(\pi/2)=2.301i$ 12.35 $-\cosh2=-3.76$

12.36 $i\cosh1=1.543i$ 12.37 $\cosh\pi$

14.1 $1 + i\pi$

14.2 $-i\pi/2$ or $3\pi i/2$

14.3 $\text{Ln}\,2 + i\pi/6$

14.4 $(1/2)\,\text{Ln}\,2 + 3\pi i/4$

14.5 $\text{Ln}\,2 + 5i\pi/4$

14.6 $-i\pi/4$ or $7\pi i/4$

14.7 $i\pi/2$

14.8 $-1,\ (1 \pm i\sqrt{3})/2$

14.9 $e^{-\pi}$

14.10 $e^{-\pi^2/4}$

14.11 $\cos(\text{Ln}\,2) + i\sin(\text{Ln}\,2) = 0.769 + 0.639i$

14.12 $-ie^{-\pi/2}$

14.13 $1/e$

14.14 $2e^{-\pi/2}[i\cos(\text{Ln}\,2) - \sin(\text{Ln}\,2)] = 0.3198i - 0.2657$

14.15 $e^{-\pi\sinh 1} = 0.0249$

14.16 $e^{-\pi/3} = 0.351$

14.17 $\sqrt{2}\,e^{-3\pi/4}e^{i(\text{Ln}\sqrt{2}+3\pi/4)} = -0.121 + 0.057i$

14.18 -1

14.19 $-5/4$

14.20 1

14.21 -1

14.22 $-1/2$

14.23 $e^{\pi/2} = 4.81$

15.1 $\pi/2 + 2n\pi \pm i\,\text{Ln}\left(2 + \sqrt{3}\right) = \pi/2 + 2n\pi \pm 1.317i$

15.2 $\pi/2 + n\pi + (i\,\text{Ln}\,3)/2$

15.3 $i(\pm\pi/3 + 2n\pi)$

15.4 $i(2n\pi + \pi/6),\ i(2n\pi + 5\pi/6)$

15.5 $\pm\left[\pi/2 + 2n\pi - i\,\text{Ln}\left(3 + \sqrt{8}\right)\right] = \pm[\pi/2 + 2n\pi - 1.76i]$

15.6 $i(n\pi - \pi/4)$

15.7 $\pi/2 + n\pi - i\,\text{Ln}(\sqrt{2} - 1) = \pi/2 + n\pi + 0.881i$

15.8 $\pi/2 + 2n\pi \pm i\,\text{Ln}\,3$

15.9 $i(\pi/3 + n\pi)$

15.10 $2n\pi \pm i\,\text{Ln}\,2$

15.11 $i(2n\pi + \pi/4),\ \ i(2n\pi + 3\pi/4)$

15.12 $i(2n\pi \pm \pi/6)$

15.13 $i(\pi + 2n\pi)$

15.14 $2n\pi + i\,\text{Ln}\,2,\ \ (2n+1)\pi - i\,\text{Ln}\,2$

15.15 $n\pi + 3\pi/8 + i\,\text{Ln}\,2)/4$

15.16 $(\text{Ln}\,2)/4 + i(n\pi + 5\pi/8)$

16.2 Motion around circle $|z| = 5$; $v = 5\omega$, $a = 5\omega^2$.

16.3 Motion around circle $|z| = \sqrt{2}$; $v = \sqrt{2}$, $a = \sqrt{2}$.

16.4 $v = \sqrt{13}$, $a = 0$

16.5 $v = |z_1 - z_2|$, $a = 0$

16.6 (a) Series: $3 - 2i$ (b) Series: $2(1 + i\sqrt{3})$

   Parallel: $5 + i$ Parallel: $i\sqrt{3}$

16.7 (a) Series: $1 + 2i$ (b) Series: $5 + 5i$

   Parallel: $3(3 - i)/5$ Parallel: $1.6 + 1.2i$

16.8 $\left[R - i(\omega CR^2 + \omega^3 L^2 C - \omega L)\right] / \left[(\omega CR)^2 + (\omega^2 LC - 1)^2\right]$; this

   simplifies to $\dfrac{L}{RC}$ if $\omega^2 = \dfrac{1}{LC}\left(1 - \dfrac{R^2 C}{L}\right)$, that is, at resonance.

16.9 (a) $\omega = \dfrac{R}{2L} \pm \sqrt{\dfrac{R^2}{4L^2} + \dfrac{1}{LC}}$ (b) $\omega = 1/\sqrt{LC}$

16.10 (a) $\omega = -\dfrac{1}{2RC} \pm \sqrt{\dfrac{1}{4R^2C^2} + \dfrac{1}{LC}}$ (b) $\omega = 1/\sqrt{LC}$

16.12 $(1 + r^4 - 2r^2\cos\theta)^{-1}$

17.1 -1 17.2 $(\sqrt{3}+i)/2$

17.3 $r = \sqrt{2}$, $\theta = 45° + 72°n : 1 + i$, $-0.642 + 1.260i$, $-1.397 - 0.221i$, $-0.221 - 1.397i$, $1.260 - 0.642i$

17.4 $i\cosh 1 = 1.54i$ 17.5 i

17.6 $-e^{-\pi^2} = -5.17 \times 10^{-5}$ or $-e^{-\pi^2} \cdot e^{\pm 2n\pi^2}$

17.7 $e^{\pi/2} = 4.81$ or $e^{\pi/2} \cdot e^{\pm 2n\pi}$

17.8 -1 17.9 $\pi/2 \pm 2n\pi$

17.10 $\sqrt{3} - 2$ 17.11 i

17.12 $-1 \pm \sqrt{2}$ 17.13 $x = 0$, $y = 4$

17.14 Circle with center $(0,\ 2)$, radius 1

17.15 $|z| < 1/e$ 17.16 $y < -2$

17.26 1   17.27 (c) $e^{-2(x-t)^2}$

17.28 $1 + \left[\dfrac{a^2 + b^2}{2ab}\right]^2 \sinh^2 b$ 17.29 $\left(-1 \pm i\sqrt{3}\right)/2$

17.30 $e^x \cos x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n 2^{n/2} \cos n\pi/4}{n!}$

$\quad\quad e^x \sin x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n 2^{n/2} \sin n\pi/4}{n!}$

Chapter 3

2.3 $\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \end{pmatrix}$, $\quad x = -3, y = 5)$

2.4 $\begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 0 & 1 \end{pmatrix}$, $\quad x = (z+1)/2, y = 1$

2.5 $\begin{pmatrix} 1 & 1/2 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $\quad$ no solution

2.6 $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$, $\quad x = 1, z = y$

2.7 $\begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$, $\quad x = -4, y = 3$

2.8 $\begin{pmatrix} 1 & -1 & 0 & -11 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $\quad x = y - 11, z = 7$

2.9 $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $\quad$ inconsistent, no solution

2.10 $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $\quad$ inconsistent, no solution

2.11 $\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{pmatrix}$, $\quad x = 2, y = -1, z = -3$

2.12 $\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$, $\quad x = -2, y = 1, z = 1$

2.13 $\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & -2 & 5/2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $\quad x = -2, y = 2z + 5/2$

2.14 $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $\quad$ inconsistent, no solution

2.15 $R = 2$ 2.16 $R = 3$
2.17 $R = 2$ 2.18 $R = 3$

3.1 -11 3.2 -721 3.3 1 3.4 2140
3.5 -544 3.6 4 3.11 0 3.12 16
3.16 $A = -(K + ik)/(K - ik), |A| = 1$
3.17 $x = \gamma(x' + vt'), t = \gamma(t' + vx'/c^2)$
3.18 $D = 3b(a + b)(a^2 + ab + b^2), z = 1$
       (Also $x = a + 2b$, $y = a - b$; these were not required.)

4.11 $-3\mathbf{i} + 8\mathbf{j} - 6\mathbf{k}, \mathbf{i} - 10\mathbf{j} + 3\mathbf{k}, 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.
4.12 $\arccos(-1/\sqrt{2}) = 3\pi/4$
4.13 $-5/3, -1, \cos\theta = -1/3$
4.14   (a) $\arccos(1/3) = 70.5°$
       (b) $\arccos(1/\sqrt{3}) = 54.7°$
       (c) $\arccos\sqrt{2/3} = 35.3°$
4.15   (a) $(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})/3$
       (b) $8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}$
       (c) Any combination of $\mathbf{i} + 2\mathbf{j}$ and $\mathbf{i} - \mathbf{k}$.
       (d) Answer to (c) divided by its magnitude.
4.17   Legs = any two vectors with dot product = 0;
       hypotenuse = their sum (or difference).
4.18 $2\mathbf{i} - 8\mathbf{j} - 3\mathbf{k}$ 4.19 $\mathbf{i} + \mathbf{j} + \mathbf{k}$
4.20 $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ 4.22   Law of cosines
4.24 $A^2 B^2$

In the following answers, note that the point and vector used may be
any point on the line and *any* vector along the line.
5.1 $\mathbf{r} = (2, -3) + (4, 3)t$ 5.2 $3/2$
5.3 $\mathbf{r} = (3, 0) + (1, 1)t$ 5.4 $\mathbf{r} = (1, 0) + (2, 1)t$
5.5 $\mathbf{r} = \mathbf{j}\, t$
5.6 $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z+5}{2};$ $\mathbf{r} = (1, -1, -5) + (1, -2, 2)t$
5.7 $\frac{x-2}{3} = \frac{y-3}{-2} = \frac{z-4}{-6};$ $\mathbf{r} = (2, 3, 4) + (3, -2, -6)t$
5.8 $\frac{x}{3} = \frac{z-4}{-5}, y = -2;$ $\mathbf{r} = (0, -2, 4) + (3, 0, -5)t$
5.9 $x = -1, z = 7;$ $\mathbf{r} = -\mathbf{i} + 7\mathbf{k} + \mathbf{j}t$
5.10 $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z+1}{6};$ $\mathbf{r} = (3, 4, -1) + (2, -3, 6)t$
5.11 $\frac{x-4}{1} = \frac{z-3}{-2}, y = -1;$ $\mathbf{r} = (4, -1, 3) + (1, 0, -2)t$
5.12 $\frac{x-5}{5} = \frac{y+4}{-2} = \frac{z-2}{1};$ $\mathbf{r} = (5, -4, 2) + (5, -2, 1)t$
5.13 $x = 3, \frac{y}{-3} = \frac{z+5}{1};$ $\mathbf{r} = 3\mathbf{i} - 5\mathbf{k} + (-3\mathbf{j} + \mathbf{k})t$
5.14 $36x - 3y - 22z = 23$ 5.15 $5x + 6y + 3z = 0$
5.16 $5x - 2y + z = 35$ 5.17 $3y - z = 5$
5.18 $x + 6y + 7z + 5 = 0$ 5.19 $x + y + 3z + 12 = 0$
5.20 $x - 4y - z + 5 = 0$
5.21 $\cos\theta = 25/\left(7\sqrt{30}\right) = 0.652, \theta = 49.3°$
5.22 $\cos\theta = 2/\sqrt{6}, \theta = 35.3°$
5.23 $\cos\theta = 4/21, \theta = 79°$
5.24 $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + (\mathbf{j} + 2\mathbf{k})t, d = 2\sqrt{6/5}$
5.25 $\mathbf{r} = (1, -2, 0) + (4, 9, -1)t, d = \left(3\sqrt{3}\right)/7$
5.26 $\mathbf{r} = (8, 1, 7) + (14, 2, 15)t, \; d = \sqrt{2/17}$
5.27 $y + 2z + 1 = 0$ 5.28 $4x + 9y - z + 27 = 0$
5.29 $2/\sqrt{6}$ 5.30 1
5.31 $5/7$ 5.32 $10/\sqrt{27}$

5.33 $\sqrt{43/15}$ 5.34 $\sqrt{11/10}$

5.35 $\sqrt{5}$ 5.36 3

5.37 Intersect at $(1, -3, 4)$ 5.38 $\arccos\sqrt{21/22} = 12.3°$

5.39 $t_1 = 1$, $t_2 = -2$, intersect at $(3, 2, 0)$, $\cos\theta = 5/\sqrt{60}$, $\theta = 49.8°$

5.40 $t_1 = -1$, $t_2 = 1$, intersect at $(4, -1, 1)$, $\cos\theta = 5/\sqrt{39}$, $\theta = 36.8°$

5.41 $\sqrt{14}$ 5.42 $1/\sqrt{5}$

5.43 $20/\sqrt{21}$ 5.44 $2/\sqrt{10}$

5.45 $d = \sqrt{2}$, $t = -1$

6.1 $AB = \begin{pmatrix} -5 & 10 \\ 1 & 24 \end{pmatrix}$ $BA = \begin{pmatrix} -2 & 8 \\ 11 & 21 \end{pmatrix}$ $A + B = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$

$A - B = \begin{pmatrix} 5 & -1 \\ 1 & 1 \end{pmatrix}$ $A^2 = \begin{pmatrix} 11 & 8 \\ 16 & 27 \end{pmatrix}$ $B^2 = \begin{pmatrix} 6 & 4 \\ 2 & 18 \end{pmatrix}$

$5A = \begin{pmatrix} 15 & 5 \\ 10 & 25 \end{pmatrix}$ $3B = \begin{pmatrix} -6 & 6 \\ 3 & 12 \end{pmatrix}$ $\det(5A) = 5^2 \det A$

6.2 $AB = \begin{pmatrix} -2 & -2 \\ 1 & 2 \end{pmatrix}$ $BA = \begin{pmatrix} -6 & 17 \\ -2 & 6 \end{pmatrix}$ $A + B = \begin{pmatrix} 1 & -1 \\ -1 & 5 \end{pmatrix}$

$A - B = \begin{pmatrix} 3 & -9 \\ -1 & 1 \end{pmatrix}$ $A^2 = \begin{pmatrix} 9 & -25 \\ -5 & 14 \end{pmatrix}$ $B^2 = \begin{pmatrix} 1 & 4 \\ 0 & 4 \end{pmatrix}$

$5A = \begin{pmatrix} 10 & -25 \\ -5 & 15 \end{pmatrix}$ $3B = \begin{pmatrix} -3 & 12 \\ 0 & 6 \end{pmatrix}$

6.3 $AB = \begin{pmatrix} 7 & -1 & 0 \\ 3 & 1 & -1 \\ 3 & 9 & 5 \end{pmatrix}$ $BA = \begin{pmatrix} 4 & -1 & 2 \\ 6 & 3 & 1 \\ 0 & 1 & 6 \end{pmatrix}$ $A + B = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 1 \\ 3 & 4 & 1 \end{pmatrix}$

$A - B = \begin{pmatrix} 0 & -1 & 2 \\ 3 & -3 & -1 \\ -3 & 6 & 1 \end{pmatrix}$ $A^2 = \begin{pmatrix} 1 & 10 & 4 \\ 0 & 1 & 6 \\ 15 & 0 & 1 \end{pmatrix}$ $B^2 = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 3 & 2 \\ 3 & 1 & -1 \end{pmatrix}$

$5A = \begin{pmatrix} 5 & 0 & 10 \\ 15 & -5 & 0 \\ 0 & 25 & 5 \end{pmatrix}$ $3B = \begin{pmatrix} 3 & 3 & 0 \\ 0 & 6 & 3 \\ 9 & -3 & 0 \end{pmatrix}$ $\det(5A) = 5^3 \det A$

6.4 $BA = \begin{pmatrix} 12 & 10 & 2 & 12 \\ 0 & 2 & 1 & -9 \\ 4 & 8 & 3 & -17 \end{pmatrix}$ $C^2 = \begin{pmatrix} 5 & 1 & 7 \\ 6 & 5 & 12 \\ -3 & -1 & -2 \end{pmatrix}$

$CB = \begin{pmatrix} 14 & 4 \\ 1 & 19 \\ 1 & -5 \end{pmatrix}$ $C^3 = \begin{pmatrix} 7 & 4 & 20 \\ 20 & 1 & 20 \\ -8 & -2 & -9 \end{pmatrix}$

$C^2 B = \begin{pmatrix} 32 & 12 \\ 53 & 7 \\ -13 & -9 \end{pmatrix}$ $CBA = \begin{pmatrix} 36 & 46 & 14 & -36 \\ 40 & 22 & 1 & 91 \\ -8 & -2 & 1 & -29 \end{pmatrix}$

6.5 $AA^T = \begin{pmatrix} 30 & -13 \\ -13 & 30 \end{pmatrix}$ $A^T A = \begin{pmatrix} 8 & 8 & 2 & 2 \\ 8 & 10 & 3 & -7 \\ 2 & 3 & 1 & -4 \\ 2 & -7 & -4 & 41 \end{pmatrix}$

$BB^T = \begin{pmatrix} 20 & -2 & 2 \\ -2 & 2 & 4 \\ 2 & 4 & 10 \end{pmatrix}$ $B^T B = \begin{pmatrix} 14 & 4 \\ 4 & 18 \end{pmatrix}$

$CC^T = \begin{pmatrix} 14 & 1 & 1 \\ 1 & 21 & -6 \\ 1 & -6 & 2 \end{pmatrix}$ $C^T C = \begin{pmatrix} 21 & -2 & -3 \\ -2 & 2 & 5 \\ -3 & 5 & 14 \end{pmatrix}$

6.8 $\quad 5x^2 + 3y^2 = 30$

6.9 $\quad AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad BA = \begin{pmatrix} 22 & 44 \\ -11 & -22 \end{pmatrix}$

6.10 $\quad AC = A\,D = \begin{pmatrix} 11 & 12 \\ 33 & 36 \end{pmatrix}$

6.13 $\quad \begin{pmatrix} 5/3 & -3 \\ -1 & 2 \end{pmatrix}$
6.14 $\quad \dfrac{1}{6}\begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix}$

6.15 $\quad -\dfrac{1}{2}\begin{pmatrix} 4 & 5 & 8 \\ -2 & -2 & -2 \\ 2 & 3 & 4 \end{pmatrix}$
6.16 $\quad \dfrac{1}{8}\begin{pmatrix} -2 & 1 & 1 \\ 6 & -3 & 5 \\ 4 & 2 & 2 \end{pmatrix}$

6.17 $\quad A^{-1} = \dfrac{1}{6}\begin{pmatrix} 2 & 2 & -1 \\ 4 & 4 & -5 \\ 8 & 2 & -4 \end{pmatrix}$
$\qquad B^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$

$\qquad B^{-1}AB = \begin{pmatrix} 3 & 1 & 2 \\ -2 & -2 & -2 \\ -2 & -1 & 0 \end{pmatrix}$
$\qquad B^{-1}A^{-1}B = \dfrac{1}{6}\begin{pmatrix} 2 & 2 & -2 \\ -4 & -4 & -2 \\ 2 & -1 & 4 \end{pmatrix}$

6.19 $\quad A^{-1} = \dfrac{1}{7}\begin{pmatrix} 1 & 2 \\ -3 & -1 \end{pmatrix}, \quad (x,y) = (5,0)$

6.20 $\quad A^{-1} = \dfrac{1}{7}\begin{pmatrix} -4 & 3 \\ 5 & -2 \end{pmatrix}, \quad (x,y) = (4,-3)$

6.21 $\quad A^{-1} = \dfrac{1}{5}\begin{pmatrix} -1 & 2 & 2 \\ -2 & -1 & 4 \\ 3 & -1 & -1 \end{pmatrix}, \quad (x,y,z) = (-2,1,5)$

6.22 $\quad A^{-1} = \dfrac{1}{12}\begin{pmatrix} 4 & 4 & 0 \\ -7 & -1 & 3 \\ 1 & -5 & 3 \end{pmatrix}, \quad (x,y,z) = (1,-1,2)$

6.30 $\quad \sin kA = A\sin k = \begin{pmatrix} 0 & \sin k \\ \sin k & 0 \end{pmatrix}, \; \cos kA = I\cos k = \begin{pmatrix} \cos k & 0 \\ 0 & \cos k \end{pmatrix},$

$\qquad e^{kA} = \begin{pmatrix} \cosh k & \sinh k \\ \sinh k & \cosh k \end{pmatrix}, \quad e^{ikA} = \begin{pmatrix} \cos k & i\sin k \\ i\sin k & \cos k \end{pmatrix}$

6.32 $\quad e^{i\theta B} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

In the following, L = linear, N = not linear.

7.1	N	7.2	L	7.3	N	7.4	L	7.5	L
7.6	N	7.7	L	7.8	N	7.9	N	7.10	N
7.11	N	7.12	L	7.13	(a) L	(b) L				
7.14	N	7.15	L	7.16	N	7.17	N		

7.22 $\quad D = 1$, rotation $\theta = -45°$ $\qquad$ 7.23 $\quad D = 1$, rotation $\theta = 210°$

7.24 $\quad D = -1$, reflection line $x + y = 0$ $\quad$ 7.25 $\quad D = -1$, reflection line $y = x\sqrt{2}$

7.26 $\quad D = -1$, reflection line $x = 2y$ $\qquad$ 7.27 $\quad D = 1$, rotation $\theta = 135°$

7.28 $\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \qquad \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$

7.29 $\quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

7.30 $R = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$;   R is a 90° rotation about

the z axis; S is a 90° rotation about the x axis.

7.31   From problem 30, $RS = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $SR = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$;

RS is a 120° rotation about $\mathbf{i} + \mathbf{j} + \mathbf{k}$; SR is a 120° rotation about $\mathbf{i} - \mathbf{j} + \mathbf{k}$.

7.32   180° rotation about $\mathbf{i} - \mathbf{k}$

7.33   120° rotation about $\mathbf{i} - \mathbf{j} - \mathbf{k}$

7.34   Reflection through the plane $y + z = 0$

7.35   Reflection through the (x, y) plane, and 90° rotation about the z axis.

8.1   In terms of basis $\mathbf{u} = \frac{1}{9}(9, 0, 7)$, $\mathbf{v} = \frac{1}{9}(0, -9, 13)$, the vectors
       are: $\mathbf{u} - 4\mathbf{v}$, $5\mathbf{u} - 2\mathbf{v}$, $2\mathbf{u} + \mathbf{v}$, $3\mathbf{u} + 6\mathbf{v}$.

8.2   In terms of basis $\mathbf{u} = \frac{1}{3}(3, 0, 5)$, $\mathbf{v} = \frac{1}{3}(0, 3, -2)$, the vectors
       are: $\mathbf{u} - 2\mathbf{v}$, $\mathbf{u} + \mathbf{v}$, $-2\mathbf{u} + \mathbf{v}$, $3\mathbf{u}$.

8.3   Basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.   8.4   Basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.

8.6 $\mathbf{V} = 3\mathbf{A} - \mathbf{B}$ 8.7 $\mathbf{V} = \frac{3}{2}(1, -4) + \frac{1}{2}(5, 2)$

8.17 $x = 0$, $y = \frac{3}{2}z$ 8.18 $x = -3y$, $z = 2y$

8.19 $x = y = z = w = 0$ 8.20 $x = -z$, $y = z$

8.21 $\begin{pmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{pmatrix} = 0$ 8.22 $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = 0$

8.23   For $\lambda = 3$, $x = 2y$; for $\lambda = 8$, $x = -2y$

8.24   For $\lambda = 7$, $x = 3y$; for $\lambda = -3$, $y = -3x$

8.25   For $\lambda = 2$: $x = 0$, $y = -3z$; for $\lambda = -3$: $x = -5y$, $z = 3y$;   .
       for $\lambda = 4$: $z = 3y$, $x = 2y$

8.26 $\mathbf{r} = (3, 1, 0) + (-1, 1, 1)z$

8.27 $\mathbf{r} = (0, 1, 2) + (1, 1, 0)x$

8.28 $\mathbf{r} = (3, 1, 0) + (2, 1, 1)z$

9.3 $A^\dagger = \begin{pmatrix} 1 & 2i & 1 \\ 0 & 2 & 1 - i \\ -5i & 0 & 0 \end{pmatrix}$, $A^{-1} = \frac{1}{10} \begin{pmatrix} 0 & 5i - 5 & -10i \\ 0 & -5i & 10 \\ -2i & -1 - i & 2 \end{pmatrix}$

9.4 $A^\dagger = \begin{pmatrix} 0 & i & 3 \\ -2i & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, $A^{-1} = \frac{1}{6} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 3 & i \\ -6 & 6i & -2 \end{pmatrix}$

9.14 $C^T B A^T$, $C^{-1} M^{-1} C$, H

10.1   (a) $d = 5$ (b) $d = 8$ (c) $d = \sqrt{56}$

10.2   The dimension of the space = the number of basis vectors listed.
       One possible basis is given; other bases consist of the same number
       of independent linear combinations of the vectors given.
       (a) $(1, -1, 0, 0)$, $(-2, 0, 5, 1)$
       (b) $(1, 0, 0, 5, 0, 1)$, $(0, 1, 0, 0, 6, 4)$, $(0, 0, 1, 0, -3, 0)$
       (c) $(1, 0, 0, 0, -3)$, $(0, 2, 0, 0, 1)$, $(0, 0, 1, 0, -1)$, $(0, 0, 0, 1, 4)$

10.3   (a) Label the vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$. Then $\cos(\mathbf{A}, \mathbf{B}) = \frac{1}{\sqrt{15}}$,
       $\cos(\mathbf{A}, \mathbf{C}) = \frac{\sqrt{2}}{3}$, $\cos(\mathbf{A}, \mathbf{D}) = \frac{3}{\sqrt{23}}$, $\cos(\mathbf{B}, \mathbf{C}) = \frac{2}{3\sqrt{15}}$,
       $\cos(\mathbf{B}, \mathbf{D}) = \sqrt{\frac{17}{690}}$, $\cos(\mathbf{C}, \mathbf{D}) = \frac{\sqrt{21}}{6\sqrt{23}}$.
       (b) $(1, 0, 0, 5, 0, 1)$ and $(0, 0, 1, 0, -3, 0)$

10.4   (a) $\mathbf{e}_1 = (0, 1, 0, 0)$, $\mathbf{e}_2 = (1, 0, 0, 0)$, $\mathbf{e}_3 = (0, 0, 3, 4)/5$
       (b) $\mathbf{e}_1 = (0, 0, 0, 1)$, $\mathbf{e}_2 = (1, 0, 0, 0)$, $\mathbf{e}_3 = (0, 1, 1, 0)/\sqrt{2}$
       (c) $\mathbf{e}_1 = (1, 0, 0, 0)$, $\mathbf{e}_2 = (0, 0, 1, 0)$, $\mathbf{e}_3 = (0, 1, 0, 2)/\sqrt{5}$

10.5   (a) $\|\mathbf{A}\| = \sqrt{43}$, $\|\mathbf{B}\| = \sqrt{41}$, |Inner product of $\mathbf{A}$ and $\mathbf{B}$| $= \sqrt{74}$
       (b) $\|\mathbf{A}\| = 7$, $\|\mathbf{B}\| = \sqrt{60}$, |Inner product of $\mathbf{A}$ and $\mathbf{B}$| $= \sqrt{5}$

11.5 $\theta = 1.1 = 63.4°$

11.11 $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$, not orthogonal

In the following answers, for each eigenvalue, the components of a
corresponding eigenvector are listed in parentheses.

11.12	4	$(1, 1)$	11.13	3	$(2, 1)$
	-1	$(3, -2)$		-2	$(-1, 2)$
11.14	4	$(2, -1)$	11.15	1	$(0, 0, 1)$
	-1	$(1, 2)$		-1	$(1, -1, 0)$
				5	$(1, 1, 0)$
11.16	2	$(0, 1, 0)$	11.17	7	$(1, 0, 1)$
	3	$(2, 0, 1)$		3	$(1, 0, -1)$
	-2	$(1, 0, -2)$		3	$(0, 1, 0)$
11.18	4	$(2, 1, 3)$	11.19	3	$(0, 1, -1)$
	2	$(0, -3, 1)$		5	$(1, 1, 1)$
	-3	$(5, -1, -3)$		-1	$(2, -1, -1)$
11.20	3	$(0, -1, 2)$	11.21	-1	$(-1, 1, 1)$
	4	$(1, 2, 1)$		2	$(2, 1, 1)$
	-2	$(-5, 2, 1)$		-2	$(0, -1, 1)$

11.22 -4 $(-4, 1, 1)$
        5   $(1, 2, 2)$
       -2 $(0, -1, 1)$

11.23 18 $(2, 2, -1)$
        9   $\Big\{$ Any two vectors orthogonal to (2,2,-1) and to each
        9   $\Big\{$ other, for example : (1,-1,0) and (1,1,4)

11.24 8 $(2, 1, 2)$
       -1 $\Big\{$ Any two vectors orthogonal to (2,1,2) and to each
       -1 $\Big\{$ other, for example : (1,0,-1) and (1,-4,1)

11.25 1 $(-1, 1, 1)$
        2   $(1, 1, 0)$
       -2 $(1, -1, 2)$

11.26 4 $(1, 1, 1)$
        1   $\Big\{$ Any two vectors orthogonal to (1,1,1) and to each
        1   $\Big\{$ other, for example : (1,-1,0) and (1,1,-2)

11.27 $D = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

11.28 $D = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$, $C = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

11.29 $D = \begin{pmatrix} 11 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \dfrac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

11.30 $D = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$, $C = \dfrac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

11.31 $D = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \dfrac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

11.32 $D = \begin{pmatrix} 7 & 0 \\ 0 & 2 \end{pmatrix}$, $C = \dfrac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$

11.41 $\lambda = 1,\ 3$; $U = \dfrac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

11.42 $\lambda = 1,\ 4$; $U = \dfrac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1-i \\ -1-i & 1 \end{pmatrix}$

11.43 $\lambda = 2,\ -3$; $U = \dfrac{1}{\sqrt{5}} \begin{pmatrix} 2 & -i \\ -i & 2 \end{pmatrix}$

11.44 $\lambda = 3,\ -7$; $U = \dfrac{1}{5\sqrt{2}} \begin{pmatrix} 5 & -3-4i \\ 3-4i & 5 \end{pmatrix}$

11.47 $U = \dfrac{1}{2} \begin{pmatrix} -1 & i\sqrt{2} & 1 \\ -1 & -i\sqrt{2} & 1 \\ \sqrt{2} & 0 & \sqrt{2} \end{pmatrix}$

11.51 Reflection through the plane $3x - 2y - 3z = 0$, no rotation

11.52 $60°$ rotation about $-\mathbf{i}\sqrt{2} + \mathbf{k}$ and reflection through the plane $z = x\sqrt{2}$

11.53 $180°$ rotation about $\mathbf{i} + \mathbf{j} + \mathbf{k}$

11.54 $-120°$ (or $240°$) rotation about $\mathbf{i}\sqrt{2} + \mathbf{j}$

11.55 Rotation $-90°$ about $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, and reflection through the plane $x - 2y + 2z = 0$

11.56 $45°$ rotation about $\mathbf{j} - \mathbf{k}$

11.58 $f(M) = \dfrac{1}{5} \begin{pmatrix} f(1) + 4f(6) & 2f(1) - 2f(6) \\ 2f(1) - 2f(6) & 4f(1) + f(6) \end{pmatrix}$

$M^4 = \dfrac{1}{5} \begin{pmatrix} 1 + 4 \cdot 6^4 & 2 - 2 \cdot 6^4 \\ 2 - 2 \cdot 6^4 & 4 + 6^4 \end{pmatrix}$ $M^{10} = \dfrac{1}{5} \begin{pmatrix} 1 + 4 \cdot 6^{10} & 2 - 2 \cdot 6^{10} \\ 2 - 2 \cdot 6^{10} & 4 + 6^{10} \end{pmatrix}$

$e^M = \dfrac{e}{5} \begin{pmatrix} 1 + 4e^5 & 2(1 - e^5) \\ 2(1 - e^5) & 4 + e^5 \end{pmatrix}$

11.59 $M^4 = 2^3 \begin{pmatrix} 1 + 2^4 & 1 - 2^4 \\ 1 - 2^4 & 1 + 2^4 \end{pmatrix}$ $M^{10} = 2^3 \begin{pmatrix} 1 + 2^{10} & 1 - 2^{10} \\ 1 - 2^{10} & 1 + 2^{10} \end{pmatrix}$,

$e^M = e^3 \begin{pmatrix} \cosh 1 & -\sinh 1 \\ -\sinh 1 & \cosh 1 \end{pmatrix}$

12.2 $3x'^2 - 2y'^2 = 24$ 12.3 $10x'^2 = 35$

12.4 $5x'^2 - 5y'^2 = 8$ 12.5 $x'^2 + 3y'^2 + 6z'^2 = 14$

12.6 $3x'^2 + \sqrt{3}y'^2 - \sqrt{3}z'^2 = 12$ 12.7 $3x'^2 + 5y'^2 - z'^2 = 60$

12.14 $y = x$ with $\omega = \sqrt{k/m}$; $y = -x$ with $\omega = \sqrt{5k/m}$

12.15 $y = 2x$ with $\omega = \sqrt{3k/m}$; $x = -2y$ with $\omega = \sqrt{8k/m}$

12.16 $y = 2x$ with $\omega = \sqrt{2k/m}$; $x = -2y$ with $\omega = \sqrt{7k/m}$

12.17 $x = -2y$ with $\omega = \sqrt{2k/m}$; $3x = 2y$ with $\omega = \sqrt{2k/(3m)}$

12.18 $y = x$ with $\omega = \sqrt{2k/m}$; $x = -5y$ with $\omega = \sqrt{16k/(5m)}$

12.19 $y = -x$ with $\omega = \sqrt{3k/m}$; $y = 2x$ with $\omega = \sqrt{3k/(2m)}$

12.21 $y = 2x$ with $\omega = \sqrt{k/m}$; $x = -2y$ with $\omega = \sqrt{6k/m}$

12.22 $y = -x$ with $\omega = \sqrt{2k/m}$; $y = 3x$ with $\omega = \sqrt{2k/(3m)}$

12.23 $y = -x$ with $\omega = \sqrt{k/m}$; $y = 2x$ with $\omega = \sqrt{k/(4m)}$

13.5 The 4's group   13.6 The cyclic group   13.7 The 4's group

13.10 If R = 90° rotation, P = reflection through the y axis, and Q = PR, then the 8 matrices of the symmetry group of the square are:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \ R^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I,$$

$$R^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -R, \ P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \ PR = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = Q,$$

$$PR^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -P, \ PR^3 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = -Q,$$

with multiplication table:

	I	R	−I	−R	P	Q	−P	−Q
I	I	R	−I	−R	P	Q	−P	−Q
R	R	−I	−R	I	−Q	P	Q	−P
−I	−I	−R	I	R	−P	−Q	P	Q
−R	−R	I	R	−I	Q	−P	−Q	P
P	P	Q	−P	−Q	I	R	−I	−R
Q	Q	−P	−Q	P	−R	I	R	−I
−P	−P	−Q	P	Q	−I	−R	I	R
−Q	−Q	P	Q	−P	R	−I	−R	I

13.11 The 4 matrices of the symmetry group of the rectangle are

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -P, \ \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

This group is isomorphic to the 4's group.

13.14

Class	I	±R	−I	±P	±Q
Character	2	0	−2	0	0

13.20 Not a group (no unit element)

13.21 SO(2) is Abelian; SO(3) is not Abelian.

For Problems 2 to 10, we list a possible basis.

14.2 e^x, $x\,e^x$, e^{-x}, or the three given functions

14.3 x, $\cos x$, $x\cos x$, $e^x \cos x$

14.4 1, x, x^3

14.5 1, $x + x^3$, x^2, x^4, x^5

14.6 Not a vector space

14.7 $(1 + x^2 + x^4 + x^6)$, $(x + x^3 + x^5 + x^7)$

14.8 1, x^2, x^4, x^6

14.9 Not a vector space; the negative of a vector with positive coefficients does not have positive coefficients.

14.10 $(1 + \frac{1}{2}x)$, $(x^2 + \frac{1}{2}x^3)$, $(x^4 + \frac{1}{2}x^5)$, $(x^6 + \frac{1}{2}x^7)$, $(x^8 + \frac{1}{2}x^9)$, $(x^{10} + \frac{1}{2}x^{11})$, $(x^{12} + \frac{1}{2}x^{13})$

15.3 (a) $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z-2}{-2}$, $\mathbf{r} = (4, -1, 2) + (1, -2, -2)t$
      (b) $x - 5y + 3z = 0$ (c) $5/7$
      (d) $5\sqrt{2}/3$ (e) $\arcsin(19/21) = 64.8°$

15.4 (a) $4x + 2y + 5z = 10$ (b) $\arcsin(2/3) = 41.8°$
      (c) $2/\sqrt{5}$ (d) $2x + y - 2z = 5$
      (e) $x = \frac{5}{2}$, $\frac{y}{2} = z$, $\mathbf{r} = \frac{5}{2}\mathbf{i} + (2\mathbf{j} + \mathbf{k})t$

15.5 (a) $y = 7$, $\frac{x-2}{3} = \frac{z+1}{4}$, $\mathbf{r} = (2, 7, -1) + (3, 0, 4)t$
      (b) $x - 4y - 9z = 0$ (c) $\arcsin \frac{33}{35\sqrt{2}} = 41.8°$
      (d) $\frac{12}{7\sqrt{2}}$ (e) $\frac{\sqrt{29}}{5}$

15.7 $A^T = \begin{pmatrix} 1 & 0 \\ -1 & i \end{pmatrix}$ $A^{-1} = \begin{pmatrix} 1 & -i \\ 0 & -i \end{pmatrix}$ $AB = \begin{pmatrix} 2 & -2 & -6 \\ 0 & 3i & 5i \end{pmatrix}$

$\overline{A} = \begin{pmatrix} 1 & -1 \\ 0 & -i \end{pmatrix}$ $B^T A^T = (AB)^T$ $B^T AC = \begin{pmatrix} 2 & 2 \\ 1-3i & 1 \\ -1-5i & -1 \end{pmatrix}$

$A^\dagger = \begin{pmatrix} 1 & 0 \\ -1 & -i \end{pmatrix}$ $B^T C = \begin{pmatrix} 0 & 2 \\ -3 & 1 \\ -5 & -1 \end{pmatrix}$ $C^{-1}A = \begin{pmatrix} 0 & -i \\ 1 & -1 \end{pmatrix}$

$A^T B^T$, BA^T, ABC, $AB^T C$, $B^{-1}C$, and CB^T are meaningless.

15.8 $A^\dagger = \begin{pmatrix} 1 & -i & 1 \\ 0 & -3 & 0 \\ -2i & 0 & -i \end{pmatrix}$ $A^{-1} = \dfrac{1}{3i}\begin{pmatrix} -3i & 0 & 6i \\ 1 & -i & -2 \\ 3 & 0 & -3 \end{pmatrix}$

15.9 $A = \begin{pmatrix} 1+\frac{(n-1)d}{nR_2} & -(n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right] \\ \frac{d}{n} & 1-\frac{(n-1)d}{nR_1} \end{pmatrix}$, $\dfrac{1}{f} = -A_{12}$

15.10 $M = \begin{pmatrix} 1-\frac{d}{f_2} & -\frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_1 f_2} \\ d & 1-\frac{d}{f_1} \end{pmatrix}$, $\dfrac{1}{f} = \dfrac{f_1+f_2-d}{f_1 f_2}$, $\det M = 1$

15.13 Area $= \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = 7/2$

15.14 $x'' = -x$, $y'' = -y$, $180°$ rotation

15.15 $x'' = -y$, $y'' = x$, $90°$ rotation of vectors or $-90°$ rotation of axes

15.16 $x'' = y$, $y'' = -x$, $z'' = z$, $90°$ rotation of (x,y) axes about the z axis, or $-90°$ rotation of vectors about the z axis

15.17 $x'' = x$, $y'' = -y$, $z'' = -z$, rotation of π about the x axis

15.18 1 $(1,1)$ 15.19 6 $(1,1)$ 15.20 1 $(1,1)$
      -2 $(0,1)$ 1 $(1,-4)$ 9 $(1,-1)$

15.21 0 $(1,-2)$ 15.22 1 $(1,0,1)$ 15.23 1 $(1,1,-2)$
      5 $(2,1)$ 4 $(0,1,0)$ 3 $(1,-1,0)$
                               5 $(1,0,-1)$ 4 $(1,1,1)$

15.24 2 $(0,4,3)$
      7 $(5,-3,4)$
      -3 $(5,3,-4)$

15.25 $C = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 1 & \sqrt{2} \end{pmatrix}$, $C^{-1} = \begin{pmatrix} \sqrt{2} & 0 \\ -1 & 1 \end{pmatrix}$

15.26 $C = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{17} \\ 1/\sqrt{2} & -4/\sqrt{17} \end{pmatrix}$, $C^{-1} = \dfrac{1}{5}\begin{pmatrix} 4\sqrt{2} & \sqrt{2} \\ \sqrt{17} & -\sqrt{17} \end{pmatrix}$

15.27 $3x'^2 - y'^2 - 5z'^2 = 15$, $d = \sqrt{5}$

15.28 $9x'^2 + 4y'^2 - z'^2 = 36$, $d = 2$

15.29 $3x'^2 + 6y'^2 - 4z'^2 = 54$, $d = 3$

15.30 $7x'^2 + 20y'^2 - 6z'^2 = 20$, $d = 1$

15.31 $\omega = (k/m)^{1/2}$, $(7k/m)^{1/2}$

15.32 $\omega = 2(k/m)^{1/2}$, $(3k/m)^{1/2}$

Chapter 4

1.1 $\partial u/\partial x = 2xy^2/(x^2+y^2)^2$, $\partial u/\partial y = -2x^2y/(x^2+y^2)^2$

1.2 $\partial s/\partial t = ut^{u-1}$, $\partial s/\partial u = t^u \ln t$

1.3 $\partial z/\partial u = u/(u^2+v^2+w^2)$

1.4 At $(0,\ 0)$, both $= 0$; at $(-2/3,\ 2/3)$, both $= -4$

1.5 At $(0,\ 0)$, both $= 0$; at $(1/4,\ \pm 1/2)$, $\partial^2 w/\partial x^2 = 6$, $\partial^2 w/\partial y^2 = 2$

1.7	$2x$	1.8	$-2x$	1.9	$2x(1+2\tan^2\theta)$
1.10	$4y$	1.11	$2y$	1.12	$2y(\cot^2\theta+2)$
1.13	$4r^2\tan\theta$	1.14	$-2r^2\cot\theta$	1.15	$r^2\sin 2\theta$
1.16	$2r(1+\sin^2\theta)$	1.17	$4r$	1.18	$2r$
1.19	0	1.20	$8y\sec^2\theta$	1.21	$-4x\csc^2\theta$
1.22	0	1.23	$2r\sin 2\theta$	1.24	0
1.7'	$-2y^4/x^3$	1.8'	$-2r^4/x^3$	1.9'	$2x\tan^2\theta\sec^2\theta$
1.10'	$2y+4y^3/x^2$	1.11'	$2yr^4/(r^2-y^2)^2$	1.12'	$2y\sec^2\theta$

1.13' $2x^2\sec^2\theta\tan\theta(\sec^2\theta+\tan^2\theta)$

1.14'	$2y^2\sec^2\theta\tan\theta$	1.15'	$2r^2\tan\theta\sec^2\theta$	1.16'	$2r\tan^2\theta$
1.17'	$4r^3/x^2-2r$	1.18'	$-2ry^4/(r^2-y^2)^2$		
1.19'	$-8r^3y^3/(r^2-y^2)^3$	1.20'	$4x\tan\theta\sec^2\theta(\tan^2\theta+\sec^2\theta)$		
1.21'	$4y\sec^2\theta\tan\theta$	1.22'	$-8r^3/x^3$		
1.23'	$4r\tan\theta\sec^2\theta$	1.24'	$-8y^3/x^3$		

2.1 $y+y^3/6-x^2y/2+x^4y/24-x^2y^3/12+y^5/120+\cdots$

2.2 $1-(x^2+2xy+y^2)/2+(x^4+4x^3y+6x^2y^2+4xy^3+y^4)/24+\cdots$

2.3 $x-x^2/2-xy+x^3/3+x^2y/2+xy^2\cdots$

2.4 $1+xy+x^2y^2/2+x^3y^3/3!+x^4y^4/4!\cdots$

2.5 $1+\frac{1}{2}xy-\frac{1}{8}x^2y^2+\frac{1}{16}x^3y^3-\frac{5}{128}x^4y^4\cdots$

2.6 $1+x+y+(x^2+2xy+y^2)/2\cdots$

2.8 $e^x\cos y = 1+x+(x^2-y^2)/2+(x^3-3xy^2)/3!\cdots$

      $e^x\sin y = y+xy+(3x^2y-y^3)/3!\cdots$

4.2	2.5×10^{-13}	4.3	14.8	4.4	12.2	4.5	14.96
4.6	9%	4.7	15%	4.8	5%	4.10	4.28 nt
4.11	3.95	4.12	2.01	4.13	5/3	4.14	0.005
4.15	8×10^{23}						

5.1	$e^{-y}\sinh t + z\sin t$	5.2	$w=1$, $dw/dp=0$
5.3	$2r(q^2-p^2)$	5.4	$(4ut+2v\sin t)/(u^2-v^2)$
5.6	$5(x+y)^4(1+10\cos 10x)$	5.7	$(1-2b-e^{2a})\cos(a-b)$

6.1 $dv/dp = -v/(ap)$, $d^2v/dp^2 = v(1+a)/(a^2p^2)$

6.2 $y'=1$, $y''=0$ 6.3 $y'=4(\ln 2-1)/(2\ln 2-1)$

6.4 $y' = y(x-1)/[x(y-1)]$, $y'' = (y-x)(y+x-2)y/[x^2(y-1)^3]$

6.5 $2x + 11y - 24 = 0$ 6.6 $1800/11^3$

6.7 $y' = 1$, $x - y - 4 = 0$ 6.8 $-8/3$

6.9 $y = x - 4\sqrt{2}$, $y = 0$, $x = 0$ 6.10 $x + y = 0$

6.11 $y'' = 4$

7.1 $dx/dy = z - y + \tan(y+z)$, $d^2x/dy^2 = \frac{1}{2}\sec^3(y+z) + \frac{1}{2}\sec(y+z) - 2$

7.2 $[2e^r \cos t - r + r^2 \sin^2 t]/[(1-r)\sin t]$

7.3 $\partial z/\partial s = z \sin s$, $\partial z/\partial t = e^{-y}\sinh t$

7.4 $\partial w/\partial u = -2(rv + s)w$, $\partial w/\partial v = -2(ru + 2s)w$

7.5 $\partial u/\partial s = (2y^2 - 3x^2 + xyt)u/(xy)$, $\partial u/\partial t = (2y^2 - 3x^2 + xys)u/(xy)$

7.6 $\partial^2 w/\partial r^2 = f_{xx}\cos^2\theta + 2f_{xy}\sin\theta\cos\theta + f_{yy}\sin^2\theta$

7.7 $(\partial y/\partial\theta)_r = x$, $(\partial y/\partial\theta)_x = r^2/x$, $(\partial\theta/\partial y)_x = x/r^2$

7.8 $\partial x/\partial s = -19/13$, $\partial x/\partial t = -21/13$, $\partial y/\partial s = 24/13$, $\partial y/\partial t = 6/13$

7.10 $\partial x/\partial s = 1/6$, $\partial x/\partial t = 13/6$, $\partial y/\partial s = 7/6$, $\partial y/\partial t = -11/6$

7.11 $\partial z/\partial s = 481/93$, $\partial z/\partial t = 125/93$

7.12 $\partial w/\partial s = w/(3w^3 - xy)$, $\partial w/\partial t = (3w - 1)/(3w^3 - xy)$

7.13 $(\partial p/\partial q)_m = -p/q$, $(\partial p/\partial q)_a = 1/(a\cos p - 1)$,

$(\partial p/\partial q)_b = 1 - b\sin q$, $(\partial b/\partial a)_p = (\sin p)(b\sin q - 1)/\cos q$

$(\partial a/\partial q)_m = [q + p(a\cos p - 1)]/(q\sin p)$

7.14   13

7.15 $(\partial x/\partial u)_v = (2yv^2 - x^2)/(2yv + 2xu)$,

$(\partial x/\partial u)_y = (x^2 u + y^2 v)/(y^2 - 2xu^2)$

7.16   (a) $\dfrac{dw}{dt} = \dfrac{3(2x+y)}{3x^2 + 1} + \dfrac{4x}{4y^3 + 1} + \dfrac{10z}{5z^4 + 1}$

(b) $\dfrac{dw}{dx} = 2x + y - \dfrac{xy}{3y^2 + x} + \dfrac{2z^2}{3z^2 - x}$

(c) $\left(\dfrac{\partial w}{\partial x}\right)_y = 2x + y - \dfrac{2z(y^3 + 3x^2 z)}{x^3 + 3yz^2}$

7.17 $(\partial p/\partial s)_t = -9/7$, $(\partial p/\partial s)_q = 3/2$

7.18 $(\partial b/\partial m)_n = a/(a-b)$, $(\partial m/\partial b)_a = 1$

7.19 $(\partial x/\partial z)_s = 7/2$, $(\partial x/\partial z)_r = 4$, $(\partial x/\partial z)_y = 3$

7.20 $(\partial u)/(\partial x)_y = 4/3$, $(\partial u/\partial x)_v = 14/5$, $(\partial x/\partial u)_y = 3/4$, $(\partial x/\partial u)_v = 5/14$

7.21 -1, -15, 2, $15/7$, $-5/2$, $-6/5$

7.26 $dy/dx = -(f_1 g_3 - f_3 g_1)/(f_2 g_3 - g_2 f_3)$

8.3 $(-1,\ 2)$ is a minimum point.   8.4 $(-1,\ -2)$ is a saddle point.

8.5 $(0,\ 1)$ is a maximum point.   8.6 $(0,\ 0)$ is a saddle point.

                                            $(-2/3,\ 2/3)$ is a maximum point.

8.8 $\theta = \pi/3$; bend up 8 cm on each side.

8.9 $l = w = 2h$ 8.10 $l = w = 2h/3$

8.11 $\theta = 30°$, $x = y\sqrt{3} = z/2$ 8.12 $d = 3$

8.13 $(4/3,\ 5/3)$ 8.15 $(1/2,\ 1/2,\ 0)$, $(1/3,\ 1/3,\ 1/3)$

8.16 $m = 5/2$, $b = 1/3$

8.17   (a) $y = 5 - 4x$ (b) $y = 0.5 + 3.35x$ (c) $y = -3 - 3.6x$

9.1 $s = l$, $\theta = 30°$ (regular hexagon)

9.2 $r : l : s = \sqrt{5} : (1 + \sqrt{5}) : 3$ 9.3   36 in by 18 in by 18 in

9.4 $4/\sqrt{3}$ by $6/\sqrt{3}$ by $10/\sqrt{3}$ 9.5 $(1/2,\ 3,\ 1)$

9.6 $V = 1/3$ 9.7 $V = d^3/(27abc)$
9.8 $(8/13,\ 12/13)$ 9.9 $A = 3ab\sqrt{3}/4$
9.10 $d = 5/\sqrt{2}$ 9.11 $d = \sqrt{6}/2$
9.12 Let legs of right triangle be a and b, height of prism $= h$;
      then $a = b$, $h = \left(2 - \sqrt{2}\,\right)a$.

10.1 $d = 1$ 10.2 $4,\ 2$
10.3 $2,\ \sqrt{14}$ 10.4 $d = 1$
10.5 $d = 1$ 10.6 $d = 2$
10.7 $\frac{1}{2}\sqrt{11}$ 10.8 $T = 8$
10.9 $\max T = 4$ at $(-1,\ 0)$ 10.10 (a) $\max T = 1/2$, $\min T = -1/2$
      $\min T = -\frac{16}{5}$ at $\left(\frac{1}{5}, \pm\frac{2}{5}\sqrt{6}\right)$ (b) $\max T = 1$, $\min T = -1/2$
                                                 (c) $\max T = 1$, $\min T = -1/2$

10.11 $\max T = 14$ at $(-1,\ 0)$ 10.12 Largest sum $= 180°$
      $\min T = 13/2$ at $(1/2,\ \pm 1)$ Smallest sum $= 3\arccos\frac{1}{\sqrt{3}}$
                                                       $= 164.2°$

10.13 Largest sum $= 3\arcsin(1/\sqrt{3}) = 105.8°$, smallest sum $= 90°$

11.1 $z = f(y + 2x) + g(y + 3x)$
11.2 $z = f(5x - 2y) + g(2x + y)$
11.3 $w = (x^2 - y^2)/4 + F(x + y) + G(x - y)$

11.6 $\dfrac{d^2 y}{dz^2} + \dfrac{dy}{dz} - 5y = 0$

11.10 $f = u - Ts$ $h = u + pv$ $g = u + pv - Ts$
      $df = -p\,dv - s dT$ $dh = Tds + vdp$ $dg = v\,dp - s\,dT$
11.11 $H = p\dot{q} - L$
11.13 (a) $(\partial s/\partial v)_T = (\partial p/\partial T)_v$ (b) $(\partial T/\partial p)_s = (\partial v/\partial s)_p$
      (c) $(\partial v/\partial T)_p = -(\partial s/\partial p)_T$

12.1 $\dfrac{\sin x}{2\sqrt{x}}$

12.2 $\dfrac{\partial s}{\partial v} = \dfrac{1 - e^v}{v} \to -1;\ \dfrac{\partial s}{\partial u} = \dfrac{e^u - 1}{u} \to 1$

12.3 $dz/dx = -\sin(\cos x)\tan x - \sin(\sin x)\cot x$
12.4 $(\sin 2)/2$
12.5 $\partial u/\partial x = -4/\pi,\ \partial u/\partial y = 2/\pi,\ \partial y/\partial x = 2$
12.6 $\partial w/\partial x = 1/\ln 3,\ \partial w/\partial y = -6/\ln 3,\ \partial y/\partial x = 1/6$
12.7 $(\partial u/\partial x)_y = -e^4,\ (\partial u/\partial y)_x = e^4/\ln 2,\ (\partial y/\partial x)_u = \ln 2$
12.8 $dx/du = e^{x^2}$ 12.9 $(\cos\pi x + \pi x\sin\pi x - 1)/x^2$
12.10 $dy/dx = (e^x - 1)/x$ 12.11 $3x^2 - 2x^3 + 3x - 6$
12.12 $(2x + 1)/\ln(x + x^2) - 2/\ln(2x)$ 12.13 0
12.14 $\pi/(4y^3)$

12.16 $n = 2,\ I = \frac{1}{4}\sqrt{\pi}\,a^{-3/2}$ $n = 4,\ I = \dfrac{1 \cdot 3}{8}\sqrt{\pi}\,a^{-5/2}$

      $n = 2m,\ I = \dfrac{1 \cdot 3 \cdot 5\ \cdots\ (2m - 1)}{2^{m+1}}\sqrt{\pi}\,a^{-(2m+1)/2}$

13.2 (a) and (b) $d = 4/\sqrt{13}$
13.3 $\sec^2\theta$
13.4 $-\csc\theta\cot\theta$
13.5 $-6x,\ 2x^2\tan\theta\sec^2\theta,\ 4x\tan\theta\sec^2\theta$
13.6 $2r\sin^2\theta,\ 2r^2\sin\theta\cos\theta,\ 4r\sin\theta\cos\theta,\ 0$
13.7 5%

13.8 π^{-1}ft $\cong 4$ inches

13.9 $dz/dt = 1 + t(2 - x - y)/z$, $z \neq 0$

13.10 $(x \ln x - y^2/x)x^y$ where $x = r \cos\theta$, $y = r \sin\theta$

13.11 $\dfrac{dy}{dx} = -\dfrac{b^2 x}{a^2 y}$, $\dfrac{d^2 y}{dx^2} = -\dfrac{b^4}{a^2 y^3}$

13.12 13

13.13 -1

13.14 $(\partial w/\partial x)_y = (\partial f/\partial x)_{s,\,t} + 2(\partial f/\partial s)_{x,\,t} + 2(\partial f/\partial t)_{x,\,s} = f_1 + 2f_2 + 2f_3$

13.15 $(\partial w/\partial x)_y = f_1 + 2xf_2 + 2yf_3$

13.17 $\sqrt{19}$

13.18 $\sqrt{26/3}$

13.19 $1/27$

13.20 At $x = -1$, $y = 20$; at $x = 1/2$, $y = -1/4$

13.21 $T(2) = 4$, $T(5) = -5$

13.22 $T(5,\ 0) = 10$, $T(2,\ \pm\sqrt{2}) = -4$

13.23 $t \cot t$

13.24 0

13.25 $-e^x/x$

13.26 $3 \sin x^3/x$

13.29 $dt = 3.9$

13.30 $2f(x,\ x) + \int_0^x \frac{\partial}{\partial x}\left[f(x,\ u) + f(u,\ x)\right] du$

Chapter 5

2.1	3	2.2	-18	2.3	4	2.4	8/3

2.5 $\quad \dfrac{e^2}{4} - \dfrac{5}{12}$ $\qquad$ 2.6 $\quad$ 2.35 $\qquad$ 2.7 $\quad$ 5/3 $\qquad$ 2.8 $\quad$ 1/2

2.9	6	2.10	5π	2.11	36	2.12	2
2.13	7/4	2.14	$4 - e(\ln 4)$	2.15	3/2	2.16	$(\ln 3)/6$
2.17	$(\ln 2)/2$	2.18	$(8\sqrt{2} - 7)/3$	2.19	32	2.20	16
2.21	131/6	2.22	5/3	2.23	9/8	2.24	9/2
2.25	3/2	2.26	4/3	2.27	32/5	2.28	1/3
2.29	2	2.30	$1 - e^{-2}$	2.31	6	2.32	$e - 1$
2.33	16/3	2.34	$8192k/15$	2.35	$216k$	2.36	1/6
2.37	7/6	2.38	-20	2.39	70	2.40	3/2
2.41	5	2.42	4	2.43	9/2	2.44	$7k/3$
2.45	$46k/15$	2.46	$8k$	2.47	16/3	2.48	$16\pi/3$
2.49	1/3	2.50	64/3				

3.2 $\quad$ (a) ρl $\qquad$ (b) $Ml^2/12$ $\qquad$ (c) $Ml^2/3$

3.3 $\quad$ (a) $M = 140$ $\quad$ (b) $\bar{x} = 130/21$ $\qquad$ (c) $I_m = 6.92M$ $\qquad$ (d) $I = 150M/7$

3.4 $\quad$ (a) $M = 3l/2$ $\quad$ (b) $\bar{x} = 4l/9$ $\qquad$ (c) $I_m = \frac{13}{162}Ml^2$ $\quad$ (d) $I = 5Ml^2/18$

3.5 $\quad$ (a) $Ma^2/3$ $\qquad$ (b) $Ma^2/12$ $\qquad$ (c) $2Ma^2/3$

3.6 $\quad$ (a) $(2, 2)$ $\qquad$ (b) $6M$ $\qquad$ (c) $2M$

3.7 $\quad$ (a) $M = 9$ $\qquad\qquad$ (b) $(\bar{x}, \bar{y}) = (2, 4/3)$

$\qquad$ (c) $I_x = 2M$, $I_y = 9M/2$ $\qquad$ (d) $I_m = 13M/18$

3.8 $\quad 2Ma^2/3$

3.9 $\quad$ (a) 1/6 $\qquad\qquad$ (b) $(1/4, 1/4, 1/4)$ $\qquad\qquad$ (c) $M = 1/24$, $\bar{z} = 2/5$

3.10 $\quad$ (a) $s = 2\sinh 1$ $\qquad$ (b) $\bar{y} = (2 + \sinh 2)/(4\sinh 1) = 1.2$

3.11 $\quad$ (a) $M = (5\sqrt{5} - 1)/6 = 1.7$

$\qquad$ (b) $\bar{x} = 0$, $M\bar{y} = (25\sqrt{5} + 1)/60 = 0.95$, $\bar{y} = (313 + 15\sqrt{5})/620 = 0.56$

3.14 $\quad V = 2\pi^2 a^2 b$, $A = 4\pi^2 ab$, where a = radius of revolving circle,

$\qquad b$ = distance to axis from center of this circle.

3.15 $\quad$ For area, $(\bar{x}, \bar{y}) = (0, \frac{4}{3}r/\pi)$; for arc, $(\bar{x}, \bar{y}) = (0, 2r/\pi)$

3.17 $\quad 4\sqrt{2}/3$ $\qquad\qquad$ 3.18 $\quad s = \left[3\sqrt{2} + \ln(1 + \sqrt{2})\right]/2 = 2.56$

3.19 $\quad 2\pi$ $\qquad\qquad$ 3.20 $\quad 13\pi/3$

3.21 $\quad s\bar{x} = \left[51\sqrt{2} - \ln(1 + \sqrt{2})\right]/32 = 2.23$, $s\bar{y} = 13/6$, s as in Problem 3.18;

$\qquad$ then $\bar{x} = 0.87$, $\bar{y} = 0.85$

3.22 $\quad (4/3, 0, 0)$

3.23 $\quad (149/130, 0, 0)$

3.24 $\quad 2M/5$

3.25 $\quad I/M$ has the same numerical value as $\bar{x}$ in 3.21.

3.26 $\quad 2M/3$ $\qquad$ 3.27 $\quad \frac{149}{130}M$ $\qquad$ 3.28 $\quad$ 13/6 $\qquad$ 3.29 $\quad$ 2 $\qquad\qquad$ 3.30 $\quad$ 32/5

4.1   (b) $\bar{x} = \bar{y} = 4a/(3\pi)$
       (c) $I = Ma^2/4$
       (e) $\bar{x} = \bar{y} = 2a/\pi$
4.2   (c) $\bar{y} = 4a/(3\pi)$
       (d) $I_x = Ma^2/4$, $I_y = 5Ma^2/4$, $I_z = 3Ma^2/2$
       (e) $\bar{y} = 2a/\pi$
       (f) $\bar{x} = 6a/5$, $I_x = 48Ma^2/175$, $I_y = 288Ma^2/175$, $I_z = 48Ma^2/25$
       (g) $A = (\frac{2}{3}\pi - \frac{1}{2}\sqrt{3})a^2$
4.3   (a), (b), or (c) $\frac{1}{2}Ma^2$
4.4   (a) $4\pi a^2$ (b) $(0,0,a/2)$ (c) $2Ma^2/3$
       (d) $4\pi a^3/3$ (e) $(0,0,3a/8)$
4.5 $7\pi/3$ 4.6 $\pi \ln 2$
4.7   (a) $V = 2\pi a^3(1 - \cos\alpha)/3$ (b) $\bar{z} = 3a(1 + \cos\alpha)/8$
4.8 $I_z = Ma^2/4$
4.10   (a) $V = 64\pi$ (b) $\bar{z} = 231/64$
4.11 12π
4.12   (c) $M = (16\rho/9)(3\pi - 4) = 9.64\rho$
            $I = (128\rho/15^2)(15\pi - 26) = 12.02\rho = 1.25M$
4.13   (b) $\pi a^2(z_2 - z_1) - \pi(z_2^3 - z_1^3)/3$ (c) $\dfrac{\frac{1}{2}a^2(z_2^2 - z_1^2) - \frac{1}{4}(z_2^4 - z_1^4)}{a^2(z_2 - z_1) - (z_2^3 - z_1^3)/3}$
4.14 $\pi(1 - e^{-1})/4$ 4.16 $u^2 + v^2$
4.17 $a^2(\sinh^2 u + \sin^2 v)$ 4.19 $\pi/4$
4.20 $1/12$ 4.22 $12(1 + 36\pi^2)^{1/2}$
4.23   Length $= (R\sec\alpha)$ times change in latitude
4.24 $\rho G\pi a/2$
4.26   (a) $7Ma^2/5$ (b) $3Ma^2/2$
4.27 $2\pi ah$ (where $h =$ distance between parallel planes)
4.28 $(0,0,a/2)$

5.1 $9\pi\sqrt{30}/5$ 5.2 $\pi\sqrt{7/5}$
5.3 $\pi(37^{3/2} - 1)/6 = 117.3$ 5.4 $\pi/\sqrt{6}$
5.5 8π for each nappe   5.6   4
5.7   4   5.8 $\left[3\sqrt{6} + 9\ln(\sqrt{2} + \sqrt{3})\right]/16$
5.9 $\pi\sqrt{2}$ 5.10 $2\pi a^2(\sqrt{2} - 1)$
5.11 $(\bar{x}, \bar{y}, \bar{z}) = (1/3, 1/3, 1/3)$ 5.12 $M = \sqrt{3}/6$, $(\bar{x}, \bar{y}, \bar{z}) = (1/2, 1/4, 1/4)$
5.13 $\bar{z} = \dfrac{\pi}{4(\pi - 2)}$ 5.14 $M = \dfrac{\pi}{2} - \dfrac{4}{3}$
5.15 $I_z/M = \dfrac{2(3\pi - 7)}{9(\pi - 2)} = 0.472$ 5.16 $\bar{x} = 0$, $\bar{y} = 1$, $\bar{z} = \dfrac{32}{9\pi}\sqrt{\dfrac{2}{5}} = 0.716$

6.1 $7\pi(2 - \sqrt{2})/3$ 6.2 $45(2 + \sqrt{2})/112$ 6.3 $15\pi/8$
6.4   (a) $\frac{1}{2}MR^2$ (b) $\frac{3}{2}MR^2$
6.5   cone: $2\pi ab^2/3$;   ellipsoid: $4\pi ab^2/3$;   cylinder: $2\pi ab^2$
6.6   (a) $\dfrac{4\pi - 3\sqrt{3}}{6}$ (b) $\bar{x} = \dfrac{5}{4\pi - 3\sqrt{3}}$, $\bar{y} = \dfrac{6\sqrt{3}}{4\pi - 3\sqrt{3}}$
6.7 $\dfrac{8\pi - 3\sqrt{3}}{4\pi - 3\sqrt{3}}M$ 6.8   (a) $5\pi/3$ (b) $27/20$
6.9 $(\bar{x}, \bar{y}) = (0, 3c/5)$
6.10   (a) $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$ (b) $\pi^2/2$ (c) $3M/8$
6.11 $\bar{z} = 3h/4$ 6.12 $(abc)^2/6$
6.13 $8a^2$ 6.14 $16a^3/3$
6.15 $I_x = 8Ma^2/15$, $I_y = 7Ma^2/15$ 6.16 $\bar{x} = \bar{y} = 2a/5$

6.17 $Ma^2/6$ 6.18 $(0, 0, 5h/6)$

6.19 $I_x = I_y = 20Mh^2/21,\ I_z = 10Mh^2/21,\ I_m = 65Mh^2/252$

6.20   (a) $\pi(5\sqrt{5} - 1)/6$ (b) $3\pi/2$

6.21 $\pi G\rho h(2 - \sqrt{2})$

6.22 $I_x = Mb^2/4,\ I_y = Ma^2/4,\ I_z = M(a^2 + b^2)/4$

6.23   (a) $(0, 0, 2c/3)$ (b) $(0, 0, 5c/7)$

6.24 $(0, 0, 2c/3)$

6.25 $\pi/2$

6.26 $\frac{1}{2}\sinh 1$

6.27 $e^2 - e - 1$

Chapter 6

3.1 $(\mathbf{A} \cdot \mathbf{B})\mathbf{C} = 6\mathbf{C} = 6(\mathbf{j} + \mathbf{k})$, $\mathbf{A}(\mathbf{B} \cdot \mathbf{C}) = -2\mathbf{A} = -2(2, -1, -1)$,
$\quad$ $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = -8$, $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = 4(\mathbf{j} - \mathbf{k})$,
$\quad$ $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = -4(\mathbf{i} + 2\mathbf{k})$

3.2 $\mathbf{B} \cdot \mathbf{C} = -16$

3.3 $(\mathbf{A} + \mathbf{B}) \cdot \mathbf{C} = -5$

3.4 $\mathbf{B} \times \mathbf{A} = -\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$, $|\mathbf{B} \times \mathbf{A}| = \sqrt{59}$, $(\mathbf{B} \times \mathbf{A}) \cdot \mathbf{C}/|\mathbf{C}| = -8/\sqrt{26}$

3.5 $\boldsymbol{\omega} = 2\mathbf{A}/\sqrt{6}$, $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{C} = \left(2/\sqrt{6}\right)(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$

3.6 $\mathbf{v} = \left(2/\sqrt{6}\right)(\mathbf{A} \times \mathbf{B}) = \left(2/\sqrt{6}\right)(\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$,
$\quad$ $\mathbf{r} \times \mathbf{F} = (\mathbf{A} - \mathbf{C}) \times \mathbf{B} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k}$,
$\quad$ $\mathbf{n} \cdot \mathbf{r} \times \mathbf{F} = [(\mathbf{A} - \mathbf{C}) \times \mathbf{B}] \cdot \mathbf{C}/|\mathbf{C}| = 8/\sqrt{26}$

3.7 (a) $11\mathbf{i} + 3\mathbf{j} - 13\mathbf{k}$ $\qquad$ (b) 3 $\qquad$ (c) 17

3.8 $4\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$, 4, -8, 4

3.9 $-9\mathbf{i} - 23\mathbf{j} + \mathbf{k}$, $1/\sqrt{21}$

3.12 $A^2 B^2$

3.15 $\mathbf{u}_1 \cdot \mathbf{u} = -\mathbf{u}_3 \cdot \mathbf{u}$, $n_1 \mathbf{u}_1 \times \mathbf{u} = n_2 \mathbf{u}_2 \times \mathbf{u}$

3.16 $\mathbf{L} = m[r^2 \boldsymbol{\omega} - (\boldsymbol{\omega} \cdot \mathbf{r})\mathbf{r}]$
$\quad$ For $\mathbf{r} \perp \boldsymbol{\omega}$, $v = |\boldsymbol{\omega} \times \mathbf{r}| = \omega r$, $L = m|r^2 \boldsymbol{\omega}| = mvr$

3.17 $\mathbf{a} = (\boldsymbol{\omega} \cdot \mathbf{r})\boldsymbol{\omega} - \omega^2 \mathbf{r}$; for $\mathbf{r} \perp \boldsymbol{\omega}$, $\mathbf{a} = -\omega^2 \mathbf{r}$, $|\mathbf{a}| = v^2/r$.

3.19 (a) $16\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$ $\qquad$ (b) $8/\sqrt{6}$

3.20 (a) $13/5$ $\qquad$ (b) 12

4.2 (a) $t = 2$
$\quad$ (b) $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$, $|\mathbf{v}| = 2\sqrt{14}$
$\quad$ (c) $(x - 4)/4 = (y + 4)/(-2) = (z - 8)/6$, $2x - y + 3z = 36$

4.3 $t = -1$, $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$, $(x - 1)/3 = (y + 1)/3 = (z - 5)/(-5)$,
$\quad$ $3x + 3y - 5z + 25 = 0$

4.5 $|d\mathbf{r}/dt| = \sqrt{2}$; $|d^2\mathbf{r}/dt^2| = 1$; path is a helix.

4.8 $d\mathbf{r}/dt = \mathbf{e}_r(dr/dt) + \mathbf{e}_\theta(rd\theta/dt)$,
$\quad$ $d^2\mathbf{r}/dt^2 = \mathbf{e}_r[d^2r/dt^2 - r(d\theta/dt)^2]$
$\qquad\qquad\qquad$ $+\mathbf{e}_\theta[rd^2\theta/dt^2 + 2(dr/dt)(d\theta/dt)]$.

4.10 $\mathbf{V} \times d\mathbf{V}/dt$

6.1 $-16\mathbf{i} - 12\mathbf{j} + 8\mathbf{k}$ $\qquad\qquad$ 6.2 $\quad -\mathbf{i}$

6.3 0 $\qquad\qquad\qquad\qquad\qquad$ 6.4 $\quad \pi e/(3\sqrt{5})$

6.5 $\boldsymbol{\nabla}\phi = \mathbf{i} - \mathbf{k}$; $-\boldsymbol{\nabla}\phi$; $\quad d\phi/ds = 2/\sqrt{13}$

6.6 $6x + 8y - z = 25$, $(x - 3)/6 = (y - 4)/8 = (z - 25)/(-1)$

6.7 $5x - 3y + 2z + 3 = 0$, $\quad \mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + (5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})t$

6.8 (a) $7/3$ $\qquad\qquad$ (b) $5x - z = 8$; $\frac{x-1}{5} = \frac{z+3}{-1}$, $\quad y = \pi/2$

6.9 (a) $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ $\qquad$ (b) $5/\sqrt{6}$ $\qquad$ (c) $\mathbf{r} = (1, 1, 1) + (2, -2, -1)t$

6.10 $\mathbf{j}$, 1, $-4/5$

6.11 $\boldsymbol{\nabla}\phi = 2x\mathbf{i} - 2y\mathbf{j}$, $\mathbf{E} = -2x\mathbf{i} + 2y\mathbf{j}$

6.12 (a) $2\sqrt{5}$, $-2\mathbf{i} + \mathbf{j}$ (b) $3\mathbf{i} + 2\mathbf{j}$ (c) $\sqrt{10}$

6.13 (a) $\mathbf{i} + \mathbf{j}$, $|\nabla\phi| = e$ (b) $-1/2$ (c) $\mathbf{i}$, $|\mathbf{E}| = 1$ (d) e^{-1}

6.14 (b) Down, at the rate $11\sqrt{2}$

6.15 (a) $4\sqrt{2}$, up   (b) 0, around the hill
      (c) $-4/\sqrt{10}$, down   (d) 8/5, up

6.17 $\mathbf{e}_r$ 6.18 $\mathbf{i}$ 6.19 $\mathbf{j}$ 6.20 $2r\mathbf{e}_r$

7.1 $\boldsymbol{\nabla}\cdot\mathbf{r} = 3$, $\boldsymbol{\nabla}\times\mathbf{r} = 0$ 7.2 $\boldsymbol{\nabla}\cdot\mathbf{r} = 2$, $\boldsymbol{\nabla}\times\mathbf{r} = 0$

7.3 $\boldsymbol{\nabla}\cdot\mathbf{V} = 1$, $\boldsymbol{\nabla}\times\mathbf{V} = 0$ 7.4 $\boldsymbol{\nabla}\cdot\mathbf{V} = 0$, $\boldsymbol{\nabla}\times\mathbf{V} = -(\mathbf{i}+\mathbf{j}+\mathbf{k})$

7.5 $\boldsymbol{\nabla}\cdot\mathbf{V} = 2(x+y+z)$, $\boldsymbol{\nabla}\times\mathbf{V} = 0$

7.6 $\boldsymbol{\nabla}\cdot\mathbf{V} = 5xy$, $\boldsymbol{\nabla}\times\mathbf{V} = \mathbf{i}xz - \mathbf{j}yz + \mathbf{k}(y^2 - x^2)$

7.7 $\boldsymbol{\nabla}\cdot\mathbf{V} = 0$, $\boldsymbol{\nabla}\times\mathbf{V} = x\mathbf{i} - y\mathbf{j} - x\cos y\mathbf{k}$

7.8 $\boldsymbol{\nabla}\cdot\mathbf{V} = 2 + x\sinh z$, $\boldsymbol{\nabla}\times\mathbf{V} = 0$ 7.9 $6y$

7.10 0   7.11 $-(x^2 + y^2)/(x^2 - y^2)^{3/2}$

7.12 $4(x + y)^{-3}$ 7.13 $2xy$

7.14 0   7.15 0

7.16 $2(x^2 + y^2 + z^2)^{-1}$ 7.18 $2\mathbf{k}$

7.19 $2/r$ 7.20 0

8.1 $-11/3$

8.2   (a) -4π (b) -16 (c) -8

8.3   (a) 5/3   (b) 1   (c) 2/3

8.4   (a) 3   (b) 8/3

8.5   (a) 86/3   (b) $-31/3$

8.6   (a) 3   (b) 3   (c) 3

8.7   (a) -2π (b) 0   (c) -2 (d) 2π

8.8 $yz - x$ 8.9 $3xy - x^3yz - z^2$

8.10 $\frac{1}{2}kr^2$ 8.11 $-y\sin^2 x$

8.12 $-(xy + z)$ 8.13 $-z^2\cosh y$

8.14 $-\arcsin xy$ 8.15 $-(x^2 + 1)\cos^2 y$

8.16 (a) $\mathbf{F}_1$; $\phi_1 = y^2z - x^2$
      (b) For $\mathbf{F}_2$:   (1) $W = 0$ (2) $W = -4$ (3) $W = 2\pi$

8.17 $\mathbf{F}_2$ conservative, $W = 0$; for $\mathbf{F}_1$, $W = 2\pi$

8.18 (a) $\pi + \pi^2/2$ (b) $\pi^2/2$

8.20 $\phi = mgz$, $\phi = -C/r$

9.2   40   9.3   14/3   9.4 $-3/2$

9.5   20   9.7 πab 9.8 24π

9.9 $(\overline{x}, \overline{y}) = (1, 1)$ 9.10 -20 9.11 2

9.12 29/3

10.1 4π 10.2 3   10.3 9π

10.4 36π 10.5 $4\pi \cdot 5^5$ 10.6 1

10.7 48π 10.8 80π 10.9 16π

10.10 27π

10.12 $\phi = \begin{cases} 0, r \le R_1 \\ (k/2\pi\epsilon_0)\ln(R_1/r), & R_1 \le r \le R_2 \\ (k/2\pi\epsilon_0)\ln(R_1/R_2), & r \ge R_2 \end{cases}$

11.1 $-3\pi a^2$	11.2 $2ab^2$	11.3 0
11.4 -12	11.5 36	11.6 45π
11.7 0	11.8 0	11.9 $32\pi/3$
11.10 -6π	11.11 24	11.12 18π
11.13 0	11.14 -8π	11.15 $-2\pi\sqrt{2}$

In the answers for Problems 18 to 22, u is arbitrary.

11.18 $\mathbf{A} = (xz - yz^2 - y^2/2)\mathbf{i} + (x^2/2 - x^2z + yz^2/2 - yz)\mathbf{j} + \boldsymbol{\nabla}u$

11.19 $\mathbf{A} = (y^2z - xy^2/2)\mathbf{i} + xz^2\mathbf{j} + x^2y\mathbf{k} + \boldsymbol{\nabla}u$

11.20 $\mathbf{A} = \mathbf{i}\sin zx + \mathbf{j}\cos zx + \mathbf{k}e^{zy} + \boldsymbol{\nabla}u$

11.21 $\mathbf{A} = \mathbf{i}y + \boldsymbol{\nabla}u$

11.22 $\mathbf{A} = \mathbf{i}(xz - y^3/3) + \mathbf{j}(-yz + x^3/3) + \mathbf{k}(x + y)z + \boldsymbol{\nabla}u$

12.1 $\sin\theta\cos\theta\,\mathbf{C}$

12.2 $\frac{1}{2}|\mathbf{B} \times \mathbf{A}|$

12.5 (a) -4π (b) -16 (c) -8

12.6 $5\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$

12.7 (a) $9\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ (b) $29/3$

12.8 $2/\sqrt{5}$

12.9 24

12.10 (a) $2\mathbf{i} + \mathbf{j}$ (b) $11/5$ (c) $2x + y = 4$

12.11 (a) $\operatorname{grad}\phi = -3y\mathbf{i} - 3x\mathbf{j} + 2z\mathbf{k}$
    (b) $-\sqrt{3}$
    (c) $2x + y - 2z + 2 = 0$, $\mathbf{r} = (1, 2, 3) + (2, 1, -2)t$

12.12 (a) $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$
    (b) 3
    (c) $3x + 2y + z = 4$, $\mathbf{r} = (0, 1, 2) + (3, 2, 1)t$
    (d) $(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})/\sqrt{14}$

12.13 (a) $6\mathbf{i} - \mathbf{j} - 4\mathbf{k}$
    (b) $53^{-1/2}(6\mathbf{i} - \mathbf{j} - 4\mathbf{k})$
    (c) same as (a)
    (d) $53^{1/2}$
    (e) $53^{1/2}$

12.14 (a) $-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$
    (b) 0

12.15 $\phi = -y^2\cosh^2 xz$

12.16 (a) $\mathbf{F}_1$ is conservative.
    (b) $W_1 = x^2z + \frac{1}{2}y^2$
    (c) and (d) 1

12.17 6

12.18 Not conservative
    (a) $1/2$
    (b) $4/3$

12.19 (a) $\mathbf{F}_1$ is conservative; $\mathbf{F}_2$ is not conservative $(\boldsymbol{\nabla} \times \mathbf{F}_2 = \mathbf{k})$
    (b) 2π
    (c) For $\mathbf{F}_1$, $V_1 = 2xy - yz - \frac{1}{2}z^2$
    (d) $W_1 = 45/2$
    (e) $W_2 = 2\pi$

12.20 π	12.21 4	12.22 108π	12.23 192π
12.24 54π	12.25 -18π	12.26 0	12.27 4
12.28 -2π	12.29 10	12.30 4	12.31 $29/3$

Chapter 7

		amplitude	period	frequency	velocity amplitude
2.1		3	$2\pi/5$	$5/(2\pi)$	15
2.2		2	$\pi/2$	$2/\pi$	8
2.3		1/2	2	1/2	$\pi/2$
2.4		5	2π	$1/(2\pi)$	5
2.5	$s = \sin 6t$	1	$\pi/3$	$3/\pi$	6
2.6	$s = 6\cos\frac{\pi}{8}\sin 2t$	$6\cos\frac{\pi}{8} = 5.54$	π	$1/\pi$	$12\cos\frac{\pi}{8} = 11.1$
2.7		5	2π	$1/(2\pi)$	5
2.8		2	4π	$1/(4\pi)$	1
2.9		2	2	1/2	2π
2.10		4	π	$1/\pi$	8
2.11	q	3	1/60	60	
	I	360π	1/60	60	
2.12	q	4	1/15	15	
	I	120π	1/15	15	

2.13 A = maximum value of θ, $\omega = \sqrt{g/l}$.

2.14 $t = 12$ 2.15 $t = 3\pi$

2.16 $t \cong 4.91 \cong 281°$ 2.18 $A = 2$, $T = 1$, $f = 1$, $v = 3$, $\lambda = 3$

2.19 $A = 1$, $T = 4$, $f = 1/4$, $v = 1/4$, $\lambda = 1$

2.20 $A = 3$, $T = 4$, $f = 1/4$, $v = 1/2$, $\lambda = 2$

2.21 $y = 20\sin\frac{\pi}{2}(x - 6t)$, $\frac{\partial y}{\partial t} = -60\pi\cos\frac{\pi}{2}(x - 6t)$

2.22 $y = 4\sin 2\pi(\frac{x}{3} - \frac{t}{6})$ 2.23 $y = \sin 880\pi(\frac{x}{350} - t)$

2.24 $y = \sin\frac{200\pi}{153}(x - 1530t)$ 2.25 $y = 10\sin\frac{\pi}{250}(x - 3\cdot 10^8 t)$

3.6 $\sin(2x + \frac{\pi}{3})$ 3.7 $-\sqrt{2}\sin(\pi x - \frac{\pi}{4})$

4.3	0	4.4	e^{-1}	4.5	$1/\pi + 1/2$	4.6	$2/\pi$
4.7	$\pi/12 - 1/2$	4.8	0	4.9	1/2	4.10	0
4.11	1/2	4.12	1/2	4.14	(a) $2\pi/3$	(b) π	
4.15	(a) 3/2	(b) 3/2		4.16	(a) π/ω	(b) 1	

5.1 to 5.11   The answers for Problems 5.1 to 5.11 are the
sine-cosine series in Problems 7.1 to 7.11.

$x \rightarrow$	-2π	$-\pi$	$-\pi/2$	0	$\pi/2$	π	2π
6.1	1/2	1/2	1	1/2	0	1/2	1/2
6.2	1/2	0	0	1/2	1/2	0	1/2
6.3	0	1/2	0	0	1/2	1/2	0
6.4	-1	0	-1	-1	0	0	-1
6.5	$-1/2$	1/2	0	$-1/2$	0	1/2	$-1/2$
6.6	1/2	1/2	1/2	1/2	1/2	1/2	1/2
6.7	0	$\pi/2$	0	0	$\pi/2$	$\pi/2$	0
6.8	1	1	$1-\frac{\pi}{2}$	1	$1+\frac{\pi}{2}$	1	1
6.9	0	π	$\pi/2$	0	$\pi/2$	π	0
6.10	π	0	$\pi/2$	π	$\pi/2$	0	π
6.11	0	0	0	0	1	0	0

6.13 and 6.14   At $x = \pi/2$, same series as in the example.

7.1 $f(x) = \dfrac{1}{2} + \dfrac{i}{\pi} \displaystyle\sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \dfrac{1}{n}\, e^{inx} = \dfrac{1}{2} - \dfrac{2}{\pi} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n} \sin nx$

7.2 $a_n = \frac{1}{n\pi} \sin \frac{n\pi}{2}, \;\; b_n = \frac{1}{n\pi}\left(1 - \cos \frac{n\pi}{2}\right), \;\; a_0/2 = c_0 = \frac{1}{4},$
$c_n = \frac{i}{2n\pi}(e^{-in\pi/2} - 1), \;\; n > 0; \; c_{-n} = \bar{c}_n$
$f(x) = \frac{1}{4} + \frac{1}{2\pi}\big[(1-i)e^{ix} + (1+i)e^{-ix} - \frac{2i}{2}(e^{2ix} - e^{-2ix})$
$\qquad\qquad\qquad - \frac{1+i}{3}e^{3ix} - \frac{1-i}{3}e^{-3ix} + \frac{1-i}{5}e^{5ix} + \frac{1+i}{5}e^{-5ix}\cdots\big]$
$\quad = \frac{1}{4} + \frac{1}{\pi}\left(\cos\, x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x \cdots\right)$
$\qquad + \frac{1}{\pi}\left(\sin x + \frac{2}{3}\sin 2x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{2}{6}\sin 6x\cdots\right)$

7.3 $a_n = -\frac{1}{n\pi}\sin\frac{n\pi}{2}, \; a_0/2 = c_0 = \frac{1}{4}$
$b_n = \frac{1}{n\pi}\left(\cos\frac{n\pi}{2} - \cos n\pi\right) = \frac{1}{n\pi}\{1, -2, 1, 0, \text{ and repeat}\}$
$c_n = \frac{i}{2n\pi}\left(e^{-in\pi} - e^{-in\pi/2}\right), \;\; n > 0; \;\; c_{-n} = \bar{c}_n$
$f(x) = \frac{1}{4} + \frac{1}{2\pi}\left[-(1+i)\,e^{ix} - (1-i)\,e^{-ix} + \frac{2i}{2}(e^{2ix} - e^{-2ix})\right.$
$\qquad\qquad \left. + \frac{1-i}{3}\,e^{3ix} + \frac{1+i}{3}\,e^{-3ix} - \frac{1+i}{5}\,e^{5ix} - \frac{(1-i)}{5}\,e^{-5ix}\cdots\right]$
$\quad = \frac{1}{4} - \frac{1}{\pi}\left(\cos\, x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x \cdots\right)$
$\qquad + \frac{1}{\pi}\left(\sin x - \frac{2}{2}\sin 2x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x\cdots\right)$

7.4 $c_0 = a_0/2 = -1/2$; for $n \neq 0$, coefficients are 2 times the
coefficients in Problem 7.3.
$f(x) = -\frac{1}{2} - \frac{1}{\pi}\big[(1+i)e^{ix} + (1-i)e^{-ix} - \frac{2i}{2}(e^{2ix} - e^{-2ix})$
$\qquad - \frac{1-i}{3}e^{3ix} - \frac{1+i}{3}e^{-3ix} + \frac{1+i}{5}e^{5ix} + \frac{1-i}{5}e^{-5ix}\cdots\big]$
$\quad = -\frac{1}{2} - \frac{2}{\pi}\left(\cos\, x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x \cdots\right)$
$\qquad + \frac{2}{\pi}\left(\sin x - \frac{2}{2}\sin 2x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x - \frac{2}{6}\sin 6x\cdots\right)$

7.5 $a_n = -\frac{2}{n\pi} \sin \frac{n\pi}{2}$ $a_0/2 = c_0 = 0$

        $b_n = \frac{1}{n\pi} \left(2 \cos \frac{n\pi}{2} - 1 - \cos n\pi\right) = -\frac{4}{n\pi}\{0, 1, 0, 0, \text{ and repeat}\}$

        $c_n = \frac{1}{2in\pi} \left(2e^{-in\pi/2} - 1 - e^{-in\pi}\right) = \frac{1}{n\pi}\{-1, 2i, 1, 0, \text{ and repeat }\}, \; n > 0$

        $c_{-n} = \overline{c}_n$

        $f(x) = -\frac{1}{\pi} \left[e^{ix} + e^{-ix} - \frac{2i}{2}\left(e^{2ix} - e^{-2ix}\right) - \frac{1}{3}(e^{3ix} + e^{-3ix})\right.$

                  $\left. + \frac{1}{5}(e^{5ix} + e^{-5ix}) - \frac{2i}{6}(e^{6ix} - e^{-6ix}) \cdots\right]$

            $= -\frac{2}{\pi}\left(\cos \; x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x \cdots\right)$

              $-\frac{4}{\pi}\left(\frac{1}{2}\sin 2x + \frac{1}{6}\sin 6x + \frac{1}{10}\sin 10x \cdots\right)$

7.6 $f(x) = \frac{1}{2} + \frac{2}{i\pi}\sum \frac{1}{n}e^{inx}$ $(n = \pm 2, \; \pm 6, \; \pm 10, \cdots)$

           $= \frac{1}{2} + \frac{4}{\pi}\sum \frac{1}{n}\sin \; nx$ $(n = 2, \; 6, \; 10, \cdots)$

7.7 $f(x) = \frac{\pi}{4} - \sum\limits_{\substack{-\infty \\ \text{odd } n}}^{\infty} \left(\frac{1}{n^2\pi} + \frac{i}{2n}\right)e^{inx} + \sum\limits_{\substack{-\infty \\ \text{even } n \neq 0}}^{\infty} \frac{i}{2n}e^{inx}$

           $= \frac{\pi}{4} - \sum\limits_{1}^{\infty} \frac{(-1)^n}{n}\sin \; nx \; - \; \frac{2}{\pi}\sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2}\cos \; nx$

7.8 $f(x) = 1 + \sum\limits_{\substack{-\infty \\ n \neq 0}}^{\infty} (-1)^n \frac{i}{n}\,e^{inx} = \; 1 + 2\sum\limits_{1}^{\infty}(-1)^{n+1}\frac{1}{n}\sin \; nx$

7.9 $f(x) = \frac{\pi}{2} - \frac{2}{\pi}\sum\limits_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{e^{inx}}{n^2} = \frac{\pi}{2} - \frac{4}{\pi}\sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos \; nx}{n^2}$

7.10 $f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum\limits_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{e^{inx}}{n^2} = \frac{\pi}{2} + \frac{4}{\pi}\sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos \; nx}{n^2}$

7.11 $f(x) = \frac{1}{\pi} + \frac{e^{ix} - e^{-ix}}{4i} - \frac{1}{\pi}\sum\limits_{\substack{-\infty \\ \text{even } n \neq 0}}^{\infty} \frac{e^{inx}}{n^2 - 1}$

           $= \frac{1}{\pi} + \frac{1}{2}\sin \; x - \frac{2}{\pi}\sum\limits_{\substack{2 \\ \text{even } n}}^{\infty} \frac{\cos nx}{n^2 - 1}$

7.13 $a_n = 2\,\text{Re}\,c_n, \; b_n = -2\,\text{Im}\,c_n, \; c_n = \frac{1}{2}(a_n - ib_n), \; c_{-n} = \frac{1}{2}(a_n + ib_n)$

8.1 $f(x) = \frac{1}{2} + \frac{i}{\pi}\sum\limits_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{1}{n}e^{in\pi x/l} = \frac{1}{2} - \frac{2}{\pi}\sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n}\sin \frac{n\pi x}{l}$

8.2 $a_n = \frac{1}{n\pi}\sin \frac{n\pi}{2}, \; b_n = \frac{1}{n\pi}\left(1 - \cos \frac{n\pi}{2}\right), \; a_0/2 = c_0 = \frac{1}{4}$

        $c_n = \frac{i}{2n\pi}(e^{-in\pi/2} - 1)$

          $= \frac{1}{2n\pi}\{1 - i, -2i, -(1 + i), 0, \text{ and repeat}\}, \; n > 0; c_{-n} = \overline{c}_n$

        $f(x) = \frac{1}{4} + \frac{1}{2\pi}\left[(1 - i)e^{i\pi x/l} + (1 + i)e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l})\right.$

                 $\left. - \frac{1+i}{3}e^{3i\pi x/l} - \frac{1-i}{3}e^{-3i\pi x/l} + \frac{1-i}{5}e^{5i\pi x/l} + \frac{1+i}{5}e^{-5i\pi x/l} \cdots\right]$

            $= \frac{1}{4} + \frac{1}{\pi}\left(\cos \frac{\pi x}{l} - \frac{1}{3}\cos \frac{3\pi x}{l} + \frac{1}{5}\cos \frac{5\pi x}{l} \cdots\right)$

              $+ \frac{1}{\pi}\left(\sin \frac{\pi x}{l} + \frac{2}{2}\sin \frac{2\pi x}{l} + \frac{1}{3}\sin \frac{3\pi x}{l} + \frac{1}{5}\sin \frac{5\pi x}{l} + \frac{2}{6}\sin \frac{6\pi x}{l} \cdots\right)$

8.3 $a_n = -\frac{1}{n\pi} \sin \frac{n\pi}{2}, \;\; a_0/2 = c_0 = \frac{1}{4}$

$b_n = \frac{1}{n\pi} \left(\cos \frac{n\pi}{2} - \cos n\pi \right) = \frac{1}{n\pi}\{1, -2, 1, 0 \text{ , and repeat}\}$

$c_n = \frac{i}{2n\pi}(e^{-in\pi} - e^{-in\pi/2})$

$= \frac{1}{2n\pi} \{-(1+i), 2i, 1-i, 0, \text{ and repeat}\}, \;\; n > 0; c_{-n} = \overline{c}_n$

$f(x) = \frac{1}{4} + \frac{1}{2\pi} \left[-(1+i)e^{i\pi x/l} - (1-i)e^{-i\pi x/l} + \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l}) \right.$

$\left. + \frac{1-i}{3}e^{3i\pi x/l} + \frac{1+i}{3}e^{-3i\pi x/l} - \frac{1+i}{5}e^{5i\pi x/l} - \frac{1-i}{5}e^{-5i\pi x/l} \cdots \right]$

$= \frac{1}{4} - \frac{1}{\pi} \left(\cos \frac{\pi x}{l} - \frac{1}{3}\cos \frac{3\pi x}{l} + \frac{1}{5}\cos \frac{5\pi x}{l} \cdots \right)$

$+ \frac{1}{\pi} \left(\sin \frac{\pi x}{l} - \frac{2}{2}\sin \frac{2\pi x}{l} + \frac{1}{3}\sin \frac{3\pi x}{l} + \frac{1}{5}\sin \frac{5\pi x}{l} - \frac{2}{6}\sin \frac{6\pi x}{l} \cdots \right)$

8.4 $c_0 = a_0/2 = -1/2$; for $n \neq 0$, coefficients are 2 times the
coefficients in Problem 8.3.

$f(x) = -\frac{1}{2} - \frac{1}{\pi} \left[(1+i)e^{i\pi x/l} + (1-i)e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l}) \right.$

$\left. - \frac{1-i}{3}e^{3i\pi x/l} - \frac{1+i}{3}e^{-3i\pi x/l} + \frac{1+i}{5}e^{5i\pi x/l} + \frac{1-i}{5}e^{-5i\pi x/l} \cdots \right]$

$= -\frac{1}{2} - \frac{2}{\pi} \left(\cos \frac{\pi x}{l} - \frac{1}{3}\cos \frac{3\pi x}{l} + \frac{1}{5}\cos \frac{5\pi x}{l} \cdots \right)$

$+ \frac{2}{\pi} \left(\sin \frac{\pi x}{l} - \frac{2}{2}\sin \frac{2\pi x}{l} + \frac{1}{3}\sin \frac{3\pi x}{l} + \frac{1}{5}\sin \frac{5\pi x}{l} - \frac{2}{6}\sin \frac{6\pi x}{l} \cdots \right)$

8.5 $a_n = -\frac{2}{n\pi} \sin \frac{n\pi}{2}, \;\; a_0 = 0,$

$b_n = \frac{1}{n\pi}(2 \cos \frac{n\pi}{2} - 1 - \cos n\pi) = -\frac{4}{n\pi}\{0, 1, 0, 0, \text{ and repeat}\}$

$c_n = \frac{1}{2in\pi}(2e^{-in\pi/2} - 1 - e^{-in\pi}) = \frac{1}{n\pi}\{-1, 2i, 1, 0, \text{ and repeat }\}, n > 0$

$c_{-n} = \overline{c}_n, \; c_0 = 0$

$f(x) = -\frac{1}{\pi} \left[e^{i\pi x/l} + e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l}) \right.$

$- \frac{1}{3}(e^{3i\pi x/l} + e^{-3i\pi x/l})$

$\left. + \frac{1}{5}(e^{5i\pi x/l} + e^{-5i\pi x/l}) - \frac{2i}{6}(e^{6i\pi x/l} - e^{-6i\pi x/l}) \cdots \right]$

$= -\frac{2}{\pi} \left(\cos \frac{\pi x}{l} - \frac{1}{3}\cos \frac{3\pi x}{l} + \frac{1}{5}\cos \frac{5\pi x}{l} \cdots \right)$

$- \frac{4}{\pi} \left(\frac{1}{2}\sin \frac{2\pi x}{l} + \frac{1}{6}\sin \frac{6\pi x}{l} + \frac{1}{10}\sin \frac{10\pi x}{l} \cdots \right)$

8.6 $f(x) = \frac{1}{2} + \frac{2}{i\pi} \sum \frac{1}{n}e^{in\pi x/l} \quad (n = \pm 2, \pm 6, \pm 10, \cdots)$

$= \frac{1}{2} + \frac{4}{\pi} \sum \frac{1}{n} \sin \frac{n\pi x}{l} \quad (n = 2, 6, 10, \cdots)$

8.7 $f(x) = \frac{l}{4} + \frac{il}{2\pi} \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n}e^{in\pi x/l} - \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{l}{n^2\pi^2}e^{in\pi x/l}$

$= \frac{l}{4} - \frac{2l}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{l} - \frac{l}{\pi} \sum_{1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{l}$

8.8 $f(x) = 1 + \frac{il}{\pi} \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n}e^{in\pi x/l} = 1 - \frac{2l}{\pi} \sum_{1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{l}$

8.9 $f(x) = \frac{l}{2} - \frac{2l}{\pi^2} \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{e^{in\pi x/l}}{n^2} = \frac{l}{2} - \frac{4l}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos n\pi x/l}{n^2}$

8.10   (a) $f(x) = i \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n}e^{inx} = -2 \sum_{1}^{\infty} \frac{(-1)^n}{n} \sin nx$

(b) $f(x) = \pi + \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{i}{n}e^{inx} = \pi - 2 \sum_{1}^{\infty} \frac{\sin nx}{n}$

8.11 (a) $f(x) = \dfrac{\pi^2}{3} + 2\displaystyle\sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \dfrac{(-1)^n}{n^2}e^{inx} = \dfrac{\pi^2}{3} + 4\sum_{1}^{\infty}\dfrac{(-1)^n}{n^2}\cos nx$

   (b) $f(x) = \dfrac{4\pi^2}{3} + 2\displaystyle\sum_{\substack{-\infty \\ n\neq 0}}^{\infty}\left(\dfrac{1}{n^2}+\dfrac{i\pi}{n}\right)e^{inx} = \dfrac{4\pi^2}{3}+4\sum_{1}^{\infty}\dfrac{\cos nx}{n^2}-4\pi\sum_{1}^{\infty}\dfrac{\sin nx}{n}$

8.12 (a) $f(x) = \dfrac{\sinh\pi}{\pi}\displaystyle\sum_{-\infty}^{\infty}\dfrac{(-1)^n(1+in)}{1+n^2}e^{inx}$

   $\qquad = \dfrac{\sinh\pi}{\pi} + \dfrac{2\sinh\pi}{\pi}\displaystyle\sum_{1}^{\infty}\dfrac{(-1)^n}{1+n^2}(\cos\ nx - n\ \sin\ nx)$

   (b) $f(x) = \dfrac{e^{2\pi}-1}{2\pi}\displaystyle\sum_{-\infty}^{\infty}\dfrac{1+in}{1+n^2}e^{inx}$

   $\qquad = \dfrac{e^{2\pi}-1}{\pi}\left[\dfrac{1}{2}+\displaystyle\sum_{1}^{\infty}\dfrac{1}{1+n^2}(\cos\ nx - n\ \sin\ nx)\right]$

8.13 (a) $f(x) = 2 + \dfrac{2}{i\pi}\displaystyle\sum_{\substack{-\infty \\ n\neq 0}}^{\infty}\dfrac{(-1)^n}{n}e^{in\pi x/2} = 2 + \dfrac{4}{\pi}\sum_{1}^{\infty}\dfrac{(-1)^n}{n}\sin\dfrac{n\pi x}{2}$

   (b) $f(x) = \dfrac{2}{i\pi}\displaystyle\sum_{\substack{-\infty \\ n\neq 0}}^{\infty}\dfrac{1}{n}e^{in\pi x/2} = \dfrac{4}{\pi}\sum_{1}^{\infty}\dfrac{1}{n}\sin\dfrac{n\pi x}{2}$

8.14 (a) $f(x) = \dfrac{8}{\pi}\displaystyle\sum_{1}^{\infty}\dfrac{n(-1)^{n+1}}{4n^2-1}\sin 2n\pi x = \dfrac{4i}{\pi}\sum_{-\infty}^{\infty}\dfrac{n(-1)^n}{4n^2-1}e^{2in\pi x}$

   (b) $f(x) = \dfrac{2}{\pi} - \dfrac{4}{\pi}\displaystyle\sum_{1}^{\infty}\dfrac{\cos 2n\pi x}{4n^2-1} = -\dfrac{2}{\pi}\sum_{-\infty}^{\infty}\dfrac{1}{4n^2-1}e^{2in\pi x}$

8.15 (a) $f(x) = \dfrac{i}{\pi}\displaystyle\sum_{\substack{-\infty \\ n\neq 0}}^{\infty}\dfrac{(-1)^n}{n}e^{in\pi x} = \dfrac{2}{\pi}\sum_{1}^{\infty}(-1)^{n+1}\dfrac{\sin n\pi x}{n}.$

   (b) $f(x) = \dfrac{4}{\pi^2}\displaystyle\sum_{\substack{-\infty \\ \text{odd } n}}^{\infty}\dfrac{1}{n^2}e^{in\pi x} = \dfrac{8}{\pi^2}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\dfrac{\cos n\pi x}{n^2}.$

   (c) $f(x) = \dfrac{-4i}{\pi^3}\displaystyle\sum_{\substack{-\infty \\ \text{odd } n}}^{\infty}\dfrac{1}{n^3}e^{in\pi x} = \dfrac{8}{\pi^3}\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\dfrac{\sin n\pi x}{n^3}.$

8.16 $f(x) = 1 - \dfrac{2}{\pi}\displaystyle\sum_{1}^{\infty}\dfrac{\sin n\pi x}{n} = 1 - \dfrac{1}{i\pi}\sum_{\substack{-\infty \\ n\neq 0}}^{\infty}\dfrac{1}{n}e^{in\pi x}$

8.17 $f(x) = \frac{3}{4} - \frac{1}{\pi}(\cos\frac{\pi x}{2} - \frac{1}{3}\cos\frac{3\pi x}{2} + \frac{1}{5}\cos\frac{5\pi x}{2}\cdots)$
   $\qquad + \frac{1}{\pi}(\sin\frac{\pi x}{2} + \frac{2}{2}\sin\pi x + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \frac{2}{6}\sin 3\pi x\cdots)$

8.18 $f(x) = \dfrac{100}{3} + \dfrac{100}{\pi^2}\displaystyle\sum_{1}^{\infty}\dfrac{1}{n^2}\cos\dfrac{n\pi x}{5} - \dfrac{100}{\pi}\sum_{1}^{\infty}\dfrac{1}{n}\sin\dfrac{n\pi x}{5}$

   $\qquad = \dfrac{100}{3} + 50\displaystyle\sum_{\substack{-\infty \\ n\neq 0}}^{\infty}\left(\dfrac{1}{n^2\pi^2}-\dfrac{1}{in\pi}\right)e^{in\pi x/5}$

8.19 $f(x) = \dfrac{1}{8} - \dfrac{1}{\pi^2}\displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty}\dfrac{1}{n^2}\cos 2n\pi x + \dfrac{1}{2\pi}\sum_{1}^{\infty}\dfrac{(-1)^{n+1}}{n}\sin 2n\pi x$

8.20 $f(x) = \dfrac{2}{3} + \displaystyle\sum_1^\infty a_n \cos \dfrac{2n\pi x}{3} + \sum_1^\infty b_n \sin \dfrac{2n\pi x}{3}, \ \text{where}$

$$a_n = \begin{cases} 0, \ n = 3k \\ \dfrac{-9}{8n^2\pi^2}, \ \text{otherwise} \end{cases} \qquad b_n = \begin{cases} -\dfrac{1}{n\pi}, \ n = 3k \\ -\dfrac{1}{n\pi} - \dfrac{3\sqrt{3}}{8n^2\pi^2}, \quad n = 3k+1 \\ -\dfrac{1}{n\pi} + \dfrac{3\sqrt{3}}{8n^2\pi^2}, \quad n = 3k+2 \end{cases}$$

9.1   (a) $\cos nx + i \sin nx$ $\qquad\qquad$ (b) $x \sinh x + x \cosh x$

9.2   (a) $\frac{1}{2}\ln|1 - x^2| + \frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|$ $\qquad$ (b) $(\cos x + x \sin x) + (\sin x + x \cos x)$

9.3   (a) $(-x^4 - 1) + (x^5 + x^3)$ $\qquad\qquad$ (b) $(1 + \cosh x) + \sinh x$

9.5 $f(x) = \dfrac{4}{\pi} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{1}{n} \sin nx$ $\qquad\qquad$ 9.6 $f(x) = \dfrac{4}{\pi} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{1}{n} \sin \dfrac{n\pi x}{l}$

9.7 $a_n = \frac{2}{n\pi}\sin\frac{n\pi}{2}, \ a_0/2 = 1/2$

$f(x) = \frac{1}{2} + \frac{2}{\pi}(\cos \frac{\pi x}{2} - \frac{1}{3}\cos\frac{3\pi x}{2} + \frac{1}{5}\cos\frac{5\pi x}{2}\cdots)$

9.8 $f(x) = \displaystyle\sum_1^\infty \dfrac{(-1)^{n+1}}{n}\sin 2nx$

9.9 $f(x) = \dfrac{1}{12} + \dfrac{1}{\pi^2}\displaystyle\sum_1^\infty \dfrac{(-1)^n}{n^2}\cos 2n\pi x$

9.10 $f(x) = \dfrac{\pi}{4} - \dfrac{2}{\pi}\displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{1}{n^2}\cos 2nx$

9.11 $f(x) = \dfrac{2\sinh\pi}{\pi}\left(\dfrac{1}{2} + \displaystyle\sum_1^\infty \dfrac{(-1)^n}{n^2 + 1}\cos nx\right)$

9.12 $f(x) = -\dfrac{2}{\pi}\displaystyle\sum_1^\infty \dfrac{1}{n}\sin n\pi x$

9.15 $f_c(x) = \dfrac{1}{2} - \dfrac{4}{\pi^2}\displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{\cos n\pi x}{n^2}$ $\qquad f_s(x) = \dfrac{2}{\pi}\displaystyle\sum_1^\infty \dfrac{(-1)^{n+1}\sin n\pi x}{n}$

9.16 $f_s = \dfrac{8}{\pi}\displaystyle\sum_{\text{odd } n=1}^\infty \dfrac{\sin nx}{n(4 - n^2)}$ $\qquad f_c = f_p = (1 - \cos 2x)/2$

9.17 $f_c(x) = \dfrac{4}{\pi}(\cos\pi x - \dfrac{1}{3}\cos 3\pi x + \dfrac{1}{5}\cos 5\pi x \cdots)$

$f_s(x) = f_p(x) = \dfrac{4}{\pi}\displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{1}{n}\sin 2n\pi x$

9.18   Even function: $a_0/2 = 1/3$,

$a_n = \frac{2}{n\pi}\sin\frac{n\pi}{3} = \frac{\sqrt{3}}{n\pi}\{1, 1, 0, -1, -1, 0, \text{ and repeat}\}$

$f_c(x) = \frac{1}{3} + \frac{\sqrt{3}}{\pi}(\cos\frac{\pi x}{3} + \frac{1}{2}\cos\frac{2\pi x}{3} - \frac{1}{4}\cos\frac{4\pi x}{3} - \frac{1}{5}\cos\frac{5\pi x}{3} + \frac{1}{7}\cos\frac{7\pi x}{3}\cdots)$

Odd function: $b_n = \frac{2}{n\pi}(1 - \cos\frac{n\pi}{3}) = \frac{1}{n\pi}\{1, 3, 4, 3, 1, 0, \text{ and repeat}\}$

$f_s(x) = \frac{1}{\pi}(\sin\frac{\pi x}{3} + \frac{3}{2}\sin\frac{2\pi x}{3} + \frac{4}{3}\sin\frac{3\pi x}{3}$

$+ \frac{3}{4}\sin\frac{4\pi x}{3} + \frac{1}{5}\sin\frac{5\pi x}{3} + \frac{1}{7}\sin\frac{7\pi x}{3}\cdots)$

9.18 continued

Function of period 3:

$a_n = \frac{1}{n\pi} \sin \frac{2n\pi}{3} = \frac{\sqrt{3}}{2n\pi}\{1, -1, 0, \text{and repeat}\}, \;\; a_0/2 = 1/3$

$b_n = \frac{1}{n\pi}(1 - \cos \frac{2n\pi}{3}) = \frac{3}{2n\pi}\{1, 1, 0, \text{and repeat}\}$

$f_p(x) = \frac{1}{3} + \frac{\sqrt{3}}{2\pi}(\cos \frac{2\pi x}{3} - \frac{1}{2}\cos \frac{4\pi x}{3} + \frac{1}{4}\cos \frac{8\pi x}{3} - \frac{1}{5}\cos \frac{10\pi x}{3} \cdots)$

$\qquad + \frac{3}{2\pi}(\sin \frac{2\pi x}{3} + \frac{1}{2}\sin \frac{4\pi x}{3} + \frac{1}{4}\sin \frac{8\pi x}{3} + \frac{1}{5}\sin \frac{10\pi x}{3} \cdots)$

9.19 $\;\; f_c(x) = f_p(x) = \dfrac{2}{\pi} - \dfrac{4}{\pi} \displaystyle\sum_1^\infty \dfrac{(-1)^n \cos 2nx}{4n^2 - 1}$

For f_s, $b_n = \dfrac{2}{\pi} \begin{cases} 0, \; n \text{ even} \\ \frac{2}{n+1}, \; n = 1 + 4k \\ \frac{2}{n-1}, \; n = 3 + 4k \end{cases}$

$f_s(x) = \frac{2}{\pi}(\sin x + \sin 3x + \frac{1}{3}\sin 5x + \frac{1}{3}\sin 7x + \frac{1}{5}\sin 9x + \frac{1}{5}\sin 11x \cdots)$

9.20 $\;\; f_c(x) = \dfrac{1}{3} + \dfrac{4}{\pi^2} \displaystyle\sum_1^\infty \dfrac{(-1)^n}{n^2} \cos n\pi x$

$f_s(x) = \dfrac{2}{\pi} \displaystyle\sum_1^\infty \dfrac{(-1)^{n+1}}{n} \sin n\pi x - \dfrac{8}{\pi^3} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{1}{n^3} \sin n\pi x$

$f_p(x) = \dfrac{1}{3} + \dfrac{1}{\pi^2} \displaystyle\sum_1^\infty \dfrac{1}{n^2} \cos 2n\pi x - \dfrac{1}{\pi} \displaystyle\sum_1^\infty \dfrac{1}{n} \sin 2n\pi x$

9.21 $\;\; f_c(x) = f_p(x) = \dfrac{1}{2} - \dfrac{4}{\pi^2} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{1}{n^2} \cos n\pi x$

$f_s(x) = \dfrac{8}{\pi^2} \left(\sin \dfrac{\pi x}{2} - \dfrac{1}{3^2}\sin \dfrac{3\pi x}{2} + \dfrac{1}{5^2}\sin \dfrac{5\pi x}{2} \cdots \right); \;\; b_n = \dfrac{8}{n^2\pi^2}\sin \dfrac{n\pi}{2}$

9.22 Even function: $a_n = -\dfrac{20}{n\pi}\sin \dfrac{n\pi}{2}$

$f_c(x) = 15 - \frac{20}{\pi}(\cos \frac{\pi x}{20} - \frac{1}{3}\cos \frac{3\pi x}{20} + \frac{1}{5}\cos \frac{5\pi x}{20} \cdots)$

Odd function:

$b_n = \frac{20}{n\pi}(\cos \frac{n\pi}{2} + 1 - 2\cos n\pi) = \frac{20}{n\pi}\{3, -2, 3, 0, \text{and repeat}\}$

$f_s(x) = \frac{20}{\pi}(3\sin \frac{\pi x}{20} - \frac{2}{2}\sin \frac{2\pi x}{20} + \frac{3}{3}\sin \frac{3\pi x}{20} + \frac{3}{5}\sin \frac{5\pi x}{20} \cdots)$

Function of period 20:

$f_p(x) = 15 - \dfrac{20}{\pi} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{1}{n}\sin \dfrac{n\pi x}{10}$

9.23 $\;\; f(x, 0) = \dfrac{8h}{\pi^2} \left(\sin \dfrac{\pi x}{l} - \dfrac{1}{3^2}\sin \dfrac{3\pi x}{l} + \dfrac{1}{5^2}\sin \dfrac{5\pi x}{l} \cdots \right)$

9.24 $\;\; f(x, 0) = \dfrac{8h}{\pi^2} \displaystyle\sum_1^\infty \dfrac{\lambda_n}{n^2}\sin \dfrac{n\pi x}{l}$ where

$\lambda_1 = \sqrt{2} - 1, \; \lambda_2 = 2, \; \lambda_3 = \sqrt{2} + 1, \; \lambda_4 = 0, \; \lambda_5 = -(\sqrt{2} + 1),$

$\lambda_6 = -2, \; \lambda_7 = -\sqrt{2} + 1, \; \lambda_8 = 0, \; ..., \; \lambda_n = 2\sin \frac{n\pi}{4} - \sin \frac{n\pi}{2}$

9.26 $\;\; f(x) = \dfrac{1}{2} - \dfrac{48}{\pi^4} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^\infty \dfrac{\cos n\pi x}{n^4}$ 9.27 $\;\; f(x) = \dfrac{8\pi^4}{15} - 48\displaystyle\sum_1^\infty (-1)^n \dfrac{\cos nx}{n^4}$

10.1 $p(t) = \sum_{1}^{\infty} a_n \cos 220 n\pi t$, $a_0 = 0$

$a_n = \frac{2}{n\pi}(\sin \frac{n\pi}{3} + \sin \frac{2n\pi}{3}) = \frac{2}{n\pi}\{\sqrt{3},\ 0,\ 0,\ 0,\ -\sqrt{3},\ 0,\ \text{and repeat}\}$

Relative intensities $= 1 : 0 : 0 : 0 : \frac{1}{25} : 0 : \frac{1}{49} : 0 : 0 : 0$

10.2 $p(t) = \sum_{1}^{\infty} b_n \sin 262 n\pi t$, where

$b_n = \frac{2}{n\pi}(1 - \cos \frac{n\pi}{3} - 3\cos n\pi + 3\cos \frac{2n\pi}{3})$

$= \frac{2}{n\pi}\{2, -3, 8, -3, 2, 0, \text{and repeat}\}$

Relative intensities $= 4 : \frac{9}{4} : \frac{64}{9} : \frac{9}{16} : \frac{4}{25} : 0$

10.3 $p(t) = \sum b_n \sin 220 n\pi t$

$b_n = \frac{2}{n\pi}(3 - 5\cos \frac{n\pi}{2} + 2\cos n\pi) = \frac{2}{n\pi}\{1, 10, 1, 0, \text{and repeat}\}$

Relative intensities $= 1 : 25 : \frac{1}{9} : 0 : \frac{1}{25} : \frac{25}{9} : \frac{1}{49} : 0 : \frac{1}{81} : 1$

10.4 $V(t) = \frac{200}{\pi}\left[1 + \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{2}{1 - n^2} \cos 120 n\pi t\right]$

Relative intensities $= 0 : 1 : 0 : \frac{1}{25} : 0 : (\frac{3}{35})^2$

10.5 $I(t) = \frac{5}{\pi}\left[1 + \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{2}{1 - n^2} \cos 120 n\pi t\right] + \frac{5}{2}\sin 120\pi t$

Relative intensities $= (\frac{5}{2})^2 : (\frac{10}{3\pi})^2 : 0 : (\frac{2}{3\pi})^2 : 0 : (\frac{2}{7\pi})^2$

$= 6.25 : 1.13 : 0 : 0.045 : 0 : 0.008$

10.6 $V(t) = 50 - \frac{400}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos 120 n\pi t$

Relative intensities $= 1 : 0 : \left(\frac{1}{3}\right)^4 : 0 : \left(\frac{1}{5}\right)^4$

10.7 $I(t) = -\frac{20}{\pi} \sum_{1}^{\infty} \frac{(-1)^n}{n} \sin 120 n\pi t$

Relative intensities $= 1 : \frac{1}{4} : \frac{1}{9} : \frac{1}{16} : \frac{1}{25}$

10.8 $I(t) = \frac{5}{2} - \frac{20}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos 120 n\pi t - \frac{10}{\pi} \sum_{1}^{\infty} \frac{(-1)^n}{n} \sin 120 n\pi t$

Relative intensities $= \left(1 + \frac{4}{\pi^2}\right) : \frac{1}{4} : \frac{1}{9}\left(1 + \frac{4}{9\pi^2}\right) : \frac{1}{16} : \frac{1}{25}\left(1 + \frac{4}{25\pi^2}\right)$

$= 1.4 : 0.25 : 0.12 : 0.06 : 0.04$

10.9 $V(t) = \frac{400}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} \sin 120 n\pi t$

Relative intensities $= 1 : 0 : \frac{1}{9} : 0 : \frac{1}{25}$

10.10 $V(t) = 75 - \frac{200}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos 120 n\pi t - \frac{100}{\pi} \sum_{1}^{\infty} \frac{1}{n} \sin 120 n\pi t$

Relative intensities as in problem 10.8

11.5 $\pi^2/8$ 11.6 $\pi^4/90$ 11.7 $\pi^2/6$

11.8 $\pi^4/96$ 11.9 $\dfrac{\pi^2}{16} - \dfrac{1}{2}$

12.2 $\quad f_s(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{1 - \cos\alpha}{\alpha} \sin\alpha x \, d\alpha$

12.3 $\quad f(x) = \displaystyle\int_{-\infty}^\infty \dfrac{1 - \cos\alpha\pi}{i\alpha\pi} e^{i\alpha x} \, d\alpha$

12.4 $\quad f(x) = \displaystyle\int_{-\infty}^\infty \dfrac{\sin\alpha\pi - \sin(\alpha\pi/2)}{\alpha\pi} e^{i\alpha x} \, d\alpha$

12.5 $\quad f(x) = \displaystyle\int_{-\infty}^\infty \dfrac{1 - e^{-i\alpha}}{2\pi i\alpha} e^{i\alpha x} \, d\alpha$

12.6 $\quad f(x) = \displaystyle\int_{-\infty}^\infty \dfrac{\sin\alpha - \alpha\cos\alpha}{i\pi\alpha^2} e^{i\alpha x} \, d\alpha$

12.7 $\quad f(x) = \displaystyle\int_{-\infty}^\infty \dfrac{\cos\alpha + \alpha\sin\alpha - 1}{\pi\alpha^2} e^{i\alpha x} \, d\alpha$

12.8 $\quad f(x) = \displaystyle\int_{-\infty}^\infty \dfrac{(i\alpha + 1)e^{-i\alpha} - 1}{2\pi\alpha^2} e^{i\alpha x} \, d\alpha$

12.9 $\quad f(x) = \dfrac{2}{\pi} \displaystyle\int_{-\infty}^\infty \dfrac{i - \cos\alpha a}{\alpha^2} e^{i\alpha x} \, d\alpha$

12.10 $f(x) = 2 \displaystyle\int_{-\infty}^\infty \dfrac{\alpha a - \sin\alpha a}{i\pi\alpha^2} e^{i\alpha x} \, d\alpha$

12.11 $f(x) = \dfrac{1}{\pi} \displaystyle\int_{-\infty}^\infty \dfrac{\cos(\alpha\pi/2)}{1 - \alpha^2} e^{i\alpha x} \, d\alpha$

12.12 $f(x) = \dfrac{1}{\pi i} \displaystyle\int_{-\infty}^\infty \dfrac{\alpha\cos(\alpha\pi/2)}{1 - \alpha^2} e^{i\alpha x} \, d\alpha$

12.13 $f_c(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{\sin\alpha\pi - \sin(\alpha\pi/2)}{\alpha} \cos\alpha x \, d\alpha$

12.14 $f_c(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{\cos\alpha + \alpha\sin\alpha - 1}{\alpha^2} \cos\alpha x \, d\alpha$

12.15 $f_c(x) = \dfrac{4}{\pi} \displaystyle\int_0^\infty \dfrac{1 - \cos\alpha a}{\alpha^2} \cos\alpha x \, d\alpha$

12.16 $f_c(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{\cos(\alpha\pi/2)}{1 - \alpha^2} \cos\alpha x \, d\alpha$

12.17 $f_s(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{1 - \cos\pi\alpha}{\alpha} \sin\alpha x \, d\alpha$

12.18 $f_s(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{\sin\alpha - \alpha\cos\alpha}{\alpha^2} \sin\alpha x \, d\alpha$

12.19 $f_s(x) = \dfrac{4}{\pi} \displaystyle\int_0^\infty \dfrac{\alpha a - \sin\alpha a}{\alpha^2} \sin\alpha x \, d\alpha$

12.20 $f_s(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{\alpha\cos(\alpha\pi/2)}{1 - \alpha^2} \sin\alpha x \, d\alpha$

12.21 $g(\alpha) = \dfrac{\sigma}{\sqrt{2\pi}} e^{-\alpha^2\sigma^2/2}$ $\qquad\qquad$ 12.24 (c) $g_c(\alpha) = \sqrt{\dfrac{\pi}{2}} e^{-|\alpha|}$

12.25 (a) $f(x) = \dfrac{1}{2\pi} \displaystyle\int_{-\infty}^\infty \dfrac{1 + e^{-i\alpha\pi}}{1 - \alpha^2} e^{i\alpha x} \, d\alpha$

12.27 (a) $f_c(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{\sin(\alpha\pi/2)}{\alpha} \cos\alpha x \, d\alpha$

$\quad$ (b) $f_s(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{1 - \cos(\alpha\pi/2)}{\alpha} \sin\alpha x \, d\alpha$

12.28 (a) $f_c(x) = \dfrac{4}{\pi} \displaystyle\int_0^\infty \dfrac{\cos 3\alpha\sin\alpha}{\alpha} \cos\alpha x \, d\alpha$

$\quad$ (b) $f_s(x) = \dfrac{4}{\pi} \displaystyle\int_0^\infty \dfrac{\sin 3\alpha\sin\alpha}{\alpha} \sin\alpha x \, d\alpha$

12.29 (a) $f_c(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{\sin 3\alpha - 2\sin 2\alpha}{\alpha} \cos \alpha x \, d\alpha$

(b) $f_s(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{2\cos 2\alpha - \cos 3\alpha - 1}{\alpha} \sin \alpha x \, d\alpha$

12.30 (a) $f_c(x) = \dfrac{1}{\pi} \displaystyle\int_0^\infty \dfrac{1 - \cos 2\alpha}{\alpha^2} \cos \alpha x \, d\alpha$

(b) $f_s(x) = \dfrac{1}{\pi} \displaystyle\int_0^\infty \dfrac{2\alpha - \sin 2\alpha}{\alpha^2} \sin \alpha x \, d\alpha$

13.2 $\quad f(x) = \dfrac{i}{2\pi} \displaystyle\sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \dfrac{1}{n} e^{2in\pi x}$

13.4 (c) $q(t) = CV\left[1 - 2(1 - e^{-1/2}) \displaystyle\sum_{-\infty}^{\infty} (1 + 4in\pi)^{-1} e^{4in\pi t/(RC)} \right]$

13.6 $\quad f(t) = \displaystyle\sum_{-\infty}^{\infty} \dfrac{(-1)^n \sin \omega\pi}{\pi(\omega - n)} e^{int}$

13.7 $\quad f(x) = \dfrac{\pi}{2} - \dfrac{4}{\pi} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n^2} \cos nx$

13.8 (a) $1/2$ (b) 1

13.9 (b) $-1/2, 0, 0, 1/2$ (c) $13/6$

13.10 (c) $0, -1/2, -2, -2$ (d) $-1, -1/2, -2, -1$

13.11 Cosine series: $a_0/2 = -3/4$,

$$a_n = \dfrac{4}{n^2\pi^2}\left(\cos\dfrac{n\pi}{2} - 1\right) + \dfrac{6}{n\pi}\sin\dfrac{n\pi}{2}$$

$$= \dfrac{4}{n^2\pi^2}\{-1, -2, -1, 0, \text{and repeat}\} + \dfrac{6}{n\pi}\{1, 0, -1, 0, \text{and repeat}\}$$

$$f_c(x) = -\dfrac{3}{4} + \left(-\dfrac{4}{\pi^2} + \dfrac{6}{\pi}\right)\cos\dfrac{\pi x}{2} - \dfrac{2}{\pi^2}\cos\pi x$$

$$- \left(\dfrac{4}{9\pi^2} + \dfrac{2}{\pi}\right)\cos\dfrac{3\pi x}{2} + \left(\dfrac{-4}{25\pi^2} + \dfrac{6}{5\pi}\right)\cos\dfrac{5\pi x}{2}\cdots$$

Sine series:

$$b_n = \dfrac{4}{n^2\pi^2}\sin\dfrac{n\pi}{2} + \dfrac{1}{n\pi}\left(4\cos n\pi - 6\cos\dfrac{n\pi}{2}\right)$$

$$= \dfrac{4}{n^2\pi^2}\{1, 0, -1, 0, \text{and repeat}\} + \dfrac{1}{n\pi}\{-4, 10, -4, -2, \text{and repeat}\}$$

$$f_s(x) = \left(\dfrac{4}{\pi^2} - \dfrac{4}{\pi}\right)\sin\dfrac{\pi x}{2} + \dfrac{5}{\pi}\sin\pi x - \left(\dfrac{4}{9\pi^2} + \dfrac{4}{3\pi}\right)\sin\dfrac{3\pi x}{2}$$

$$- \dfrac{1}{2\pi}\sin 2\pi x + \left(\dfrac{4}{25\pi^2} - \dfrac{4}{5\pi}\right)\sin\dfrac{5\pi x}{2} + \dfrac{5}{3\pi}\sin 3\pi x\cdots$$

Exponential series of period 2:

$$f_p(x) = -\dfrac{3}{4} - \displaystyle\sum_{\substack{-\infty \\ \text{odd } n}}^{\infty}\left(\dfrac{1}{n^2\pi^2} + \dfrac{5i}{2n\pi}\right)e^{in\pi x} + \dfrac{i}{2\pi}\displaystyle\sum_{\substack{-\infty \\ \text{even } n \neq 0}}^{\infty}\dfrac{1}{n}e^{in\pi x}$$

13.12 $f = 90$

13.13 (a) $f_s(x) = \displaystyle\sum_1^\infty \dfrac{\sin nx}{n}$ (b) $\pi^2/6$

13.14 (a) $f(x) = \dfrac{1}{3} + \dfrac{4}{\pi^2}\displaystyle\sum_1^\infty \dfrac{\cos n\pi x}{n^2}$ (b) $\pi^4/90$

13.15 $g(\alpha) = \dfrac{\cos 2\alpha - 1}{i\pi\alpha}, \quad f(x) = \dfrac{2}{\pi} \displaystyle\int_0^\infty \dfrac{\cos 2\alpha - 1}{\alpha} \sin \alpha x \, d\alpha, \quad -\pi/4$

13.16 $f(x) = \dfrac{8}{\pi} \displaystyle\int_0^\infty \dfrac{\cos \alpha \sin^2(\alpha/2)}{\alpha^2} \cos \alpha x \, d\alpha, \quad \pi/8$

13.19 $\displaystyle\int_0^\infty |f(x)|^2 \, dx = \int_0^\infty |g_c(\alpha)|^2 \, d\alpha = \int_0^\infty |g_s(\alpha)|^2 \, d\alpha$

13.20 $g(\alpha) = \dfrac{2}{\pi} \dfrac{\sin^2 \alpha a}{\alpha^2}, \quad \pi a^3/3 \qquad\qquad$ 13.23 $\pi^2/8$

Chapter 8

1.4 $x = k^{-1}gt + k^{-2}g(e^{-kt} - 1)$

1.5 $x = -A\omega^{-2}\sin\omega t + v_0 t + x_0$

1.6   (a) 15 months   (b) $t = 30(1 - 2^{-1/3}) = 6.19$ months

1.7 $x = (c/F)[(m^2 c^2 + F^2 t^2)^{1/2} - mc]$

2.1 $y = mx,\ m = 3/2$	2.2 $(1 - x^2)^{1/2} + (1 - y^2)^{1/2} = C,\ C = \sqrt{3}$
2.3 $\ln y = A(\csc x - \cot x),\ A = \sqrt{3}$	2.4 $x^2(1 + y^2) = K,\ K = 25$
2.5 $y = axe^x,\ a = 1/e$	2.6 $2y^2 + 1 = A(x^2 - 1)^2,\ A = 1$
2.7 $y^2 = 8 + e^{K - x^2},\ K = 1$	2.8 $y(x^2 + C) = 1,\ C = -3$
2.9 $ye^y = ae^x,\ a = 1$	2.10 $y + 1 = ke^{x^2/2},\ k = 2$
2.11 $(y - 2)^2 = (x + C)^3,\ C = 0$	2.12 $xye^y = K,\ K = e$
2.13 $y \equiv 1,\ y \equiv -1,\ x \equiv 1,\ x \equiv -1$	2.14 $y \equiv 0$
2.15 $y \equiv 2$	2.16 $4y = (x + C)^2,\ C = 0$

2.17 $x = (t - t_0)^2/4$

2.19   (a) $I/I_0 = e^{-0.5} = 0.6$ for $s = 50$ft

        Half value thickness $= (\ln 2)/\mu = 69.3$ft

     (b) Half life $T = (\ln 2)/\lambda$

2.20   (a) $q = q_0 e^{-t/(RC)}$ (b) $I = I_0 e^{-(R/L)t}$ (c) $\tau = RC,\ \tau = L/R$

     Corresponding quantities are $a,\ \lambda = (\ln 2)/T,\ \mu,\ 1/\tau$.

2.21 $N = N_0 e^{Kt}$

2.22 $N = N_0 e^{Kt} - (R/K)(e^{Kt} - 1)$ where $N_0 = $ number of bacteria at $t = 0$,

     $KN = $ rate of increase, $R = $ removal rate.

2.23 $T = 100[1 - (\ln r)/(\ln 2)]$

2.24 $T = 100(2r^{-1} - 1)$

2.26   (a) $k = $ weight divided by terminal speed.

     (b) $t = g^{-1} \cdot$ (terminal speed) $\cdot (\ln 100)$; typical terminal

     speeds are 0.02 to 0.1 cm/sec, so t is of the order of 10^{-4} sec.

2.27 $t = 10(\ln\frac{5}{13})/(\ln\frac{3}{13}) = 6.6$ min

2.28 $66°$	2.29 $t = 100\ \ln\frac{9}{4} = 81.1$min
2.30 $A = Pe^{It/100}$	2.31 $ay = bx$
2.32 $x^2 + 2y^2 = C$	2.33 $x^2 + ny^2 = C$
2.34 $x^2 - y^2 = C$	2.35 $x(y - 1) = C$

3.1 $y = \frac{1}{2}e^x + Ce^{-x}$	3.2 $y = 1/(2x) + C/x^3$
3.3 $y = (\frac{1}{2}x^2 + C)e^{-x^2}$	3.4 $y = \frac{1}{3}x^{5/2} + Cx^{-1/2}$
3.5 $y(\sec x + \tan x) = x - \cos x + C$	3.6 $y = (x + C)/(x + \sqrt{x^2 + 1})$
3.7 $y = \frac{1}{3}(1 + e^x) + C(1 + e^x)^{-2}$	3.8 $y = \frac{1}{2}\ln x + C/\ln x$
3.9 $y(1 - x^2)^{1/2} = x^2 + C$	3.10 $y\ \cosh x = \frac{1}{2}e^{2x} + x + C$
3.11 $y = 2(\sin x - 1) + Ce^{-\sin x}$	3.12 $x = (y + C)\cos y$

3.13 $\quad x = \frac{1}{2}e^y + Ce^{-y}$

3.14 $\quad x = y^{2/3} + Cy^{-1/3}$

3.15 $\quad S = \frac{1}{2} \times 10^7 \left[(1 + 3t/10^4) + (1 + 3t/10^4)^{-1/3}\right]$, where S = number of pounds of salt, and t is in hours.

3.16 $\quad I = Ae^{-Rt/L} + V_0(R^2 + \omega^2 L^2)^{-1}(R\cos\omega t + \omega L\sin\omega t)$

3.17 $\quad I = Ae^{-t/(RC)} - V_0\omega C(\sin\omega t - \omega RC\cos\omega t)/(1 + \omega^2 R^2 C^2)$

3.18 $\quad$ RL circuit: $I = Ae^{-Rt/L} + V_0(R + i\omega L)^{-1}e^{i\omega t}$
$\quad$ RC circuit: $I = Ae^{-t/RC} + i\omega V_0 C(1 + i\omega RC)^{-1}e^{i\omega t}$

3.19 $\quad N_2 = N_0\lambda t e^{-\lambda t}$

3.20 $\quad N_3 = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} + c_3 e^{-\lambda_3 t}$, where
$$c_1 = \frac{\lambda_1\lambda_2 N_0}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)}, \; c_2 = \frac{\lambda_1\lambda_2 N_0}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)}, \; c_3 = \frac{\lambda_1\lambda_2 N_0}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)}$$

3.21 $\quad N_n = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} + \cdots$, where
$$c_1 = \frac{\lambda_1\lambda_2...\lambda_{n-1}N_0}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)...(\lambda_n - \lambda_1)}, \; c_2 = \frac{\lambda_1\lambda_2...\lambda_{n-1}N_0}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)...(\lambda_n - \lambda_2)},$$
etc. (all $\lambda's$ different).

3.22 $\quad y = x + 1 + Ke^x$ $\qquad\qquad$ 3.23 $\quad x = 2\pi^{-1/2}e^{-y^2}\int_k^y e^{u^2}du$

4.1 $\quad y^{1/3} = x - 3 + Ce^{-x/3}$ $\qquad$ 4.2 $\quad y^{1/2} = \frac{1}{3}x^{5/2} + Cx^{-1/2}$

4.3 $\quad y^3 = 1/3 + Cx^{-3}$ $\qquad$ 4.4 $\quad x^2 e^{3y} + e^x - \frac{1}{3}y^3 = C$

4.5 $\quad x^2 - y^2 + 2x(y + 1) = C$

4.6 $\quad 4\sin x\cos y + 2x - \sin 2x - 2y - \sin 2y = C$

4.7 $\quad x = y(\ln x + C)$ $\qquad$ 4.8 $\quad y^2 = 2Cx + C^2$

4.9 $\quad y^2 = Ce^{-x^2/y^2}$ $\qquad$ 4.10 $\quad xy = Ce^{x/y}$

4.11 $\quad \tan\frac{1}{2}(x + y) = x + C$ $\qquad$ 4.12 $\quad x\sin(y/x) = C$

4.13 $\quad y^2 = -\sin^2 x + C\sin^4 x$ $\qquad$ 4.14 $\quad y = -x^{-2} + K(x - 1)^{-1}$

4.15 $\quad y = -x^{-1}\ln(C - x)$ $\qquad$ 4.16 $\quad y^2 = C(C \pm 2x)$

4.17 $\quad 3x^2 y - y^3 = C$ $\qquad$ 4.18 $\quad x^2 + (y - k)^2 = k^2$

4.19 $\quad r = Ae^{-\theta}, r = Be^{\theta}$

4.25 $\quad$ (a) $y = \dfrac{C + x^2}{x^2(C - x^2)}$ $\quad$ (b) $y = \dfrac{x(C + e^{4x})}{C - e^{4x}}$ $\quad$ (c) $y = \dfrac{e^x(C - e^{2x})}{C + e^{2x}}$

5.1 $\quad y = Ae^x + Be^{-2x}$

5.2 $\quad y = (Ax + B)e^{2x}$

5.3 $\quad y = Ae^{3ix} + Be^{-3ix}$ or other forms as in (5.24)

5.4 $\quad y = e^{-x}(Ae^{ix} + Be^{-ix})$ or equivalent forms (5.17), (5.18)

5.5 $\quad y = (Ax + B)e^x$

5.6 $\quad y = Ae^{4ix} + Be^{-4ix}$ or other forms as in (5.24)

5.7 $\quad y = Ae^{3x} + Be^{2x}$ $\qquad\qquad$ 5.8 $\quad y = A + Be^{-5x}$

5.9 $\quad y = Ae^{2x}\sin(3x + \gamma)$ $\qquad$ 5.10 $\quad y = A + Be^{2x}$

5.11 $\quad y = (A + Bx)e^{-3x/2}$ $\qquad$ 5.12 $\quad y = Ae^{-x} + Be^{x/2}$

5.19 $\quad y = Ae^{-ix} + Be^{-(1+i)x}$ $\qquad$ 5.20 $\quad y = Ae^{-x} + Be^{ix}$

5.22 $\quad y = Ae^x + Be^{-3x} + Ce^{-5x}$

5.23 $\quad y = Ae^{ix} + Be^{-ix} + Ce^x + De^{-x}$

5.24 $\quad y = Ae^{-x} + Be^{x/2}\sin\left(\frac{1}{2}x\sqrt{3} + \gamma\right)$

5.25 $\quad y = A + Be^{2x} + Ce^{-3x}$

5.26 $\quad y = Ae^{5x} + (Bx + C)e^{-x}$

5.27 $\quad y = Ax + B + (Cx + D)e^x + (Ex^2 + Fx + G)e^{-2x}$

5.28 $\quad y = e^x(A\sin x + B\cos x) + e^{-x}(C\sin x + D\cos x)$

5.29 $\quad y = (A + Bx)e^{-x} + Ce^{2x} + De^{-2x} + E\sin(2x + \gamma)$

5.30 $\quad y = (Ax + B)\sin x + (Cx + D)\cos x + (Ex + F)e^x + (Gx + H)e^{-x}$

5.34 $\quad \theta = \theta_0 \cos \omega t, \ \omega = \sqrt{g/l}$

5.35 $\quad T = 2\pi\sqrt{R/g} \cong 85$ min.

5.36 $\quad \omega = 1/\sqrt{LC}$

5.38 $\quad$ overdamped: $R^2 C > 4L$; critically damped: $R^2 C = 4L$;
underdamped: $R^2 C < 4L$.

5.40 $\quad \ddot{y} + \dfrac{16\pi}{15}\dot{y} + \dfrac{4\pi^2}{9}y = 0, \ y = e^{-8\pi t/15}\left(A\sin\dfrac{2\pi t}{5} + B\cos\dfrac{2\pi t}{5}\right)$

6.1 $\quad y = Ae^{2x} + Be^{-2x} - \frac{5}{2}$ $\qquad$ 6.2 $\quad y = (A + Bx)e^{2x} + 4$

6.3 $\quad y = Ae^{x} + Be^{-2x} + \frac{1}{4}e^{2x}$ $\qquad$ 6.4 $\quad y = Ae^{-x} + Be^{3x} + 2e^{-3x}$

6.5 $\quad y = Ae^{ix} + Be^{-ix} + e^{x}$ $\qquad$ 6.6 $\quad y = (A + Bx)e^{-3x} + 3e^{-x}$

6.7 $\quad y = Ae^{-x} + Be^{2x} + xe^{2x}$ $\qquad$ 6.8 $\quad y = Ae^{4x} + Be^{-4x} + 5xe^{4x}$

6.9 $\quad y = (Ax + B + x^2)e^{-x}$ $\qquad$ 6.10 $\quad y = (A + Bx)e^{3x} + 3x^2e^{3x}$

6.11 $\quad y = e^{-x}(A\sin 3x + B\cos 3x) + 8\sin 4x - 6\cos 4x$

6.12 $\quad y = e^{-2x}[A\sin(2\sqrt{2}\,x) + B\cos(2\sqrt{2}\,x)] + 5(\sin 2x - \cos 2x)$

6.13 $\quad y = (Ax + B)e^{x} - \sin x$

6.14 $\quad y = e^{-2x}(A\sin 3x + B\cos 3x) - 3\cos 5x$

6.15 $\quad y = e^{-6x/5}[A\sin(8x/5) + B\cos(8x/5)] - 5\cos 2x$

6.16 $\quad y = A\sin 3x + B\cos 3x - 5x\cos 3x$

6.17 $\quad y = A\sin 4x + B\cos 4x + 2x\sin 4x$

6.18 $\quad y = e^{-x}(A\sin 4x + B\cos 4x) + 2e^{-4x}\cos 5x$

6.19 $\quad y = e^{-x/2}(A\sin x + B\cos x) + e^{-3x/2}(2\cos 2x - \sin 2x)$

6.20 $\quad y = Ae^{-2x}\sin(2x + \gamma) + 4e^{-x/2}\sin(5x/2)$

6.21 $\quad y = e^{-3x/5}[A\sin(x/5) + B\cos(x/5)] + (x^2 - 5)/2$

6.22 $\quad y = A + Be^{-x/2} + x^2 - 4x$

6.23 $\quad y = A\sin x + B\cos x + (x - 1)e^{x}$

6.24 $\quad y = (A + Bx + 2x^3)e^{3x}$

6.25 $\quad y = Ae^{3x} + Be^{-x} - (\frac{4}{3}x^3 + x^2 + \frac{1}{2}x)e^{-x}$

6.26 $\quad y = A\sin x + B\cos x - 2x^2\cos x + 2x\sin x$

6.33 $\quad y = A\sin(x + \gamma) + x^3 - 6x - 1 + x\sin x + (3 - 2x)e^{x}$

6.34 $\quad y = Ae^{3x} + Be^{2x} + e^{x} + x$

6.35 $\quad y = A\sinh x + B\cosh x + \frac{1}{2}x\cosh x$

6.36 $\quad y = A\sin x + B\cos x + x^2\sin x$

6.37 $\quad y = (A + Bx)e^{x} + 2x^2e^{x} + (3 - x)e^{2x} + x + 1$

6.38 $\quad y = A + Be^{2x} + (3x + 4)e^{-x} + x^3 + 3(x^2 + x)/2 + 2xe^{2x}$

6.41 $\quad y = e^{-x}(A\cos x + B\sin x) + \dfrac{1}{4}\pi + \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{4(n^2 - 2)\cos nx - 8n\sin nx}{\pi n^2(n^4 + 4)}$

6.42 $\quad y = A\cos 3x + B\sin 3x + \dfrac{1}{36} + \dfrac{2}{\pi^2}\displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{\cos n\pi x}{n^2(n^2\pi^2 - 9)} + \dfrac{1}{\pi}\displaystyle\sum_{\substack{1 \\ \text{all } n}}^{\infty} \dfrac{(-1)^n\sin n\pi x}{n(n^2\pi^2 - 9)}$

7.1 $\quad y = 2A\tanh(Ax + B)$, or $y = 2A\tan(B - Ax)$,
or $y(x + a) = 2$, or $y = C$.
(a) $y \equiv 5$ $\qquad\qquad\qquad$ (b) $y(x + 1) = 2$
(c) $y = \tan(\frac{\pi}{4} - \frac{x}{2}) = \sec x - \tan x$ $\quad$ (d) $y = 2\tanh x$

7.3 $\quad y = a(x + b)^2$, or $y = C$

7.4 $\quad x^2 + (y - b)^2 = a^2$, or $y = C$

7.5 $\quad y = b + k^{-1}\cosh k(x - a)$

7.8 $\quad v = (v_0^2 - 2gR + 2gR^2/r)^{1/2}$, $r_{\max} = 2gR^2/(2gR - v_0^2)$,
escape velocity $= \sqrt{2gR}$

7.10 $x = \sqrt{1+t^2}$

7.11 $x = (1-3t)^{1/3}$

7.12 $t = \int_1^x u^2 (1-u^4)^{-1/2} du$

7.13 $t = (\omega\sqrt{2})^{-1} \int (\cos\theta)^{-1/2} d\theta$

7.16 (a) $y = Ax + Bx^{-3}$ (b) $y = Ax^2 + Bx^{-2}$

     (c) $y = (A + B\ln x)/x^3$ (d) $y = Ax\cos(\sqrt{5}\ln x) + Bx\sin(\sqrt{5}\ln x)$

7.17 $y = Ax^4 + Bx^{-4} + x^4 \ln x$

7.18 $y = Ax + Bx^{-1} + \frac{1}{2}\left(x + x^{-1}\right)\ln x$

7.19 $y = x^3(A + B\ln x) + x^3(\ln x)^2$

7.20 $y = x^2(A + B\ln x) + x^2(\ln x)^3$

7.21 $y = A\sqrt{x}\sin\left(\frac{\sqrt{3}}{2}\ln x + \gamma\right) + x^2$

7.22 $y = A\cos\ln x + B\sin\ln x + x$

7.23 $R = Ar^n + Br^{-n}$, $n \neq 0$; $R = A\ln r + B$, $n = 0$

     $R = Ar^l + Br^{-l-1}$

7.25 $x^{-1} - 1$ 7.26 $x^2 - 1$ 7.27 $x^3 e^x$

7.28 $x^{1/3} e^x$ 7.29 $xe^{1/x}$ 7.30 $(x-1)\ln x - 4x$

8.8 $e^{-2t} - te^{-2t}$ 8.9 $e^t - 3e^{-2t}$

8.10 $\frac{1}{3}e^t \sin 3t + 2e^t \cos 3t$ 8.11 $\frac{4}{7}e^{-2t} + \frac{3}{7}e^{t/3}$

8.12 $3\cosh 5t + 2\sinh 5t$ 8.13 $e^{-2t}(2\sin 4t - \cos 4t)$

8.17 $2a(3p^2 - a^2)/(p^2 + a^2)^3$ 8.21 $2b(p + a)/[(p + a)^2 + b^2]^2$

8.22 $[(p + a)^2 - b^2]/[(p + a)^2 + b^2]^2$ 8.23 $y = te^{-2t}(\cos t - \sin t)$

8.25 $e^{-p\pi/2}/(p^2 + 1)$ 8.26 $\cos(t - \pi)$, $t > \pi$; 0, $t < \pi$

8.27 $-v(p^2 + v^2)^{-1}e^{-px/v}$

9.2 $y = e^t(3 + 2t)$ 9.3 $y = e^{-2t}(4t + \frac{1}{2}t^2)$

9.4 $y = \cos t + \frac{1}{2}(\sin t - t\cos t)$ 9.5 $y = -\frac{1}{2}t\cos t$

9.6 $y = \frac{1}{6}t^3 e^{3t} + 5te^{3t}$ 9.7 $y = 1 - e^{2t}$

9.8 $y = t\sin 4t$ 9.9 $y = (t + 2)\sin 4t$

9.10 $y = 3t^2 e^{2t}$ 9.11 $y = te^{2t}$

9.12 $y = \frac{1}{2}(t^2 e^{-t} + 3e^t - e^{-t})$ 9.13 $y = \sinh 2t$

9.14 $y = te^{2t}$ 9.15 $y = 2\sin 3t + \frac{1}{6}t\sin 3t$

9.16 $y = \frac{1}{6}t\sin 3t + 2\cos 3t$ 9.17 $y = 2$

9.18 $y = 2e^{-2t} - e^{-t}$ 9.19 $y = e^{2t}$

9.20 $y = 2t + 1$ 9.21 $y = e^{3t} + 2e^{-2t}\sin t$

9.22 $y = 2\cos t + \sin t$ 9.23 $y = \sin t + 2\cos t - 2e^{-t}\cos 2t$

9.24 $y = (5 - 6t)e^t - \sin t$ 9.25 $y = (3 + t)e^{-2t}\sin t$

9.26 $y = te^{-t}\cos 3t$ 9.27 $y = t + (1 - e^{4t})/4$, $z = \frac{1}{3} + e^{4t}$

9.28 $\begin{cases} y = t\cos t - 1 \\ z = \cos t + t\sin t \end{cases}$ 9.29 $\begin{cases} y = e^t \\ z = t + e^t \end{cases}$

9.30 $y = t - \sin 2t$ 9.31 $y = t$

     $z = \cos 2t$ $z = e^t$

9.32 $\begin{cases} y = \sin 2t \\ z = \cos 2t - 1 \end{cases}$ 9.33 $\begin{cases} y = \sin t - \cos t \\ z = \sin t \end{cases}$

9.34 $3/13$ 9.35 $10/26^2$

9.36 $\arctan(2/3)$ 9.37 $15/8$

9.38 $4/5$ 9.39 $\ln 2$

9.40 1 9.41 $\arctan(1/\sqrt{2})$

9.42 $\pi/4$

10.3 $\frac{1}{2}t \sinh t$

10.4 $\dfrac{e^{-at} + e^{-bt}[(a-b)t - 1]}{(b-a)^2}$

10.5 $\dfrac{b(b-a)te^{-bt} + a[e^{-bt} - e^{-at}]}{(b-a)^2}$

10.6 $\dfrac{e^{-at} - \cosh bt + (a/b)\sinh bt}{a^2 - b^2}$

10.7 $\dfrac{a\cosh bt - b\sinh bt - ae^{-at}}{a^2 - b^2}$

10.8 $\dfrac{e^{-at}}{(b-a)(c-a)} + \dfrac{e^{-bt}}{(a-b)(c-b)} + \dfrac{e^{-ct}}{(a-c)(b-c)}$

10.9 $(2t^2 - 2t + 1 - e^{-2t})/4$

10.10 $(1 - \cos at - \frac{1}{2}at\sin at)/a^4$

10.11 $\dfrac{\cos at - \cos bt}{b^2 - a^2}$

10.12 $\dfrac{1}{b^2 - a^2}\left(\dfrac{\cos bt}{b^2} - \dfrac{\cos at}{a^2}\right) + \dfrac{1}{a^2 b^2}$

10.13 $(e^{-t} + \sin t - \cos t)/2$

10.14 $e^{-3t} + (t-1)e^{-2t}$

10.15 $\dfrac{1}{14}e^{3t} + \dfrac{1}{35}e^{-4t} - \dfrac{1}{10}e^{t}$

10.17 $y = \begin{cases} (\cosh at - 1)/a^2, & t > 0 \\ 0, & t < 0 \end{cases}$

11.1 $y = \begin{cases} t - 2, & t > 2 \\ 0, & t < 2 \end{cases}$

11.7 $y = \begin{cases} (t - t_0)e^{-(t-t_0)}, & t > t_0 \\ 0, & t < t_0 \end{cases}$

11.8 $y = \begin{cases} e^{-2(t-t_0)}\sin(t - t_0), & t > t_0 \\ 0, & t < t_0 \end{cases}$

11.9 $y = \begin{cases} \frac{1}{3}e^{-(t-t_0)}\sin 3(t - t_0), & t > t_0 \\ 0, & t < t_0 \end{cases}$

11.10 $y = \begin{cases} \frac{1}{3}\sinh 3(t - t_0), & t > t_0 \\ 0, & t < t_0 \end{cases}$

11.11 $y = \begin{cases} \frac{1}{2}[\sinh(t - t_0) - \sin(t - t_0)], & t > t_0 \\ 0, & t < t_0 \end{cases}$

11.13 (a) $5\delta(x - 2) + 3\delta(x + 7)$ (b) $3\delta(x + 5) - 4\delta(x - 10)$

11.15 (a) 1   (b) 0   (c) -3 (d) $\cosh 1$

11.21 (a) 8   (b) $\phi(|a|)/(2|a|)$ (c) $1/2$ (d) 1

11.23 (a) $\delta(x + 5)\delta(y - 5)\delta(z)$, $\delta(r - 5\sqrt{2})\delta(\theta - \frac{3\pi}{4})\delta(z)/r$,
            $\delta(r - 5\sqrt{2})\delta(\theta - \frac{\pi}{2})\delta(\phi - \frac{3\pi}{4})/(r\sin\theta)$

        (b) $\delta(x)\delta(y + 1)\delta(z + 1)$, $\delta(r - 1)\delta(\theta - \frac{3\pi}{2})\delta(z + 1)/r$,
            $\delta(r - \sqrt{2})\delta(\theta - \frac{3\pi}{4})\delta(\phi - \frac{3\pi}{2})/(r\sin\theta)$

        (c) $\delta(x + 2)\delta(y)\delta(z - 2\sqrt{3})$, $\delta(r - 2)\delta(\theta - \pi)\delta(z - 2\sqrt{3})/r$,
            $\delta(r - 4)\delta(\theta - \frac{\pi}{6})\delta(\phi - \pi)/(r\sin\theta)$

        (d) $\delta(x - 3)\delta(y + 3)\delta(z + \sqrt{6})$, $\delta(r - 3\sqrt{2})\delta(\theta - \frac{7\pi}{4})\delta(z + \sqrt{6})/r$,
            $\delta(r - 2\sqrt{6})\delta(\theta - \frac{2\pi}{3})\delta(\phi - \frac{7\pi}{4})/(r\sin\theta)$

11.25 (a) and (b) $F''(x) = \delta(x) - 2\delta'(x)$ (c) $G''(x) = \delta(x) + 5\delta'(x)$

12.2 $y = \dfrac{\sin\omega t - \omega t\cos\omega t}{2\omega^2}$

12.3 $y = \dfrac{\sin\omega t - \omega\cos\omega t + \omega e^{-t}}{\omega(1 + \omega^2)}$

12.6 $y = (\cosh at - 1)/a^2, \ t > 0$

12.7 $y = \dfrac{a(\cosh at - e^{-t}) - \sinh at}{a(a^2 - 1)}$

12.8 $y = \begin{cases} 1 - e^{-t} - te^{-t}, & 0 < t < a \\ (t + 1 - a)e^{a-t} - (t + 1)e^{-t}, & t > a \end{cases}$

12.11 $y = -\frac{1}{3}\sin 2x$

12.12 $y = \cos x\ln\cos x + (x - \frac{\pi}{2})\sin x$

12.13 $y = \begin{cases} x - \sqrt{2}\,\sin x, & x < \pi/4 \\ \frac{\pi}{2} - x - \sqrt{2}\,\cos x, & x > \pi/4 \end{cases}$

12.15 $y = x\sinh x - \cosh x \ln\cosh x$ 12.16 $y = -x\ln x - x - x(\ln x)^2/2$

12.17 $y = -\frac{1}{4}\sin^2 x$ 12.18 $y = x^2/2 + x^4/6$

13.1 $y = -\frac{1}{3}x^{-2} + Cx$	linear 1st order
13.2 $(\ln y)^2 - (\ln x)^2 = C$	separable
13.3 $y = A + Be^{-x}\sin(x + \gamma)$	3rd order linear
13.4 $r = (A + Bt)e^{3t}$	2nd order linear, $a = b$
13.5 $x^2 + y^2 - y\sin^2 x = C$	exact
13.6 $y = Ae^{-x}\sin(x + \gamma)$	2nd order linear, complex a, b, c
$+2e^x + 3xe^{-x}\sin x$	
13.7 $3x^2y^3 + 1 = Ax^3$	Bernoulli, or integrating factor $1/x^4$
13.8 $y = x(A + B\ln x) + \frac{1}{2}x(\ln x)^2$	Cauchy
13.9 $y(e^{3x} + Ce^{-2x}) + 5 = 0$	Bernoulli
13.10 $u - \ln u + \ln v + v^{-1} = C$	separable
13.11 $y = 2x\ln x + Cx$	linear 1st order, or homogeneous
13.12 $y = A\ln x + B + x^2$	y missing, or Cauchy
13.13 $y = Ae^{-2x}\sin(x + \gamma) + e^{3x}$	2nd order linear, complex a, b
13.14 $y = Ae^{-2x}\sin(x + \gamma)$	2nd order linear, complex a, b, c
$+xe^{-2x}\sin x$	
13.15 $y = (A + Bx)e^{2x} + 3x^2e^{2x}$	2nd order linear, $c = a = b$
13.16 $y = Ae^{2x} + Be^{3x} - xe^{2x}$	2nd order linear, $c = a \neq b$
13.17 $y^2 + 4xy - x^2 = C$	exact, or homogeneous
13.18 $x = (y + C)e^{-\sin y}$	linear 1st order for $x(y)$
13.19 $(x + y)\sin^2 x = K$	separable with $u = x + y$
13.20 $y = Ae^x\sin(2x + \gamma)$	2nd order linear, complex a, b, c
$+x+\frac{2}{5} + e^x(1 - x\cos 2x)$	
13.21 $x^2 + \ln(1 - y^2) = C$	separable, or Bernoulli
13.22 $y = (A + Bx)e^{2x} + C\sin(3x + \gamma)$	4th order linear
13.23 $r = \sin\theta[C + \ln(\sec\theta + \tan\theta)]$	1st order linear
13.24 $y^2 = ax^2 + b$	separable after substitution

13.25 $x^3y = 2$ 13.26 $y = x^2 + x$ 13.27 $y = 2e^{2x} - 1$

13.28 $y^2 + 4(x - 1)^2 = 9$ 13.29 62 min more

13.30 $y = \frac{1}{6}g[(1 + t)^2 + 2(1 + t)^{-1} - 3]$; at $t = 1$, $y = g/3$, $v = 7g/12$, $a = 5g/12$

13.31 $v = \sqrt{2k/(ma)}$ 13.32 1:23 p.m.

13.33 In both (a) and (b), the temperature of the mixture at time t
     is $T_a(1 - e^{-kt}) + (n + n')^{-1}(nT_0 + n'T_0')e^{-kt}$.

13.36 (a) $v = u\ln(m_0/m)$ 13.38 $\ln\sqrt{a^2 + p^2} - \ln p$

13.39 $2pa/(p^2 - a^2)^2$ 13.40 5/27

13.41 $(\tanh 1 - \operatorname{sech}^2 1)/4 = 0.0854$ 13.42 $te^{-at}(1 - \frac{1}{2}at)$

13.43 $(\sin at + at\cos at)/(2a)$

13.44 $(3\sin at - 3at\cos at - a^2t^2\sin at)/(8a^5)$

13.46 e^{-x}: $g_s(\alpha) = \sqrt{\dfrac{2}{\pi}}\dfrac{\alpha}{1 + \alpha^2}$, $g_c(\alpha) = \sqrt{\dfrac{2}{\pi}}\dfrac{1}{1 + \alpha^2}$

     xe^{-x} : $g_s(\alpha) = \sqrt{\dfrac{2}{\pi}}\dfrac{2\alpha}{(1 + \alpha^2)^2}$, $g_c(\alpha) = \sqrt{\dfrac{2}{\pi}}\dfrac{1 - \alpha^2}{(1 + \alpha^2)^2}$

13.47 $y = A\sin t + B\cos t + \sin t\ \ln(\sec t + \tan t) - 1$

13.48 $y = A\sin t + B\cos t + (t\sin t - t^2\cos t)/4$

Chapter 9

2.1 $(y - b)^2 = 4a^2(x - a^2)$ 2.2 $x^2 + (y - b)^2 = a^2$

2.3 $ax = \sinh(ay + b)$ 2.4 $ax = \cosh(ay + b)$

2.5 $y = ae^x + be^{-x}$ or $y = A\cosh(x + B)$, etc.

2.6 $x + a = \frac{4}{3}(y^{1/2} - 2b)(b + y^{1/2})^{1/2}$

2.7 $e^x \cos(y + b) = C$ 2.8 $K^2 x^2 - (y - b)^2 = K^4$

2.9 $x = ay^2 + b$ 2.10 $y = Ax^{3/2} - \ln x + B$

3.1 $dx/dy = C(y^3 - C^2)^{-1/2}$ 3.2 $dx/dy = Cy^2(1 - C^2 y^4)^{-1/2}$

3.3 $x^4 y'^2 = C^2(1 + x^2 y'^2)^3$ 3.4 $\dfrac{dx}{dy} = \dfrac{C}{y(y^4 - C^2)^{1/2}}$

3.5 $y^2 = ax + b$ 3.6 $x = ay^{3/2} - \frac{1}{2}y^2 + b$

3.7 $y = K\sinh(x + C) = ae^x + be^{-x}$, etc., as in Problem 2.5

3.8 $r\cos(\theta + \alpha) = C$ 3.9 $\cot\theta = A\cos(\phi - \alpha)$

3.10 $s = be^{at}$ 3.11 $a(x + 1) = \cosh(ay + b)$

3.12 $(x - a)^2 + y^2 = C^2$ 3.13 $(x - a)^2 = 4K^2(y - K^2)$

3.14 $r = be^{c\theta}$

3.15 $r\cos(\theta + \alpha) = C$, in polar coordinates; or, in rectangular coordinates, the straight line $x\cos\alpha - y\sin\alpha = C$.

3.17 Intersection of the cone with $r\cos\left(\dfrac{\theta + C}{\sqrt{2}}\right) = K$

3.18 Geodesics on the sphere: $\cot\theta = A\cos(\phi - \alpha)$. (See Problem 3.9)
Intersection of $z = ax + by$ with the sphere: $\cot\theta = a\cos\phi + b\sin\phi$.

4.4 42.2 min; 5.96 min

4.5 $x = a(1 - \cos\theta)$, $y = a(\theta - \sin\theta) + C$

4.6 $x = a(\theta - \sin\theta) + C$, $y = 1 + a(1 - \cos\theta)$

4.7 $x = a(1 - \cos\theta) - \frac{5}{2}$, $y = a(\theta - \sin\theta) + C$

5.2 $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - V(r, \theta, z)$
$m(\ddot{r} - r\dot{\theta}^2) = -\partial V/\partial r$
$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = -(1/r)(\partial V/\partial\theta)$
$m\ddot{z} = -\partial V/\partial z$
Note: The equations in 5.2 and 5.3 are in the form
$m\mathbf{a} = \mathbf{F} = -\boldsymbol{\nabla}V$.

5.3 $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2) - V(r, \theta, \phi)$
$m(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\,\dot{\phi}^2) = -\partial V/\partial r$
$m(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\,\dot{\phi}^2) = -(1/r)(\partial V/\partial\theta)$
$m(r\sin\theta\,\ddot{\phi} + 2r\cos\theta\,\dot{\theta}\dot{\phi} + 2\sin\theta\,\dot{r}\dot{\phi}) = -(1/r\sin\theta)(\partial V/\partial\phi)$

5.4 $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - \cos\theta)$
$l\ddot{\theta} + g\sin\theta = 0$

5.5 $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$
$m\ddot{x} + kx = 0$

5.6 $L = \frac{1}{2}m(r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2) - mgr\cos\theta$
$\begin{cases} a\ddot{\theta} - a\sin\theta\cos\theta\,\dot{\phi}^2 - g\sin\theta = 0 \\ (d/dt)(\sin^2\theta\,\dot{\phi}) = 0 \end{cases}$

5.8 $L = \frac{1}{2}m(2\dot{r}^2 + r^2\dot{\theta}^2) - mgr$
$2\ddot{r} - r\dot{\theta}^2 + g = 0$
$(d/dt)(r^2\dot{\theta}) = 0$

5.9 $L = \frac{1}{2}m(2\dot{r}^2 + r^2\dot{\theta}^2) - mgr$
$2\ddot{r} - r\dot{\theta}^2 + g = 0$
$r^2\dot{\theta} = \text{const.}$

5.10 $L = \frac{1}{2}m_1(4\dot{r}^2 + r^2\dot{\theta}^2) + 2m_2\dot{r}^2 - m_1gr\sqrt{3} + m_2g(l - 2r)$
$4(m_1 + m_2)\ddot{r} - m_1r\dot{\theta}^2 + m_1g\sqrt{3} + 2m_2g = 0$
$r^2\dot{\theta} = \text{const.}$

5.11 $L = \frac{1}{2}(m + Ia^{-2})\dot{z}^2 - mgz$ (If z is taken as positive down,
$(ma^2 + I)\ddot{z} + mga^2 = 0$ change the signs of z and $\ddot{z}$.)

5.12 $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \left[\frac{1}{2}k(r - r_0)^2 - mgr\cos\theta\right]$
$\ddot{r} - r\dot{\theta}^2 + \frac{k}{m}(r - r_0) - g\cos\theta = 0, \; \frac{d}{dt}(r^2\dot{\theta}) + gr\sin\theta = 0$

5.13 $L = \frac{1}{2}m[\dot{r}^2(1 + 4r^2) + r^2\dot{\theta}^2] - mgr^2$
$\ddot{r}(1 + 4r^2) + 4r\dot{r}^2 - r\dot{\theta}^2 + 2gr = 0, \; r^2\dot{\theta} = \text{const.}$
If $z = \text{const.}$, then $r = \text{const.}$, so $\dot{\theta} = \sqrt{2g}$

5.14 $L = M\dot{x}^2 + Mgx\sin\alpha, \quad 2M\ddot{x} - Mg\sin\alpha = 0$

5.15 $L = \frac{1}{2}(M + Ia^{-2})\dot{x}^2 + Mgx\sin\alpha, \; (M + Ia^{-2})\ddot{x} - Mg\sin\alpha = 0$
Since smaller I means greater acceleration, objects reach the
bottom in order of increasing I.

5.16 $L = \frac{1}{2}m(l + a\theta)^2\dot{\theta}^2 - mg[a\sin\theta - (l + a\theta)\cos\theta]$
$(l + a\theta)\ddot{\theta} + a\dot{\theta}^2 + g\sin\theta = 0$

5.17 $L = \frac{1}{2}(M + m)\dot{X}^2 + \frac{1}{2}m(l^2\dot{\theta}^2 + 2l\cos\theta\,\dot{X}\dot{\theta}) + mgl\cos\theta$
$(M + m)\dot{X} + ml\cos\theta\,\dot{\theta} = \text{const.}$
$\frac{d}{dt}(l\dot{\theta} + \cos\theta\,\dot{X}) + g\sin\theta = 0$

5.18 $x + y = x_0 + y_0 + a\theta, \; L = m(\dot{x}^2 + \dot{y}^2 + \dot{x}\dot{y}) + mgy$
$\dot{x} = -\frac{1}{3}gt, \; \dot{y} = \frac{2}{3}gt, \; a\dot{\theta} = \frac{1}{3}gt$

5.19 $x = y$ with $\omega = \sqrt{g/l}; \quad x = -y$ with $\omega = \sqrt{3g/l}$

5.20 $x = y$ with $\omega = \sqrt{g/l}; \quad x = -y$ with $\omega = \sqrt{7g/l}$

5.21 $L = ml^2[\dot{\theta}^2 + \frac{1}{2}\dot{\phi}^2 + \dot{\theta}\dot{\phi}\cos(\theta - \phi)] + mgl(2\cos\theta + \cos\phi)$
$2\ddot{\theta} + \ddot{\phi}\cos(\theta - \phi) + \dot{\phi}^2\sin(\theta - \phi) + \frac{2g}{l}\sin\theta = 0$
$\ddot{\phi} + \ddot{\theta}\cos(\theta - \phi) - \dot{\theta}^2\sin(\theta - \phi) + \frac{g}{l}\sin\phi = 0$

5.22 $L = l^2[\frac{1}{2}M\dot{\theta}^2 + \frac{1}{2}m\dot{\phi}^2 + m\dot{\theta}\dot{\phi}\cos(\theta - \phi)] + gl(M\cos\theta + m\cos\phi)$
$M\ddot{\theta} + m\ddot{\phi}\cos(\theta - \phi) + m\dot{\phi}^2\sin(\theta - \phi) + \frac{Mg}{l}\sin\theta = 0$
$\ddot{\phi} + \ddot{\theta}\cos(\theta - \phi) - \dot{\theta}^2\sin(\theta - \phi) + \frac{g}{l}\sin\phi = 0$

5.23 $\phi = 2\theta$ with $\omega = \sqrt{2g/(3l)}; \quad \phi = -2\theta$ with $\omega = \sqrt{2g/l}$

5.24 $\phi = \frac{3}{2}\theta$ with $\omega = \sqrt{3g/(5l)}; \quad \phi = -\frac{3}{2}\theta$ with $\omega = \sqrt{3g/l}$

5.25 $\phi = \sqrt{M/m}\,\theta$ with $\omega^2 = \frac{g}{l}\dfrac{1}{1 + \sqrt{m/M}}$
$\phi = -\sqrt{M/m}\,\theta$ with $\omega^2 = \frac{g}{l}\dfrac{1}{1 - \sqrt{m/M}}$

6.1	catenary	6.2	circle	6.3	circular cylinder
6.4	catenary	6.5	circle	6.6	circle

8.2 $\quad I = \displaystyle\int \frac{x^2 y'^2}{\sqrt{1+y'^2}}\, dx, \quad x^2(2y' + y'^3) = K(1+y'^2)^{3/2}$

8.3 $\quad I = \displaystyle\int \frac{y\,dy}{\sqrt{x'^2 + 1}}, \quad x'^2 y^2 = C^2(1+x'^2)^3$

8.4 $\quad I = \displaystyle\int \sqrt{r^2 + r^4 \theta'^2}\, dr, \quad \frac{dr}{d\theta} = Kr\sqrt{r^4 - K^2}$

8.5 $\quad y = ae^{bx}$

8.6 $\quad (x-a)^2 + (y+1)^2 = C^2$

8.7 $\quad (y-b)^2 = 4a^2(x + 1 - a^2)$

8.8 $\quad$ Intersection of $r = 1 + \cos\theta$ with $z = a + b\sin(\theta/2)$

8.9 $\quad$ Intersection of the cone with $r\cos(\theta\sin\alpha + C) = K$

8.10 $\quad$ Intersection of $y = x^2$ with $az = b[2x\sqrt{4x^2 + 1} + \sinh^{-1} 2x] + c$

8.11 $\quad r = K\sec^2 \frac{\theta + c}{2}$ $\qquad\qquad$ 8.12 $\quad e^y \cos(x - a) = K$

8.13 $\quad (x + \frac{3}{2})^2 + (y - b)^2 = c^2$ $\qquad$ 8.14 $\quad (x - a)^2 = 4K^2(y + 2 - K^2)$

8.15 $\quad y + c = \frac{3}{2}K\left[x^{1/3}\sqrt{x^{2/3} - K^2} + K^2 \cosh^{-1}(x^{1/3}/K)\right]$

8.16 $\quad$ Hyperbola: $r^2 \cos(2\theta + \alpha) = K$ or $(x^2 - y^2)\cos\alpha - 2xy\sin\alpha = K$

8.17 $\quad K \ln r = \cosh(K\theta + C)$

8.18 $\quad$ Parabola: $(x - y - C)^2 = 4K^2(x + y - K^2)$

8.19 $\quad m(\ddot{r} - r\dot{\theta}^2) + kr = 0,\ r^2\dot{\theta} = $ const.

8.20 $\quad m(\ddot{r} - r\dot{\theta}^2) + K/r^2 = 0,\ r^2\dot{\theta} = $ const.

8.21 $\quad \ddot{r} - r\dot{\theta}^2 = 0,\ r^2\dot{\theta} = $ const., $\ddot{z} + g = 0$

8.22 $\quad \frac{1}{r} \cdot m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - r^2\sin\theta\cos\theta\ \dot{\phi}^2) = -\frac{1}{r}\frac{\partial V}{\partial \theta} = F_\theta = ma_\theta$
$\qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\ \dot{\phi}^2$

8.23 $\quad L = \frac{1}{2}ma^2\dot{\theta}^2 - mga(1 - \cos\theta),\ a\ddot{\theta} + g\sin\theta = 0,$
$\qquad \theta$ measured from the downward direction.

8.25 $\quad l = 2\sqrt{\pi A}$

8.26 $\quad r = Ae^{b\theta}$

8.27 $\quad \dfrac{dr}{d\theta} = r\sqrt{K^2(1 + \lambda r)^2 - 1}$

8.28 $\quad r^2\dot{\theta} = $ const., $|\mathbf{r} \times m\mathbf{v}| = mr^2\dot{\theta} = $ const., $\dfrac{dA}{dt} = \dfrac{1}{2}r^2\dot{\theta} = $ const.

Chapter 10

4.4 $I = \dfrac{2}{15}\begin{pmatrix} \pi & -1 & 0 \\ -1 & \pi & 0 \\ 0 & 0 & \pi \end{pmatrix}$ Principal moments: $\dfrac{2}{15}(\pi - 1, \pi, \pi + 1)$; principal

axes along the vectors: $(1, 1, 0), (0, 0, 1), (1, -1, 0)$.

4.5 $I = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & -2 & 4 \end{pmatrix}$ Principal moments: $(2, 4, 6)$; principal axes along the

vectors: $(0, 1, 1), (1, 0, 0), (0, 1, -1)$.

4.6 $I = \begin{pmatrix} 9 & 0 & -3 \\ 0 & 6 & 0 \\ -3 & 0 & 9 \end{pmatrix}$ Principal moments: $(6, 6, 12)$; principal axes along the

vectors: $(1, 0, -1)$ and any two orthogonal vectors in the plane $z = x$, say
$(0, 1, 0)$ and $(1, 0, 1)$.

4.7 $I = \dfrac{1}{120}\begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ Principal moments: $\left(\dfrac{1}{60}, \dfrac{1}{24}, \dfrac{1}{24}\right)$; principal axes

along the vectors: $(1, 1, 1)$ and any two orthogonal vectors in the plane
$x + y + z = 0$, say $(1, -1, 0)$ and $(1, 1, -2)$.

5.5   1 if $j\ k = m\ n$ (6 cases); -1 if $j\ k = n\ m$ (6 cases);   0 otherwise

5.6   (a) 3   (b) 0   (c) 2   (d) -2 (e) -1 (f) -1

5.7   (a) $\delta_{kq}\delta_{ip} - \delta_{kp}\delta_{iq}$ (b) $\delta_{ap}\delta_{bq} - \delta_{aq}\delta_{bp}$

6.9 to 6.14 $\mathbf{r}$, $\mathbf{v}$, $\mathbf{F}$, $\mathbf{E}$ are vectors; $\boldsymbol{\omega}$, $\boldsymbol{\tau}$, $\mathbf{L}$, $\mathbf{B}$ are pseudovectors; T is a scalar.

6.15   (a) vector   (b) pseudovector   (c) vector

6.16   vector (if $\mathbf{V}$ is a vector); pseudovector (if $\mathbf{V}$ is a pseudovector)

8.1 $h_r = 1, \quad h_\theta = r, \quad h_\phi = r\sin\theta$

$d\mathbf{s} = \mathbf{e}_r dr + \mathbf{e}_\theta r d\theta + \mathbf{e}_\phi r \sin\theta d\phi$

$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$\mathbf{a}_r = \mathbf{i}\sin\theta\cos\phi + \mathbf{j}\sin\theta\sin\phi + \mathbf{k}\cos\theta = \mathbf{e}_r$

$\mathbf{a}_\theta = \mathbf{i}r\cos\theta\cos\phi + \mathbf{j}r\cos\theta\sin\phi - \mathbf{k}r\sin\theta = r\mathbf{e}_\theta$

$\mathbf{a}_\phi = -\mathbf{i}r\sin\theta\sin\phi + \mathbf{j}r\sin\theta\cos\phi = r\sin\theta\mathbf{e}_\phi$

8.2 $d^2\mathbf{s}/dt^2 = \mathbf{e}_r(\ddot{r} - r\dot{\theta}^2) + \mathbf{e}_\theta(r\ddot{\theta} + 2\dot{r}\dot{\theta}) + \mathbf{e}_z\ddot{z}$

8.3 $d\mathbf{s}/dt = \mathbf{e}_r\dot{r} + \mathbf{e}_\theta r\dot{\theta} + \mathbf{e}_\phi r\sin\theta\dot{\phi}$

$d^2\mathbf{s}/dt^2 = \mathbf{e}_r(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\,\dot{\phi}^2)$

$\qquad + \mathbf{e}_\theta(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\dot{\phi}^2)$

$\qquad + \mathbf{e}_\phi(r\sin\theta\,\ddot{\phi} + 2r\cos\theta\,\dot{\theta}\dot{\phi} + 2\sin\theta\,\dot{r}\dot{\phi})$

8.4 $\mathbf{V} = -r\mathbf{e}_\theta + \mathbf{k}$

8.5 $\quad \mathbf{V} = \mathbf{e}_r \cos\theta - \mathbf{e}_\theta \sin\theta - \mathbf{e}_\phi r \sin\theta$

8.6 $\quad h_u = h_v = (u^2 + v^2)^{1/2}, \quad h_z = 1$

$d\mathbf{s} = (u^2 + v^2)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + \mathbf{e}_z dz$

$dV = (u^2 + v^2) \ du \ dv \ dz$

$\mathbf{a}_u = \mathbf{i}u + \mathbf{j}v = (u^2 + v^2)^{1/2}\mathbf{e}_u$

$\mathbf{a}_v = -\mathbf{i}v + \mathbf{j}u = (u^2 + v^2)^{1/2}\mathbf{e}_v$

$\mathbf{a}_z = \mathbf{k} = \mathbf{e}_z$

8.7 $\quad h_u = h_v = a(\sinh^2 u + \sin^2 v)^{1/2}, \ h_z = 1$

$d\mathbf{s} = a(\sinh^2 u + \sin^2 v)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + \mathbf{e}_z dz$

$dV = a^2(\sinh^2 u + \sin^2 v) \ du \ dv \ dz$

$\mathbf{a}_u = \mathbf{i}a \sinh u \cos v + \mathbf{j}a \cosh u \sin v = h_u\mathbf{e}_u$

$\mathbf{a}_v = -\mathbf{i}a \cosh u \sin v + \mathbf{j}a \sinh u \cos v = h_v\mathbf{e}_v$

$\mathbf{a}_z = \mathbf{k} = \mathbf{e}_z$

8.8 $\quad h_u = h_v = (u^2 + v^2)^{1/2}, \ \ h_\phi = uv$

$d\mathbf{s} = (u^2 + v^2)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + uv\mathbf{e}_\phi d\phi$

$dV = uv \ (u^2 + v^2) \ du \ dv \ d\phi$

$\mathbf{a}_u = \mathbf{i}v \cos\phi + \mathbf{j}v \sin\phi + \mathbf{k}u = h_u\mathbf{e}_u$

$\mathbf{a}_v = \mathbf{i}u \cos\phi + \mathbf{j}u \sin\phi - \mathbf{k}v = h_v\mathbf{e}_v$

$\mathbf{a}_\phi = -\mathbf{i}uv \sin\phi + \mathbf{j}uv \cos\phi = h_\phi\mathbf{e}_\phi$

8.9 $\quad h_u = h_v = \ a(\cosh u + \cos v)^{-1}$

$d\mathbf{s} = a(\cosh u + \cos v)^{-1}(\mathbf{e}_u du + \mathbf{e}_v dv)$

$dA = a^2(\cosh u + \cos v)^{-2} \ du \ dv$

$\mathbf{a}_u = (h_u^2/a)[\mathbf{i}(1 + \cos v \cosh u) - \mathbf{j}\sin v \sinh u] = h_u\mathbf{e}_u$

$\mathbf{a}_v = (h_v^2/a)[\mathbf{i}\sinh u \sin v + \mathbf{j}(1 + \cos v \cosh u)] = h_v\mathbf{e}_v$

8.11 $\quad d\mathbf{e}_u/dt = (u^2 + v^2)^{-1}(u\dot{v} - v\dot{u})\mathbf{e}_v$

$d\mathbf{e}_v/dt = (u^2 + v^2)^{-1}(v\dot{u} - u\dot{v})\mathbf{e}_u$

$d\mathbf{s}/dt = (u^2 + v^2)^{1/2}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v}) + \mathbf{e}_z\dot{z}$

$d^2\mathbf{s}/dt^2 = \mathbf{e}_u(u^2 + v^2)^{-1/2}[(u^2 + v^2)\ddot{u} + u(\dot{u}^2 - \dot{v}^2) + 2v\dot{u}\dot{v}]$

$\qquad\qquad + \mathbf{e}_v(u^2 + v^2)^{-1/2}[(u^2 + v^2)\ddot{v} + v(\dot{v}^2 - \dot{u}^2) + 2u\dot{u}\dot{v}] + \mathbf{e}_z\ddot{z}$

8.12 $\quad d\mathbf{e}_u/dt = (\sinh^2 u + \sin^2 v)^{-1}(\dot{v}\sinh u \cosh u - \dot{u}\sin v \cos v)\mathbf{e}_v$

$d\mathbf{e}_v/dt = (\sinh^2 u + \sin^2 v)^{-1}(\dot{u}\sin v \cos v - \dot{v}\sinh u \cosh u)\mathbf{e}_u$

$d\mathbf{s}/dt = a(\sinh^2 u + \sin^2 v)^{1/2}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v}) + \mathbf{e}_z\dot{z}$

$d^2\mathbf{s}/dt^2 = \mathbf{e}_u a(\sinh^2 u + \sin^2 v)^{-1/2}$

$\qquad\qquad \times [(\sinh^2 u + \sin^2 v)\ddot{u} + (\dot{u}^2 - \dot{v}^2)\sinh u \cosh u + 2\dot{u}\dot{v}\sin v \cos v]$

$\qquad\qquad + \mathbf{e}_v a(\sinh^2 u + \sin^2 v)^{-1/2}[(\sinh^2 u + \sin^2 v)\ddot{v}$

$\qquad\qquad + (\dot{v}^2 - \dot{u}^2)\sin v \cos v + 2\dot{u}\dot{v}\sinh u \cosh u] + \mathbf{e}_z\ddot{z}$

8.13 $\quad d\mathbf{e}_u/dt = (u^2 + v^2)^{-1}(u\dot{v} - v\dot{u})\mathbf{e}_v + (u^2 + v^2)^{-1/2}v\dot{\phi}\mathbf{e}_\phi$

$d\mathbf{e}_v/dt = (u^2 + v^2)^{-1}(v\dot{u} - u\dot{v})\mathbf{e}_u + (u^2 + v^2)^{-1/2}u\dot{\phi}\mathbf{e}_\phi$

$d\mathbf{e}_\phi/dt = -(u^2 + v^2)^{-1/2}(v\mathbf{e}_u + u\mathbf{e}_v)\dot{\phi}$

$d\mathbf{s}/dt = (u^2 + v^2)^{1/2}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v}) + \mathbf{e}_\phi uv\dot{\phi}$

$d^2\mathbf{s}/dt^2 = \mathbf{e}_u(u^2 + v^2)^{-1/2}[(u^2 + v^2)\ddot{u} + u(\dot{u}^2 - \dot{v}^2) + 2v\dot{u}\dot{v} - uv^2\dot{\phi}^2]$

$\qquad\qquad + \mathbf{e}_v(u^2 + v^2)^{-1/2}[(u^2 + v^2)\ddot{v} + v(\dot{v}^2 - \dot{u}^2) + 2u\dot{u}\dot{v} - u^2v\dot{\phi}^2]$

$\qquad\qquad + \mathbf{e}_\phi(uv\ddot{\phi} + 2v\dot{u}\dot{\phi} + 2u\dot{v}\dot{\phi})$

8.14 $\quad d\mathbf{e}_u/dt = -(\cosh u + \cos v)^{-1}(\dot{u}\sin v + \dot{v}\sinh u)\mathbf{e}_v$

$d\mathbf{e}_v/dt = (\cosh u + \cos v)^{-1}(\dot{u}\sin v + \dot{v}\sinh u)\mathbf{e}_u$

$d\mathbf{s}/dt = a(\cosh u + \cos v)^{-1}(\mathbf{e}_u\dot{u} + \mathbf{e}_v\dot{v})$

$d^2\mathbf{s}/dt^2 = \mathbf{e}_u a(\cosh u + \cos v)^{-2}[(\cosh u + \cos v)\ddot{u} + (\dot{v}^2 - \dot{u}^2)\sinh u + 2\dot{u}\dot{v}\sin v]$

$\qquad\qquad + \mathbf{e}_v a(\cosh \ u + \cos v)^{-2}[(\cosh u + \cos v)\ddot{v} + (\dot{v}^2 - \dot{u}^2)\sin v - 2\dot{u}\dot{v}\sinh u]$

9.3   See 8.2

9.5 $\boldsymbol{\nabla} U = \mathbf{e}_r \dfrac{\partial U}{\partial r} + \mathbf{e}_\theta \left(\dfrac{1}{r} \dfrac{\partial U}{\partial \theta} \right) + \mathbf{e}_\phi \left(\dfrac{1}{r \sin \theta} \dfrac{\partial U}{\partial \phi} \right)$

$\boldsymbol{\nabla} \cdot \mathbf{V} = \dfrac{1}{r^2} \dfrac{\partial}{\partial r} \left(r^2 V_r \right) + \dfrac{1}{r \sin \theta} \dfrac{\partial}{\partial \theta} \left(\sin \theta \, V_\theta \right) + \dfrac{1}{r \sin \theta} \dfrac{\partial V_\phi}{\partial \phi}$

$\nabla^2 U = \dfrac{1}{r^2} \dfrac{\partial}{\partial r} \left(r^2 \dfrac{\partial U}{\partial r} \right) + \dfrac{1}{r^2 \sin \theta} \dfrac{\partial}{\partial \theta} \left(\sin \theta \dfrac{\partial U}{\partial \theta} \right) + \dfrac{1}{r^2 \sin^2 \theta} \dfrac{\partial^2 U}{\partial \phi^2}$

$\boldsymbol{\nabla} \times \mathbf{V} = \dfrac{1}{r \sin \theta} \left[\dfrac{\partial}{\partial \theta} (\sin \theta \, V_\phi) - \dfrac{\partial V_\theta}{\partial \phi} \right] \mathbf{e}_r$

$\qquad\qquad + \dfrac{1}{r \sin \theta} \left[\dfrac{\partial V_r}{\partial \phi} - \sin \theta \dfrac{\partial}{\partial r} (rV_\phi) \right] \mathbf{e}_\theta + \dfrac{1}{r} \left[\dfrac{\partial}{\partial r} (rV_\theta) - \dfrac{\partial V_r}{\partial \theta} \right] \mathbf{e}_\phi$

9.6   See 8.11   9.7   See 8.12   9.8   See 8.13   9.9   See 8.14

9.10   Let $h = (u^2 + v^2)^{1/2}$ represent the u and v scale factors.

$\boldsymbol{\nabla} U = h^{-1} \left(\mathbf{e}_u \dfrac{\partial U}{\partial u} + \mathbf{e}_v \dfrac{\partial U}{\partial v} \right) + \mathbf{k} \dfrac{\partial U}{\partial z}$

$\boldsymbol{\nabla} \cdot \mathbf{V} = h^{-2} \left[\dfrac{\partial}{\partial u} (hV_u) + \dfrac{\partial}{\partial v} (hV_v) \right] + \dfrac{\partial V_z}{\partial z}$

$\nabla^2 U = h^{-2} \left(\dfrac{\partial^2 U}{\partial u^2} + \dfrac{\partial^2 U}{dv^2} \right) + \dfrac{\partial^2 U}{\partial z^2}$

$\boldsymbol{\nabla} \times \mathbf{V} = \left(h^{-1} \dfrac{\partial V_z}{\partial v} - \dfrac{\partial V_v}{\partial z} \right) \mathbf{e}_u$

$\qquad\qquad + \left(\dfrac{\partial V_u}{\partial z} - h^{-1} \dfrac{\partial V_z}{\partial u} \right) \mathbf{e}_v + h^{-2} \left[\dfrac{\partial}{\partial u} (hV_v) - \dfrac{\partial}{\partial v} (hV_u) \right] \mathbf{e}_z$

9.11   Same as 9.10 with $h = a(\sinh^2 u + \sin^2 v)^{1/2}$

9.12   Let $h = (u^2 + v^2)^{1/2}$

$\boldsymbol{\nabla} U = h^{-1} \left(\mathbf{e}_u \dfrac{\partial U}{\partial u} + \mathbf{e}_v \dfrac{\partial U}{\partial v} \right) + (uv)^{-1} \dfrac{\partial U}{\partial \phi} \mathbf{e}_\phi$

$\boldsymbol{\nabla} \cdot \mathbf{V} = \dfrac{1}{uh^2} \dfrac{\partial}{\partial u} (uhV_u) + \dfrac{1}{vh^2} \dfrac{\partial}{\partial v} (vhV_v) + \dfrac{1}{uv} \dfrac{\partial V_\phi}{\partial \phi}$

$\nabla^2 U = \dfrac{1}{h^2 u} \dfrac{\partial}{\partial u} \left(u \dfrac{\partial U}{\partial u} \right) + \dfrac{1}{h^2 v} \dfrac{\partial}{\partial v} \left(v \dfrac{\partial U}{\partial v} \right) + \dfrac{1}{u^2 v^2} \dfrac{\partial^2 U}{\partial \phi^2}$

$\boldsymbol{\nabla} \times \mathbf{V} = \left[\dfrac{1}{hv} \dfrac{\partial}{\partial v} (vV_\phi) - \dfrac{1}{uv} \dfrac{\partial V_v}{\partial \phi} \right] \mathbf{e}_u$

$\qquad\qquad + \left[\dfrac{1}{uv} \dfrac{\partial V_u}{\partial \phi} - \dfrac{1}{hu} \dfrac{\partial}{\partial u} (uV_\phi) \right] \mathbf{e}_v + \dfrac{1}{h^2} \left[\dfrac{\partial}{\partial u} (hV_v) - \dfrac{\partial}{\partial v} (hv_u) \right] \mathbf{e}_\phi$

9.13   Same as 9.10 if $h = a(\cosh u + \cos v)^{-1}$ and terms involving either
z derivatives or V_z are omitted. Note, however, that $\boldsymbol{\nabla} \times \mathbf{V}$ has
only a z component if $\mathbf{V} = \mathbf{e}_u V_u + \mathbf{e}_v V_v$ where V_u and V_v are functions
of u and v.

9.14 $h_u = [(u+v)/u]^{1/2}, \;\; h_v = [(u+v)/v]^{1/2}$
$\mathbf{e}_u = h_u^{-1} \mathbf{i} + h_v^{-1} \mathbf{j}, \;\; \mathbf{e}_v = -h_v^{-1} \mathbf{i} + h_u^{-1} \mathbf{j}$
$m \left[h_u \ddot{u} - h_u^{-1} (u\dot{v} - v\dot{u})^2 / (2u^2 v) \right] = -h_u^{-1} \partial V / \partial u = F_u$
$m \left[h_v \ddot{v} - h_v^{-1} (u\dot{v} - v\dot{u})^2 / (2uv^2) \right] = -h_v^{-1} \partial V / \partial v = F_v$

9.15 $h_u = 1, \; h_v = u(1 - v^2)^{-1/2}$
$\mathbf{e}_u = \mathbf{i} v + \mathbf{j}(1 - v^2)^{1/2}, \; \mathbf{e}_v = \mathbf{i}(1 - v^2)^{1/2} - \mathbf{j} v$
$m[\ddot{u} - u\dot{v}^2 / (1 - v^2)] = -\partial V / \partial u = F_u$
$m[(u\ddot{v} + 2\dot{u}\dot{v})(1 - v^2)^{-1/2} + uv\dot{v}^2 (1 - v^2)^{-3/2}] = -h_v^{-1} \partial V / \partial v = F_v$

9.16 $r^{-1}, \; 0, \; 0, \; r^{-1} \mathbf{e}_z$

9.17 $2r^{-1}, \; r^{-1} \cot \theta, \; r^{-1} \mathbf{e}_\phi, \; r^{-1} (\mathbf{e}_r \cot \theta - \mathbf{e}_\theta)$

9.18 $-r^{-1}\mathbf{e}_\theta,\ \ r^{-1}\mathbf{e}_r,\ \ 3$

9.19 $2\mathbf{e}_\phi,\ \ \mathbf{e}_r\cos\theta - \mathbf{e}_\theta\sin\theta,\ \ 3$

9.20 $r^{-1},\ \ r^{-3},\ \ 0$

9.21 $2r^{-1},\ \ 6,\ \ 2r^{-4},\ \ -k^2 e^{ikr\cos\theta}$

11.4   Vector

11.5 $ds^2 = du^2 + h_v^2 dv^2,\ h_u = 1,\ h_v = u(2v - v^2)^{-1/2},$
         $dA = u(2v - v^2)^{-1/2} du\, dv,\ d\mathbf{s} = \mathbf{e}_u\, du + h_v \mathbf{e}_v dv,$
         $\mathbf{e}_u = \mathbf{i}(1 - v) + \mathbf{j}(2v - v^2)^{1/2},\ \ \mathbf{e}_v = -\mathbf{i}(2v - v^2)^{1/2} + \mathbf{j}(1 - v)$
         $\mathbf{a}_u = \mathbf{e}_u = \mathbf{a}^u,\ \ \mathbf{a}_v = h_v \mathbf{e}_v,\ \ \mathbf{a}^v = \mathbf{e}_v/h_v$

11.6 $m\left(\ddot{u} - \dfrac{u\dot{v}^2}{v(2 - v)}\right) = -\dfrac{\partial V}{\partial u} = F_u$

         $m\left(\dfrac{u\ddot{v} + 2\dot{u}\dot{v}}{[v(2 - v)]^{1/2}} + \dfrac{u\dot{v}^2(v - 1)}{[v(2 - v)]^{3/2}}\right) = -u^{-1}[v(2 - v)]^{1/2}\dfrac{\partial V}{\partial v} = F_v$

11.7 $\boldsymbol{\nabla} U = \mathbf{e}_u \partial U/\partial u + \mathbf{e}_v u^{-1}\sqrt{v(2 - v)}\,\partial U/\partial v$

         $\boldsymbol{\nabla}\cdot\mathbf{V} = u^{-1}\partial(uV_u)/\partial u + u^{-1}\sqrt{v(2 - v)}\,\partial V_v/\partial v$

         $\nabla^2 U = \dfrac{1}{u}\dfrac{\partial}{\partial u}\left(u\dfrac{\partial U}{\partial u}\right) + \dfrac{1}{u^2}\sqrt{v(2 - v)}\dfrac{\partial}{\partial V}\left(\sqrt{v(2 - v)}\dfrac{\partial U}{\partial V}\right)$

11.8 $u^{-1},\ \ u^{-1}\mathbf{k},\ \ 0$

Chapter 11

3.2	$3/2$	3.3	$9/10$	3.4	$25/14$
3.5	$32/35$	3.6	72	3.7	8
3.8	$\Gamma(5/3)$	3.9	$\Gamma(5/4)$	3.10	$\Gamma(3/5)$
3.11	1	3.12	$\Gamma(2/3)/3$	3.13	$3^{-4}\Gamma(4) = 2/27$
3.14	$-\Gamma(4/3)$	3.15	$\Gamma(2/3)/4$	3.17	$\Gamma(p)$

7.1 $\quad \frac{1}{2}B(5/2, 1/2) = 3\pi/16$

7.2 $\quad \frac{1}{2}B(5/4, 3/4) = \pi\sqrt{2}/8$

7.3 $\quad \frac{1}{3}B(1/3, 1/2)$

7.4 $\quad \frac{1}{2}B(3/2, 5/2) = \pi/32$

7.5 $\quad B(3, 3) = 1/30$

7.6 $\quad \frac{1}{3}B(2/3, 4/3) = 2\pi\sqrt{3}/27$

7.7 $\quad \frac{1}{2}B(1/4, 1/2)$

7.8 $\quad 4\sqrt{2}B(3, 1/2) = 64\sqrt{2}/15$

7.10 $\quad \frac{4}{3}B(1/3, 4/3)$

7.11 $\quad 2B(2/3, 4/3)/B(1/3, 4/3)$

7.12 $\quad (8\pi/3)B(5/3, 1/3) = 32\pi^2\sqrt{3}/27$

7.13 $\quad I_y/M = 8B(4/3, 4/3)/B(5/3, 1/3)$

8.1 $\quad B(1/2, 1/4)\sqrt{2l/g} = 7.4163\sqrt{l/g} \quad \left(\text{Compare } 2\pi\sqrt{l/g}.\right)$

8.2 $\quad \frac{1}{4}\sqrt{35/11}\,B(1/2, 1/4) = 2.34 \text{ sec}$

8.3 $\quad t = \pi\sqrt{a/g}$

10.2 $\quad \Gamma(p, x) \sim x^{p-1}e^{-x}[1 + (p-1)x^{-1} + (p-1)(p-2)x^{-2} + \cdots]$

10.3 $\quad \text{erfc}\,(x) = \Gamma\left(1/2, x^2\right)/\sqrt{\pi}$

10.5 $\quad$ (a) $E_1(x) = \Gamma(0, x)$

$\qquad$ (b) $\Gamma(0, x) \sim x^{-1}e^{-x}[1 - x^{-1} + 2x^{-2} - 3!\,x^{-3} + \cdots]$

10.6 $\quad$ (a) $\text{Ei}(\ln x)$ $\quad$ (b) $\text{Ei}(x)$ $\quad$ (c) $-\text{Ei}(\ln x)$

11.4 $\quad 1/\sqrt{\pi}$ $\qquad\qquad$ 11.5 $\quad 1$ $\qquad\qquad\qquad$ 11.10 $\quad e^{-1}$

12.1 $\quad K = F(\pi/2, k) = (\pi/2)\left[1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1\cdot3}{2\cdot4}\right)^2 k^4 + \cdots\right]$

$\qquad E = E\left(\pi/2, k\right) = (\pi/2)\left[1 - \left(\frac{1}{2}\right)^2 k^2 - \left(\frac{1}{2\cdot4}\right)^2 \cdot 3k^4 - \left(\frac{1\cdot3}{2\cdot4\cdot6}\right)^2 \cdot 5k^6 \cdots\right]$

Caution : For the following answers, see the text warning about elliptic integral notation just after equations (12.3) and in Example 1.

12.4 $\quad K(1/2) \cong 1.686$

12.5 $\quad E(1/3) \cong 1.526$

12.6 $\quad \frac{1}{3}F\left(\frac{\pi}{3}, \frac{1}{3}\right) \cong 0.355$

12.7 $\quad 5E\left(\frac{5\pi}{4}, \frac{1}{5}\right) \cong 19.46$

12.8 $\quad 7E\left(\frac{\pi}{3}, \frac{2}{7}\right) \cong 7.242$

12.9 $\quad F\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right) \cong 0.542$

12.10 $\quad \frac{1}{2}F\left(\frac{\pi}{4}, \frac{1}{2}\right) \cong 0.402$

12.11 $\quad F\left(\frac{3\pi}{8}, \frac{3}{\sqrt{10}}\right) + K\left(\frac{3}{\sqrt{10}}\right) \cong 4.097$

12.12 $\quad 10E\left(\frac{\pi}{6}, \frac{1}{10}\right) \cong 5.234$

12.13 $\quad 3E\left(\frac{\pi}{6}, \frac{2}{3}\right) + 3E\left(\arcsin\frac{3}{4}, \frac{2}{3}\right) \cong 3.96$

12.14 $12E\left(\frac{\sqrt{5}}{3}\right) \cong 15.86$ 12.15 $2\left[E\left(\frac{\sqrt{3}}{2}\right) - E\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)\right] \cong 0.585$

12.16 $2\sqrt{2}\,E\left(1/\sqrt{2}\right) \cong 3.820$

12.23 $T = 8\sqrt{\frac{a}{5g}}\,K\left(1/\sqrt{5}\right)$; for small vibrations, $T \cong 2\pi\sqrt{\frac{2a}{3g}}$

13.7 $\Gamma(4) = 3!$ 13.8 $\frac{\sqrt{\pi}}{2}\,\mathrm{erf}(1)$

13.9 $2\,E\left(\sqrt{3}/2\right) \simeq 2.422$ 13.10 $\sqrt{2}\,K\left(2^{-1/2}\right) \simeq 2.622$

13.11 $\frac{1}{5}F\left(\arcsin\frac{3}{4}, 4/5\right) \cong 0.1834$ 13.12 $2^{-1/2}K\left(2^{-1/2}\right) \cong 1.311$

13.13 $-\,\mathrm{sn}\,u\,\mathrm{dn}\,u$ 13.14 $\sqrt{\pi}/2\,\mathrm{erfc}(1/\sqrt{2})$

13.15 $\Gamma(7/2) = 15\sqrt{\pi}/8$ 13.16 $\sqrt{\pi}$

13.17 $\frac{1}{2}B(5/4, 7/4) = 3\pi\sqrt{2}/64$ 13.18 $\Gamma(3/4)$

13.19 $\frac{1}{2}\sqrt{\pi}\,\mathrm{erfc}\,5$ 13.20 $\frac{1}{2}\,B(1/2, 7/4)$

13.21 $5^4 B(2/3, 13/3) = (5/3)^5\left(14\pi/\sqrt{3}\right)$

13.22 $4E(1/2) - 2\,E(\pi/8, 1/2) \cong 5.089$ 13.23 $109!!\sqrt{\pi}/2^{55}$

13.24 $-2^{55}\sqrt{\pi}/109!!$ 13.25 $2^{28}\sqrt{\pi}/55!!$

Chapter 12

<div>

1.1 $y = a_1 x e^x$ 1.2 $y = a_0 e^{x^3}$

1.3 $y = a_1 x$ 1.4 $y = a_0 \cos 2x + a_1 \sin 2x$

1.5 $y = a_0 \cosh x + a_1 \sinh x$ 1.6 $y = (A + Bx)e^x$

1.7 $y = Ax + Bx^3$ 1.8 $y = a_0(1 + x) + a_2 x^2$

1.9 $y = a_0(1 - x^2) + a_1 x$ 1.10 $y = (A + Bx)e^{x^2}$

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2.1   See Problem 5.3   2.4 $Q_0 = \dfrac{1}{2} \ln \dfrac{1+x}{1-x}, \quad Q_1 = \dfrac{x}{2} \ln \dfrac{1+x}{1-x} - 1$

3.2 $xe^x + 10e^x$ 3.3 $(30 - x^2) \sin x + 12x \cos x$

3.4 $-x \sin x + 25 \cos x$ 3.5 $(x^2 - 200x + 9900)e^{-x}$

4.3   See Problem 5.3

5.1   See Problem 5.3

5.3 $P_0(x) = 1$ $P_4(x) = (35x^4 - 30x^2 + 3)/8$

         $P_1(x) = x$ $P_5(x) = (63x^5 - 70x^3 + 15x)/8$

         $P_2(x) = (3x^2 - 1)/2$ $P_6(x) = (231x^6 - 315x^4 + 105x^2 - 5)/16$

         $P_3(x) = (5x^3 - 3x)/2$

5.8 $5P_0 - 2P_1$ 5.9 $2P_2 + P_1$

5.10 $\frac{8}{35}P_4 + \frac{4}{7}P_2 + \frac{1}{5}P_0$ 5.11 $\frac{2}{5}(P_1 - P_3)$

5.12 $\frac{8}{5}P_4 + 4P_2 - 3P_1 + \frac{12}{5}P_0$ 5.13 $\frac{8}{63}P_5 + \frac{4}{9}P_3 + \frac{3}{7}P_1$

8.1 $N = \sqrt{\pi/2}, \quad \sqrt{2/\pi} \cos nx$ 8.2 $N = \sqrt{2/5}, \quad \sqrt{5/2}\, P_2(x)$

8.3 $N = 2^{1/2}, \quad 2^{-1/2}xe^{-x/2}$ 8.4 $N = \pi^{1/4}, \quad \pi^{-1/4}e^{-x^2/2}$

8.5 $N = \frac{1}{2}\pi^{1/4}, \quad 2\pi^{-1/4}xe^{-x^2/2}$

9.1 $\frac{3}{2}P_1 - \frac{7}{8}P_3 + \frac{11}{16}P_5 \cdots$ 9.2 $\frac{1}{4}P_0 + \frac{1}{2}P_1 + \frac{5}{16}P_2 - \frac{3}{32}P_4 \cdots$

9.3 $5P_2 + P_0$ 9.4 $\frac{\pi}{8}(3P_1 + \frac{7}{16}P_3 + \frac{11}{64}P_5 \cdots)$

9.5 $\frac{1}{2}P_0 - \frac{5}{8}P_2 + \frac{3}{16}P_4 \cdots$ 9.6 $P_0 + \frac{3}{8}P_1 - \frac{20}{9}P_2 \cdots$

9.7 $\frac{1}{3}P_0 + \frac{2}{5}P_1 - \frac{11}{42}P_2 \cdots$

9.8 $\frac{1}{2}(1 - a)P_0 + \frac{3}{4}(1 - a^2)P_1 + \frac{5}{4}a(1 - a^2)P_2 + \frac{7}{16}(1 - a^2)(5a^2 - 1)P_3 \cdots$

9.9 $P_n' = \sum (2l + 1)P_l$, where the sum is over odd l from 1 to $n - 1$
        when n is even, and over even l from 0 to $n - 1$, when n is odd.

9.10 $2P_2 + P_1$ 9.11 $\frac{8}{5}P_4 + 4P_2 - 3P_1 + \frac{12}{5}P_0$

9.12 $\frac{2}{5}(P_1 - P_3)$ 9.13 $\frac{1}{5}P_0 + \frac{4}{7}P_2 = \frac{3}{35}(10x^2 - 1)$

9.14 $\dfrac{1}{2}P_0 + \dfrac{5}{8}P_2 = \dfrac{3}{16}(5x^2 + 1)$ 9.15 $-\dfrac{15}{\pi^2}P_2 = -\dfrac{15}{2\pi^2}(3x^2 - 1)$

10.4 $\sin\theta$ 10.5 $\sin\theta\,(35\cos^3\theta - 15\cos\theta)/2$
10.6 $15\sin^2\theta\cos\theta$

11.1 $y = b_0\cos x/x^2$ 11.2 $y = Ax^{-3} + Bx^3$
11.3 $y = Ax^{-3} + Bx^2$ 11.4 $y = Ax^{-2} + Bx^3$
11.5 $y = A\cos(2x^{1/2}) + B\sin(2x^{1/2})$
11.6 $y = Ae^{-x} + Bx^{2/3}[1 - 3x/5 + (3x)^2/(5\cdot 8) - (3x)^3/(5\cdot 8\cdot 11) + \cdots]$
11.7 $y = Ax^2(1 + x^2/10 + x^4/280 + \cdots) + Bx^{-1}(1 - x^2/2 - x^4/8 - \cdots)$
11.8 $y = A(x^{-1} - 1) + Bx^2(1 - x + 3x^2/5 - 4x^3/15 + 2x^4/21 + \cdots)$
11.9 $y = A(1 - 3x^6/8 + 9x^{12}/320 - \cdots) + Bx^2(1 - 3x^6/16 + 9x^{12}/896 - \cdots)$
11.10 $y = A[1 + 2x - (2x)^2/2! + (2x)^3/(3\cdot 3!) - (2x)^4/(3\cdot 5\cdot 4!) + \cdots]$
$$+ Bx^{3/2}[1 - 2x/5 + (2x)^2/(5\cdot 7\cdot 2!) - (2x)^3/(5\cdot 7\cdot 9\cdot 3!) + \cdots]$$
11.11 $y = Ax^{1/6}[1 + 3x^2/2^5 + 3^2x^4/(5\cdot 2^{10}) + \cdots]$
$$+ Bx^{-1/6}[x + 3x^3/2^6 + 3^2x^5/(7\cdot 2^{11}) + \cdots]$$
11.12 $y = e^x(A + Bx^{1/3})$

15.9 $5^{-3/2}$

16.1 $y = x^{-3/2}Z_{1/2}(x)$ 16.2 $y = x^{1/2}Z_{1/4}(x^2)$
16.3 $y = x^{-1/2}Z_1(4x^{1/2})$ 16.4 $y = x^{1/6}Z_{1/3}(4x^{1/2})$
16.5 $y = xZ_0(2x)$ 16.6 $y = x^{1/2}Z_1(x^{1/2})$
16.7 $y = x^{-1}Z_{1/2}(x^2/2)$ 16.8 $y = x^{1/2}Z_{1/3}(\frac{2}{3}x^{3/2})$
16.9 $y = x^{1/3}Z_{2/3}(4\sqrt{x})$ 16.10 $y = xZ_{2/3}(2x^{3/2})$
16.11 $y = x^{-2}Z_2(x)$ 16.12 $y = x^{1/4}Z_{1/2}(\sqrt{x})$
16.14 $y = Z_3(2x)$ 16.15 $y = Z_2(5x)$
16.16 $y = Z_1(4x)$ 16.17 $y = Z_0(3x)$

17.7 (a) $y = x^{1/2}I_1(2x^{1/2})$ (b) $y = x^{1/2}I_{1/6}(x^3/3)$
Note that the factor i is not needed since any multiple of y is a solution.
17.9 $\frac{d}{dx}[x^pI_p(x)] = x^pI_{p-1}(x)$
$\frac{d}{dx}[x^{-p}I_p(x)] = x^{-p}I_{p+1}(x)$
$I_{p-1}(x) - I_{p+1}(x) = \frac{2p}{x}I_p(x)$
$I_{p-1}(x) + I_{p+1}(x) = 2I_p'(x)$
$I_p'(x) = -\frac{p}{x}I_p(x) + I_{p-1}(x) = \frac{p}{x}I_p(x) + I_{p+1}(x)$

18.9 Amplitude increases; outward swing takes longer.
18.10 $y = (Ax + B)^{1/2}J_{1/3}[\frac{2}{3A}(Ax + B)^{3/2}]$
18.11 1.7 m for steel, 0.67 m for lead

19.2 $\frac{1}{2}\sin^2\alpha$

19.3 $\int_0^1 x^2 j_n(\alpha x)j_n(\beta x)\,dx = \begin{cases} 0, & \alpha \neq \beta \\ \frac{1}{2}j_{n-1}^2(\alpha), & \alpha = \beta \end{cases}$ where $j_n(\alpha) = j_n(\beta) = 0$.

19.6 $\int_0^1 \cos^2(\alpha_n x)dx = \int_0^1 \frac{1}{2}\pi\alpha_n x N_{1/2}^2(\alpha_n x)\,dx = \frac{1}{2}$, where $\alpha_n = (n + \frac{1}{2})\pi$

20.1 1/6   20.2 1   20.3 $4/\pi$
20.4 $-1/(\pi p)$ 20.5 1/2   20.6 $-1/(2n+1)$
20.7 $h_n^{(1)}(x) \sim \frac{1}{x}e^{i[x-(n+1)\pi/2]}$ 20.8 $h_n^{(2)}(x) \sim \frac{1}{x}e^{-i[x-(n+1)\pi/2]}$
20.9 $h_n^{(1)}(ix) \sim -\frac{1}{i^n}\frac{1}{x}\,e^{-x}$ 20.10 $h_n^{(2)}(ix) \sim i^n\frac{1}{x}e^x$

21.1 $y = Ax + B\left(x\sinh^{-1}x - \sqrt{x^2+1}\right)$

21.2 $y = A(1+x) + Bxe^{1/x}$

21.3 $y = A(1 - \frac{2}{x}) + B(1 + \frac{2}{x})e^{-x}$

21.4 $y = Ax - Be^x$

21.5 $y = A(x-1) + B[(x-1)\ln x - 4]$

21.6 $y = A\sqrt{x} + B[\sqrt{x}\ln x + x]$

21.7 $y = A\frac{x}{1-x} + B[\frac{x}{1-x}\ln x + \frac{1+x}{2}]$

21.8 $y = A(x^2 + 2x) + B[(x^2 + 2x)\ln x + 1 + 5x - x^3/6 + x^4/72 + \cdots]$

21.9 $y = Ax^2 + B[x^2\ln x - x^3 + x^4/(2\cdot 2!) - x^5/(3\cdot 3!) + x^6/(4\cdot 4!) + \cdots]$

21.10 $y = Ax^3 + B(x^3\ln x + x^2)$

22.4 $H_0(x) = 1$ $H_3(x) = 8x^3 - 12x$

       $H_1(x) = 2x$ $H_4(x) = 16x^4 - 48x^2 + 12$

       $H_2(x) = 4x^2 - 2$ $H_5(x) = 32x^5 - 160x^3 + 120x$

22.13 $L_0(x) = 1$

       $L_1(x) = 1 - x$

       $L_2(x) = (2 - 4x + x^2)/2!$

       $L_3(x) = (6 - 18x + 9x^2 - x^3)/3!$

       $L_4(x) = (24 - 96x + 72x^2 - 16x^3 + x^4)/4!$

       $L_5(x) = (120 - 600x + 600x^2 - 200x^3 + 25x^4 - x^5)/5!$

       Note: The factor $1/n!$ is omitted in most quantum mechanics books but is included as here in most reference books.

22.20 $L_0^k(x) = 1$

       $L_1^k(x) = 1 + k - x$

       $L_2^k(x) = \frac{1}{2}(k+1)(k+2) - (k+2)x + \frac{1}{2}x^2$

22.28 $f_1 = xe^{-x/2}$

       $f_2 = xe^{-x/4}(2 - \frac{x}{2})$

       $f_3 = xe^{-x/6}(3 - x + \frac{x^2}{18})$

22.30 $R_n = -x^n D x^{-n},\ L_n = x^{-n-1} D x^{n+1}$

23.6 $P_{2n+1}^1(0) = (2n+1)P_{2n}(0) = \dfrac{(-1)^n(2n+1)!!}{2^n n!}$

23.9   For $n \le l$, $\int_{-1}^{1} x P_l(x) P_n(x)\,dx = \begin{cases} \dfrac{2l}{(2l-1)(2l+1)}, & n = l-1 \\ 0, & \text{otherwise} \end{cases}$

23.18 (a) $y = Z_0(e^x)$ (b) $y = Z_p(e^{x^2}/2)$

23.23 $T_0 = 1,\ T_1 = x,\ T_2 = 2x^2 - 1$

23.30 $\pi/6$

Chapter 13

2.1 $\quad T = \dfrac{20}{\pi} \sum\limits_{1}^{\infty} \dfrac{(-1)^{n+1}}{n} \, e^{-n\pi y/10} \sin \dfrac{n\pi x}{10}$

2.2 $\quad T = \dfrac{200}{\pi} \left(\sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} -2 \sum\limits_{\substack{2 \\ n=2+4k}}^{\infty} \right) \dfrac{1}{n} \, e^{-n\pi y/20} \sin \dfrac{n\pi x}{20}$

2.3 $\quad T = \dfrac{4}{\pi} \sum\limits_{\substack{2 \\ \text{even } n}}^{\infty} \dfrac{n}{n^2-1} \, e^{-ny} \sin nx$

2.4 $\quad T = \dfrac{120}{\pi^2} \left(e^{-\pi y/30} \sin \dfrac{\pi x}{30} - \dfrac{1}{9} \, e^{-3\pi y/30} \sin \dfrac{3\pi x}{30} + \dfrac{1}{25} \, e^{-5\pi y/30} \sin \dfrac{5\pi x}{30} \cdots \right)$

2.7 $\quad T = \dfrac{4}{\pi} \sum\limits_{\substack{2 \\ \text{even } n}}^{\infty} \dfrac{n}{(n^2-1)\sinh n} \sinh n(1-y) \sin nx$

2.8 $\quad T = \sum\limits_{1}^{\infty} \dfrac{b_n}{\sinh \frac{4n\pi}{3}} \sinh \dfrac{n\pi}{30}(40-y) \sin \dfrac{n\pi x}{30}$

$\quad$ where $b_n = \dfrac{200}{n\pi} \left(1 - \cos \dfrac{n\pi}{3} \right) = \dfrac{100}{n\pi}(1,\ 3,\ 4,\ 3,\ 1,\ 0,\ \text{and repeat})$

2.9 $\quad T = \dfrac{200}{\pi} \left(\sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} -2 \sum\limits_{\substack{2 \\ n=2+4k}}^{\infty} \right) \dfrac{1}{n \sinh \frac{n\pi}{2}} \sinh \dfrac{n\pi}{20}(10-y) \sin \dfrac{n\pi x}{20}$

2.10 $\quad T = \sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{400}{n\pi \sinh n\pi} \sinh \dfrac{n\pi}{10}(10-y) \sin \dfrac{n\pi x}{10}; \quad T(5,5) \cong 25°$

2.11 $\quad T = \sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{400}{n\pi \sinh n\pi} \left[\sinh \dfrac{n\pi}{10}(10-y) \sin \dfrac{n\pi x}{10} + \sinh \dfrac{n\pi}{10}(10-x) \sinh \dfrac{n\pi y}{10} \right]$

2.12 $\quad T = \sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{400}{n\pi \sinh 3n\pi} \sinh \dfrac{n\pi}{10}(30-y) \sin \dfrac{n\pi x}{10}$

$\quad + \sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{400}{n\pi \sinh(n\pi/3)} \sinh \dfrac{n\pi}{30}(10-x) \sin \dfrac{n\pi y}{30}$

2.13 $\quad T(x,y) = \dfrac{20}{\pi} \sum\limits_{1}^{\infty} \dfrac{(-1)^{n+1}}{n \sinh 2n\pi} \sinh \dfrac{n\pi}{10}(20-y) \sin \dfrac{n\pi x}{10}$

$\quad + \dfrac{40}{\pi} \sum\limits_{1}^{\infty} \dfrac{(-1)^{n+1}}{n \sinh \frac{n\pi}{2}} \sinh \dfrac{n\pi}{20}(10-x) \sin \dfrac{n\pi y}{20}$

2.14   For $f(x) = x - 5$: $T = -\dfrac{40}{\pi^2} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n^2} \cos \dfrac{n\pi x}{10} e^{-n\pi y/10}$

For $f(x) = x$: Add 5 to the answer just given.

2.15   For $f(x) = 100$, $T = 100 - 10y/3$

For $f(x) = x$, $T = \dfrac{1}{6}(30 - y) - \dfrac{40}{\pi^2} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n^2 \sinh 3n\pi} \sinh \dfrac{n\pi}{10}(30 - y) \cos \dfrac{n\pi x}{10}$

3.2 $u = \dfrac{400}{\pi} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n} e^{-(n\pi\alpha/10)^2 t} \sin \dfrac{n\pi x}{10}$

3.3 $u = 100 - \dfrac{100x}{l} - \dfrac{400}{\pi} \displaystyle\sum_{\substack{2 \\ \text{even } n}}^{\infty} \dfrac{1}{n} e^{-(n\pi\alpha/l)^2 t} \sin \dfrac{n\pi x}{l}$

3.4 $u = \dfrac{40}{\pi} \left(\displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} - 2 \displaystyle\sum_{\substack{2 \\ n=2+4k}}^{\infty} \right) \dfrac{1}{n} e^{-(n\pi\alpha/10)^2 t} \sin \dfrac{n\pi x}{10}$

3.5 $u = 100 + 400 \displaystyle\sum_{1}^{\infty} b_n e^{-(n\pi\alpha/2)^2 t} \sin \dfrac{n\pi x}{2}$ where $b_n = \begin{cases} 0, & \text{even } n \\ \dfrac{2}{n^2\pi^2} - \dfrac{1}{n\pi}, & n = 1 + 4k \\ \dfrac{-2}{n^2\pi^2} - \dfrac{1}{n\pi}, & n = 3 + 4k \end{cases}$

3.6   Add to $(3.15): u_f = 20 + 30x/l$. Note: Any linear function added to both u_0 and u_f leaves the Fourier series unchanged.

3.7 $u = \dfrac{l}{2} - \dfrac{4l}{\pi^2} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n^2} \cos \dfrac{n\pi x}{l} e^{-(n\pi\alpha/l)^2 t}$

3.8 $u = 50x + \dfrac{200}{\pi} \displaystyle\sum_{1}^{\infty} \dfrac{(-1)^n}{n} e^{-(n\pi\alpha/2)^2 t} \sin \dfrac{n\pi x}{2}$

3.9 $u = 100 - \dfrac{400}{\pi} \displaystyle\sum_{0}^{\infty} \dfrac{(-1)^n}{2n + 1} e^{-[(2n+1)\pi\alpha/4]^2 t} \cos \left(\dfrac{2n + 1}{4} \pi x \right)$

3.11 $E_n = \dfrac{n^2 \hbar^2}{2m}$, $\quad \Psi(x, t) = \dfrac{4}{\pi} \displaystyle\sum_{\text{odd } n} \dfrac{\sin nx}{n} e^{-iE_n t/\hbar}$

3.12 $E_n = \dfrac{n^2 \pi^2 \hbar^2}{2m}$, $\quad \Psi(x, t) = \dfrac{8}{\pi} \displaystyle\sum_{\text{odd } n} \dfrac{\sin n\pi x}{n(4 - n^2)} e^{-iE_n t/\hbar}$

4.2 $y = \dfrac{8h}{\pi^2} \displaystyle\sum_{n=1}^{\infty} B_n \sin \dfrac{n\pi x}{l} \cos \dfrac{n\pi vt}{l}$, where $B_1 = \sqrt{2} - 1$, $B_2 = \dfrac{1}{2}$,

$B_3 = \dfrac{1}{9}(\sqrt{2} + 1)$, $B_4 = 0, \cdots$, $B_n = (2 \sin n\pi/4 - \sin n\pi/2)/n^2$

4.3 $y = \dfrac{16h}{\pi^2} \displaystyle\sum_{1}^{\infty} B_n \sin \dfrac{n\pi x}{l} \cos \dfrac{n\pi vt}{l}$ where $B_n = \left(2 \sin \dfrac{n\pi}{8} - \sin \dfrac{n\pi}{4} \right) / n^2$

4.4 $y = \dfrac{8h}{\pi^2} \displaystyle\sum_{1}^{\infty} \dfrac{1}{n^2} \sin \dfrac{n\pi}{2} \sin \dfrac{2n\pi x}{l} \cos \dfrac{2n\pi vt}{l}$

4.5 $y = \dfrac{8hl}{\pi^3 v} \displaystyle\sum_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n^3} \sin \dfrac{n\pi}{2} \sin \dfrac{n\pi x}{l} \sin \dfrac{n\pi vt}{l}$

4.6 $\quad y = \dfrac{4hl}{\pi^2 v} \sum\limits_{\substack{1 \\ \text{odd } n}}^{\infty} \dfrac{1}{n^2} \sin\dfrac{n\pi}{2} \sin\dfrac{n\pi w}{l} \sin\dfrac{n\pi x}{l} \sin\dfrac{n\pi vt}{l}$

4.7 $\quad y = \dfrac{9hl}{\pi^3 v} \sum\limits_{1}^{\infty} \dfrac{1}{n^3} \sin\dfrac{n\pi}{3} \sin\dfrac{n\pi x}{l} \sin\dfrac{n\pi vt}{l}$

4.8 $\quad y = \dfrac{4l}{\pi^2 v}\left[\dfrac{1}{3}\sin\dfrac{\pi x}{l}\sin\dfrac{\pi vt}{l} + \dfrac{\pi}{16}\sin\dfrac{2\pi x}{l}\sin\dfrac{2\pi vt}{l} - \sum\limits_{n=3}^{\infty}\dfrac{\sin(n\pi/2)}{n(n^2-4)}\sin\dfrac{n\pi x}{l}\sin\dfrac{n\pi vt}{l}\right]$

4.9   1. $n = 1$, $\nu = v/(2l)$
       2. $n = 2$, $\nu = v/l$
       3. $n = 3$, $\nu = 3v/(2l)$, and $n = 4$, $\nu = 2v/l$, have nearly equal intensity.
       4. $n = 2$, $\nu = v/l$
       5, 6, 7, 8. $n = 1$, $\nu = v/(2l)$

4.11   The basis functions for a string pinned at $x = 0$, free at $x = l$, and
      with zero initial string velocity, are $y = \sin\dfrac{(n+\frac{1}{2})\pi x}{l}\cos\dfrac{(n+\frac{1}{2})\pi vt}{l}$.
      The solutions for Problems 2, 3, 4, parts (a) and (b) are:

        (a) $\quad y = \sum\limits_{0}^{\infty} a_n \cos\dfrac{(n+\frac{1}{2})\pi x}{l}\cos\dfrac{(n+\frac{1}{2})\pi vt}{l}$

        (b) $\quad y = \sum\limits_{0}^{\infty} b_n \sin\dfrac{(n+\frac{1}{2})\pi x}{l}\cos\dfrac{(n+\frac{1}{2})\pi vt}{l}$

      where the coefficients are:

        2(a) $\quad a_n = \dfrac{128h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{16}\cos\dfrac{(2n+1)\pi}{8}$

        2(b) $\quad b_n = \dfrac{128h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{16}\sin\dfrac{(2n+1)\pi}{8}$

        3(a) $\quad a_n = \dfrac{256h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{32}\cos\dfrac{(2n+1)\pi}{16}$

        3(b) $\quad b_n = \dfrac{256h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{32}\sin\dfrac{(2n+1)\pi}{16}$

        4(a) $\quad a_n = \dfrac{256h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{16}\sin\dfrac{(2n+1)\pi}{8}\sin\dfrac{(2n+1)\pi}{4}$

        4(b) $\quad b_n = \dfrac{256h}{(2n+1)^2\pi^2}\sin^2\dfrac{(2n+1)\pi}{16}\sin\dfrac{(2n+1)\pi}{8}\cos\dfrac{(2n+1)\pi}{4}$

4.12   With $b_n = \dfrac{8}{n^3\pi^3}$, odd n, the six solutions on $(0, 1)$ are:
      1. Temperature in semi-infinite plate: $T = \sum b_n e^{-n\pi y}\sin n\pi x$
      2. Temperature in finite plate of height H:
$$T = \sum \dfrac{b_n}{\sinh(n\pi H)}\sinh n\pi(H - y)\sin n\pi x$$
      3. 1-dimensional heat flow: $u = \sum b_n e^{-(n\pi\alpha)^2 t}\sin n\pi x$
      4. Particle in a box: $\Psi = \sum b_n \sin n\pi x\, e^{-iE_n t/\hbar}$, $\quad E_n = \dfrac{\hbar^2 n^2\pi^2}{2m}$
      5. Plucked string: $y = \sum b_n \sin n\pi x \cos n\pi vt$
      6. String with initial velocity: $y = \sum \dfrac{b_n}{n\pi v}\sin n\pi x \sin n\pi vt$

4.13 With $b_n = \dfrac{16}{n\pi(4 - n^2)}$, n odd, the six solutions on $(0, \pi)$ are

     1. $T = \sum b_n e^{-ny} \sin nx$

     2. $T = \sum \dfrac{b_n}{\sinh nH} \sinh n(H - y) \sin nx$

     3. $u = \sum b_n e^{(-n\alpha)^2 t} \sin nx$

     4. $\Psi = \sum b_n \sin nx\, e^{-iE_n t/\hbar}, \quad E_n = \dfrac{\hbar^2 n^2}{2m}$

     5. $y = \sum b_n \sin nx \cos nvt$

     6. $y = \sum \dfrac{b_n}{nv} \sin nx \sin nvt$

4.14 Same as 4.12 with $b_n = \dfrac{12(-1)^{n+1}}{n^3 \pi^3}$, all n, on $(0, 1)$

5.1 (a) $u \cong 9.76°$ (b) $u \cong 9.76°$

5.2 (a) $\displaystyle\sum_{m=1}^{\infty} \dfrac{2}{k_m J_2(k_m)} J_1(k_m r)\, e^{-k_m z} \sin\theta, \; k_m = $ zeros of J_1

     (b) $\displaystyle\sum_{m=1}^{\infty} \dfrac{2a}{k_m J_2(k_m)} J_1(k_m r/a)\, e^{-k_m z/a} \sin\theta, \; k_m = $ zeros of J_1

     $u(r = 1, z = 1, \theta = \pi/2) \cong 0.211$

5.3 (a) $u = \displaystyle\sum_{m=1}^{\infty} \dfrac{200}{k_m J_1(k_m)\sinh(10k_m)} J_0(k_m r) \sinh k_m(10 - z), \; k_m = $ zeros of J_0

     (b) $u = \displaystyle\sum_{m=1}^{\infty} \dfrac{200}{k_m J_1(k_m)\sinh(k_m H/a)} J_0(k_m r/a) \sinh \dfrac{k_m(H - z)}{a},$

     $k_m = $ zeros of J_0

5.4 $u = \displaystyle\sum_{m=1}^{\infty} \dfrac{200}{k_m J_1(k_m)} J_0(k_m r/a) e^{-(k_m \alpha/a)^2 t}, \; k_m = $ zeros of J_0

5.5 $\displaystyle\sum_{m=1}^{\infty} \dfrac{200a}{k_m J_2(k_m)} J_1(k_m r/a) e^{-(k_m \alpha/a)^2 t} \sin\theta, \; k_m = $ zeros of J_1

5.6 $a_{mn} = \dfrac{2}{\pi a^2 J_{n+1}^2(k_{mn})} \displaystyle\int_0^a \int_0^{2\pi} f(r, \theta) J_n(k_{mn} r/a) \cos n\theta\, r\, dr\, d\theta$

     $b_{mn} = \dfrac{2}{\pi a^2 J_{n+1}^2(k_{mn})} \displaystyle\int_0^a \int_0^{2\pi} f(r, \theta) J_n(k_{mn} r/a) \sin n\theta\, r\, dr\, d\theta$

5.7 $u = \dfrac{400}{\pi} + \displaystyle\sum_{\text{odd } n} \dfrac{1}{n I_0(3n\pi/20)} I_0\left(\dfrac{n\pi r}{20}\right) \sin \dfrac{n\pi z}{20}$

5.8 $u = 40 + \displaystyle\sum_{m=1}^{\infty} \dfrac{120}{k_m J_1(k_m)} J_0(k_m r) e^{-k_m^2 \alpha^2 t}, \;$ where $J_0(k_m) = 0$

5.9 $u = \dfrac{1600}{\pi^2} \displaystyle\sum_{\text{odd } m} \sum_{\text{odd } n} \dfrac{\sin(n\pi x/10)\sin(m\pi y/10)\sinh[\pi(n^2 + m^2)^{1/2}(10 - z)/10]}{mn \sinh[\pi(n^2 + m^2)^{1/2}]}$

5.10 $u = \dfrac{6400}{\pi^3} \displaystyle\sum_{\text{odd } n} \sum_{\text{odd } m} \sum_{\text{odd } p} \dfrac{1}{nmp} \sin \dfrac{n\pi x}{l} \sin \dfrac{m\pi y}{l} \sin \dfrac{p\pi z}{l} e^{-(\alpha\pi/l)^2(n^2 + m^2 + p^2)t}$

5.11 $R = r^n, \, r^{-n}, \, n \neq 0; \; R = \ln r,$ const., $n = 0$

     $R = r^l, \, r^{-l-1}$

5.12 $u = 50 + \dfrac{200}{\pi} \displaystyle\sum_{\text{odd } n} \left(\dfrac{r}{a}\right)^n \dfrac{\sin n\theta}{n}$

5.13 $u = \dfrac{400}{\pi} \displaystyle\sum_{\text{odd}n} \dfrac{1}{n} \left(\dfrac{r}{10}\right)^{4n} \sin 4n\theta$

5.14 $\quad u = \dfrac{50 \ln r}{\ln 2} + \dfrac{200}{\pi} \displaystyle\sum_{\text{odd } n} \dfrac{r^n - r^{-n}}{n(2^n - 2^{-n})} \sin n\theta$

5.15 $\quad u = 50\left(1 - \dfrac{\ln r}{\ln 2}\right) - \dfrac{200}{\pi} \displaystyle\sum_{\text{odd } n} \dfrac{1}{n\left(2^n - 2^{-n}\right)}\left[\left(\dfrac{r}{2}\right)^n - \left(\dfrac{r}{2}\right)^{-n}\right]\sin n\theta$

6.2 $\quad$ The first six frequencies are ν_{10}, $\nu_{11} = 1.593\nu_{10}$, $\nu_{12} = 2.135\nu_{10}$
$\quad\quad \nu_{20} = 2.295\nu_{10}$, $\nu_{13} = 2.652\nu_{10}$, $\nu_{21} = 2.917\nu_{10}$.

6.4 $\quad \nu_{nm} = \dfrac{v}{2}\sqrt{\left(\dfrac{l}{a}\right)^2 + \left(\dfrac{m}{b}\right)^2 + \left(\dfrac{n}{c}\right)^2}$

6.5 $\quad z = \dfrac{64l^4}{\pi^6} \displaystyle\sum_{\text{odd } m}\sum_{\text{odd } n} \dfrac{1}{n^3 m^3} \sin\dfrac{n\pi x}{l}\sin\dfrac{m\pi y}{l}\cos\dfrac{\pi v(m^2 + n^2)^{1/2}t}{l}$

6.6 $\quad \Psi_n = \sin\dfrac{n_x\pi x}{l}\sin\dfrac{n_y\pi y}{l}e^{-iE_n t/\hbar}, \quad E_n = \dfrac{\pi^2\hbar^2(n_x{}^2 + n_y{}^2)}{2ml^2}$

6.7 $\quad$ See Problem 6.3. Some other examples of degeneracy:

$\quad\quad (n_x, n_y) = (1,8),\ (8,1),\ (4,7),\ (7,4)$, giving $E_n = 65\dfrac{\pi^2\hbar^2}{2ml^2}$;

$\quad\quad$ similarly $2^2 + 9^2 = 6^2 + 7^2 = 85$; $2^2 + 11^2 = 5^2 + 10^2 = 125$, etc.

6.8 $\quad \Psi_{mn} = J_n(k_{mn}r)\left\{\begin{array}{c}\sin n\theta \\ \cos n\theta\end{array}\right\}e^{-iE_{mn}t/\hbar}, \quad E_{mn} = \dfrac{\hbar^2 k_{mn}^2}{2ma^2}$

7.1 $\quad u = 7P_0(\cos\theta) + 20r^2 P_2(\cos\theta) + 8r^4 P_4(\cos\theta)$

7.2 $\quad u = \dfrac{2}{5}rP_1(\cos\theta) - \dfrac{2}{5}r^3 P_3(\cos\theta)$

7.3 $\quad u = -2P_0(\cos\theta) + rP_1(\cos\theta) + 2r^2 P_2(\cos\theta)$

7.4 $\quad u = -2P_0(\cos\theta) + 3rP_1(\cos\theta) + 2r^2 P_2(\cos\theta) + 2r^3 P_3(\cos\theta)$

7.5 $\quad u = \dfrac{1}{2}P_0(\cos\theta) + \dfrac{5}{8}r^2 P_2(\cos\theta) - \dfrac{3}{16}r^4 P_4(\cos\theta)\cdots$

7.6 $\quad u = \dfrac{\pi}{8}[3rP_1(\cos\theta) + \dfrac{7}{16}r^3 P_3(\cos\theta) + \dfrac{11}{64}r^5 P_5(\cos\theta)\cdots]$

7.7 $\quad u = \dfrac{1}{4}P_0(\cos\theta) + \dfrac{1}{2}rP_1(\cos\theta) + \dfrac{5}{16}r^2 P_2(\cos\theta) - \dfrac{3}{32}r^4 P_4(\cos\theta)\cdots$

7.8 $\quad u = 25[P_0(\cos\theta) + \dfrac{9}{4}rP_1(\cos\theta) + \dfrac{15}{8}r^2 P_2(\cos\theta) + \dfrac{21}{64}r^3 P_3(\cos\theta)\cdots]$

7.9 $\quad u = r^2 P_2^1(\cos\theta)\sin\phi$

7.10 $\quad u = \dfrac{1}{15}r^3 P_3^2(\cos\theta)\cos 2\phi - rP_1(\cos\theta)$

7.11 $\quad u = 200[(3/4)]rP_1(\cos\theta) - (7/16)r^3 P_3(\cos\theta) + (11/32)r^5 P_5(\cos\theta) + \cdots]$

7.12 $\quad u = \dfrac{3}{4}rP_1(\cos\theta) + \dfrac{7}{24}r^3 P_3(\cos\theta) - \dfrac{11}{192}r^5 P_5(\cos\theta)\cdots$

7.13 $\quad u = E_0(r - a^3/r^2)P_1(\cos\theta)$

7.14 $\quad u = 100[(1 - r^{-1})P_0(\cos\theta)$

$\quad\quad + \dfrac{3}{7}(r - r^{-2})P_1(\cos\theta) - \dfrac{7}{127}(r^3 - r^{-4})P_3(\cos\theta)\cdots]$

7.15 $\quad u = 100 + \dfrac{200a}{\pi r}\displaystyle\sum_1^\infty \dfrac{(-1)^n}{n}\sin\dfrac{n\pi r}{a}\,e^{-(\alpha n\pi/a)^2 t}$

$\quad\quad = 100 + 200\displaystyle\sum_{n=1}^\infty (-1)^n j_0(n\pi r/a)e^{-(\alpha n\pi/a)^2 t}$

7.17 $\quad \Psi_n = \sin\dfrac{n_x\pi x}{l}\sin\dfrac{n_y\pi y}{l}\sin\dfrac{n_z\pi z}{l}e^{-iE_n t/\hbar}, E_n = \dfrac{\pi^2\hbar^2(n_x{}^2 + n_y{}^2 + n_z{}^2)}{2ml^2}$

7.19 $\quad \Psi(r,\theta,\phi) = j_l(\beta r)P_l^m(\cos\theta)e^{\pm im\phi}e^{-iEt/\hbar}$,

$\quad\quad$ where $\beta = \sqrt{2ME/\hbar^2}$, $\beta a = $ zeros of j_l, $E = \dfrac{\hbar^2}{2Ma^2}$ (zeros of $j_l)^2$.

7.20 $\quad \psi_n(x) = e^{-\alpha^2 x^2/2}H_n(\alpha x),\ \alpha = \sqrt{m\omega/\hbar}$

7.21 $\quad \psi_n(x) = e^{-\alpha^2(x^2+y^2+z^2)/2} H_{n_x}(\alpha x) H_{n_y}(\alpha y) H_{n_z}(\alpha z), \quad \alpha = \sqrt{m\omega/\hbar},$
$\quad E_n = (n_x + \frac{1}{2} + n_y + \frac{1}{2} + n_z + \frac{1}{2})\hbar\omega = (n + \frac{3}{2})\hbar\omega.$

$\quad$ Degree of degeneracy of E_n is $C(n+2, n) = \dfrac{(n+2)(n+1)}{2}, \quad n = 0$ to $\infty.$

7.22 $\quad \Psi(r, \theta, \phi) = R(r) Y_l^m(\theta, \phi), \; R(r) = r^l e^{-r/(na)} L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right), \; E_n = -\dfrac{Me^4}{2\hbar^2 n^2}$

8.3 $\quad$ The second terms in (8.20) and (8.21) are replaced by
$$-q\sum_l \frac{a^l r^l}{R^{2l+1}} P_l(\cos\theta) = \frac{-qR/a}{\sqrt{r^2 - 2(rR^2/a)\cos\theta + (R^2/a)^2}}$$
$\quad$ Image charge $-qR/a$ at $(0, 0, R^2/a)$

8.4 $\quad$ Let K = line charge per unit length. Then
$$V = -K\ln(r^2 + a^2 - 2ra\cos\theta) + K\ln a^2 - K\ln R^2$$
$$+ K\ln\left[r^2 + \left(\frac{R^2}{a}\right)^2 - 2\frac{R^2}{a}r\cos\theta\right]$$

8.5 $\quad K$ at $(a, 0), \; -K$ at $(R^2/a, 0)$

9.2 $\quad u = \dfrac{200}{\pi}\displaystyle\int_0^\infty k^{-2}(1 - \cos 2k)e^{-ky}\cos kx \; dk$

9.4 $\quad u(x,t) = \dfrac{200}{\pi}\displaystyle\int_0^\infty \frac{1 - \cos k}{k} e^{-k^2\alpha^2 t}\sin kx \; dk$

9.7 $\quad u(x,t) = 100\,\text{erf}\left(\dfrac{x}{2\alpha\sqrt{t}}\right) - 50\,\text{erf}\left(\dfrac{x-1}{2\alpha\sqrt{t}}\right) - 50\,\text{erf}\left(\dfrac{x+1}{2\alpha\sqrt{t}}\right)$

10.1 $\quad T = \dfrac{2}{\pi}\displaystyle\sum_1^\infty \frac{1}{n\sinh 2n\pi}\sinh n\pi(2 - y)\sin n\pi x$

10.2 $\quad T = \dfrac{2}{\pi}\displaystyle\sum_1^\infty \frac{1}{n\sinh 2n\pi}\sinh n\pi y\sin n\pi x$

10.3 $\quad T = \dfrac{1}{4}(2 - y) + \dfrac{4}{\pi^2}\displaystyle\sum_{\text{odd } n} \frac{1}{n^2\sinh 2n\pi}\sinh n\pi(2 - y)\cos n\pi x$

10.4 $\quad T = 20 + \dfrac{40}{\pi}\displaystyle\sum_{\text{odd } n}\frac{1}{n\sinh\frac{3n\pi}{5}}\sinh\frac{n\pi y}{5}\sin\frac{n\pi x}{5}$
$$+ \dfrac{40}{\pi}\displaystyle\sum_{\text{odd } n}\frac{1}{n\sinh\frac{5n\pi}{3}}\sinh\frac{n\pi(5 - x)}{3}\sin\frac{n\pi y}{3}$$

10.5 $\quad u = 20 - \dfrac{80}{\pi}\displaystyle\sum_{\text{odd } n}\frac{1}{n}e^{-(n\pi\alpha/l)^2 t}\sin\frac{n\pi x}{l}$

10.6 $\quad u = 20 - \dfrac{80}{\pi}\displaystyle\sum_{n=0}^\infty \frac{(-1)^n}{2n+1}e^{-[(2n+1)\pi\alpha/(2l)]^2 t}\cos\left(\frac{2n+1}{2l}\pi x\right)$

10.7 $\quad u = \dfrac{1}{4}y + \dfrac{4}{\pi^2}\displaystyle\sum_{\text{odd } n}\frac{1}{n^2\sinh 2n\pi}\sinh n\pi y\cos n\pi x$

10.8 $\quad u = 20 - x - \dfrac{40}{\pi}\displaystyle\sum_{\substack{\text{even } n \\ 2}}^\infty \frac{1}{n}e^{-(n\pi\alpha/10)^2 t}\sin\frac{n\pi x}{10}$

10.9 $\quad y = \dfrac{8l^2}{\pi^3}\displaystyle\sum_{\text{odd } n}\frac{1}{n^3}\cos\frac{n\pi vt}{l}\sin\frac{n\pi x}{l}$

10.10 $\quad u = \dfrac{1600}{\pi^2}\displaystyle\sum_{\text{odd } n}\sum_{\text{odd } m}\frac{1}{nmI_n(3m\pi/20)}I_n\left(\frac{m\pi r}{20}\right)\sin n\theta\sin\frac{m\pi z}{20}$

10.12 $u = \dfrac{400}{\pi} \sum_{\text{odd } n} \dfrac{1}{n} \left(\dfrac{r}{a}\right)^{2n} \sin 2n\theta$

10.14 Same as 9.12

10.15 $u = \dfrac{400}{\pi} \sum_{\text{odd } n} \dfrac{1}{n} \left(\dfrac{r}{10}\right)^{6n} \sin 6n\theta = \dfrac{200}{\pi} \arctan \dfrac{2(10r)^6 \sin 6\theta}{10^{12} - r^{12}}$

10.16 $v\sqrt{5}/(2\pi)$

10.17 $\nu_{mn}, \ n \neq 0;$ the lowest frequencies are:
$\nu_{11} = 1.59\nu_{10}, \ \nu_{12} = 2.14\nu_{10}, \ \nu_{13} = 2.65\nu_{10}, \ \nu_{21} = 2.92\nu_{10}, \ \nu_{14} = 3.16\nu_{10}$

10.18 $\nu_{mn}, \ n = 3, 6, \cdots;$ the lowest frequencies are:
$\nu_{13} = 2.65 \ \nu_{10}, \ \nu_{23} = 4.06 \ \nu_{10}, \ \nu_{16} = 4.13 \ \nu_{10}, \ \nu_{33} = 5.4 \ \nu_{10}$

10.19 $u = E_0 \left(r - \dfrac{a^2}{r}\right) \cos\theta$

10.20 $\nu = \dfrac{v\lambda_l}{2\pi a}$ where λ_l = zeros of j_l, a = radius of sphere, v = speed of sound

10.21 $u = \tfrac{2}{3}P_0(\cos\theta) + \tfrac{3}{5}rP_1(\cos\theta) - \tfrac{2}{3}r^2 P_2(\cos\theta) + \tfrac{2}{5}r^3 P_3(\cos\theta)$

10.22 $u = 1 - \tfrac{1}{2}rP_1(\cos\theta) + \tfrac{7}{8}r^3 P_3(\cos\theta) - \tfrac{11}{16}r^5 P_5(\cos\theta)\cdots$

10.23 $u = 100 \sum_{\text{odd } l} (a_l r^l + b_l r^{-l-1}) P_l(\cos\theta)$ where

$a_l = \dfrac{2A+1}{2A^2 - 1}c_l, \quad b_l = -\dfrac{2A(A+1)}{2A^2 - 1}c_l, \quad A = 2^l,$

$c_l = (2l+1)\int_0^1 P_l(x)\,dx$ (Chapter 12, Problem 9.1).

The first few terms are

$u = (107.1r - 257.1r^{-2})P_1(\cos\theta)$
$-(11.7r^3 - 99.2r^{-4})P_3(\cos\theta) + (2.2r^5 - 70.9r^{-6})P_5(\cos\theta)\cdots$

10.24 $T = A + \sum_{\text{odd } n} \dfrac{4(D-A)}{n\pi \sinh(n\pi b/a)} \sinh \dfrac{n\pi}{a}(b-y) \sin \dfrac{n\pi x}{a}$

$+ \sum_{\text{odd } n} \dfrac{4(C-A)}{n\pi \sinh(n\pi a/b)} \sinh \dfrac{n\pi}{b}(a-x) \sin \dfrac{n\pi y}{b}$

$+ \sum_{\text{odd } n} \dfrac{4(B-A)}{n\pi \sinh(n\pi b/a)} \sinh \dfrac{n\pi y}{a} \sin \dfrac{n\pi x}{a}$

10.26 $\nu = \dfrac{v}{2\pi} \sqrt{(k_{mn}/a)^2 + \lambda^2}$ where k_{mn} is a zero of J_n

10.27 $\nu = \dfrac{v}{2} \sqrt{\left(\dfrac{n}{a}\right)^2 + \left(\dfrac{m}{b}\right)^2 + \left(\dfrac{\lambda}{\pi}\right)^2}$

10.28 $u(x,y) = \dfrac{200}{\pi} \int_0^\infty \dfrac{\sin k}{k \cosh k} \cos kx \cosh ky \, dk$

Chapter 14

1.1 $u = x^3 - 3xy^2$, $v = 3x^2y - y^3$ 1.2 $u = x$, $v = y$

1.3 $u = x$, $v = -y$ 1.4 $u = (x^2 + y^2)^{1/2}$, $v = 0$

1.5 $u = x$, $v = 0$ 1.6 $u = e^x \cos y$, $v = e^x \sin y$

1.7 $u = \cos y \cosh x$, $v = \sin y \sinh x$

1.8 $u = \sin x \cosh y$, $v = \cos x \sinh y$

1.9 $u = x/(x^2 + y^2)$, $v = -y/(x^2 + y^2)$

1.10 $u = (2x^2 + 2y^2 + 7x + 6)/[(x+2)^2 + y^2]$, $v = y/[(x+2)^2 + y^2]$

1.11 $u = 3x/[x^2 + (y-2)^2]$, $v = (-2x^2 - 2y^2 + 5y - 2)/[x^2 + (y-2)^2]$

1.12 $u = x(x^2 + y^2 + 1)/[(x^2 - y^2 + 1)^2 + 4x^2y^2]$,

        $v = y(1 - x^2 - y^2)/[(x^2 - y^2 + 1)^2 + 4x^2y^2]$

1.13 $u = \ln(x^2 + y^2)^{1/2}$, $v = 0$ 1.14 $u = x(x^2 + y^2)$, $v = y(x^2 + y^2)$

1.15 $u = e^x \cos y$, $v = -e^x \sin y$ 1.16 $u = 0$, $v = 4xy$

1.17 $u = \cos x \cosh y$, $v = \sin x \sinh y$

1.18 $u = \pm 2^{-1/2}[(x^2 + y^2)^{1/2} + x]^{1/2}$, $v = \pm 2^{-1/2}[(x^2 + y^2)^{1/2} - x]^{1/2}$,

        where the $\pm$ signs are chosen so that uv has the sign of y.

1.19 $u = \ln(x^2 + y^2)^{1/2}$, $v = \arctan(y/x)$ [angle is in the quadrant

        of the point (x, y)].

1.20 $u = x^2 - y^2 - 4xy - x - y + 3$, $v = 2x^2 - 2y^2 + 2xy + x - y$

1.21 $u = e^{-y} \cos x$, $v = e^{-y} \sin x$

In 2.1 to 2.24, A = analytic, N = not analytic

2.1	A	2.2	A	2.3	N	2.4	N
2.5	N	2.6	A	2.7	A	2.8	A
2.9	A, $z \neq 0$	2.10	A, $z \neq -2$	2.11	A, $z \neq 2i$	2.12	A, $z \neq \pm i$
2.13	N	2.14	N	2.15	N	2.16	N
2.17	N	2.18	A, $z \neq 0$	2.19	A, $z \neq 0$	2.20	A
2.21	A	2.22	N	2.23	A, $z \neq 0$	2.24	N

2.34 $-z - \frac{1}{2}z^2 - \frac{1}{3}z^3 \cdots$, $|z| < 1$

2.35 $1 - (z^2/2!) + (z^4/4!) \cdots$, all z

2.36 $1 + \frac{1}{2}z^2 - \frac{1}{8}z^4 \cdots$, $|z| < 1$

2.37 $z - \frac{1}{3}z^3 + \frac{2}{15}z^5 \cdots$, $|z| < \pi/2$

2.38 $-\frac{1}{2}i + \frac{1}{4}z + \frac{1}{8}iz^2 - \frac{1}{16}z^3 \cdots$, $|z| < 2$

2.39 $(z/9) - (z^3/9^2) + (z^5/9^3) \cdots$, $|z| < 3$

2.40 $1 + z + z^2 + z^3 \cdots$, $|z| < 1$

2.41 $1 + iz - z^2/2 - iz^3/3! + z^4/4! \cdots$, all z

2.42 $z + z^3/3! + z^5/5! \cdots$, all z

2.48	Yes, $z \neq 0$	2.49	No	2.50	Yes, $z \neq 0$	2.51	Yes
2.52	No	2.53	Yes, $z \neq 0$	2.54	$-iz$	2.55	$-iz^3$
2.56	$-iz^2/2$	2.57	$(1-i)z$	2.58	$\cos z$	2.59	e^z
2.60	$2\ln z$	2.61	$1/z$	2.62	$-ie^{iz}$	2.63	$-i/(1-z)$

3.1 $\frac{1}{2}+i$ 3.2 $-(2+i)/3$ 3.3 0 3.4 $i\pi/2$

3.5 -1 3.6 $-1,\,-1$ 3.7 $\pi(1-i)/8$ 3.8 $i/2$

3.9 1 3.10 $(2i-1)e^{2i}$ 3.11 $2\pi i$

3.12   (a) $\frac{5}{3}(1+2i)$ (b) $\frac{1}{3}(8i+13)$ 3.16 0

3.17   (a) 0 (b) $i\pi$ 3.18 $i\pi\sqrt{3}/6$ 3.19 $16i\pi$

3.20   (a) 0 (b) $-17\pi i/4$ 3.22 $-i\pi\sqrt{3}/108$

3.23 $72i\pi$ 3.24 $-17i\pi/96$

4.3   For $0<|z|<1$: $\frac{1}{2}z^{-1}+\frac{3}{4}+\frac{7}{8}z+\frac{15}{16}z^2\cdots$; $R(0)=\frac{1}{2}$

   For $1<|z|<2$: $-(\cdots z^{-4}+z^{-3}+z^{-2}+\frac{1}{2}z^{-1}+\frac{1}{4}+\frac{1}{8}z+\frac{1}{16}z^2+\frac{1}{32}z^3\cdots)$

   For $|z|>2$: $z^{-3}+3z^{-4}+7z^{-5}+15z^{-6}\cdots$

4.4   For $0<|z|<1$: $-\frac{1}{4}z^{-1}-\frac{1}{2}-\frac{11}{16}z-\frac{13}{16}z^2\cdots$; $R(0)=-\frac{1}{4}$

   For $1<|z|<2$: $\cdots+z^{-3}+z^{-2}+\frac{3}{4}z^{-1}+\frac{1}{2}+\frac{5}{16}z+\frac{3}{16}z^2\cdots$

   For $|z|>2$: $z^{-4}+5z^{-5}+17z^{-6}+49z^{-7}\cdots$

4.5   For $0<|z|<2$: $\frac{1}{2}z^{-3}-\frac{1}{4}z^{-2}-\frac{1}{8}z^{-1}-\frac{1}{16}-\frac{1}{32}z-\frac{1}{64}z^2$; $R(0)=-\frac{1}{8}$

   For $|z|>2$: $z^{-3}+z^{-4}+2z^{-5}+4z^{-6}+8z^{-7}\cdots$

4.6   For $0<|z|<1$: $z^{-2}-2z^{-1}+3-4z+5z^2\cdots$; $R(0)=-2$

   For $|z|>1$: $z^{-4}-2z^{-5}+3z^{-6}\cdots$

4.7   For $|z|<1$: $2-z+2z^2-z^3+2z^4-z^5\cdots$; $R(0)=0$

   For $|z|>1$: $z^{-1}-2z^{-2}+z^{-3}-2z^{-4}\cdots$

4.8   For $|z|<1$: $-5+\frac{25}{6}z-\frac{175}{36}z^2\cdots$; $R(0)=0$

   For $1<|z|<2$: $-5(\cdots+z^{-3}-z^{-2}+z^{-1}+\frac{1}{6}z+\frac{1}{36}z^2+\frac{7}{216}z^3\cdots)$

   For $2<|z|<3$: $\cdots+3z^{-3}+9z^{-2}-3z^{-1}+1-\frac{1}{3}z+\frac{1}{9}z^2-\frac{1}{27}z^3\cdots$

   For $|z|>3$: $30(z^{-3}-2z^{-4}+9z^{-5}\cdots)$

4.9   (a) regular   (b) pole of order 3

   (c) pole of order 2   (d) pole of order 1

4.10   (a) simple pole   (b) pole of order 2

   (c) pole of order 2   (d) essential singularity

4.11   (a) regular   (b) pole of order 2

   (c) simple pole   (d) pole of order 3

4.12   (a) pole of order 3   (b) pole of order 2

   (c) essential singularity   (d) pole of order 1

6.1 $z^{-1}-1+z-z^2\cdots$; $R=1$

6.2 $(z-1)^{-1}-1+(z-1)-(z-1)^2\cdots$; $R=1$

6.3 $z^{-3}-\frac{1}{6}z^{-1}+\frac{1}{120}z\cdots$; $R=-\frac{1}{6}$

6.4 $z^{-2}+(1/2!)+(z^2/4!)\cdots$; $R=0$

6.5 $\frac{1}{2}e[(z-1)^{-1}+\frac{1}{2}+\frac{1}{4}(z-1)\cdots]$; $R=\frac{1}{2}e$

6.6 $z^{-1}-(1/3!)z^{-3}+(1/5!)z^{-5}\cdots$; $R=1$

6.7 $\frac{1}{4}\left[\left(z-\frac{1}{2}\right)^{-1}-1+(1-\pi^2/2)\left(z-\frac{1}{2}\right)+\cdots\right]$, $R=\frac{1}{4}$

6.8 $1/2-(z-\pi)^2/4!+(z-\pi)^4/6!-\cdots$, $R=0$

6.9 $-\left[(z-2)^{-1}+1+(z-2)+(z-2)^2+\cdots\right]$; $R=-1$

6.14 $R(-2/3)=1/8$, $R(2)=-1/8$ 6.15 $R(1/2)=1/3$, $R(4/5)=-1/3$

6.16 $R(0)=-2$, $R(1)=1$ 6.17 $R(1/2)=5/8$, $R(-1/2)=-3/8$

6.18 $R(3i)=\frac{1}{2}-\frac{1}{3}i$ 6.19 $R(\pi/2)=1/2$

6.20 $R(i)=1/4$ 6.21 $R[\sqrt{2}\,(1+i)]=\sqrt{2}\,(1-i)/16$

6.22 $R(i\pi)=-1$ 6.23 $R(2i/3)=-ie^{-2/3}/12$

6.24 $R(0)=2$ 6.25 $R(0)=2$

6.26 $R(e^{2\pi i/3})=\frac{1}{6}(i\sqrt{3}-1)e^{-\pi\sqrt{3}}$ 6.27 $R(\pi/6)=-1/2$

6.28 $R(3i) = -\frac{1}{16} + \frac{1}{24}i$ 6.29 $R(\ln 2) = 4/3$

6.30 $R(0) = 1/6!$ 6.31 $R(0) = 9/2$

6.32 $R(2i) = -3ie^{-2}/32$ 6.33 $R(\pi) = -1/2$

6.34 $R(0) = -7,\ R(1/2) = 7$ 6.35 $R(i) = 0$

6.14′ $\pi i/4$ 6.15′ 0 6.16′ $-2\pi i$ 6.17′ $\pi i/2$

6.18′ 0 6.19′ 0 6.20′ 0 6.21′ 0

6.22′ 0 6.23′ $-\frac{\pi}{3}\sinh\frac{2}{3}$ 6.24′ $4\pi i$ 6.25′ $4\pi i$

6.26′ $-\frac{2}{3}\pi i(1 + \cosh\pi\sqrt{3} + i\sqrt{3}\sinh\pi\sqrt{3}\,)$

6.27′ $-\pi i$ 6.28′ $\frac{1}{4}\pi i$ 6.29′ $5\pi i/2$ 6.30′ $\pi i/360$

6.31′ $9\pi i$ 6.32′ 0 6.33′ 0 6.34′ 0

6.35′ 0 6.36 $-\frac{1}{4}$ 6.37 $R(-n) = (-1)^n/n!$

7.1 $\pi/6$ 7.2 $\pi/2$ 7.3 $2\pi/3$ 7.4 $2\pi/9$

7.5 $\pi/(1-r^2)$ 7.6 $2\pi/3^{3/2}$ 7.7 $\pi/6$ 7.8 $\pi/18$

7.9 $2\pi/|\sin\alpha|$ 7.10 π 7.11 $3\pi/32$ 7.12 $\pi\sqrt{2}/8$

7.13 $\pi/10$ 7.14 $-(\pi/e)\sin 2$ 7.15 $\pi e^{-4/3}/12$ 7.16 $\pi e^{-2/3}/18$

7.17 $(\pi/e)(\cos 2 + 2\sin 2)$ 7.18 $\frac{1}{2}\pi e^{-\pi\sqrt{3}/2}$ 7.19 $\pi e^{-3}/54$

7.20 $\pi e^{-1/3}/9$ 7.22 $-\pi/2$ 7.23 $\pi/8$ 7.24 π

7.25 $\pi/36$ 7.26 $-\pi/2$ 7.27 $\pi/4$ 7.28 $\pi/4$

7.29 $\pi/2$ for $a > 0$, 0 for $a = 0$, $-\pi/2$ for $a < 0$

7.30 $\pi/(2\sqrt{2}\,)$ 7.31 $\pi/3$ 7.32 $\frac{3}{16}\pi\sqrt{2}$ 7.33 $\pi\sqrt{2}/2$

7.34 $\pi/2$ 7.35 $2\pi(2^{1/3}-1)/\sqrt{3}$ 7.36 $-\pi^2\sqrt{2}$

7.38 $\pi\cot p\pi$ 7.39 2 7.40 $\pi^2/4$ 7.41 $(2\pi)^{1/2}/4$

7.45   One negative real, one each in quadrants I and IV

7.46   One negative real, one each in quadrants II and III

7.47   One negative real, one each in quadrants I and IV

7.48   Two each in quadrants I and IV

7.49   Two each in quadrants I and IV

7.50   Two each in quadrants II and III

7.51 $4\pi i,\ 8\pi i$ 7.52 πi

7.53 πi 7.54 $8\pi i$

7.55 $\cosh t\cos t$ 7.56 $(\sinh t - \sin t)/2$

7.57 $1 + \sin t - \cos t$ 7.58 $(\cos 2t + \cosh 2t)/2$

7.59 $2e^t\cos t\sqrt{3} + e^{-2t}$ 7.60 $t + e^{-t} - 1$

7.61 $\frac{1}{3}(\cosh 2t + 2\cosh t\cos t\sqrt{3})$ 7.62 $1 - 4te^{-t}$

7.63 $(\cosh t - \cos t)/2$ 7.64 $\frac{2}{3}\sinh 2t - \frac{1}{3}\sinh t$

7.65 $(\cos 2t + 2\sin 2t - e^{-t})/5$

8.3   Regular, $R = -1$ 8.4   Regular, $R = -2$

8.5   Regular, $R = -1$ 8.6   Simple pole, $R = -5$

8.7   Simple pole, $R = -2$ 8.8   Regular, $R = 0$

8.9   Regular, $R = 0$ 8.10   Regular, $R = 2$

8.11   Regular, $R = -1$ 8.12   Regular, $R = -2$

8.14 $-2\pi i$ 8.15 πi

9.1 $x^2 = \frac{1}{2}[u + (u^2 + v^2)^{1/2}],\ y^2 = \frac{1}{2}[-u + (u^2 + v^2)^{1/2}]$

9.2 $u = y/2,\ v = -(x+1)/2$

9.3 $u = x/(x^2 + y^2),\ v = -y/(x^2 + y^2)$

9.4 $u = e^x\cos y,\ v = e^x\sin y$

9.5 $u = (x^2 + y^2 - 1)/[x^2 + (y+1)^2],\ v = -2x/[x^2 + (y+1)^2]$

9.7 $u = \sin x\cosh y,\ v = \cos x\sinh y$

9.8 $u = \cosh x\cos y,\ v = \sinh x\sin y$

10.4 $T = 200\pi^{-1} \arctan(y/x)$ 10.5 $V = 200\pi^{-1} \arctan(y/x)$

10.6 $T = 100y/(x^2 + y^2)$; isothermals $y/(x^2 + y^2) = $ const.;
      flow lines $x/(x^2 + y^2) = $ const.

10.7   Streamlines $xy = $ const.; $\Phi = (x^2 - y^2)V_0$, $\Psi = 2xy\mathbf{V}_0$, $\mathbf{V} = (2\mathbf{i}x - 2\mathbf{j}y)V_0$

10.9   Streamlines $y - y/(x^2 + y^2) = $ const.

10.10 $\cos x \sinh y = $ const.

10.11 $(x - \coth u)^2 + y^2 = \operatorname{csch}^2 u$
      $x^2 + (y + \cot v)^2 = \csc^2 v$

10.12 $T = (20/\pi) \arctan[2y/(1 - x^2 - y^2)]$, arc tan between $\pi/2$ and $3\pi/2$

10.13 $V = \dfrac{V_2 - V_1}{\pi} \arctan \dfrac{2y}{1 - x^2 - y^2} + \dfrac{3V_1 - V_2}{2}$, arc tan between $\pi/2$ and $3\pi/2$

10.14 $\phi = \dfrac{1}{2}V_0 \ln \dfrac{(x + 1)^2 + y^2}{(x - 1)^2 + y^2}$

      $\psi = V_0 \arctan \dfrac{2y}{1 - x^2 - y^2}$,   arc tan between $\pi/2$ and $3\pi/2$.

      $V_x = \dfrac{2V_0(1 - x^2 + y^2)}{(1 - x^2 + y^2)^2 + 4x^2y^2}$, $V_y = \dfrac{-4V_0xy}{(1 - x^2 + y^2)^2 + 4x^2y^2}$

11.1 $\ln(1 + z)$ 11.2 $-i\ln(1 + z)$

11.5 $R(i) = (1 - i\sqrt{3})/4$ 11.6 $R(-1/2) = i/(6\sqrt{2})$
      $R(-i) = -1/2$ $R(e^{i\pi/3}/2) = R(e^{5\pi i/3}/2) = -i/(6\sqrt{2})$

11.7 $R(i) = \pi/4$, $R(-i) = R(e^{3\pi i/2}) = -3\pi/4$

11.8 $R(1/2) = 1/2$ 11.9 $-1/6$

11.10 -1 11.12 $1/2$

11.13 (a) $1/96$ (b) -5 (c) $-1/80$ (d) $1/2$

11.14 (a) 2 (b) $-\sin 5$ (c) $1/16$ (d) -2π

11.15 $\pi/6$ 11.16 $-\pi/6$

11.17 $\pi(e^{-1/2} - \frac{1}{6}e^{-3})/35$ 11.18 $\pi e^{-\pi/2}/4$

11.19 $3(2^{-1} - e^{-\pi})/(10\pi)$ 11.20 $\pi(e^{-1} + \sin 1)/2$

11.28 π 11.29 $\pi^3/8$ (Caution: $-\pi^3/8$ is wrong.)

11.31 One in each quadrant

11.32 One negative real, one each in quadrants II and III

11.33 One each in quadrants I and IV, two each in II and III

11.34 Two each in quadrants I and IV, one each in II and III

11.40 $\dfrac{2a^2p}{p^4 + 4a^4}$ 11.41 $\pi^2/8$

Chapter 15

1.1	1/10, 1/9		1.2	3/8, 1/8, 1/4
1.3	1/3, 5/9		1.4	1/2, 1/52, 2/13, 7/13
1.5	1/4, 3/4, 1/3, 1/2		1.6	27/52, 16/52, 15/52
1.7	9/26, 1/2, 1/13		1.8	9/100, 1/10, 3/100, 1/10
1.9	3/10, 1/3		1.10	3/8

2.12   (a) 3/4   (b) 1/5   (c) 2/3   (d) 3/4   (e) 3/7

2.14   (a) 3/4   (b) 25/36   (c) 37, 38, 39, 40

2.15   (a) 1/6   (b) 1/2   (c) 1/3   (d) 1/3   (e) 1/9

2.17   (a) 3 to 9 with $p(5) = p(7) = 2/9$; others, $p = 1/9$
       (b) 5 and 7   (c) 1/3

2.18   (a) 1/2, 1/2   (b) 1/2, 1/4, 1/4   (c) Not a sample space

2.19   1/3, 1/3; 1/7, 1/7

3.3 2^{-6}, 2^{-3}, 2^{-3}

3.4   (a) 8/9, 1/2   (b) 3/5, 1/11, 2/3, 2/3, 6/13

3.5   1/33, 2/9

3.6   4/13, 1/52

3.10   (a) 1/6   (b) 2/3   (c) $P(A) = P(B) = 1/3$, $P(A+B) = 1/2$, $P(AB) = 1/6$

3.11   1/8

3.12   (a) 1/49   (b) 68/441   (c) 25/169   (d) 15 times   (e) 44/147

3.13   (a) 1/4   (b) 25/144, 1/16, 1/16

3.14 $n > 3.3$, so 4 tries are needed.

3.15   (a) 1/3   (b) 1/7

3.16   9/23

3.17   (a) 39/80, 5/16, 1/5, 11/16   (b) 374/819   (c) 185/374

3.18   (a) 15/34   (b) 2/15

3.19   1/3

3.20   5/7, 2/7, 11/14

3.21   2/3, 1/3

3.22   6/11, 5/11

4.1   (a) $P(10, 8)$ (b) $C(10, 8)$ (c) 1/45

4.3   3, 7, 31, $2^n - 1$

4.4 1.98×10^{-3}, 4.95×10^{-4}, 3.05×10^{-4}, 1.39×10^{-5}

4.5 2^8, 2^{-8}, 7/32   4.6   15

4.7   1/26   4.8   1/221, 1/33, 1/17

4.9 $25/102,\ 25/77,\ 49/101,\ 12/25$ 4.11 $0.097,\ 0.37,\ 0.67;\ 13$

4.12   5   4.14 $n!/n^n$

4.17   MB: 16,   FD: 6,   BE: 10   4.18   MB: 125,   FD: 10,   BE: 35

4.21 $C(n+2,n)$ 4.22   0.135   4.23   0.30

5.1 $\mu = 0,\ \sigma = \sqrt{3}$ 5.2 $\mu = 7,\ \sigma = \sqrt{35/6}$

5.3 $\mu = 2,\ \sigma = \sqrt{2}$ 5.4 $\mu = 1,\ \sigma = \sqrt{21}/2$

5.5 $\mu = 1,\ \sigma = \sqrt{7/6}$ 5.6 $\mu = 3,\ \sigma = \sqrt{284/13} = 4.67$

5.7 $\mu = 3(2p-1),\ \sigma = 2\sqrt{3p(1-p)}$ 5.8 $E(x) = \$12.25$

5.12 $E(x) = 7$ 5.15 $\bar{x} = 3(2p-1)$

5.17   Problem 5.2: $E(x^2) = 329/6,\ \ \sigma^2 = 35/6$

Problem 5.6: $E(x^2) = 401/13,\ \ \sigma^2 = 284/13$

Problem 5.7: $E(x^2) = 24p^2 - 24p + 9,\ \ \sigma^2 = 12p(1-p)$

6.1   (a) $f(x) = \pi^{-1}(a^2 - x^2)^{-1/2}$ (c) $\bar{x} = 0,\ \sigma = a/\sqrt{2}$

6.2 $e^{-2} = 0.135$

6.3 $f(h) = 1/(2\sqrt{l}\sqrt{l-h}\,)$

6.4 $f(x) = \alpha e^{-\alpha^2 x^2}/\sqrt{\pi},\ \ \bar{x} = 0,\ \ \sigma = 1/(\alpha\sqrt{2}\,)$

6.5 $f(t) = \lambda e^{-\lambda t},\ \ F(t) = 1 - e^{-\lambda t},\ \ \bar{t} = 1/\lambda,\ \ \text{half life} = \bar{t}\ln 2$

6.6 $F(r) = r^2,\ \ f(r) = 2r,\ \ \bar{r} = 2/3,\ \ \sigma = \sqrt{2}/6$

6.7   (a) $F(s) = 2[1 - \cos(s/R)],\ \ f(s) = (2/R)\sin(s/R)$

(b) $F(s) = [1 - \cos(s/R)]/[1 - \cos(1/R)] \cong s^2,$

$f(s) = R^{-1}[1 - \cos(1/R)]^{-1}\sin(s/R) \cong 2s$

6.8 $f(r) = 3r^2;\ \bar{r} = 3/4,\ \sigma = \sqrt{3/80} = 0.19$

6.9 $f(r) = 4a^{-3}r^2 e^{-2r/a}$

	n	Exactly 7 h	At most 7 h	At least 7 h	Most probable number of h	Expected number of h
7.1	7	0.0078	1	0.0078	3 or 4	7/2
7.2	12	0.193	0.806	0.387	6	6
7.3	15	0.196	0.500	0.696	7 or 8	15/2
7.4	18	0.121	0.240	0.881	9	9

7.5   0.263

8.3 $\mu = 0,\ \ \sigma^2 = kT/m,\ \ f(v) = \dfrac{1}{\sqrt{2\pi kT/m}}e^{-mv^2/(2kT)}$

In (8.11) to (8.20), the first number is the binomial result and the second number is the normal approximation using whole steps at the ends as in Example 2.

8.11   0.0796, 0.0798   8.12   0.03987, 0.03989

8.13   0.9598, 0.9596   8.14   0.9546, 0.9546

8.15   0.03520, 0.03521   8.16   0.4176, 0.4177

8.17   0.0770, 0.0782   8.18   0.372, 0.376

8.19   0.0946, 0.0967   8.20   0.462, 0.455

8.25   C: 38.3%, B and D: 24.2%, A and F: 6.7%

In $\mu + \frac{1}{2}\sigma$ and $\mu + \frac{3}{2}\sigma$, change $\frac{1}{2}$ to 0.5244, and $\frac{3}{2}$ to 1.2816.

9.3   Number of particles: 0   1   2   3   4   5
       Number of intervals: 406   812   812   541   271   108

9.4 $P_0 = 0.018$, $P_1 = 0.073$, $P_4 = 0.195$

9.5 $P_0 = 0.37$, $P_1 = 0.37$, $P_2 = 0.18$, $P_3 = 0.06$

9.6   Exactly 5: 64 days. Fewer than 5: 161 days. Exactly 10: 7 days. More than 10: 5 days. Just 1: 12 days. None at all: 2 or 3 days

9.7   0.238   9.8   3, 10, 3

9.9 $P_2 = 0.022$, $P_6 = P_7 = 0.149$, $P_{n>10} = 0.099$

9.11   Normal: 0.08, Poisson: 0.0729, (binomial: 0.0732)

10.8 $\bar{x} = 5$, $\bar{y} = 1$, $s_x = 0.122$, $s_y = 0.029$,
       $\sigma_x = 0.131$, $\sigma_y = 0.030$, $\sigma_{mx} = 0.046$, $\sigma_{my} = 0.0095$,
       $r_x = 0.031$, $r_y = 0.0064$,
       $\overline{x+y} = 6$ with $r = 0.03$, $\overline{xy} = 5$ with $r = 0.04$,
       $\overline{x^3 \sin y} = 105$ with $r = 2.00$, $\overline{\ln x} = 1.61$ with $r = 0.006$

10.9 $\bar{x} = 100$ with $r = 0.47$, $\bar{y} = 20$ with $r = 0.23$,
       $\overline{x-y} = 80$ with $r = 0.5$, $\overline{x/y} = 5$ with $r = 0.06$,
       $\overline{x^2 y^3} = 8 \cdot 10^7$ with $r = 2.9 \cdot 10^6$, $\overline{y \ln x} = 92$ with $r = 1$

10.10 $\bar{x} = 6$ with $r = 0.062$, $\bar{y} = 3$ with $r = 0.067$,
       $\overline{2x-y} = 9$ with $r = 0.14$, $\overline{y^2 - x} = 3$ with $r = 0.4$,
       $\overline{e^y} = 20$ with $r = 1.3$, $\overline{x/y^2} = 0.67$ with $r = 0.03$

11.1   (a) 11/30   (b) 19.5 cents   (c) 6/11   (d)7/11

11.2   (b) $E(x) = 5$, $\sigma = \sqrt{3}$ (c) 0.0767   (d) 0.0807   (e) 0.0724

11.3   20/47

11.4   5/8

11.6   MB: 25   FD: 10   BE: 15

11.7 $\bar{x} = 1/4$, $\sigma = \sqrt{3}/4$

11.8   (b) $\bar{x} = 4/3$, $\sigma = 2/3$ (c) 1/5

11.9   (a) x: 0   1   2
       $p : 55/72 = 0.764$ $16/72 = 0.222$ $1/72 = 0.139$
       (b) $17/72 = 0.236$
       (c) $6/17 = 0.353$
       (d) $\bar{x} = 1/4$, $\sigma = \sqrt{31}/12 = 0.463$

11.10 (a) 0.7979, 0.7979   (b) 0.9123, 0.9123

11.11 (a) 0.0347, 0.0352   (b) 0.559, 0.562

11.12 (a) 0.00534, 0.00540   (b) 0.503, 0.500

11.13 30, 60   11.14 1

11.15 binomial: 0.2241,   normal: 0.195,   Poisson: 0.2240

11.16 (a)   binomial: 0.0439,   normal: 0.0457,   Poisson: 0.0446
       (b)   binomial: 0.0946,   normal: 0.0967,   Poisson: 0.0846

11.17 $\bar{x} = 2$ with $r = 0.073$, $\bar{y} = 1$ with $r = 0.039$, $\overline{x-y} = 1$ with $r = 0.08$,
       $\overline{xy} = 2$ with $r = 0.11$, $\overline{x/y^3} = 2$ with $r = 0.25$

11.18 $\bar{x} = 5$ with $r = 0.134$, $\bar{y} = 60$ with $r = 0.335$, $\overline{x+y} = 65$ with $r = 0.36$,
       $\overline{y/x} = 12$ with $r = 0.33$, $\overline{x^2} = 25$ with $r = 1.3$