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9. ~~F0~~ need graph with C.V.

2-13. solution

$$u^*=\frac{uL}{\nu}\quad v^*=\frac{vL}{\nu}\quad x^*=\frac{x}{L}\quad y^*=\frac{y}{L}\quad T^*=\frac{T-T_1}{T_0-T_1}$$

$$\Rightarrow u=\frac{u^*\nu}{L}.\ v=\frac{v^*\nu}{L}\ .x=x^*L\ .\ y=y^*L\quad T-T_1=T^*(T_0-T_1)$$

Then

① $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{\nu}{L^2}\frac{\partial u^*}{\partial x^*}+\frac{\nu}{L^2}\frac{\partial v^*}{\partial y^*}=0\ \Rightarrow\ \frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$$

② $$u\frac{\partial u}{\partial x}+\cancel{\nu}\,v\frac{\partial u}{\partial y}=g\beta(T-T_1)+\nu\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}\right)$$

$$\Rightarrow\frac{u^*\cdot\nu}{L}\frac{\nu}{L^2}\cdot\frac{\partial u^*}{\partial x^*}+\frac{v^*\nu}{L}\frac{\nu}{L^2}\cdot\frac{\partial u^*}{\partial y^*}=g\beta T^*(T_0-T_1)+\nu\left(\frac{\nu}{L^3}\cdot\frac{\partial^2u^*}{\partial x^{*2}}+\frac{\nu}{L^3}\cdot\frac{\partial^2u^*}{\partial y^{*2}}\right)$$

$$\Rightarrow u^*\cdot\frac{\partial u^*}{\partial x^*}+v^*\cdot\frac{\partial u^*}{\partial y^*}=\frac{L^3}{\nu^2}g\beta T^*(T_0-T_1)+\left(\frac{\partial^2u^*}{\partial x^{*2}}+\frac{\partial^2u^*}{\partial y^{*2}}\right)$$

$$\because\ \nu=\frac{\mu}{\rho}\quad\Rightarrow\quad\frac{L^3}{\nu^2}g\beta T^*(T_0-T_1)=\frac{\rho^2L^2g\beta(T_0-T_1)}{\mu^2}=Gr$$

$$\Rightarrow u^*\frac{\partial u^*}{\partial x^*}+v^*\cdot\frac{\partial u^*}{\partial y^*}=Gr\cdot T^*+\frac{\partial^2u^*}{\partial x^*}+\frac{\partial^2u^*}{\partial y^*}$$

③ $$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}\right)$$

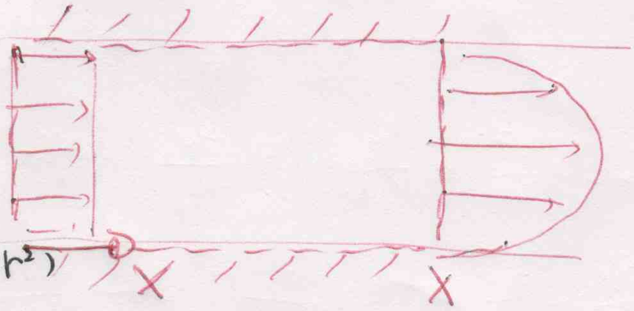
$$\rho c_p\left[\frac{\nu\cdot u^*}{L}\cdot\frac{\partial(T_1+T^*(T_0-T_1))}{\partial(x^*L)}+\frac{\nu\cdot v^*}{L}\cdot\frac{\partial(T_1+T^*(T_0-T_1))}{\partial(y^*L)}\right]=k\cdot\left[\frac{\partial^2(T^*(T_0-T_1)+T_1)}{\partial(L^2\cdot x^{*2})}+\frac{\partial^2(T^*(T_0-T_1)+T_1)}{\partial(L^2y^{*2})}\right]$$

$$\Rightarrow\frac{\rho c_p\nu}{k}\left(u^*\frac{\partial T^*}{\partial x^*}+v^*\cdot\frac{\partial T^*}{\partial y^*}\right)=\frac{\partial^2T^*}{\partial x^{*2}}+\frac{\partial^2T^*}{\partial y^{*2}}$$

$$\because\ \nu=\frac{\mu}{\rho}\qquad\frac{\rho c_p\cdot\nu}{k}=\frac{\mu c_p}{k}=Pr$$

$$\Rightarrow Pr\left(u^*\cdot\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}\right)=\frac{\partial^2T^*}{\partial x^{*2}}+\frac{\partial^2T^*}{\partial y^{*2}}$$

2-14. solution:



Continuity $$U_0\pi r_0^2=\int_0^{r_0}2\pi r\,u(r)\,dr=\frac{\pi c r_0^4}{2}$$

$$\Rightarrow C=\frac{2U_0}{r_0^2}\ \Rightarrow\ u(r)=\frac{2U_0}{r_0^2}(r_0^2-r^2)$$

$$F_x=\iint_{CS}\vec{V}(\rho\vec{V}\cdot d\vec{A})+\frac{\partial}{\partial t}\iiint_{CV}\vec{V}\rho\,dVol=\frac{1}{3}\rho U_0^2\pi r_0^2+0$$

$$\because\ F_x=P_0A-P_xA-Drag\ \Rightarrow\ Drag=P_0A-P_xA-F_x$$

$$=\pi r_0^2\left(P_0-P_x-\frac{1}{3}\rho U_0^2\right)$$