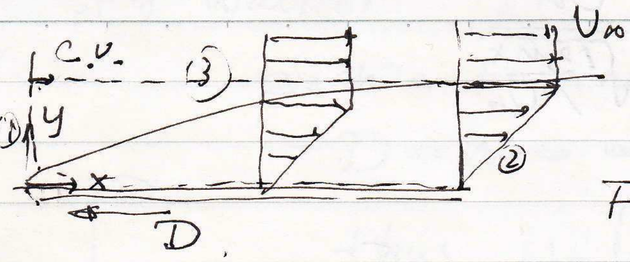
Boundary layer

1.



$u/U_\infty = y/\delta \quad y<\delta$

Find $\tau_w(x)$, c_f, θ, δ^*

① ~~Assume~~ Assume, steady. ~~incomp~~ incompressible

$$\cancel{\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall} + \int_{cs} \vec{V}\rho\, d\vec{A} = 0$$

$$-\rho A U_\infty + \int_0^\delta \rho u\, dy + \dot{m}_3 = 0$$

$$\cancel{\frac{\partial}{\partial t}\int u \rho\, d\forall} + \int_{cs} u \rho \vec{V}\cdot d\vec{A} = \Sigma F_x$$

$$-D = -\rho A U_\infty^2 + \int_0^\delta \rho u^2 dy + \dot{m}_3 U_\infty$$

$$-D = -\rho A U_\infty^2 + \int_0^\delta \rho \frac{U_\infty^2}{\delta^2} y^2 dy + \left(\rho A U_\infty - \int_0^\delta \rho u\, dy\right) U_\infty$$

$$-D = -\rho A U_\infty^2 + \rho \frac{U_\infty^2}{\delta^2}\frac{\delta^3}{3} + \rho A U_\infty^2 - \rho \frac{\delta^2}{2}\frac{U_\infty^2}{\delta}$$

$$D = \frac{1}{6}\rho U_\infty^2 \delta$$

$$\tau_w = \frac{dD}{dx} = \frac{\rho U_\infty^2}{6}\frac{d\delta}{dx}$$

Shear stress on the wall ~~at~~ can also be given by $\tau_w = \mu \frac{du}{dy} = \mu \frac{d}{dy}\left(\frac{U_\infty}{\delta} y\right) = \mu \frac{U_\infty}{\delta}$

Thus $\quad \frac{\rho U_\infty^2}{6}\frac{d\delta}{dx} = \mu \frac{U_\infty}{\delta}$

$$\rho U_\infty \delta\, d\delta = 6 \mu\, dx$$

$$\Rightarrow \frac{\delta^2}{2} = \frac{6\mu}{\rho U_\infty} x, \quad \Rightarrow \frac{\delta^2}{x^2} = \frac{12\mu}{\rho U_\infty x}$$