已知 $u=yt$，$v=xt$，$w=0$ 求

1. 加速度场 2. 迹线及流线 3. 散度及旋度

4. 速度的拉格朗日描述.

解 1. 求加速度场 $u=yt$，$v=xt$，$w=0$ 为欧拉描述

质点加速度为 $\frac{Du}{Dt}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=y+xt^2$

$$\frac{Dv}{Dt}=x+yt^2 \quad , \quad \frac{Dw}{Dt}=0$$

也可认为某微团 t时刻经过 x, y，它的加速度为

$$\frac{du}{\partial t}=\frac{\partial(yt)}{\partial t}=\frac{\partial y}{\partial t}t+y=xt^2+y$$

$$\frac{\partial v}{\partial t}=\frac{\partial(xt)}{\partial t}=\frac{\partial x}{\partial t}t+x=ut+x=yt^2+y$$

2. 迹线 $dx/dt=u=yt$

$dy/dt=v=xt$

$dz/dt=w=0$

$$\left\{\begin{array}{l}\frac{dy}{dx}=\frac{x}{y} \Rightarrow x^2-y^2=C_1 \\ z=C_2 \\ \cancel{dz=0}\end{array}\right.$$

流线. $\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}$

$$\frac{dx}{yt}=\frac{dy}{xt} \Rightarrow x^2-y^2=C_1$$

3 散度 $\mathrm{div}\,\vec{V}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

旋度 $\nabla\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yt & xt & 0\end{vmatrix}=\vec{k}\left(\frac{\partial yt}{\partial y}-\frac{\partial xt}{\partial x}\right)=0$

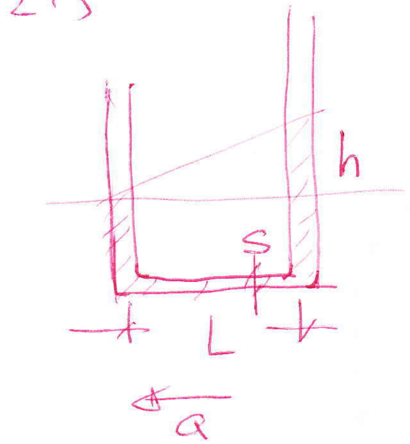
4. 速度拉格朗日描述 $\frac{dx}{dt}=u=yt$，$\frac{dy}{dt}=v=xt$，$\frac{dz}{dt}=0$

$\frac{d(x+y)}{dt}=(x+y)t$ ，$x+y=C_1e^{\frac{t^2}{2}}$

$\frac{d(x-y)}{dt}=-(x-y)t$. $x-y=C_2e^{-t^2/2}$

$x=\frac{1}{2}C_1e^{t^2/2}+\frac{1}{2}C_2e^{-t^2/2}$，$u=\frac{dx}{dt}=\frac{t}{2}\left(C_1e^{\frac{t^2}{2}}+C_2e^{-\frac{t^2}{2}}\right.$

2.5

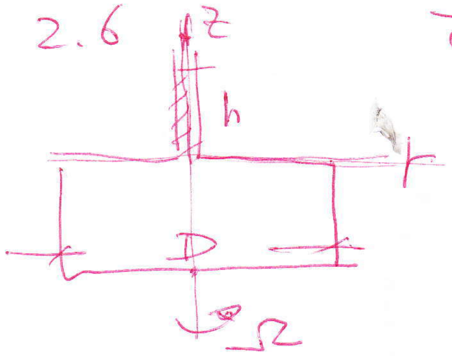


求a.

$$\rho g h S = \rho S L a$$

$$a = \frac{h}{L} g.$$

2.6



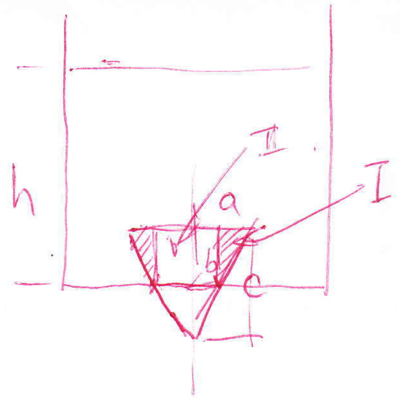
求 $z=0$ 平面合力.

$$p = \frac{1}{2}\rho\omega^2 r^2 + \rho g h$$

$$F = \int_0^{D/2} p \, 2\pi r \, dr$$

$$F = \rho \frac{\pi \Omega^2 D^4}{64} + \rho g h \frac{\pi D^2}{4}$$

2.14



求总合力

I部分合力 $F_1 = \frac{1}{3}\pi a^2 c \rho g - \frac{1}{3}\pi b^2 c \frac{b}{a} \rho g - \pi b^2 \frac{a-b}{a} \cdot c \rho g$.

向上

II部分合力 $F_2 = \pi b^2 \left(h - \frac{a-b}{a} c\right) \rho g$

向下.

合力 $F = F_2 - F_1$

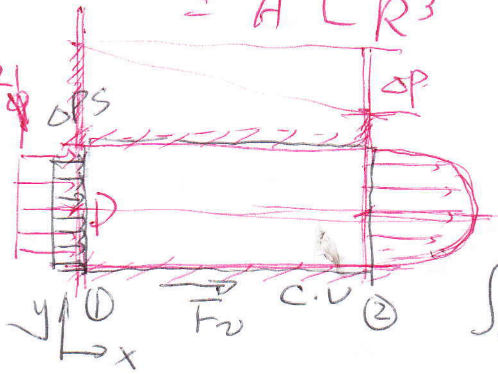
3.8 证 $V_i=\frac{Ax_i}{R^3}$ $i=1,2,3$ 满足连续性方程，$R^2=x^2+y^2+z^2$

$$\operatorname{div}\vec{V}=\partial(Ax/R^3)/\partial x+\partial(Ay/R^3)/\partial y+\partial(Az/R^3)/\partial z$$

$$=A\left[\frac{1}{R^3}-3\frac{x}{R^4}\frac{x}{R}+\frac{1}{R^3}-3\frac{y}{R^4}\frac{y}{R}+\frac{1}{R^3}-3\frac{z}{R^4}\frac{z}{R}\right]$$

$$=A\left[\frac{3}{R^3}-3\frac{(x^2+y^2+z^2)}{R^5}\right]=0$$

3.21



$u=U(1-\frac{2r}{D})^{1/7}$

$\int_{cs}\vec{V}\cdot d\vec{A}=0$ 质量守恒

$$\frac{\pi D^2}{4}U_1=\int_0^{D/2}U(1-\frac{2r}{D})^{1/7}\,2\pi r\,dr$$

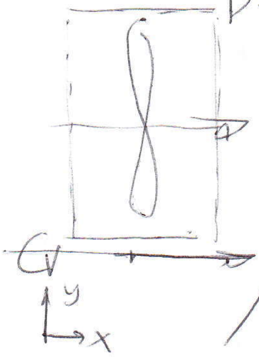
$$U=\frac{60}{49}U_1$$

$\sum\vec{F}=\int_{cs}\vec{V}\rho\vec{V}\cdot d\vec{A}$ 动量方程

$$\Delta p\frac{\pi D^2}{4}+F_v=\int_0^{D/2}U^2(1-\frac{2r}{D})^{2/7}\rho\,2\pi r\,dr$$ — 原来的动量

$$F_v=\frac{\pi D^2}{4}\left(-\Delta p+\frac{1}{49}\rho U_1^2\right)$$

4.13



$D=0.3m$

$\dot{Q}=60m^3/min$

错了

μmg

$\sum\vec{F}=\int_{cs}\vec{V}\rho\vec{V}\cdot d\vec{A}$

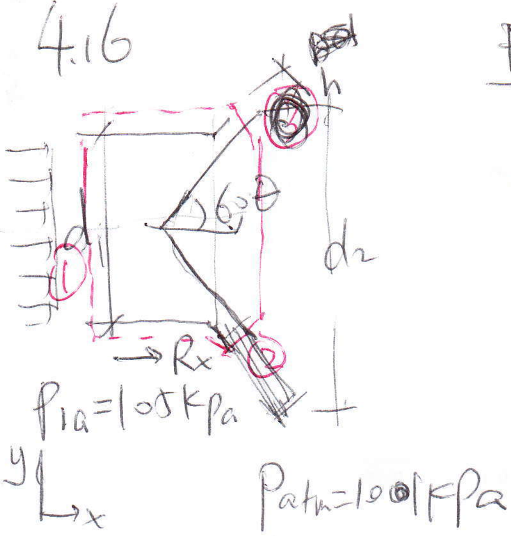
$\cancel{\mu mg\ G=\rho\dot{Q}\frac{\dot{Q}}{\pi D^4/4}}$

$\cancel{\mu G=\frac{4\rho\dot{Q}^2}{\mu\pi D^4}}$

$$\mu G=\rho\dot{Q}\left(\frac{\dot{Q}}{0.9\pi D^2/4}-\frac{\dot{Q}}{1.1\pi D^2/4}\right)$$

$$G=\frac{4\rho\dot{Q}^2}{\mu\pi D^2}\left(\frac{1}{0.9}-\frac{1}{1.1}\right)=\frac{4\times1.2\times1}{0.1\times3.14\times0.3^2}$$

4.16



$$\frac{P_1}{\rho}+\frac{V_1^2}{2}=\frac{P_2}{\rho}+\frac{V_2^2}{2}$$

$$V_1\frac{\pi d_1^2}{4}=V_2\,\pi d_2 h$$

得. V_1, V_2

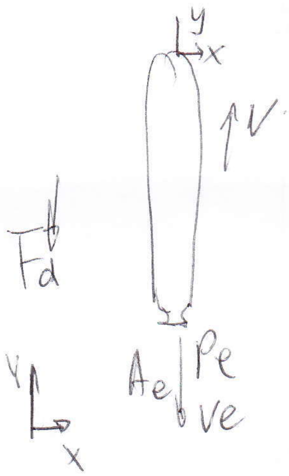
$$\sum\vec{F}=\int_{cs}\vec{V}\,\rho\vec{V}\cdot d\vec{A}$$

$$(P_{1a}-P_{atm})\frac{\pi d_1^2}{4}+R_x=\underline{V_2\cos\theta}\;\underline{\pi d_2 h}\;\underline{V_2}$$

$-\rho V_1 S_1 V_1$

可得. R_x

4.25 M_0, P_{atm}, g, $\dot{m}_f$, V_e, P_e



$$-(M-\dot{m}t)g-D+\Delta P A_e=(M-\dot{m}t)a+(-\dot{m}V_e$$

$$a=\frac{\dot{m}V_e+\Delta P A_e-D}{M-\dot{m}t}-g$$

推力 $T-D-Mg=Ma$

$$T=M(a+g)+D$$

$$du=\frac{\partial u}{\partial t}dt+\frac{\partial u}{\partial y}dy$$