

Setting:

Best arm  $arm_1 \sim \text{Bernoulli}(\mu_1)$ . Pulled  $m$  times. Success:  $a_1$ . Fail:  $b_1$ .  $a_1 + b_1 = m$

Second best arm  $arm_2 \sim \text{Bernoulli}(\mu_2)$ . Pulled  $n$  times. Success:  $a_2$ . Fail:  $b_2$ .  $a_2 + b_2 = n$

So, new samples will be  $u_1 \sim \text{Beta}(a_1, b_1)$ ,  $u_2 \sim \text{Beta}(a_2, b_2)$ .

According to a close form derived by Evan Miller,

$$\mathbb{P}(u_1 > u_2) = \sum_{i=0}^{a_1-1} \frac{B(a_2 + i, b_1 + b_2)}{(b_1 + i)B(1 + i, b_1)B(a_2, b_2)}$$

The monotonicity is not obvious. I tried 3 methods. They all end up with some form  $\int f(a_2)g(a_1, b_1, b_2)$  where  $f(a_2)$  is not always greater or less than 1.