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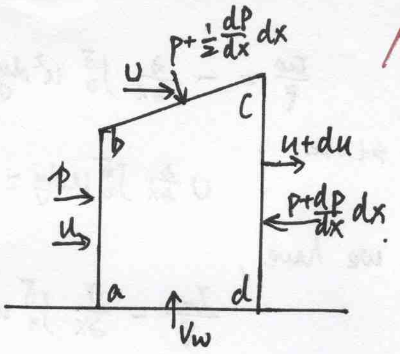
P331 4-19 ~~Soul~~

Solution.



Assumption: ① Steady flow

② 2-D flow.

Continuity equation:

$$\frac{\partial}{\partial t}\int_{cv}\rho\, dv + \int_{cs}\rho\vec{V}\cdot d\vec{A} = 0.$$

$$\frac{\partial}{\partial t}\int_{cv}\rho\, dv = 0.$$

$$\Rightarrow \int_{cs}\rho\vec{V}\cdot d\vec{A} = \dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} + \dot{m}_{da} = 0.$$

$$\Rightarrow \dot{m}_{bc} = -\dot{m}_{ab} - \dot{m}_{cd} - \dot{m}_{da}$$

$$= -\left(-\int_0^\delta \rho u\, dy\right) - \left[\int_0^\delta \rho u\, dy + \frac{\partial}{\partial x}\left(\int_0^\delta \rho u\, dy\right)dx\right] - \left[-\rho\left(v_w + \frac{1}{2}dv_w\right)dx\right]$$

$$= -\frac{\partial}{\partial x}\left(\int_0^\delta \rho u\, dy\right)dx + \rho\left(v_w + \frac{1}{2}dv_w\right)dx.$$

X-Momentum Equation:

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{cv} u\rho\, dv + \int_{cs} u\rho\vec{V}\cdot d\vec{A}$$

Body force $F_{Bx}=0$, and $\frac{\partial}{\partial t}\int_{cv} u\rho\, dv = 0$, ~~so we have~~

$$F_{sx} = F_{ab} + F_{cd} + F_{bc} + F_{da}$$

$$= p\delta + \left[-\left(p+\frac{dp}{dx}dx\right)\cdot(\delta + d\delta)\right] + \left(p + \frac{1}{2}\frac{dp}{dx}dx\right)\cdot d\delta + \left[-\left(\tau_w + \frac{1}{2}d\tau_w\right)\cdot dx\right]$$

$$= -\frac{dp}{dx}\delta\, dx - \frac{1}{2}\frac{dp}{dx}dx\, d\delta - \tau_w dx - \frac{1}{2}d\tau_w\, dx$$

$$\int_{cs} u\rho\vec{V}\cdot d\vec{A} = \int_{ab} u\rho\vec{V}\cdot d\vec{A} + \int_{bc} u\rho\vec{V}\cdot d\vec{A} + \int_{cd} u\rho\vec{V}\cdot d\vec{A} + \int_{da} u\rho\vec{V}\cdot d\vec{A}$$

$$= \left(-\int_0^\delta u\rho u\, dy\right) + U\dot{m}_{bc} + \left[\int_0^\delta u\rho u\, dy + \frac{\partial}{\partial x}\left(\int_0^\delta u\rho u\, dy\right)dx\right] + 0$$

$$= \frac{\partial}{\partial x}\left(\int_0^\delta u\rho u\, dy\right)dx + U\frac{\partial}{\partial x}\left(\int_0^\delta \rho u\, dy\right)dx + \rho U v_w dx + \frac{1}{2}\rho U dv_w dx$$

so,

$$-\frac{dp}{dx}\delta\, dx - \frac{1}{2}\frac{dp}{dx}dx\, d\delta - \tau_w dx - \frac{1}{2}d\tau_w dx = \frac{\partial}{\partial x}\left(\int_0^\delta u\rho u\, dy\right)dx + U\frac{\partial}{\partial x}\left(\int_0^\delta \rho u\, dy\right) + \rho U v_w dx + \frac{1}{2}\rho U dv_w dx.$$

Neglecting the terms of small quantities of second order and diving the equation by dx, we obtain

$$-\frac{dp}{dx}\delta - \tau_w = \frac{\partial}{\partial x}\left(\int_0^\delta u\rho u\, dy\right) + U\frac{\partial}{\partial x}\int_0^\delta \rho u\, dy + \rho U v_w \qquad (*)$$

Applying the Bernoulli equation $p + \frac{1}{2}\rho U^2 + \rho g y = const$ to the inviscid flow outside the boundary layer, we have

$$\frac{dp}{dx} = -\rho U\frac{dU}{dx}$$

Noting that $\delta = \int_0^\delta dy$, we can rewritten the equation (*) as

$$\frac{\tau_w}{\rho} = -\frac{\partial}{\partial x}\int_0^\delta u^2 dy + U\frac{\partial}{\partial x}\int_0^\delta u\,dy + \frac{dU}{dx}\int_0^\delta U\,dy - UV_w$$

since

$$U\frac{\partial}{\partial x}\int_0^\delta u\,dy = \frac{\partial}{\partial x}\int_0^\delta uU\,dy - \frac{dU}{dx}\int_0^\delta u\,dy.$$

we have,

$$\frac{\tau_w}{\rho} = \frac{\partial}{\partial x}\int_0^\delta u(U-u)\,dy + \frac{dU}{dx}\int_0^\delta (U-u)\,dy - UV_w$$

i.e.
$$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + U\delta^*\frac{dU}{dx} - UV_w$$

尽管没有摊开和懂做
但依然很好!
Bravo.

P_{333} 4-37. Solution.

For the symmilarity of the problem, we just consider the half of the problem.

Assumptions: ① Steady flow,
② $V_w = 0$.

x- Momentum equation (4-120)

$$\frac{\tau_w}{\rho} = \frac{\partial}{\partial x}\int_0^\infty u(U(x)-u)\,dy + \frac{dU(x)}{dx}\int_0^\infty (U(x)-u)\,dy$$

substituting $\tau_w = \mu\frac{\partial u}{\partial y}|_{y=0}$ and $u = U(x)\left(\frac{2y}{\delta(x)} - \frac{y^2}{\delta^2(x)}\right)$ into the above equation,

we obtain
$$\frac{2\nu U(x)}{\delta(x)} = \frac{d}{dx}\left(U^2(x)\cdot\frac{2}{15}\delta(x)\right) + \frac{dU(x)}{dx}\cdot U(x)\cdot\frac{\delta(x)}{3} \qquad (*)$$

Conservation of mass

$$\int_0^H U_0\,dy = \int_0^\delta u\,dy + \int_\delta^H U(x)\,dy$$

Substituting the expressions for u, we have

$$U_0 H = U(x)\left(H - \tfrac{1}{3}\delta(x)\right) \qquad (**)$$

$$\Rightarrow U(x) = \frac{U_0 H}{H - \frac{1}{3}\delta(x)}.$$

substituting the expressions for $U(x)$ into Eq. (*), we have

$$\frac{\delta(x)\,U_0 H\left(2H + \frac{7}{3}\delta(x)\right)}{30\nu\left(H - \frac{1}{3}\delta(x)\right)^2}\frac{d\delta(x)}{dx} = 1.$$

With integrating the above equation and $\delta(0)=0$, we obtain,

$$\frac{7U_0H}{10\nu}\delta(x) + \frac{81U_0H^3}{10\nu(3H-\delta(x))} + \frac{24U_0H^2}{5\nu}\ln(3H-\delta) = x + \frac{27U_0H^2}{10\nu} + \frac{24U_0H^2}{5\nu}\ln(3H)$$

From the above equation, we can obtain $U(x)$, and then we can obtain $\delta(x)$ from the equation (**) by using $U(x)$.

At $x = x_L$, $\delta(x) = H$, and substituting it into the above equation, we obtain.

$$x_L = \cancel{\frac{41U_0H^3}{10\nu(3H-}}\ \frac{41U_0H^2}{20\nu} + \frac{24U_0H^2}{5\nu}\ln\frac{2}{3}$$