Engineering Mathematics II, Mid-term Exam #1
Differential Equations
Civil Engineering 06, Oct 26 2007

Question 1:[5 marks] Explain the Dirichlet, Newmann and Robin types of boundary conditions, each using one sentence.

Question 2:[15 marks] Classify the following partial differential equation

$$xu_{xx} - (sin^2 y + 1)u_{yy} = x^2 u$$

, defined over $-\infty < x < \infty$ and $-\infty < y < \infty$, as parabolic, hyperbolic or elliptic. If the equation is of mixed type, identify the relevant regions and give classification within each region.

Question 3:[15 marks] Show the following equation is linear or nonlinear

$$u_{xx} + 5u_{yy} - xu - e^x = 0$$

Question 4:[15 marks] Determine the Fourier coefficients of the periodic function, $f(x)$.

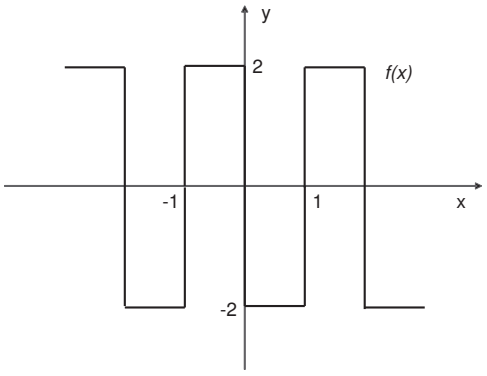

Figure 1: Figure for question 4.

Question 5:[15 marks] The temperature changes in an infinite long wall was governed by the diffusion equation

$$u_t = \alpha^2 u_{xx}$$

, where u is the temperature, α is the thermal conductivity of the wall, x is the spatial coordinate and t is time. Obtain a set of ordinary differential equations using the method of separation of variables.

Question 6:[15 marks] Solve the non-homogeneous first order ordinary differential equation

$$x\frac{dy}{dx} + y = \frac{1}{x}$$

in the domain $x > 0$ using variation of parameters method.

Question 7:[10 marks] Determine the fundamental period and the Fourier coefficients of the periodic function

$$f(x) = 2cosx + 4sinx + cos2x + 2sin2x + 2cos4x + sin4x + \frac{1}{2}sin\frac{\pi}{2}$$

, where $x \in (-\infty, \infty)$.

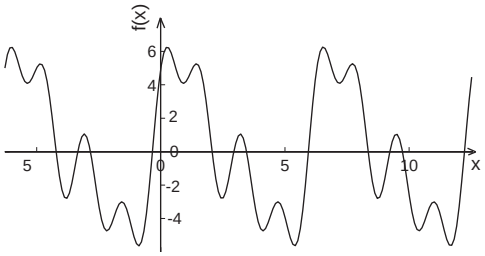

Figure 2: Figure for question 7.

Question 8:[10 marks] The motion of a simple spring-mass system with friction damping is governed by equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

, where the three terms represent inertia, friction force and spring force, respectively. Here, m is $0.5kg$, c is $3Ns/m$ and k is $4N/m$. Find the solution if the mass was pulled slowly to $x = 0.1m$ and then released. There is no deformation in the spring when the block is at the origin of x axis.

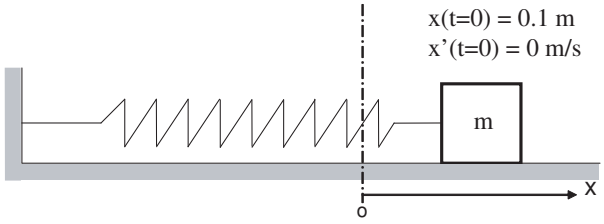

Figure 3: Figure for question 8.