Engineering Mathematics II, Mid-term Exam #2

Function of Complex Variables

Civil Engineering 06, Dec 17 2007

*Only partial marks will be given if there is no detailed process in the solution.

Question 1:[10 marks] Evaluate the general and the principle values of $log(-3i)$.

Question 2:[10 marks] Evaluate $(-2-\sqrt{3}i)^{3/4}$.

Question 3:[15 marks] Determine $f'(z)$ for $f(z)=x+isiny$, state where $f(z)$ is analytic.

Question 4:[10 marks] Determine the conformality of $f(z)=sinz$ at $z=0$.

Question 5:[20 marks] Evaluate the following two integrals, where C is the counter-clockwise circle $|z|=3$, using any methods

$$I_1=\oint_C \frac{z^2-1}{z^2+1}e^z dz,$$

$$I_2=\oint_C \frac{dz}{z(z-2)(z-4)}.$$

Question 6:[10 marks] Obtain the Taylor series about $z=0$ for $f(z)=\frac{z^3}{2-iz}$, give the region of convergence.

Question 7:[10 marks] Determine the range of $f(z)=\frac{1}{z-1}$ for the domain given in Fig1.

Question 8:[10 marks] Determine the range of $f(z)=e^z$ for the domain given in Fig2.

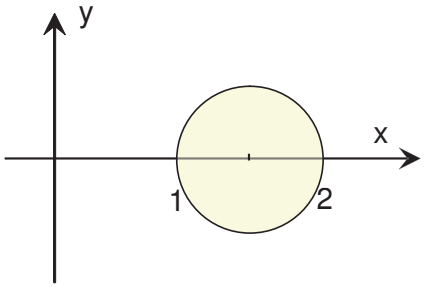

Figure 1

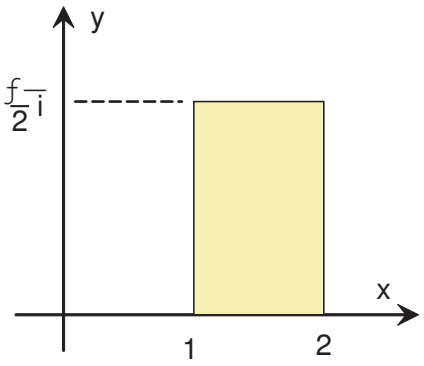

Figure 2

Question 9:[5 marks] Evaluate the series

$$-i-1+i+1-i-1+i+1-i-1+i+1\cdots\cdots$$