Investigation of the Large Scale Flow Structures in Dual Planar Attaching Jets

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An experimental investigation was performed to characterize the large scale structures present in turbulent offset attaching jets with a lower velocity co-flowing jet. The results showed that the nature of the structures present in the jets changed as the ratio of the lower jet velocity to the upper jet velocity changed. When the lower jet velocity was small, the lower jet was entrained into the upper jet before it attached to the surface. In these cases, the structures that developed in the inner shear layer were most prominent near the reattachment point, before a lower frequency motion that may be related to the outer shear layer of the jet became dominant. When the lower jet velocity was higher, there was a significant periodic motion in the near field that seemed to be associated with a flapping of the inner shear layer of the upper jet. This significantly increased the vertical fluctuating velocity in this region, but the fluctuations seemed to be damped as the inner shear layer approached the wall and did not have a large impact on the heat transfer or fluctuating pressure in this region where the upper jet interacted with the wall.

Nomenclature

- $C_{p'}$ Fluctuating wall pressure coefficient, $2p'/\rho U_U^2$
- c_n Specific heat of air, J/KgK
- f Frequency, Hz
- F_{pp} Power spectrum of fluctuating pressure, pa^2/Hz
- H_c Height of the splitter plate between the two jets, m
- H_i Height of the upper jet, m
- P Static pressure, pa
- p' Rms value of the fluctuating wall pressure, pa
- Re Reynolds number, $U_U H_i / \nu$
- St Stanton number, $h/\rho c_p U_U$
- U_L Flow rate averaged velocity of the lower jet at the exit, m/s
- U_U Flow rate averaged velocity of the upper jet at the exit, m/s
- u' RMS value of the horizontal component of the velocity fluctuations, m/s
- v' RMS value of the vertical component of the velocity fluctuations, m/s
- X_r Attachment length, m
- x Spatial coordinate in the streamwise direction, m
- y Spatial coordinate in the vertical direction, m
- z Spatial coordinate in the cross-stream direction, m
- ϕ Phase angle of cross spectra
- ν Kinematic viscosity of air, m^2/s
- ρ Density of air, kg/m^3
- ρ_{pp} Correlation coefficient of the fluctuating wall pressure
- ρ_{pu} Correlation coefficient of the fluctuating pressure and fluctuating streamwise velocity
- ρ_{pu} Correlation coefficient of the fluctuating pressure and fluctuating vertical velocity

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I. Introduction

Offset attaching planar jets, where planar jets exit near a wall, can be found in a number of practical applications. In these flows, the planar jet initially traveling parallel to an adjacent wall curves toward the wall as it develops downstream and eventually attaches to the wall due to the Coanda effect at a downstream position that depends on the initial distance of the jet from the wall.¹⁵ The interaction of the flow with the wall produces a maximum in the static wall pressure and convective heat transfer rate.^{12,15} The development of the Reynolds averaged flow field in the turbulent offset planar jet have been characterized analytically, ¹⁹ experimentally, ^{15,18} and numerically using RANS models¹⁷ for cases when the distance of the lower edge of the jet from the wall was greater than the jet height.

Gao et al.⁹ and Li et al.¹³ recently examined the development of offset attaching jets with a second lower velocity co-flowing jet issuing between the main jet and the wall, typical of the cooling jets used in the blow film manufacturing process. They found that the lower speed jet was entrained into the main jet before the main jet attached to the wall making this process more gradual and reducing the maximum in the static and fluctuating pressure. This was consistent with the previous work of Tanaka et al.^{21–23} who examined the development of a radial attaching jet discharged from a cylindrical nozzle when air was blown or sucked though a gap in the corner of the recirculation region. Tanaka et al. found that a suction flow shifted the location of the reattachment point upstream and increased the maximum in the static pressure, while a blowing flow had the opposite effect. This effect was more pronounced for higher added flow rates or for attaching jets with smaller offset distances. Forliti and Strykowski⁵ also consider the effect of a suction flow on the flow over a backward facing step and found that the turbulence in the shear layer increased when suction was applied.

Gao and Ewing⁷ also considered the development of the large-scale structures produced by the cooling jets used in the blow film manufacturing process. They found that the characteristic frequency of the wall pressure fluctuations changed as the flow evolved downstream. In particular, the main jet produced fluctuations with frequencies at $fH_j/U_U \approx 0.2$ when it initially interacted with the wall. The contributions from these fluctuations gradually decreased as the flow evolved downstream and another low frequency peak at $fH_j/U_U \approx 0.05$ emerged in the pressure spectra measured at $x/H_j > 8$. This seemed to be associated with the structures in the outer shear layer of the main jet. This investigation⁷ was limited to cases when the ratio of the lower jet velocity to the main jet velocity was less than 0.3, and it was not performed for a generic geometry.

The objective of this investigation is to characterize the development of the large scale structures in coflowing attaching jets for a range of different velocity ratios. The experiments were performed using a more generic geometry where both jets exited long channels. The experimental facility and methodology used in the investigation are discussed first. Some measurement of the mean flow are then presented to illustrate the differences in the mean flow. More details on the Reynolds averaged flow field, the heat transfer and skin friction can be found in the accompanying paper at this conference.⁸ The development of the large-scale structures in the flows are then characterized using measurements of the fluctuating wall pressure and the correlation between the fluctuating wall pressure and the velocity fluctuations in the flow.

II. Experimental Methodology

The development of the jets was examined using the facility shown in Fig. 1. The two jets in this facility exited long channels designed so that flows at the exit would be nearly fully developed. The air flow, supplied from a 10 HP blower, was split into a 122cm by 72.4cm by 45.7cm settling chamber attached to the upper channel, and a 81.3cm by 40.6cm by 22.9cm settling chamber attached to the lower channel. The hose connections included gate valves that were used here to adjust the flow rate to the jets. Both settling chambers included layers of foam and perforated plates to condition the flow. The width of both channels were 74.3cm. The height of the upper channel was 3.8cm so the ratio of the upper jet width to its height is 19.5, while the height of the lower jet was 1.8cm so the aspect ratio of this jet was 41. The lower edge of the upper jet was 3.8 cm above the wall, so ratio of the offset distance of the jet to the jet height, H_s/H_j , was 1. The height of the splitter plate between the two jet, H_c , for most of the measurements was 2.0 cm, so the lower jet height was 1.8 cm. Measurements were also performed for a second case when the height of

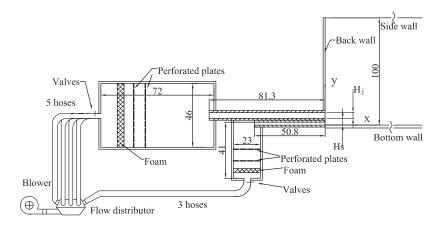


Figure 1. Schematic of the dual attaching planar jet facility with an offset ratio $H_s/H_j=1.0$.

the splitter plate was $3.1\,cm$, so the height of the lower jet was $0.7\,cm$.

The flow exiting the channels developed over a 1.8m long plate that was mounted parallel to the channels. The flow was confined using a 80cm high back wall and two 100cm high and 180cm long side walls. The profiles measured along the both channel outlets were uniform over the central region of the jet from $-6 \le z/H_j \le 6$, with variations in mean and rms velocity less than 2% and 5%, respectively. The effect of side walls on the two dimensionality of the jets was also checked by measuring the boundary layer on the side walls as the flow evolved downstream. The thickness of these layers grew to approximately $7H_i$ at $x/H_i = 20$.

The development of the flow field in the offset jets was measured using a cross-wire probe and an in-house hot-wire anemometry system. The measurements were performed using a probe with wire diameter of 5μ m and length of $1.5\,mm$. The cross-wire probe was calibrated in a separate round jet facility where the jet exited a contoured nozzle using a modified cosine calibration technique. The flow field was measured by moving the hot-wire probe on a computer controlled traverse that could be positioned with an accuracy less than $0.05\,mm$. At each point, the output signal from the anemometry system was sampled using a 14-bit A/D board at a frequency of 4096Hz for a total time of 50 seconds. The uncertainties for mean velocity and rms velocity measurements due to sample size evaluated following the approach in Bruun are less than 1% and 3%, 19 times out of 20, respectively. The ambient air temperature during the measurements was measured using a RTD temperature sensor with a resolution of 0.1° C. The temperature varied less than $\pm 1^{\circ}$ C in all cases, and the effect of the temperature on the velocity measurements was compensated using the technique proposed by Beuther.

The static pressure on the bottom wall was measured using a series of pressure taps mounted along the jet centerline of the wall. The fluctuating pressure was measured using 16 Panasonic WM-60B microphones, that have a flat response for 20Hz to 5000Hz. The microphones were mounted directly into blind cavities drilled from the bottom of the wall on a line 0.75 cm off the centerline of the jet. The microphones sense the flow through a 1 mm-diameter, 5 mm-long pinhole drilled through the wall to the top of the cavity. Measurements indicated that the lowest resonance frequency was approximately 2 000 Hz and the spectra was largely unaffected for frequencies less than 1 800 Hz. The microphones were calibrated in situ using a piston phone at 1 000 Hz. The signals from the microphones were simultaneously acquired with the 14-bit A/D board in 100 blocks of 4096 data points at a frequency of 4096Hz. The uncertainties for the static and fluctuating wall pressure due to the sample size were less 2% and 5%, respectively, 19 times out of 20. The response of these microphones were checked using an acoustic chamber against a reference microphone and were found to be within $50\mu s$. The development of structures was also characterized by simultaneously sampling the signals from cross-wire and the microphones in 100 blocks of 4096 data points at a frequency of 4096Hz.

The measurements were performed here for cases where the average velocity of the upper jet at the exit, U_U , was 18m/s that corresponded to a Reynolds number of 43 000, and the ratio of the velocities of the two jets $U_L/U_U = 0$ to 0.8. The lower jet was sealed when the velocity ratio of 0 was studied.

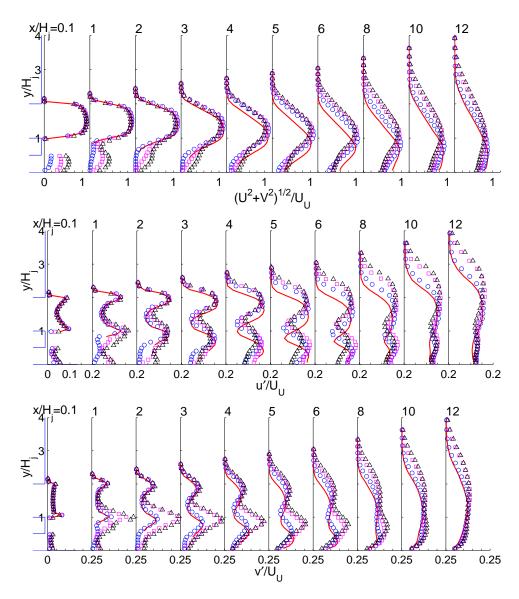
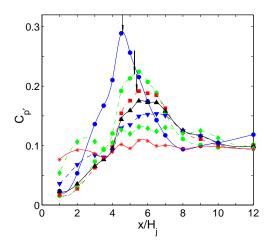


Figure 2. The profiles of the mean and fluctuating velocities for jets with $U_L/U_U = -0 \circ 0.2 \square 0.4$ and $\triangle 0.6$.

III. Results and Discussions

The distributions of the mean velocity, u', and v' in the jets with $U_L/U_U=0$ to 0.8 are shown in Fig. 2. The results show that the velocity of the lower jet had a significant effect on the development of the main jet. When the lower jet velocity was $0.2U_U$, the lower jet separated from the wall and was entrained by the upper jet. The resulting jet attached to the wall at $x/H_j \approx 5.4$ further downstream than when the co-flowing jet was not present. The lower jet did not separate from the wall in a mean sense when the outlet velocity velocity was $0.4U_U$ or larger. In these cases, there was a significant increase in v' in the inner shear layer of the outer jet. In fact, the value of v' was larger than u' in the wake behind the splitter plate indicating that the presence of the lower jet has a dramatic impact on the vertical fluctuations in the inner shear layer in the main jet. The vertical fluctuations were attenuated when the upper jet interacted with the wall. Somewhat surprisingly, the large increase amplification in the vertical fluctuating velocity did not have a large impact on the fluctuating pressure or the Stanton numbers in the region where the upper jet interacted with the wall shown in Fig. 3.8 The fluctuating wall pressure and the Stanton number did increase near jet exit at $x/H_j < 4$ as the lower jet velocity was increased, but it was not clear whether this was related to the change in the lower jet velocity or to the increase in the fluctuations in the inner shear layer of the outer jet. Further



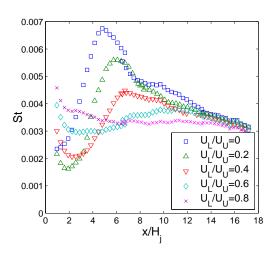


Figure 3. Distributions of (a) the fluctuating wall pressure coefficient for the cases with velocity ratios $U_L/U_U = \bullet$ 0, \circ 0.1 \Box 0.2 \triangle 0.3 \triangledown 0.4 \lozenge 0.6 * 0.8. and (b) the Stanton number for $U_L/U_U = \Box$ 0 \triangle 0.2 \triangledown 0.4 \lozenge 0.6 and \times 0.8.

details on the development of the Reynolds averaged flow field, heat transfer, and skin friction can be found in Gao $et\ al.^8$

The spectra of the fluctuating wall pressure for different cases are shown in Fig. 4. The spectra for frequencies less than 20Hz are dotted here because they are below the flat response region of the microphones. The spectra in the offset jet near the jet exit had low frequency oscillations at $fX_r/U_a \approx 0.1$ similar to those observed in the flow over a backward facing step¹⁰ that are thought to be associated with the flapping of the recirculating flow region. These low frequency oscillations near the outlet were apparent when the lower jet velocity was $0.2U_L$, but not for larger lower jet velocities. Further downstream, there were motions with a higher characteristic at $fH_i/U_U \approx 0.2$. In the jets with $U_L/U_U = 0$ and 0.2 the characteristic frequency decreased as the flow evolved downstream and attached to the wall. The characteristic frequency at the reattachment location was $fX_R/U_U \approx 1.0$ consistent with the results in other reattaching shear layer flows.^{2,4,10} When the velocity ratio was increased to 0.4, was still a small peak at $fH_j/U_U \approx 0.2$, but there is another sharp peak at $fH_i/U_U \approx 0.44$ indicating that there were highly periodic motions in the jet. This peak was more prominent when the velocity ratio was increased to 0.6. In all cases, the higher frequency oscillations decrease as the flow evolves downstream, and lower frequency peaks at $fH_i/U_U \approx 0.04 - 0.06$ begin to emerge. As it will be shown later, these lower frequency oscillations are associated with the largescale structures that form in the wall jet downstream of the initial interaction between the upper jet and the wall. The low frequency peak begins to emerge at $x/H_i \approx 7$ in the offset jet with $U_L/U_U = 0$ and gradually shifts downstream as the lower jet velocity increases indicating that the transition of the flow to a wall jet flow is delayed when the lower jet velocity increases. The characteristic frequency of the motions are lower when the lower jet velocity was higher than 0.4, that may be because the characteristic width of the flow is larger.

The periodic motions in the offset jets with the large lower jet velocities seemed to be associated with the wake behind the splitter plate between the jets. This was examined by repeating the measurements for a case when the splitter plate thickness was increased to 3.1cm. The spectra of the fluctuating wall pressure for both splitter plates shown in Fig. 5 had a sharp peak at $fH_c/U_U \approx 0.24$ when the velocity ratio was 0.4 and 0.6 indicating the motions are indeed associated with the wake downstream of the splitter plate. As it will be shown later, however, the effect of this seems to be confined to the inner shear layer of the upper jet or one side of this wake.

The two-point and two-time correlations of the fluctuating pressure for cases with the reference microphones at $x/H_j=2$, 5, 8 and 12 are shown in Figs. 6 and 7. In the cases when $U_L/U_U=0$ to 0.2, the correlations of the fluctuating wall pressure upstream of the reattachment were similar. The convection velocity of the structures vary as the flow attached to the surface, and then decreased significantly after the flow attached to the wall typical of offset jets. The wall pressure fluctuations were very well correlated near the jet exit for the jets with $U_L/U_U=0.4$ and 0.6 due to the periodic motions generated in these flows. The

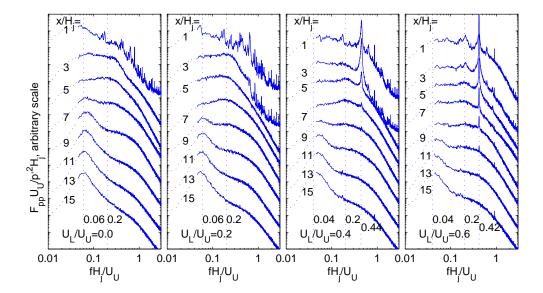


Figure 4. The spectra of the fluctuating wall pressure for jets with velocity ratios of 0, 0.2, 0.4 and 0.6.

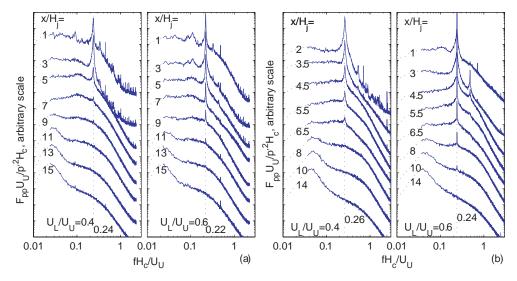


Figure 5. The spectra of the fluctuating wall pressure for jets with velocity ratios of 0.4 and 0.6 and a splitter plate thickess of (a) 2.0cm (b) 3.1cm.

convection velocity of the fluctuating pressure is relatively uniform until is decreases at $x/H_j \approx 10$, when pressure fluctuations change to the lower frequency motions.

The difference in the development of the motions in the two cases can be seen in the phase of the cross-spectra measured about $x/H_j=5$ and 8 in the jets with $U_L/U_U=0.2$ and 0.6 shown in Fig. 8. The phase of the cross-spectra changes approximately linearly with frequency indicating that the turbulent motions present in the flow are being convected over the surface, as expected. In the jet with $U_L/U_U=0.2$ (and 0 not shown here), the fluctuating wall pressure at $x/H_j=5$ ($x/X_r\approx 1$) is out of phase with the upstream fluctuations with frequencies less than $fH_j/U_U\approx 0.1$ are often associated with a flapping of the reattaching flow, but in phase with the upstream fluctuations with frequencies higher than $fH_j/U_U\approx 0.2$ ($fX_r/U_a\approx 1.0$) that are associated with the structures in the shear layer. The fluctuations at $x/H_j=5$ with frequencies greater than $fH_j/U_U\approx 0.2$ are in phase with the fluctuations downstream at $x/H_j=7$, indicating the structures in the reattaching inner shear layer are convected along the surface. They seem out of phase with the fluctuations at these frequencies at $x/H_j=8$, suggesting is an another transition in advection of the large-scale motions near this point that was also observed in the two-point correlation. Thereafter, the fluctuations with frequencies less than $fH_j/U_U\approx 0.1$ that are in phase with the fluctuations

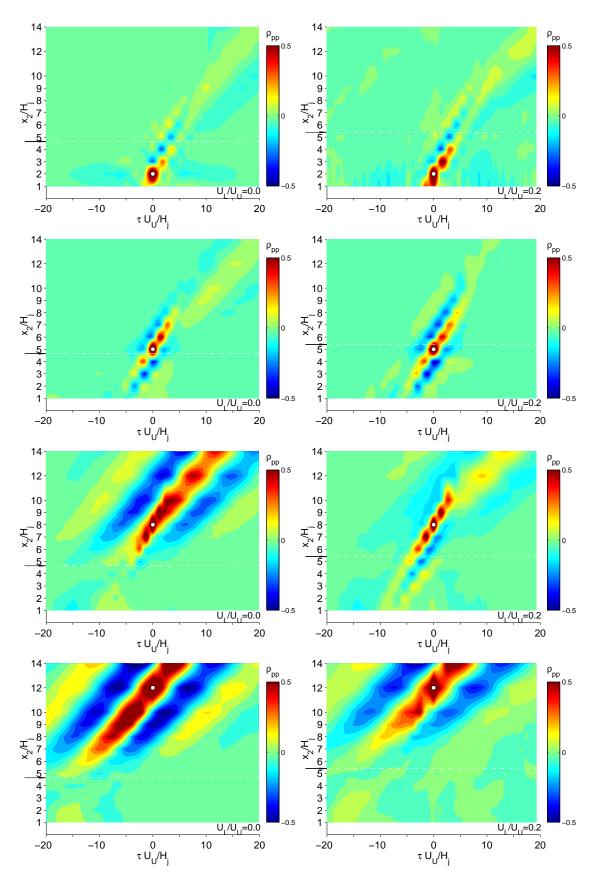


Figure 6. The two-time two-point correlation coefficient of fluctuating wall pressure, ρ_{pp} , for $x_1/H_j=2$, 5, 8 and 12 for jets with $U_L/U_U=0$ and 0.2.

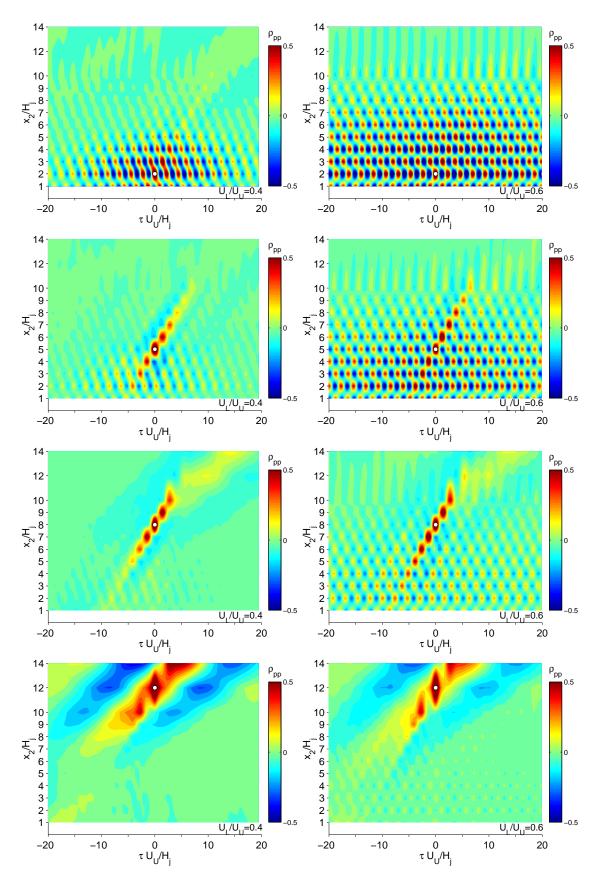


Figure 7. The two-point and two-time correlation coefficient of fluctuating wall pressure, ρ_{pp} , for $x_1/H_j=2$, 5, 8 and 12 for jets with $U_L/U_U=0.4$ and 0.6.

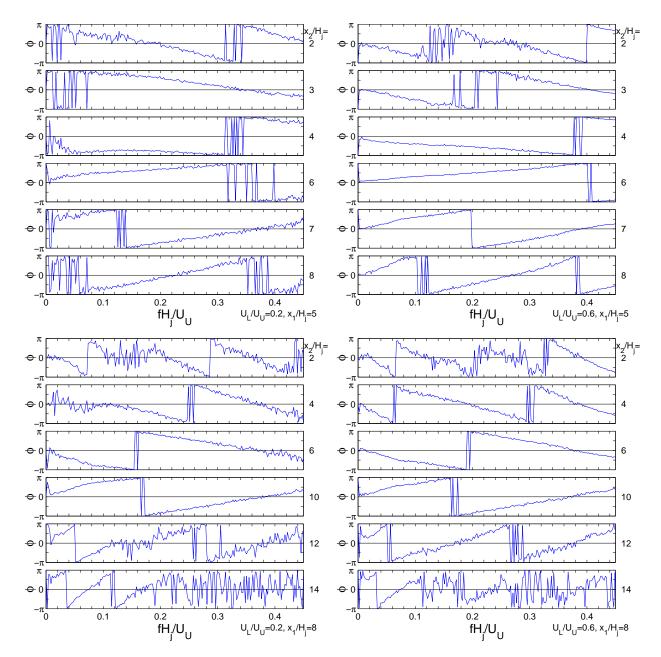


Figure 8. The phase angle of the fluctuating wall pressure cross-spectra about $x_1/H_j=5$ and 0.8 in the jets with $U_L/U_U=0.2$ and 0.6.

at $x/H_j = 8$ suggesting that the low frequency motions are being convected over the surface. These motions seemed to be associated with the motions in the outer shear layer, suggesting that this change in the phase was due to interaction between the outer shear layer and the wall. This will be examined further using measurements of the fluctuating pressure and the velocity.

In the jet with the velocity ratio $U_L/U_U=0.6$, the pressure fluctuations at $x/H_j=5$ were in phase with the fluctuations at all frequencies both upstream and downstream of this point up to $x/H_j=8$ indicating that the unsteady motions are convected along the surface. The relationship between the fluctuations at $x/H_j=8$ and those further downstream differ for frequencies greater than $fH_j/U_a\approx 0.2$, and those with frequencies less than this value. This is likely associated with the inner shear of the outer jet interacting with the wall. Only the motions with the lower frequencies at $x/H_j=14$ are in phase with those at $x/H_j=8$ due to the change of the motions present in the flow to lower frequency wall jet structures similar to the

cases with the lower velocity ratios.

The development of the structures in the flows can be examined using the correlation of the fluctuating wall pressure and fluctuating velocity in the flow. The results for the jets with a velocity ratio of 0.6 are shown in Fig. 9. The location of the two jet outlets are shown on the left in this figure to show where the motions are relative to these outlets. The figures are the two-time correlation at different downstream location, and the horizontal axis is the time separation and should not be misinterpreted as separation in the streamwise direction. It is clear that the highly periodic motions near the exit are associated with the shear layer of the inner jet. The time scale suggests they are associated with the wake behind the splitter plate and there are periodic motions in this region, but the fluctuations also have significant impact on the inner region of the main jet. The correlations between the fluctuating velocity and pressure decrease as the flow evolves downstream and the peak in the pressure spectra decrease. The periodicity of these correlations decreased significantly between $x/H_j = 6$ and 10 where the motions appear to interact with the wall, indicating that this interaction attenuates the periodic motions or at least their periodicity. There is evidence of a short time scale motion near the wall until $x/H_j = 12$ and even at $x/H_j = 14$. The characteristic frequency of these motions are not the same as the periodic motions but seem to be motions with a frequency of $fH_i/U_U=0.2$. At $x/H_j = 14$ the dominant motions seem to be a lower frequency more periodic motion consistent with the pressure spectra. These motions are similar to those observed in planar wall jets and it is thought that these may be formed due to the interaction between the outer shear layer and the wall. This will examined in a later investigation by using a stochastic estimation technique to approximate the velocity in the jet from the pressure velocity correlations and the simultaneous measurements of the fluctuating pressure along the wall.

The measurements of the correlation between the fluctuating wall pressure and fluctuating velocity were repeated for several other cases. One of particular interest was the case with a velocity ratio of 0.3, that seemed close to the transition between the two types of flows; i.e. there was evidence of a small mean recirculating flow region before the upper jet interacts with the surface. The pressure velocity correlations in this case are shown in Figure 10. The measurements near the outlet show evidence of a weak periodic motions in the inner shear layer. Further downstream, before the jet attaches to the wall the fluctuating wall pressure and velocity were correlated for a shorter time. The correlation from these motions are dominant until $x/H_j = 8$ and persist at $x/H_j = 10$ indicating that the structures in the reattaching shear layer persist until the lower frequency motions dominate. The characteristic frequency of the motions that are only correlated for a short time was different from the periodic motions. Evidence of this can be seen in the pressure spectra for the case with a velocity ratio $U_L/U_U = 0.4$, where there is a small peak in the spectra at $fH_j/U_a \approx 0.2$ similar to the lower velocity ratio jets, in addition to the sharp peak at $fH_j/U_a \approx 0.44$. The peak at $fH_j/U_a \approx 0.2$ also shifts in frequency as the flow evolves downstream indicating they were not a subharmonic of the periodic motions, so that the periodic motions seem to be a flapping of the jet shear layer.

IV. Concluding Remarks

An experimental investigation was performed to characterize the nature of the large-scale structures present in the flow field of offset attaching jets with a co-flowing jet. The results showed that the nature of the structures changed as the velocity ratio increased due to a change in the role of the lower jet. When the lower jet velocity was small, the lower jet was entrained into the upper jet before it attached to the surface. In this case, the wall pressure fluctuations were similar to the offset jet and other attaching shear flows. In particular, there was evidence of low frequency fluctuations near the jet exit that are often associated with the flapping of the recirculating flow region. The fluctuations near the reattachment point seemed to be associated with the inner shear layer of the main jet attaching to the surface. The large-scale structures then transition to a lower frequency more periodic motion as the flow evolved further downstream that were typical of the motions in planar wall jets.

When the velocity of the lower jet was increased there was evidence of highly periodic motions that significantly increased the vertical component of the fluctuating velocity in the inner shear layer of the upper jet. These motions were damped as the inner shear layer approach the wall. The measurements suggested that these periodic motions were not necessarily forced motions in the inner shear layer of the main jet, but instead seemed to be due to a flapping of the inner layer, although further investigation of the measurements is required to confirm this. One of the interesting aspects of this flow is that the large increase in the vertical velocity fluctuations did not seem to have a significant impact on the heat transfer in the region where

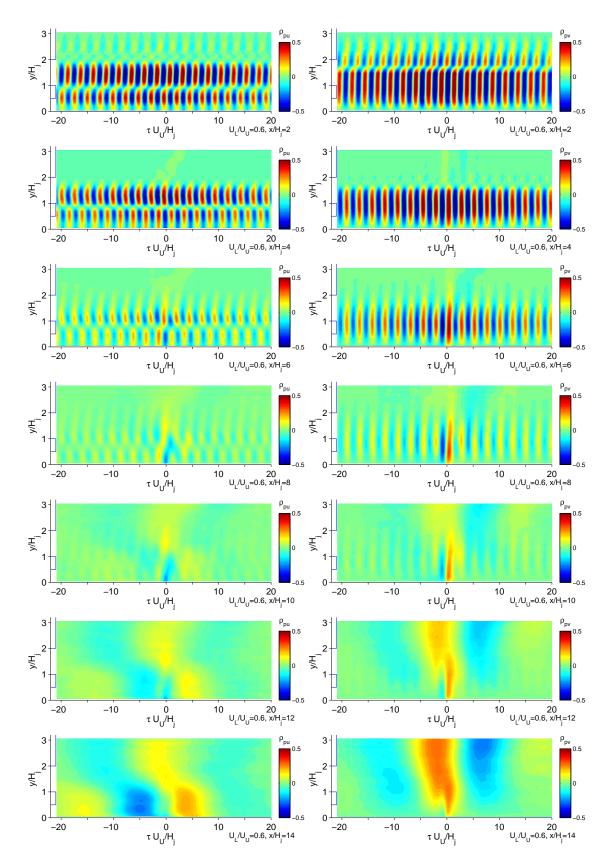


Figure 9. Correlation coefficient of fluctuating wall pressure coefficient and the streamwise and vertical component of the fluctuating velocity, ρ_{pv} and ρ_{pv} , for $x/H_j=2$ to 14 in jets with $U_L/U_U=0.6$.

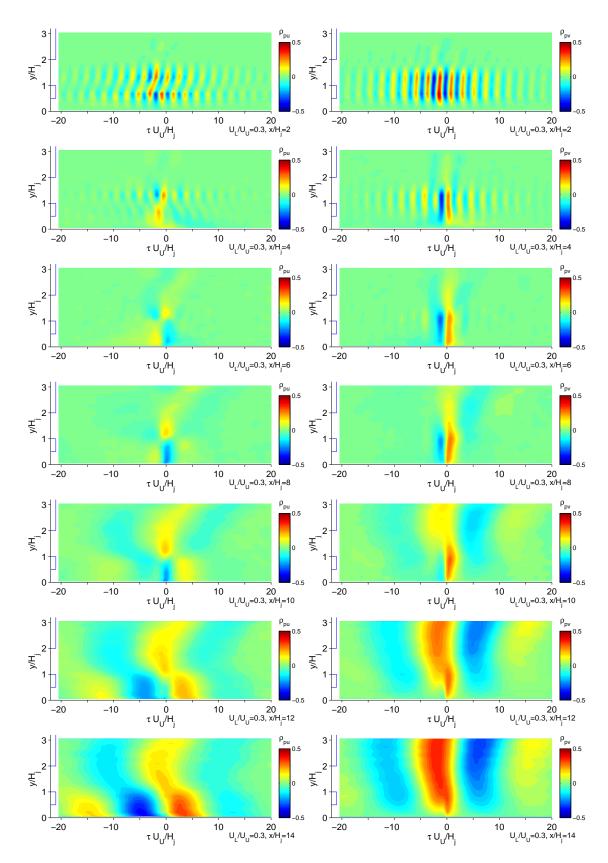


Figure 10. Correlation coefficient of fluctuating wall pressure and the streamwise and vertical component of the fluctuating velocity, ρ_{pv} and ρ_{pv} , for $x/H_j=2$ to 14 in jets with $U_L/U_U=0.3$.

the upper jet attached to the wall, providing further evidence that these periodic motions were damped before they interacted with the wall. This nature of this interaction will be investigate further in future investigations.

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