湍流基础讲座5

湍流实验数据分析处理

高南

2007年12月27日

内容

- ✓单传感器测量分析
  - 统计分布
  - 基本统计量
  - 自相关性
  - 谱分布
- ✓多传感器信号分析
  - 交叉谱
  - 相关性等

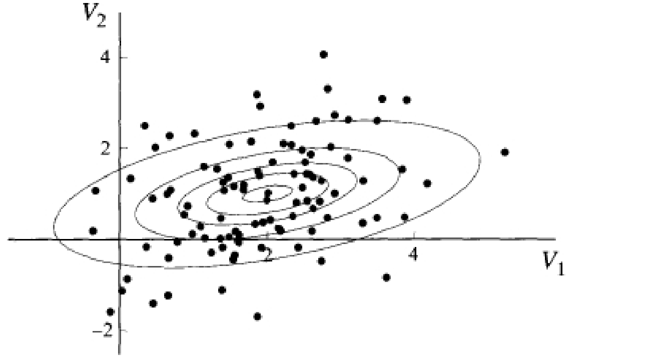
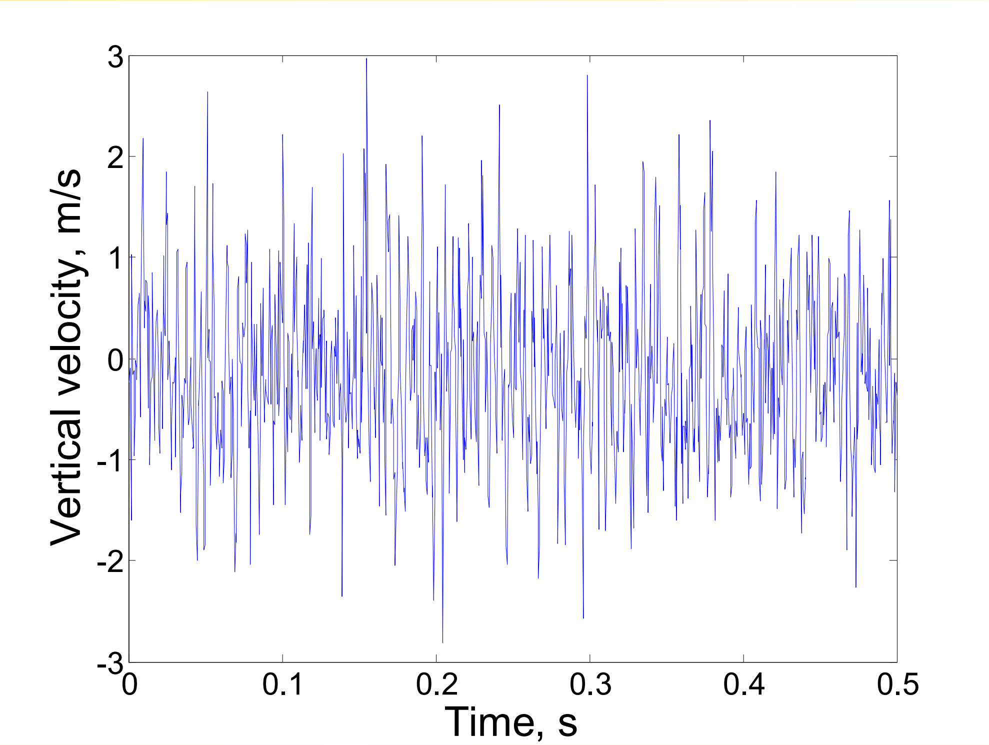


Fig. 3.18. A scatter plot and constant-probability density lines in the V_1–V_2 plane for joint-normal random variables (U_1, U_2) with $\langle U_1\rangle = 2$, $\langle U_2\rangle = 1$, $\langle u_1^2\rangle = 1$, $\langle u_2^2\rangle = \frac{5}{16}$, and $\rho_{12} = 1/\sqrt{5}$.

取自A Pope, Turbulence

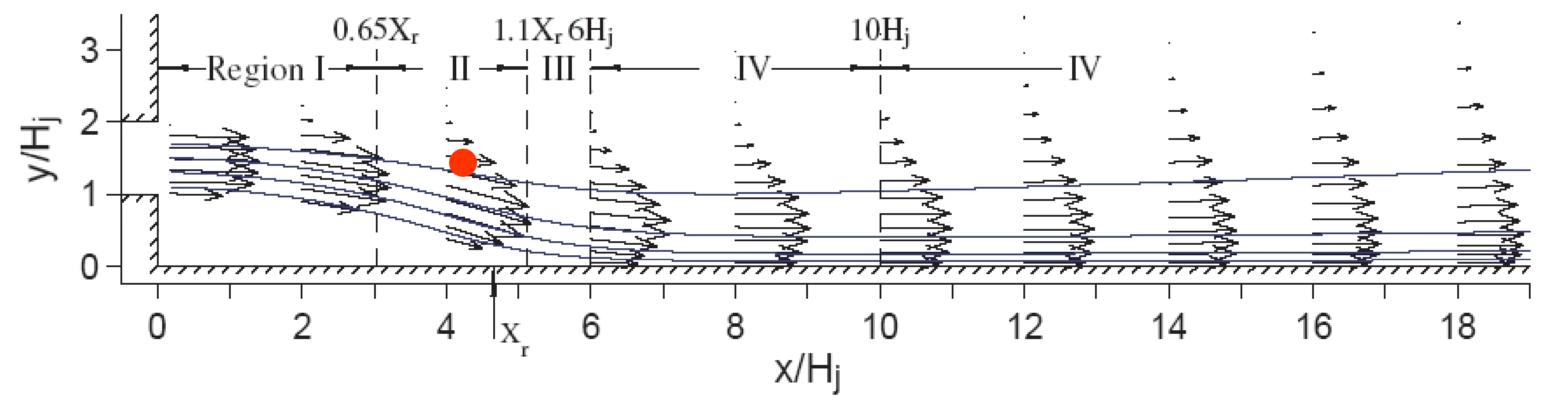
热线风速仪测量的流速信号



平面射流垂直于来流方向的速度
2006, 高南

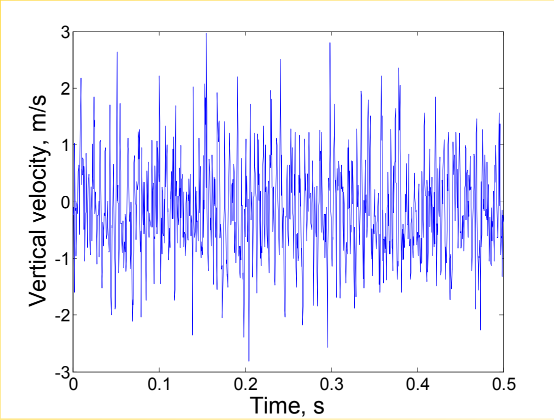
- 如何描述这段信号
  - 平均速度，脉动强度，强度变化频率(周期)

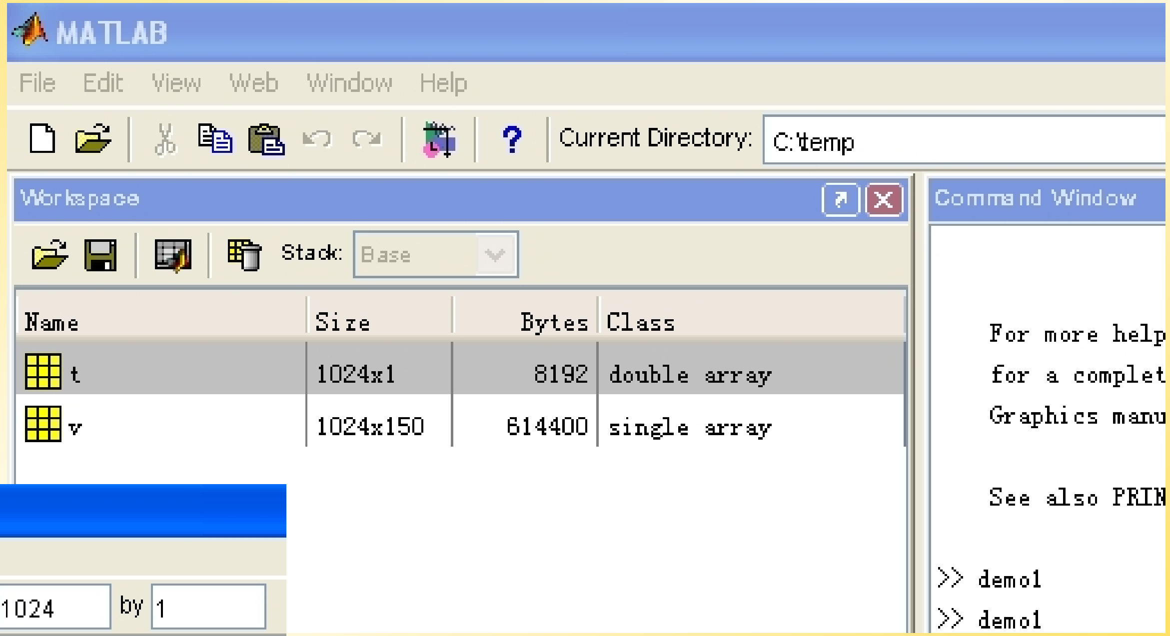
实验介绍

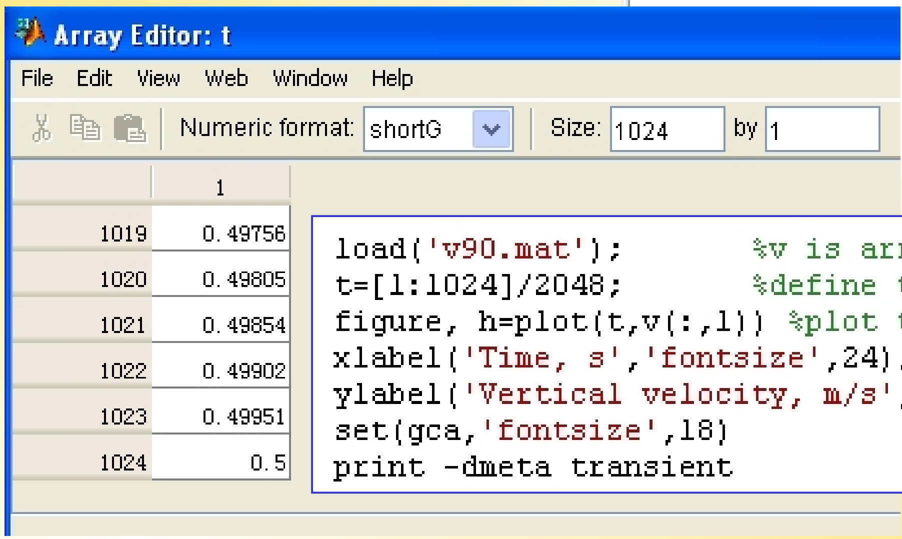


- McMaster University, Hamilton, Canada
- 平面附壁射流，H_j=H_s=3.8cm, Re~44000
- X型热线风速仪，
- 采样频率2048Hz,每组采样时间0.5秒，共150组
- 祥见Experiments in Fluids (2007) 42:941–954

流速信号 Matlab

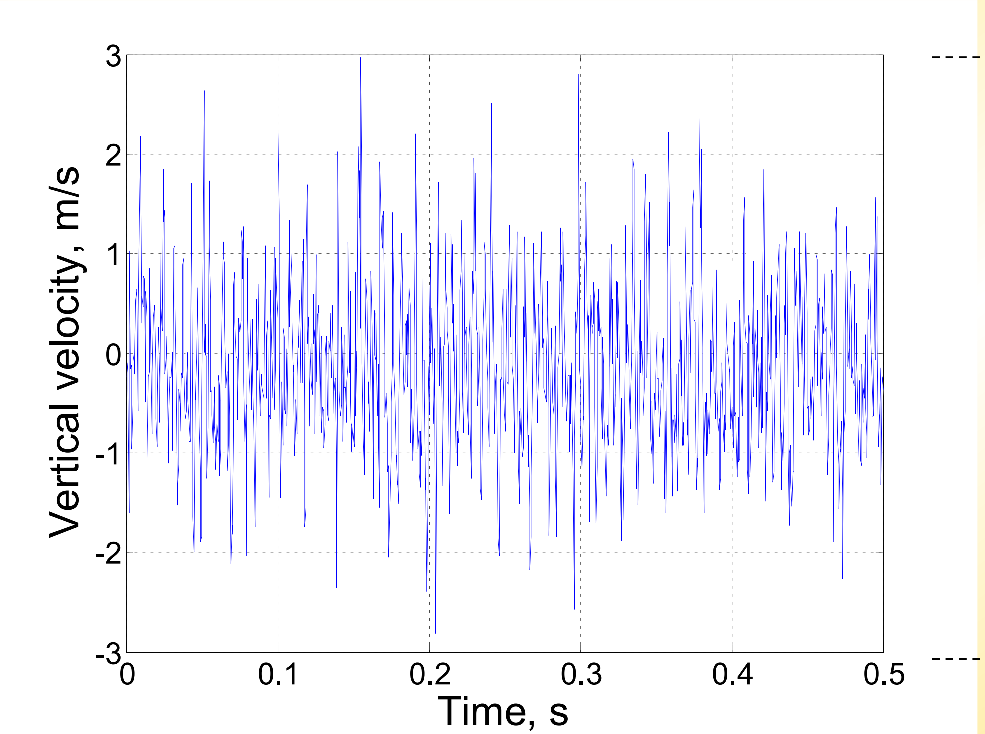


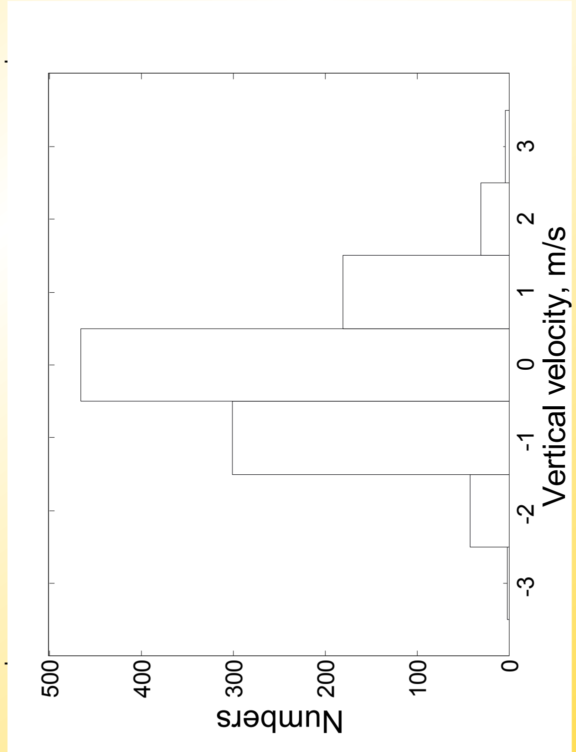





```
load('v90.mat');        %v is array, 150 blocks of vertical velocities, size=1024x150
t=[1:1024]/2048;        %define time, sample frequency 2048Hz, total time 0.5 second
figure, h=plot(t,v(:,1)) %plot transient vertical velocity for block 1
xlabel('Time, s','fontsize',24);
ylabel('Vertical velocity, m/s','fontsize',24);
set(gca,'fontsize',18)
print -dmeta transient
```

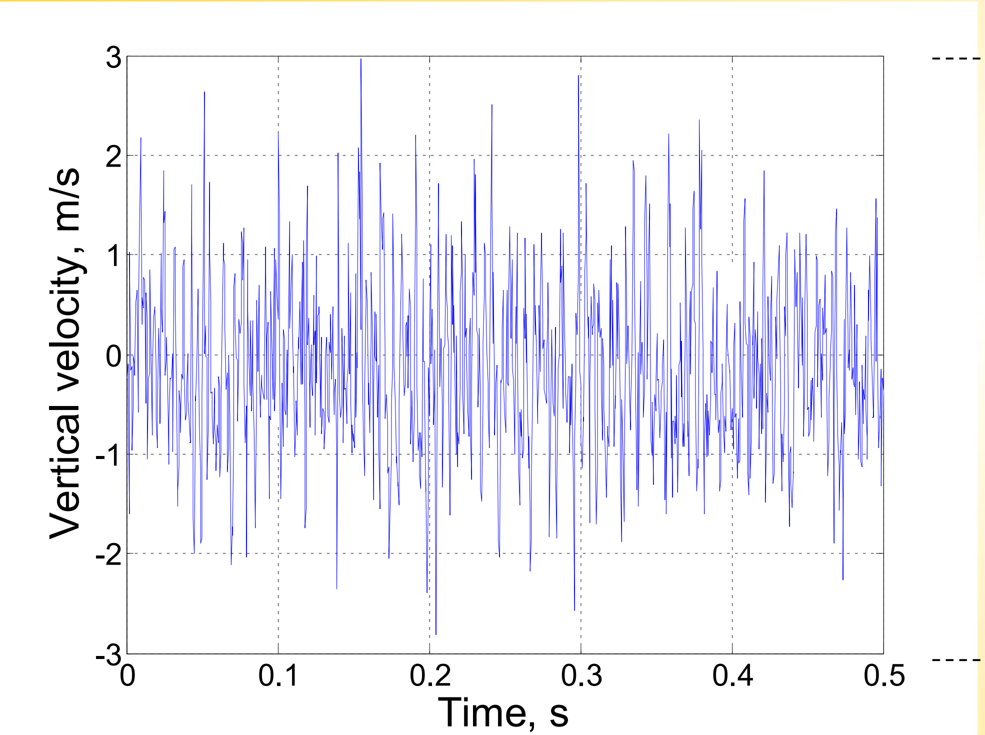
累积概率直方图

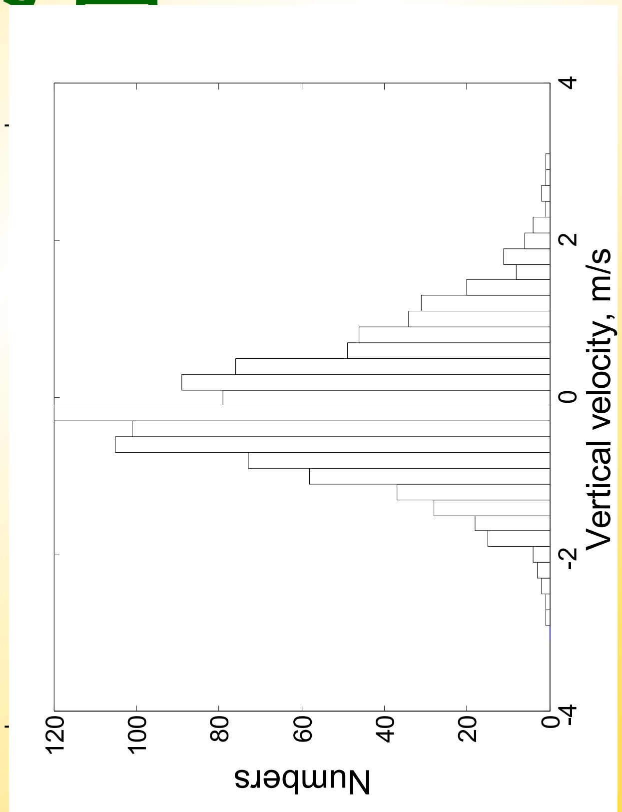





```
figure,
hist(v(:,1),[-3:1:3]); % histogram
ylabel('Numbers','fontsize',24);
xlabel('Vertical velocity, m/s','fontsize',24);
set(gca,'fontsize',18)
print -dmeta hist1
```

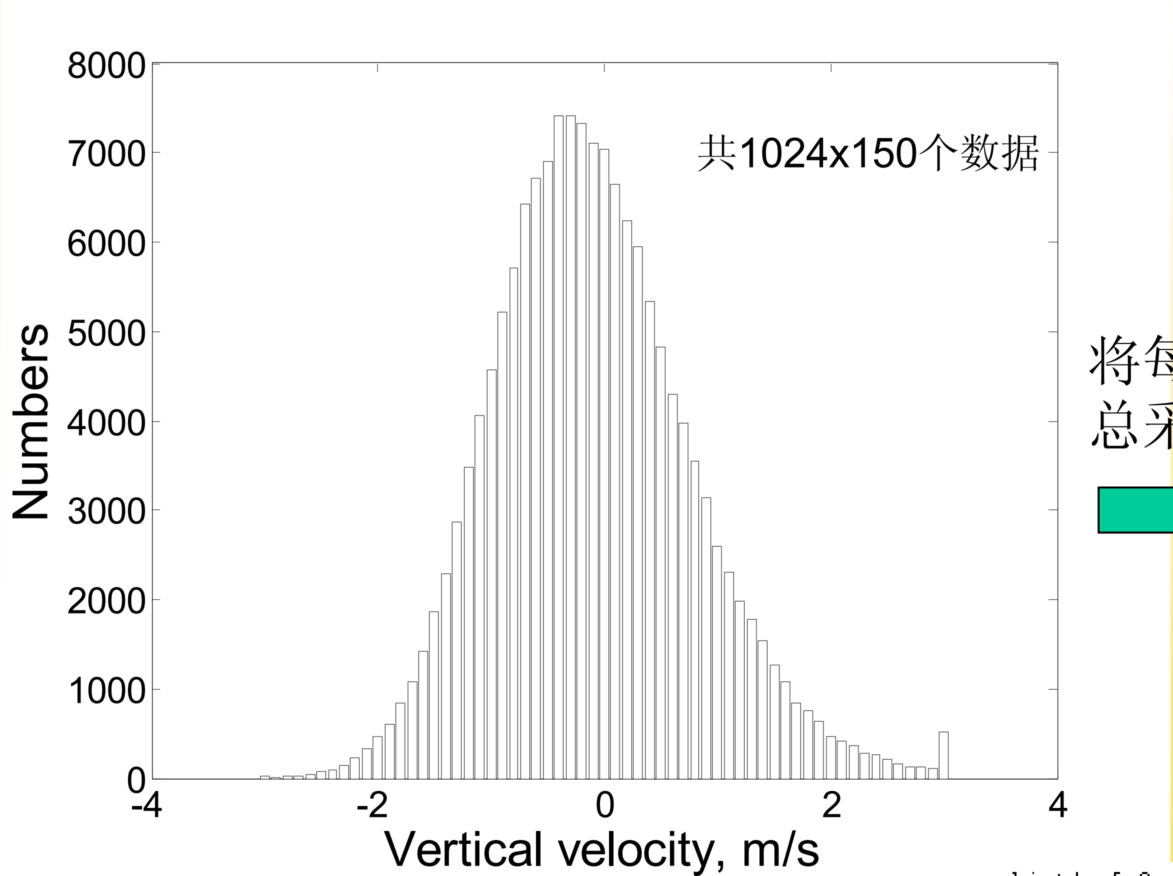
累积概率直方图






```
figure,
hist(v(:,1),[-3:.1:3]);% histogram
ylabel('Numbers','fontsize',24);
xlabel('Vertical velocity, m/s','fontsize',24);
set(gca,'fontsize',18)
print -dmeta hist2
```

累积概率直方图 共150组数据



将每段数值除以
总采样数1024x150

```
a=hist(v,[-3:.1:3]);% histogram
b=sum(a');
figure,
bar([-3:.1:3],b);
xlabel('Vertical velocity, m/s','fontsize',24);
ylabel('Numbers','fontsize',24);
set(gca,'fontsize',18)
print -dmeta hist3
```

概率密度函数
Probability Density Function (PDF)

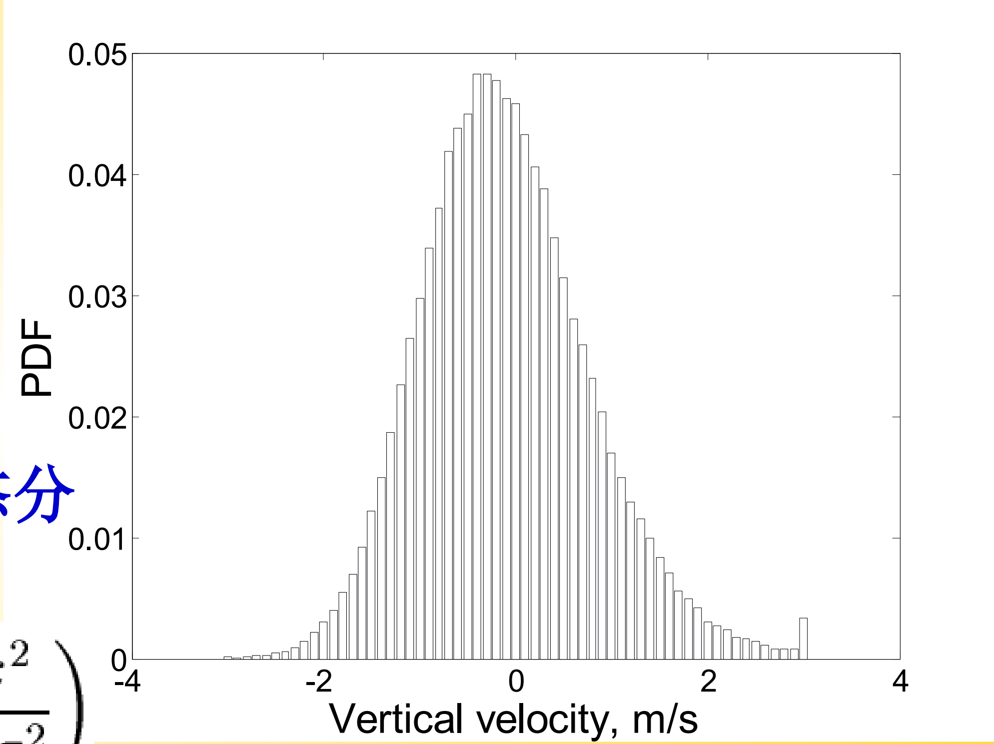
✓ 积分结果为1

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

✓ 高斯分布

- "没有间歇性的正态分布"(张兆顺)

$$p(x) = \frac{1}{(2\pi\sigma^2)^{1/2}} exp\left(-\frac{x^2}{2\sigma^2}\right)$$



概率密度函数
Probability Density Function (PDF)

- ✓ “如果知道湍流场任何一点概率密度，我们就完全掌握了它的性质” (张兆顺)
  - 不大可能，或者说十分困难
- ✓ 更多的是需要利用统计规律来描述流场
  - 统计矩 **Moments**
    - 一阶矩：平均值 **Mean**
    - 二阶矩：均方差 **variance** 或者 **mean square**
    - 高阶矩：**Skewness, Kurtosis**
  - 相关性 **Correlation**

统计矩 Moment

✓定义式

$$\langle v^n \rangle = \int_{-\infty}^{\infty} \tilde{v}^n p(\tilde{v}) d\tilde{v} = \frac{\sum_{i=1}^{N} \tilde{v}_i^n}{N}$$

- 一阶n=1 平均值 Mean

$$V = \langle v \rangle = \int_{-\infty}^{\infty} \tilde{v} p(\tilde{v}) d\tilde{v}$$

- 二阶n=2 均方差

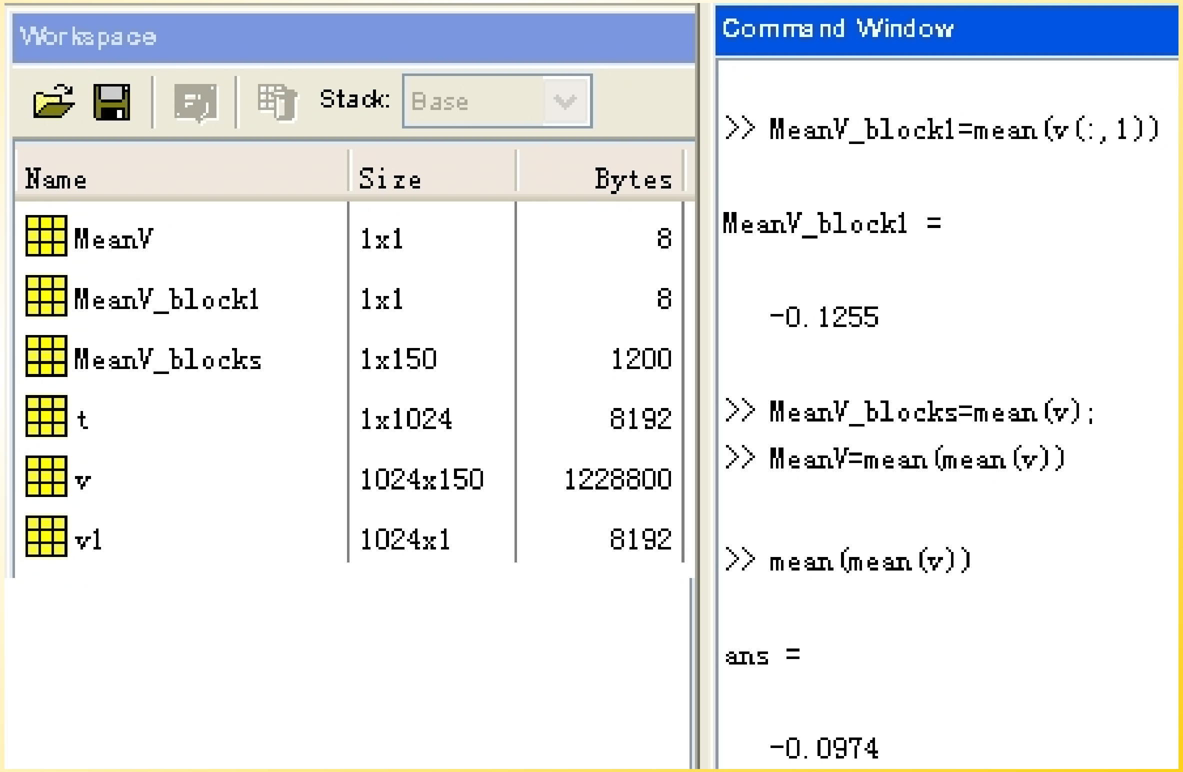
$$\sigma^2 = \langle v^2 \rangle = \int_{-\infty}^{\infty} \tilde{v}^2 p(\tilde{v}) d\tilde{v}$$

平均值 Mean

- ✓ 平均速度

$$\langle v \rangle = \frac{\sum_{i=1}^{N} \widetilde{v_i}}{N}$$

- ✓ **Matlab commend: mean**



均方差 Variance, σ^2

- 又称mean square, $\sigma^2 = \langle v^2 \rangle = \frac{\sum_{i=1}^{N} \tilde{v}_i^2}{N}$
- 其平方根为标准差， standard deviation,或者称为均方根root mean square (rms)，脉动速度v'

$$\sigma = \langle v^2 \rangle^{1/2} = \sqrt{\frac{\sum_{i=1}^{N} \tilde{v}_i^2}{N}} = v'$$

- 湍流度 Turbulence intensity

$$\frac{\langle v^2 \rangle^{1/2}}{\langle v \rangle} \quad or \quad \frac{v'}{V}$$

用Matlab计算均方差 σ^2

Stack: Base

Name	Size	Bytes	Class
MeanV	1x1	8	double array
MeanV_block1	1x1	8	double array
MeanV_blocks	1x150	1200	double array
Rms_block1	1x1	8	double array
VarV	1x1	8	double array
VarV_block1	1x1	8	double array
VarV_blocks	1x150	1200	double array
ans	1x1	8	double array
t	1x1024	8192	double array
v	1024x150	1228800	double array
v1	1024x1	8192	double array

```
>> VarV_block1=var(v(:,1))

VarV_block1 =

    0.7046

>> VarV_blocks=var(v);
>> VarV=mean(var(v))

VarV =

    0.7891

>> Rms_block1=sqrt(VarV_block1)

Rms_block1 =

    0.8394

>> std(v(:,1))

ans =

    0.8394
```

误差分析-平均速度
Experimental Uncertainties (Error)

✓平均速度的误差 $\varepsilon = \dfrac{\sigma}{\langle v \rangle N_s}$

- 基于95%置信度

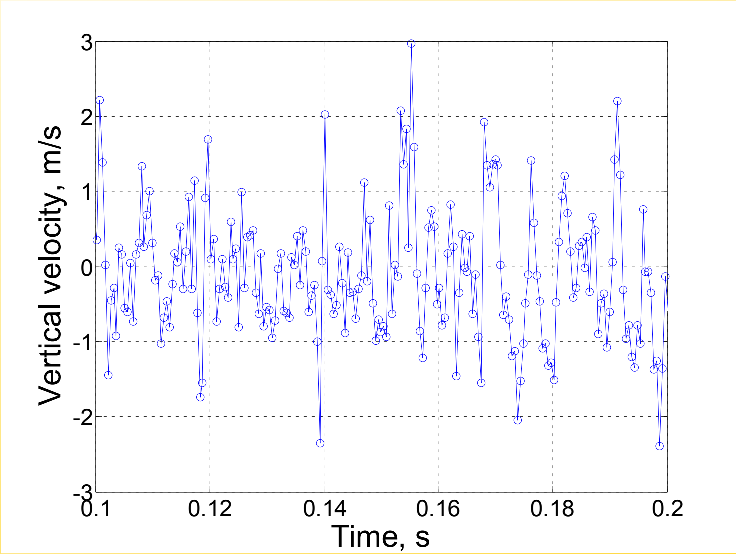
✓N_s为独立的采样次数 $N_s = \dfrac{T_s}{2T}$

- 这里T_s为采样时间，T为信号特征周期
- 如果T~0.01s,T_s=0.5s

<v>~-0.1m/s,σ~0.9m/s

Ns~25, ε ~±35%

- T_s=0.5x150s, ε ~±0.2%



误差分析-脉动流速(rms)

Experimental Uncertainties (Error)

✓平均速度的误差

- 基于95%置信度

$$\varepsilon = \frac{1}{\sqrt{N_s}}$$

✓如果T~0.01s,T_s=0.5s, <v>~-0.1m/s, σ ~0.9m/s, 得到 Ns~25, ε ~±20%

✓T_s=0.5x150s, ε ~±0.14%

相关性 Correlation

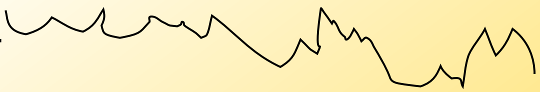
- ✓ 量化 两组信号的关系(互相关)；或者一组信号自身随时间变化的大小(自相关)
- ✓ 定义：相关系数-任何时刻完全一样ρ_{12}=1

1 

2 

3 

任何时刻‘一样’但相反， ρ_{13}=-1

4 

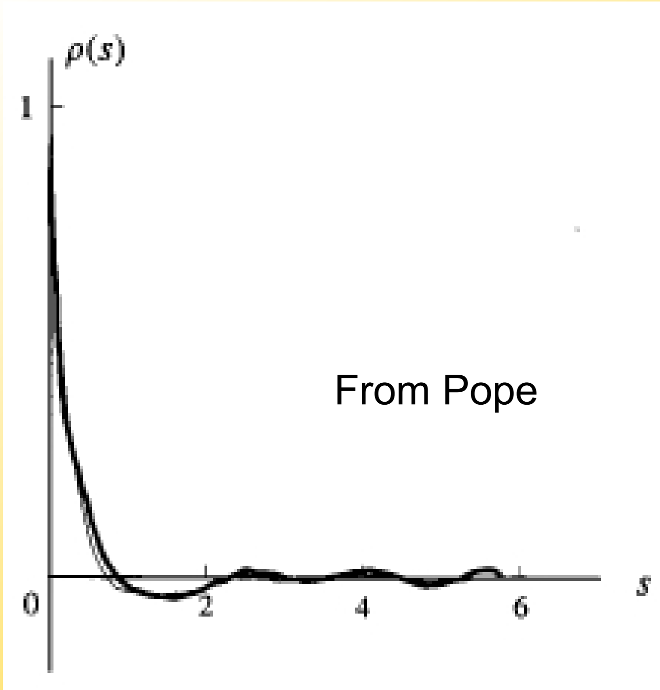
不相关 ρ_{14}=>0

自相关系数 Autocorrelation Coef.

R_{vv}相关函数

- 自相关系数 $$\rho_{vv}(\tau)=\frac{R_{vv}}{v'^2}=\frac{\langle\tilde{v}(t)\tilde{v}(t+\tau)\rangle}{v'^2}$$

- 当τ 为零， ρ_{vv} 为1
- τ 增加， ρ_{vv} 逐渐减少



相关函数R的定义

$$R_{vv}(\tau) = \langle \tilde{v}(t)\tilde{v}(t+\tau) \rangle$$

	V		V
	1		1
1	-0.23474	1	-0.23474
2	-0.094609	2	-0.094609
3	-0.3653	3	-0.3653
4	-1.6	4	-1.6
5	1.0277	5	1.0277
6	-0.16433	6	-0.16433
7	-0.11887	7	-0.11887
8	-0.96311	8	-0.96311
9	-0.52107	9	-0.52107
10	-0.020203	10	-0.020203
11	-0.095545	11	-0.095545
12	-0.20945	12	-0.20945
13	0.53103	13	0.53103
14	0.61706	14	0.61706
15	0.64573	15	0.64573
16	-0.58137	16	-0.58137
17	0.34439	17	0.34439
18	1.3597	18	1.3597
19	2.1774	19	2.1774
20	1.8354	20	1.8354
21	0.29639	21	0.29639
22	0.56406	22	0.56406
23	0.45985	23	0.45985

$$\tau = 0$$

	V		V
	1		1
1	-0.23474	1	-0.23474
2	-0.094609	2	-0.094609
3	-0.3653	3	-0.3653
4	-1.6	4	-1.6
5	1.0277	5	1.0277
6	-0.16433	6	-0.16433
7	-0.11887	7	-0.11887
8	-0.96311	8	-0.96311
9	-0.52107	9	-0.52107
10	-0.020203	10	-0.020203
11	-0.095545	11	-0.095545
12	-0.20945	12	-0.20945
13	0.53103	13	0.53103
14	0.61706	14	0.61706
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19	2.1774	19	2.1774
20	1.8354	20	1.8354
21	0.29639	21	0.29639
22	0.56406	22	0.56406
23	0.45985	23	0.45985

$$\tau = \Delta t = T_s/S$$

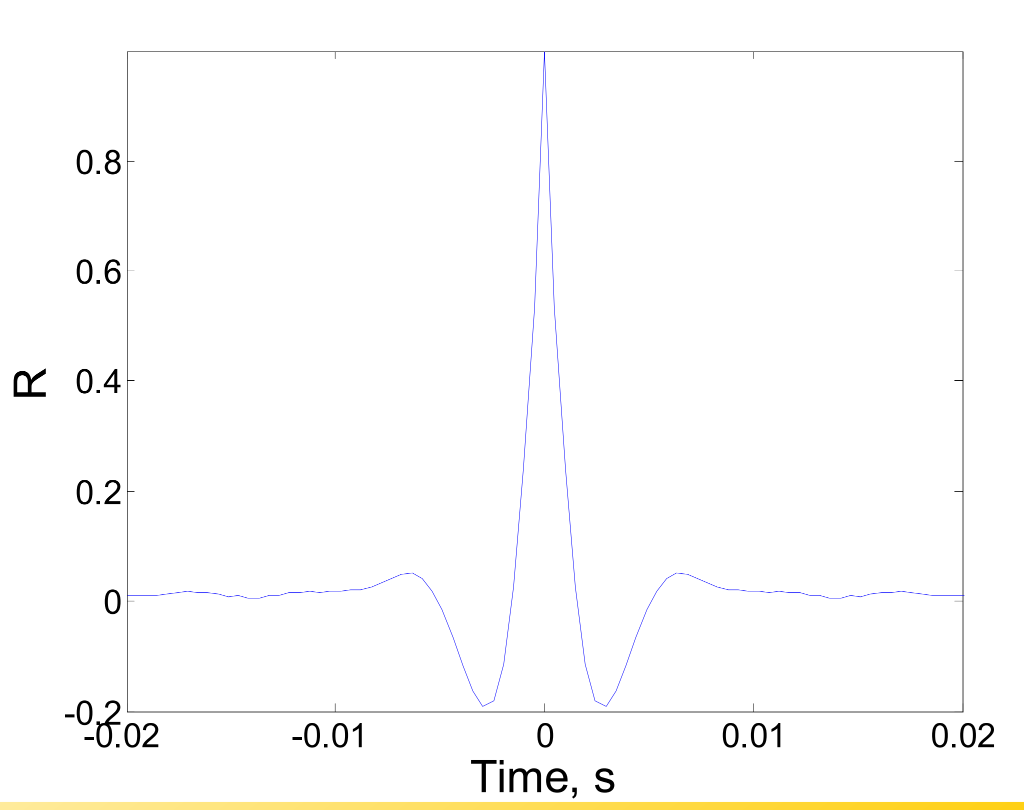
	V		V
	1		1
1	-0.23474	1	-0.23474
2	-0.094609	2	-0.094609
3	-0.3653	3	-0.3653
4	-1.6	4	-1.6
5	1.0277	5	1.0277
6	-0.16433	6	-0.16433
7	-0.11887	7	-0.11887
8	-0.96311	8	-0.96311
9	-0.52107	9	-0.52107
10	-0.020203	10	-0.020203
11	-0.095545	11	-0.095545
12	-0.20945	12	-0.20945
13	0.53103	13	0.53103
14	0.61706	14	0.61706
15	0.64573	15	0.64573
16	-0.58137	16	-0.58137
17	0.34439	17	0.34439
18	1.3597	18	1.3597
19	2.1774	19	2.1774
20	1.8354	20	1.8354
21	0.29639	21	0.29639
22	0.56406	22	0.56406
23	0.45985	23	0.45985

$$\tau = 2\Delta t = 2T_s/S$$

相关性 计算

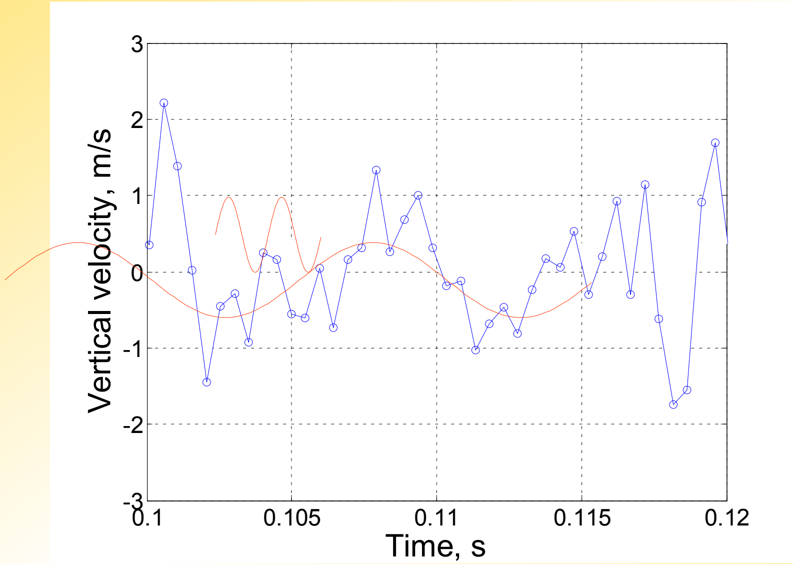
✓通过Matlab 函数 xcorr

```
cr=zeros(1024*2-1,1);
t=[-1023:1023]/2048;     %sample frequency 2048H
for block=1:150;
    c=xcorr(v(:,block))/max(xcorr(v(:,block)));
    cr=cr+c;
end
cr=cr/150;
figure, plot(t,cr)
axis([-0.02 .02 -.2 1])
xlabel('Time, s','fontsize',24);
ylabel('R','fontsize',24);
set(gca,'fontsize',18)
print -dmeta R
```



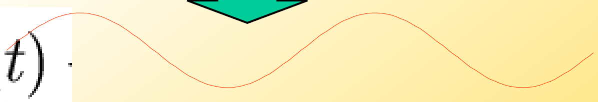
信号 分解

✓ 另一种视角看问题

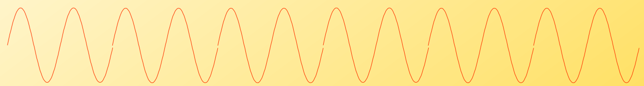


原信号

$$\tilde{v}(t) \approx a_1 cos(2\pi f_1 t) + b_1 sin(2\pi f_1 t) \cdot$$



$$+ a_2 cos(2\pi f_2 t) + b_2 sin(2\pi f_2 t)$$



✓ **v(t) 被转化成了系数a_n,b_n**

信号 分解

✓ 通过傅立叶变换确定系数a_n, b_n

$$\mathcal{F}\left(\tilde{v}(t)\right) = F(f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{v}(t)e^{-i2\pi f t}dt$$

- 因为 $e^{-i2\pi f t} = cos(2\pi f t) - isin(2\pi f t)$
- 得到的 $F(f)$为复数(v为偶函数的时候出外)， 实部代表a,虚部代表b
- $|F(f)|^2$ 被称为能量谱，‘代表’ $a_n^2 + b_n^2$
- 能量谱表示了能量在各个时间尺度的分布

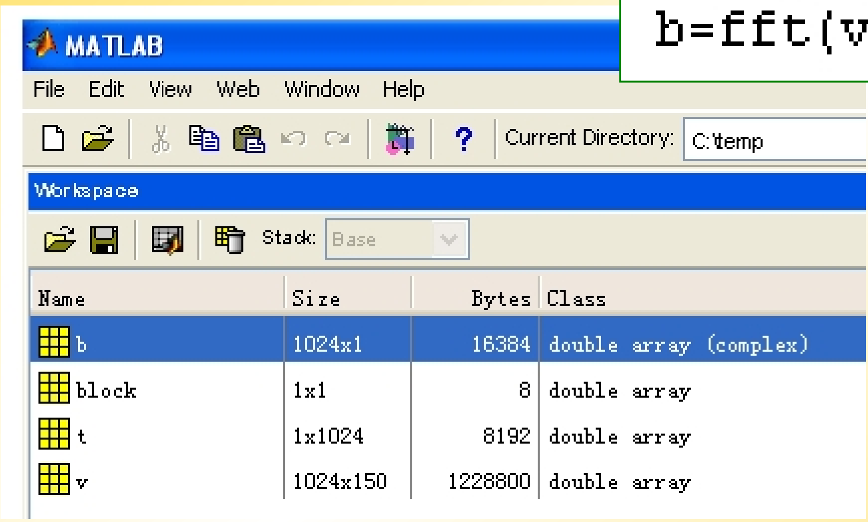
$$|F(f)|^2 = \frac{F(f)F^*(f)}{N^2}$$

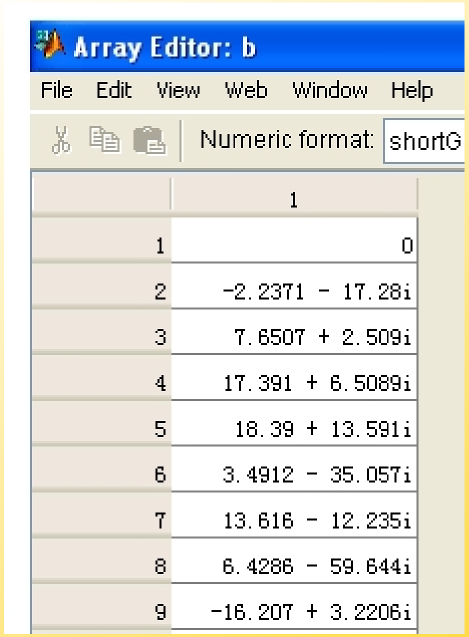
能量谱power spectrum计算方法 I

✓ 快速傅立叶变换Fast Fourier Transform

- 利用Matlab FFT函数

```
block=1;
b=fft(v(:,block)-mean(v(:,block)));
```





能量谱计算方法II

power spectrum

$$|F(f)|^2 = \frac{F(f)F^*(f)}{N^2}$$

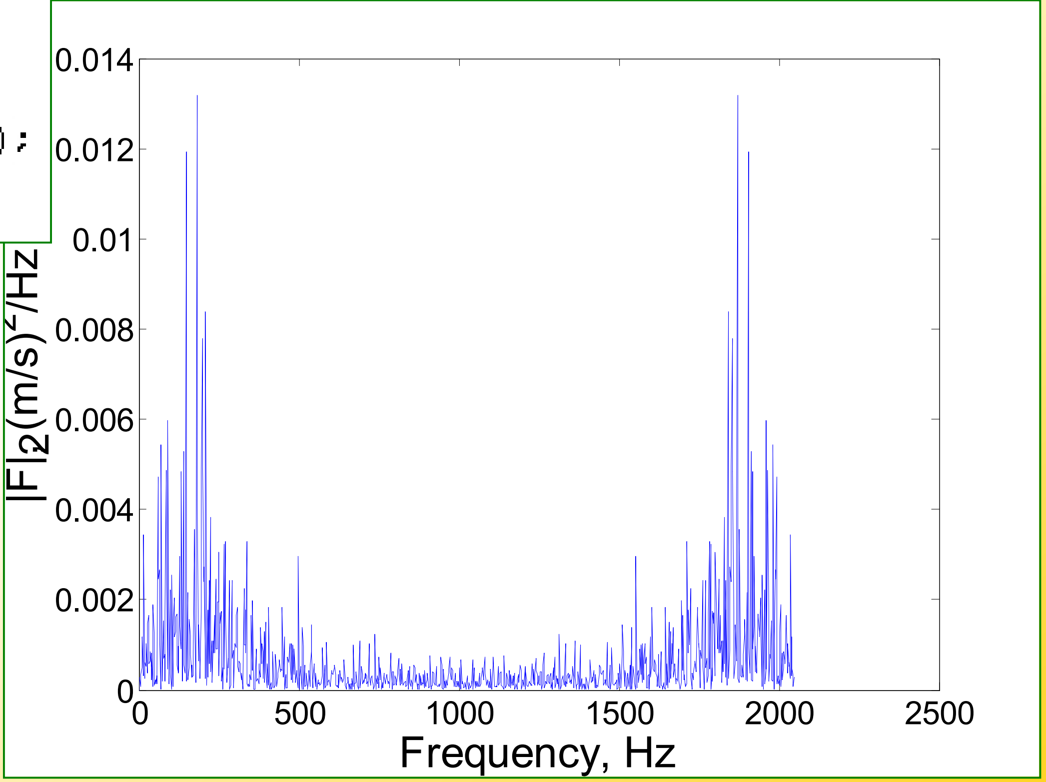
```
block=1;
N=1024;                %N sample number
b=fft(v(:,block)-mean(v(:,block)));
F=b.*conj(b)/N^2;
f=[1:N]*(1/0.5);       %0.5 sample time
figure, plot(f,F)
xlabel('Frequency, Hz','fontsize',24);
ylabel('|F|, (m/s)^2/Hz','fontsize',24);
set(gca,'fontsize',18)
print -dmeta spec1
```

b

	1
1	0
2	38.333 + 1.6137i
3	-4.8734 - 2.144i
4	23.67 - 7.8949i
5	12.01 + 17.82i
6	-8.8278 + 31.693i
7	57.59 - 8.9153i
8	-13.109 + 9.5148i
9	-15.893 + 19.326i
10	23.615 + 6.0247i

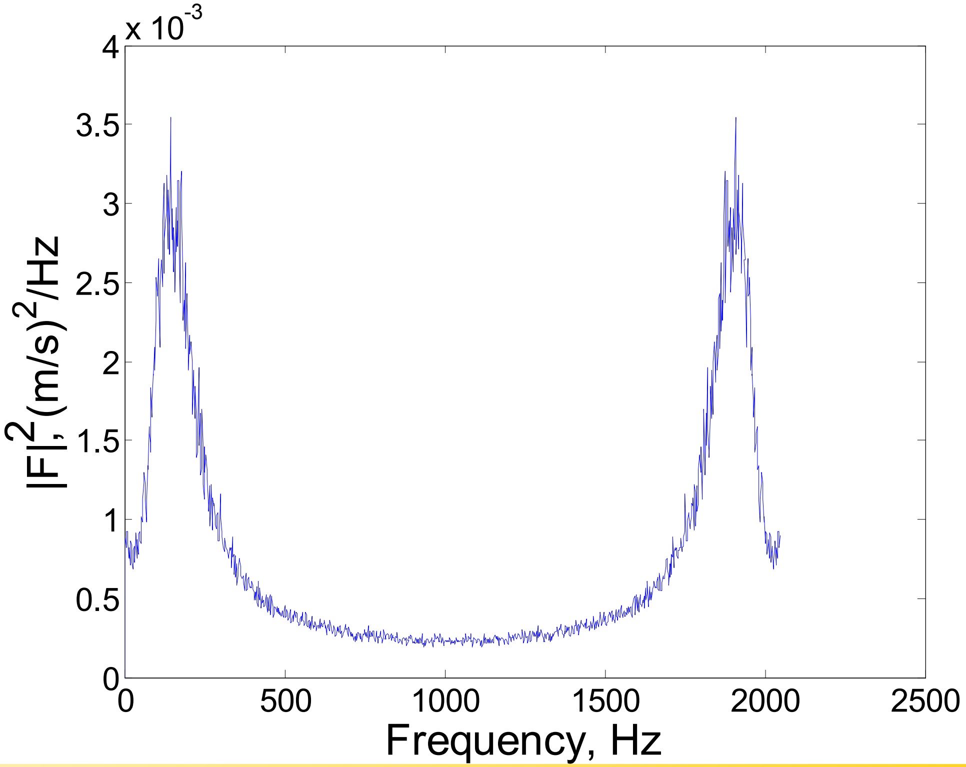
conj(b)

	1
1	0
2	38.333 - 1.6137i
3	-4.8734 + 2.144i
4	23.67 + 7.8949i
5	12.01 - 17.82i
6	-8.8278 - 31.693i
7	57.59 + 8.9153i
8	-13.109 - 9.5148i
9	-15.893 - 19.326i
10	23.615 - 6.0247i



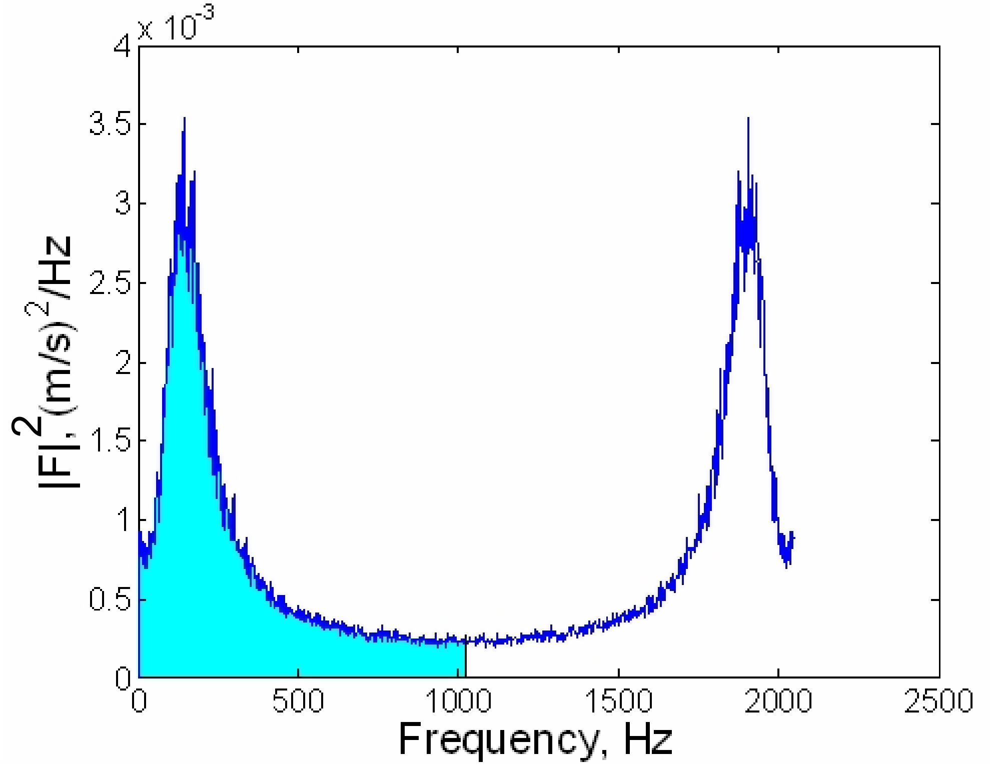
150组数据的平均 能量谱

```
Fs=zeros(1024,1);
N=1024;                %N sample number
for block=1:150;
    b=fft(v(:,block)-mean(v(:,block)));
    F=b.*conj(b)/N^2;
    Fs=Fs+F;
end
Fs=Fs/150;
f=[1:N]*(1/0.5);% 0.5 is the sample time
figure, plot(f,Fs)
xlabel('Frequency, Hz','fontsize',24);
ylabel('|F|, (m/s)^2/Hz','fontsize',24);
set(gca,'fontsize',18)
print -dmeta spec2
```



能量谱积分—> 均方差(Variance) σ²

$$\langle v^2 \rangle = \int_{-\infty}^{\infty} F_{vv}(f)\,df$$




```
>> sum(Fs(1:1024/2))*(1/0.5)

ans =

    0.7881

>> mean(var(v))

ans =

    0.7891
```

频谱逆变换 -> 自相关函数

$$\mathcal{F}\left(\tilde{v}(t)\right) = F(f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{v}(t)e^{-i2\pi f t}dt$$

$$\mathcal{F}^{-1}\left(F(f)\right) = R(\tau) = \int_{-\infty}^{\infty} F(f)e^{i2\pi f\tau}d\tau$$

多传感器信号分析— 交叉谱

cross spectrum

- 单独一点信号的能量谱表示了脉动能量在各个频率(时间尺度)的分布
- 交叉谱反映两个相同时段的不同时间序列$v_1(t)$和$v_2(t)$在频域变化上的相互关系

The cross spectra $\Phi(x_1, x_2, f)$ was given by

$$\overline{\hat{p}(x_1, f)\hat{p}^*(x_2, f')} = \Phi(x_1, x_2, f)\delta(f - f'), \tag{3.13}$$

where δ is the Dirac delta function, $\hat{p}$ is the Fourier transform of the transient wall pressure, and $*$ represents the complex conjugate. These spectra were computed by averaging the results for different blocks

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$$\Phi(x_1, x_2, f) = \frac{\overline{\hat{p}(x_1, f)\hat{p}^*(x_2, f)}}{T}, \tag{3.14}$$

改p帽为F

交叉谱的 模 和 相位角

coherence and phase angle

The nature of the relationship between the pressure at the two points was also examined using the coherence of the pressure fluctuations measured at two locations given by,

$$\gamma_{pp}^{2}(x_1, x_2, f) = \frac{|\Phi(x_1, x_2, f)|^2}{\Phi(x_1, x_1, f)\Phi(x_2, x_2, f)}, \tag{3.15}$$

and the phase angle of the cross spectrum, $\theta_{pp}(x_1, x_2, f)$ given by

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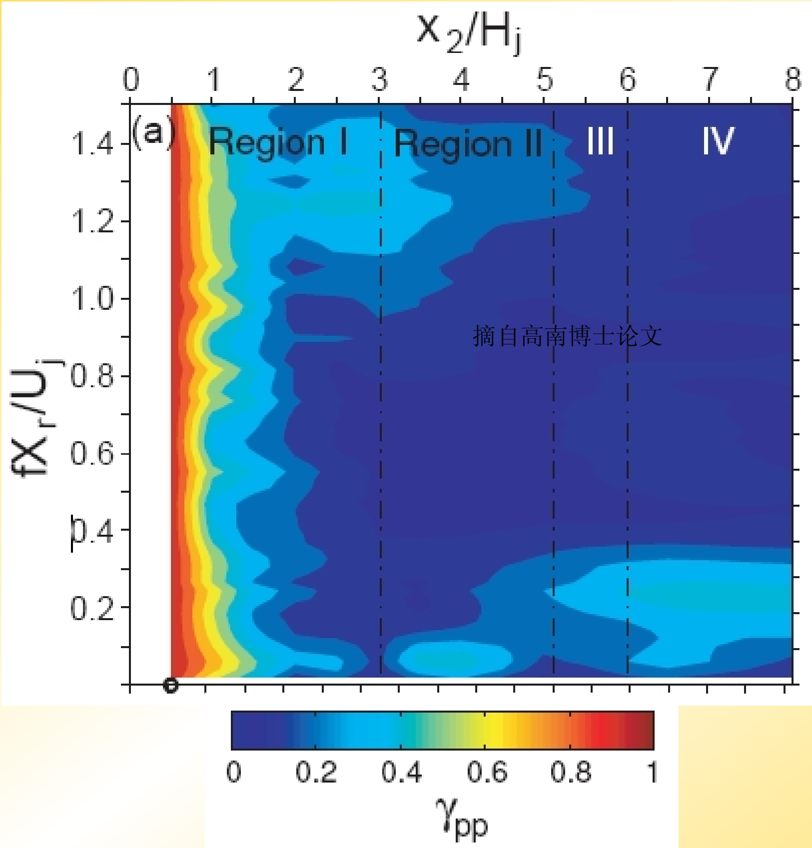
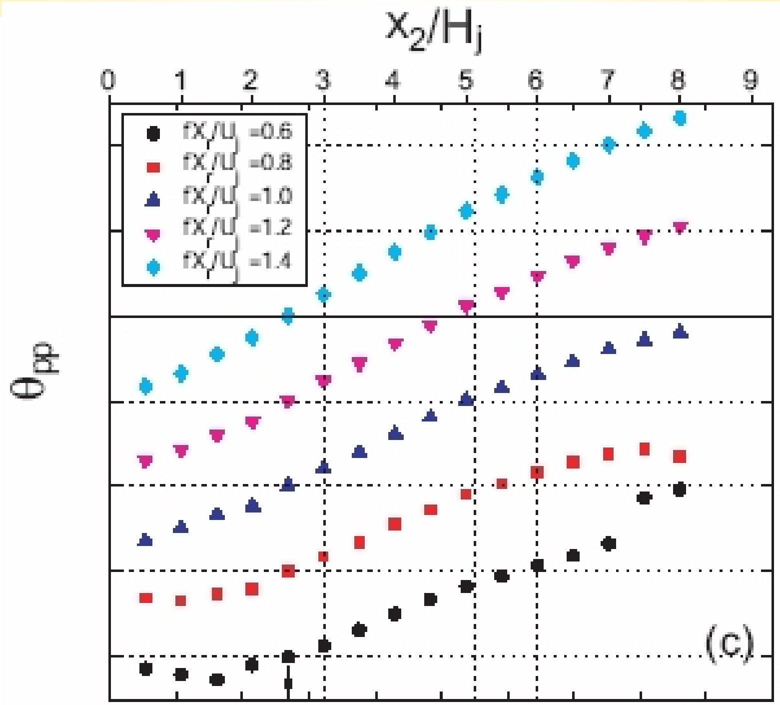
$$\theta_{pp}(x_1, x_2, f) = tan^{-1}\left(\frac{img\left(\Phi(x_1, x_2, f)\right)}{real\left(\Phi(x_1, x_2, f)\right)}\right). \tag{3.16}$$

交叉谱的 模 和 相位角
coherence and phase angle





交叉谱 逆变换 互相关函数

the cross correlation $R_{pp}(x_1, x_2, \tau)$ was computed by inverse Fourier transforming the cross spectra, *i.e.* ,

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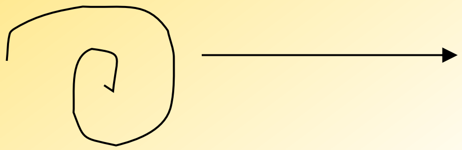
$$R_{pp}(x_1, x_2, \tau) = \int_{-\infty}^{\infty} \Phi(x_1, x_2, f) e^{i2\pi f\tau} df.$$

交叉谱 逆变换—》互相关函数



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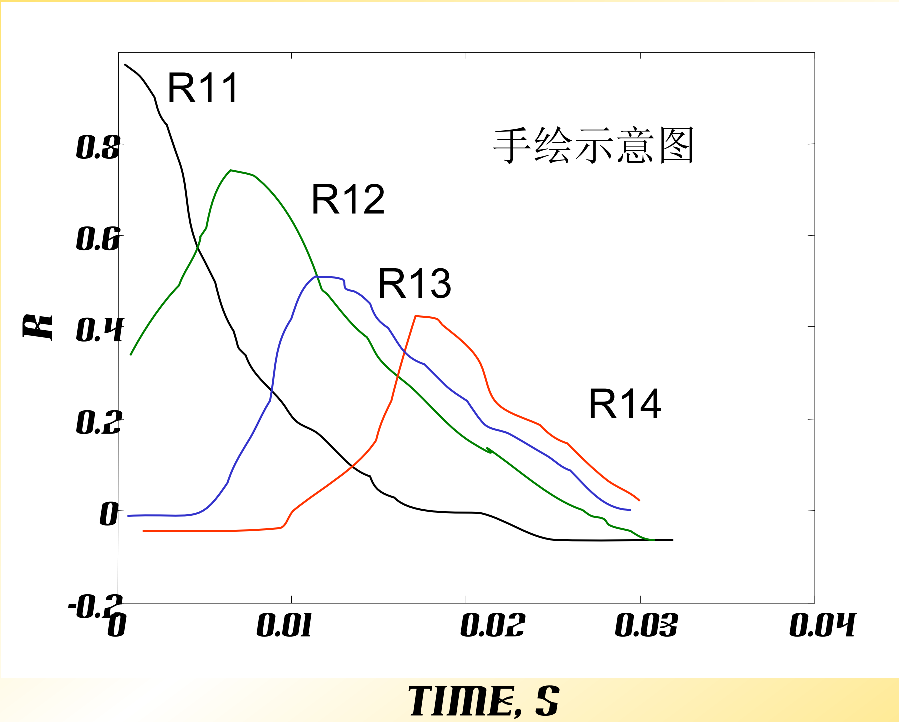

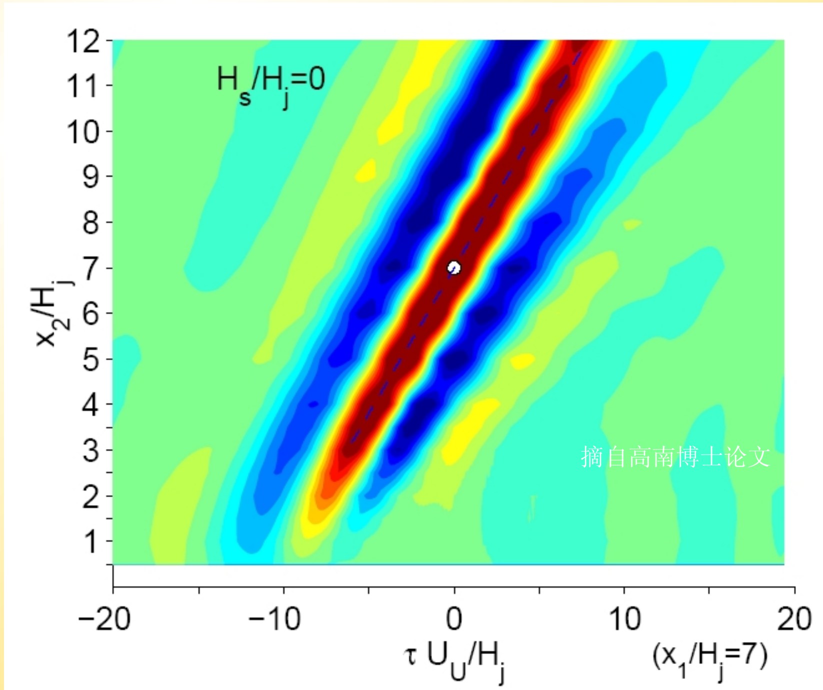

壁面压强互相关系数