

Problem 1

- (a) False. Suppose you have a Regret, a Burn, and 5 other cards in your hand, and you have one stack of Buffer (with no block). If the Regret comes first, you Buffer the Regret and lose 2 HP, but if the Burn comes first, you Buffer the Burn and lose 7 HP.
- (b) False. Suppose you have Thousand Cuts and Mental Fortress in play against the Heart. You and the Heart both have 1 HP. You play an Inner Peace. If Thousand Cuts comes before Mental Fortress, the Thousand Cuts trigger ends the fight, you take Beat of Death, and you die. If Mental Fortress comes before Thousand Cuts, you get the block first, and you win. (This is only true when the Heart is about to die – if the Thousand Cuts trigger is not lethal, you live regardless of order.)

An alternative solution I didn't think of, but some exam-takers did: the order you play all of {Battle Hymn, Creative AI, Hello World, Infinite Blades, and Magnetism} do matter with respect to each other, if you have enough stacks to overflow the 10 card hand limit.

- (c)
 - i. False. The one that appears earlier in the relic bar goes first. This matters if you play an Attack that procs Pellets.
 - ii. False. The one that appears earlier in the relic bar goes first. This matters if you have No Draw from Battle Trance or Bullet Time and you play a card that procs Pellets with Ink Bottle on 9.
 - iii. True. Clockwork Souvenir always procs before Gremlin Visage, because Souvenir calls `addToTop` and Visage calls `addToBot`.
 - iv. False. Unintuitively, the one that appears later in the relic bar goes first! This is because both relics call `addToTop`, so the one that triggers later will have its effect resolve first. So, if you got Mutagenic first, the effect happens after Souvenir and eats the artifact; if you got Souvenir first, you keep the artifact (but it will still cancel the Strength down at the end of your turn unless you use it on something else during your turn).
 - v. False. Ninja Scroll always procs before Bag of Preparation, because Ninja Scroll is activated by an `atBattleStartPreDraw` trigger and Bag of Preparation is activated by an `atBattleStart` trigger. (This one I would expect the most people to actually know, since it is a real consideration that picking up Ninja Scroll turns Bag of Prep into a brick, assuming you still have Ring of the Snake.)
 - vii-ix. False. All three of Bag of Marbles, Red Mask, and Twisted Funnel call `addToBot`, so they proc in the order they appear in the relic bar. This matters for enemies with Artifact.
 - x. True. The reason it might matter is due to rounding, and indeed prices are rounded after each discount, but the Courier discount is always applied before the Membership Card discount regardless of order in the relic bar. (Interestingly, the only price that is *not* rounded after each discount is the card removal price.)
 - xi. False. Both relics are activated by an `atPreBattle` trigger, so they proc in the order they appear in the relic bar.

(d) Ironclad needs an upgrade against Cultist, and for all the other matchups the base deck wins.

The lowest possible Cultist HP is 50, and the lowest possible Jaw Worm HP is 42.

For Ironclad vs. Cultist, turn 1 is Bash-Strike, turn 2 is Defend-Defend-Strike, and turn 3 (which must kill, because Cultist is hitting for 11) is 3x Strike. This adds to $(8 + 9) + 9 + (6 + 6 + 6) = 44$, which is not enough. However, either a Strike+ or a Defend+ is enough (the Strike+ adds 4 extra damage on turn 1 and 3 extra damage on turn 3, or the Defend+ adds 6 extra damage on turn 2 by playing an extra Strike).

For Ironclad vs. Jaw Worm, the base deck can do it: turn 1 (Chomp for 12) is 3x Defend, turn 2 (Bellow) is 3x Strike, turn 3 (Thrash for 12) is 3x Defend, turn 4 (Bellow) is Bash-Strike, and turn 5 is 3x Strike, for a total of $(6 + 6 + 6) + (8 + 9 - 5) + (9 + 9 + 9 - 9) = 48$. (Thanks to Simon via Telegram and Maunto_Rukasu via Reddit for pointing out that the original answer key incorrectly claimed this one was impossible!)

For Silent vs. Cultist, the base deck wins. Turn 1 is 3x Strike, turn 2 is Defend-Strike-Strike-Neutralize, and turn 3 is 3x Strike, for a total of $(6 + 6 + 6) + (6 + 6 + 3) + (6 + 6 + 6) = 51$.

For Silent vs. Jaw Worm, the base deck again wins: turn 1 (Chomp for 12) is Strike-Defend-Defend-Neutralize, turn 2 (Bellow) is 3x Strike, turn 3 (Chomp for 17) is Defend-Defend-Survivor, and turn 4 is 3x Strike. This is 7 Strikes and 1 Neutralize, which is 45.

For Defect vs. Cultist, the base deck easily wins. Turn 1 is Zap-Dualcast-Strike, turn 2 is Defend-Defend-Strike, and turn 3 is Strike-Strike-Dualcast, for a total of $(3 + 16 + 6) + (3 + 6) + (6 + 6 + 16) = 62$.

For Defect vs. Jaw Worm, the base deck wins on turn 3: turn 1 (Chomp for 12) is 3x Defend, turn 2 (Bellow) is Strike-Strike-Zap, and turn 3 is Strike-Strike-Dualcast. This is $3 + (6 + 6 + 3 + 3) + (6 + 6 + 16 + 3 - 9) = 43$.

Watcher's base deck beats both fights on turn 2 without the Miracle (lol). Turn 1 is Vigilance-(Strike for Cultist, Defend for Jaw Worm), and turn 2 is Eruption-Strike-Strike-Strike. This deals $9 + 12 + 12 + 12 = 45$, plus an extra 6 to Cultist from the turn 1 Strike.

As a note of trivia, if you ask the other question and assume pessimal instead of optimal luck, Silent has a forced loss to Jaw Worm¹.

(e) True. Chosen, Dark Embrace, Medkit.

Two alternative solutions I didn't think of, credit to my very cool friend Alex²: Hello World + Warped Tongs vs. Snecko (to eventually get double 0 cost Hologram+), and Recycle + Dead Branch vs. anyone (to eventually build whatever infinite you want). Burning Pact (for example) also works for the latter, but I don't know of a solution for either Silent or Watcher (since they don't have targeted Exhaust).

(f) False. If you kill a burning elite with Black Star, your potion chance for that fight is forcibly set to 0 (even with Ornithopter), to avoid overflowing the rewards screen (which already has gold, relic, relic, green key, card).

You can also do this with the Act 2 Colosseum event: go into the first fight with 100% potion chance, and since it doesn't have a rewards screen it can never drop a potion (but will still affect your potion chance based on whether it would have dropped one).

¹<https://docs.google.com/spreadsheets/u/0/d/1qTj1LX58sbaNXnmRrnbonwNLhnQFyIHkDxysKqGnKg/htmlview>

²this method of attribution specifically requested

(g) “T” means true (you live). “F” means false (you die). “*” means you die if the damage is lethal to the enemy, but you live otherwise.

	Spiker with 3 Thorns	The Guardian with 3 Sharp Hide	Corrupt Heart with 3 Beat Of Death
Feed	T	T	T
Feel No Pain	F	F	F
Iron Wave	T	T	T
Rage	F	T	T
Reaper	F	F	T
3x After Image	F	T	T
Tough Bandages	F	F	*
evoke Frost	F	*	*
Mental Fortress	F	F	*
Nirvana	F	F	*
Talk to the Hand+	T	T	T
Wallop	T	T	T
Bite+	F	T	T
Ornamental Fan	F	T	T
The Abacus	F	F	*

Here is the same table, but sorted in a way that reveals more structure:

	Spiker with 3 Thorns	The Guardian with 3 Sharp Hide	Corrupt Heart with 3 Beat Of Death
Feed	T	T	T
Iron Wave	T	T	T
Talk to the Hand+	T	T	T
Wallop	T	T	T
Rage	F	T	T
3x After Image	F	T	T
Bite+	F	T	T
Ornamental Fan	F	T	T
Reaper	F	F	T
evoke Frost	F	*	*
Tough Bandages	F	F	*
Mental Fortress	F	F	*
Nirvana	F	F	*
The Abacus	F	F	*
Feel No Pain	F	F	F

In most cases, the cells with T are the ones that are “nice” to the player (i.e. what you would want), but note that for the healing-based sources sometimes you want the other order. For example, if you are at full HP and you Bite a Spiker, you still get healing value because it happens after the Thorns.

As if this wasn’t complicated enough, here are some more fun facts:

- For evoking Frost, Tough Bandages, Mental Fortress, Nirvana, and The Abacus: if you play them for lethal against Guardian or Heart and the resulting self damage does *not* kill you, you never even see the result animation. (Contrast with e.g. Feel No Pain, where the enemy dies and you take damage but then you see yourself gain the block before the rewards screen appears.)
- Against the Heart, playing a Buffer that evokes a lethal lightning orb (via Storm) blocks the Beat of Death, but you keep the Buffer icon!
- There are more possibilities for relevant interactions not captured here – for example, Sadistic Nature Leg Sweep or Tingsha Bandages for lethal against the Heart (both always block), or Bird Faced Urn vs. Beat of Death (Urn heals first), or Thousand Cuts (happens after evoking Frost, but before Feel No Pain, and order matters for Mental Fortress (see Problem 1b)).

- (h) False. The full description of what happens is very complicated, but here is at least a concrete example: If Reptomancer is on 1 HP and there are three full HP daggers, then a Reaper that does 15 damage will give you the full amount of healing, whereas a Reaper that does 16 damage will give you no healing at all.

The reason this happens is that the game plays an animation of little orbs flying into your character when you play Reaper, and the heal effect only comes after the animation finishes. The number of orbs (and therefore the duration of the animation) scales with the amount of damage you heal for; specifically, each enemy produces $\lfloor d/2 \rfloor$ orbs, where d is the amount of HP stolen. In the Reptomancer fight specifically, if the animation takes too long, the combat rewards screen can appear before the event queue reaches the “heal” action.

All of the following things are things that might have been true at some point in the game’s version history, or maybe even are still true now, but might be myths, and I have been unable to reproduce myself. Please let me know if you successfully replicate any of these in the current version, and tell me how!

- Feel No Pain or Charon’s Ashes affecting the event queue and therefore changing the behavior
- whether Fast Mode is on having an effect
- only getting healing from the daggers physically closer to you in some scenarios

Alternative answer: Necronomicon and Malleable.

- (i) True. Specifically, Entangled (you can’t play the attack to trigger pellets). Partial credit for mentioning Blasphemer or Surrounded, which are technically buffs.

The original intent of this was to be False because of Draw Reduction, which triggers before you can play any cards. But in fact you can pellets the status to prevent it from triggering again next turn.

- (j) True. Start on A10 Silent (13 cards). Boss swap into Calling Bell (14 cards), which is Ceramic Fish + The Courier + Old Coin. Take a card from the fight (15 cards). Floor 2 is a shop with Membership Card and a min price on sale common card, which has a billion min price common cards underneath it. Each card effectively costs 9 (price of 18 but refunds 9 from Ceramic Fish), except the on sale one which is free. You can afford to buy 33 more cards, because you started with $99 + 300 + 20 + 9 = 428$ gold, and spent $114 + 30 \times 9 = 411$. This is 49 cards total. (You have 17 gold left over, but this is not enough to buy a 50th card even though you would technically end up with positive gold thanks to Ceramic Fish.)

- (k) False. Each act has 8 floors, a chest, and 5 more floors. The first 5 can’t be elites, and elites can’t be consecutive, so you can see up to 5 elites in Act 1. You furthermore can never see the same elite twice in a row, so at most 3 of these can be triple Sentries. (Wing Boots don’t help, because even with 3 charges you can only get up to 6 elites in Act 1, which is still at most 3 of the same one.) You can get a bonus triple Sentries from the Dead Adventurer event, for a total of 12 Sentries in Act 1.

The only other source of Sentries is the Sentry + Spheric Guardian fight in Act 2’s hard pool. If you take all combats in Act 2, you get 2 easy pool fights followed by 11 hard pool fights. Seeing a particular hard pool fight prevents it from appearing in the next *two* combats, so you can encounter this fight at most 4 times. Therefore, you can see at most 16 Sentries over the course of a run.

- (l) i. False. ii. True. iii. False. iv. True. v. False. vi. True. vii. False. viii. True.

Interestingly, the wiki is currently wrong about Ranwid, and says he can’t ask for a bottled card. I’m curious if this will trap anyone, so I won’t fix it until after releasing this answer key :P

- (m) i. True, Dazed-Feel No Pain-Juggernaut. ii. True, poison. iii. False? iv. True, Judgment.

Fun fact: This used to be True on all characters via The Boot, since if you did 1 damage it would only count for 1 against Invincible but would reduce the Heart’s HP by 5, allowing a turn 1 kill. This has since been patched.

Problem 2

- (a) Infinity. This is the “St. Petersburg paradox” (https://en.wikipedia.org/wiki/St._Petersburg_paradox).
- (b) [question was removed]
- (c) $71 \frac{2}{9}$ (or approximately 71.222). You have exact lethal with Dualcast, Trip, Ball Lightning, Rebound+. An extra 2 card plays can come from Hologram Trip, and the only way you can get more is by drawing Claw+ or Aggregate. (Recycle looks like it might work, but there’s no usable target that still lets you kill this turn. You also have to play the Trip before the Rebound+ for lethal, so Rebound+ Trip doesn’t work either.)
- (d) 35/48 (or approximately 0.729). The key observation is that Orichalcum plus the Frost orb from Rainbow gives 12 block, which is just barely enough to survive one Repulsor and one Exploder. To kill a shape, it takes either a Lightning evoke (for 12) plus any other damage, or two Lightning passives (they do damage in increments of 7, and the Shivas sum to 6 so cannot help).

Therefore, the only real decision is whether to play the Body Slam for 0 damage. If you do, you get 4 block from Ornamental Fan, and you can redraw the Body Slam from Ink Bottle. With Pen Nib, this makes the Body Slam do 7 damage, effectively a targetable Lightning passive. The problem is that this disables Orichalcum, so now you only have enough block to survive two Repulsors and you must kill both Exploders. Furthermore, you have to commit to this decision after your first card play, since a card cannot draw itself off Ink Bottle (so playing Body Slam with Ink Bottle on 9 doesn’t redraw it).

The solution is as follows. First, play Rainbow. Then, here are the possible outcomes of where the two Lightnings hit and their respective probabilities:

- One Repulsor and one Exploder ($p = \frac{1}{2}$). Play Shiv on both and win.
- One on each Repulsor ($p = \frac{1}{8}$). Play Body Slam, then play Shiv on both Repulsors, then Body Slam the front Exploder, bringing it to 7. You now win with probability $\frac{1}{2}$, since you need the Lightning passive to hit the front Exploder.
- One on each Exploder ($p = \frac{1}{8}$). Play Shiv on both and win.
- Double hit on either Exploder ($p = \frac{1}{8}$). Play Body Slam, then play both Shivas and Body Slam on the remaining Exploder, bringing it to either 1 or 2. You now win with probability $\frac{1}{3}$, since you need the Lightning passive to hit the remaining Exploder.
- Double hit on either Repulsor ($p = \frac{1}{8}$). You just die, since the only way to kill an Exploder is via the Body Slam line, and then you break Orichalcum so can’t survive the other Exploder.

So, overall win probability with optimal play is $\frac{1}{2} + \frac{1}{8} \cdot \frac{1}{2} + \frac{1}{8} + \frac{1}{8} \cdot \frac{1}{3} = \frac{35}{48}$.

- (e) 7/18 (or approximately 0.389). All of the commentary in the first two paragraphs of the solution to (2d) still applies, but now you only get to see one evoke before committing to your line.

The solution is as follows. First, play Rainbow.

- If the Lightning evoke hits a Repulsor, it is no longer possible to kill both Exploders, so you are not allowed to Body Slam. Play a Shiv to kill the Repulsor, a Shiv on the back Exploder, and end turn. You live if both lightnings hit the same Exploder, which happens with probability $\frac{2}{9}$.
- If the Lightning evoke hits an Exploder, you should play the Body Slam. Then play a Shiv to kill the Exploder, drawing Body Slam via Ink Bottle; play another Shiv on the back Exploder (or anywhere if it’s dead), gaining 4 Block via Ornamental Fan; and play the Pen-Nibbed Body Slam on the living Exploder. You now have two chances for lightning passives to kill the living Exploder, which happens with probability $\frac{5}{9}$. (The line without Body Slam requires both lightnings to hit the same shape, which only happens $\frac{3}{9}$ of the time.)

Since the Lightning evoke hits an Exploder with probability $\frac{1}{2}$, we conclude that the overall probability of survival with optimal play is $\frac{1}{2} \cdot \frac{2}{9} + \frac{1}{2} \cdot \frac{5}{9} = \frac{7}{18}$.

Problem 3

Here is a table showing which relics let you survive which combats.

	A	B	C	D	E	F	G	H
I1	✓	✓					✓	
I2						✓		
S1	✓	✓		✓			✓	
S2			✓		✓			
D1		✓	✓			✓	✓	
D2			✓		✓	✓		
W1	✓		✓		✓	✓		
W2				✓	✓			

This is a bipartite matching problem, which given the above adjacency matrix is very easy to solve. (In fact, it is always true that a bipartite matching problem guaranteed to have a unique solution is “very easy to solve,” in a formal sense: there will always be either a row or a column with only one checkmark that you can assign, and after doing so repeat the process until all assignments are made.) The unique solution is:

$$\begin{array}{cccccccc} (\text{I1}) \text{ B} & (\text{I2}) \text{ F} & (\text{S1}) \text{ D} & (\text{S2}) \text{ C} & (\text{D1}) \text{ H} & (\text{D2}) \text{ G} & (\text{W1}) \text{ A} & (\text{W2}) \text{ E} \\ (\text{A}) \text{ W1} & (\text{B}) \text{ I1} & (\text{C}) \text{ S2} & (\text{D}) \text{ S1} & (\text{E}) \text{ W2} & (\text{F}) \text{ I2} & (\text{G}) \text{ D2} & (\text{H}) \text{ D1} \end{array}$$

I will now explain the table row by row.

Combat I1. There is basically only one thing you can do: play Power Through, then Reckless Charge+, then Wound Wound Dazed (redrawing both). The only decision is whether to play Fire Breathing or not.

If you skip Fire Breathing, you get 3 plays of Power Through and 4 plays of Reckless Charge+ for 45 block and 40 damage. If you play Fire Breathing, you get 2 plays of Power Through and 3 plays of Reckless Charge+, but also 2 procs of Fire Breathing from drawing Dazed, for 30 block and 42 damage.

Obviously, you have to kill the Cultist. With Vajra, you get +4 damage in the former case (exactly not enough) but +3 damage in the latter case (exactly enough), and you can therefore win by playing Fire Breathing. With Letter Opener, you get +5 damage in the former case (barely enough) but no extra damage in the latter case, and you can therefore win by skipping Fire Breathing. And of course Lizard Tail lets you win next turn (take the Fire Breathing line). None of the other relics help.

Combat I2. With no block cards, there’s certainly not enough damage to kill fast enough against enemies that attack every turn (Demon Form Vault notwithstanding). The offensive relics also don’t do nearly enough to rectify this, so the only hope is playing The Bomb and Vault and surviving one turn of damage. Orichalcum lets you survive by blocking the first two hits and Buffering the third. Lizard Tail does not – you Buffer the first hit, die once to the second hit, and die again to the big hit (which is for more than half your HP, the amount that Lizard Tail heals to).

Combat S1. This is another one where there is basically only one decision: what do you Burst? If Corpse Explosion+, then you get $18 + 12 = 30$ single target damage. If Blade Dance+, then you get $9 + 24 = 33$ single target damage.

You clearly have to kill all the Darklings. With Vajra, you get 4 extra damage in the former case (exactly not enough to hit 35), and 8 extra damage in the latter case (exactly enough to hit 41 and explode for 59). With Letter Opener, you get 5 extra damage in the former case (exactly enough to hit 35 and double explode for 50×2), and 5 extra damage in the latter case (can single explode for 50, but that doesn’t kill the third one, so you still die). The Boot is a better Vajra, and Lizard Tail lets you take two turns to play either line. None of the other relics help.

Combat S2. It is fairly clearly telegraphed that you want either a big Malaise+ to block or a big Skewer+ for lethal.

The line for Malaise+ would be Concentrate on Tactician, After Image, Skewer (8 energy); Neutralize on Deca (9 energy); Malaise+ on Deca. This would give Deca –10 Strength and Weak, which makes the attack a survivable 7×2 . But Deca has one too much Artifact for this to work.

The line for Skewer+ would be Concentrate on Tactician, After Image, Malaise (8 energy); Neutralize on Deca (9 energy); Skewer+ on Deca. This would do $6 + 13 \times 9 \times 2 = 240$ damage, which is not enough.

With Vajra, you get $7 + 14 \times 9 \times 2 = 259$ damage on the Skewer+ line, which is exactly 1 damage short. With Gremlin Horn, you can instead play Neutralize on Donu and then Skewer+ for one more energy; this does $13 \times 10 \times 2 = 260$ damage, which is exactly enough to kill. With Hand Drill, the Malaise+ line works since Neutralize strips an extra artifact by breaking block. None of the other relics do anything productive.

Combat D1. The following line optimizes damage:

```
-2 Streamline+
    FTL+
-2 All for One
    FTL+
+3 TURBO
    FTL+
-1 Streamline+
-2 All for One
+3 TURBO
    FTL+
    Streamline+
-1 Hologram
-2 All for One
    FTL+
    Streamline+
```

(You end with a spare TURBO+ in hand, but nothing to spend the energy on.) This is 4 plays of Streamline+, 3 plays of All for One, and 5 plays of FTL+. You need to kill each Spiker with a single attack, which in the current state must be a Streamline+. This leaves $2 \cdot 20 + 3 \cdot 10 + 5 \cdot 6 = 100$ damage for the Spheric Guardian. If you do not kill it this turn, you will die of Regret.

Vajra doesn't help you kill Spikers any more efficiently, so you get 10 bonus damage on Spheric Guardian, which is not enough. Letter Opener does let you spent All for One instead of Streamline+ on the Spikers, so you get 20 extra damage on Spheric Guardian (plus the 5 from Letter Opener), which is enough. Gremlin Horn is clearly enough; an example line is below.

```
-1 Streamline+
-0 Streamline+
    Streamline+
    FTL+
    Streamline+
+3 TURBO+
-2 All for One
    FTL+
    Streamline+
+3 TURBO+
-1 Hologram
-2 All for One
    FTL+
    Streamline+
```

You are only doing 9 damage into real HP, so neither The Boot nor Hand Drill work. Lizard Tail lets you survive the Regret damage and kill next turn (or in 2 turns, if you get the worst possible draw of Void Void Regret TURBO+ All for One).

As for Tungsten Rod, you need to kill the Spikers and make sure you never draw a hand like Void Void Void Regret All for One (with no cards in discard). One way to guarantee this is to play FTL, All for One, Streamline, TURBO, Hologram-Streamline, Streamline on turn 1, killing both Spikers and only adding one Void. Then you maintain the invariant of a maximum of one Void (if you don't draw it, you drew everything else, so just play FTL). You can always play All for One and FTL before TURBO to avoid having to play multiple TURBOs, so Tungsten Rod works.

Combat D2. The randomness of Thunder Strike and orbs is misleading – there's just a fixed pool of HP, and you can only deal damage in increments of 7 (except for Ball Lightning+). The only place the extra 3 from Ball Lightning+ is useful is on the 17 HP one, so hit that one to bring the enemy HP pool down to 7 copies of 7 and then play Thunder Strike for 5. The lightning evoke did another 7, so there is one enemy with 7 HP left at the end of your turn. The lightning passives will do 4 more damage.

As it turns out, even though there's only 3 HP left, Vajra does not help, because the living enemy only got hit once and all the other effects of the Strength were wasted. (Another way of seeing this is that if any enemy had gotten hit 3 times, it would already be dead.) Gremlin Horn gives the energy to play an extra Zap before all of this (and target whoever was hit with Ball Lightning+), which does 7 bonus damage from Zap plus 7 bonus damage from the extra Thunder Strike charge and wins. Orichalcum and Lizard Tail allow surviving the attack from the one remaining enemy. None of the other relics help.

Combat W1. If you play Thunderclap first, you deal 3 damage to one and 15 damage to the other. If you play Eruption first, you deal 6 damage to one and 14 damage to the other.

With Vajra, the Eruption first on the Spear works to kill both. With Gremlin Horn, the Thunderclap first works to kill the Spear and draw the Flurry for 6 more damage to kill the Shield. Orichalcum and Lizard Tail allow you to play Thunderclap first, kill the Spear, and survive this turn to kill next turn. None of the other relics help.

Combat W2. You are currently dying by 5, and are 19 short of lethal (Just Lucky is bad to play thanks to Malleable, so the most damage is Eruption for 9 and then Windmill Strike for 37). With Hand Drill, you can play Just Lucky to give Writhing Mass 3 block, play Eruption to add Vulnerable, and then play Windmill Strike with triple damage. This does $3 + (9 - 3) + (20 \cdot 3 - 4) = 65$, which is exact. With Orichalcum, you can pass your turn and kill with Stone Calendar + Windmill Strike next turn. Lizard Tail does nothing thanks to Mark of the Bloom. Nothing else gives nearly enough damage or block.

Problem 4

- (a) i. The Strength/draw stuff is a decoy; the ramp is good enough that you just play cards in order. You can deal 109 damage. The unique optimal line (verified by computer search) is:

- drink Bottled Alpha
- Flex (Alpharamp 3)
- Headbutt ($9+2+3 = 14$ damage, Alpharamp 6)
- Heavy Blade ($14+6+6 = 26$ damage, Alpharamp 9)
- Limit Break+ (Alpharamp 12)
- Pummel ($8+16+12 = 36$ damage, Alpharamp 15)
- Clash ($14+4+15 = 33$ damage)

Extra credit: if the Flex were upgraded, the Strength scaling becomes good enough, and you could deal 145 damage with the following line:

- Flex
- Limit Break+
- Headbutt on Limit Break+ ($9+8 = 17$ damage)
- Pommel Strike ($9+8 = 17$ damage)
- drink Bottled Alpha
- Limit Break+ (Alpharamp 3)
- Pummel ($8+64+3 = 75$ damage, Alpharamp 6)
- Clash ($14+16+6 = 36$ damage)

- ii. You can get 85 block. The unique optimal line (verified by computer search) is:

- Backflip, Escape Plan (8 block, draw 3)
- drink Bottled Alpha

- Neutralize (Alpharamp 3)
- Deflect ($4+3 = 7$ block, Alpharamp 0)
- Deflect (4 block, Alpharamp 3)
- Dodge and Roll ($4+3 = 7$ block, Alpharamp 6)
- Heel Hook (Alpharamp 9)
- Leg Sweep ($11+9 = 20$ block, Alpharamp 12)
- Survivor discarding Footwork ($8+12 = 20$ block, Alpharamp 15)
- Deflect ($4+15 = 19$ block)

Extra credit: if the Footwork were upgraded, you could get 90 block by playing Footwork first and skipping Dodge and Roll (to have something for Survivor to discard).

- (b) The number 50 may look arbitrary, but was carefully chosen to defeat any naive strategy that misses a ramp with a non-Attack on the first 7 plays without doing something else to compensate. For instance, if the below Ironclad strategy used some attack card instead of True Grit+, it would reach exactly $0 + 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 7 + 7 + 7 = 49$ bonus damage.

- i. Ironclad: The starting hand is Havoc+, Pummel, Rampage, Reckless Charge, Strike, Sword Boomerang, True Grit+. The draw pile is Offering+, Clumsy, and 4 Body Slam+s, with the Offering+ on top. Playing these 7 cards in alphabetical order, then the 4 Body Slam+s, bonus damage from Alpharamp 1 is $0 + 0 + 1 + 2 + 3 + 4 + 5 + 0 + 13 + 13 + 13 + 13 = 67$.

Commentary: The existence of Body Slam makes this much easier for Ironclad. Other options for exploiting Alpharamp include Dual Wield, Double Tap, and Exhume.

- ii. Silent: The deck is Backstab, Blade Dance+, Endless Agony, Grand Finale, Masterful Stab, Neutralize, Slice. Playing these 7 cards in alphabetical order, then the 4 Shivas, then the duplicated Endless Agony, bonus damage from Alpharamp 1 is $0 + 0 + 2 + 3 + 4 + 5 + 6 + 7 + 7 + 7 + 7 + 7 = 55$.
 Commentary: Unsurprisingly, Shivas are the way to go for Silent. In fact, I don't think Silent has access to any other cards that generate cards (non-randomly) or pull/play directly from the draw or discard piles.

- iii. Defect: The starting hand is Beam Cell, Boot Sequence, Claw, Go for the Eyes, Melter, Seek, Strike. The draw pile is one Scrape and four Claws. Playing these 7 cards in alphabetical order (Seeking for Scrape), then the Scrape, then the 4 Claws, bonus damage from Alpharamp 1 is $0 + 0 + 2 + 3 + 4 + 0 + 6 + 7 + 7 + 7 + 7 + 7 = 50$.

Commentary: Seek is just being used as "draw 1" here, circumventing the draw prevention mechanic. Hologram could also fetch a draw card that no longer says Alpharamp, and All for One also works to recover cards.

- iv. Watcher: The starting hand is Blasphemy, Bowling Bash, Brilliance, Consecrate, Follow-Up, Just Lucky, and Omniscience+. The draw pile, in order, is 3 copies of Clumsy, then Consecrate, then another Clumsy, then a Weave, then another Consecrate, then a Cut Through Fate. Playing these 7 cards in alphabetical order (choosing Cut Through Fate for Omniscience and always Scrying all cards), then the two Consecrates and the Weave (returned to hand by the last Scry), bonus damage from Alpharamp 1 is $0 + 1 + 2 + 3 + 4 + 5 + 0 + 7 + 7 + 7 + 7 + 7 = 50$.

Commentary: This might actually be optimal; it took me a lot of trying to barely hit 50. The main problem is that the only "cheating" cards are Flurry, Weave, Carve, and Omni, and the first three aren't good enough. So you need to play Omniscience, but that consumes all your energy, and Watcher doesn't have enough 0 cost attacks. The solution is to spend the first card play (which can't benefit from Alpharamp anyway) on Blasphemy for energy, which is conveniently alphabetically early. The other problem is that Omniscience spends two card plays, so it is required to come last and target two attacks. Fortunately, Cut Through Fate is OP and lets you get the last 3 card plays. The particular arrangement of the draw pile is also relevant, as you need to make sure the Weave can never be double-played off of Just Lucky.

(c) Yes, you have exact lethal. The unique kill (verified by computer search) is:

- Tranquility
- Alpha, swapping Carve Reality with Tantrum
- Tantrum [now under the name Carve Reality] (9 damage, Alpharamp 2)
- Indignation (Alpharamp 4)
- Just Lucky ($3 \times 3 + 4 = 13$ damage, Alpharamp 6)
- Sands of Time ($20 \times 3 + 6 = 66$ damage, Alpharamp 8)
- Carve Reality [now under the name Tantrum] ($6 \times 3 + 8 = 26$ damage, Alpharamp 10)
- Smite ($12 \times 3 + 10 = 46$ damage)

(d) Answers vary.

Problem 5

There are many solutions for each; here are some examples.

(a) Bird-Faced Urn: You have 9/20 HP facing a single 11 HP Sneaky Gremlin. Your hand is Inflame and Strike, and you already have 3 Strength from Red Skull. Your Strike currently deals 9, so you need to play Inflame to hit the gremlin for lethal. With Bird-Faced Urn, playing Inflame heals you above half, and Red Skull turns off, leaving you with only 2 Strength. Without Bird-Faced Urn, Inflame goes to 5 Strength, and then Strike kills.

(b) Mummified Hand: You have 6 HP facing an Exploder with 9 HP about to explode and a Spiker with 20 HP and 5 Thorns currently buffing. Your hand is Inflame, Inflame, and Whirlwind, and you have 3 energy. In order to kill the Exploder, you have to Whirlwind for at least 9 damage. But Whirlwind for 2 energy or more will die to the Spiker. With Mummified Hand, playing both Inflames only spends 1 energy (because the first makes the second free), so you fail to Whirlwind for 1. Without Mummified Hand, you can play both Inflames and then Whirlwind for 1 to kill the Exploder (and you can see with Frozen Eye that you can kill the Spiker next turn).

Simpler solution: Necronomicon.

(c) Blue Candle: You have 1 HP facing a single 10 HP Sneaky Gremlin. Your hand is two copies of Havoc+, and you see with Frozen Eye that your draw pile is Clumsy followed by Bludgeon. With Blue Candle, you cannot play a Havoc+, because this deals 1 damage to you for playing a curse and you die. Without Blue Candle, Havoc+ observes that the curse is unplayable and exhausts it, and then you can play the second Havoc+ for lethal.

(d) Orange Pellets: You have 1 HP facing a single 10 HP Sneaky Gremlin. You have Snecko Eye, your hand is a 0 cost Hologram, and the only card in your discard pile is a 0 cost Skim. You have 3 energy and have played a power and attack this turn, but not a skill. You see with Frozen Eye that your draw pile is Meteor Strike, Wound, Wound. If you play Hologram, the Confused debuff clears and the Skim draws a 5 cost Meteor Strike. Without Orange Pellets, you can Hologram-Skim into a Meteor Strike that costs at most 3 and kill.

(e) Bloody Idol: This one is really complicated.

- Relics: Bloody Idol, Frozen Eye, Prismatic Shard, Red Skull, Unceasing Top, Ice Cream, Pen Nib (on 9), Centennial Puzzle (has not activated this fight)
- Hand: Wish
- Draw pile (in order): Bludgeon, Feed, 7x Burn, Bludgeon+
- 7 energy, 1/10 HP, 3 Strength (because of Red Skull), 1 Weak
- Enemies: Centurion with 56 HP attacking for 14, Mystic with -8 Strength (from Disarms) and 30 HP attacking for 1

First off, there is no way to deal with Centurion by blocking, so you must kill it. Accounting for Strength, Weak, and Pen Nib, Bludgeon does 52 and Feed does 9.

If you Wish for Strength, Bludgeon now does 57 and kills Centurion, but then you die to Mystic's 1 damage attack.

If you Wish for Plated Armor, Bludgeon and Feed together kill Centurion (healing 3 from Feed), but then your Plated Armor blocks the Burn and Mystic's attack. Without Centennial Puzzle, you draw a hand of 5 Burns, which deal 10 damage, exact lethal against your 6 Plated Armor and 4 HP.

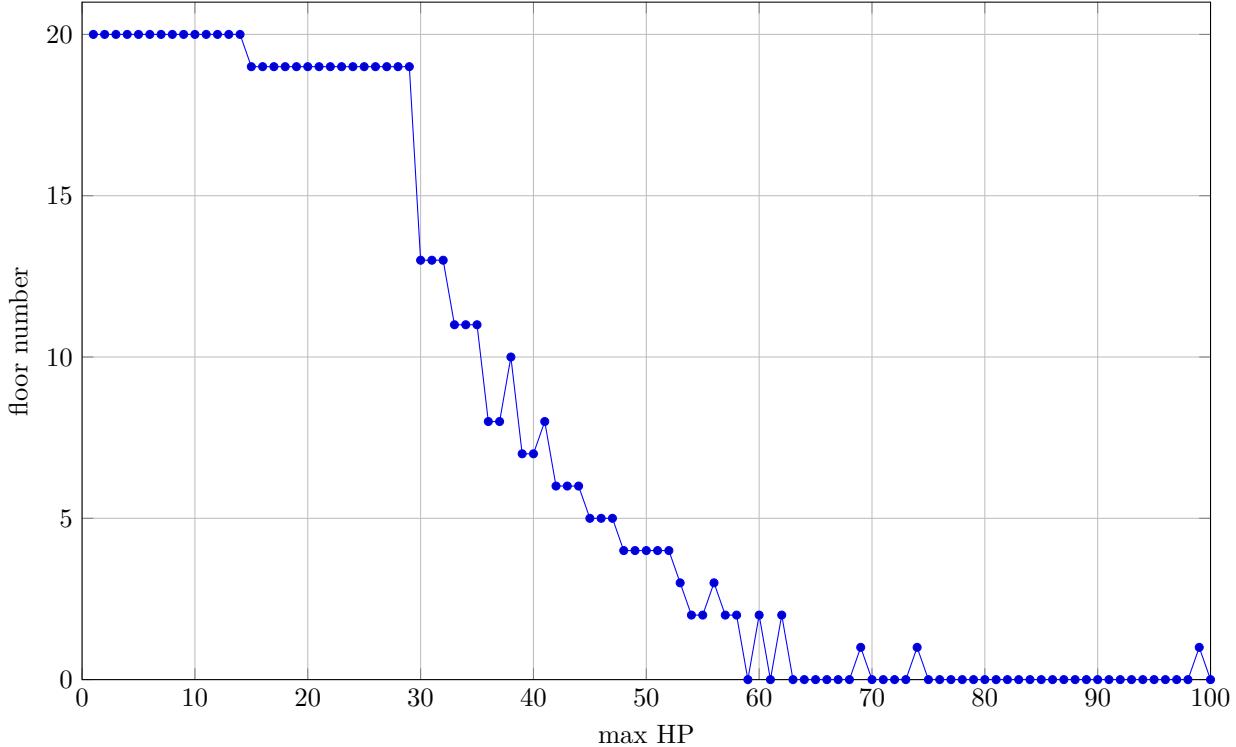
If you Wish for gold, Bloody Idol heals you above Red Skull range, and now Bludgeon for 48 and then Feed for 7 are not enough to kill Centurion.

However, without Bloody Idol, you can successfully Wish for gold, kill Centurion and heal 3 off Feed, then take 2 from a Burn (drawing from Centennial) and 1 from Mystic, going back to 1 HP. Your hand next turn is 6 Burns and a Bludgeon+, which is enough to kill Mystic.

Simpler solution found by many exam-takers: Red Skull and 2 copies of Hand of Greed. (Somehow I forgot that card existed...)

Problem 6

Here is what I believe to be an approximation of the actual graph:



This was generated by a script that is probably not exhaustive, but probably close enough. Here are the properties I care about:

- The limiting behavior. Specifically, there's a value H^+ for which $\forall h \geq H^+, f(h) = \begin{cases} 1 & h \equiv 4 \pmod{5} \\ 0 & \text{otherwise.} \end{cases}$

(I am pretty confident that I am correct about the precise value, which is 95, but honestly just drawing it as “usually 0” for big numbers is fine.) The reason is that if the “3 potions” reward from Neow gives you Entropic Brew, it is possible to repeatedly generate Fruit Juice plus another Entropic Brew, thereby increasing your max HP by 5 arbitrarily many times. But this keeps your max HP the same mod 5, and there are suitable characters and ascension levels for only four of the five residue classes:

- $70 + 5n$ max HP by starting as Silent on Ascension 14-
- $66 + 5n$ max HP by starting as Silent on Ascension 14+
- $72 + 5n$ max HP by starting as Watcher on Ascension 14-
- $68 + 5n$ max HP by starting as Watcher on Ascension 14+

For values that are 4 mod 5, you have to take some max HP modification on floor 0 (such as the “ -17 max HP for reward” bonus from Neow as Silent on Ascension 14+, which brings you to 59), but then you can’t find an Entropic Brew until the floor 1 combat reward.

- The value of $f(1)$. Again, I’m pretty sure mine is correct, but maybe I’m missing something and 19 is possible. Here’s my strategy: Start as high ascension Silent and take -17 max HP from Neow (59). Take the -10% max HP from the Golden Idol event (54). Add a Parasite from Wheel of Change and transform it (51). Now add 6 more Parasites: two from Match & Keep on high ascension, one from Wheel of Change, one from Hypnotizing Colored Mushrooms, one from Duplication Shrine, and one from Dolly’s Mirror at a shop. Finally, add any other curse (e.g. Pain from Ominous Forge). Take Astrolabe as your boss relic and transform the two non-Parasite curses, leaving you with 8 Parasites. Finally, take Apparitions on floor 19 (25), and remove all Parasites at Divine Fountain on floor 20 (1).
- The general shape between these two extremes. Obviously, it should be roughly linear. I don’t care that much about the details. Probably don’t explicitly have anything at e.g. 14 on the y axis though (that’s the pre-boss campfire, where you could technically Peace Pipe a Parasite or Dig for a Mango but surely that will never be optimal).