Fast Dictionary Attacks on Passwords Using Time-Space Tradeoff

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Overview

Goal

Ultimate goal: given a cipher text c recover the password k such that $H(k) = c$

Time-Space Tradeoff

Full look up table		Brute-force attack
Time		**Time**
$O(1)$	?????	$O(\lvert\mathcal{K}\rvert)$
Space		**Space**
$O(\lvert\mathcal{K}\rvert)$		$O(1)$

Time-Space Tradeoff

In 1980 Martin Hellman answes this question by contributing a Time-Space tradeoff for the problem of cryptanalysis.

- Precompute: m “chains” of length t
- Store only the first and last items in each chain

$$\begin{array}{cc} & \overset{t}{\longrightarrow} \\ m\big\downarrow & \begin{bmatrix} k^1_{1,1} \xrightarrow{f_1} \xrightarrow{f_1} \cdots \xrightarrow{f_1} k^1_{1,t} \\ k^1_{m,1} \xrightarrow{f_1} \xrightarrow{f_1} \cdots \xrightarrow{f_1} k^1_{m,t} \end{bmatrix} \\ & \overset{t}{\longrightarrow} \\ m\big\downarrow & \begin{bmatrix} k^2_{1,1} \xrightarrow{f_2} \xrightarrow{f_2} \cdots \xrightarrow{f_2} k^2_{1,t} \\ k^2_{m,1} \xrightarrow{f_2} \xrightarrow{f_2} \cdots \xrightarrow{f_2} k^2_{m,t} \end{bmatrix} \\ & \vdots \qquad\qquad \vdots \\ & \overset{t}{\longrightarrow} \\ m\big\downarrow & \begin{bmatrix} k^{t-1}_{1,1} \xrightarrow{f_{t-1}} \xrightarrow{f_{t-1}} \cdots \xrightarrow{f_{t-1}} k^{t-1}_{1,t} \\ k^{t-1}_{m,1} \xrightarrow{f_{t-1}} \xrightarrow{f_{t-1}} \cdots \xrightarrow{f_{t-1}} k^{t-1}_{m,t} \end{bmatrix} \\ & \overset{t}{\longrightarrow} \\ m\big\downarrow & \begin{bmatrix} k^t_{1,1} \xrightarrow{f_t} \xrightarrow{f_t} \cdots \xrightarrow{f_t} k^t_{1,t} \\ k^t_{m,1} \xrightarrow{f_t} \xrightarrow{f_t} \cdots \xrightarrow{f_t} k^t_{m,t} \end{bmatrix} \end{array}$$

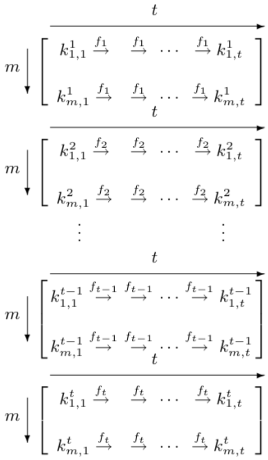
Chain Generation

The function $f(k)$ maps from one key to another. Where:

$$f(k) = R[H(k)]$$

where

- $H(k)$ is a hash function that maps from key k to ciphertext c
- $R(c)$ is a Reduction function mapping from a ciphertext c back to a value $k \in \mathcal{K}$
- Example reduction function: drop last n characters/bits



Chain Generation

Example (Chain Generation)

4-character passwords

6-character password sets

$$pass \rightarrow^{H} FE4gT6 \rightarrow^{R} ofie \rightarrow^{H}$$
$$FP03u2 \rightarrow^{R} ueyf \cdots \rightarrow^{R} lswq$$

$$\begin{array}{cc} & t \\ m\downarrow & \begin{bmatrix} k^1_{1,1} \xrightarrow{f_1} & \xrightarrow{f_1} & \cdots & \xrightarrow{f_1} k^1_{1,t} \\ k^1_{m,1} \xrightarrow{f_1} & \xrightarrow{f_1} & \cdots & \xrightarrow{f_1} k^1_{m,t} \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & t \\ m\downarrow & \begin{bmatrix} k^2_{1,1} \xrightarrow{f_2} & \xrightarrow{f_2} & \cdots & \xrightarrow{f_2} k^2_{1,t} \\ k^2_{m,1} \xrightarrow{f_2} & \xrightarrow{f_2} & \cdots & \xrightarrow{f_2} k^2_{m,t} \end{bmatrix} \end{array}$$

$$\vdots \qquad\qquad \vdots$$

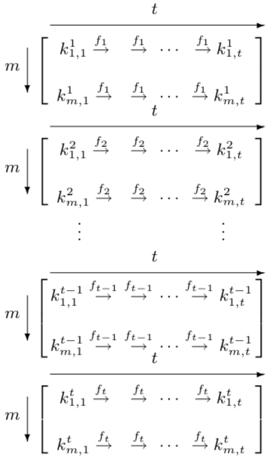
$$\begin{array}{cc} & t \\ m\downarrow & \begin{bmatrix} k^{t-1}_{1,1} \xrightarrow{f_{t-1}} & \xrightarrow{f_{t-1}} & \cdots & \xrightarrow{f_{t-1}} k^{t-1}_{1,t} \\ k^{t-1}_{m,1} \xrightarrow{f_{t-1}} & \xrightarrow{f_{t-1}} & \cdots & \xrightarrow{f_{t-1}} k^{t-1}_{m,t} \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & t \\ m\downarrow & \begin{bmatrix} k^t_{1,1} \xrightarrow{f_t} & \xrightarrow{f_t} & \cdots & \xrightarrow{f_t} k^t_{1,t} \\ k^t_{m,1} \xrightarrow{f_t} & \xrightarrow{f_t} & \cdots & \xrightarrow{f_t} k^t_{m,t} \end{bmatrix} \end{array}$$

Key Recovery

Given a cipher text c we need to recover the key k that generates the ciphertext

$$c = H(k)$$



Dictionary Attacks

Markov Chains

- Markov chains are commonly used in natural language processing. Most notably speech recognition systems.
- Markov models have been used to generate passwords for users.
- Given a set of states $S = \{s_1, s_2, \cdots, s_n\}$ there is some probability p_{ij} that denotes the probability of transitioning from state s_i to state s_j
- The probability of transitioning to the next state depends only on the current state

Markov Chains

The order of a Markov chain of order n is defined as:

$$P(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \cdots, X_1 = x_1) =$$
$$P(X_n = x_n | X_{n-1} = x_{n-1}, \cdots, X_{n-m} = x_{n-m})$$

Zero-order Markov Chain:

$$P(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \cdots, X_1 = x_1) =$$
$$P(X_n = x_n)$$

First-order Markov Chain:

$$P(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \cdots, X_1 = x_1) =$$
$$P(X_n = x_n | X_{n-1} = x_{n-1})$$

Zero-order Markov Model

In a zero-order Markov model, each character is generated given its underlying proability distribution. This is based on the frequency of the letter in the users natural language. Formally the zero-order model can be written as:

$$P(\alpha) = \prod_{x \in \alpha} \mathcal{V}(x)$$

where: where

- $P(\cdot)$ is the markovian proability distrubuion
- α is a string of characacters
- $\mathcal{V}(\cdot)$ is the frequency of a letter occuring in English
- x is an individual character

First-order Markov Model

In a First-order Markov model, each ordered pair is assigned a proability and each character is generated by looking at the pervious character. The first-order markov model can be written as:

$$P(x_1 x_2 x_3 \cdots x_n) = \mathcal{V}(x_1) \prod_{i=1}^{n-1} \mathcal{V}(x_{i+1}|x_i)$$

where: where

- $P(\cdot)$ is the markovian proability distrubuion
- x_i are individual characters
- $\mathcal{V}(\cdot)$ is the frequency of a letter or ordered pair occuring in English

Markov Dictionary

A probability distribution is not a dictionary. To create a dictionary, discretize the probabilities into two levels using a threshold θ

Zero-order dictionary

$$\mathcal{D}_{\mathcal{V},\theta} = \{\alpha : \prod_{x \in \alpha} \mathcal{V}(x) \geq \theta\}$$

First-order dictionary

$$\mathcal{D}_{\mathcal{V},\theta} = \{x_1 x_2 \cdots x_n : \mathcal{V}(x_1) \prod_{i=1}^{n-1} \mathcal{V}(x_{i+1} | x_i) \geq \theta\}$$

Markov Dictionary

The zero-order model is better for abrivtations and acrynyms. For example, a user picks their favorite song lyric and the first letter of each word creates their password.

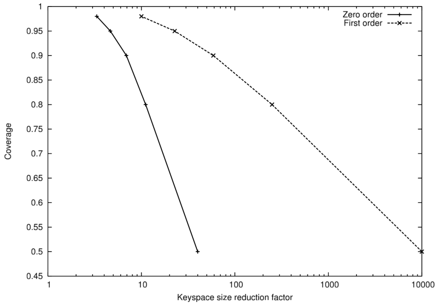


Figure: Convergence vs reduction in Keyspace size ($|\mathcal{K}|$) for 8-character sequences

Deterministic Finite Automaton

- A DFA or Deterministic Finite Automaton is a finite state machine that accepts or rejects a string
- A regular expression can be constructed from a DFA
- Humans are not random with how they use numerals and special characters
  - Numbers tend to be at the end of a password: password1
  - Capital letters are typically at the begining of a password: Password
  - there are typically more lowercase letters in passwords than uppercase letters, numerals, or special characters

Dictionary using a DFA

An improved dictionary is one where strings are both accepted by a Markovian filter and accepted by at least one DFA from some set of DFA's. The updated dictionary is defined as:

$$\mathcal{D}_{\mathcal{V},\theta,\langle M_i \rangle} = \{\alpha : \prod_{x \in \alpha} \mathcal{V}(x) \geq \theta, and \, \exists \, i : M_i accepts \alpha\}$$

where

- A is the set of 26 uppercase characters
- a is the set of 26 lowercase characters
- n is the set if 10 numerals
- s is the set of 5 special characters $\{space, hyphen, underscore, period, comma\}$

Indexing Algorithms

- The goal is to create an algorithm that will efficiently enumerate the passwords in a given password space. Given i as input return the i^{th}
- In the rainbow attack the reduction function maps from ciphertext space to $\{0, 1, \cdots, |\mathcal{K} - 1|\}$
- Composed with a mapping from $\{0, 1, \cdots, |\mathcal{K} - 1|\}$ to a key in $\mathcal{K}$.
- Makes no assumption about keyspace other than its size
- Use the rainbow attack with a "smart" way to choose the keyspace

Dictionary Modification

Modify the dictionary to only consider fixed length strings. This allows for different threshold values θ for each length.

$$\mathcal{D}_{\mathcal{V},\theta,\ell} = \{\alpha : |\alpha| = \ell and \prod_{x \in \alpha} \mathcal{V}(x) \geq \theta\}$$

Discretization

The algorithm also needs to discretize the probability distribution of the strings. First, turn the dictionary into a sum rather than a product.

Transform the product

$$\prod_{x \in \alpha} \mathcal{V}(x) \geq \theta$$

$$\log(\prod_{x \in \alpha} \mathcal{V}(x)) \geq \log(\theta)$$

$$\log(\mathcal{V}(x_1)\mathcal{V}(x_2) \cdots \mathcal{V}(x_n)) \geq \log(\theta)$$

$$\log(\mathcal{V}(x_1)) + \log(\mathcal{V}(x_2)) + \cdots + log(\mathcal{V}(x_n)) \geq \log(\theta)$$

Discretization

To arrive at a discrete version of the modified dictionary:

$$\mathcal{D}_{\mathcal{V},\theta,\ell} = \{\alpha : |\alpha| = \ell \mathit{and} \sum_{x\in\alpha} \mu(x) \geq \lambda\}$$

Where where

- $\mu(x) = \log(\mathcal{V}(x))$
- $\lambda = \log(\theta)$

Discretization

- $\mu(x) = \log(\mathcal{V}(x))$
- Discretize the values of the μ function to the nearest multiple of some μ_0
- Narayanan and Shmatikov use a μ_0 that yields approximately 1000 different discrete values

Zero-Order Markovian Dictionary

First-Order Markovian Dictionary

DFA Dictionary

Any Keyspace $\mathcal{K}$

Hybrid Markovian/DFA

Multiple Keyspaces

Optimizations

Experiment

- Measure coverage of rainbow attack vs hybrid attack
- 142 real user passwords
- 6-Character alphanumeric sequences for the rainbow attack ($|\mathcal{K}| = 36^6 \approx 2 * 10^9$)
- 70 regular expressions

Results

Category	Count	Rainbow	Hybrid
Length at most 5	63	29	63
Length 6	21	10	17
Length 7	18	0	10
Length 8, A* or a*	9	0	6
Others	31	0	0
Total	142	39(27.5%)	96(67.6%)
only length ≥ 6	79	10(12.7%)	33(41.8%)

Figure: Passwords recovered in Hybrid attack vs. Rainbow attack

Conclusions

- One of many attacks targeting human weakness
- Some possible defences against dictionary attacks for human memorable passwords
  - Graphical passwords
  - Biometric information
- Are these actually safer?

Analysis