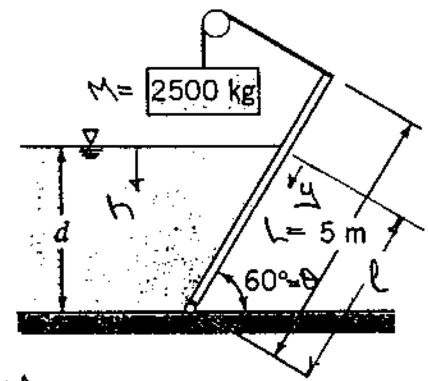
Problem 3.58



Given: Gate shown has width $b = 3\,m$; mass of gate is negligible.
Gate is in equilibrium

Find: Water depth, d

Solution:

Basic equation: $\frac{dp}{dh} = \rho g \qquad \Sigma M_z = 0$

Computing equations: $F_R = p_c A$; $y' = y_c + \frac{I_{\hat{x}\hat{x}}}{y_c A}$; $I_{\hat{x}\hat{x}} = \frac{bL^3}{12}$

Assumptions: (1) static liquid (2) ρ = constant
(3) p_{atm} acts at free surface and on underside of gate.

Then on integrating $dp = \rho g dh$, we obtain $p = \rho g h$

$F_R = p_c A = \rho g h_c A \qquad h_c = \frac{d}{2}, \; A = b \times \frac{d}{\sin\theta}$

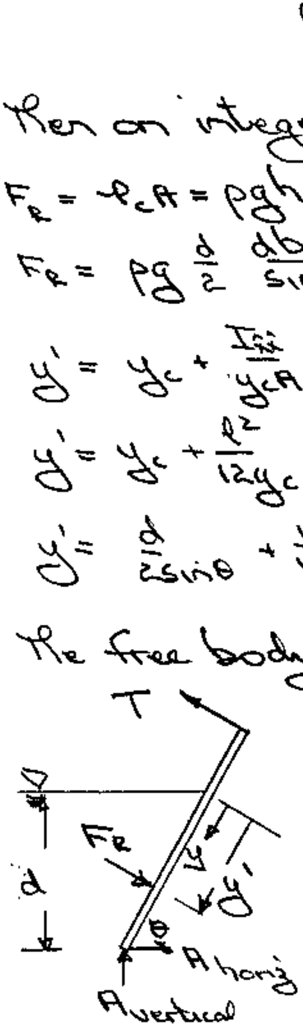
$F_R = \rho g \frac{d}{2} \frac{db}{\sin\theta} = \frac{\rho g b d^2}{2\sin\theta}$

$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{y_c A} = y_c + \frac{1}{12}\frac{b\ell^3}{y_c \ell b}$ where ℓ is length of gate in contact with the water

$y' = y_c + \frac{\ell^2}{12 y_c} \qquad \ell = \frac{d}{\sin\theta}, \; y_c = \frac{\ell}{2} = \frac{d}{2\sin\theta}$

$y' = \frac{d}{2\sin\theta} + \frac{1}{12}\left(\frac{d}{\sin\theta}\right)^2 \frac{2\sin\theta}{d} = \frac{d}{2\sin\theta} + \frac{d}{6\sin\theta} = \frac{2d}{3\sin\theta}$

The free body diagram of the gate is as shown.



Summing moments about A

$\Sigma M_z = 0 = TL - (\ell - y')F_R \qquad T = Mg$

$MgL = (\ell - y')F_R = \left(\frac{d}{\sin\theta} - \frac{2d}{3\sin\theta}\right)\frac{\rho g b d^2}{2\sin\theta}$

$MgL = \frac{1}{3}\frac{d}{\sin\theta} \times \frac{\rho g b d^2}{2\sin\theta} = \frac{\rho g b d^3}{6 \sin^2\theta}$

$d^3 = \frac{6\sin^2\theta\, ML}{\rho b}$

$d = \left[6 \times \sin^2 60° \times 2500\,kg \times 5\,m \times \frac{m^3}{999\,kg} \times \frac{1}{3\,m}\right]^{1/3} = 2.66\,m$ ← d