3-39　　　　　　　　　　　　　　　　　　　　21003079 徐**

连续性方程 $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

动量方程 $\begin{cases} x\text{方向}: u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \\ y\text{方向}: u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial \hat{p}}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \end{cases}$

然后分析每个量的量级.

$\frac{\partial u}{\partial x}\approx\frac{\Delta u}{\Delta x}\approx\frac{U}{L}$, $\frac{\partial u}{\partial y}\approx\frac{\Delta u}{\Delta y}\approx\frac{U}{h}$

$\frac{\partial^2 u}{\partial x^2}\approx\frac{\Delta u}{(\Delta x)^2}\approx\frac{U}{L^2}$, $\frac{\partial^2 u}{\partial y^2}\approx\frac{\Delta u}{(\Delta y)^2}\approx\frac{U}{h^2}$

而 $h\ll L$，故只剩 $\frac{\partial^2 u}{\partial y^2}$，在第一个动量方程中.

$$\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x}=\nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{1}{\rho\nu}\frac{\partial \hat{p}}{\partial x}$$

积分两次可得 $u=\frac{1}{2\mu}\frac{\partial \hat{p}}{\partial x}y^2+Cy+D$

边界条件 $u(y=0)=U$,

$u(y=h(x))=0$.

$$\Rightarrow u=\frac{1}{2\mu}\frac{\partial \hat{p}}{\partial x}y(y-h)+U\left(1-\frac{y}{h}\right)$$

把连续性方程改写为 $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$

$$\int_0^h\frac{\partial u}{\partial x}dy=\int_0^h-\frac{\partial v}{\partial y}dy=-v(h)+v(0)=0$$

把u代入这个式子.

得 $\frac{\partial\left(h^3\frac{\partial \hat{p}}{\partial x}\right)}{\partial x} = 6\mu U \frac{\partial h}{\partial x}$.

代入 $h = h_L + (h_0 - h_L)\left(1-\frac{x}{L}\right)^2$.

设 $p_1 = \frac{\hat{p} - p_\infty}{\mu UL/h_0^2} \Rightarrow \hat{p} = \frac{\mu UL}{h_0^2}p_1 + p_\infty$ 代入原方程得

$$\frac{\partial \hat{p}}{\partial x} = \frac{\mu UL}{h_0^2}\frac{\partial p_1}{\partial x},$$

$$\frac{\partial\left(h^3 \frac{\mu UL}{h_0^2}\frac{\partial p_1}{\partial x}\right)}{\partial x} = 6\mu U\frac{\partial h}{\partial x},$$

此微分方程的边界条件是 $\hat{p}(0)=\hat{p}(L)=p_\infty$,

$$\Rightarrow p_1(0)=p_1(L)=0.$$

$$\Rightarrow h^3 p_1'' + 3h^2h'p_1' = \frac{6h_0^2}{L}h',$$

令 $x_1 = \frac{x}{L}$, ∴ $\frac{dp_1}{dx} = \frac{dp_1}{L\,dx_1}$, $\frac{d^2p_1}{dx^2} = \frac{d^2p_1}{L^2dx_1^2}$,

$$\Rightarrow \frac{h^3}{L^2}\frac{dp_1}{dx_1^2} + \frac{3h^2h'}{L}\frac{dp_1}{dx_1} = \frac{6h_0^2}{L}h',$$

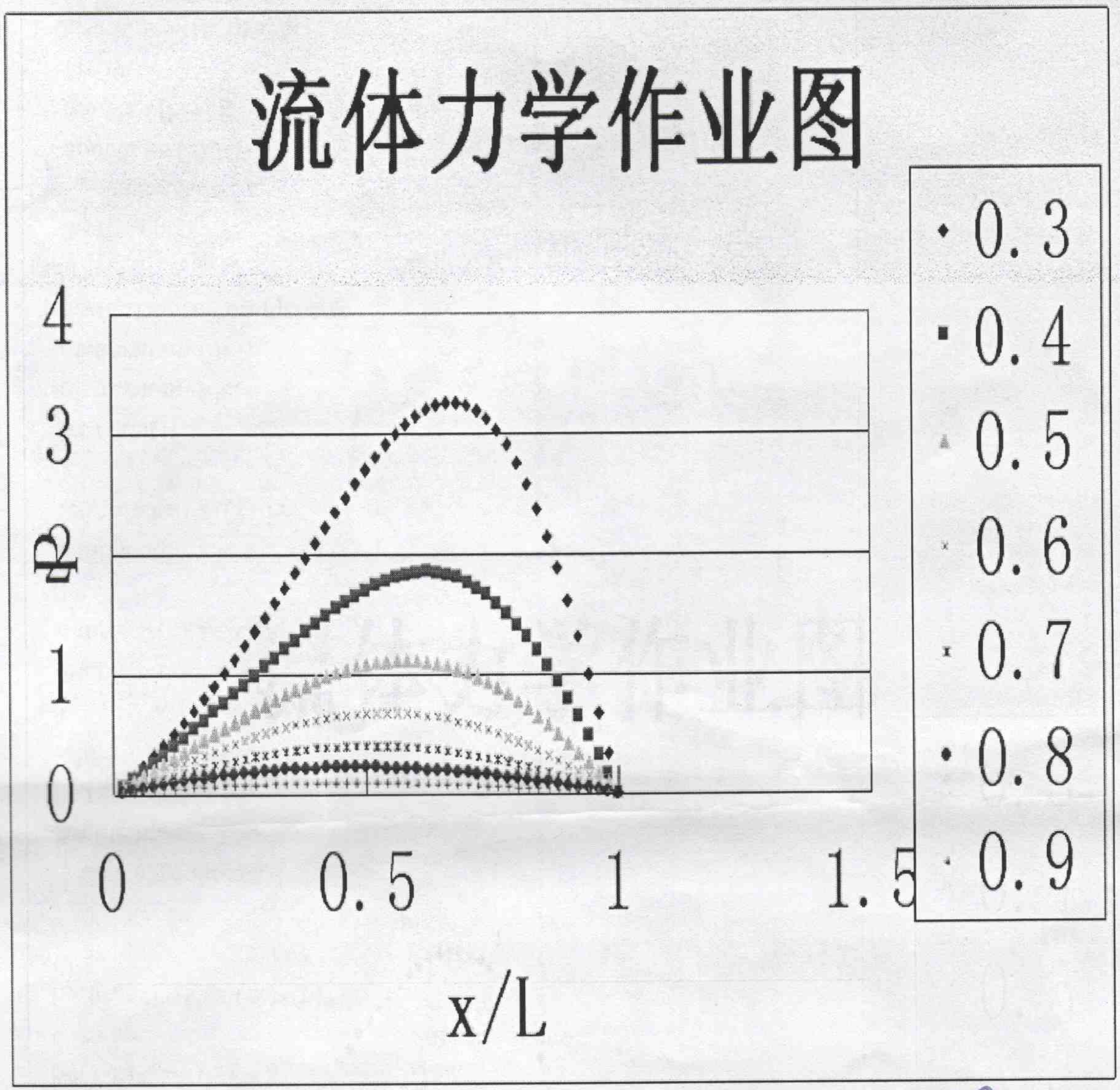
对方程进行无量纲化后的新方程为

$p_1 = \frac{\hat{p} - p_\infty}{\mu UL/h_0^2}$, $x_1 = \frac{x}{L}$, ...

$$\Rightarrow -\left[\frac{h_L}{h_0} + \left(1-\frac{h_L}{h_0}\right)(1-x_1)^2\right]^3 p_1''$$

$$+6\left[1-\frac{h_L}{h_0}\right]\left[\frac{h_L}{h_0}+\left(1-\frac{h_L}{h_0}\right)(1-x_1)^2\right]^2(1-x_1)p_1'$$

$$= 12(1-h_L/h_0)(1-x_1),$$ 边界条件 $x_1=0,1$ 时 $p_1=0$.



差分法，步长0.02

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