For any model $\mathcal{I}$ of $\mathcal{O} = \{A \sqsubseteq \exists r.B,\ B \sqsubseteq C\}$, we need to prove $\mathcal{I}$ is a model of $A \sqsubseteq \exists r.C$. i.e, $\mathcal{I} \models A \sqsubseteq \exists r.C$

$\mathcal{I} \in Mod(\mathcal{O})$ means $\mathcal{I} \models A \sqsubseteq \exists r.B$ and $\mathcal{I} \models B \sqsubseteq C$

This means $A^{\mathcal{I}} \subseteq (\exists r.B)^{\mathcal{I}}$ and $B^{\mathcal{I}} \subseteq C^{\mathcal{I}}$

$A^{\mathcal{I}} \subseteq (\exists r.B)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid$ there is $e \in \Delta^{\mathcal{I}}$, s.t. $(d,e) \in r^{\mathcal{I}}$ and $e \in B^{\mathcal{I}}\}$

Since $B^{\mathcal{I}} \subseteq C^{\mathcal{I}}$, so, for each $e \in B^{\mathcal{I}}$, $e \in C^{\mathcal{I}}$.

So $\forall x \in \{d \in \Delta^{\mathcal{I}} \mid$ there is $e \in \Delta^{\mathcal{I}}$, s.t. $(d,e) \in r^{\mathcal{I}}$ and $e \in B^{\mathcal{I}}\}$

$x \in \{d \in \Delta^{\mathcal{I}} \mid$ there is $e \in \Delta^{\mathcal{I}}$, s.t. $(d,e) \in r^{\mathcal{I}}$ and $e \in C^{\mathcal{I}}\}$

$= (\exists r.C)^{\mathcal{I}}$

Therefore, $A^{\mathcal{I}} \subseteq (\exists r.C)^{\mathcal{I}}$, that is $\mathcal{I} \models A \sqsubseteq \exists r.C$. $\square$