**Initial Conditions:**

| | | | |
|---|---|---|---|
| Air fraction in tank: | $\alpha =$ | 0.5 | --- |
| Tank pressure: | $p_0 =$ | 200 | kPa (gage) |
| Volume increment: | $\Delta\alpha =$ | 0.02 | --- |

Compute reference parameters:

**Calculated Parameters:**

| | | | |
|---|---|---|---|
| Jet area: | $A_j =$ | 7.07E-06 | $m^2$ |
| Tank volume: | $V_t =$ | 9.62E-05 | $m^3$ |
| Initial air volume: | $V_0 =$ | 4.81E-05 | $m^3$ |
| Initial water mass: | $M_0 =$ | 0.0481 | kg |

(These are used in the spreadsheet below.)

Then decrease the water fraction in the tank by $\Delta\alpha$:

**Calculated Results:**

| Water Fraction, $V_w/V_t$ (---) | Gage Pressure, $p$ (kPa) | Water Mass, $M_w$ (kg) | Jet Speed, $V_j$ (m/s) | Flow Rate, $dm/dt$ (kg/s) | Time Interval, $\Delta t$ (s) | Current Time, $t$ (s) | "Rocket" Accel., $a$ (m/s$^2$) | "Rocket" Speed, $U$ (m/s) |
|---|---|---|---|---|---|---|---|---|
| 0.50 | 200 | 0.0481 | 20.0 | 0.141 | 0 | 0 | 48.7 | 0 |
| 0.48 | 184 | 0.0461 | 19.2 | 0.135 | 0.0139 | 0.0139 | 47.5 | 0.668 |

The computation is made as follows:

(1) Decrease $\alpha$ by $\Delta\alpha$

(2) Compute $p$ from $p = p_0\left(\frac{V_0}{V}\right)^n$

$$p = (200+101.325)\,kPa\left(\frac{0.50}{0.52}\right)^{1.4} - 101.325 = 183.9 \text{ kPa (gage)}$$

(3) Use Bernoulli to calculate jet speed

$$V_j = \sqrt{\frac{2\Delta p}{\rho}} = \left[2 \times 183.9\times10^3 \frac{N}{m^2} \times \frac{m^3}{999\,kg} \times \frac{kg\cdot m}{N\cdot s^2}\right]^{\frac{1}{2}} = 19.10 \text{ m/s}^*$$

(4) Calculate water mass using $\alpha$.

(5) Use conservation of mass to compute mass flow rate

$$\dot{m} = \rho V_j A_j = 999\frac{kg}{m^3} \times 19.10\frac{m}{s} \times 7.07\times10^{-6} m^2 = 0.1349 \text{ kg/s}$$

(6) Use the average mass flow rate during the interval to approximate $\Delta t$:

$$\Delta t = \frac{\Delta m}{dm/dt} = \frac{\Delta m}{\bar{\dot{m}}} = (0.0481-0.0461)\,kg \times \frac{s}{0.138\,kg} = 0.01449 \text{ s}^*$$

(7) Use momentum to compute acceleration (note $M = M_w + M_t$):

$$a_{rf_x} = \frac{\dot{m}V_j}{M} = 0.135\frac{kg}{s} \times 19.2\frac{m}{s} \times \frac{1}{0.0461+0.0100\,kg} = 46.2 \text{ m/s}^{2*}$$

(8) Finally use average acceleration to get speed

$$U = U_0 + \bar{a}\Delta t = 0 + 48.1\frac{m}{s^2} \times 0.0139\,s = 0.669 \text{ m/s}^*$$

* Note effect of roundoff error.