Problem 6.58

Given: Steady flow of water through elbow in horizontal plane.

Find: (a) Gage pressure at ①.
(b) x component of force exerted by elbow on supply pipe.

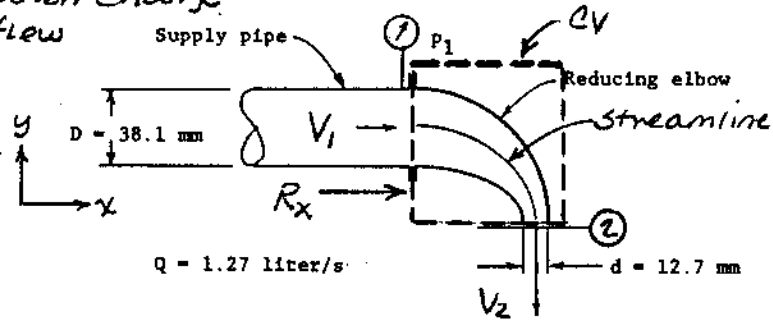
Solution: Apply Bernoulli and momentum equations using streamline and CV shown.

Basic equation: $\frac{p_1}{\rho} + \frac{V_1^2}{2} + g\cancel{z_1} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g\cancel{z_2}$

$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$

($F_{Bx} = 0$ (6); $\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall = 0$ (1))

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Neglect friction
(4) Flow along a streamline
(5) Neglect elevation change
(6) Horizontal flow
(7) $p_2 = p_{atm}$
(8) Uniform flow at each section



Then $p_{1\,gage} = \frac{\rho}{2}(V_2^2 - V_1^2)$ ...

From continuity,

$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2}$

$V_1 = \frac{4}{\pi} \times 1.27 \frac{L}{s} \times \frac{1}{(0.0381)^2 m^2} \times \frac{m^3}{1000 L} = 1.11\ m/s$

and $V_1 A_1 = V_2 A_2$

$V_2 = V_1 \frac{A_1}{A_2} = V_1 \left(\frac{D_1}{D_2}\right)^2 = 1.11 \frac{m}{s}\left(\frac{38.1}{12.7}\right)^2 = 9.99\ m/s$

Thus

$p_{1g} = \frac{1}{2} \times 999 \frac{kg}{m^3}\left[(9.99)^2 - (1.11)^2\right]\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 49.2\ kPa\,(gage)$ p_{1g}

From momentum

$R_x + p_{1g} A_1 = u_1\{-\dot{m}\} + u_2\{+\dot{m}\} = -\dot{m} V_1 = -\rho Q V_1$

$u_1 = V_1 \quad u_2 = 0$

$R_x = -p_{1g} A_1 - \dot{m} V_1 = -49.2 \times 10^3 \frac{N}{m^2} \times \frac{\pi}{4}(0.0381)^2 m^2 - 999 \frac{kg}{m^3} \times 0.00127 \frac{m^3}{s} \times 1.11 \frac{m}{s} \times \frac{N \cdot s^2}{kg \cdot m}$

$R_x = -57.5\ N$ (force on CV)

The force on the supply pipe is

$K_x = -R_x = 57.5\ N$ (on pipe to right) K_x