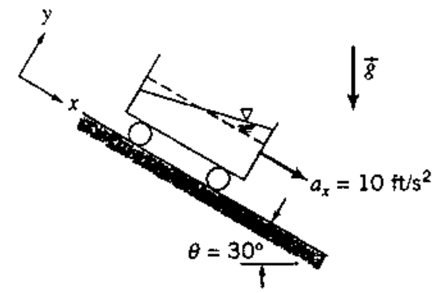
Problem *3.100



Given: Rectangular container of water undergoing constant acceleration as shown

Determine: The slope of the free surface

Solution:

Basic equation: $-\nabla P + \rho\vec{g} = \rho\vec{a}$

Writing the component equations

$$\left.\begin{aligned} -\frac{\partial P}{\partial x} + \rho g_x &= \rho a_x \\ -\frac{\partial P}{\partial y} + \rho g_y &= \rho a_y \\ -\frac{\partial P}{\partial z} + \rho g_z &= \rho a_z \end{aligned}\right\} \quad \left.\begin{aligned} &\text{For given coordinates} \\ &a_y = a_z = 0 \\ &g_y = -g\cos\theta \\ &g_x = g\sin\theta \\ &g_z = 0 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} \frac{\partial P}{\partial x} &= \rho g\sin\theta - \rho a_x \\ \frac{\partial P}{\partial y} &= -\rho g\cos\theta \\ \frac{\partial P}{\partial z} &= 0 \end{aligned}$$

From the component equations we conclude that $P = P(x,y)$

Then

$$dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy$$

Along the free surface P = constant and $dP = 0$. Hence

$$\left.\frac{dy}{dx}\right)_{surface} = -\frac{\partial P/\partial x}{\partial P/\partial y} = \frac{\rho g\sin\theta - \rho a_x}{\rho g\cos\theta}$$

$$= \frac{g\sin\theta - a_x}{g\cos\theta}$$

$$= \frac{32.2(0.5)\ \text{ft/s}^2 - 10\ \text{ft/s}^2}{32.2\,(0.866)\ \text{ft/s}^2}$$

$$\left.\frac{dy}{dx}\right)_{surface} = 0.22$$