Monte Carlo simulations for the unitary Bose gas

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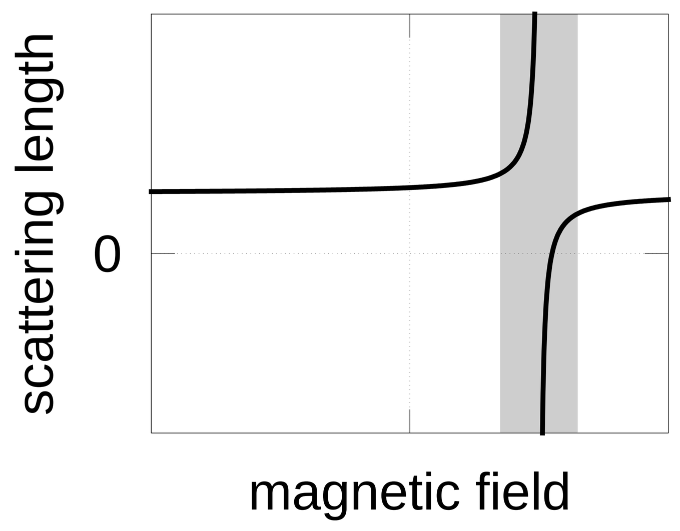
Abstract

Ultracold atomic gases are used to explore quantum phenomena in controlled and tunable set-ups, often simpler than their solid-state counterpart. An ongoing experimental and theoretical effort concentrates on the understanding of strongly-interacting systems of fermionic and bosonic atoms, including the unitary Bose gas. We study a model of this system, which displays the usual normal-gas and superfluid-gas phases and an additional liquid phase.

Physical system: unitary Bose gas

Ultracold atomic gases

- Controllable quantum systems.
- Scattering length: single parameter for low-energy interactions.
- Feshbach resonance: tune interactions via a magnetic field.



Efimov effect and unitary Bose gas

- Large scattering length $\to$ universal behavior.
- Efimov effect (nuclear physics): three-body bound states with no bound pair.
- Kraemer *et al.* (2006): Efimov signature in unitary Bose gas.
- Experimental challenges: three-atoms recombinations, particle losses. Makotyn *et al.* (2014): *metastable* unitary Bose gas.

Theoretical model

- Pair interactions: zero-range limit, with infinite scattering length.
- Three-body hard-core repulsion

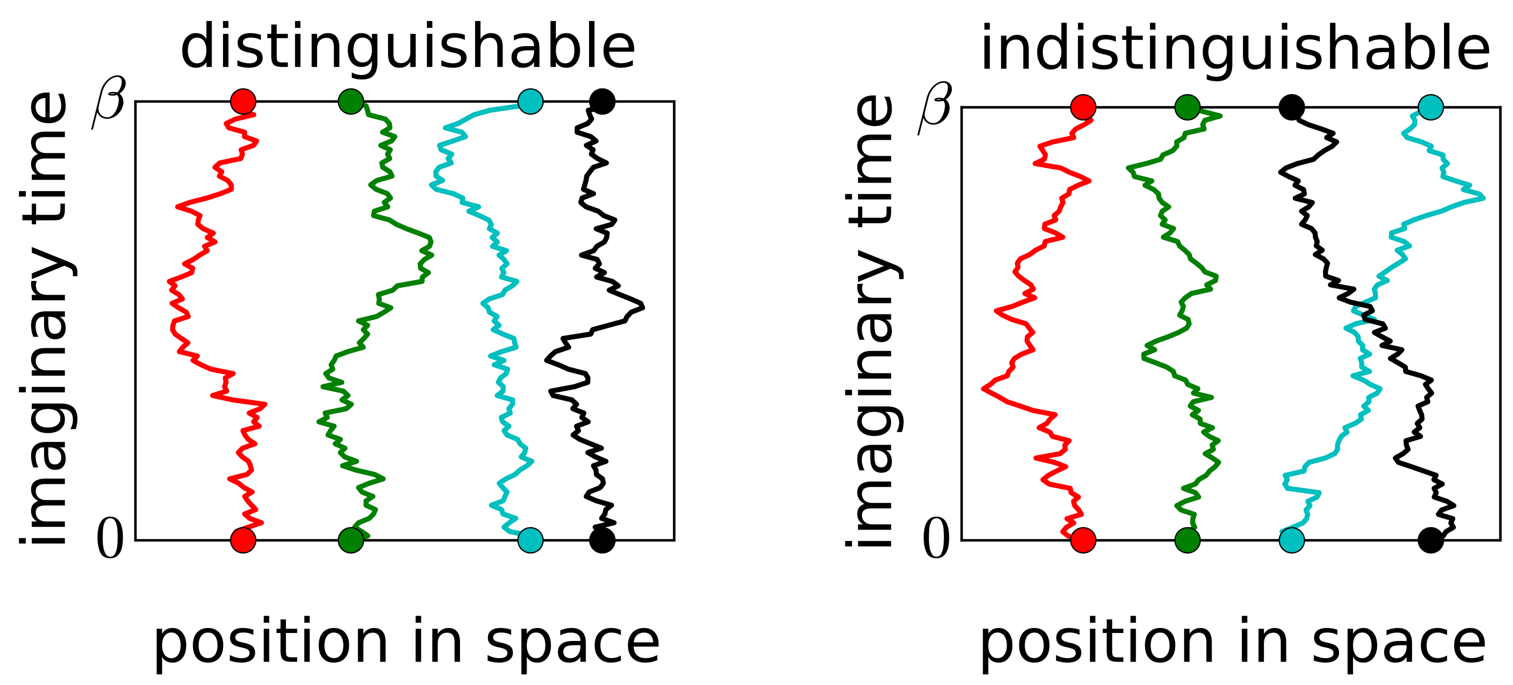
$$V_3(R_{ijk}) = \begin{cases} \infty & \text{if } R_{ijk} < R_0 \\ 0 & \text{if } R_{ijk} > R_0 \end{cases},$$

with $R_{ijk}^2 = (r_{ij}^2 + r_{jk}^2 + r_{ki}^2)/3$, for any triplet (i, j, k).

Method: path-integral Monte Carlo

Path-integral representation

- Quantum particle $\leftrightarrow$ polymer (size $\lambda \sim 1/\sqrt{T}$).
- N indistinguishable particles $\leftrightarrow$ N polymers + permutations.



- $\lambda \ll$ typical interparticle distance $\to$ normal gas.
- $\lambda \gg$ typical interparticle distance $\to$ quantum gas, long permutation cycles, superfluid phase (bosons).

Monte Carlo sampling

- Probability $P(X)$ for configuration X.
- Algorithm $\leftrightarrow$ rules to produce a sequence of configurations distributed according to $P(X)$.
- Measurement of an observable $\leftrightarrow$ average along the sequence.

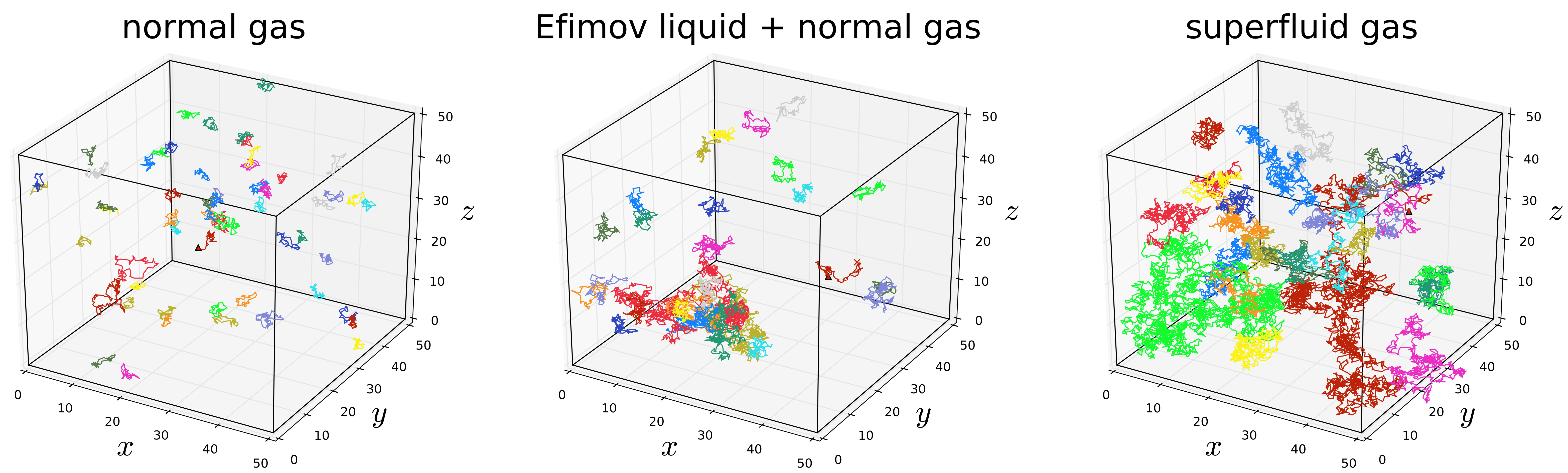
Phases and phase transitions

Small R_0: normal gas $\leftrightarrow$ Efimov liquid

- Von Stecher (2010): clusters as ground state of $N \lesssim 15$ unitary bosons.
- Piatecki and Krauth (2014): Efimov liquid of trapped unitary Bose gas at finite temperature.
- Homogeneous system + NVT ensemble $\to$ phase separation.
- High temperature: normal-gas.
  Low temperature: normal gas + liquid droplet.

Large R_0: normal gas $\leftrightarrow$ superfluid gas

- Homogeneous box + periodic boundary conditions: superfluid fraction related to winding number.
- Small systems: smooth transition between zero/finite superfluid fraction.
- Finite-size-scaling to identify the critical temperature.
- Superfluid gas $\neq$ Efimov liquid: different densities, continuous/discontinuous transition to normal gas.



References

- T. Kraemer *et al.*, Nature **440** (2006), 315.
- D. Makotyn *et al.*, Nat. Phys. **10** (2014), 116.
- J. von Stecher, J. Phys. B: At. Mol. Opt. Phys. **43** (2010), 101002.
- S. Piatecki and W. Krauth, Nat. Commun. **5** (2014), 3503.