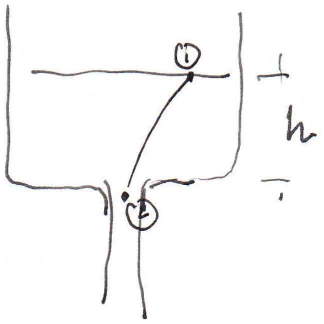
Dimensional Analysis



	$\dot{m}$ ~	f(V,	g,	h,	D,	μ,	σ,	ρ
M	1					1	1	1
L		1	1	1	1	−1	0	−3
T	−1	−1	−2			−1	−2	0

Choose ρ, V, D three variables

$$\Pi_1 = \frac{\dot{m}}{\rho V D^2}$$

$$\Pi_2 = \frac{\mu}{\rho V D}$$

$$\Pi_3 = \frac{\sigma}{\rho V^2 D}$$

$$\Pi_4 = \frac{h}{D}$$

$$\Pi_5 = \frac{gD}{V^2}$$

, by observation, we found gh is a better way to normalize V^2

you can prove this by applying Bernoulli between (1) and

$$\cancelto{0}{\frac{P_1}{\rho g}} + \cancelto{0}{\frac{V_1^2}{2g}} + z_1 = \cancelto{0}{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + z_2.$$

gage pressure zero　　water level drops slow.　　gage pressure zero

Thus.

$$\frac{\dot{m}}{\rho V D^2} \sim f\left(\frac{gh}{V^2}, \frac{h}{D}, \frac{\mu}{\rho V D}, \frac{\sigma}{\rho V^2 D}\right)$$

For dynamic simularity

$$\Pi_1: \quad \frac{\dot{m}_{model}}{\rho V_{model} D^2_{model}} = \frac{\dot{m}_{prototyp}}{\rho V_{prototype} D_{prototype}}$$

$$\frac{\dot{m}_m}{\dot{m}_p} = \left(\frac{V_m}{V_p}\right)\left(\frac{D_m}{D_p}\right)^2$$

also Π_4: $\dfrac{h_m}{D_m} = \dfrac{h_p}{D_p}$, $\dfrac{h_m}{h_p} = \dfrac{D_m}{D_p} = \dfrac{1}{5}$

$$\Pi_5: \quad \frac{g \cancel{D_m} h_m}{V_m^2} = \frac{g h_p}{V_p^2}, \quad \frac{V_m}{V_p} = \sqrt{\frac{h_m}{h_p}} = \frac{1}{\sqrt{5}}$$

thus $\dfrac{\dot{m}_m}{\dot{m}_p} = \left(\dfrac{1}{5}\right)^2 \dfrac{1}{\sqrt{5}} = 0.0179$

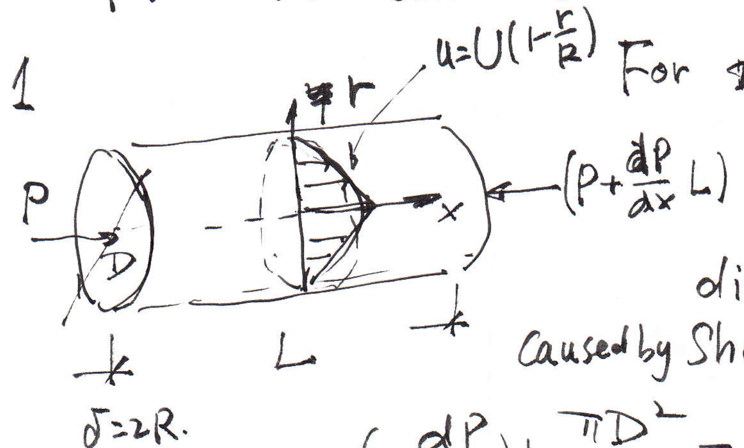
For full dynamic simularity, also requires

$$\Pi_2: \quad \frac{\mu}{\rho V_m D_m} = \frac{\mu}{\rho V_p D_p}, \quad \frac{V_m}{V_p} = \frac{D_p}{D_m} = 5$$

can not satisfy.

Flow in Conduit

1

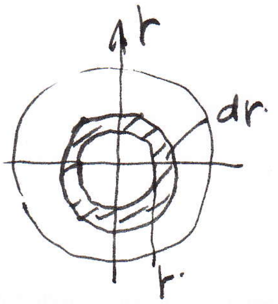


For a fluid in a circular pipe force caused by the pressure difference in the streamwise direction is balanced by the force caused by Shear stress on the wall

$$\left(-\frac{dP}{dx}\right)L\frac{\pi D^2}{4} = \tau_w \pi D L$$

$$\frac{dP}{dx} = -\frac{4\tau_w}{D}$$

average velocity $\overline{V} = \dfrac{\int_0^R u\, 2\pi r\, dr}{\pi R^2}$



$$\overline{V} = \frac{\int_0^R U(1-r/R)\, 2\pi r\, dr}{\pi R^2}$$

$$= 2\pi U\left(\frac{R^2}{2} - \frac{R^2}{3}\right) / \pi R^2$$

$$= U/3$$

$$\tau_w = \mu \frac{du}{dr}\Big|_{r=R} = \mu\frac{U}{R} = 2\mu\frac{U}{D}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho\overline{V}^2} = \frac{2\mu U}{D\frac{1}{2}\rho\frac{U^2}{9}} = \frac{36\mu}{\rho U D}$$