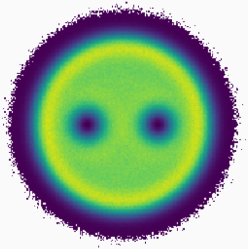
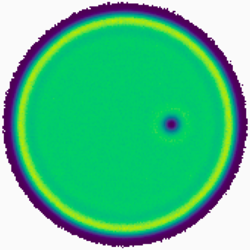
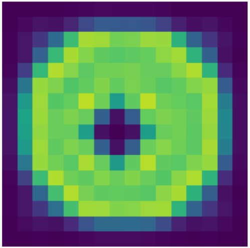
A scheme to detect anyons in cold atoms

Tommaso Comparin

January 27th, 2020 – GdT MaCon

PHYSICAL REVIEW LETTERS **120,** 230403 (2018)

Time-of-Flight Measurements as a Possible Method to Observe Anyonic Statistics

R. O. Umucalılar,[1] E. Macaluso,[2] T. Comparin,[2] and I. Carusotto[2]

PHYSICAL REVIEW LETTERS **123,** 266801 (2019)

Fusion Channels of Non-Abelian Anyons from Angular-Momentum and Density-Profile Measurements

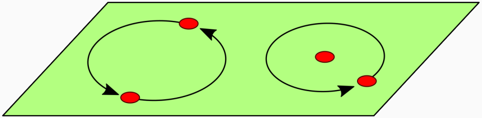
E. Macaluso,[1] T. Comparin,[1] L. Mazza,[2] and I. Carusotto[1]

Charge and statistics of lattice quasiholes from density measurements: a Tree Tensor Network study

E. Macaluso,[1] T. Comparin,[1,2] R. O. Umucalılar,[3] M. Gerster,[4] S. Montangero,[5] M. Rizzi,[6,7] and I. Carusotto[1]

Anyons?

Exchange and braiding of quantum particles



Exchange (left) and braiding (right)

Two indistinguishable particles in two dimensions:

- $\Psi(\mathbf{r}_2, \mathbf{r}_1) = +\Psi(\mathbf{r}_1, \mathbf{r}_2) \longleftrightarrow$ bosons
- $\Psi(\mathbf{r}_2, \mathbf{r}_1) = -\Psi(\mathbf{r}_1, \mathbf{r}_2) \longleftrightarrow$ fermions
- $\Psi(\mathbf{r}_2, \mathbf{r}_1) = e^{i\varphi_{\mathrm{st}}}\Psi(\mathbf{r}_1, \mathbf{r}_2) \longleftrightarrow$ anyons, with $\varphi_{\mathrm{st}} \in [0, 2\pi)$

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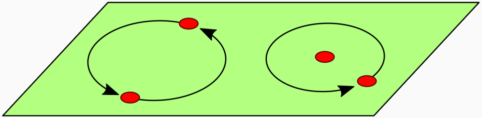

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Statistical phase $\varphi_{\rm st}$ (0 for bosons, π for fermions).

Braiding phase $\varphi_{\rm br} = 2\varphi_{\rm st}$ (two exchanges + translations).

Anyons in condensed matter

Quantum Hall states:

- 2D electrons in transverse magnetic field (25-100 mK, 10 T).
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- 2D electrons in transverse magnetic field (25-100 mK, 10 T).
- Topological properties (quantization of resistivity, chiral edge modes, Chern number, robustness against disorder..).
- They should host anyonic quasiparticle excitations[1].
- How to detect anyonic statistics? Mainly interference schemes (see talk by Aurélien Grabsch). No unambiguous results yet.

[1] The system is made of fermions, the anyons are the excitations.

Artificial quantum Hall states

- On-going work on engineered systems of ultracold atomic gases[1], superconducting qubits[2], photons[3].
- Common features: "clean&tunable", several accessible observables (notably density profiles).

[1] Bloch, "Quantum simulations come of age", Nat. Phys. 14, 1159 (2018).
[2] Roushan *et al.*, "Chiral ground-state currents of interacting photons in a synthetic magnetic field", Nat. Phys. 13, 146 (2017).
[3] Clark *et al.*, "Observation of Laughlin states made of light", arXiv:1907.05872.

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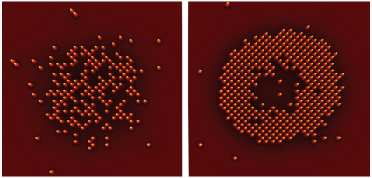
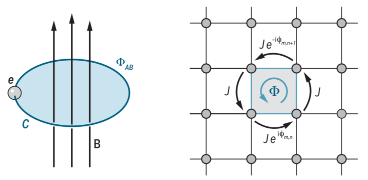
- Common features: "clean&tunable", several accessible observables (notably density profiles).
- Examples: quantum gas microscope and synthetic gauge fields.





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- Generation of anyons is under control (in theory).
- Early works on detection: we need adiabatic time evolution.
- This work: we only need static density-profile measurements.

Laughlin state &
Braiding through rotations

Laughlin state

$$\Psi_{\mathrm{L}}(\{z_j\}) \propto \prod_{j=1}^{N} e^{-|z_j|^2/4l_{\mathrm{B}}^2} \prod_{k<j}(z_j - z_k)^M$$

- Ansatz for N particles in 2D (coordinates: $z_k = x_k + iy_k$).
- Defined in lowest Landau level (transverse magnetic field B, magnetic length $l_{\mathrm{B}} \propto 1/\sqrt{B}$).
- Quantum Hall state at magnetic filling $\nu = N/N_\phi = 1/M$.

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- M odd: approximate ground state for 2D electrons.
- $M = 2$: exact ground state for 2D bosons with short-range repulsion (filling $\nu = 1/2$).

Quasihole Laughlin state

$$\Psi_{\mathrm{QH}}(\{z_j\};\{\eta_\mu\}) \propto \Psi_{\mathrm{L}}(\{z_j\}) \times \prod_{j=1}^{N}\prod_{\mu}(z_j-\eta_\mu)$$

- Quasiholes[4]: excitations localized at $z = \eta_1, \eta_2, \ldots$
- Fractional charge $(1/M)$.
- Anyonic statistics $(\varphi_{\mathrm{br}} = 2\pi/M)$.

[4] Today: Quasiholes = Anyons.

Quasihole Laughlin state

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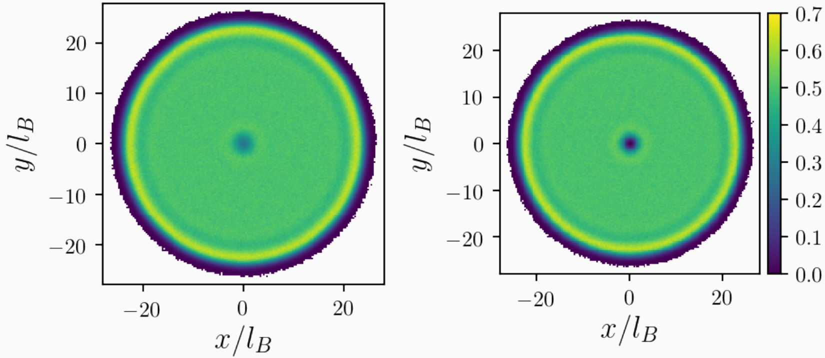
How to stabilize quasiholes? Ψ_{QH} becomes the ground state in the presence of (external) repulsive potentials at $\eta_1, \eta_2, \ldots$

[4] Today: Quasiholes = Anyons.

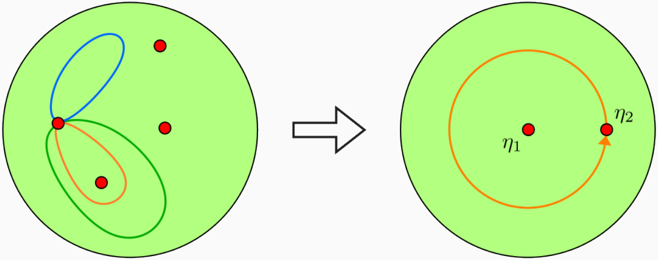
How do these states look like?



Density profile $n(z)$ of Ψ_{QH}, for $N = 150$ bosons ($M = 2$).

Left: one quasihole at $\eta_1 = 0$.
Right: two quasiholes at $\eta_1 = \eta_2 = 0$.

Braiding through rigid rotations – 1/2



- Braiding via quasihole dynamics: $\eta_\mu = \eta_\mu(t)$ for $0 \leq t \leq T$ (adiabaticity $\rightarrow$ large T).
- Replace braiding with rigid rotations of anyons.

Braiding through rigid rotations – 2/2

- Time-dependent wave function:

$$\Psi_t(\{z_j\}) \equiv \Psi(\{z_j\};\{\eta_\mu(t)\})$$

- In general[5], at final time T:

$$\Psi_T(\{z_j\}) = e^{i\alpha}\Psi_0(\{z_j\})$$

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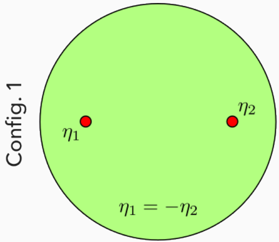
- For rotations:

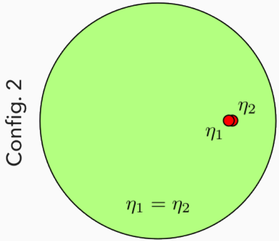
$$\alpha = \frac{2\pi}{\hbar}\langle\Psi_0|\hat{L}_z|\Psi_0\rangle + (\text{dynamical phase})$$

Important: only static ($t = 0$) observables!

[5] Assume: closed path of $\eta_\mu(t)$, Abelian anyons.

Extracting braiding phase





- Phase α has topological and non-topological contributions.
- Remove non-topological part by subtracting a non-braiding case.

$$\boxed{\varphi_{\mathrm{br}} = \frac{2\pi}{\hbar}\left[\langle \hat{L}_z \rangle_{\eta_2=-\eta_1} - \langle \hat{L}_z \rangle_{\eta_2=\eta_1}\right]}$$

Connection with the density profile

For states in the lowest Landau level, angular momentum corresponds to average squared radius[6],

$$\frac{\langle \hat{L}_z \rangle}{N\hbar} = \frac{\langle r^2 \rangle}{2 l_{\mathrm{B}}^2} - 1,$$

and then to density profile:

$$\langle r^2 \rangle \propto \int_{\mathbb{C}} dz \, n(z) \, |z|^2 .$$

[6] Ho&Mueller, "Rotating Spin-1 Bose Clusters", Phys. Rev. Lett. 89, 050401 (2002).

From global to local observables

Last braiding phase expression:

$$\varphi_{\mathrm{br}} = \frac{N\pi}{\hbar l_{\mathrm{B}}^2} \left[\langle r^2 \rangle_{\eta_2 = -\eta_1} - \langle r^2 \rangle_{\eta_2 = \eta_1} \right]$$

$\rightarrow$ highly sensitive to noise (statistical, in η_μ, thermal).

But $\eta_1 = \pm\eta_2$ profiles only differ in small regions..

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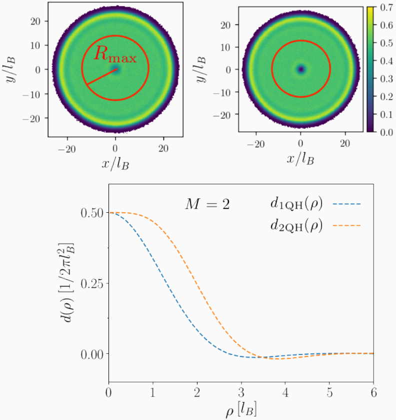
After some algebra:

$$\boxed{\varphi_{\mathrm{br}} = \frac{\pi}{l_{\mathrm{B}}^2} \int_{|\rho|<R_{\max}} d\rho\, |\rho|^2 \left[d_{2\mathrm{QH}}(\rho) - 2d_{1\mathrm{QH}}(\rho) \right]}$$

[$d_{k\mathrm{QH}}(\rho) = n_{\mathrm{bulk}} - n_{k\mathrm{QH}}(\eta+\rho)$: density depletion for k quasiholes at η]

$\Rightarrow$ We only need the depletion profiles for one/two quasiholes!

Example of results



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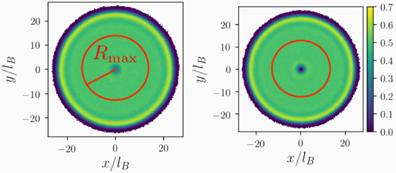
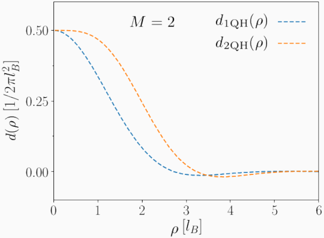
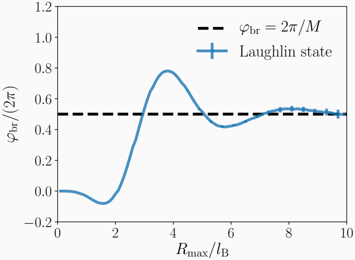






It works!

(i.e. we can measure φ_{br})

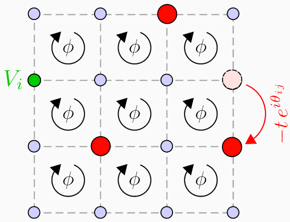
This was all for a specific Ansatz (Laughlin state).

Does it hold for the ground state of an interesting Hamiltonian?

Harper-Hofstadter model

Harper-Hofstadter model for lattice bosons

$$H = -t \sum_{\langle i,j \rangle} e^{i\theta_{ij}} \hat{a}_i^\dagger \hat{a}_j + \sum_i V_i \hat{n}_i + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$$



- N bosons on a $L \times L$ lattice.
- Complex nearest-neighbor hopping $\leftrightarrow$ magnetic flux ϕ per plaquette.
- External potentials.
- On-site repulsion: $U = \infty$.

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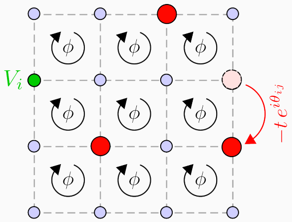

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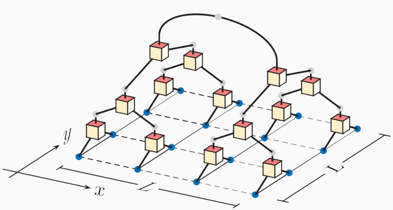
Realistic model for artificial quantum systems.

Ground state: superfluid, charge-density wave, Laughlin ($M = 2$).

How to find the ground state?

- Tree Tensor Network Ansatz[7].
- Accuracy controlled via bond dimension D.
- Only works for small N.



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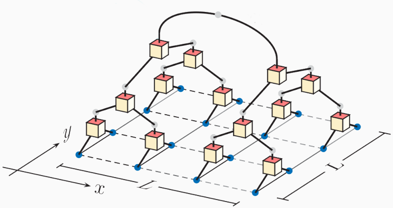
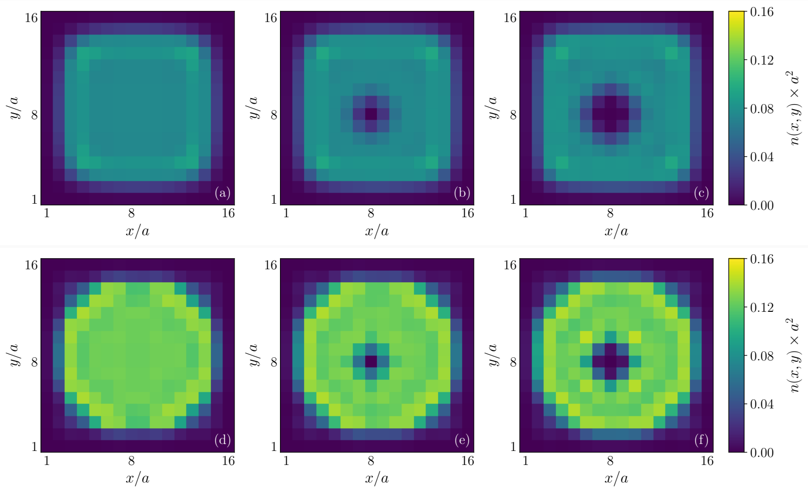


Two finite-size cases ($N = 12, 18$), with D up to 500.

V_i potentials: harmonic trapping and localized repulsive potentials.

[1] Gerster *et al.*, "Fractional quantum Hall effect in the interacting Hofstadter model via tensor networks", Phys. Rev. B 96, 195123 (2017).

Density profiles $n(x, y)$



Rows: cases 1 and 2. Columns: 0/1/2 quasiholes.

Braiding phase of quasiholes

- Discretize expression for φ_{br} through depletions.
- Warning: $\langle \hat{L}_z \rangle$ is not well defined!

[8] Details in [Macaluso *et al.*, Phys. Rev. Research (in press), arXiv 1910.05222].

Braiding phase of quasiholes

- Discretize expression for φ_{br} through depletions.
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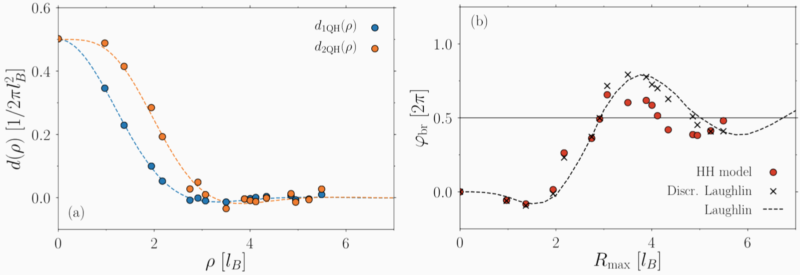
- Depletion curves are similar to discretized Laughlin.
- φ_{br} detection scheme “works”[8] (caveat: finite size effects!).



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Detecting anyons in continuum

- Braiding phase linked to density profile (via $\langle \hat{L}_z \rangle$ and $\langle r^2 \rangle$).
- Abelian quasiholes of Laughlin state[9].
- Not shown: Non-Abelian quasiholes of Moore-Read state[10].

[9] Umucalilar *el al.*, Phys. Rev. Lett. 120, 230403 (2018).
[10] Macaluso *el al.*, Phys. Rev. Lett. 123, 266801 (2019).
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Harper-Hofstadter lattice model[11]

- Accurate ground states via Tree Tensor Networks.
- We can stabilize single/double quasiholes.
- $\varphi_{\rm br}$ expression: not justified but numerically (almost) valid.

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[9] Umucalilar *el al.*, Phys. Rev. Lett. 120, 230403 (2018).
[10] Macaluso *el al.*, Phys. Rev. Lett. 123, 266801 (2019).
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