1. Prove $(\vec{V}\cdot\nabla)\vec{V} = \nabla\frac{V^2}{2} - \vec{V}\times(\nabla\times\vec{V})$

$$\vec{V}\times(\nabla\times\vec{V}) = \varepsilon_{ijk} V_j \varepsilon_{kpq} \partial_p V_q$$
$$= \varepsilon_{ijk}\varepsilon_{kpq} V_j \partial_p V_q$$
$$= (\delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}) V_j \partial_p V_q$$
$$= V_q \partial_i V_q - V_j \partial_j V_i$$
$$= \partial_i\left(\frac{V_q^2}{2}\right) - V_j \partial_j V_i$$
$$= \nabla\frac{V^2}{2} - (\vec{V}\cdot\nabla)\vec{V}$$

2. Prove $\nabla\cdot\vec{\omega} = 0$

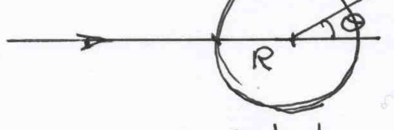
$$\nabla\cdot\vec{\omega} = \partial_i \varepsilon_{ijk}\partial_j V_k$$
$$= \varepsilon_{ijk}\partial_i\partial_j V_k$$

ε_{ijk} antisymmetric
$\partial_i\partial_j$ symmetric $\Rightarrow \nabla\cdot\vec{\omega} = 0$

or. $$\varepsilon_{ijk}\partial_i\partial_j V_k = \partial_1\partial_2 V_3 + \partial_2\partial_3 V_1 + \partial_3\partial_1 V_2 - \partial_3\partial_2 V_1 - \partial_2\partial_1 V_3 - \partial_1\partial_3 V_2 = 0$$

3. Strain rate cylinder flow.

$$\begin{cases} V_r = U_\infty\left(1 - \frac{R^2}{r}\right)\cos\theta \\ V_\theta = -U_\infty\left(1 + \frac{R^2}{r^2}\right)\sin\theta. \end{cases}$$



cylinder.

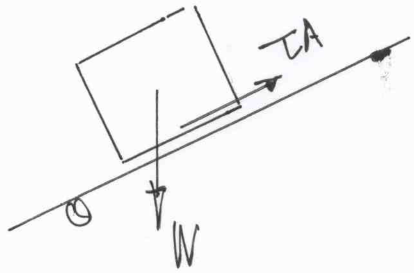
Strain Rates

$$\varepsilon_{rr} = \frac{dV_r}{dr} = \frac{2R^2U_\infty}{r^3}\cos(2\pi) = \frac{2RU_\infty}{r^3}$$

$$\varepsilon_{r\theta} = \frac{dV_r}{r\,d\theta} + \frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} = -\frac{U_\infty(1 - R^2/r)\sin(2\pi)}{r} + 3U_\infty\frac{R^3}{r^2}\sin(2\pi) + U_\infty\left(1 + \frac{R^2}{r^2}\right)\sin(2\pi) = 0$$

Infinite time.

4.



Plate, area A, weigh ~~W~~ mg
oil film thickness h
Viscosity μ.

Find (a) Final V_f

(b) time to $0.99 V_f$

assume Velocity Profile linear in gap

(a)
$$mg\sin\theta = \tau A$$
$$mg\sin\theta = \mu\frac{V_f}{h}A \Rightarrow V_f = \frac{mg\sin\theta\, h}{\mu A}.$$

(b)
$$m\frac{dV}{dt} = mg\sin\theta - \mu\frac{V}{h}A$$
$$\frac{dV}{dt} + \frac{\mu A}{hm}V + (-g\sin\theta) = 0$$

Let $a = \mu A/hm$, $b = -g\sin\theta$. $\boxed{\dot{V} + aV + b = 0}$

equation has solution in form of
$$V = ce^{-at} - b/a$$
, c is a constant.

when $t=0$. $V=0$. $\Rightarrow c = b/a$

$$t = -\frac{1}{a}\ln\left(\frac{a}{b}V+1\right)$$

$$t_{0.99} = -\frac{1}{a}\ln\left(\frac{a}{b}0.99V_f+1\right)$$