

First Slay the Spire Exam (version 2)

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info/errata/submissions: <https://tck.mn/stsexam>

Your answers must be well-supported by argument. Even a perfectly correct answer will be given a low score unless accompanied by an explanation.

Assume all questions refer to a normal, unmodded, non-daily, non-custom, non-endless game on the latest version unless stated otherwise. For questions that ask whether something is “possible,” you may assume you have full control over all RNG. If character or ascension level are not specified, you may choose. You may not use glitches to your advantage in any problem.

I have done my best to make all the screenshots possible to achieve in an actual game, but please don’t overanalyze them (e.g. deck size and exhaust pile size are likely incoherent, and some backgrounds are wrong).

Problem 1. Answer true or false for each question.

- (a) The order of cards in your hand never has any observable mechanical effect. In other words, from any reachable game state, the player can never benefit from swapping the positions of two cards in their hand.
- (b) The order of your own buffs never has any observable mechanical effect. In other words, from any reachable game state, the player can never benefit from swapping the positions of two buffs under their character.
- (c) The order of the given relics with respect to each other in your relic bar never has any observable mechanical effect. In other words, from any reachable game state with the two relics adjacent in the relic bar, the player can never benefit from swapping their positions.
 - i. Orange Pellets and Duality
 - ii. Orange Pellets and Ink Bottle
 - iii. Clockwork Souvenir and Gremlin Visage
 - iv. Clockwork Souvenir and Mutagenic Strength
 - v. Bag of Preparation and Ninja Scroll
 - vi. Bag of Marbles and Red Mask
 - vii. Bag of Marbles and Twisted Funnel
 - viii. Red Mask and Twisted Funnel
 - ix. Membership Card and The Courier
 - x. Nuclear Battery and Symbiotic Virus
- (d) It is possible to win the given floor 1 fight on Ascension 20 with no Neow bonus without taking damage. If not, also answer whether it is possible with the Neow bonus “Upgrade a card.”
 - i. Ironclad vs. Cultist
 - ii. Ironclad vs. Jaw Worm
 - iii. Silent vs. Cultist
 - iv. Silent vs. Jaw Worm
 - v. Defect vs. Cultist
 - vi. Defect vs. Jaw Worm
 - vii. Watcher vs. Cultist
 - viii. Watcher vs. Jaw Worm

- (e) There exists a choice of (1) a character, (2) a single unupgraded card of that character's color, (3) a single relic, and (4) a combat in the game; such that if you add (2) and (3) to the starter deck and immediately enter fight (4), then with probability 1, you can eventually go infinite. Here, "go infinite" means that you are in a concrete game state where, for any positive integer n , you can demonstrate a line that deterministically plays n cards without ending your turn. You may pretend that you have infinite HP during setup – but not while executing the infinite – and you may also pretend that the enemies have infinite HP while executing the infinite (in case your infinite does damage).

Note that some possible solutions ruled out by these stipulations are Pandora's Box (there is nonzero probability you just whiff), Toolbox into Magnetism (again there is only one chance to see Magnetism which might fail), or Deva Form + Unceasing Top (there is no concrete game state that can play n cards for arbitrary n).

- (f) If your first six combats of an act do not drop a potion and you enter and win the seventh (non-boss) combat, it is guaranteed to drop a potion. (Assume no Smoke Bombs are used.)
- (g) In each scenario, suppose you have 1 HP and no block. Answer whether you survive if you play an attack triggering the given source of block or healing, in the case of Reaper or Wallop assuming the enemy has at least 3 HP and is not damage capped. (Fill in each cell of the table.)

	Spiker with 3 Thorns	The Guardian with 3 Sharp Hide	Corrupt Heart with 3 Beat Of Death
Feed	i.	ii.	iii.
Feel No Pain	iv.	v.	vi.
Iron Wave	vii.	viii.	ix.
Rage	x.	xi.	xii.
Reaper	xiii.	xiv.	xv.
3x After Image	xvi.	xvii.	xviii.
Tough Bandages	xix.	xx.	xxi.
evoke Frost	xxii.	xxiii.	xxiv.
Mental Fortress	xxv.	xxvi.	xxvii.
Nirvana	xxviii.	xxix.	xxx.
Talk to the Hand+	xxxi.	xxxii.	xxxiii.
Wallop	xxxiv.	xxxv.	xxxvi.
Bite+	xxxvii.	xxxviii.	xxxix.
Ornamental Fan	xl.	xli.	xlii.
The Abacus	xliii.	xliv.	xlv.

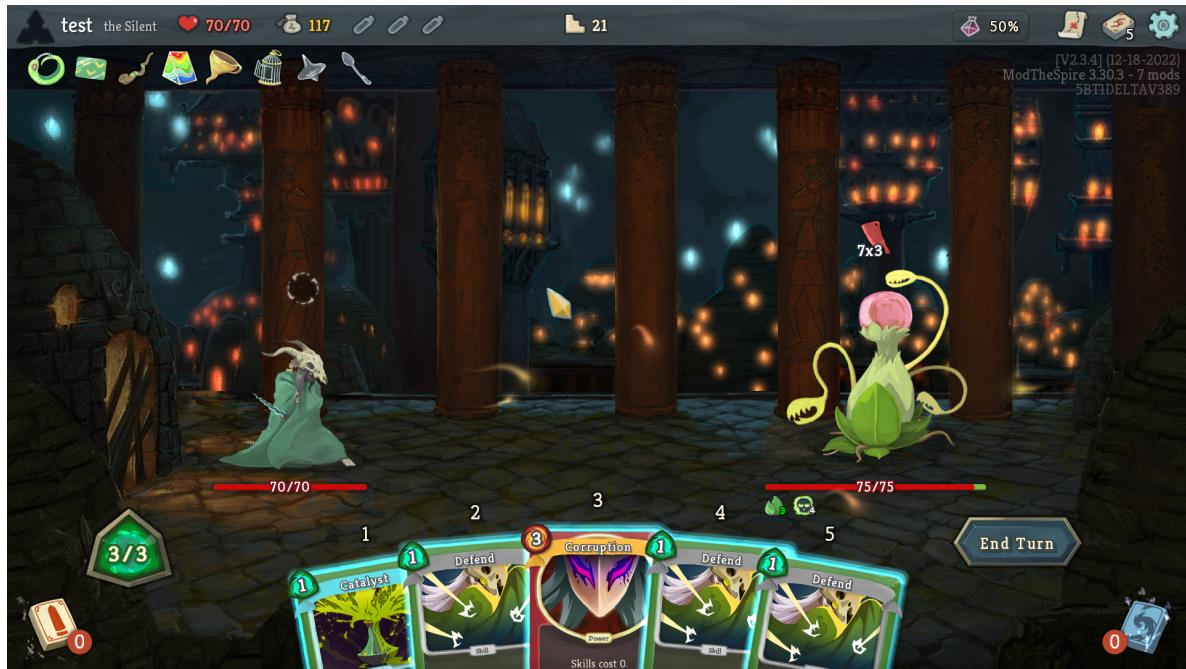
- (h) The amount of HP gained by playing Reaper is a nondecreasing function in Strength (i.e. having more Strength never means you heal strictly less) from an otherwise identical position.
- (i) There is exactly one debuff which having Orange Pellets can never be useful against.
- (j) It is possible to have 46 cards in your deck on floor 2. (You may count cards added on floor 2 – in other words, we are talking about the moment you go from floor 2 to floor 3.)
- (k) It is possible to see 17 individual Sentries total, in one run.
- (l) It is possible to remove a bottled card via the given relic or event.
- i. A Note For Yourself
 - ii. Astrolabe
 - iii. Bonfire Spirits
 - iv. Empty Cage
 - v. Falling
 - vi. N'lOTH (give the bottle, then try to remove the card at a shop)
 - vii. Transmogrifier
 - viii. We Meet Again! (Ranwid)

(m) On Ascension 20, if you are playing as the given character (without Prismatic Shard) and the Corrupt Heart has 201 HP at the start of your turn, then it is possible for it to die before it starts to perform its next action. (This means that e.g. thorns doesn't count, because the Heart has started its attack action before it dies to the thorns.)

- i. Ironclad
- ii. Silent
- iii. Defect
- iv. Watcher

Problem 2. Compute the expected value of the given quantity, assuming you are playing an optimal strategy to maximize its expected value. There are no mods that affect gameplay.

- (a) The amount of poison on Snake Plant immediately before the end of your current turn. (Assume that the poison number is unbounded and cannot overflow.)



- (b) *Problem temporarily removed, as I made a major mistake in constructing it that will require significant reworking.*

- (c) The sum of the quantities [HP at end of fight] and [number on Ink Bottle at end of fight]. Apologies for the Spanish cards; this is from a real run I played when I had changed my language for fun. The cards in hand are Ball Lightning (7 damage), Hologram+ (5 block), Trip, Dualcast, and Rebound+ (12 damage).



- (d) The quantity [1 if you live, 0 if you die]. In other words, compute your win probability.



- (e*) [extra credit] As above, but with 4 orb slots.

Problem 3. Find the unique matching of the 8 relics below to the 8 fights on the following page (using each relic exactly once) such that you have a provable win in all scenarios. Then, prove that this matching is the only solution. (Assume the Letter Opener is on 0.)

(A)



(B)



(C)



(D)



(E)



(F)



(G)



(H)



(I1)



(I2)



(S2) The enemies have Ascension 9 HP but Ascension 0 attack patterns; please ignore this discrepancy and assume the numbers shown are correct. All cards in draw and discard are Grand Finale+.

(S1)



(D1) No cards have been played this turn.

(D2) The current number on Thunder Strike is 4.



(W1) The card in the draw pile is Flurry of Blows.

(W2) The enemy has already added a Parasite to your deck. All cards in draw and discard are Just Lucky.



Problem 4. In this problem, we define a new keyword called **Alpharamp**. This keyword never natively appears on cards, but it can be added to cards via mid-combat effects.

After you play a card with Alpharamp n , one of three things happens. If you do not have the Alpharamp buff, you gain n stacks of it, associated with the name of the card granting it. If you have the Alpharamp buff and the name of the card associated with it comes strictly earlier in the alphabet than the card you just played, you gain n additional stacks of it and the name associated with it changes to the card you just played. If you have the Alpharamp buff and the name of the card associated with it is later in the alphabet than or the same as the card you just played, you lose the buff.

The Alpharamp keyword has three other special properties. First, when you play a card with the Alpharamp keyword, the keyword is immediately removed. This happens before any other effects are resolved; so for example, if you Burst an Alpharamp card, the Alpharamp effect only happens once. Secondly, all cards lose the Alpharamp keyword at the end of your turn, even cards in your draw/discard/exhaust piles. Finally, if the Alpharamp keyword is present on a card, any effect that says “draw (n) cards” on it is negated. So a Pummel Strike would still do damage, but would have no other effect.

What does the Alpharamp buff do? If you have n stacks of Alpharamp and you play an Attack, you deal n extra damage to all enemies targeted by the attack. This is a flat damage bonus and not a Strength-like effect, so hitting an enemy with a multihit attack still only gives a bonus of n . Since this is not attack damage, it is not affected by Strength, Vulnerable, Wrath, and so on. If you have n stacks of Alpharamp and you play a Skill that grants block, you gain n extra block. Again, since this effects comes from the buff and not the card, it is not affected by Dexterity, No Block (from Panic Button), and so on.

The Alpharamp that comes from a card play never applies to that card play itself. For example, if you play a Strike with Alpharamp 1 followed by a Twin Strike with Alpharamp 1, the Strike does normal damage and the Twin Strike does 1 extra damage. However, the Alpharamp gained from a card does apply to cards played as a result of that card. For instance, if you play a Havoc with Alpharamp 1 into a normal (non-Alpharamp) Strike followed by a Twin Strike with Alpharamp 1, both attacks do 1 extra damage.

I have made no effort to make the screenshots below realistic; please disregard obvious discrepancies.

- (a) The potion “Bottled Alpha” says: Give **Alpharamp 3** to all cards in your hand.

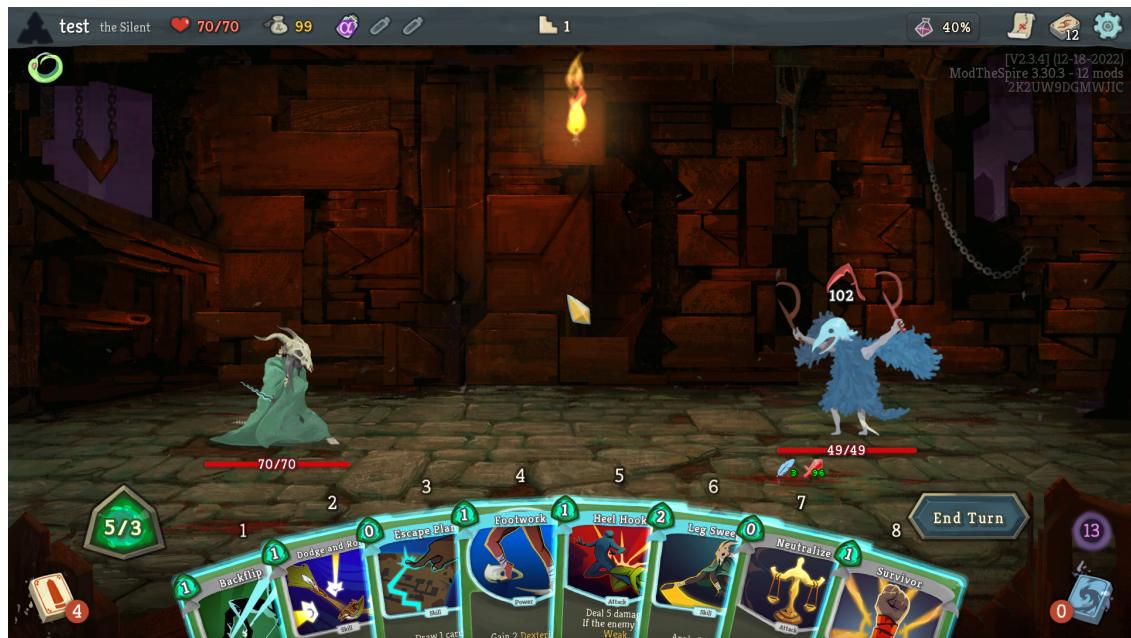
For each subproblem, determine...

- i. ... the maximum damage you can deal this turn. The card in the draw pile is a Wound.



i*. [extra credit] ... as above, but if Flex is upgraded.

ii. ... the maximum block you can gain this turn. All cards in the draw pile are Deflects.



ii*. [extra credit] ... as above, but if Footwork is upgraded.

- (b) The relic “Initial Alpha” says: Give **Alpharamp 1** to all cards in your hand on the first turn of each combat.

Suppose your starter relic is replaced with Bag of Preparation, and you have no other relics and no potions. For each of the four characters, construct a deck and draw order with the following properties:

- The maximum number of cards it is possible to play on turn 1 (without Initial Alpha) is 12 or fewer.
- If you had Initial Alpha, you could deal at least 50 additional damage against turn 1 of Transient (compared to the maximum amount of damage it would be possible to deal without it).
- The maximum number of card plays and damage dealt in both cases do not rely on randomness (except the draw order you have predetermined).
- Your deck has no colorless cards or cards from other characters.

(Note: I believe this to be much more difficult for Watcher than for the other three characters.)

- (c) The card “Alpha” says... wait, no, that’s not right. Whatever – pretend the card “Alpha” said: Give **Alpharamp 2** to all cards in your hand. Swap the names of two cards in your hand. Exhaust.

Do you have lethal this turn?



- (d*) [extra credit] Why is this mechanic a catastrophically bad game design decision?

Problem 5. For each relic, construct a position in a fight where you provably die, but if you had the option to discard the relic, you provably win the fight. (RNG state is not part of the position – “provably” means “independent of RNG.”)

- (a) Bird-Faced Urn
- (b) Mummified Hand
- (c) Blue Candle
- (d) Orange Pellets
- (e) Bloody Idol

Problem 6*. [extra credit] Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be the partial function that sends a natural number n to the minimum floor number $f(n)$ such that it is possible for the player to have n max HP on floor $f(n)$. Note that this function is not total; for instance, $f(0)$ is not defined because it is never possible to have 0 max HP. Sketch a graph of $f(n)$ (it does not have to be precise, but you should show that you understand its behavior).