Open-Ended Problem Statement: Several toy manufacturers sell water "rockets" that consist of plastic tanks to be partially filled with water and then pressurized with air. Upon release, the compressed air forces water out the nozzle rapidly, propelling the rocket. You are asked to help specify optimum conditions for this water-jet propulsion system. To simplify the analysis, consider horizontal motion only. Perform the analysis and design needed to define the acceleration performance of the compressed air/water-propelled rocket. Identify the fraction of tank volume that initially should be filled with compressed air to achieve optimum performance (i.e., maximum speed from the water charge). Describe the effect of varying the initial air pressure in the tank.

Discussion: The process may be modeled as a polytropic expansion of the trapped air which forces water out the jet nozzle, causing the "rocket" to accelerate. The polytropic exponent may be varied to model anything from an isothermal expansion process ($n = 1$) to an adiabatic expansion process ($n = k$), which is more likely to be an accurate model for the sudden expansion of the air.

Speed of the water jet leaving the "rocket" is proportional to the square root of the pressure difference between the tank and atmosphere.

Qualitatively it is apparent that the smaller the initial volume fraction of trapped air, the larger will be the expansion ratio of the air, and the more rapid will be the pressure reduction as the air expands. This will cause the water jet speed to drop rapidly. The combination of low water jet speed and relatively large mass of water will produce sluggish acceleration.

Increasing the initial volume fraction of air will reduce the expansion ratio, so higher pressure will be maintained longer in the tank and the water jet will maintain higher speed longer. This combined with the relatively small mass of water in the tank will produce rapid acceleration.

If the initial volume fraction of air is too large, all water will be expended before the air pressure is reduced significantly. In this situation, some of the stored energy of the air will be dissipated in a relatively ineffective air jet. Consequently, for any initial pressure in the tank, there is an optimum initial air fraction.

This problem cannot be solved in closed form because of the varying air pressure, mass flow rate, and mass of water in the tank;. it can only be solved numerically. One possible integration scheme is to increment time and solve for all properties of the system at each instant. The drawback to this scheme is that the water is unlikely to be exhausted at an even increment of time. A second scheme is to increment the volume of water remaining and solve for properties using the average flow rate during the interval. This scheme is outlined below.

Model the air/water jet-propelled "rocket" using the CV and coordinates shown.

First choose dimensions and mass of "rocket" to be simulated:
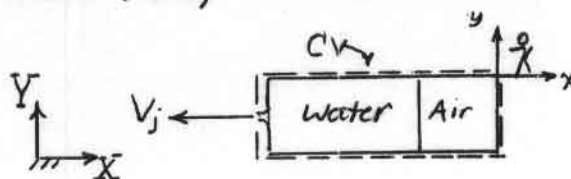
Input Data:

Jet diameter:	$D_j =$	0.003	m
Tank diameter:	$D_t =$	0.035	m
Tank length:	$L =$	0.1	m
Tank mass:	$M_t =$	0.01	kg
Polytropic exponent:	$n =$	1.4	---

Next choose initial conditions for the simulation (see sample calculations below):