

Contents

1	Exercise 4.1	2
2	Exercise 4.2	4
3	Exercise 4.3	6
4	Exercise 4.4	8
5	Exercise 4.5	10

1 Exercise 4.1

```
X <- matrix(c(1,1,0,0, 0,0,1,1), nrow=4)
A <- matrix(c(1,0, 0,2), nrow=2)
M <- matrix(c(rep(c(1,1,0,0), 2),
               rep(c(0,0,2,2), 2)),
            nrow=4)
X; A; M
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    1    0
## [3,]    0    1
## [4,]    0    1
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    0    0
## [2,]    1    1    0    0
## [3,]    0    0    2    2
## [4,]    0    0    2    2
```

```
X %*% A %*% t(X) == M
```

```
##      [,1] [,2] [,3] [,4]
## [1,] TRUE TRUE TRUE TRUE
## [2,] TRUE TRUE TRUE TRUE
## [3,] TRUE TRUE TRUE TRUE
## [4,] TRUE TRUE TRUE TRUE
```

```
all(X %*% A %*% t(X) == M)
```

```
## [1] TRUE
```

Note that X is a numeric matrix and so can only have numeric entries. $((i == 2*j) \mid (i == 2*j - 1))$ is a boolean (T or F), and so is converted into a number (1 or 0) when inserted into the numeric matrix.

```

# Create a zero matrix
X=matrix(rep(0, 4*2), nrow=4, ncol=2)
# Change entries selectively
for (i in c(1:4)) {
  for (j in c(1:2)) {
    X[i,j] = ((i==2*j) | (i==2*j-1))
  }
}
A = diag(c(1:2))
M = X %*% A %*% t(X)

X; A; M

```

```

##      [,1] [,2]
## [1,]    1    0
## [2,]    1    0
## [3,]    0    1
## [4,]    0    1

```

```

##      [,1] [,2]
## [1,]    1    0
## [2,]    0    2

```

```

##      [,1] [,2] [,3] [,4]
## [1,]    1    1    0    0
## [2,]    1    1    0    0
## [3,]    0    0    2    2
## [4,]    0    0    2    2

```

```

X = matrix(0, nrow=8, ncol=4)
for (i in c(1:8)) {
  for (j in c(1:4)) {
    X[i,j] = ((i==2*j) | (i==2*j-1))
  }
}
A = diag(1:4)
N = X %*% A %*% t(X)

N

```

```

##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
## [1,]    1    1    0    0    0    0    0    0
## [2,]    1    1    0    0    0    0    0    0
## [3,]    0    0    2    2    0    0    0    0

```

```
## [4,]    0    0    2    2    0    0    0    0
## [5,]    0    0    0    0    3    3    0    0
## [6,]    0    0    0    0    3    3    0    0
## [7,]    0    0    0    0    0    0    4    4
## [8,]    0    0    0    0    0    0    4    4
```

2 Exercise 4.2

```
painlevel <- function(score) {
  if (score < 5) {
    if (score < 2) {
      result <- "no pain"
    } else {
      result <- "minor pain"
    }
  } else if (score < 8) {
    result <- "moderate pain"
  } else {
    result <- "severe pain"
  }
  return(result)
}
painlevel(2)
```

```
## [1] "minor pain"
```

```
painlevel(7)
```

```
## [1] "moderate pain"
```

```
painlevel(9)
```

```
## [1] "severe pain"
```

`painlevel(2)` will give an output of “minor pain”, `painlevel(7)` will give an output of “moderate pain”, `painlevel(9)` will give an output of “severe pain”.

Again, nested if statements are terrible. Reorganising the code makes this much more clear:

```
painlevel <- function(score) {
  if (score < 2) {
    return("no pain")
  }
}
```

```

} else if (score < 5) {
  return ("minor pain")
} else if (score < 8) {
  return ("moderate pain")
} else {
  return ("severe pain")
}
}

```

Using `painlevel()`, one option is to use a for loop:

```

painvector <- function(score) {
  result <- NULL
  for (i in 1:length(score)) {
    result[i] = painlevel(score[i])
  }
  result
}
painvector(c(2,7,9))

```

```
## [1] "minor pain"      "moderate pain" "severe pain"
```

Another approach, instead of dealing with values one by one, is to use functions that handle vectors. In other words, we will *vectorise* this function.

There are several options, the most basic of which uses the `ifelse()` function.

A simple demonstration of the `ifelse()` function should suffice: given a boolean vector `test`, each value in `test` will evaluate to either a value in `yes` (TRUE values) or `no` (FALSE values).

```

# ifelse(test, yes, no)
ifelse(c(T, F, T), c(1, 2, 3), c(4, 5, 6))

```

```
## [1] 1 5 3
```

In this case, with multiple values, only a nested `ifelse()` will work.

```

painlevel <- function(score) {
  results <- ifelse(score < 2, "no pain",
                    ifelse(score < 5, "minor pain",
                            ifelse(score < 8, "moderate pain", "severe
↪ pain"))))
  results
}
painlevel(c(2,7,9))

```

```
## [1] "minor pain"      "moderate pain" "severe pain"
```

Another function to solve this problem is the `cut()` function. Again, to understand how to use these functions, look up the documentation and some examples.

```
painlevel <- function(score) {
  cut(score, c(0, 2, 5, 8, 10),
       c("none", "minor", "moderate", "severe"),
       include.lowest=T, right=F)
}

painlevel(c(2,7,9))
```

```
## [1] minor      moderate severe
## Levels: none minor moderate severe
```

3 Exercise 4.3

The suggested solution makes use of the implicit repeating of rows for Treatment and Period until the number of entries (3 and 9 respectively) matches the number of entries for Subject (27 entries).

```
n=27
Subject = rep(1001:(1001+n-1), rep(3, n))
Treatment <- c("Placebo", "Low Dose", "High Dose")
Period = c(1,2,3,3,1,2,2,3,1)
data.frame(Subject, Treatment, Period)
```

```
## # A tibble: 81 x 3
##   Subject Treatment Period
##   <int> <chr>      <dbl>
## 1    1001 Placebo      1
## 2    1001 Low Dose     2
## 3    1001 High Dose    3
## 4    1002 Placebo      3
## 5    1002 Low Dose     1
## 6    1002 High Dose    2
## 7    1003 Placebo      2
## 8    1003 Low Dose     3
## 9    1003 High Dose    1
## 10   1004 Placebo      1
## # i 71 more rows
```

I find it more comforting to specify the data for each row:

```
n = 27
Subject <- rep(1001:(1001+n-1), rep(3, n))

treatments <- c("Placebo", "Low Dose", "High Dose")
Treatment <- rep(treatments, n)

Period = rep(c(1,2,3,3,1,2,2,3,1), n/3)

ex4.3 <- data.frame(Subject, Treatment, Period)
ex4.3
```

```
## # A tibble: 81 x 3
##   Subject Treatment Period
##   <int> <chr>      <dbl>
## 1    1001 Placebo      1
## 2    1001 Low Dose     2
## 3    1001 High Dose    3
## 4    1002 Placebo      3
## 5    1002 Low Dose     1
## 6    1002 High Dose    2
## 7    1003 Placebo      2
## 8    1003 Low Dose     3
## 9    1003 High Dose    1
## 10   1004 Placebo      1
## # i 71 more rows
```

Out of scope, but a generalised way to make the sequence for any n, assuming the sequence 1-2-3, 3-1-2, 2-3-1:

```
n = 4
Period = rep(0:2, n)
Period
```

```
## [1] 0 1 2 0 1 2 0 1 2 0 1 2
```

```
seq(0, 3*n-1)
```

```
## [1] 0 1 2 3 4 5 6 7 8 9 10 11
```

```
seq(0, 3*n-1) %/% 3 # floor division
```

```
## [1] 0 0 0 1 1 1 2 2 2 3 3 3
```

```
Period = Period - seq(0, 3*n-1) %/% 3
Period
```

```
## [1] 0 1 2 -1 0 1 -2 -1 0 -3 -2 -1
```

```
Period %% 3
```

```
## [1] 0 1 2 2 0 1 1 2 0 0 1 2
```

```
Period %% 3 + 1
```

```
## [1] 1 2 3 3 1 2 2 3 1 1 2 3
```

```
Period = (rep(0:2, n) - seq(0, 3*n-1) %/% 3) %% 3 + 1
Period
```

```
## [1] 1 2 3 3 1 2 2 3 1 1 2 3
```

4 Exercise 4.4

```
CA_result <- data.frame(
  Student_ID = 101:110,
  Test1 = c(78, 87, 67, 53, 89, 57, 83, 92, 74, 82),
  Test2 = c(68, 53, 47, 56, 75, 69, 48, 84, 83, 68),
  Test3 = c(73, 67, 75, 89, 46, 75, 67, 88, 80, 75),
  Test4 = c(93, 73, 67, 73, 52, 86, 86, 87, 53, 65),
  Test5 = c(67, 70, 91, 56, 75, 56, 81, 81, 54, 77)
)

str(CA_result)
```

```
## 'data.frame': 10 obs. of 6 variables:
## $ Student_ID: int 101 102 103 104 105 106 107 108 109 110
## $ Test1 : num 78 87 67 53 89 57 83 92 74 82
## $ Test2 : num 68 53 47 56 75 69 48 84 83 68
```



```
## $ Test3      : num  73 67 75 89 46 75 67 88 80 75
## $ Test4      : num  93 73 67 73 52 86 86 87 53 65
## $ Test5      : num  67 70 91 56 75 56 81 81 54 77
```

The `apply()` function takes in an *iterable object*, a *margin* or *axis*, and a *function* to apply to each entry in the margin.

For example, we have a data frame, which is 2-dimensional. Recall the indexing notation: using a comma within the indexing results in indexing over rows and columns, i.e. the 1st and 2nd dimensions of the dataframe respectively.

Specifying `MARGIN = 1` will iterate over the first dimension (the rows). In this example, it takes the mean across each row.

`MARGIN = 2` will iterate over the second dimension (the columns).

```
CA_result[,2:6]
```

```
## # A tibble: 10 x 5
##   Test1 Test2 Test3 Test4 Test5
##   <dbl> <dbl> <dbl> <dbl> <dbl>
## 1     78     68     73     93     67
## 2     87     53     67     73     70
## 3     67     47     75     67     91
## 4     53     56     89     73     56
## 5     89     75     46     52     75
## 6     57     69     75     86     56
## 7     83     48     67     86     81
## 8     92     84     88     87     81
## 9     74     83     80     53     54
## 10    82     68     75     65     77
```

```
apply(CA_result[,2:6], 1, mean)
```

```
## [1] 75.8 70.0 69.4 65.4 67.4 68.6 73.0 86.4 68.8 73.4
```

You may also have tried to apply the mean function over the row directly, but received errors:

```
CA_result[1, 2:6]
```

```
## # A tibble: 1 x 5
##   Test1 Test2 Test3 Test4 Test5
##   <dbl> <dbl> <dbl> <dbl> <dbl>
## 1     78     68     73     93     67
```

```
mean(CA_result[1, 2:6])
```

```
## Warning in mean.default(CA_result[1, 2:6]): argument is not numeric or logical
## returning NA
```

```
## [1] NA
```

The reason is likely because `mean()` can only act on numeric or logical vectors, but the slice across 1 row is represented as a data frame, not a vector.

To make it work, we use `as.numeric()` to convert the data frame into a vector of numerics.

```
class(CA_result[1, 2:6])
```

```
## [1] "data.frame"
```

```
as.numeric(CA_result[1, 2:6])
```

```
## [1] 78 68 73 93 67
```

```
mean(as.numeric(CA_result[1, 2:6]))
```

```
## [1] 75.8
```

```
best3_mean <- function(scores) {
  mean(sort(scores, decreasing=T)[1:3])
}
```

```
apply(CA_result[,2:6], 1, best3_mean)
```

```
## [1] 81.33333 76.66667 77.66667 72.66667 79.66667 76.66667 83.33333 89.00000
## [9] 79.00000 78.00000
```

5 Exercise 4.5

Making use of the `prod()` function multiplies all entries in a vector, similar to the `sum()` function adding all entries.

Modulo ensures the input is an integer (since it is remainder 0 or 1).

```
f <- function(n) {  
  if (n %% 2 == 0 & n > 0) {  
    x <- seq(2, n, 2)  
    return(prod(x))  
  } else if (n %% 2 == 1 & n > 0) {  
    x <- seq(1, n, 2)  
    return(prod(x))  
  }  
}  
  
f(6); f(7); f(-1); f(1.5)
```

```
## [1] 48
```

```
## [1] 105
```

There is some overlapping code in both the odd and even cases. This can actually be shortened down more, with a change in `seq()` to count down towards 1 or 2 instead of up:

```
f <- function(n) {  
  if (floor(n) == n & n > 0) {  
    x <- seq(n, 1, -2)  
    return(prod(x))  
  }  
}  
  
f(6); f(7); f(-1); f(1.5)
```

```
## [1] 48
```

```
## [1] 105
```

With a while loop:

```
f <- function(n) {  
  if (floor(n) == n & n > 0) {  
    result = 1  
    while (n > 0) {  
      result = result * n  
      n = n - 2  
    }  
    return(result)  
  }  
}  
  
f(6); f(7); f(-1); f(1.5)
```

```
## [1] 48
```

```
## [1] 105
```

With a for loop:

```
f <- function(n) {  
  if (floor(n) == n & n > 0) {  
    result = 1  
    for (i in seq(n, 1, -2)) result = result * i  
    return(result)  
  }  
}  
f(6); f(7); f(-1); f(1.5)
```

```
## [1] 48
```

```
## [1] 105
```