Assignment #4, Fluid Mechanics 2011

due on Nov. 14, 2011

τ: deviatoric stress tensor; $\mathbf{S}$: rate of strain tensor; $\mathbf{v}$: velocity vector

1: Show that a Newtonian fluid (fluid with constant viscosity) in incompressible flow obeys the relation

$$\nabla \cdot \tau = \mu \nabla^2 \mathbf{v}.$$

2: Show that for a Newtonian fluid the viscous dissipation, Φ, is given by

$$\tau : \nabla \mathbf{v} = -\frac{2}{3}\mu(\nabla \cdot \mathbf{v})^2 + 2\mu \mathbf{S} : \mathbf{S}.$$

Note, symbol $\mathbf{T} : \mathbf{S}$ stands for the inner product of two tensors $T_{ij}S_{ji}$

3: Water flow steadily from a large open reservoir through a short length of pipe and a nozzle with cross-section area of $A = 4\,cm^2$. A well-insulated 10 kW heater surrounds the pipe. Make proper assumptions and find the temperature rise of the water. (hints: use Bernoulli and energy equations, look up the constants you need from textbooks or internet.)