Semantic Web and Ontology : Exercises

(A 2-page A4 format document is authorised)

Exercise n^o 1

Translate the following sentences into RDF Turtle syntax or DL axioms.

1. Mary is a woman.
2. Every mother is a woman.
3. Mary is John's wife.
4. Mothers are women who are also parents.
5. At least one child of a grandparent has also a child.

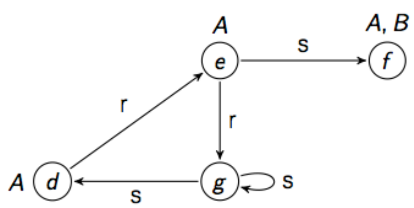
Exercise n^o 2

Give the negation norm forms (NNF) for the following three concepts :

$$\neg(\exists r.\neg(\forall s.\forall t.(A \sqcup (B \sqcap C)))), \neg(\neg(\forall r.(A \sqcup \exists.B))), \forall r.\neg(A \sqcap \exists r.\neg B \sqcap C).$$

Exercise n^o 3

Consider the following description logic interpretation I represented in the form of a graph :



Question 1. Write down the definition of $I = (\Delta^I, \cdot^I)$ corresponding to the graph.

Question 2. For each of the following $\mathcal{ALC}$ concepts C, list all the elements x of Δ^I such that $x \in C^I$:

1. $A \sqcap B$
2. $\neg(B \sqcup (\exists s.A))$
3. $\neg\exists r.(A \sqcap B)$

Exercise n^o 4

Using Tableau algorithm, check the satisfiability of the following concepts C with respect to the given TBoxes. If C is satisfiable, give the interpretation corresponding to your tableau construction.

1. TBox is $\emptyset$ and C is $(A \sqcup B \sqcup \forall R.B) \sqcap (\neg A \sqcap \neg B \sqcap \exists R.\neg B)$.
2. TBox is $\emptyset$ and C is $(\forall R.\forall S.\forall R.\exists S.A) \sqcap (\exists R.\exists S.\exists R.\forall S.\neg A)$.

Exercise n^o 5

1. Is the ontology $O = (TBox, ABox)$ consistent, where $TBox = \{A \sqsubseteq B, B \sqsubseteq \exists r.\top, \exists r.\top \sqsubseteq D, B \sqcap D \sqsubseteq \bot\}$ and $ABox = \emptyset$? If yes, please give a model of O ; Otherwise, justify your answer.
2. The same question as the previous one for $O' = O \cup \{A(a)\}$.
3. Which of the following subsumptions are always true ? Justify your answer by the semantics of $\mathcal{ALC}$.
   - $\neg(A \sqcap B) \sqsubseteq \neg A \sqcup \neg B$
   - $\forall s.A \sqcap \forall s.B \sqsubseteq \forall s.(A \sqcap B)$
   - $\exists s.(\neg A \sqcap B) \sqsubseteq \exists s.(\neg A) \sqcap \exists s.B$

Exercise n^o 6

Prove the following results :

- **Lemma 1. If C is unsatisfiable w.r.t. an ontology $\mathcal{O}$, and $\mathcal{O} \subseteq \mathcal{O}'$, then C is unsatisfiable w.r.t. $\mathcal{O}'$.**
- **Lemma 2. $C \sqsubseteq D$ if and only if the concept $C \sqcap \neg D$ is unsatisfiable.**
- $\forall r.A \sqcap \exists r.\neg A$ is unsatisfiable (w.r.t. any ontology)
  (Hint : you can prove the conclusion by semantics or using CSat algorithm together with Lemma 1.)
- $\exists r.(A \sqcap B)$ is subsumed by $\exists r.A$ (w.r.t. any ontology)
  (Hint : you can prove the conclusion by semantics, or using CSat algorithm to test the satisfiability of the concept $\exists r.(A \sqcap B) \sqcap \neg\exists r.A$ wrt. empty TBox and then using Lemma 1.)
- $\{A \sqsubseteq \exists r.B, B \sqsubseteq C\} \models A \sqsubseteq \exists r.C$ (Hint : since CSat algorithm we learned in the lecture is only for empty TBox, we need to prove this by semantics.)