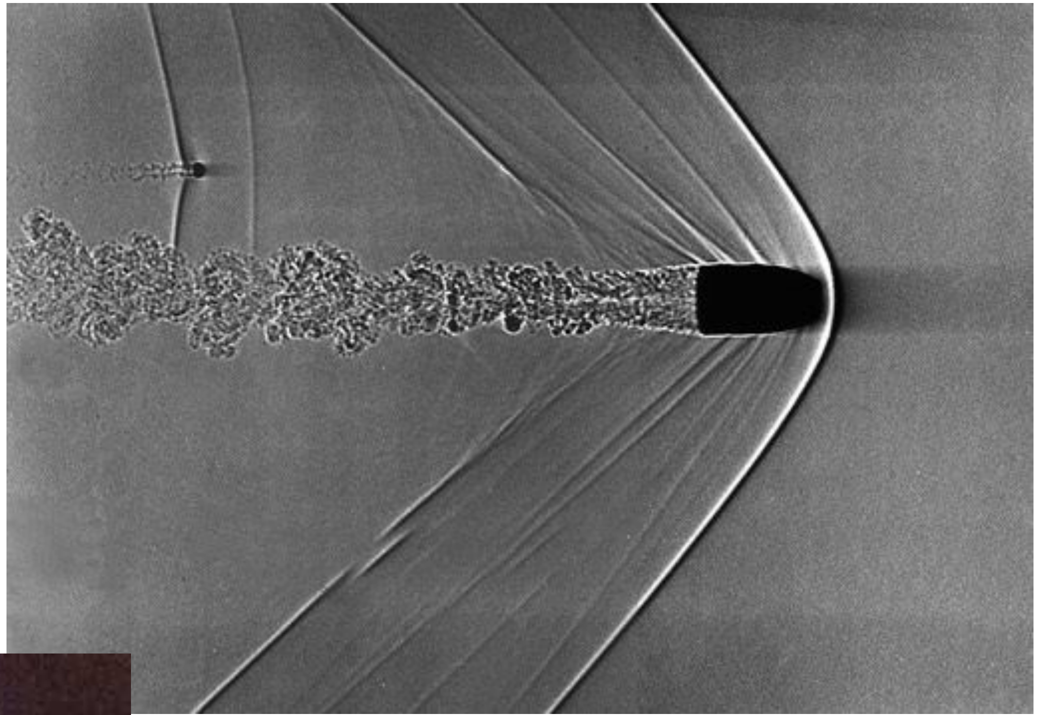
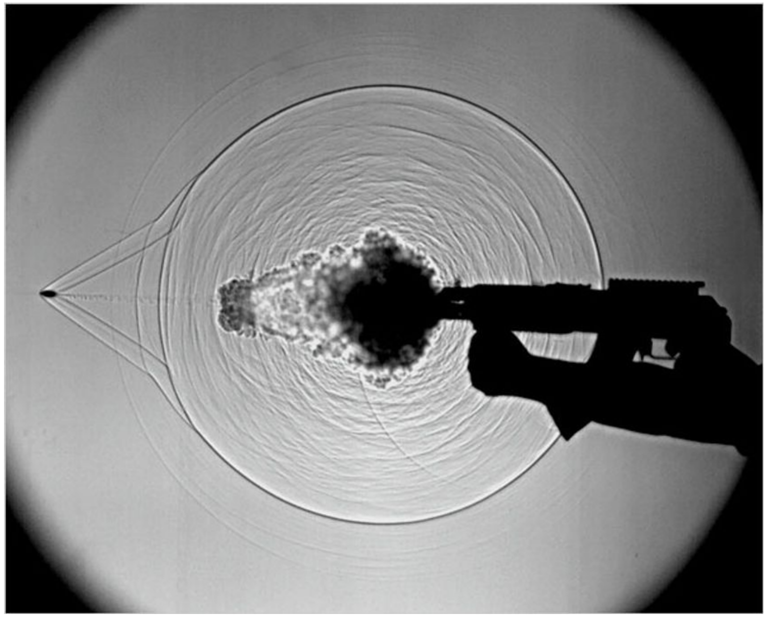
Compressible Flow

Gao,Nan

2013-12-19

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Main Topics

- Basic Equations for
  One-Dimensional Compressible Flow 一维可压缩流动方程
- Isentropic Flow of an Ideal Gas
  – Area Variation 变截面积的理想气体等熵流动
- Supersonic Channel Flow with Shocks 有激波的管道流动
- Oblique Shocks and Expansion Waves 斜激波和膨胀波
- 图片取自Fox and McDonald, "Fluid Mechanics"

大连理工大学空气动力学实验室

Ideal gas, sound speed 理想气体声速

$$c^2 = \frac{dp}{d\rho}$$

$$\frac{p}{\rho^k} = \text{constant}$$

$$c = \sqrt{kRT}$$

$$c = \sqrt{kRT}$$

$$c = \sqrt{1.4 \times 287 \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \times 288\ \text{K} \times \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2}} = 340\ \text{m/s} \xleftarrow{c_{\text{air (288 K)}}}$$

Isentropic process等熵过程

- 声波压缩空气是“等熵过程”
  - No heat transfer 没有传热
  - No friction 没有摩擦阻力
  - No shock wave 没有激波
  - Smooth variation, fully reversible 变化过程可逆

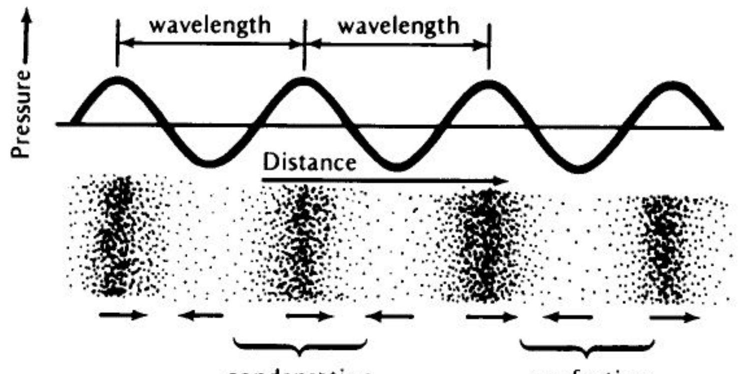
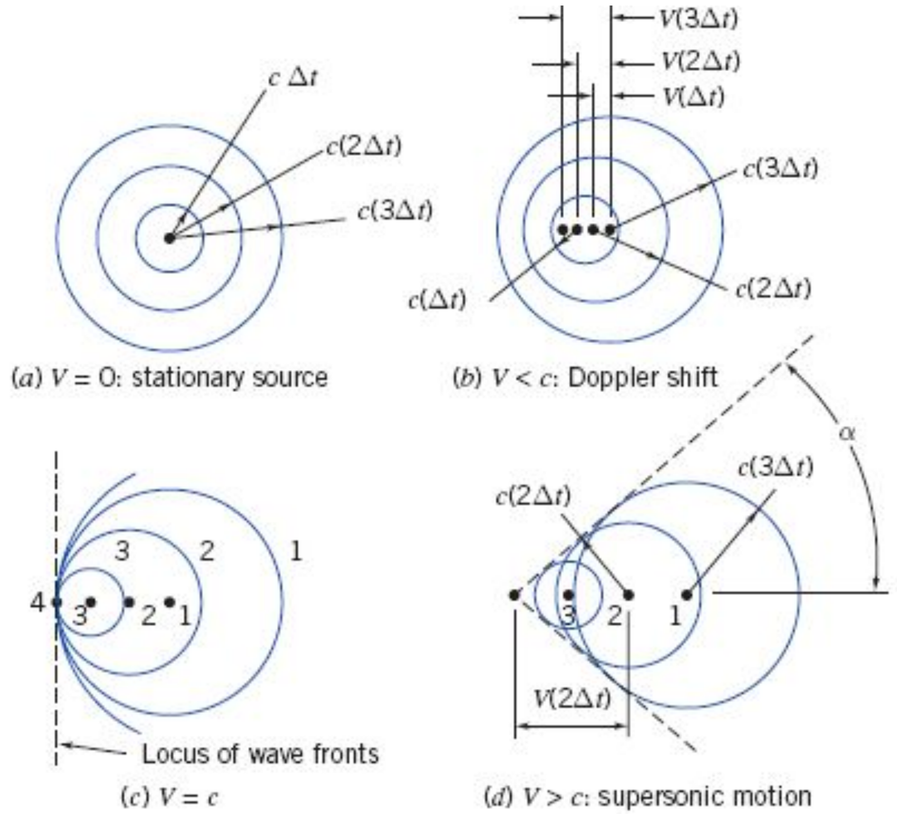
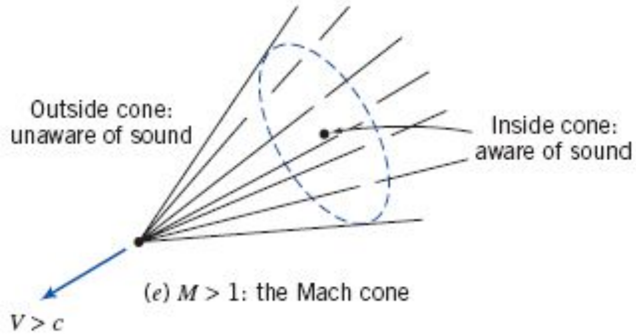


Figure 1

Mach Cone 马赫锥





Stagnation Condition 滞点

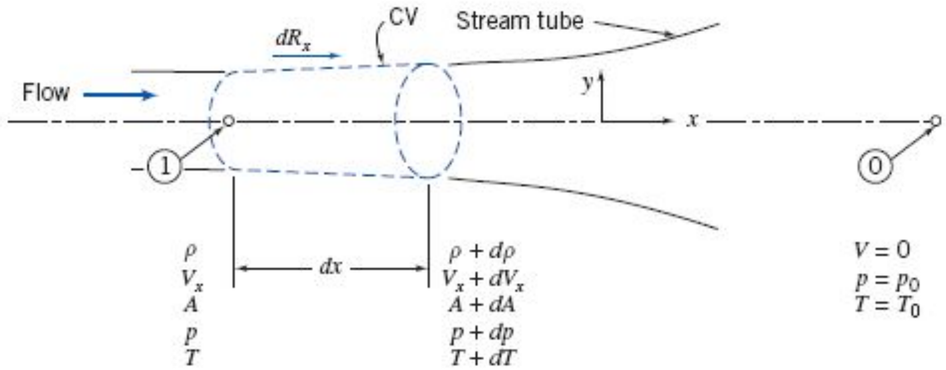


Fig. 12.5 Compressible flow in an infinitesimal stream tube.

$$\frac{dp}{\rho} + d\left(\frac{V_x^2}{2}\right) = 0$$

$$\frac{p}{\rho^k} = \text{constant}$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}\frac{V^2}{c^2}\right]^{k/(k-1)}$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{k-1}{2}M^2\right]^{1/(k-1)}$$

Basic Equations for One-Dimensional Compressible Flow

- Control Volume

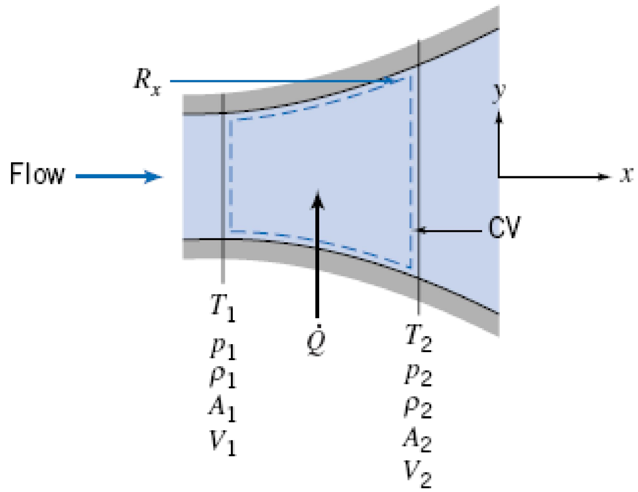


Fig. 13.1 Control volume for analysis of a general one-dimensional flow.

Basic Equations for One-Dimensional Compressible Flow

- Continuity

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \rho V A = \dot{m} = \text{constant}$$

✓ **Momentum**

$$R_x + p_1 A_1 - p_2 A_2 = \dot{m} V_2 - \dot{m} V_1$$

Basic Equations for One-Dimensional Compressible Flow

- First Law of Thermodynamics

$$\frac{\delta Q}{dm} + h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

✓ **Second Law of Thermodynamics**

$$\dot{m}(s_2 - s_1) \geq \int_{CS} \frac{1}{T}\left(\frac{\dot{Q}}{A}\right) dA$$

Basic Equations for One-Dimensional Compressible Flow

- Equation of State

$$p = \rho R T$$

✓ **Property Relations**

$$\Delta h = h_2 - h_1 = c_p \Delta T = c_p(T_2 - T_1)$$

$$\Delta s = s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Isentropic Flow of an Ideal Gas – Area Variation

- Basic Equations for Isentropic Flow

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \rho V A = \dot{m} = \text{constant}$$

$$R_x + p_1 A_1 - p_2 A_2 = \dot{m} V_2 - \dot{m} V_1$$

$$h_{0_1} = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h_{0_2} = h_0$$

$$s_2 = s_1 = s$$

$$p = \rho R T$$

$$\Delta h = h_2 - h_1 = c_p \Delta T = c_p (T_2 - T_1)$$

$$\frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k} = \frac{p}{\rho^k} = \text{constant}$$

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) = 0$$

$$dp = -\rho V\,dV$$

$$\frac{dp}{\rho V^2} = -\frac{dV}{V}$$

$$\rho A V = \text{constant}$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

$$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho}$$

$$\frac{dA}{A} = \frac{dp}{\rho V^2} - \frac{d\rho}{\rho}$$

$$\frac{dA}{A} = \frac{dp}{\rho V^2}\left[1 - \frac{V^2}{dp/d\rho}\right]$$

for an isentropic process, $dp/d\rho = \partial p/\partial\rho)_s = c^2$, so

$$\frac{dA}{A} = \frac{dp}{\rho V^2}\left[1 - \frac{V^2}{c^2}\right] = \frac{dp}{\rho V^2}[1 - M^2]$$

$$\frac{dp}{\rho V^2} = \frac{dA}{A}\frac{1}{[1 - M^2]}$$

$$\frac{dV}{V} = -\frac{dA}{A}\frac{1}{[1 - M^2]}$$

Isentropic Flow of an Ideal Gas – Area Variation

- Subsonic, Supersonic, and Sonic Flows

$$\frac{dV}{V} = -\frac{dA}{A}\frac{1}{\left[1 - M^2\right]}$$

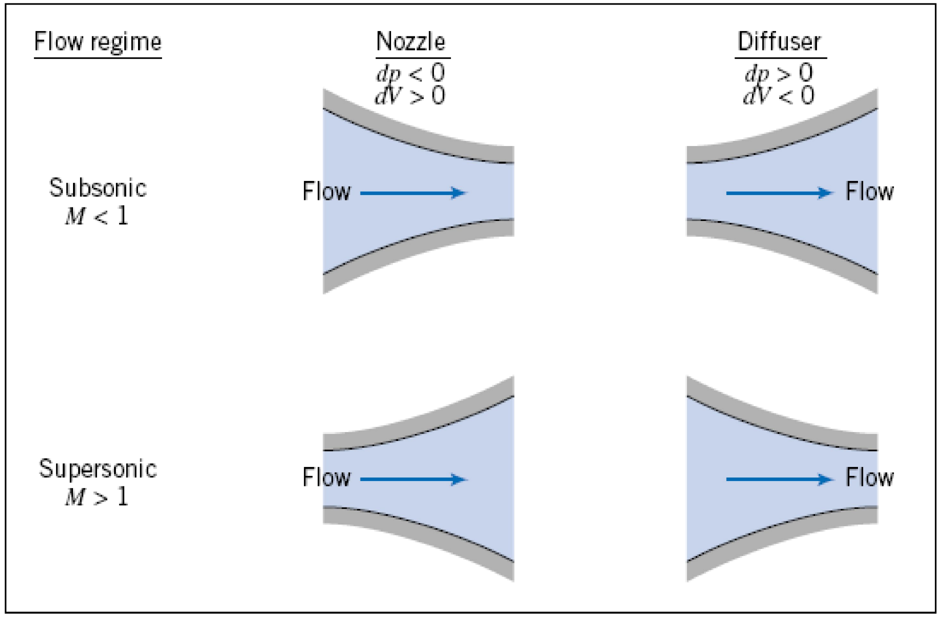


Fig. 13.3 Nozzle and diffuser shapes as a function of initial Mach number.

Isentropic Flow of an Ideal Gas – Area Variation

- Property Relations

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{k-1}{2}M^2\right]^{1/(k-1)}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)}$$

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow in a Converging Nozzle

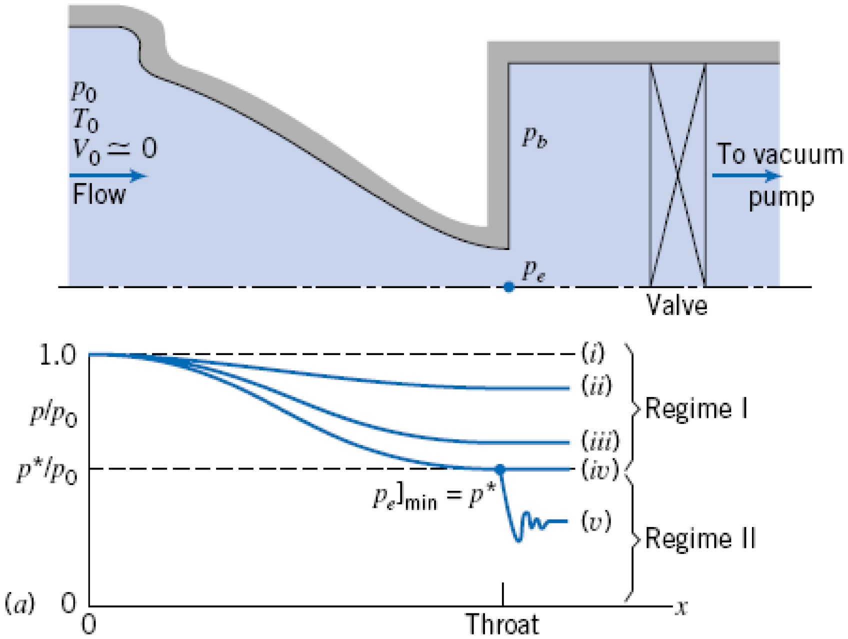


Fig. 13.6 Converging nozzle operating at various back pressures.

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow in a Converging Nozzle

$$\left.\frac{p_e}{p_0}\right|_{\text{choked}} = \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\dot{m}_{\text{choked}} = A_e p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}$$

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow in a Converging-Diverging Nozzle

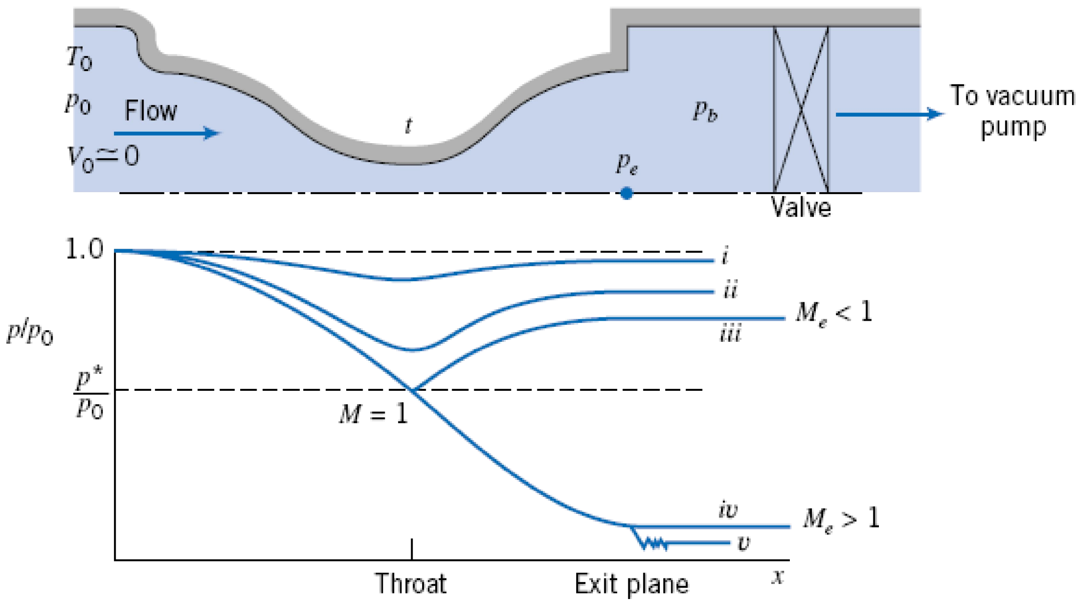


Fig. 13.8 Pressure distributions for isentropic flow in a converging-diverging nozzle.

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow in a Converging-Diverging Nozzle

$$\dot{m}_{\text{choked}} = A_t p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1} \right)^{(k+1)/2(k-1)}$$

Normal Shocks

- Control Volume

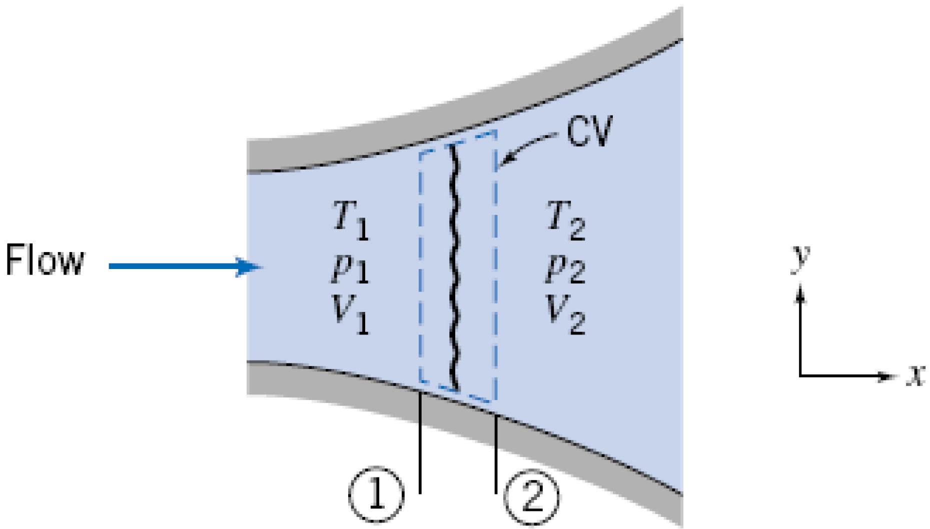


Fig. 13.17 Control volume used for analysis of normal shock.

Normal Shocks

- Basic Equations for a Normal Shock

$$\rho_1 V_1 = \rho_2 V_2 = \frac{\dot{m}}{A} = \text{constant}$$

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

$$h_{0_1} = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h_{0_2}$$

$$s_2 > s_1$$

$$p = \rho R T$$

$$\Delta h = h_2 - h_1 = c_p \Delta T = c_p (T_2 - T_1)$$

$$\Delta s = s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Normal Shocks

- Normal Shock Relations

$$M_2^2 = \frac{M_1^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1} M_1^2 - 1}$$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left[\dfrac{\dfrac{k+1}{2} M_1^2}{1 + \dfrac{k-1}{2} M_1^2} \right]^{k/(k-1)}}{\left[\dfrac{2k}{k+1} M_1^2 - \dfrac{k-1}{k+1} \right]^{1/(k-1)}}$$

Normal Shocks

- Normal Shock Relations (Continued)

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{k-1}{2} M_1^2\right)\left(kM_1^2 - \frac{k-1}{2}\right)}{\left(\frac{k+1}{2}\right)^2 M_1^2}$$

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_1^2 - \frac{k-1}{k+1}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{\frac{k+1}{2} M_1^2}{1 + \frac{k-1}{2} M_1^2}$$

Supersonic Channel Flow with Shocks

- Flow in a Converging-Diverging Nozzle

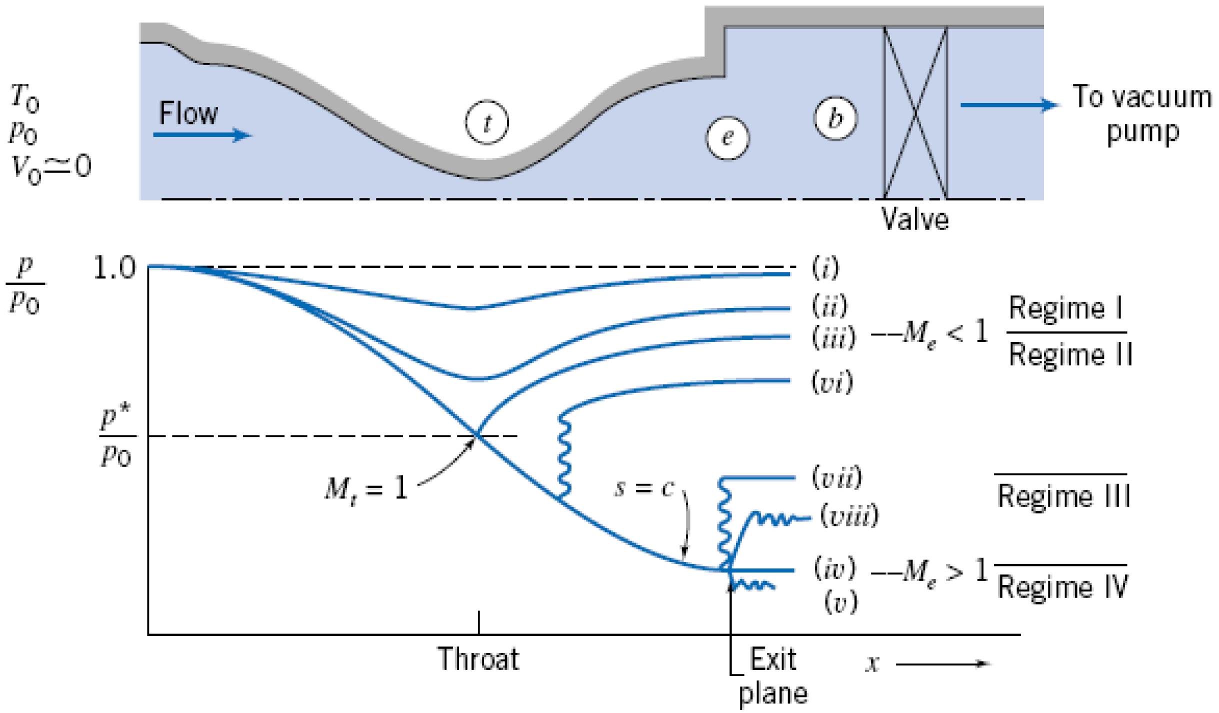
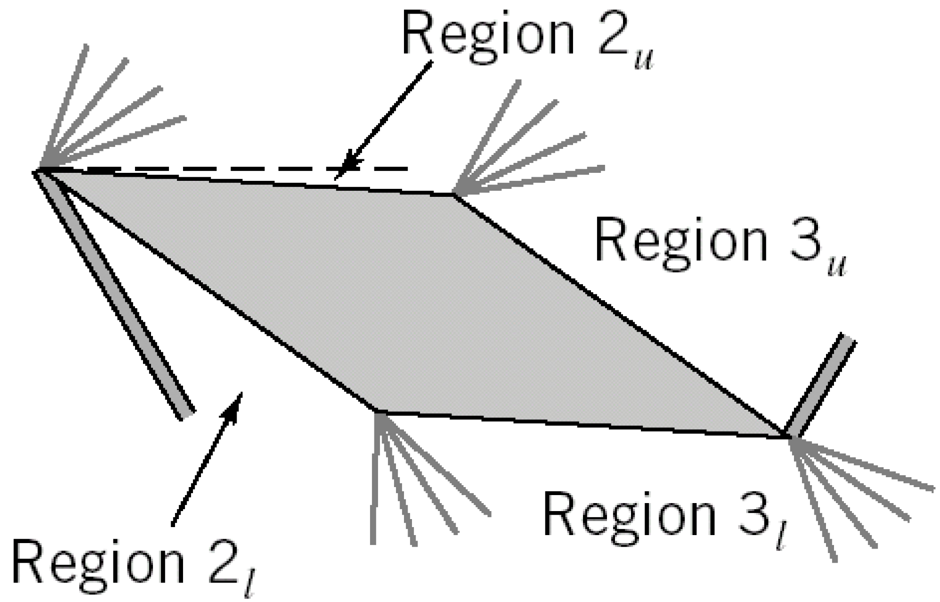


Fig. 13.20 Pressure distributions for flow in a converging-diverging nozzle for different back pressures.

Oblique Shocks and Expansion Waves

- Typical Application



Oblique Shocks and Expansion Waves

- Mach Angle and Oblique Shock Angle

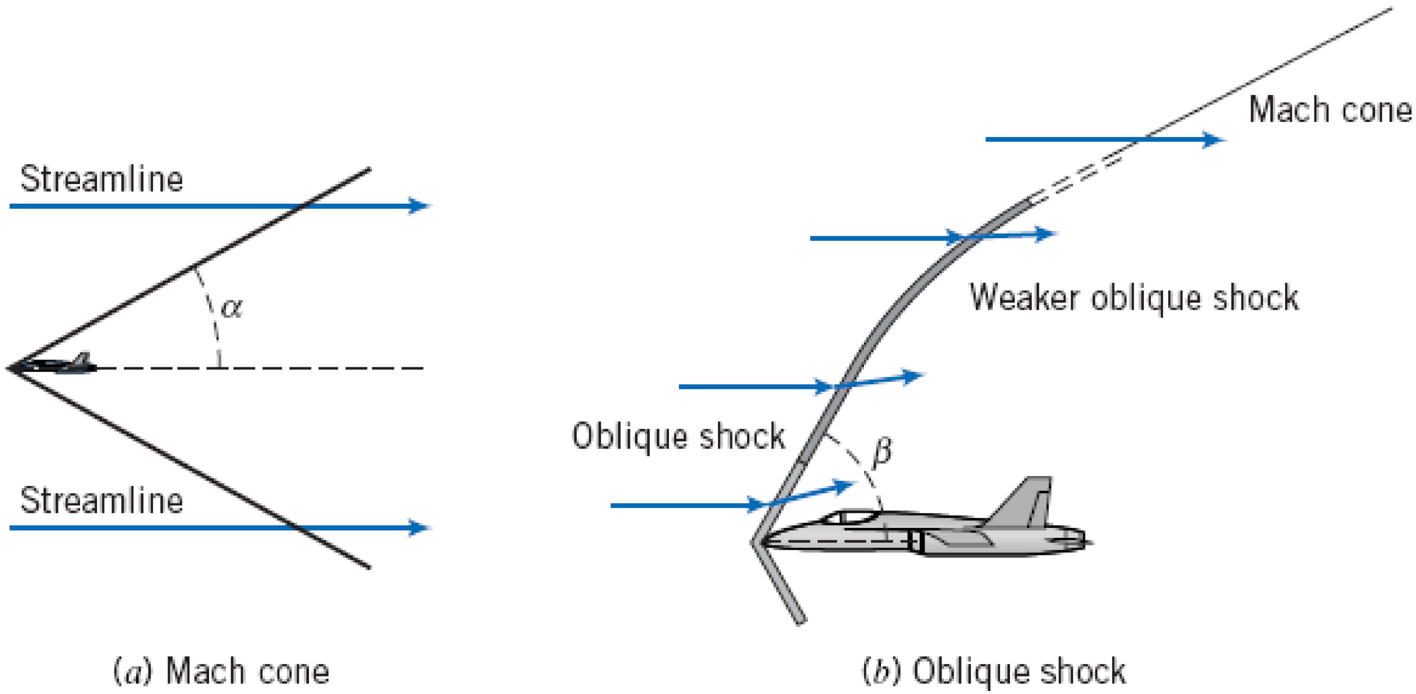
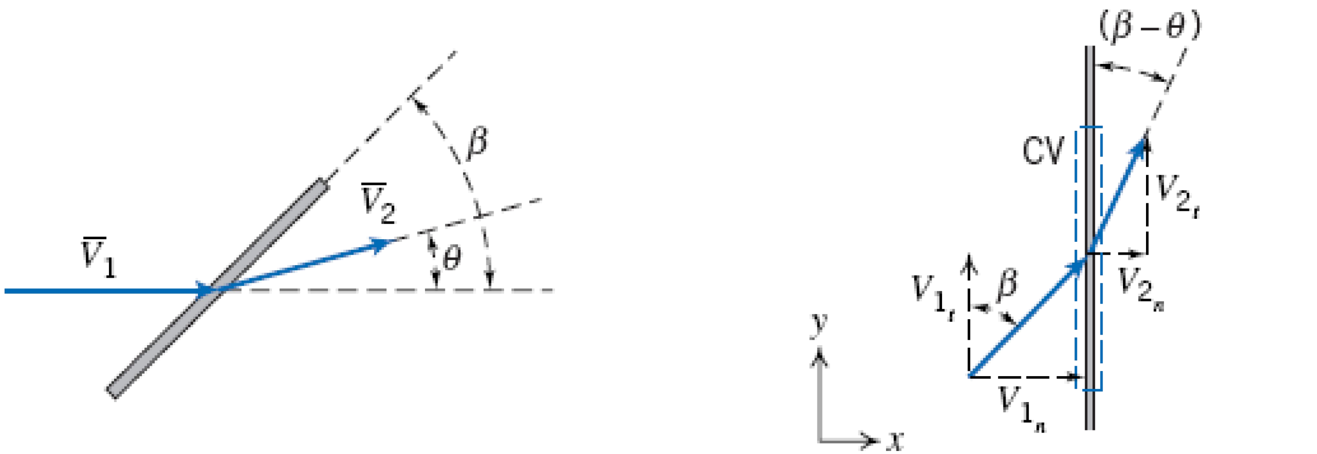


Fig. 13.26 Mach cone and oblique shock generated by airplane.

Oblique Shocks and Expansion Waves

- Oblique Shock: Control Volume



(*a*) Oblique shock (*b*) Oblique shock in shock coordinates

Fig. 13.28 Oblique shock control volume.

Oblique Shocks and Expansion Waves

- Oblique Shock: Useful Formulas

$$V_{1_n} = V_1 \sin\beta \qquad M_{1_n} = \frac{V_{1_n}}{c_1} = M_1 \sin\beta$$

$$V_{2_n} = V_2 \sin(\beta - \theta) \qquad M_{2_n} = \frac{V_{2_n}}{c_2} = M_2 \sin(\beta - \theta)$$

Oblique Shocks and Expansion Waves

- Oblique Shock Relations

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1} M_{1_n}^2 - 1}$$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left[\dfrac{\dfrac{k+1}{2} M_{1_n}^2}{1 + \dfrac{k-1}{2} M_{1_n}^2}\right]^{k/(k-1)}}{\left[\dfrac{2k}{k+1} M_{1_n}^2 - \dfrac{k-1}{k+1}\right]^{1/(k-1)}}$$

Oblique Shocks and Expansion Waves

- Oblique Shock Relations (Continued)

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{k-1}{2} M_{1_n}^2\right)\left(k M_{1_n}^2 - \frac{k-1}{2}\right)}{\left(\frac{k+1}{2}\right)^2 M_{1\,n}^2}$$

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_{1_n}}{V_{2_n}} = \frac{\frac{k+1}{2} M_{1_n}^2}{1 + \frac{k-1}{2} M_{1_n}^2}$$

Oblique Shocks and Expansion Waves

- Oblique Shock: Deflection Angle

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2\,(k + \cos 2\beta) + 2}$$

Oblique Shocks and Expansion Waves

- Oblique Shock: Deflection Angle

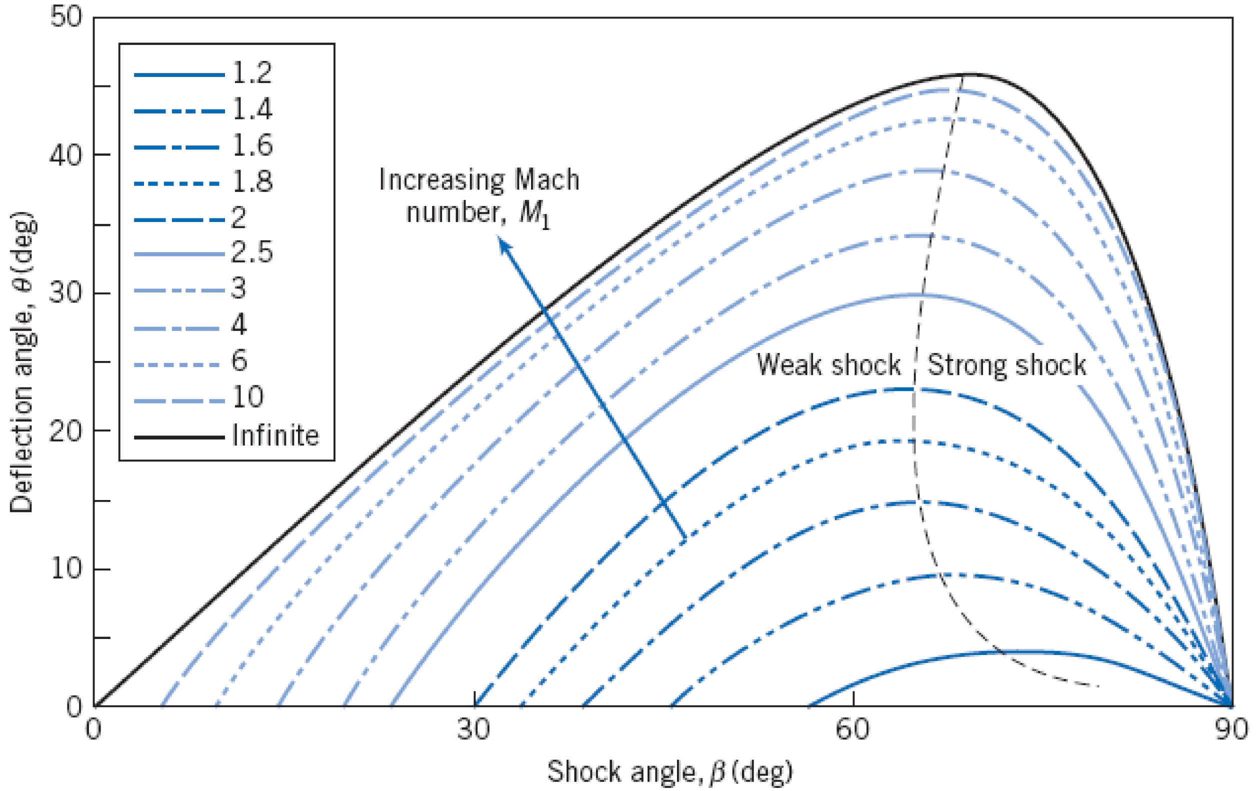
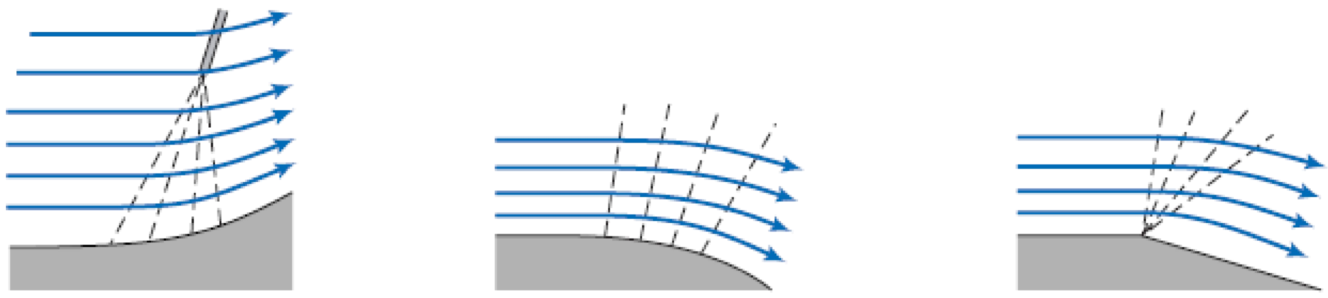


Fig. 13.29 Oblique shock deflection angle.

Oblique Shocks and Expansion Waves

- Expansion and Compression Waves



(*a*) Isentropic compression waves (*b*) Isentropic expansion waves (*c*) Isentropic expansion at a corner

Fig. 13.30 Isentropic compression and expansion waves.

Oblique Shocks and Expansion Waves

- Expansion Wave: Control Volume

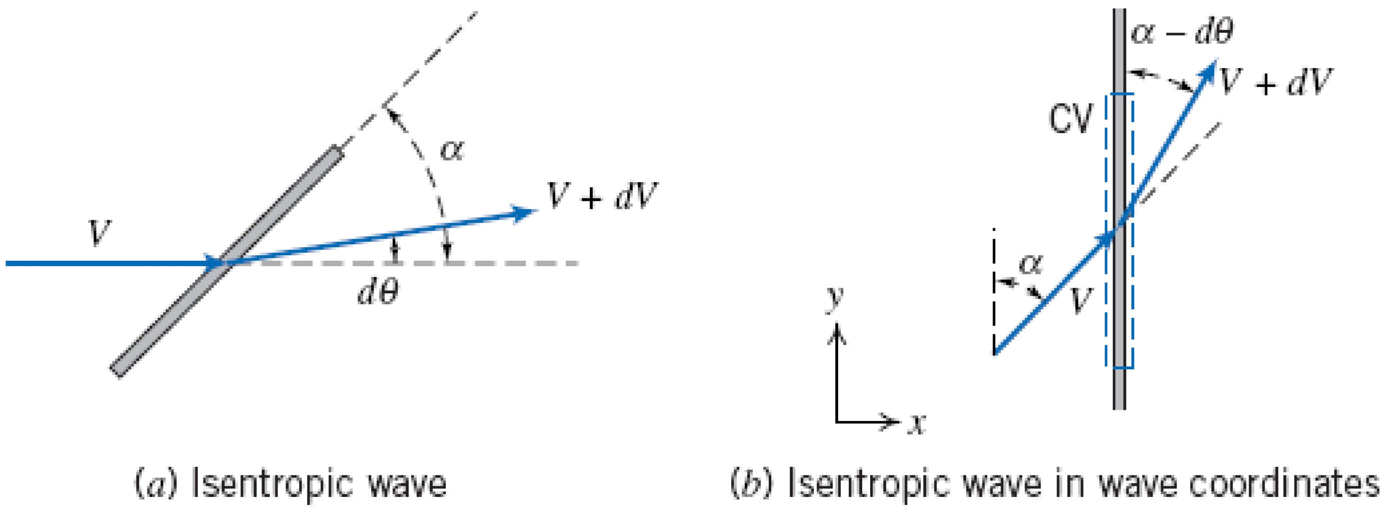


Fig. 13.31 Isentropic wave control volume.

Oblique Shocks and Expansion Waves

- Expansion Wave:
  Prandtl-Meyer Expansion Function

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right)$$

$$\text{Deflection} = \omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$$

Oblique Shocks and Expansion Waves

- Expansion Wave: Isentropic Relations

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{k-1}{2}M^2\right]^{1/(k-1)}$$