

速度测量

高南

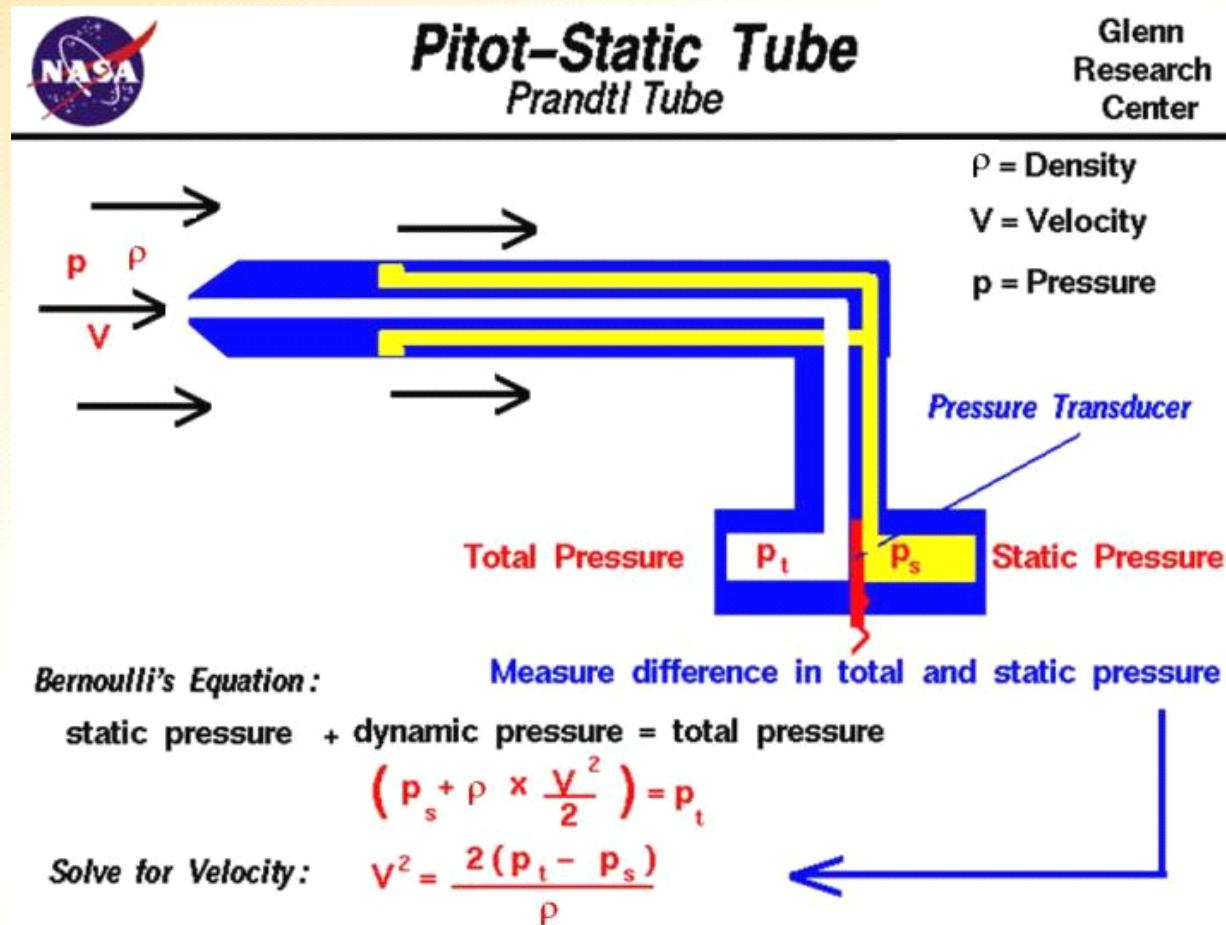
2013年4月3日

内容

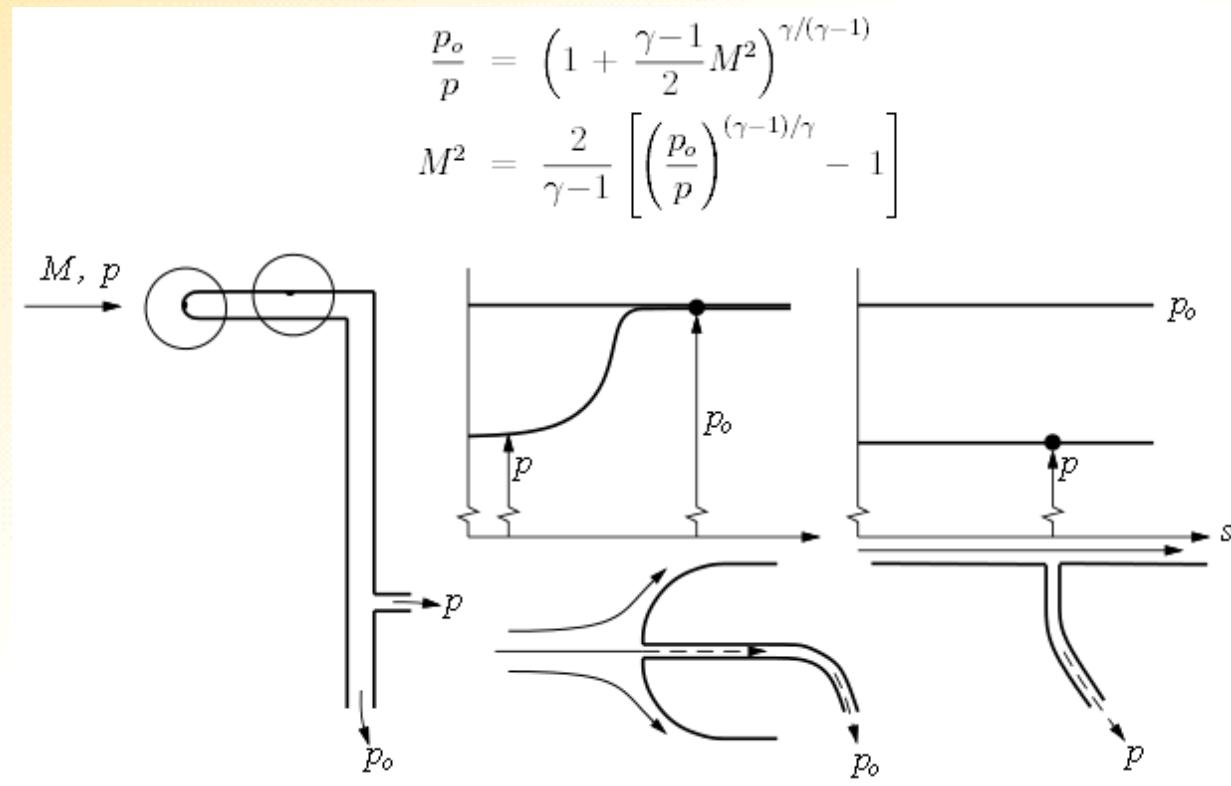
- ✓ 毕托管
- ✓ 激光粒子测速仪
- ✓ 激光多普勒测速仪
- ✓ 数据采集和处理

部分内容下载自**Dantec, National instrument
Validyne, Omega, Panasonic**网站

毕托管 平均压强

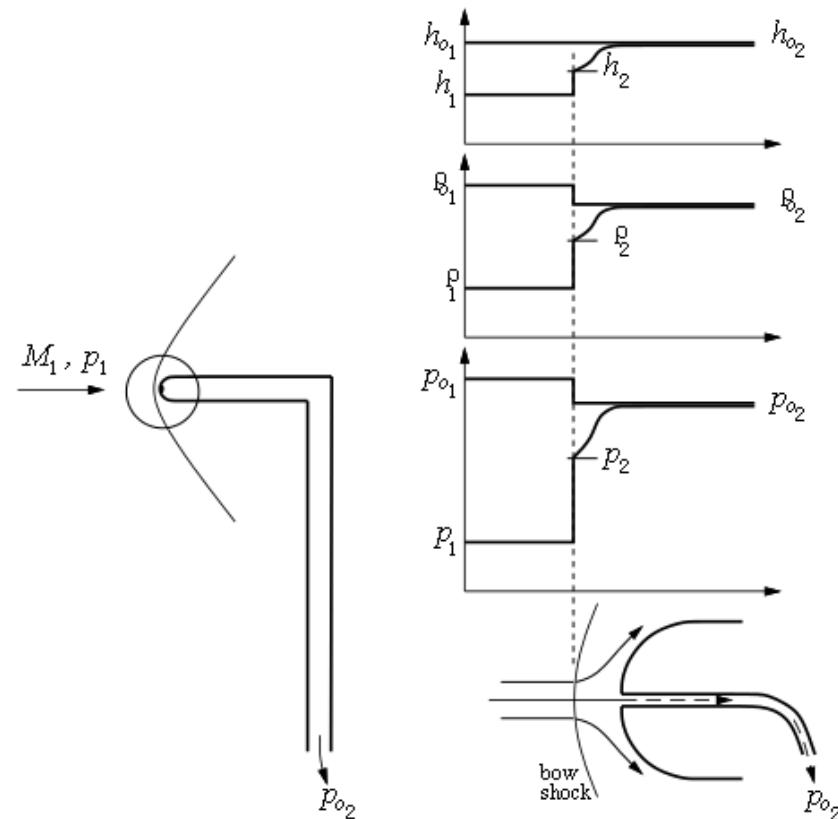


亚音速 毕托管



<http://web.mit.edu/16.unified/www/SPRING/fluids/Spring2008/LectureNotes/f16.pdf>

超音速 毕托管



Rayleigh Pitot tube formula.

$$\frac{p_{o2}}{p_1} = \frac{p_{o2} p_2}{p_2 p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)} \right)^{\gamma/(\gamma-1)} \frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1}$$

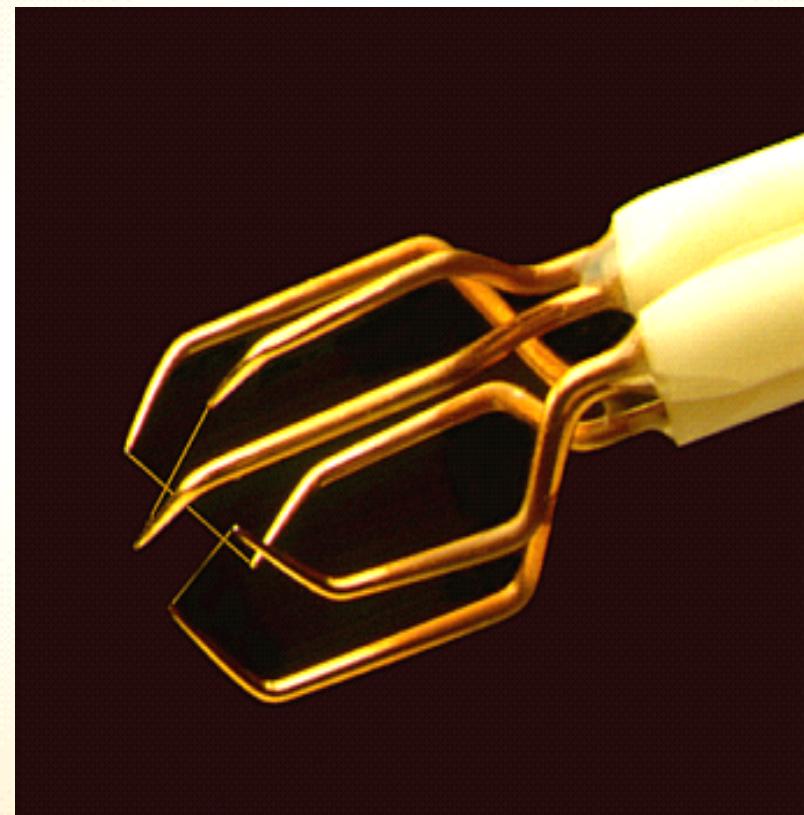
<http://web.mit.edu/16.unified/www/SPRING/fluids/Spring2008/LectureNotes/f16.pdf>



热线风速仪

Hot-Wire Anemometry

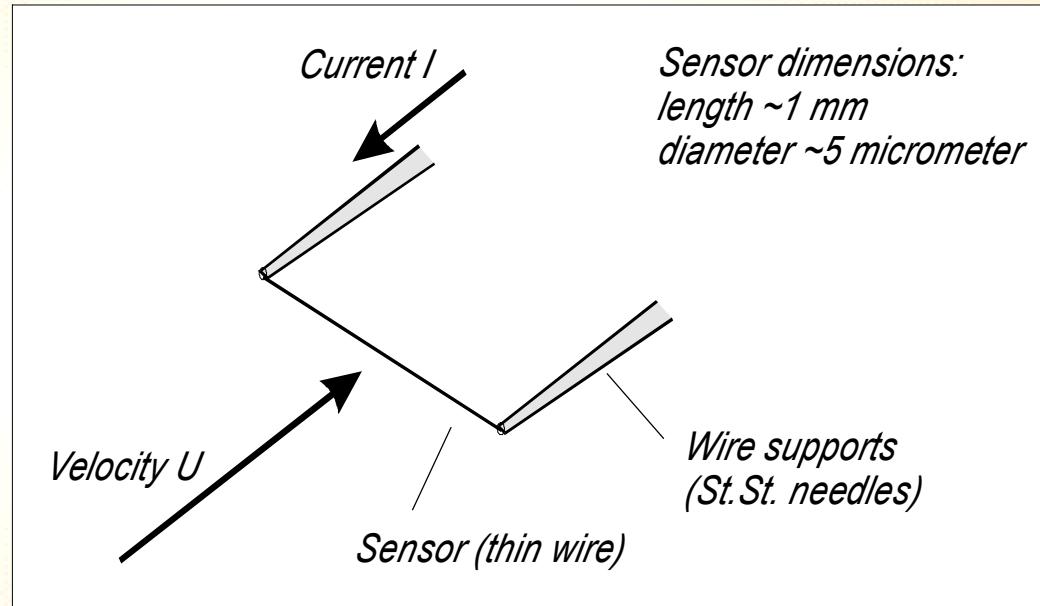
- ✓ 测量流场的平均和脉动速度
- ✓ 修改过的电路可以测量平均和脉动温度



热线工作原理

Principles of operation

- ✓ 细小的金属丝(一般为钨丝或者白金丝)处于来流速度 U 的流场中.
- ✓ 当电流通过金属丝, 会生成热 ($P R_w$). 当进入平衡状态, 这些热被对流换热带走



恒温测量系统

Constant Temperature Anemometer CTA

✓ 原理：

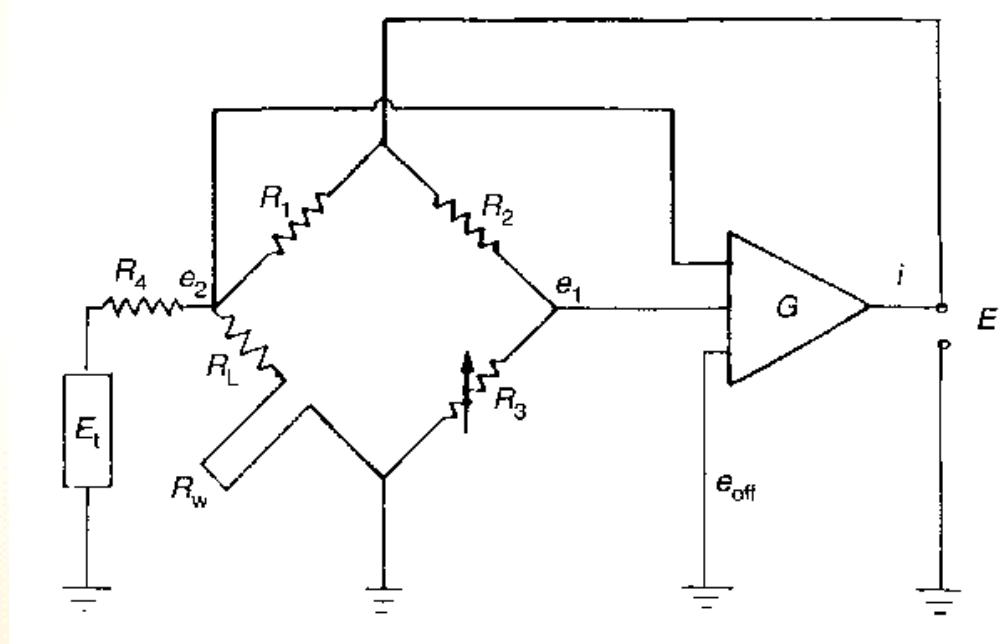
传感器的电阻被放大器维持恒定

✓ 优点(对比恒流测量系统)：

- 简单易用
- 高频率响应
- 低噪
- 被广泛接收

✓ 缺点：

- 价格高于恒流系统



控制方程 Governing equation I

$$\frac{dE}{dt} = W - H$$

✓ Governing Equation:

E= 金属丝热能 thermal energy stored in wire

$$E = C_w m T$$

C_w= 金属丝比热 heat capacity of wire

m= 金属丝质量 mass of wire

W= 发热量 power generated by Joule heating

$$W = P R_w$$

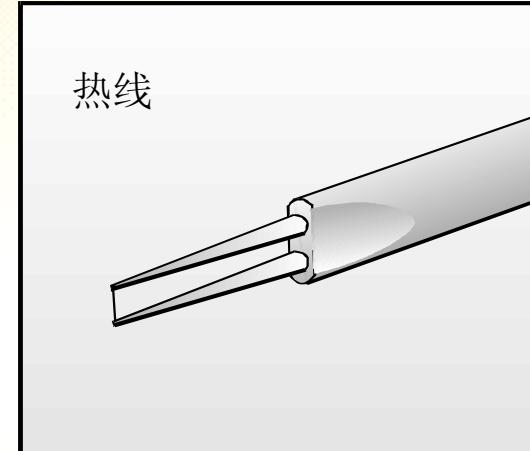
而 **R_w**=R_w(T_w)

H=换热量 heat transferred to surroundings

探头分类

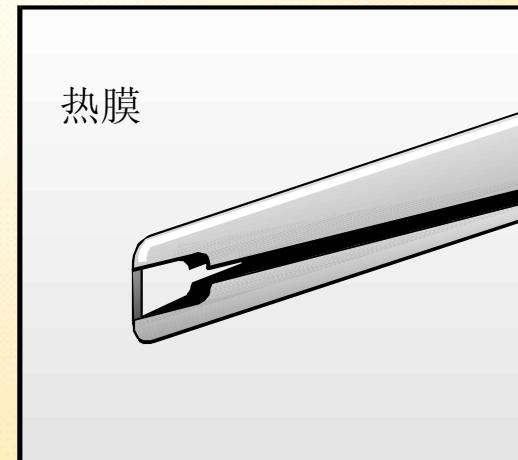
✓ 热线探头

- 尽可能先选用热线
- 价廉，响应快，可以维修



✓ 热膜探头

- 当测量水流速或者测量环境较脏的时候，选用热膜代替热线
- 响应速度较慢，不能修复
- 绝缘，不受化学、机械干扰

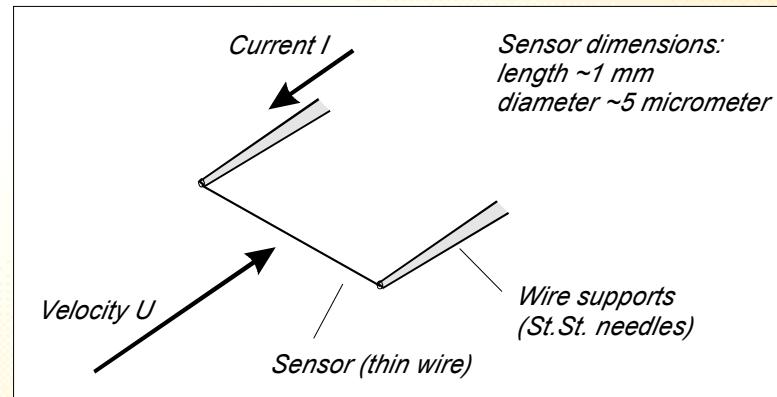


对探头的要求

为了达到较高的频率响应，总体来说，金属丝的热容量(**heat capacity**)应该越小越好。

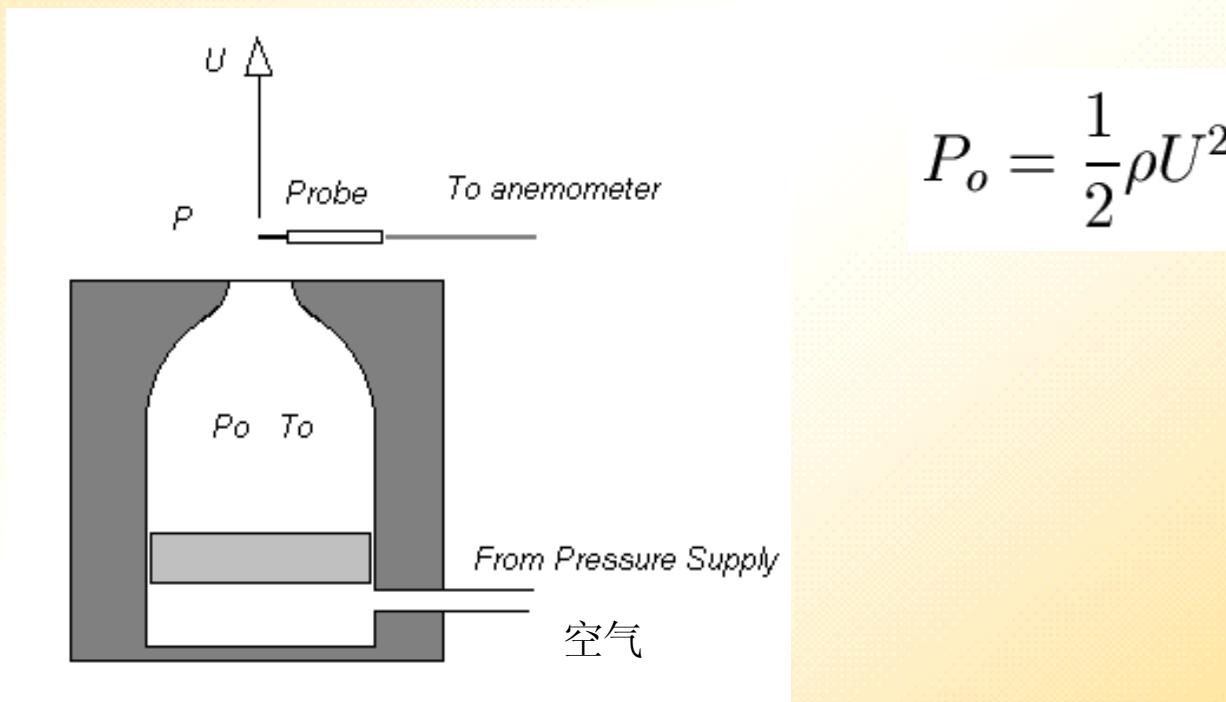
其他要点：

- 越小越好 (测量的空间解析力<探头大小)
- 长径比 (l/d) 应该大约**100**，两端的影响才可以忽略不计
- 金属丝应该耐高温，这样可以在高温度下运行实验，降低噪音影响
- 同样温度变化，金属丝电阻变化越大越好，这样也可以减低噪音

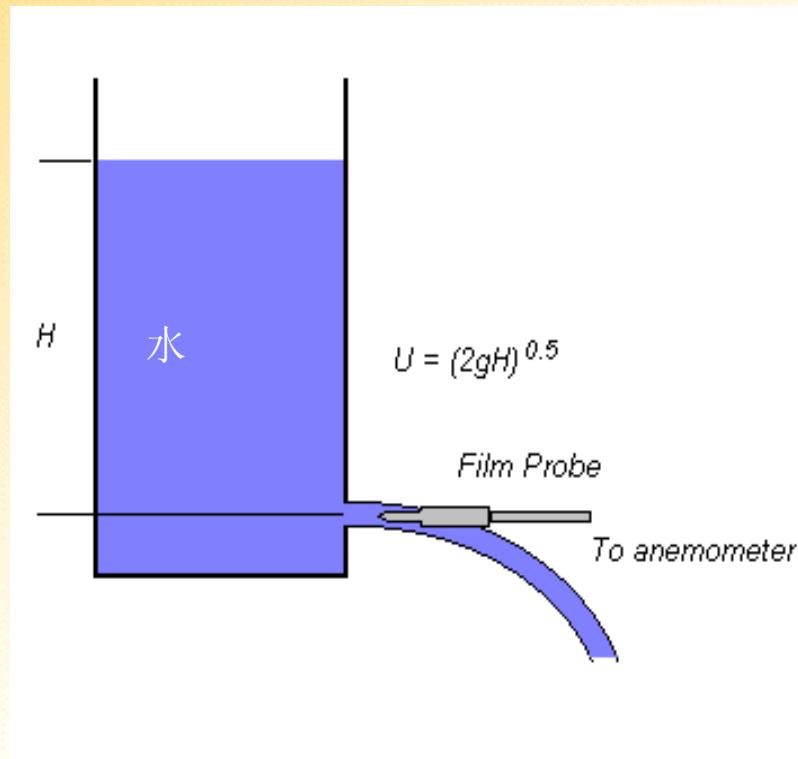


热线的标定

- ✓ 标定-找到流场速度和传感器输出的一个函数关系 $\mathbf{U}=\mathbf{f}(\mathbf{E})$



热膜标定

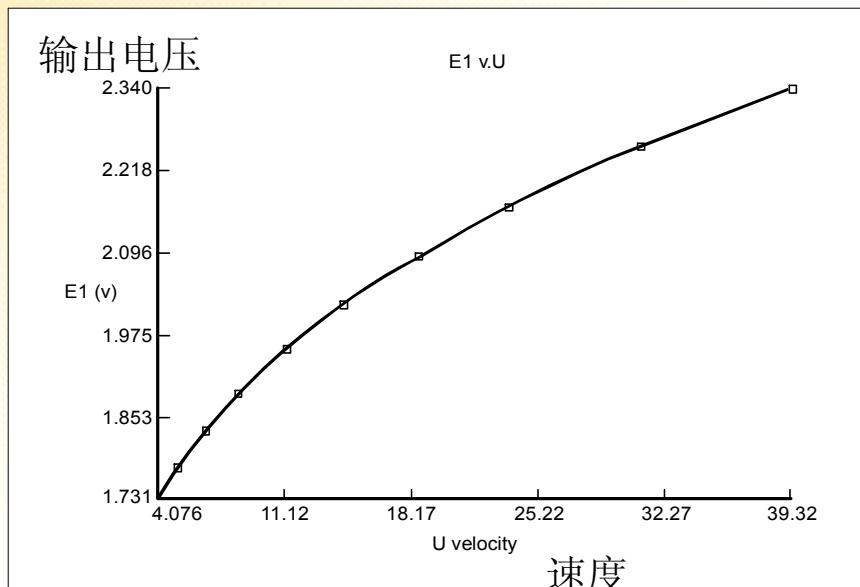


- 热膜在水中的标定
 - 如图所示，用一个容器形成射流，估算流速
 - 把探头放到滑车上在水里拖动



典型的标定曲线

✓ 热线标定曲线



回归曲线 **Curve fit (velocity U as function of output voltage E):**

$$U = C_0 + C_1 E + C_2 E^2 + C_3 E^3 + C_4 E^4$$

应对方法:探头被污染

✓ 热线容易被以下物质污染

- 灰尘 **dust particles**
- 油滴 **oil vapours**
- 化学物质 **chemicals**

✓ 导致

- 敏感度下降，标定曲线便宜
- 热线响应速度下降

✓ 解决办法 **Cure**

- 清理热线 **Clean the sensor**
- 重新标定 **Recalibrate**

应对方法：环境温度发生变化

- 环境温度变化的影响

传热大小正比于探头和环境的温差

$$E^2 = (T_w - T_a)(A + B \cdot U)$$

如果 T_a 发生变化：

- 传热发生变化
- 流体性质发生变化

在空气中测量：

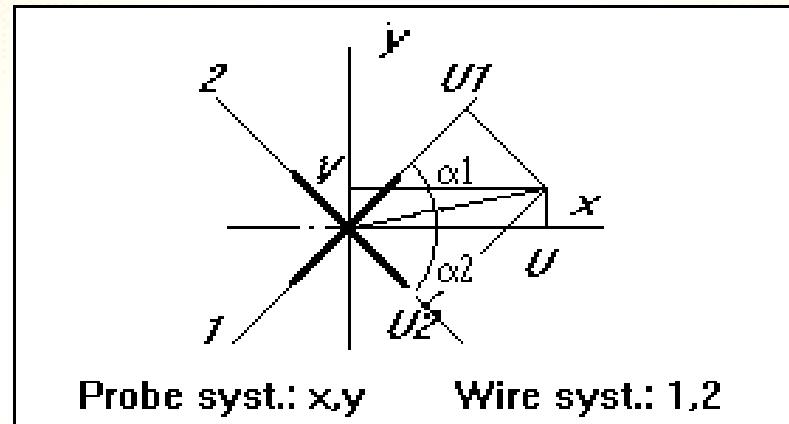
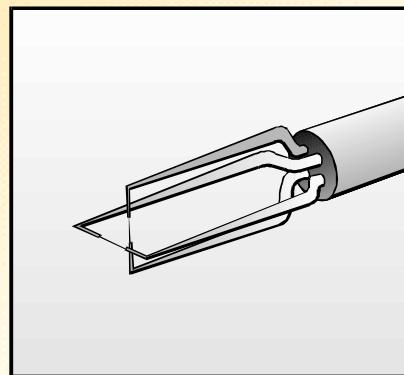
- 探头温度较高
- 空气性质变化较小

水的性质变化相对较大，探头温度较低，所以温度变化在液体中测量影响更大：

X型探头测量二维流场

X型探头 (两线和探头的轴成 $\pm 45^\circ$):

- 利用角度变化，热线的输出会发生变化的现象
- 这样速度被分解成 (U, V)



where U_1 and U_2 in wire coordinate system are found by solving:

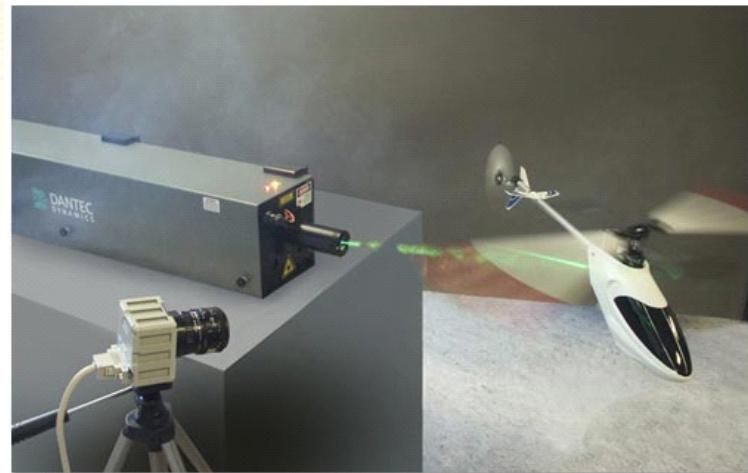
利用照相的方法测量流场速度

- ✓ 连续拍摄两张照片
- ✓ 间隔时间已知
- ✓ 距离通过图片求得
- ✓ 可得物体移动速度
- ✓ 这就是**PIV**基本原理



Particle Image Velocimetry (PIV)

- ✓ PIV利用电子拍照，数据处理等现代方法
- ✓ 是一个全流场测量技术，它在一个平面内给出一个瞬时的流场速度
- ✓ 一般测量两个速度分量
- ✓ 如果使用**stereoscopic**方法可以测量这个平面的三维速度



PIV 原理

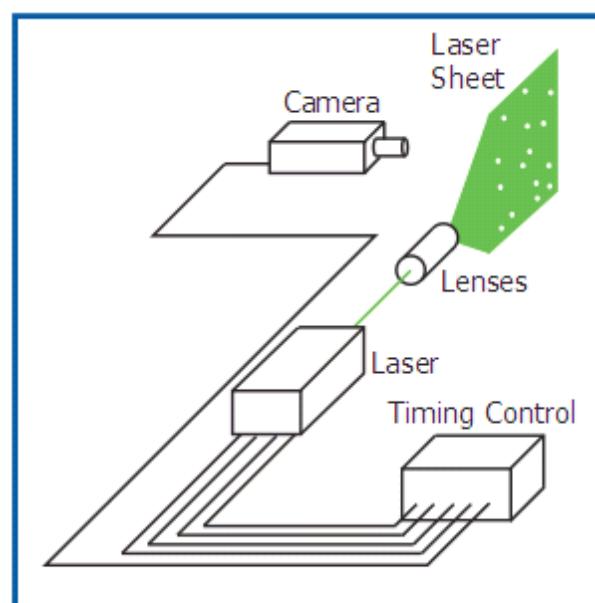


Figure 1. The components of a basic PIV system.

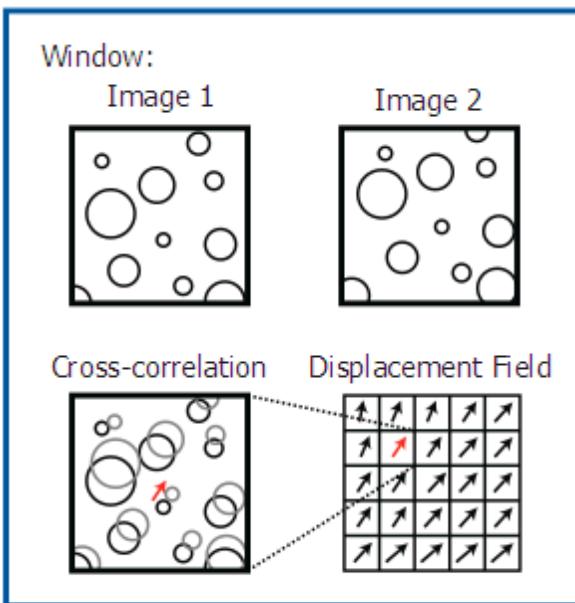


Figure 2. PIV processing determines the displacement field.

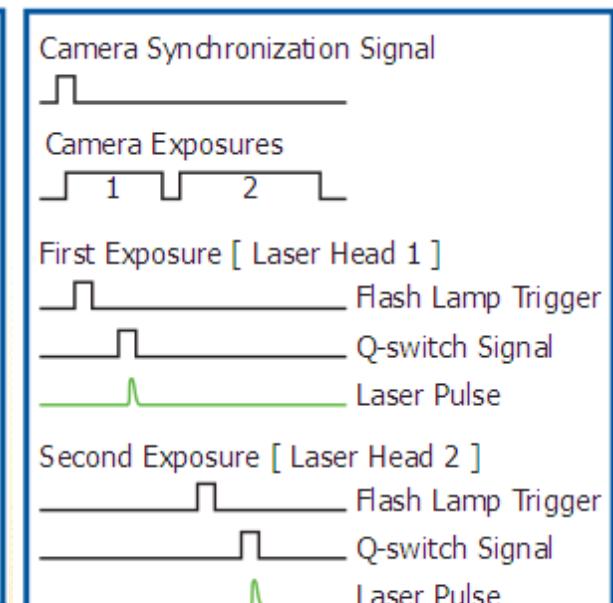
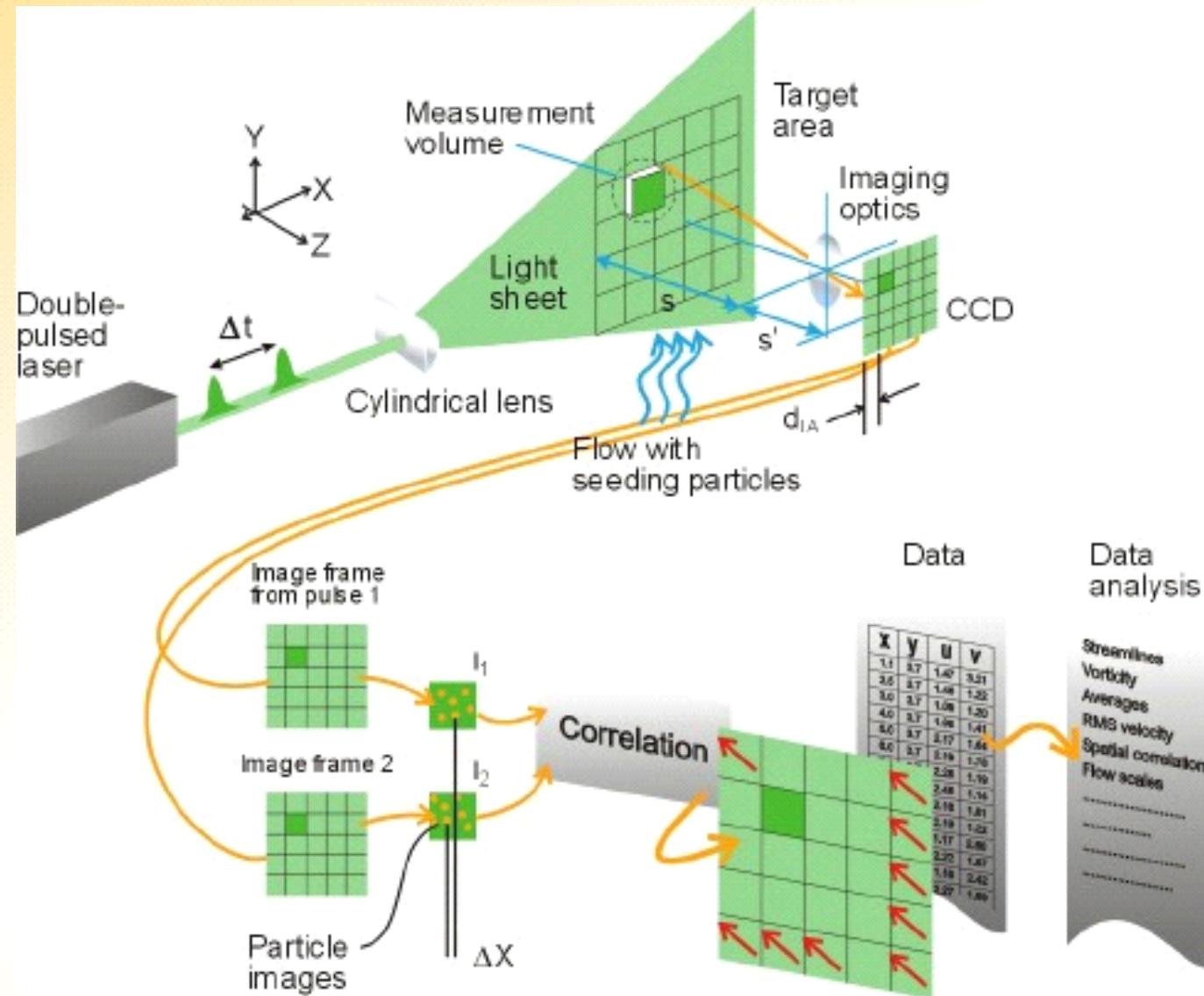
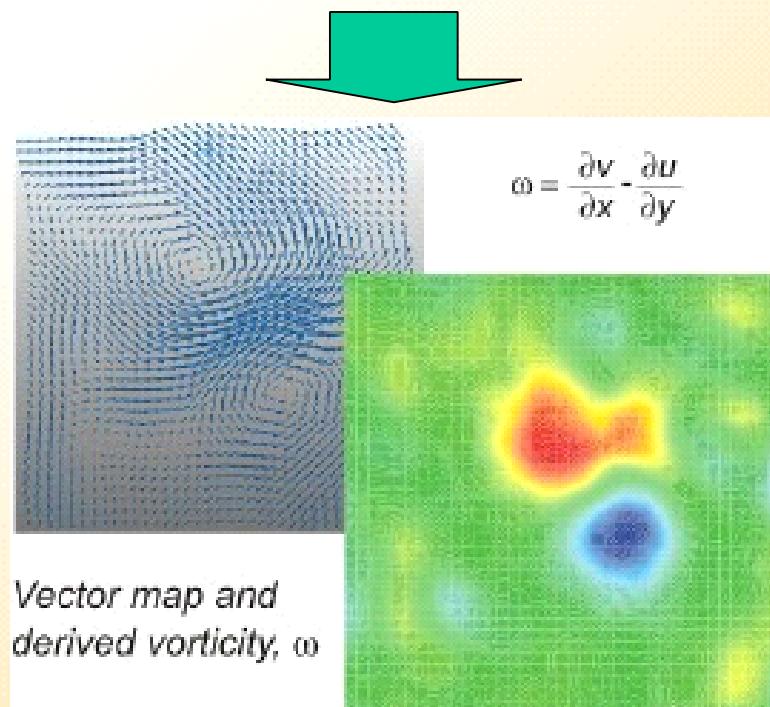
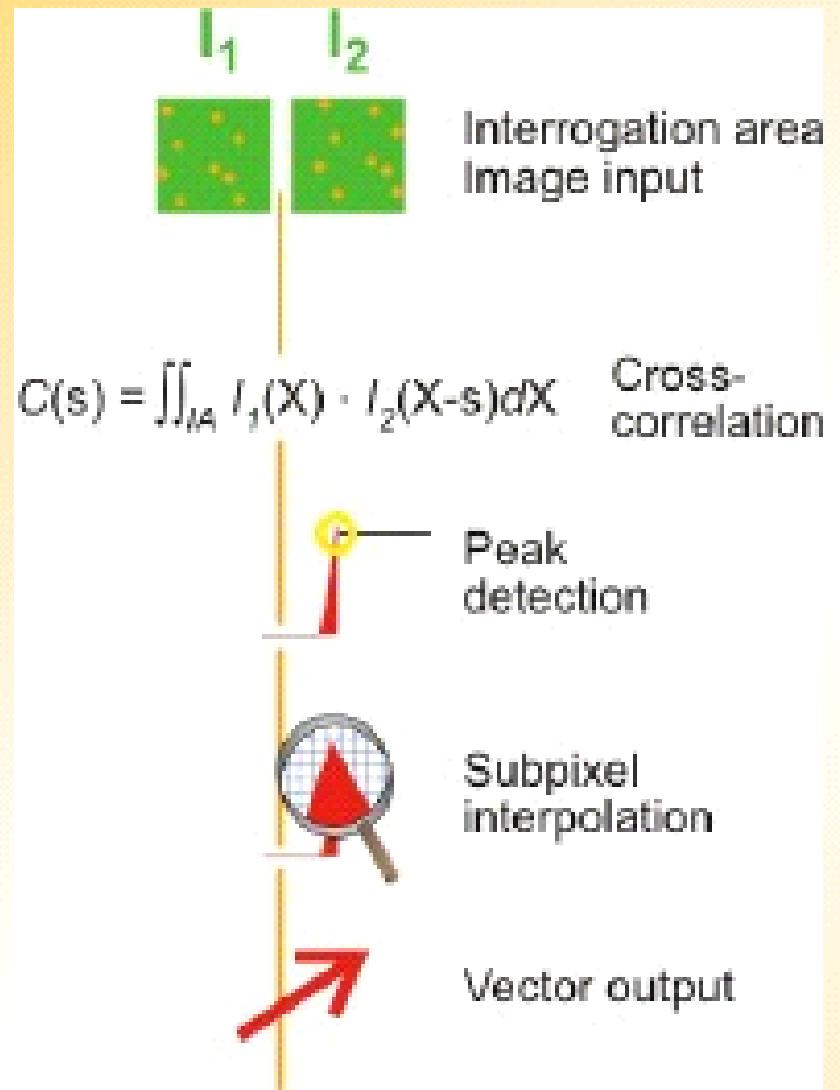


Figure 3. Timing signals required for full control of a PIV system.

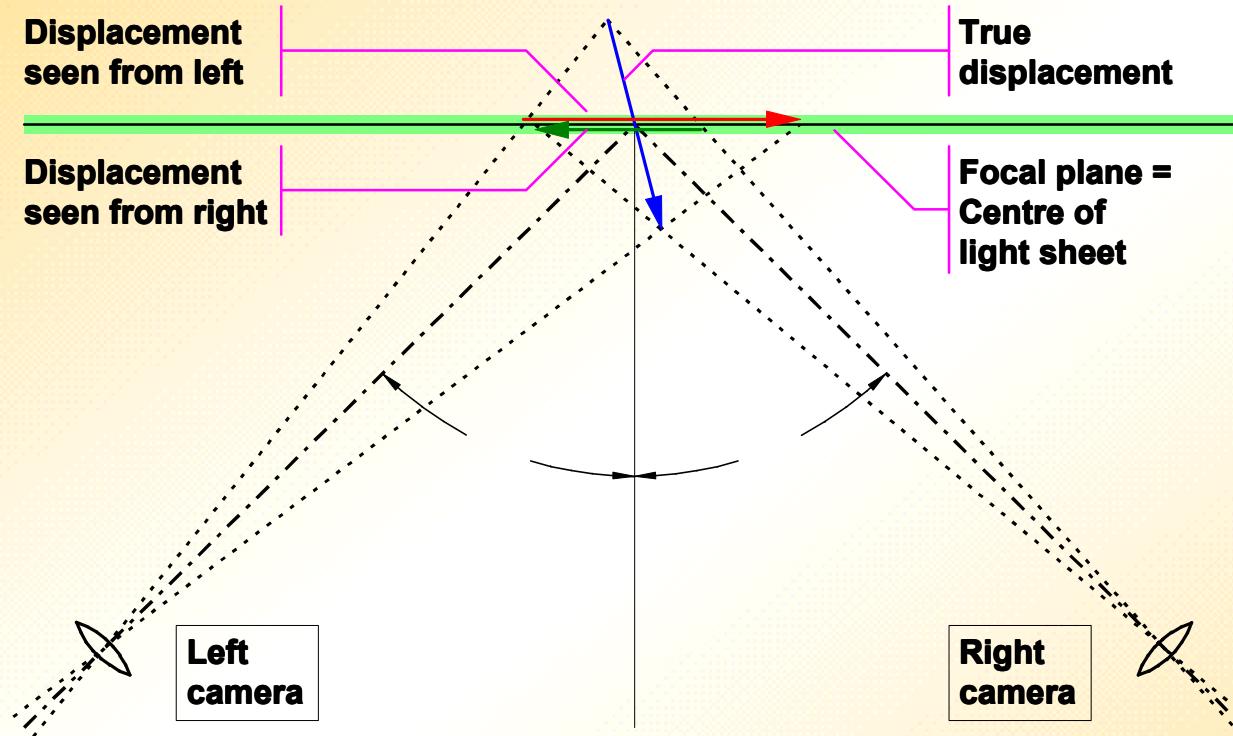
PIV测量过程



PIV数据处理



Fundamentals of stereo vision



True 3D displacement ($\Delta X, \Delta Y, \Delta Z$) is estimated from a pair of 2D displacements ($\Delta x, \Delta y$) as seen from left and right camera respectively

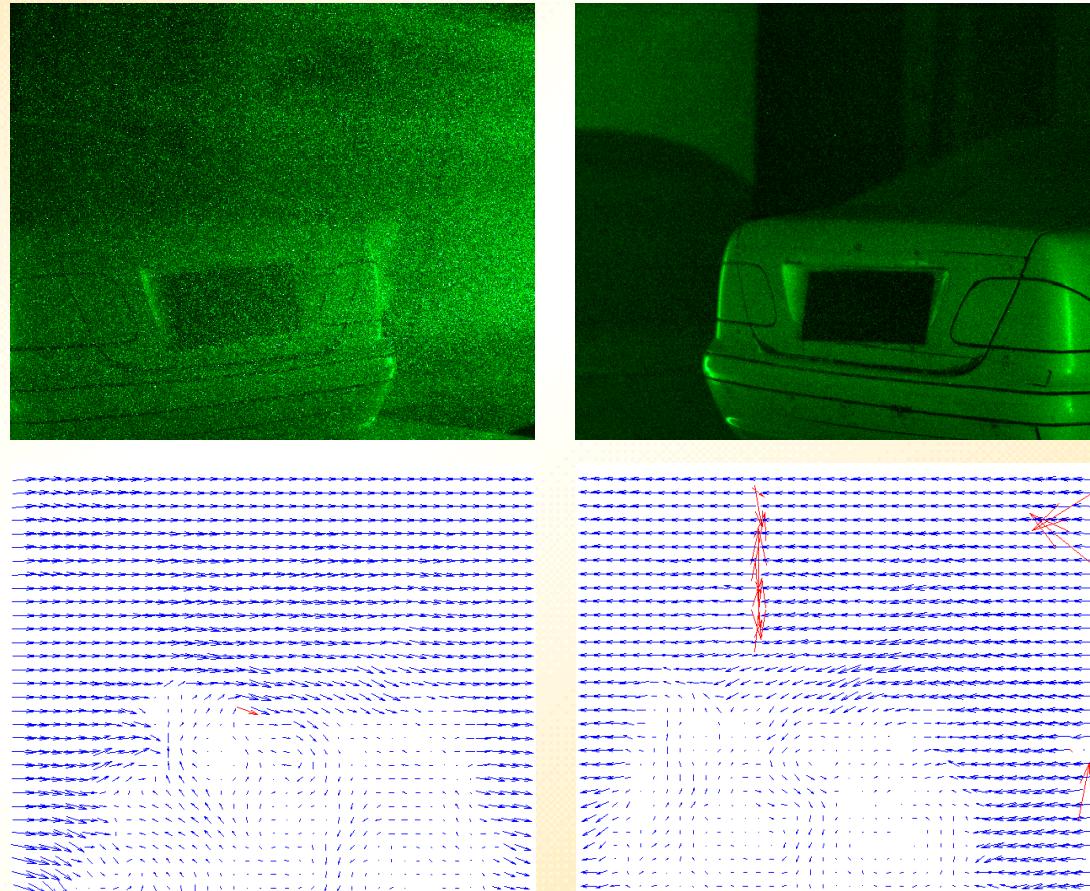
Left / Right 2D vector maps

Left & Right camera images are recorded simultaneously.

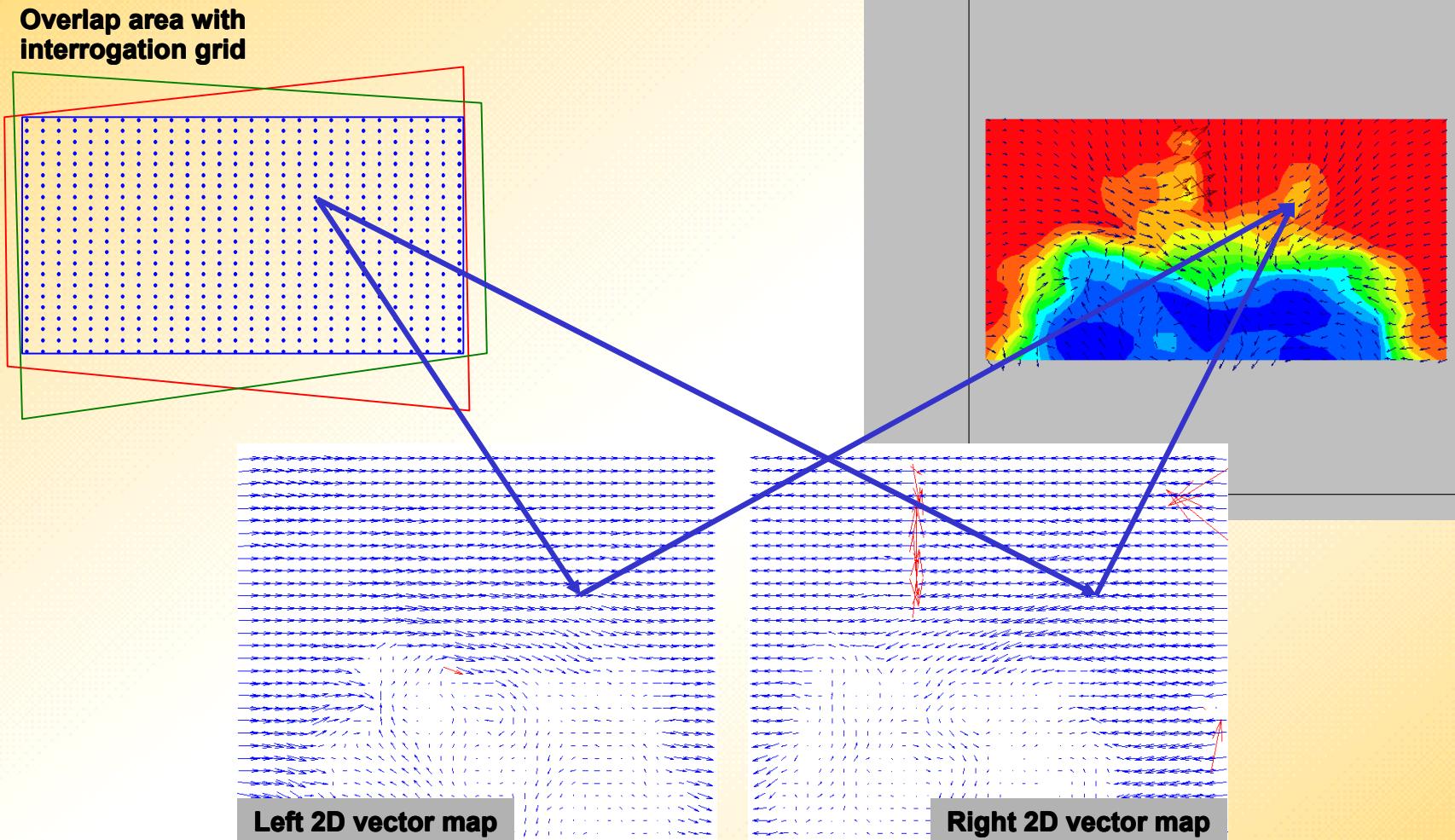
Conventional PIV processing produce 2D vector maps representing the flow field as seen from left & right.

The vector maps are re-sampled in points corresponding to the interrogation grid.

Combining left / right results, 3D velocities are estimated.



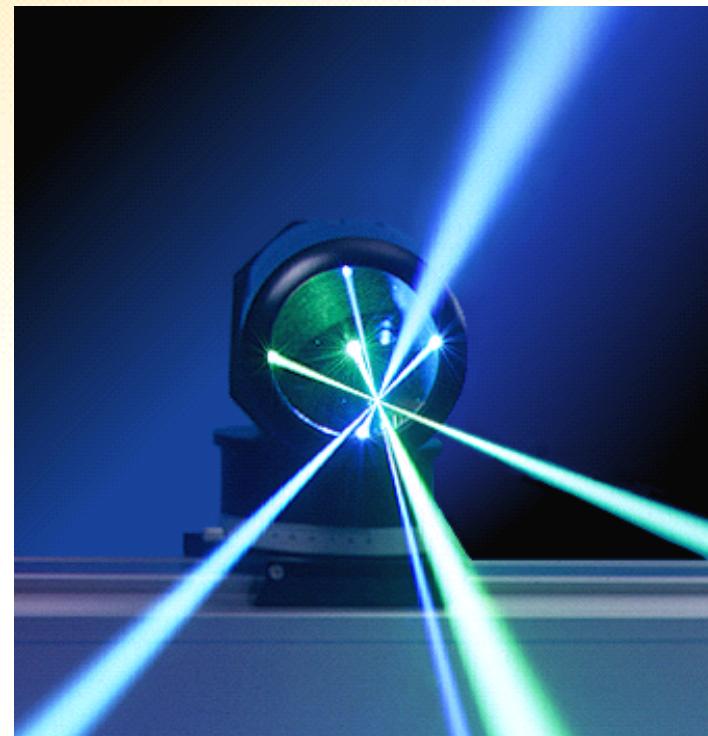
3D reconstruction



Laser Doppler Anemometry

激光多普勒测量仪

Introduction to principles and applications



只选用了原稿的一部分
加入了中文 **2007-12-20**

特点 Characteristics of LDA

- ✓ Yeh and Cummins in 1964 首创
- ✓ 流体力学速度研究 (gas, liquid)
- ✓ 可以测到三维
- ✓ 不影响流场 Non-intrusive measurements (optical technique)
- ✓ 无须标定 (no calibration required)
- ✓ 高精度 Very high accuracy
- ✓ 高分辨率 Very high spatial resolution due to small measurement volume
- ✓ 需要示踪粒子 Tracer particles are required

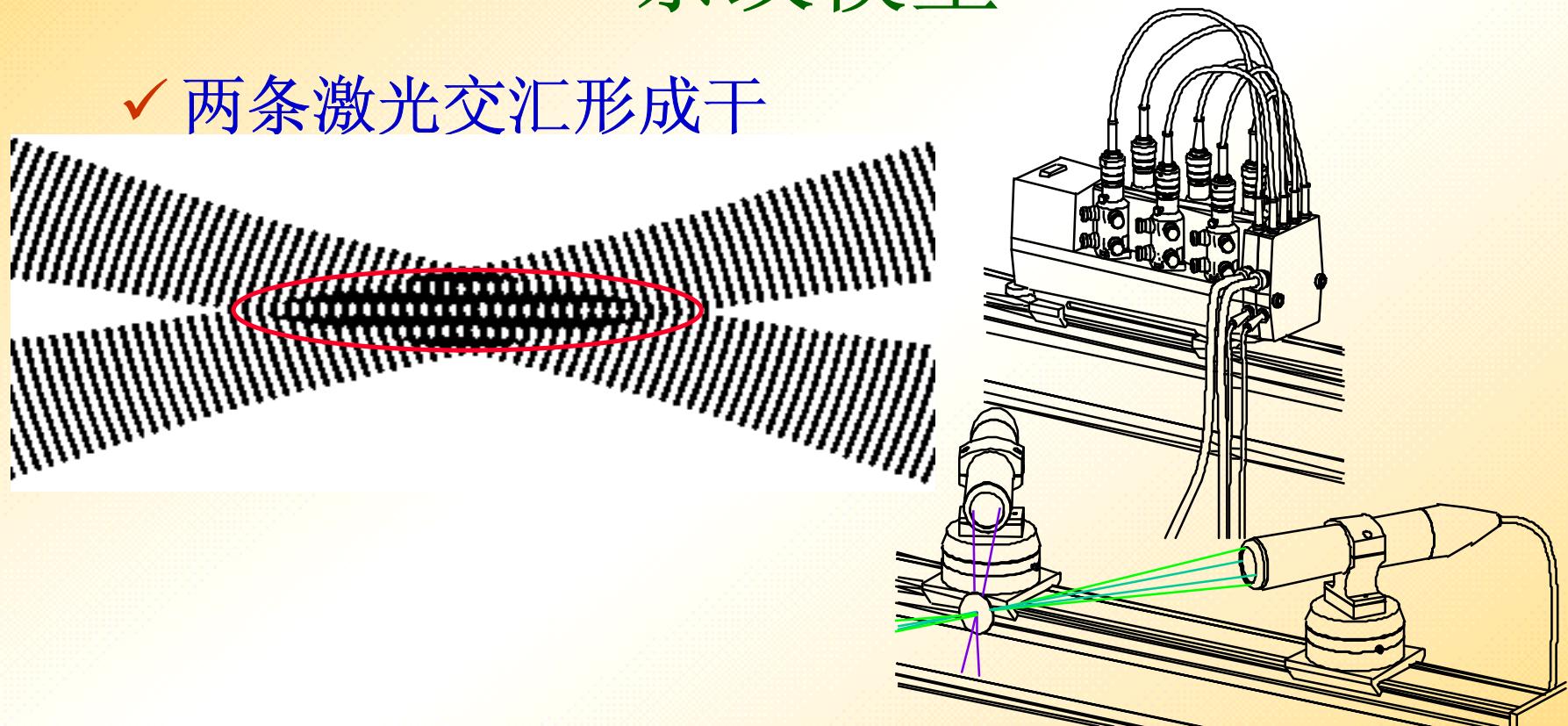
用途 Applications of LDA

- ✓ 层流，湍流 **Laminar and turbulent flows**
- ✓ 空气动力学 **Investigations on aerodynamics**
- ✓ 超音速 **Supersonic flows**
- ✓ 燃气轮机，汽车 **Turbines, automotive etc.**
- ✓ 液体流动 **Liquid flows**
- ✓ 液体表面速度，振动研究 **Surface velocity and vibration measurement**
- ✓ 高温环境 **Hot environments (flames, plasma etc.)**
- ✓ 粒子速度 **Velocity of particles**
- ✓ 以及很多，很多... **etc., etc., etc.**

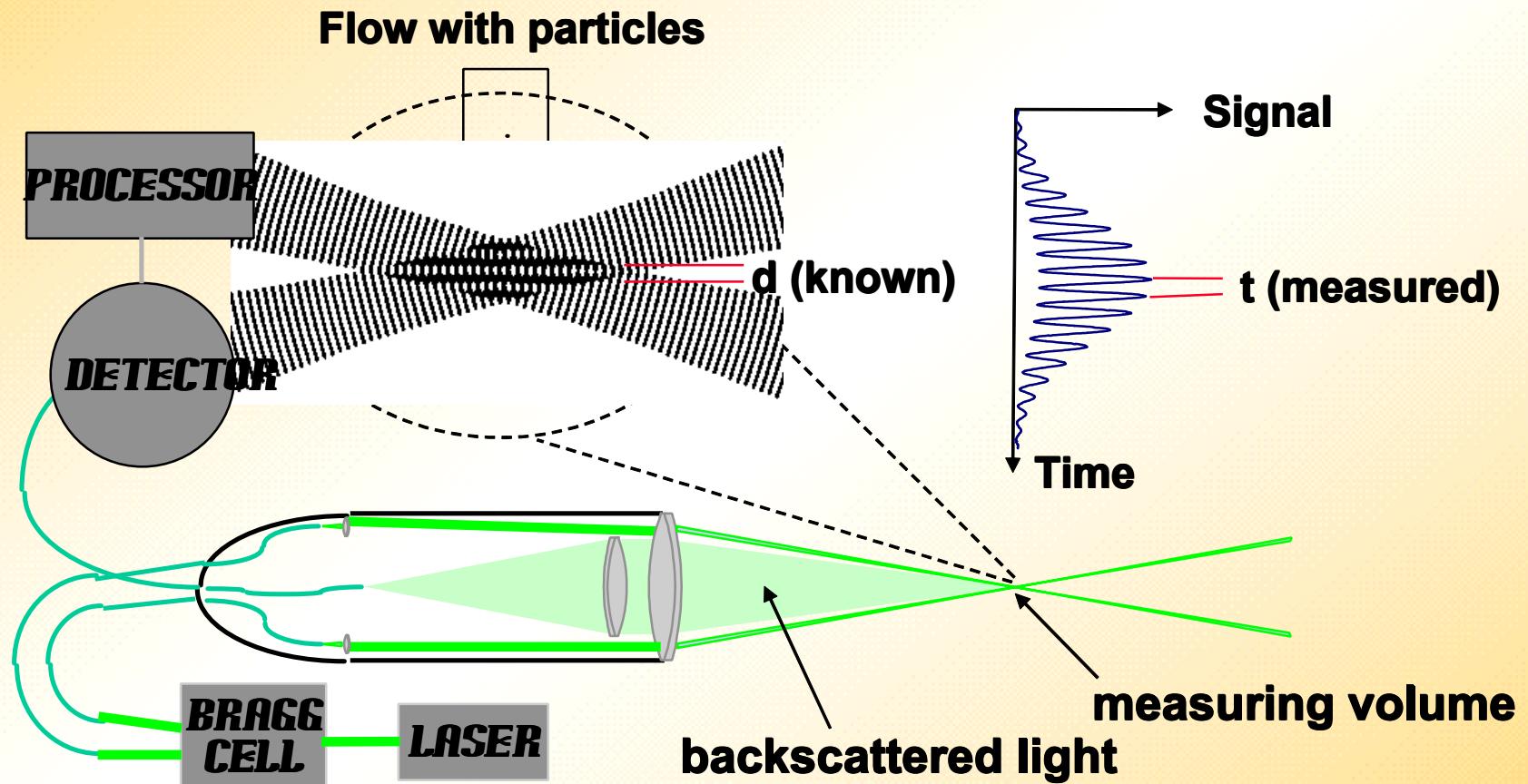
LDA - Fringe model

条纹模型

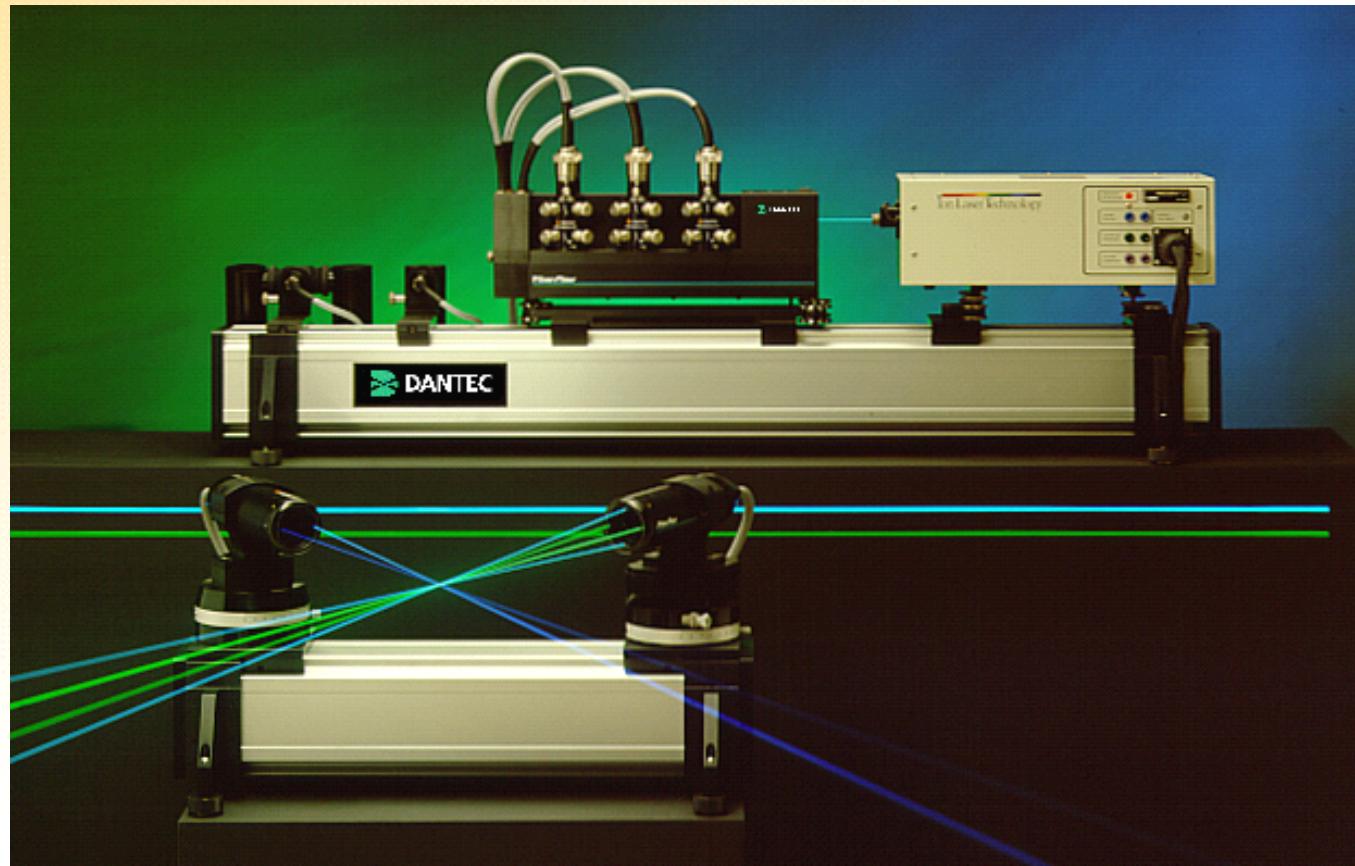
✓ 两条激光交汇形成干涉条纹



$$\text{Velocity} = \frac{\text{distance}}{\text{time}}$$



LDA Fibre optical system

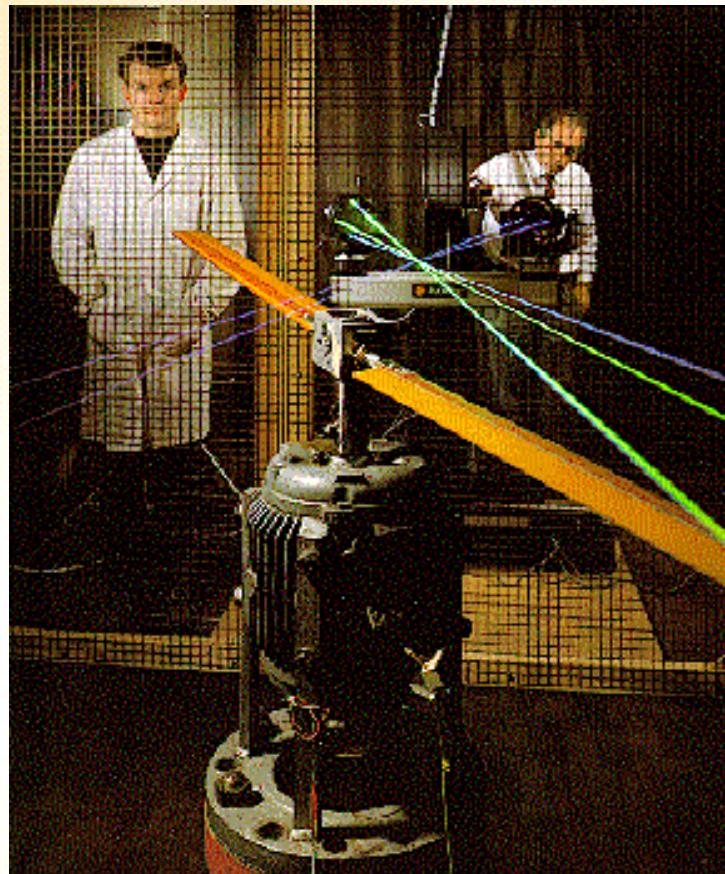


Seeding: ability to follow flow

对示踪粒子的要求

Particle	Fluid	Diameter (μm)	
		f = 1 kHz	f = 10 kHz
Silicone oil	atmospheric air	2.6	0.8
TiO₂	atmospheric air	1.3	0.4
MgO	methane-air flame (1800 K)	2.6	0.8
TiO₂	oxygen plasma (2800 K)	3.2	0.8

Measurement of air flow around a helicopter rotor model in a wind tunnel



**Photo courtesy
of University
of Bristol, UK**

Measurement of water flow inside a pump model

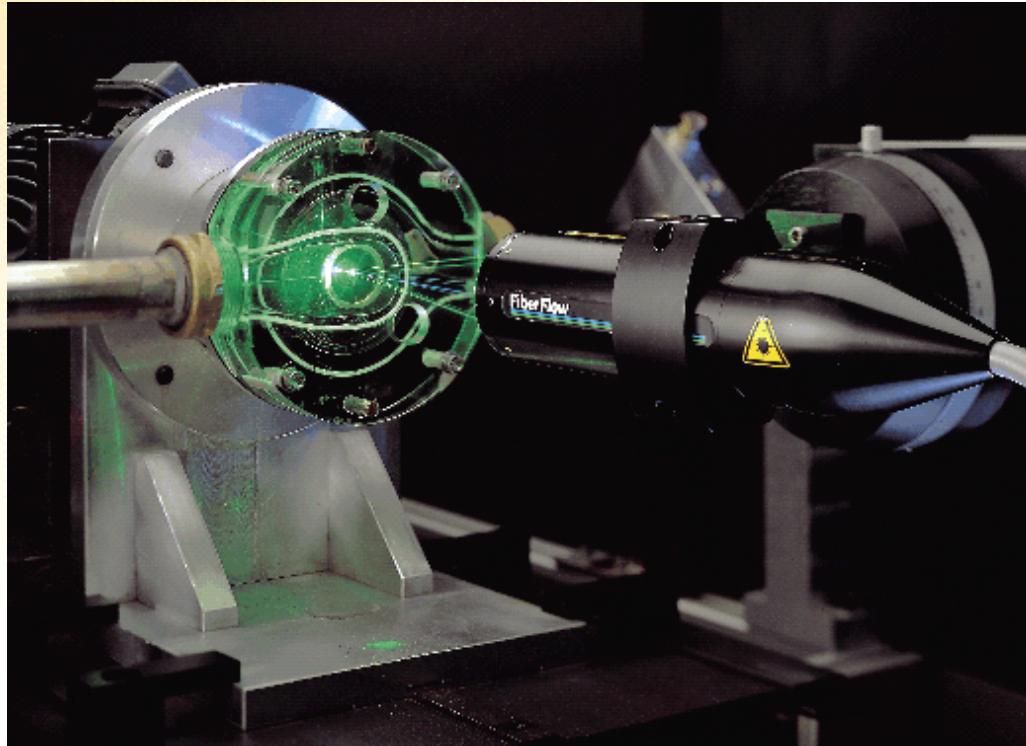
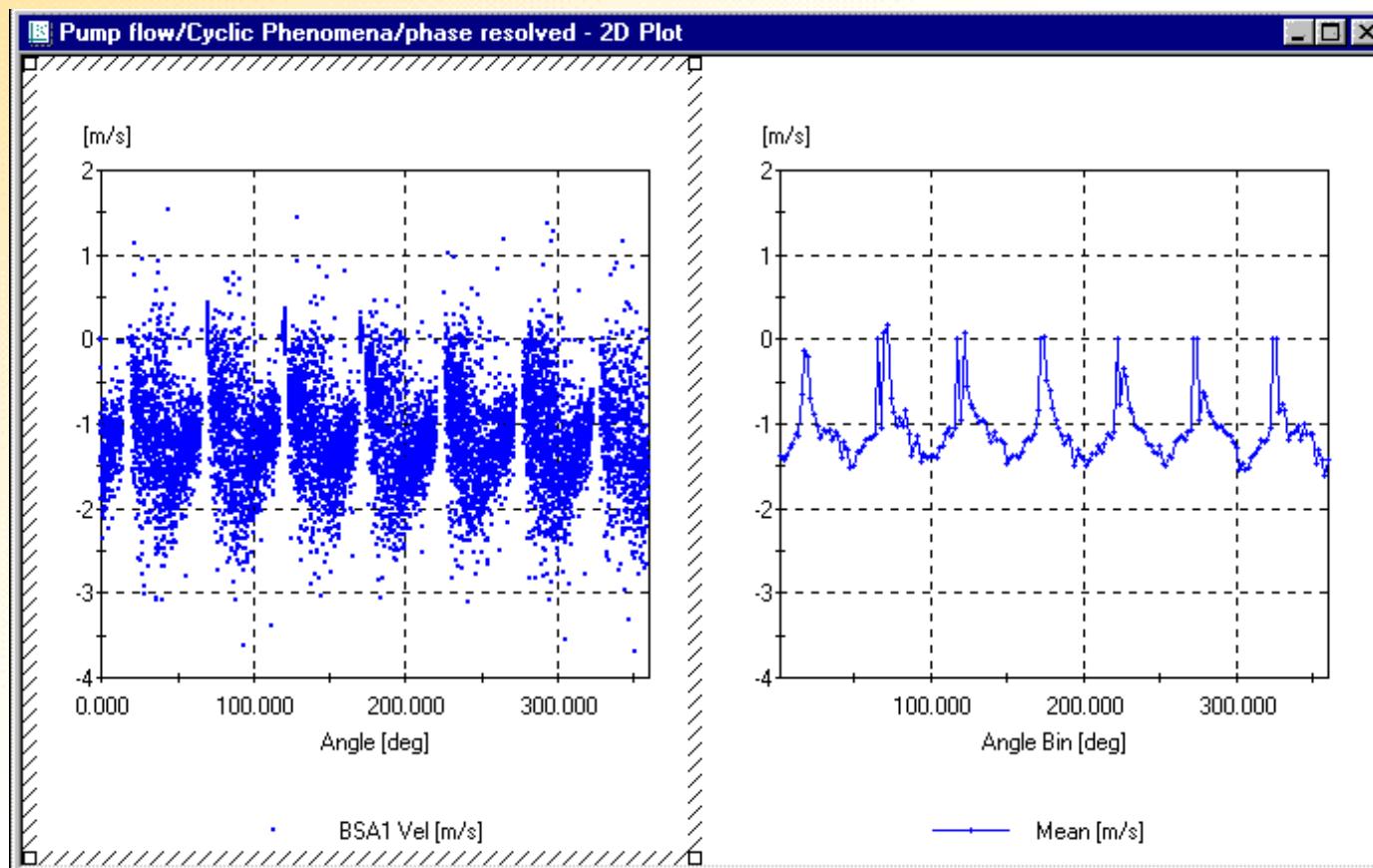
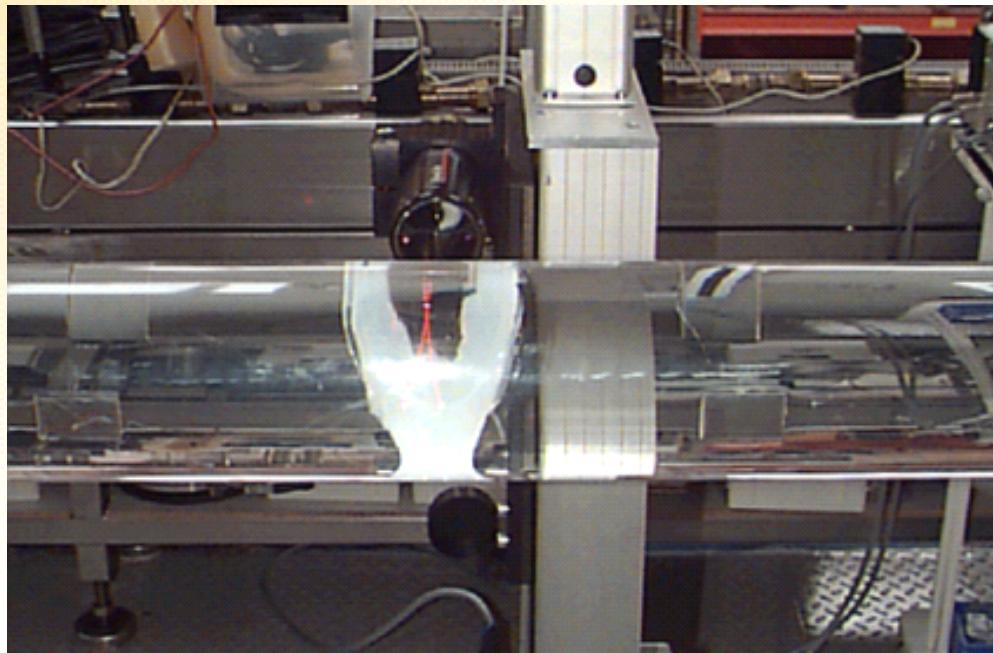


Photo courtesy of Grundfos A/S, DK

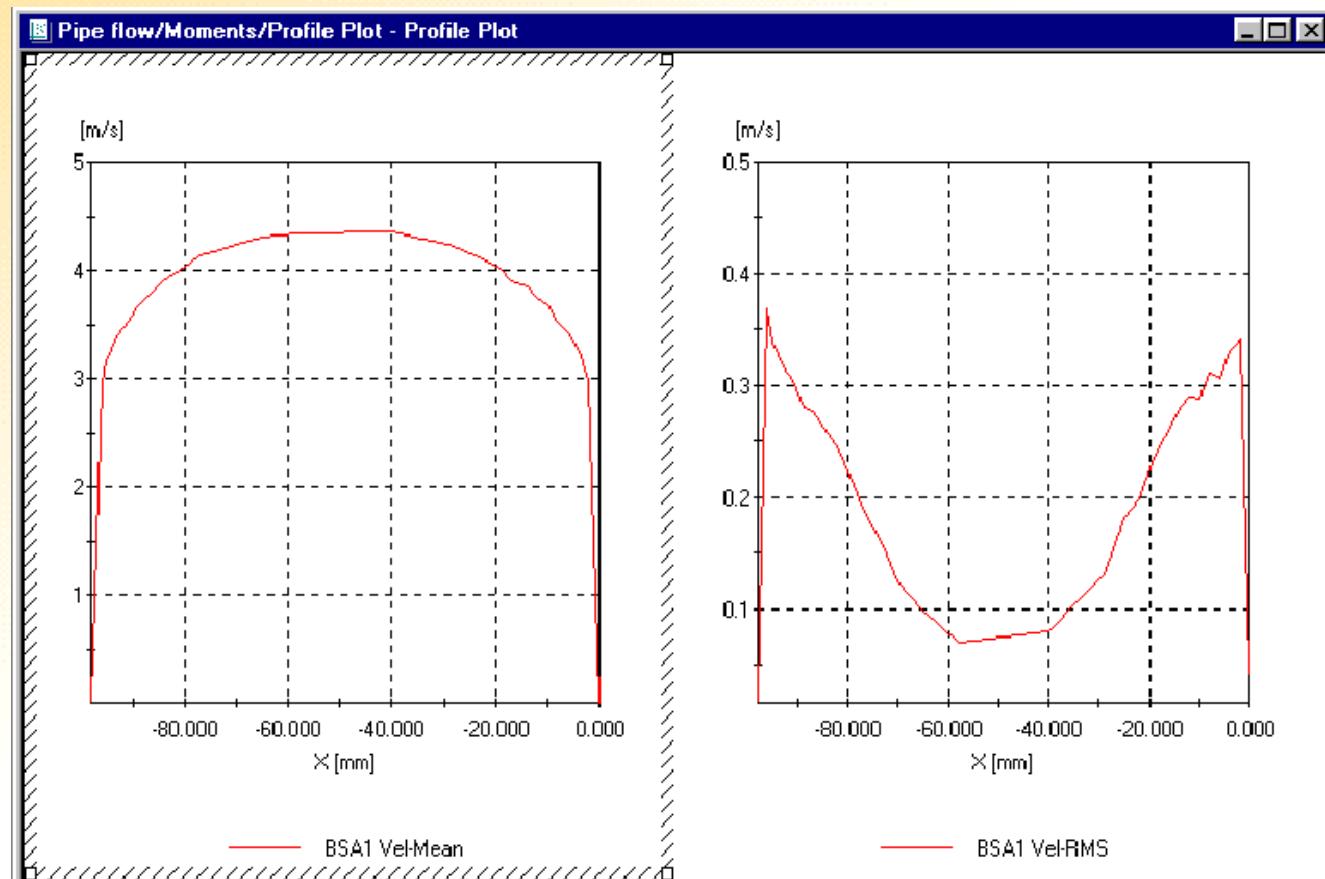
Phase resolved and phase averaged data



Measurement of velocity profiles in a water pipe



Velocity profile, fully developed turbulent pipe flow



Measurement of flow field around a 1:5 scale car model in a wind tunnel

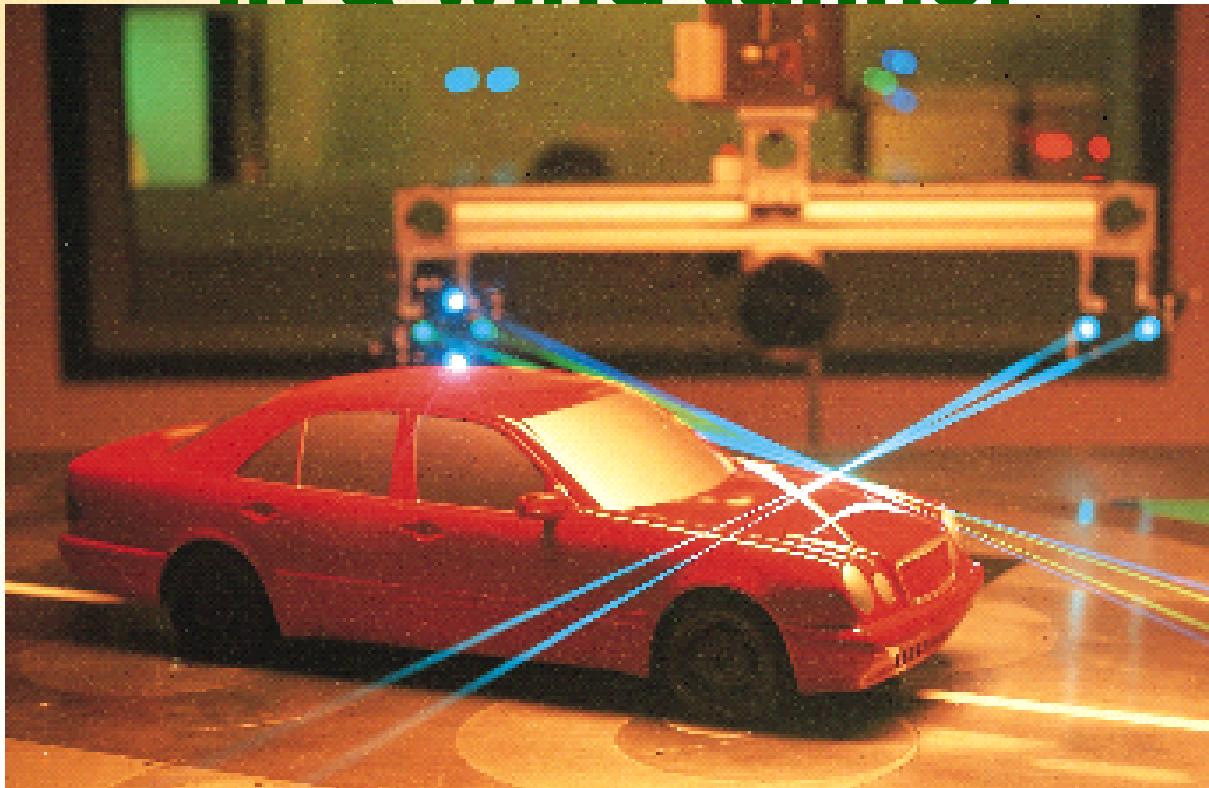


Photo courtesy of Mercedes-Benz, Germany

Measurement of wake flow around a ship model in a towing tank



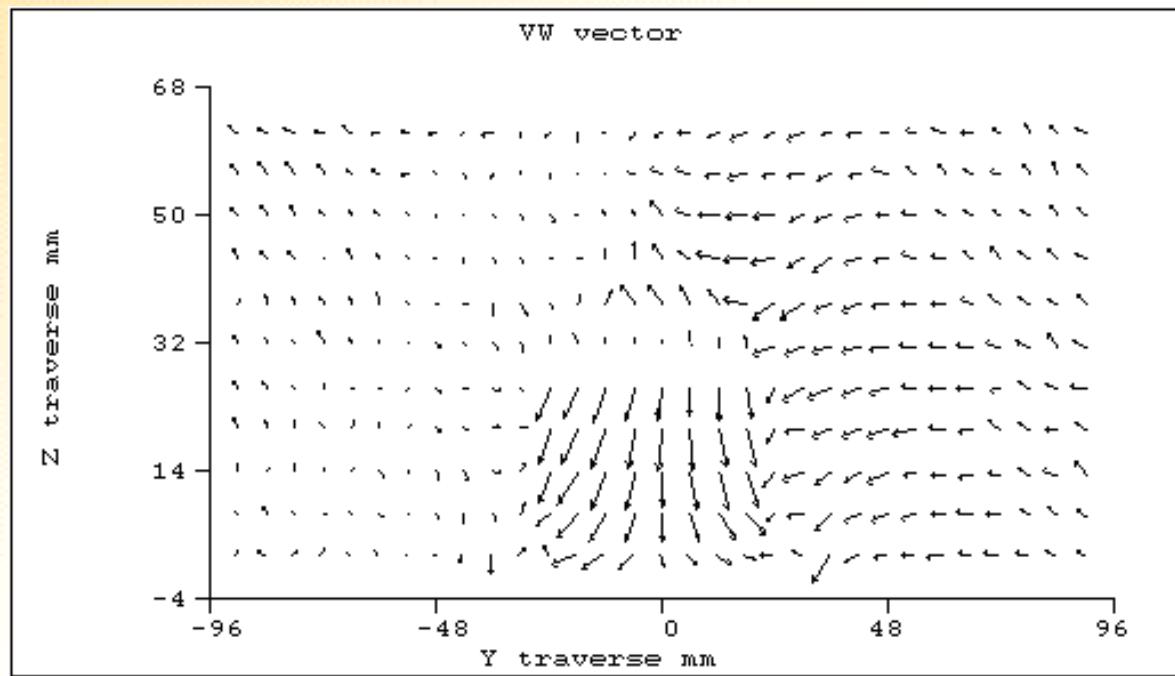
Photo courtesy of Marin, the Netherlands

Measurement of air flow field around a ship model in a wind tunnel

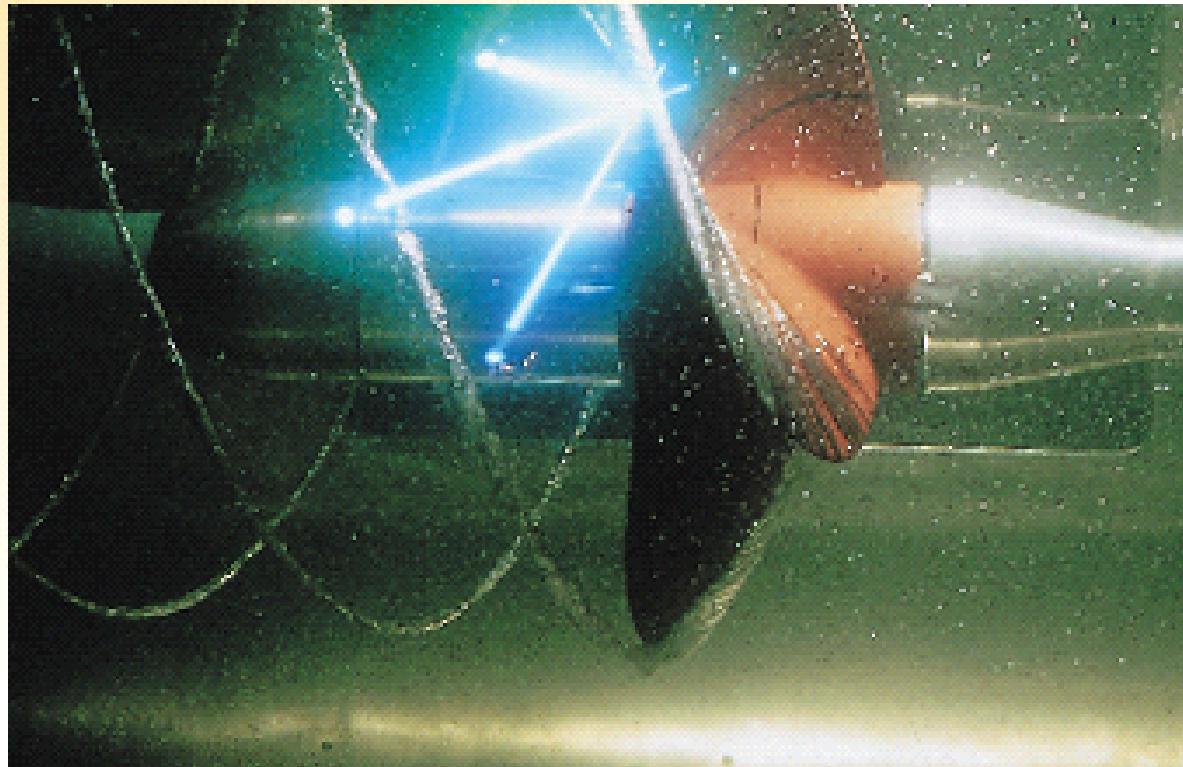


Photo courtesy of University of Bristol, UK

Wake flow field behind hangar



Measurement of flow around a ship propeller in a cavitation tank



Measurement of flow in a valve model

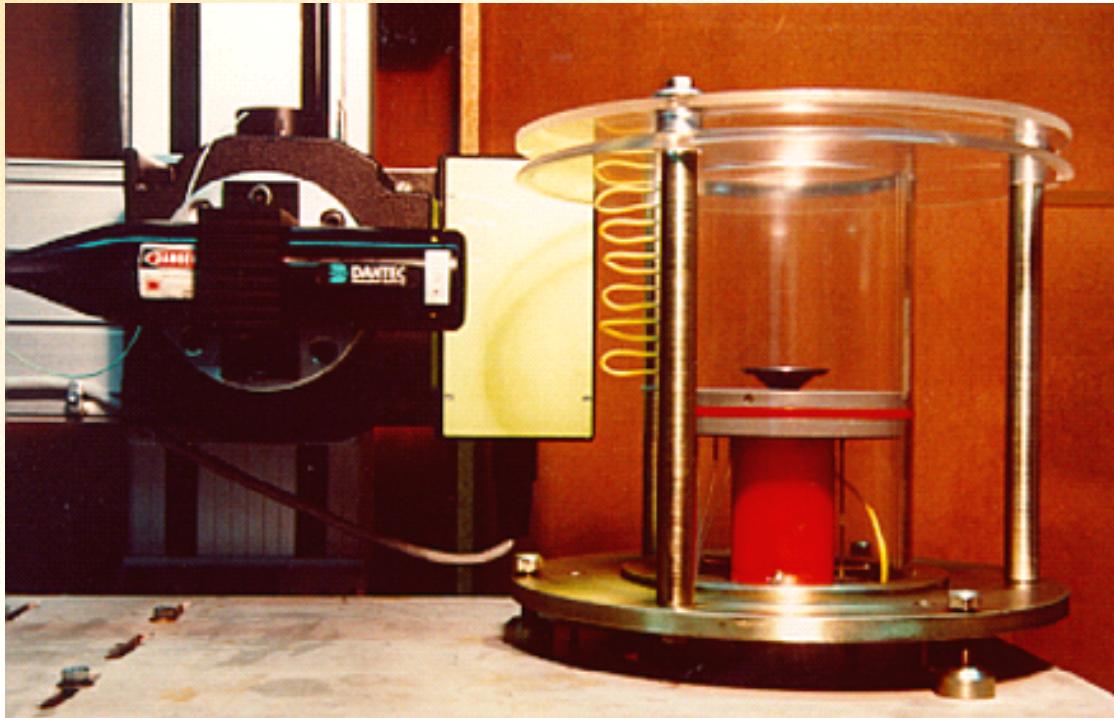
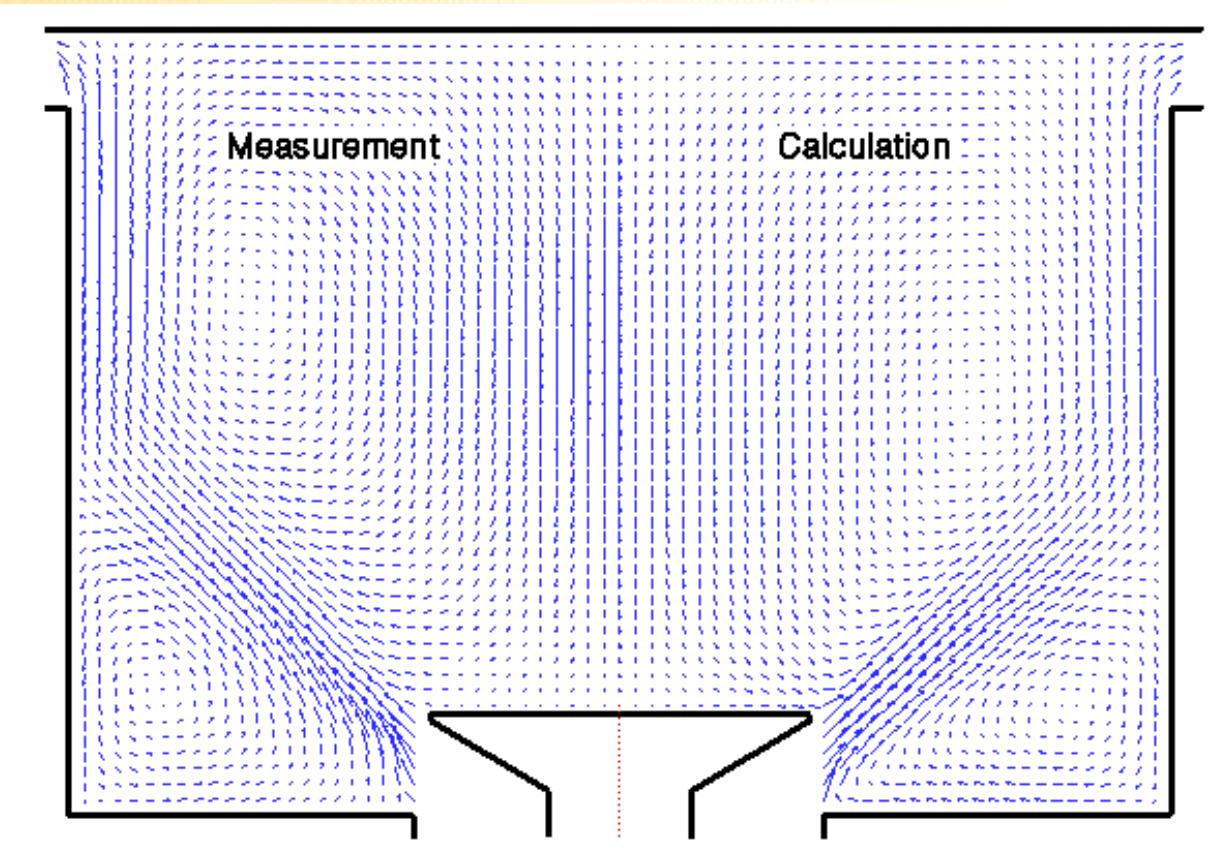


Photo courtesy of Westsächsische Hochschule Zwickau, Germany

Comparison of Measurements and CFD results

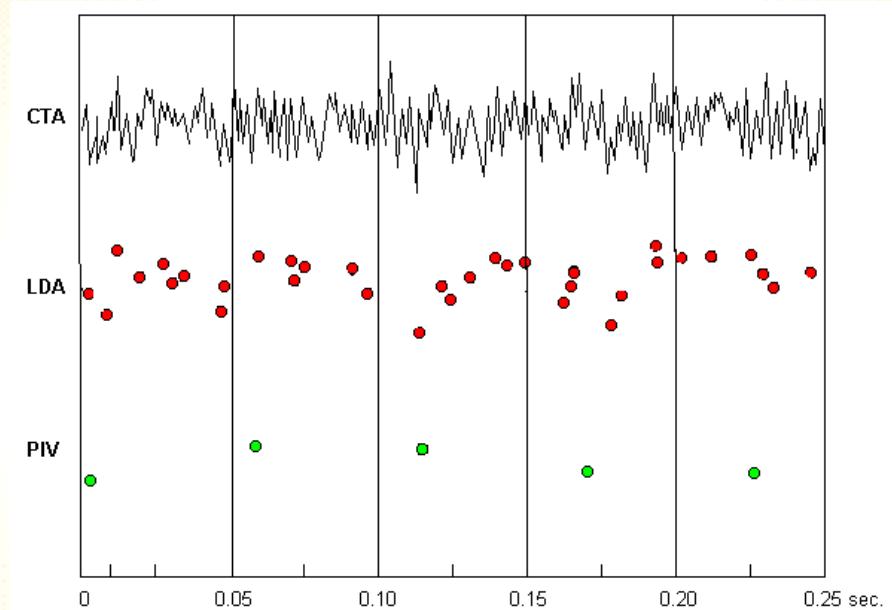


热线,LDA,PIV信号对比

Anemometer signal output

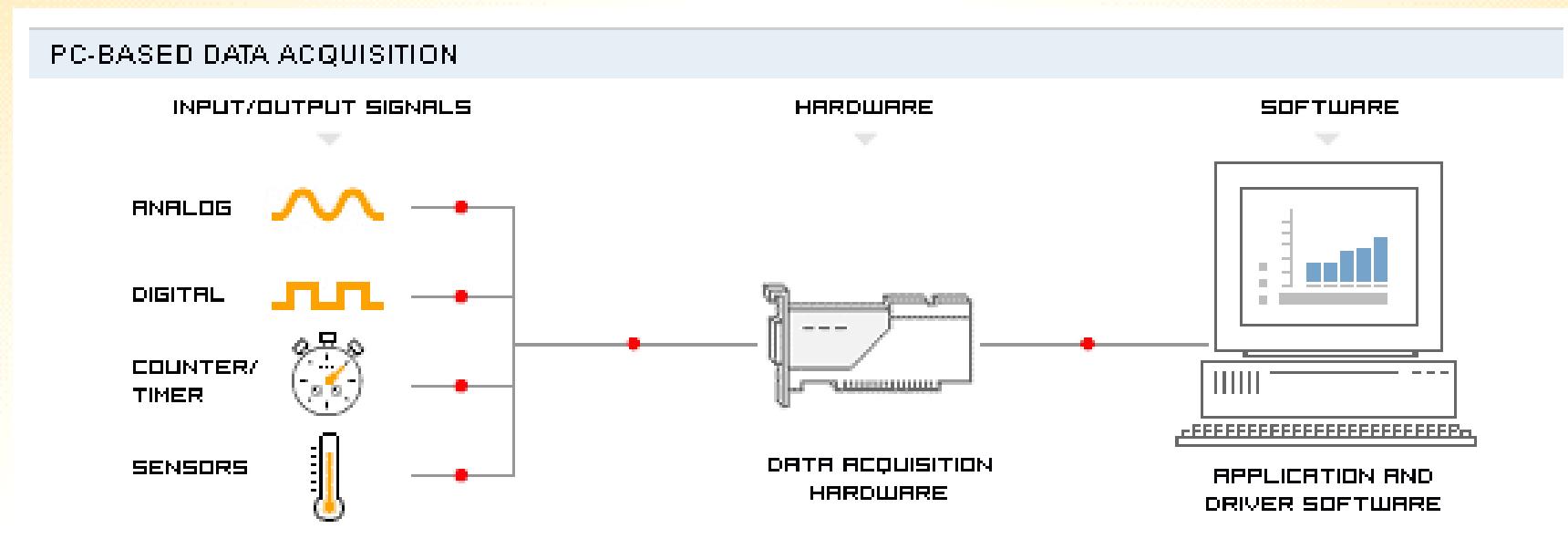
热线风速仪的输出
信号是连续的

LDA输出信号是随
机的,而**PIV**是由
数字相机曝光时间
决定的



数据采集

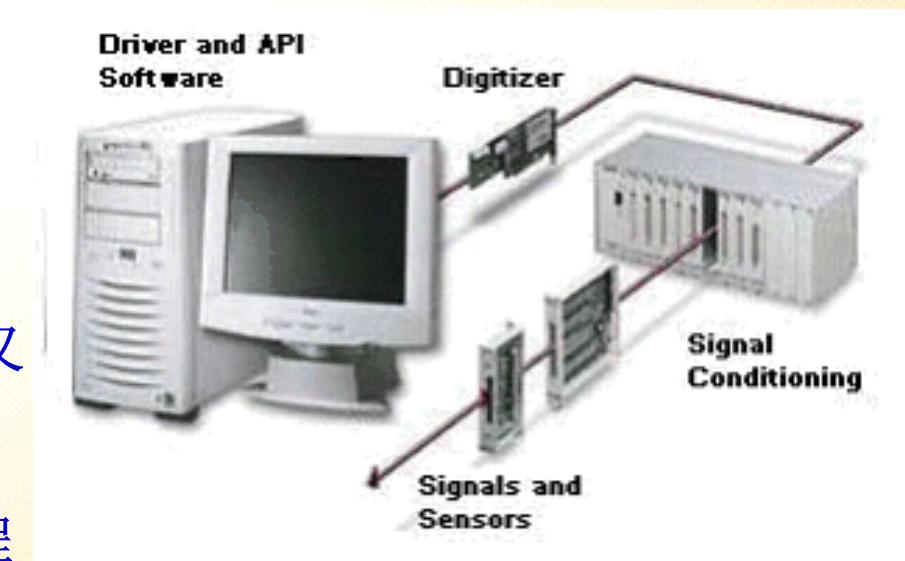
- ✓ 各种信号通过计算机记录下来的系统
- ✓ 把模拟信号(**-5V**到**+5V**或者其他范围)转换成数字信号,**Analog-Digital(AD)**, 反过来称**DA**
- ✓ www.ni.com National Instrument



采集过程

传感器-信号处理-模拟数字**AD**转换-记录

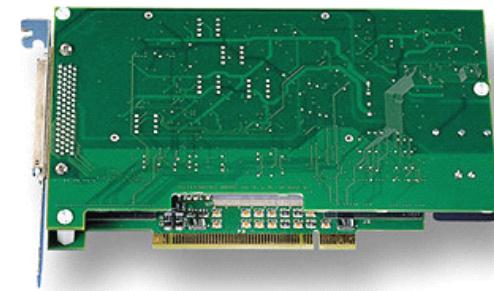
- ✓ 传感器输出的信号量级
 - 应力应变片:微伏
 - 热电偶:毫伏
 - 热线风速仪:伏
- ✓ 信号到**DAQ**前预处理
 - 放大**gain**, 移动**offset**
- ✓ 模数转换**AD**
 - 使用转换卡**Data Acquisition Card(DAQ)** 又称**digitizer**
- ✓ 通过软件记录数据
 - 一般通过软件**Labview**编程



数据采集卡DAQ Card

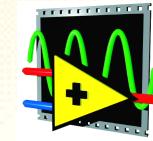
✓ 采集卡

- 适用方法,**PCI,PCMCIA,USB**等
- 通道数,**16AD,2DA**
- 位数:**12,14,16bits**, 决定精度, **10V/2¹²**
- 采样速度:**10MHz-1GHz**
- 输入范围, **+ -5V**等
- 价格, **1500-15000RMB**

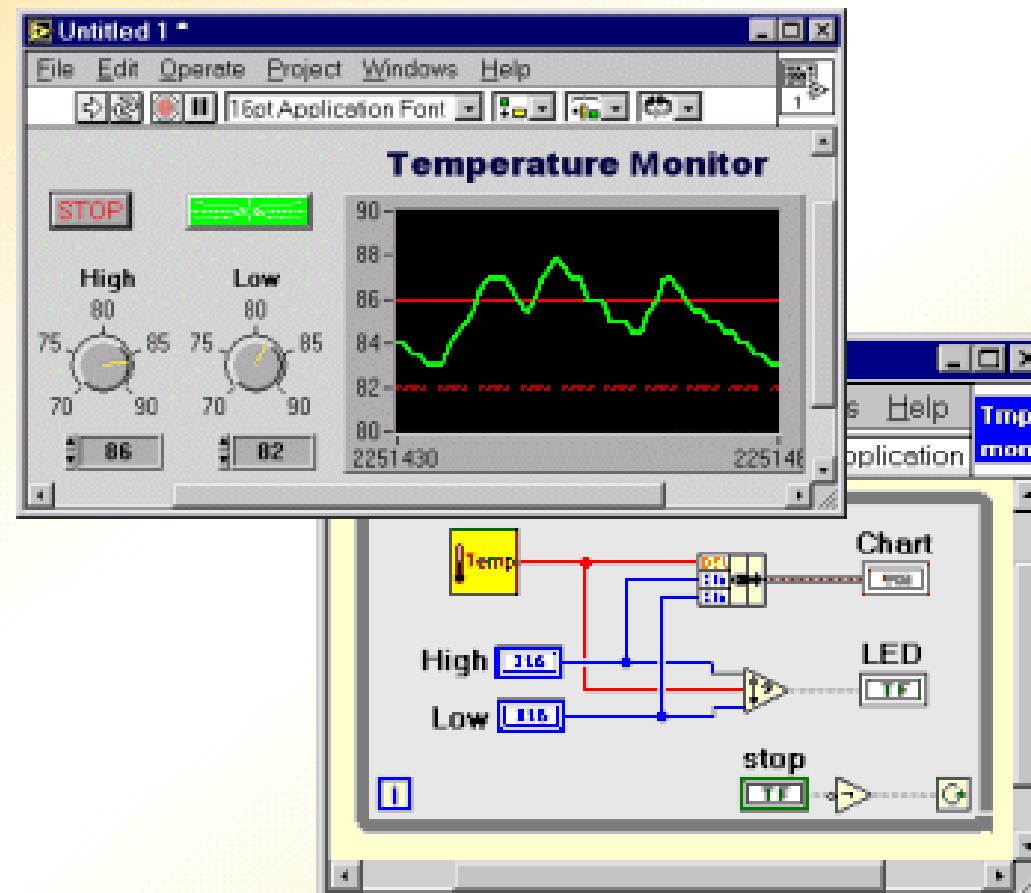


Family	Bus	Analog Inputs	Input Resolution	Max Sampling Rate	Input Range	Analog Outputs
NI 6071E	PCI, PXI	64 SE/32 DI	12 bits	1.25 MS/s	±0.05 to ±10 V	2
NI 6070E	PCI, PXI, FireWire	16 SE/8 DI	12 bits	1.25 MS/s	±0.05 to ±10 V	2
NI 6062E	PCMCIA	16 SE/8 DI	12 bits	500 kS/s	±0.05 to ±10 V	2
NI 6052E	PCI, PXI, FireWire	16 SE/8 DI	16 bits	333 kS/s	±0.05 to ±10 V	2
NI 6040E	PCI, PXI	16 SE/8 DI	12 bits	500 kS/s	±0.05 to ±10 V	2
NI 6033E	PCI	64 SE/32 DI	16 bits	100 kS/s	±0.1 to ±10 V	0

与DAQ卡通信方法:



NATIONAL INSTRUMENTS™
LabVIEW™



湍流基础讲座5

湍流实验数据分析处理

高南

2007年12月27日

内容

✓ 单传感器测量分析

- 统计分布
- 基本统计量
- 自相关性
- 谱分布

✓ 多传感器信号分析

- 交叉谱
- 相关性等

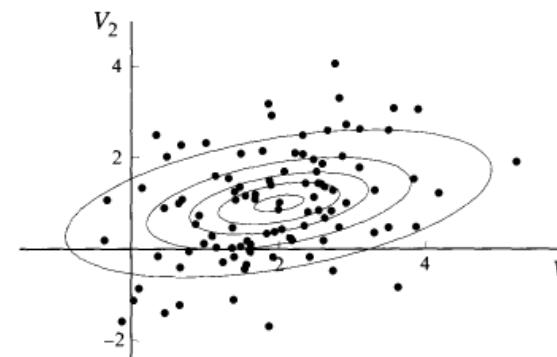
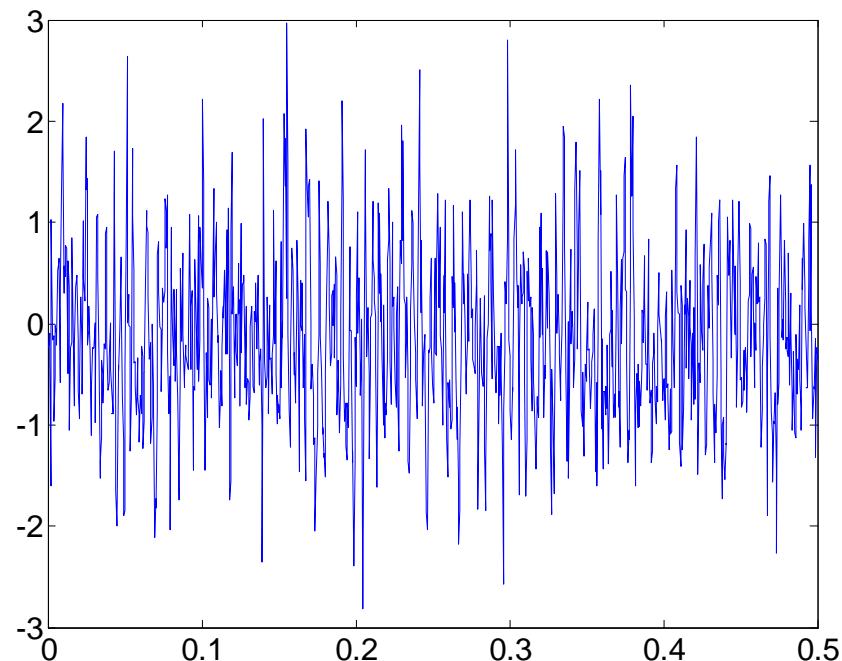


Fig. 3.18. A scatter plot and constant-probability density lines in the V_1 - V_2 plane for joint-normal random variables (U_1, U_2) with $\langle U_1 \rangle = 2$, $\langle U_2 \rangle = 1$, $\langle u_1^2 \rangle = 1$, $\langle u_2^2 \rangle = \frac{3}{16}$, and $\rho_{12} = 1/\sqrt{5}$.

取自A Pope, Turbulence

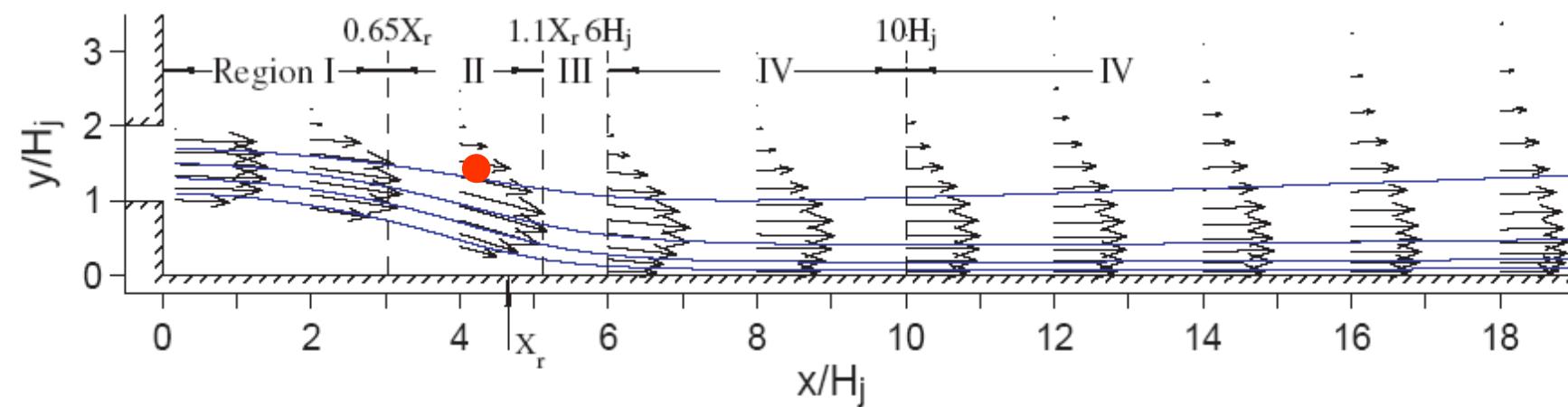
热线风速仪测量的流速信号



- ✓ 如何描述这段信号
 - 平均速度, 脉动强度, 强度变化频率(周期)

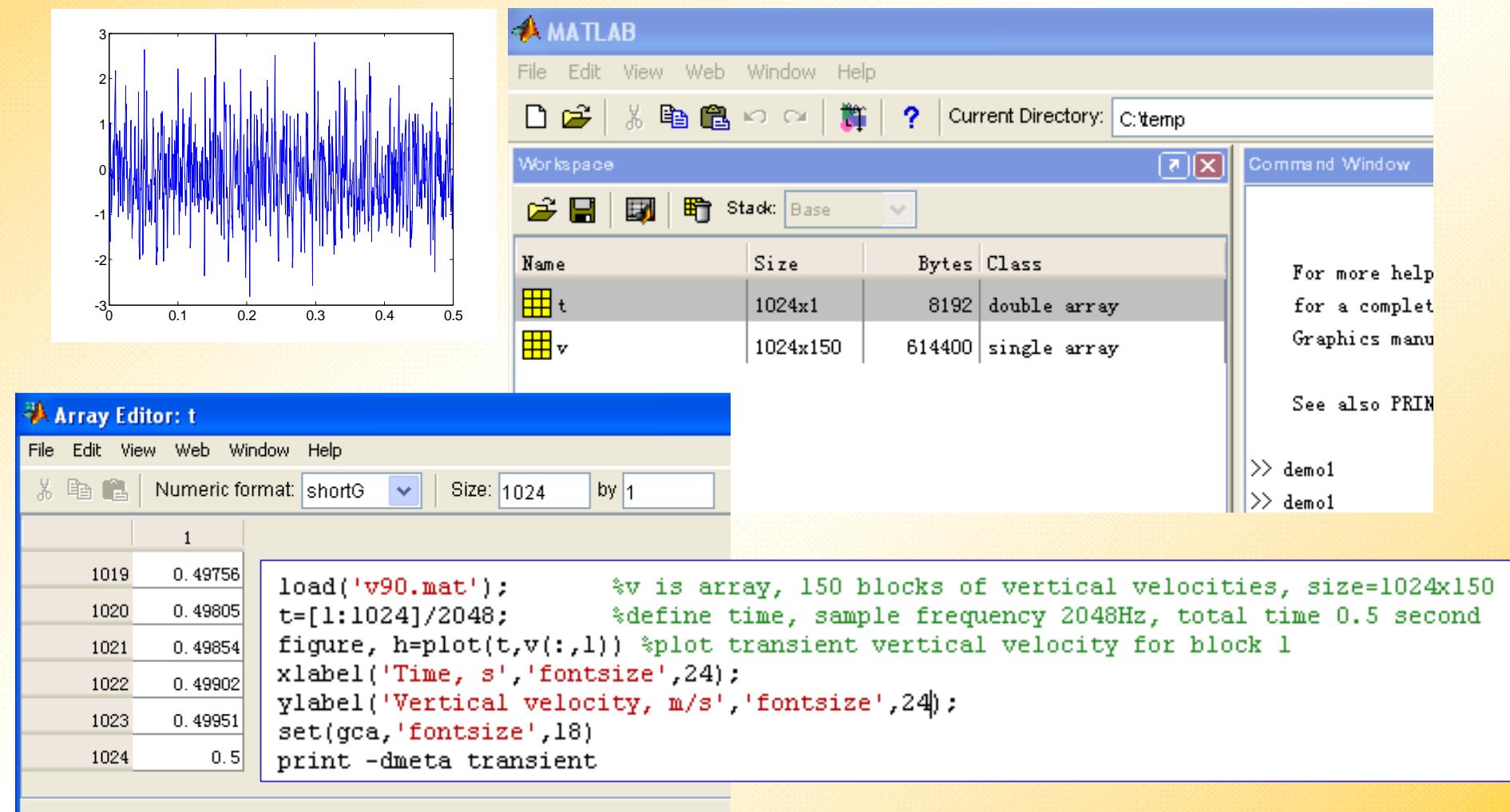
平面射流垂直于来流方向的速度
2006, 高南

实验介绍

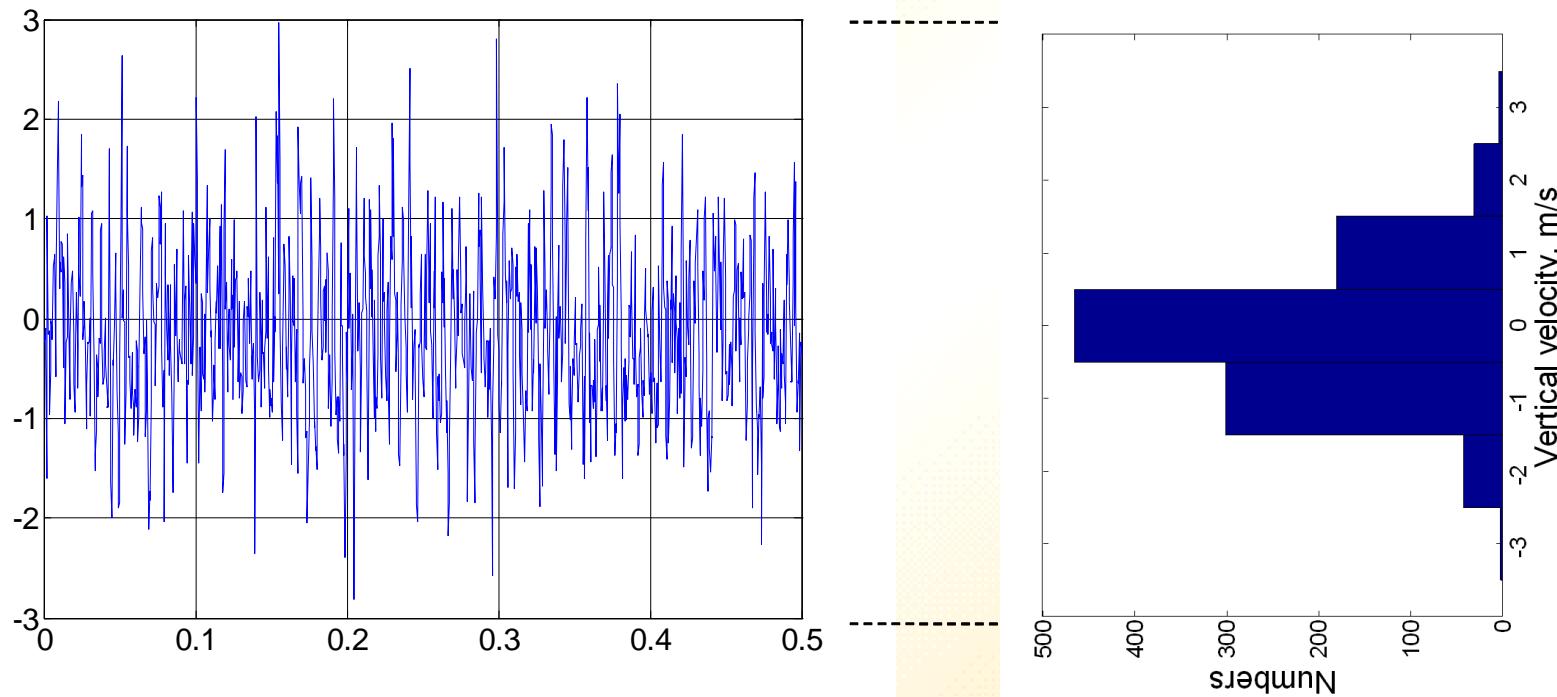


- ✓ McMaster University, Hamilton, Canada
- ✓ 平面附壁射流, $H_j=H_s=3.8\text{cm}$, $Re\sim 44000$
- ✓ X型热线风速仪,
- ✓ 采样频率**2048Hz**, 每组采样时间**0.5秒**, 共**150组**
- ✓ 祥见Experiments in Fluids (2007) 42:941–954

流速信号 Matlab

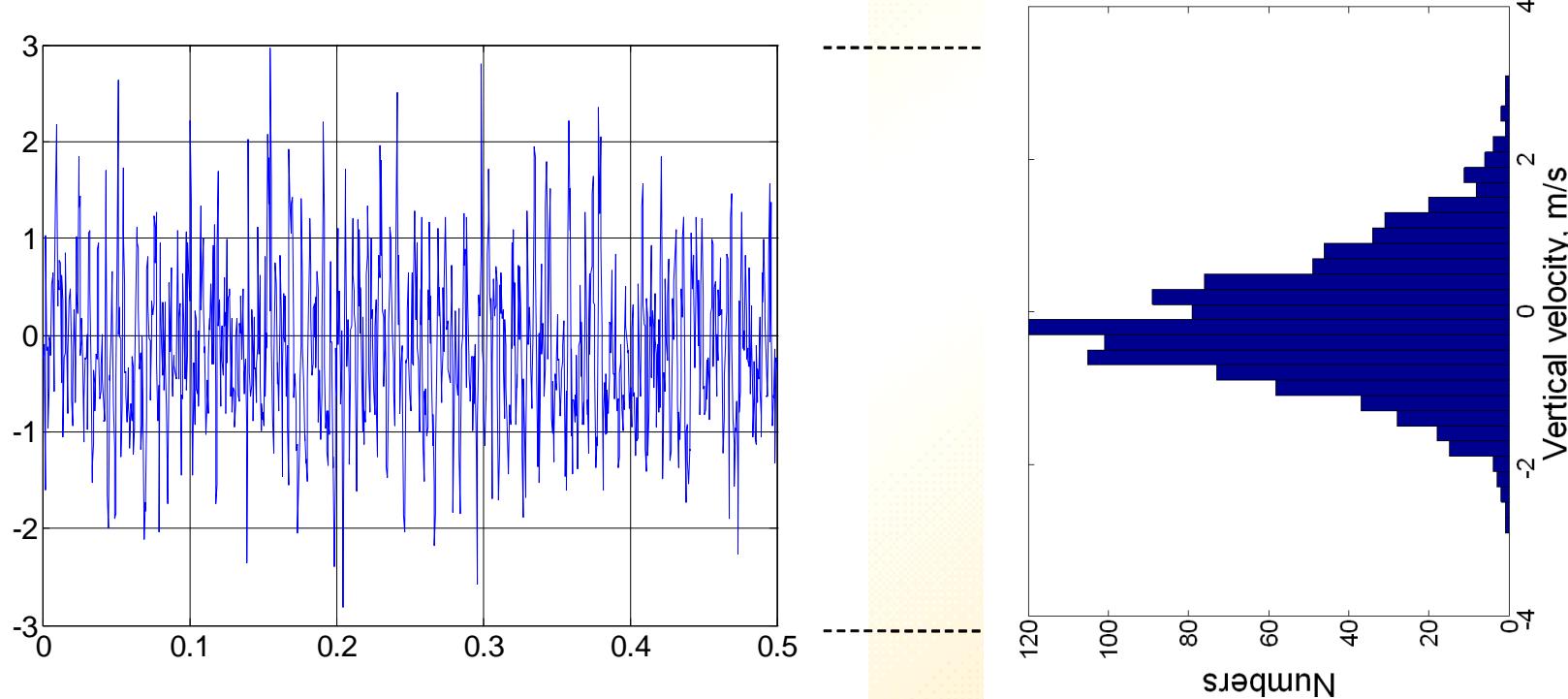


累积概率直方图



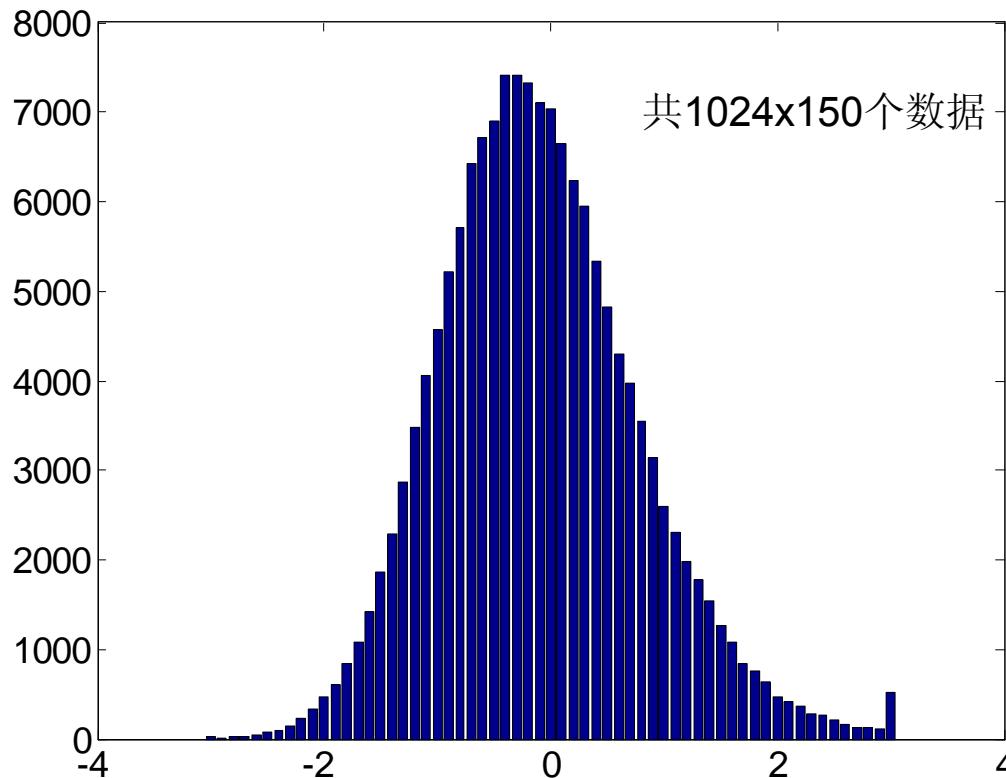
```
figure,  
hist(v(:,1),[-3:1:3]); % histogram  
ylabel('Numbers','fontsize',24);  
xlabel('Vertical velocity, m/s','fontsize',24);  
set(gca,'fontsize',18)  
print -dmeta hist1
```

累积概率直方图



```
figure,  
hist(v(:,1),[-3:.1:3]);% histogram  
ylabel('Numbers','fontsize',24);  
xlabel('Vertical velocity, m/s','fontsize',24);  
set(gca,'fontsize',18)  
print -dmeta hist2
```

累积概率直方图 共**150**组数据



将每段数值除以
总采样数1024x150



```
a=hist(v,[-3:.1:3]);% histogram  
b=sum(a');  
figure,  
bar([-3:.1:3],b);  
xlabel('Vertical velocity, m/s','fontsize',24);  
ylabel('Numbers','fontsize',24);  
set(gca,'fontsize',18)  
print -dmeta hist3
```

概率密度函数

Probability Density Function (PDF)

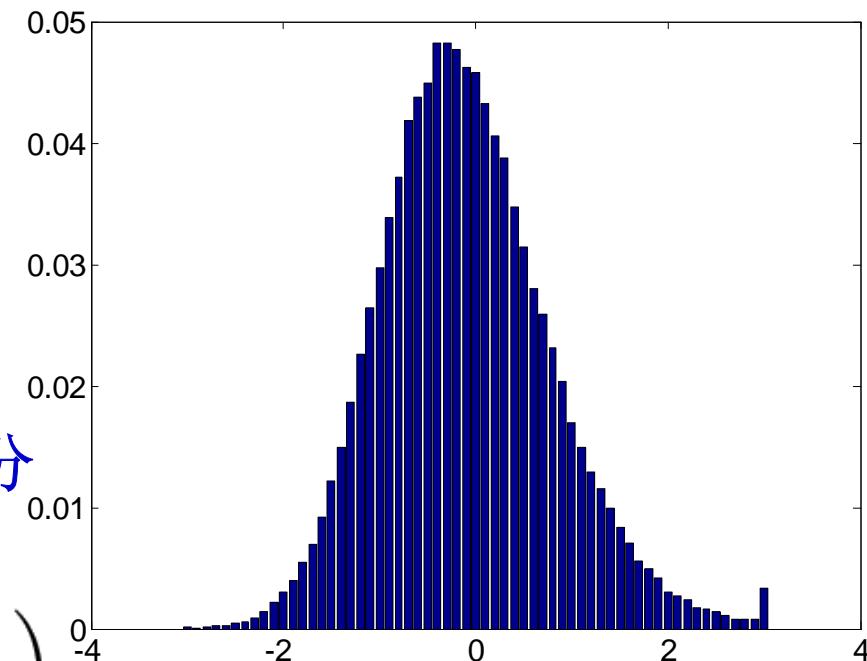
✓ 积分结果为1

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

✓ 高斯分布

- “没有间歇性的正态分布”(张兆顺)

$$p(x) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



概率密度函数

Probability Density Function (PDF)

- ✓ “如果知道湍流场任何一点概率密度，我们就完全掌握了它的性质” (张兆顺)
 - 不大可能，或者说十分困难
- ✓ 更多的是需要利用统计规律来描述流场
 - 统计矩 **Moments**
 - 一阶矩：平均值 **Mean**
 - 二阶矩：均方差 **variance** 或者 **mean square**
 - 高阶矩：**Skewness, Kurtosis**
 - 相关性 **Correlation**

统计矩 **Moment**

✓ 定义式

$$\langle v^n \rangle = \int_{-\infty}^{\infty} \tilde{v}^n p(\tilde{v}) d\tilde{v} = \frac{\sum_{i=1}^N \tilde{v}_i^n}{N}$$

- 一阶**n=1** 平均值 **Mean**

$$V = \langle v \rangle = \int_{-\infty}^{\infty} \tilde{v} p(\tilde{v}) d\tilde{v}$$

- 二阶**n=2** 均方差

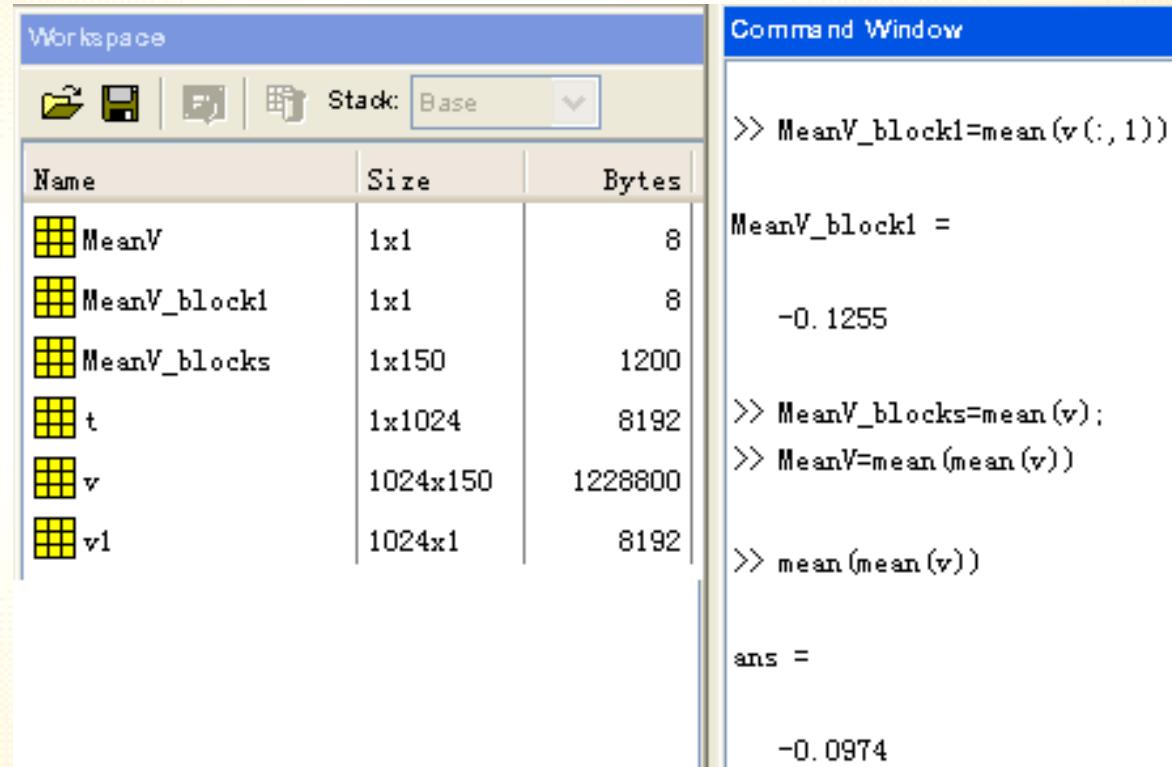
$$\sigma^2 = \langle v^2 \rangle = \int_{-\infty}^{\infty} \tilde{v}^2 p(\tilde{v}) d\tilde{v}$$

平均值 Mean

✓ 平均速度

✓ Matlab command: mean

$$\langle v \rangle = \frac{\sum_{i=1}^N \tilde{v}_i}{N}$$



均方差 Variance, σ^2

- ✓ 又称**mean square**,
- ✓ 其平方根为标准差, **standard deviation**, 或者称为均方根**root mean square (rms)**, 脉动速度**v'**

$$\sigma = \langle v^2 \rangle^{1/2} = \sqrt{\frac{\sum_{i=1}^N \tilde{v}_i^2}{N}} = v'$$

- ✓ 湍流度 **Turbulence intensity**

$$\frac{\langle v^2 \rangle^{1/2}}{\langle v \rangle} \quad or \quad \frac{v'}{V}$$

用Matlab计算均方差 σ^2

The screenshot shows the MATLAB environment with the workspace and command window.

Workspace:

Name	Size	Bytes	Class
MeanV	1x1	8	double array
MeanV_block1	1x1	8	double array
MeanV_blocks	1x150	1200	double array
Rms_block1	1x1	8	double array
VarV	1x1	8	double array
VarV_block1	1x1	8	double array
VarV_blocks	1x150	1200	double array
ans	1x1	8	double array
t	1x1024	8192	double array
v	1024x150	1228800	double array
v1	1024x1	8192	double array

Command Window:

```
>> VarV_block1=var(v(:,1))

VarV_block1 =
    0.7046

>> VarV_blocks=var(v);
>> VarV=mean(var(v))

VarV =
    0.7891

>> Rms_block1=sqrt(VarV_block1)

Rms_block1 =
    0.8394

>> std(v(:,1))

ans =
    0.8394
```

误差分析-平均速度

Experimental Uncertainties (Error)

✓ 平均速度的误差

- 基于**95%**置信度

✓ **N_s**为独立的采样次数 $N_s = \frac{T_s}{2T}$

- 这里**T_s**为采样时间，**T**为信号特征周期

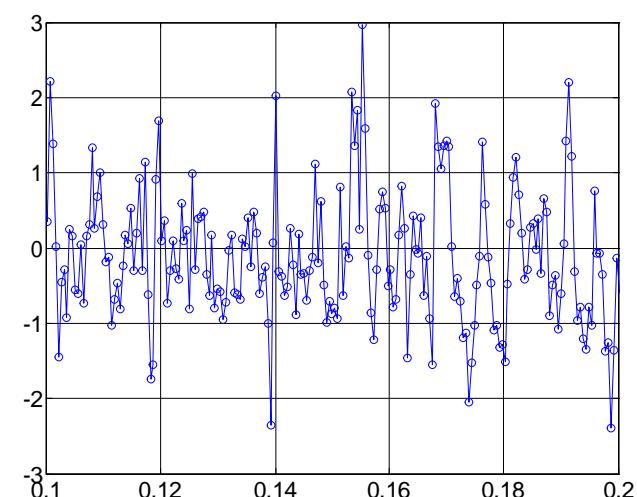
• 如果**T~0.01s, T_s=0.5s**

<v>~-0.1m/s, σ~0.9m/s

N_s~25, ε ~±35%

• **T_s=0.5x150s, ε ~±0.2%**

$$\varepsilon = \frac{\sigma}{\langle v \rangle N_s}$$



误差分析-脉动流速(rms)

Experimental Uncertainties (Error)

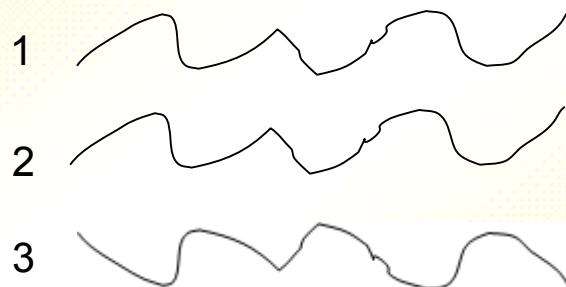
- ✓ 平均速度的误差
 - 基于**95%**置信度

$$\varepsilon = \frac{1}{\sqrt{N_s}}$$

- ✓ 如果 $T \sim 0.01\text{s}$, $T_s = 0.5\text{s}$, $\langle v \rangle \sim -0.1\text{m/s}$, $\sigma \sim 0.9\text{m/s}$, 得到 $N_s \sim 25$, $\varepsilon \sim \pm 20\%$
- ✓ $T_s = 0.5 \times 150\text{s}$, $\varepsilon \sim \pm 0.14\%$

相关性 Correlation

- ✓ 量化 两组信号的关系(互相关); 或者一组信号自身随时间变化的大小(自相关)
- ✓ 定义: 相关系数-任何时刻完全一样 $\rho_{12}=1$



任何时刻‘一样’但相反, $\rho_{13}=-1$



不相关 $\rho_{14}>0$

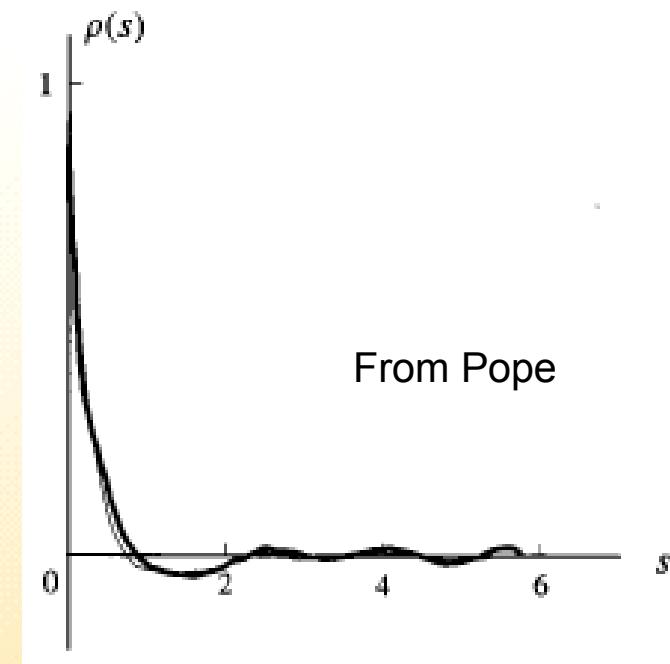
自相关系数 Autocorrelation Coef.

R_w 相关函数

✓ 自相关系数

$$\rho_{vv}(\tau) = \frac{R_{vv}}{v'^2} = \frac{\langle \tilde{v}(t)\tilde{v}(t + \tau) \rangle}{v'^2}$$

- ✓ 当 τ 为零, ρ_{vv} 为 1
- ✓ τ 增加, ρ_{vv} 逐渐减少



相关函数R的定义

$$R_{vv}(\tau) = \langle \tilde{v}(t)\tilde{v}(t + \tau) \rangle$$

V	V	V	V	V	V
1	1	1	1	1	1
1 -0.23474	1 -0.23474	1 -0.23474	1 -0.23474	1 -0.23474	1 -0.23474
2 -0.094609	2 -0.094609	2 -0.094609	2 -0.094609	2 -0.094609	2 -0.094609
3 -0.3653	3 -0.3653	3 -0.3653	3 -0.3653	3 -0.3653	3 -0.3653
4 -1.6	4 -1.6	4 -1.6	4 -1.6	4 -1.6	4 -1.6
5 1.0277	5 1.0277	5 1.0277	5 1.0277	5 1.0277	5 1.0277
6 -0.16433	6 -0.16433	6 -0.16433	6 -0.16433	6 -0.16433	6 -0.16433
7 -0.11887	7 -0.11887	7 -0.11887	7 -0.11887	7 -0.11887	7 -0.11887
8 -0.96311	8 -0.96311	8 -0.96311	8 -0.96311	8 -0.96311	8 -0.96311
9 -0.52107	9 -0.52107	9 -0.52107	9 -0.52107	9 -0.52107	9 -0.52107
10 -0.020203	10 -0.020203	10 -0.020203	10 -0.020203	10 -0.020203	10 -0.020203
11 -0.095545	11 -0.095545	11 -0.095545	11 -0.095545	11 -0.095545	11 -0.095545
12 -0.20945	12 -0.20945	12 -0.20945	12 -0.20945	12 -0.20945	12 -0.20945
13 0.53103	13 0.53103	13 0.53103	13 0.53103	13 0.53103	13 0.53103
14 0.61706	14 0.61706	14 0.61706	14 0.61706	14 0.61706	14 0.61706
15 0.64573	15 0.64573	15 0.64573	15 0.64573	15 0.64573	15 0.64573
16 -0.58137	16 -0.58137	16 -0.58137	16 -0.58137	16 -0.58137	16 -0.58137
17 0.34439	17 0.34439	17 0.34439	17 0.34439	17 0.34439	17 0.34439
18 1.3597	18 1.3597	18 1.3597	18 1.3597	18 1.3597	18 1.3597
19 2.1774	19 2.1774	19 2.1774	19 2.1774	19 2.1774	19 2.1774
20 1.8354	20 1.8354	20 1.8354	20 1.8354	20 1.8354	20 1.8354
21 0.29639	21 0.29639	21 0.29639	21 0.29639	21 0.29639	21 0.29639
22 0.56406	22 0.56406	22 0.56406	22 0.56406	22 0.56406	22 0.56406
23 0.45985	23 0.45985	23 0.45985	23 0.45985	23 0.45985	23 0.45985

$$\tau = 0$$

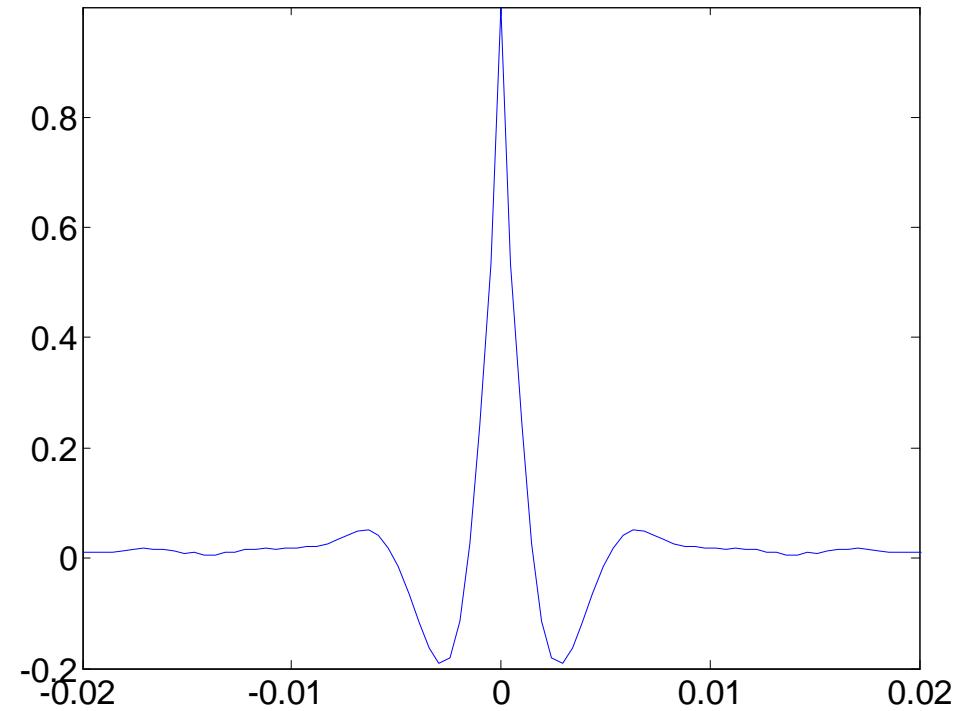
$$\tau = \Delta t = T_s/S$$

$$\tau = 2\Delta t = 2T_s/S$$

相关性 计算

✓ 通过**Matlab** 函数 **xcorr**

```
cr=zeros(1024*2-1,1);
t=[-1023:1023]/2048; %sample frequency 2048Hz
for block=1:150;
    c=xcorr(v(:,block))/max(xcorr(v(:,block)));
    cr=cr+c;
end
cr=cr/150;
figure, plot(t,cr)
axis([-0.02 .02 -.2 1])
xlabel('Time, s','fontsize',24);
ylabel('R','fontsize',24);
set(gca,'fontsize',18)
print -dmeta R
```



信号 分解

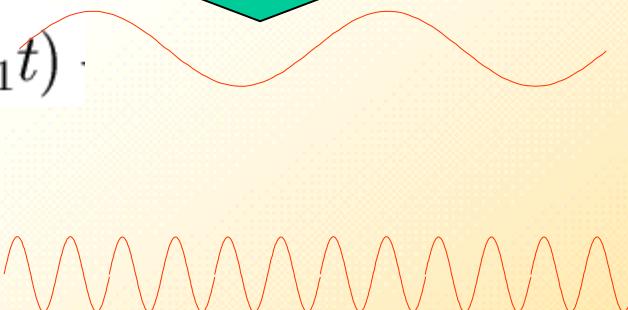
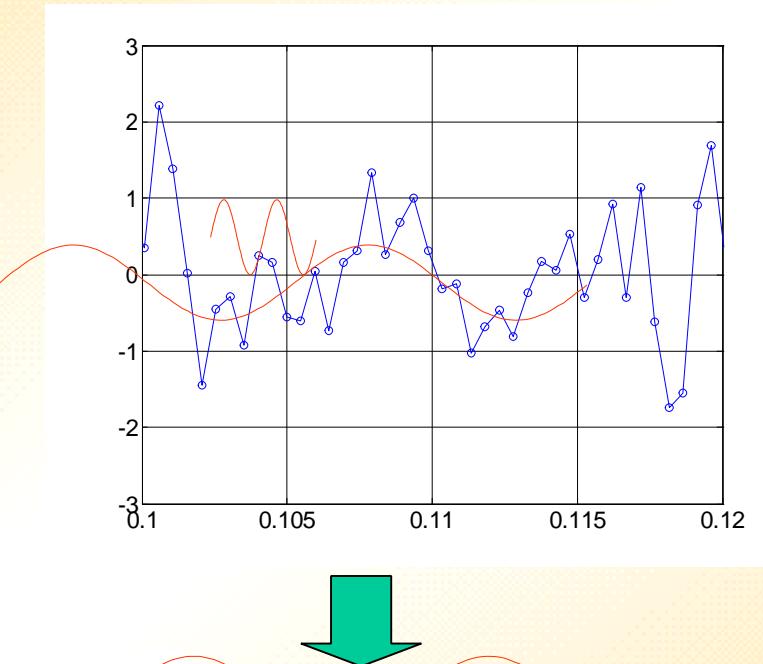
✓ 另一种视角看问题

原信号

$$\tilde{v}(t) \approx a_1 \cos(2\pi f_1 t) + b_1 \sin(2\pi f_1 t)$$

$$+ a_2 \cos(2\pi f_2 t) + b_2 \sin(2\pi f_2 t)$$

✓ **v(t)** 被转化成了系数 **a_n, b_n**



信号 分解

✓ 通过傅立叶变换确定系数 **a_n, b_n**

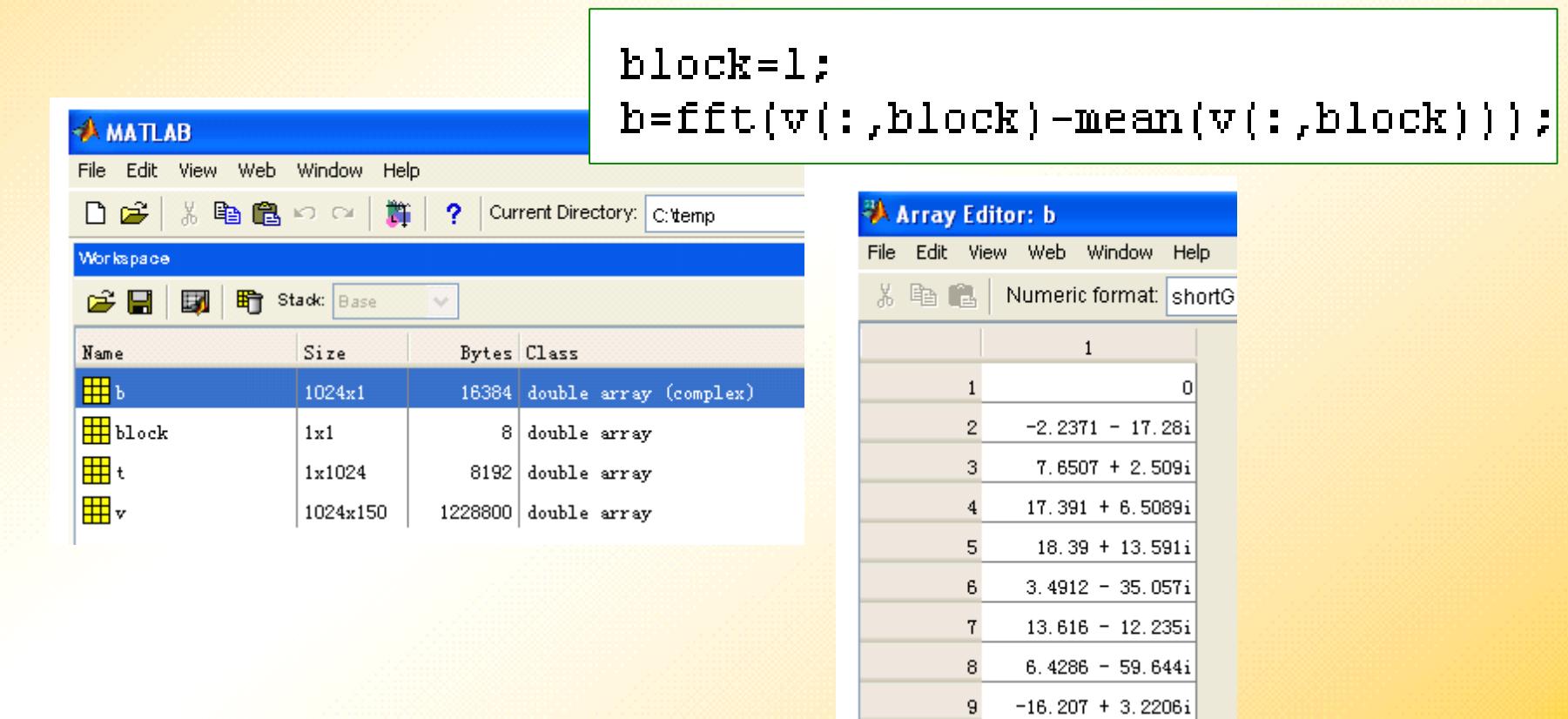
$$\mathcal{F}(\tilde{v}(t)) = F(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{v}(t) e^{-i2\pi ft} dt$$

- 因为 $e^{-i2\pi ft} = \cos(2\pi ft) - i\sin(2\pi ft)$
- 得到的 $F(f)$ 为复数 (**v** 为偶函数的时候除外), 实部代表 **a**, 虚部代表 **b**
- $|F(f)|^2$ 被称为能量谱, ‘代表’ $a_n^2 + b_n^2$
- 能量谱表示了能量在各个时间尺度的分布

$$|F(f)|^2 = \frac{F(f)F^*(f)}{N^2}$$

能量谱**power spectrum**计算方法 I

- ✓ 快速傅立叶变换**Fast Fourier Transform**
 - 利用**Matlab FFT**函数



能量谱计算方法II

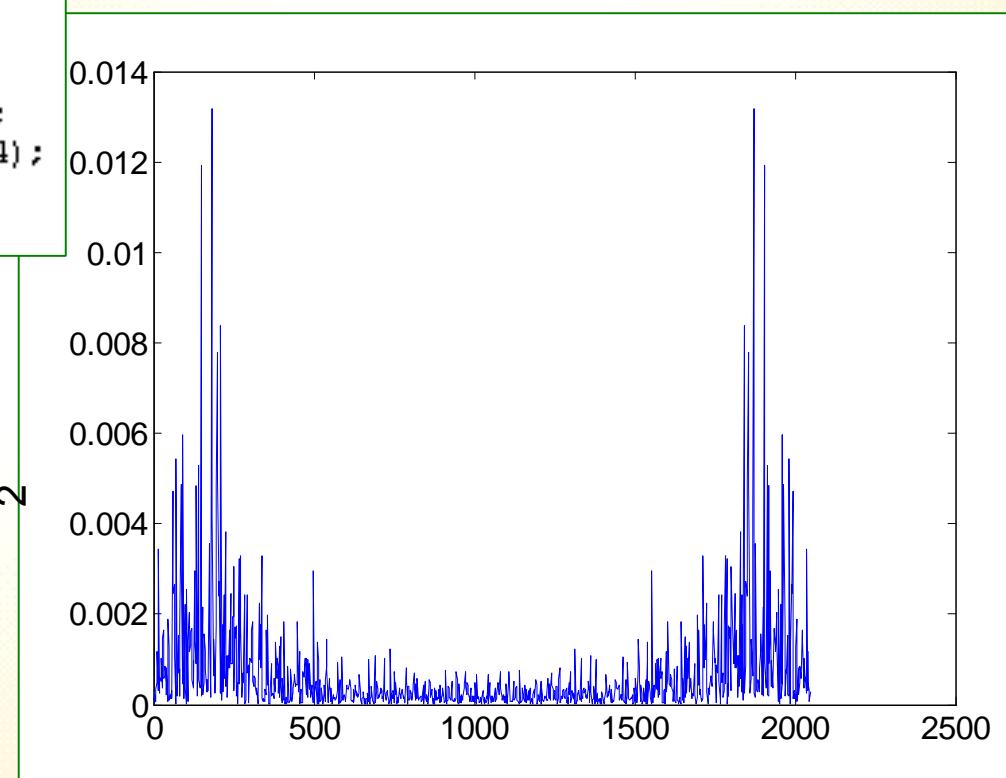
power spectrum

$$|F(f)|^2 = \frac{F(f)F^*(f)}{N^2}$$

```
block=1;
N=1024; %N sample number
b=fft(v(:,block))-mean(v(:,block));
F=b.*conj(b)/N^2;
f=[1:N]^(1/0.5); %0.5 sample time
figure, plot(f,F)
xlabel('Frequency, Hz','fontsize',24);
ylabel('|F|, (m/s)^2/Hz','fontsize',24);
set(gca,'fontsize',18)
print -dmeta spec1
```

b	
1	0
2	38.333 + 1.6137i
3	-4.8734 - 2.144i
4	23.67 - 7.8949i
5	12.01 + 17.82i
6	-8.8278 + 31.693i
7	57.59 - 8.9153i
8	-13.109 + 9.5148i
9	-15.893 + 19.326i
10	23.615 - 6.0247i

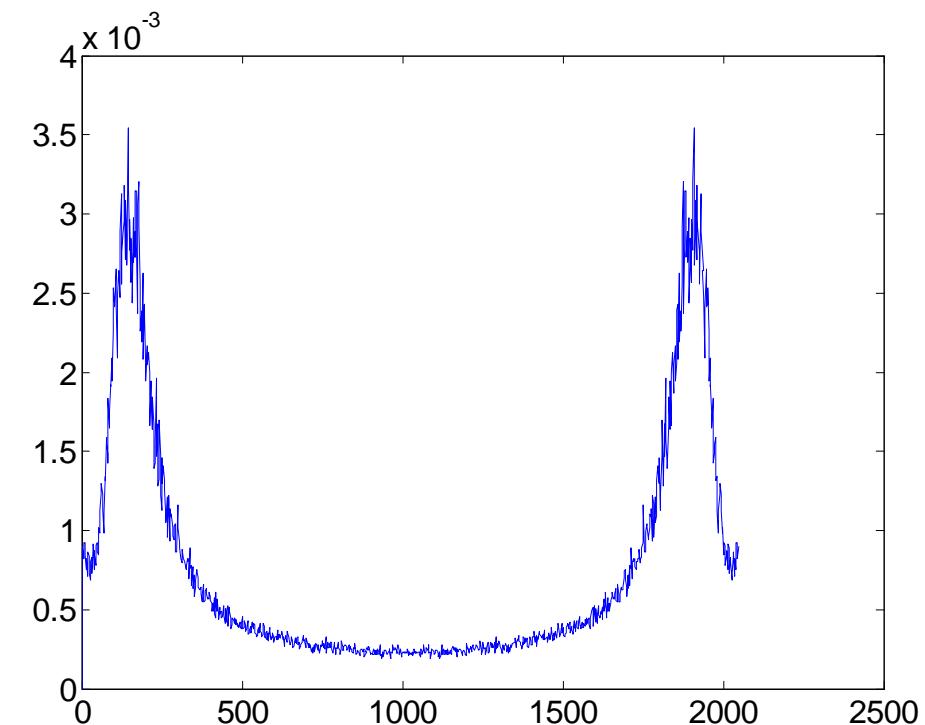
conj(b)	
1	0
2	38.333 - 1.6137i
3	-4.8734 + 2.144i
4	23.67 + 7.8949i
5	12.01 - 17.82i
6	-8.8278 - 31.693i
7	57.59 + 8.9153i
8	-13.109 - 9.5148i
9	-15.893 - 19.326i
10	23.615 + 6.0247i



150组数据的平均 能量谱

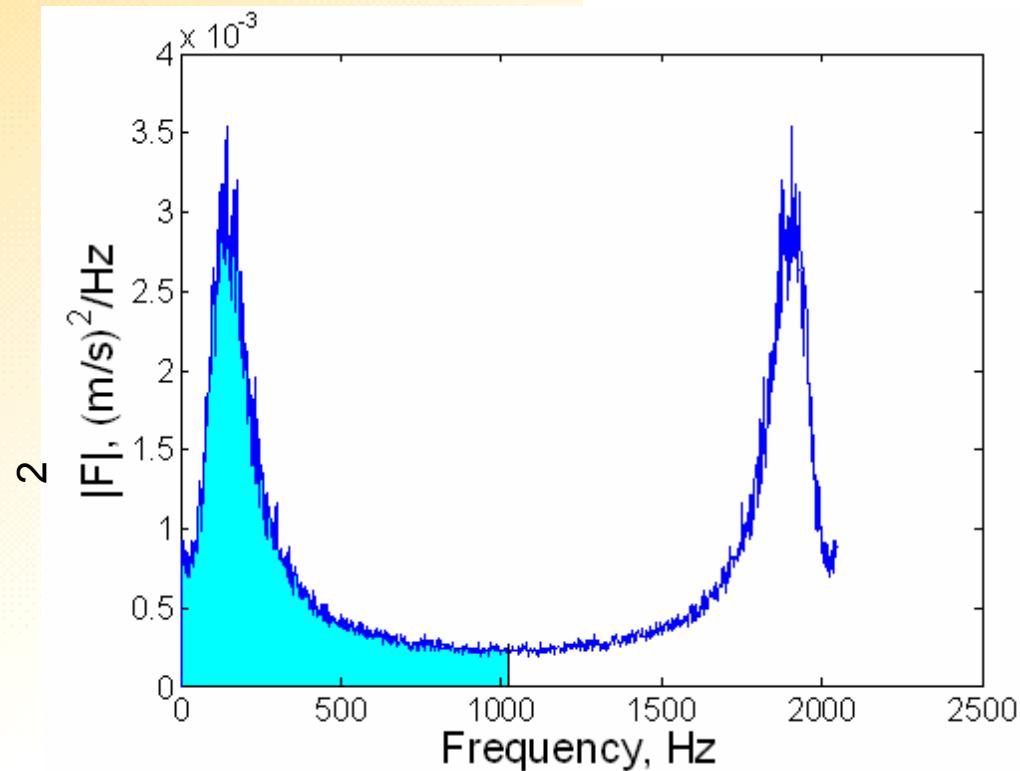
```
Fs=zeros(1024,1);
N=1024; %N sample number
for block=1:150;
    b=fft(v(:,block))-mean(v(:,block)));
    F=b.*conj(b)/N^2;
    Fs=Fs+F;
end
Fs=Fs/150;
f=[1:N]*(1/0.5);% 0.5 is the sample time
figure, plot(f,Fs)
xlabel('Frequency, Hz','fontsize',24);
ylabel('|F|, (m/s)^2/Hz','fontsize',24);
set(gca,'fontsize',18)
print -dmeta spec2
```

2



能量谱积分 → 均方差(Variance) σ^2

$$\langle v^2 \rangle = \int_{-\infty}^{\infty} F_{vv}(f) df$$



```
>> sum(Fs(1:1024/2))*(1/0.5)
```

ans =

0.7881

```
>> mean(var(v))
```

ans =

0.7891

频谱逆变换 \rightarrow 自相关函数

$$\mathcal{F}(\tilde{v}(t)) = F(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{v}(t) e^{-i2\pi f t} dt$$

$$\mathcal{F}^{-1}(F(f)) = R(\tau) = \int_{-\infty}^{\infty} F(f) e^{i2\pi f \tau} d\tau$$

多传感器信号分析— 交叉谱

cross spectrum

- ✓ 单独一点信号的能量谱表示了脉动能量在各个频率(时间尺度)的分布
- ✓ 交叉谱反映两个相同时段的不同时间序列 $\mathbf{v}_1(t)$ 和 $\mathbf{v}_2(t)$ 在频域变化上的相互关系

The cross spectra $\Phi(x_1, x_2, f)$ was given by

$$\overline{\hat{p}(x_1, f)\hat{p}^*(x_2, f')} = \Phi(x_1, x_2, f)\delta(f - f'), \quad (3.13)$$

where δ is the Dirac delta function, \hat{p} is the Fourier transform of the transient wall pressure, and $*$ represents the complex conjugate. These spectra were computed by averaging the results for different blocks

摘自高南博士论文

$$\Phi(x_1, x_2, f) = \frac{\overline{\hat{p}(x_1, f)\hat{p}^*(x_2, f)}}{T}, \quad \text{改p帽为F} \quad (3.14)$$

交叉谱的 模 和 相位角

coherence and phase angle

The nature of the relationship between the pressure at the two points was also examined using the coherence of the pressure fluctuations measured at two locations given by,

$$\gamma_{pp}^2(x_1, x_2, f) = \frac{|\Phi(x_1, x_2, f)|^2}{\Phi(x_1, x_1, f)\Phi(x_2, x_2, f)}, \quad (3.15)$$

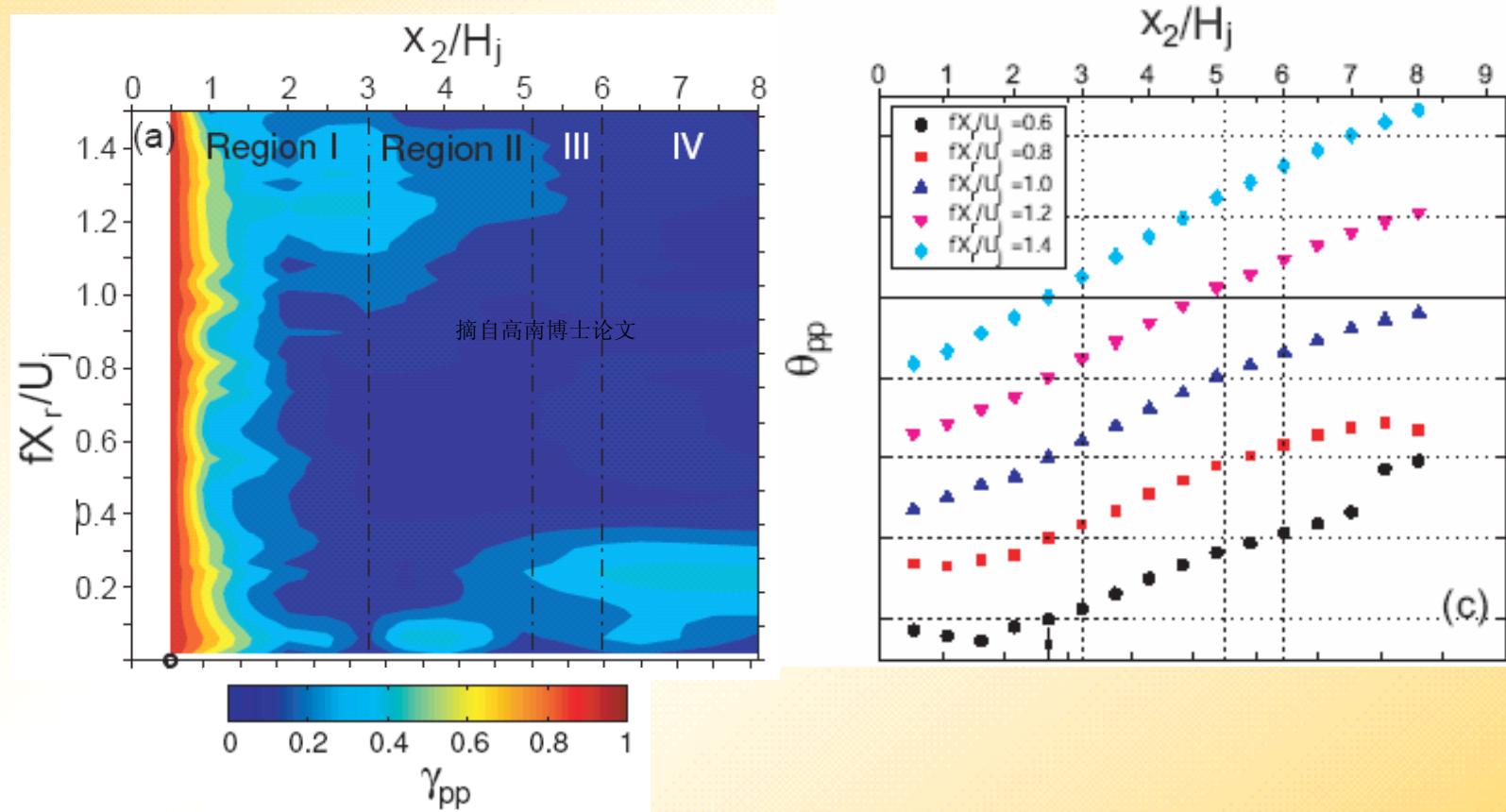
and the phase angle of the cross spectrum, $\theta_{pp}(x_1, x_2, f)$ given by

摘自高南博士论文

$$\theta_{pp}(x_1, x_2, f) = \tan^{-1} \left(\frac{\text{img}(\Phi(x_1, x_2, f))}{\text{real}(\Phi(x_1, x_2, f))} \right). \quad (3.16)$$

交叉谱的 模 和 相位角

coherence and phase angle



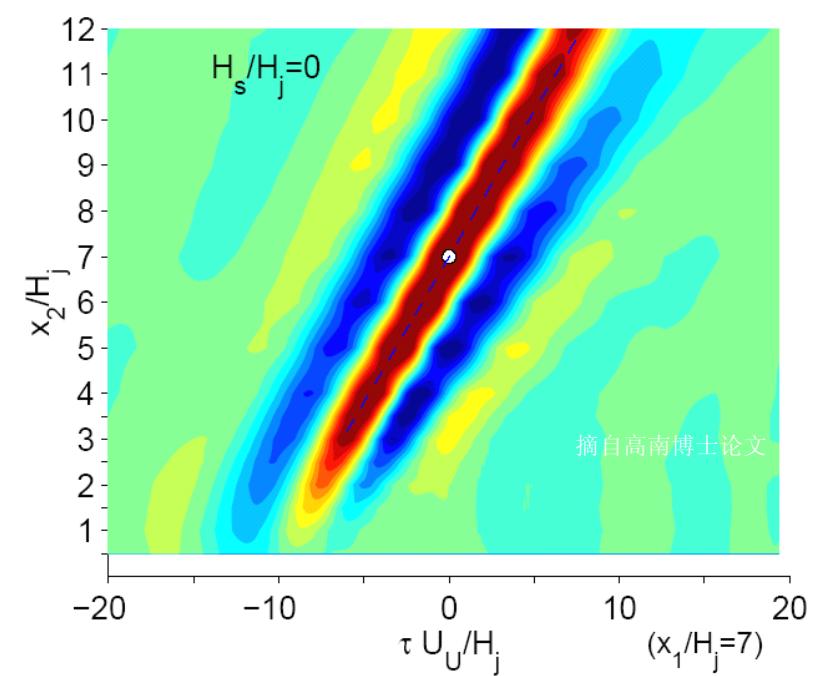
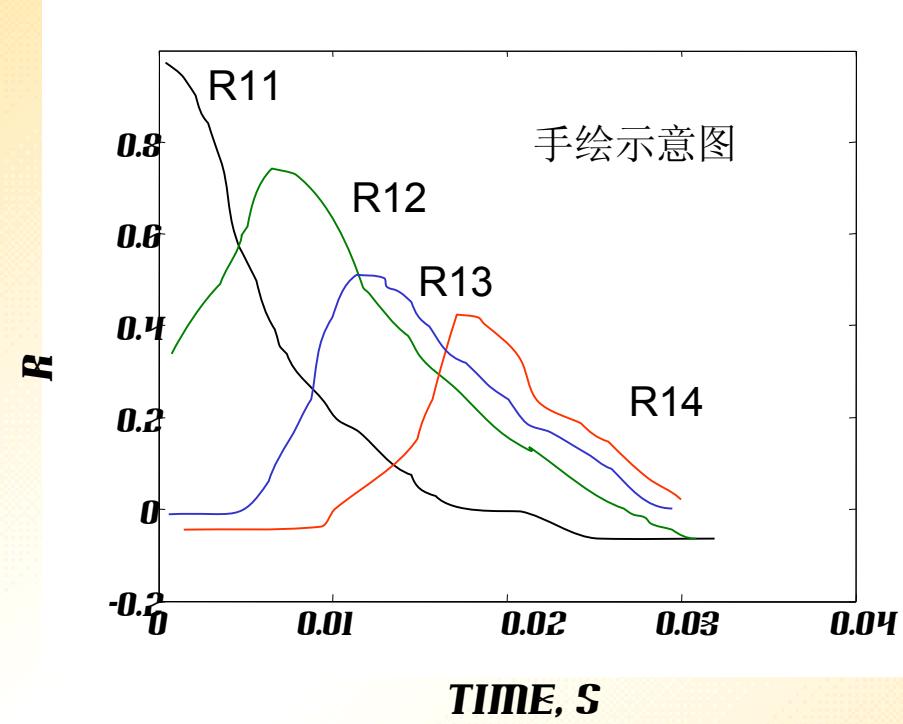
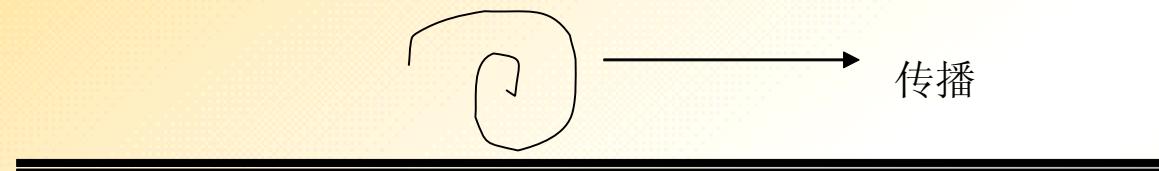
交叉谱 逆变换 互相关函数

the cross correlation $R_{pp}(x_1, x_2, \tau)$ was computed by inverse Fourier transforming the cross spectra, *i.e.* ,

摘自高南博士论文

$$R_{pp}(x_1, x_2, \tau) = \int_{-\infty}^{\infty} \Phi(x_1, x_2, f) e^{i2\pi f \tau} df.$$

交叉谱 逆变换 → 互相关函数



壁面压强互相关系数