MODELING GROWTH AND DISTURBANCE WITH DYNAMIC MODELS

WAYS TO USE DYNAMIC MODELS

Goal: To provide insight into environmental systems, and how they respond to intervention/disturbance/change

- predict how something we care above will change over space and/or time (*system evolution*)
- sensitivity analysis of output (*output generated by solving differential equation to see system evolution in time or space*)
  - output response to variation in inputs/parameters
  - quantifying how summary measures respond to variation in inputs/parameters

DISTURBANCE

- Sometimes we are interested in how dynamics of a system (through time or space) are influenced by a force of change
  - Press - continuous through time; slowly increasing or decreasing
  - Pulse - intervals/one-time - could be a shock to the system

Examples?

MODELLING DISTURBANCE

Disturbance is **exogenous**, (modelled outside of the model and then input)
Disturbance is integrated into the model **endogenous**

QUESTIONS TO ASK

Exogenous or Endogenous implementation

- do you care about feedbacks between the system state and disturbance frequency or severity
  - fire is a disturbance that influences vegetation; but vegetation influences likelihood of future fire (YES)
  - high intensity precipitation (flood events) might be a disturbance; but streamflow response probably doesn't impact likelihood of future high intensity precipitation (NO)
- do you have a model of the disturbance
  - could you integrate this model into your existing dynamic model

Exogenous is easier to implement; and may allow you to use output from another model even if there are feedbacks it might still be useful to see what happens after a disturbance
Endogenous allows for a more complete integration of the disturbance

MODELLING DISTURBANCE - AN EXAMPLE

What would a model of forest growth look like if we harvest a forest on a regular basis
Some possible options

1. Disturbance is **exogenous** (outside of the model)
   - look at post-harvest recovery
2. Disturbance is integrated into the model **endogenous**
   - a fixed harvest every year
   - proportional harvest (% of current stock)

We can build this on a forest growth model

LETS CONSIDER FOREST GROWTH USING OUR SIMPLE GROWTH MODEL

Modeling carbon growth in a forest

- r is growth rate
- K is maximum carbon capacity of the forest

We can actually re-use our model - if we assume forest growth follows a logistic growth curve (e.g exponential growth towards a carrying capacity)
Let’s consider harvesting as **exogenous** first - simply change the starting condition

EXOGENEOUS MODEL

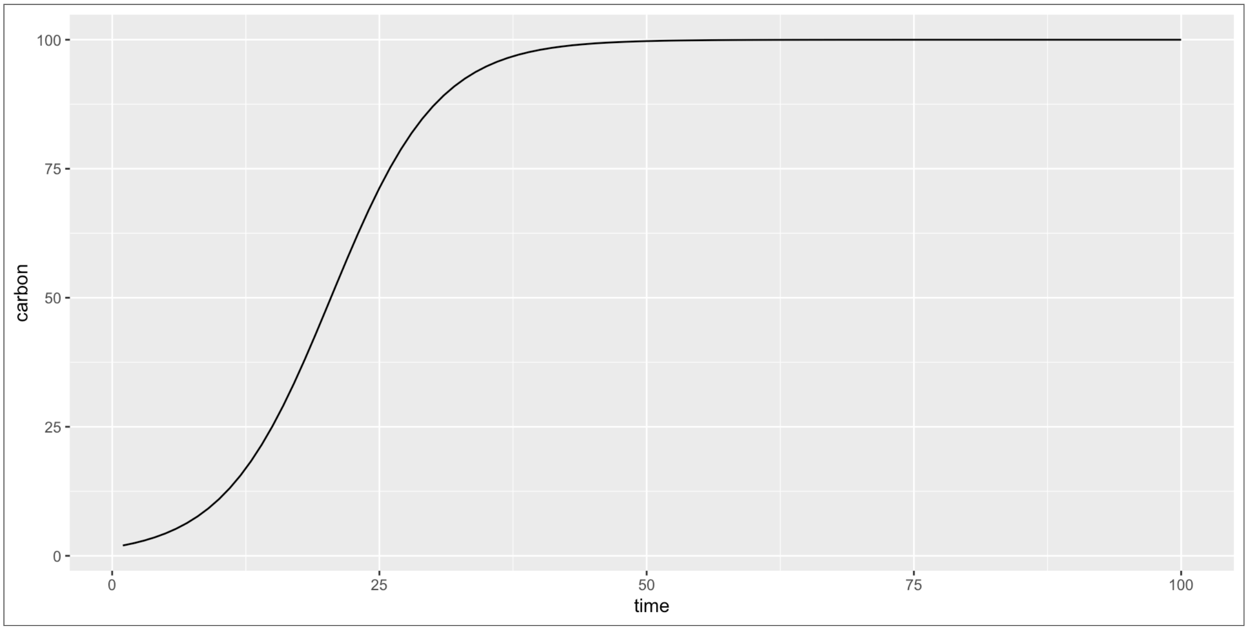
```r
source(here("R/dexppopK.R"))

dexppopK
```

```
function (time, P, parms)
{
    dexpop <- parms$r * P * (1 - P/parms$K)
    return(list(dexpop))
}
```

```r
# set parameters
forestparms <- list(K = 100, r = 0.2)

# create a time sequence
tm <- seq(from = 1, to = 100)

# start a small forest
postfire_initial_carbon <- 2

# watch it grow
forest <- ode(y = postfire_initial_carbon, times = tm, dexppopK, pa
```

EXOGENEOUS MODEL - PLOT

```
colnames(forest) <- c("time", "carbon")
ggplot(as.data.frame(forest), aes(time, carbon)) +
  geom_line()
```



ENDOGENOUS EXAMPLE - REGULAR HARVEST

Lets consider a regular harvesting of the forest -annual proportional harvest
How would you change the differential equation to implement this - try it!

- modify *dexppopK*
- rerun starting with mature conditions (30kgC)
- harvest 10% of the forest every year
- run the model for 500 years
- try changing the harvesting rate to see what happens

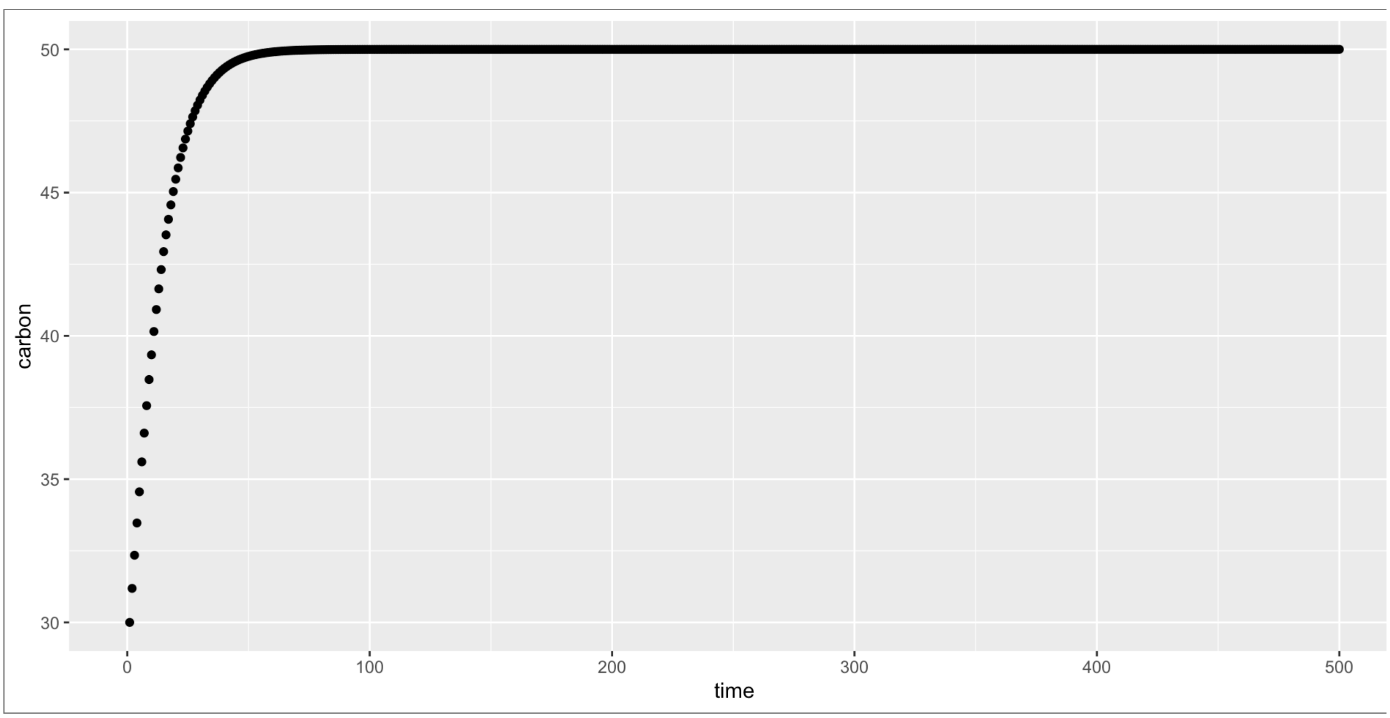
MY ANSWER

We can include harvesting in our forest with a new parameter **harv** (proportional harvesting rate))
I also changed the variable names to be more meaningful

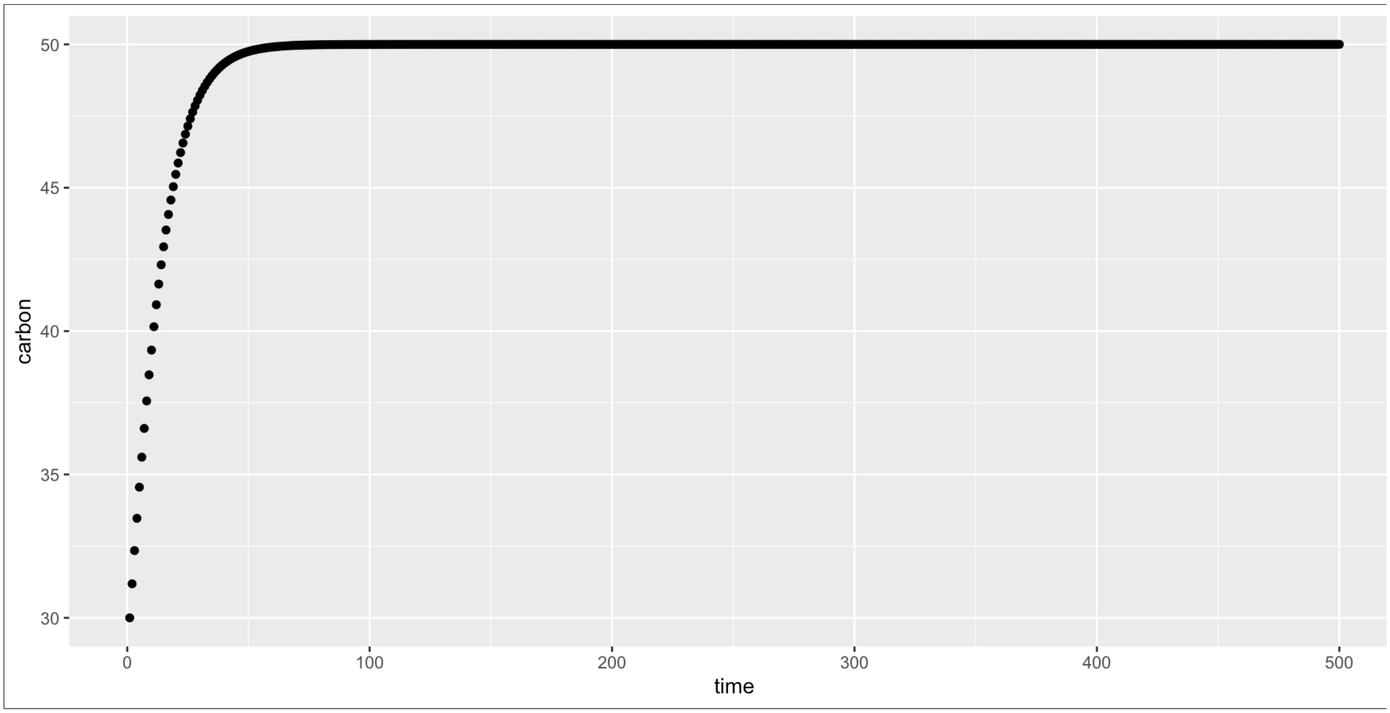
```
# fixed amount per year
source(here("R/dharvest.R"))
dharvest
```

```
function (Time, biomass, parms)
{
    db <- parms$r * biomass * (1 - biomass/parms$K) - parms$harv *
        biomass
    return(list(db))
}
```

```
# try it out
tm <- seq(from = 1, to = 500)
Cinitial <- 30
gps <- list(harv = 0.1, K = 100, r = 0.2)
res <- ode(Cinitial, tm, dharvest, gps)
colnames(res) <- c("time", "carbon")

ggplot(as.data.frame(res), aes(time, carbon)) +
  geom_point()
```



WHAT DOES THIS LOOK LIKE



HOW DOES HARVESTING RATE EFFECT CARBON

Compute trajectories of forest biomass (or carbon) over time - given different harvesting rates

- create a vector of different harvesting rates that we want to try
- write a wrapper function to run our model and extract trajectories
- run the wrapper function for our harvesting rates
- graph

MY CODE

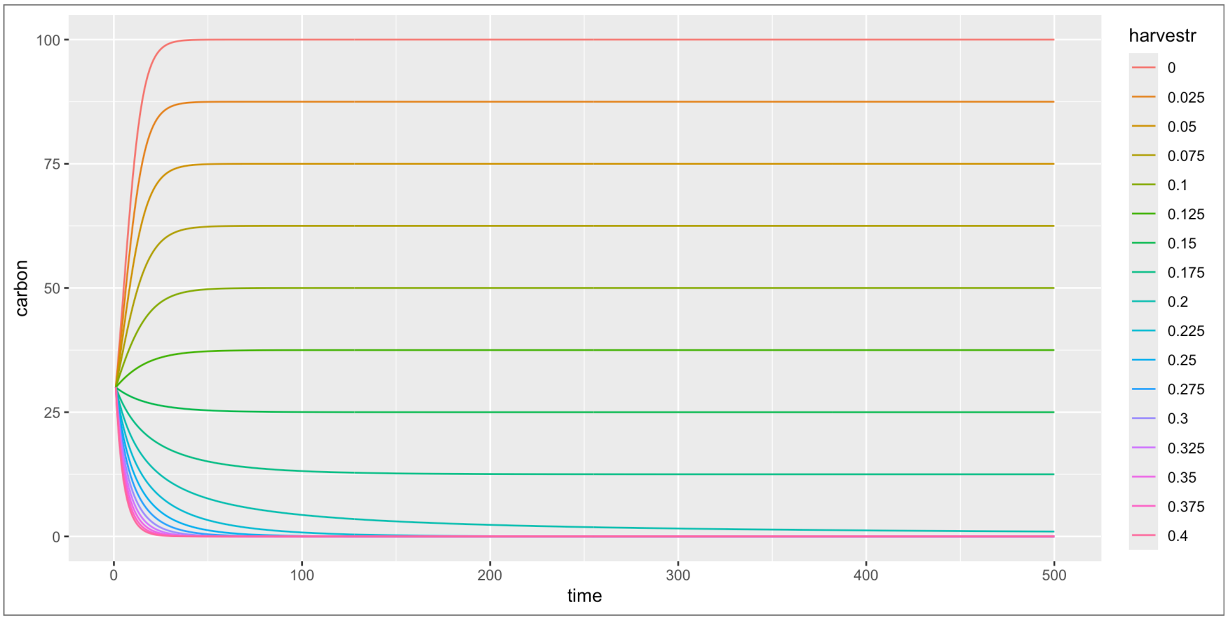
```
# what if we harvest a a much greater rates
# lets vary the harvest rates from 0.0 to 0.4
harvestr <- seq(from = 0.0, to = 0.4, by = 0.025)

# save all of the trajectories
# use a wrapper function to just return the carbon trajectories fro
getcarbon <- function(Cinitial, tm, harv, K, r, hfunc) {
  gps <- list(harv = harv, K = K, r = r)
  res <- ode(Cinitial, tm, hfunc, gps)
  colnames(res) <- c("time", "carbon")
  res <- as.data.frame(res)
  return(carbon = res$carbon)
}

# apply this function to all harvest values
res <- harvestr %>% map_dfc(~ getcarbon(Cinitial = Cinitial, tm = t
# rows are time, columns are carbon for each harvest scenario
colnames(res) <- harvestr
res <- as.data.frame(res)
```

```
         0    0.025     0.05    0.075      0.1    0.125     0.15
0.175
1 30.00000 30.00000 30.00000 30.00000 30.00000 30.00000 30.00000
30.00000
2 34.35985 33.53827 32.73592 31.95234 31.18711 30.43991 29.71018
28.99766
3 39.00030 37.22369 35.52436 33.89924 32.34530 30.85973 29.43969
28.08257
4 43.84888 41.00637 38.33489 35.82562 33.46995 31.25972 29.18691
27.24404
5 48.81781 44.83069 41.13608 37.71711 34.55716 31.64015 28.95045
26.47307
6 53.81022 48.63856 43.89691 39.56035 35.60360 32.00148 28.72905
25.76204
       0.2    0.225     0.25    0.275      0.3    0.325     0.35
0.375
```

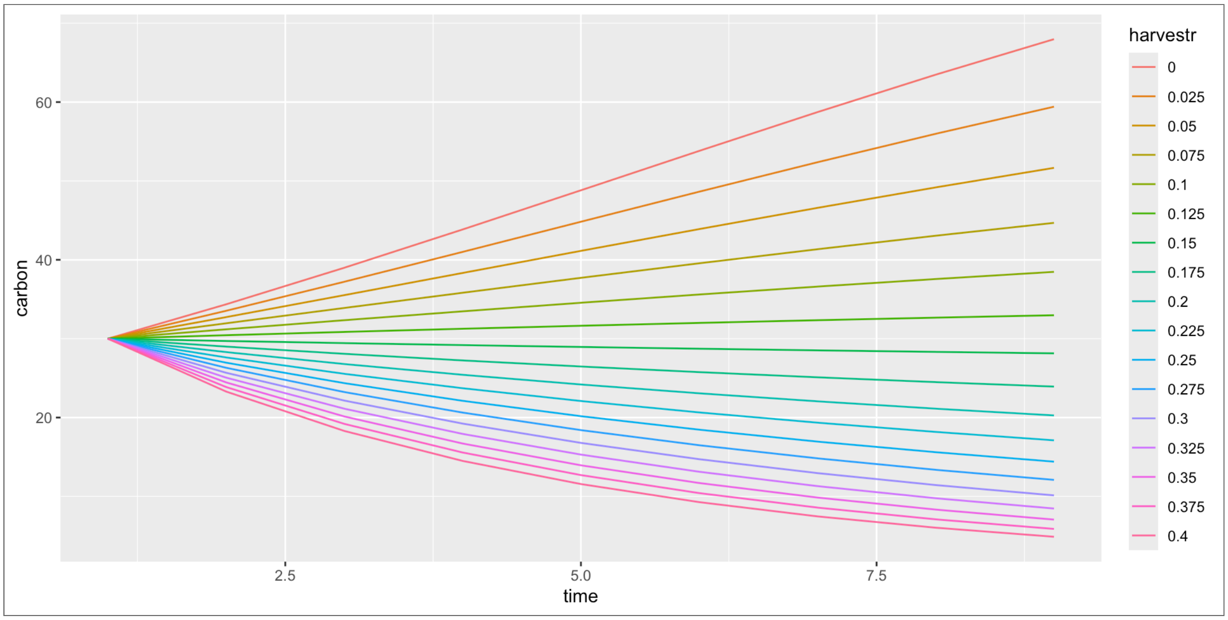
PLOT THE RESULTS

- Notice that stable forest carbon changes with harvest rates
- Some forests are not stable - (or forest size goes to zero)

```
# put in to a form where we can plot
resl <- gather(res, key = "harvestr", value = "carbon", -time)
ggplot(resl, aes(time, carbon, col = harvestr)) +
  geom_line()
```




```
# notice that stable forest value changes with harvest rates

# notes that some forests are not stable - (or forest size goes to

# see this at the beginning - plot the first 10 years
ggplot(subset(resl, time < 10), aes(time, carbon, col = harvestr))
  geom_line()
```


harvestr
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
0.275
0.3
0.325
0.35
0.375
0.4
carbon
60
40
20
time
2.5
5.0
7.5

PARAMETER INTERACTIONS

How do different harvest rates interact with different growth rates (and/or) carrying capacity?
Common "type" of question - how does disturbance interact with the properties (parameters) of the system?
Explore on your own
Try different combination of growth and harvest rates…

- Notice how carbon growth changes…
- What does stability mean -
- Can you find parameter sets that lead to a stable non-zero forest

SENSITIVITY ANALYSIS

We could do some sensitivity analysis to see how harvest rate, and growth rate interact
Sensitivity of what? What do we care about?

- where forest ends up after 10 years and 50 years (short and long planning horizons)

We will design a function to compute metrics and a wrapper to make this easy

```
# fixed amount per year
source(here("R/dharvest.R"))

# compute our two metrics of interest - harvest after 10 and 50 yea
# make it general in case we want to use other times
compute_short_long_C <- function(carbontime, short = 10, long = 50)
  Cshort <- carbontime %>%
    subset(time == short) %>%
    select(C)
  Clong <- carbontime %>%
    subset(time == long) %>%
    select(C)
  # use as numeric to clean up
  return(list(Cshort = as.numeric(Cshort), Clong = as.numeric(Clong
}

# now lets get our sample parameters
# lets assume a uniform distribution of harvest rates
# and of normal growth rates
```

```
             r      harv
[1,] 0.2682309 0.2635548
[2,] 0.3185244 0.3731608
[3,] 0.3492284 0.2330287
[4,] 0.3145324 0.2483109
[5,] 0.2511035 0.1232881
[6,] 0.2905072 0.1270780
```

```
# do a quick test
# try it out
tm <- seq(from = 1, to = 50)
Cinitial <- 30
parms <- list(r = sens_forest$X[1, 1], harv = sens_forest$X[1, 2],

res <- ode(y = Cinitial, times = tm, func = dharvest, parms = parms
res <- as.data.frame(res)
colnames(res) <- c("time", "C")

compute_short_long_C(res)
```

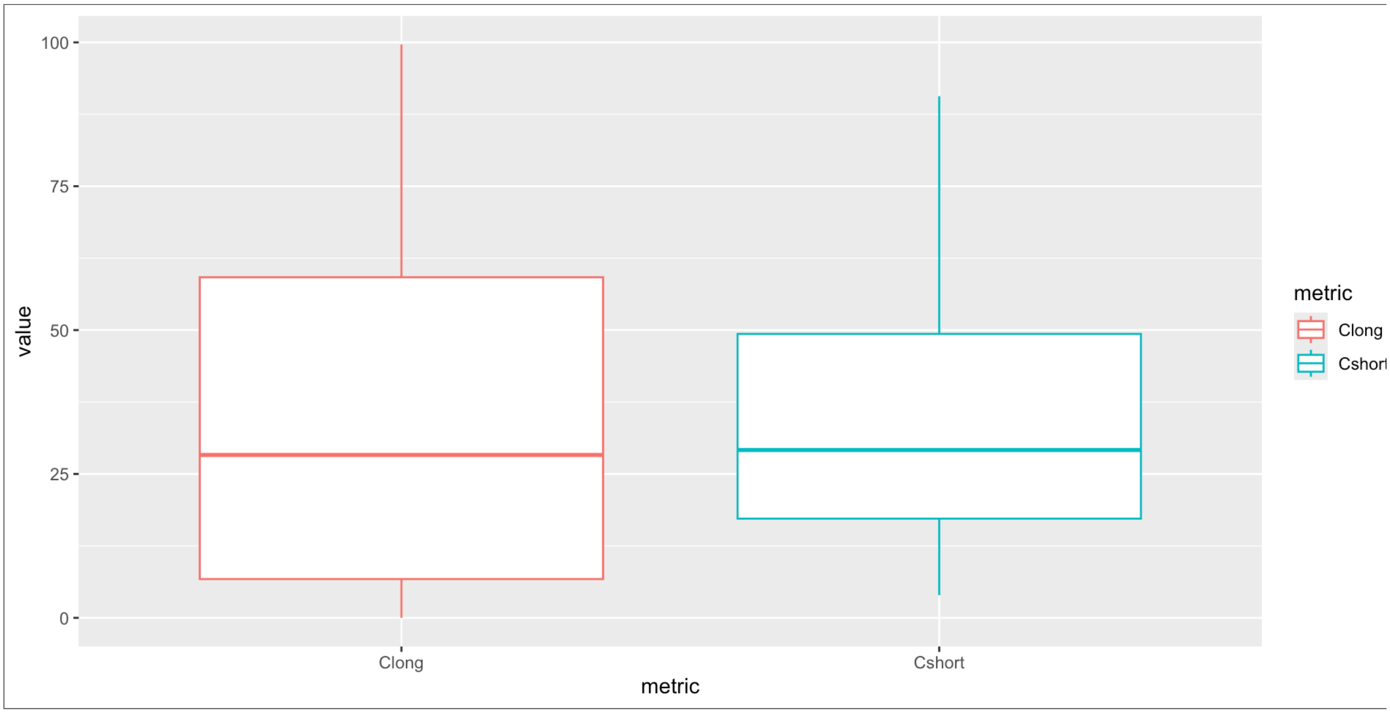
```
$Cshort
[1] 17.98579

$Clong
[1] 6.945886
```

BUILD OUR WRAPPER FUNCTION AND RUN FOR ALL THE PARAMETER SETS

```
# use a wrapper function to just return the carbon trajectories
p_wrapper <- function(r, harv, K, Cinitial, simtimes, odefunc, metr
  parms <- list(r = r, K = K, harv = harv)
  result <- ode(y = Cinitial, times = simtimes, func = odefunc, par
  result <- as.data.frame(result)
  colnames(result) <- c("time", "C")
  # get metrics
  metrics <- metricfunc(result)
  return(metrics)
}

# notice how we added in K, a parameter that we are NOT varying

# try it out
tm <- seq(from = 1, to = 50)
Cinitial <- 30


allresults <- as.data.frame(sens_forest$X) %>% pmap(p_wrapper, K =
```

```
    Cshort      Clong
1 17.98579  6.9458859
2 10.92509  0.7846803
3 32.04461 33.2610547
4 25.19360 21.3015204
5 41.70437 50.8339843
6 46.83899 56.2401154
```

GRAPH THE RESULTS



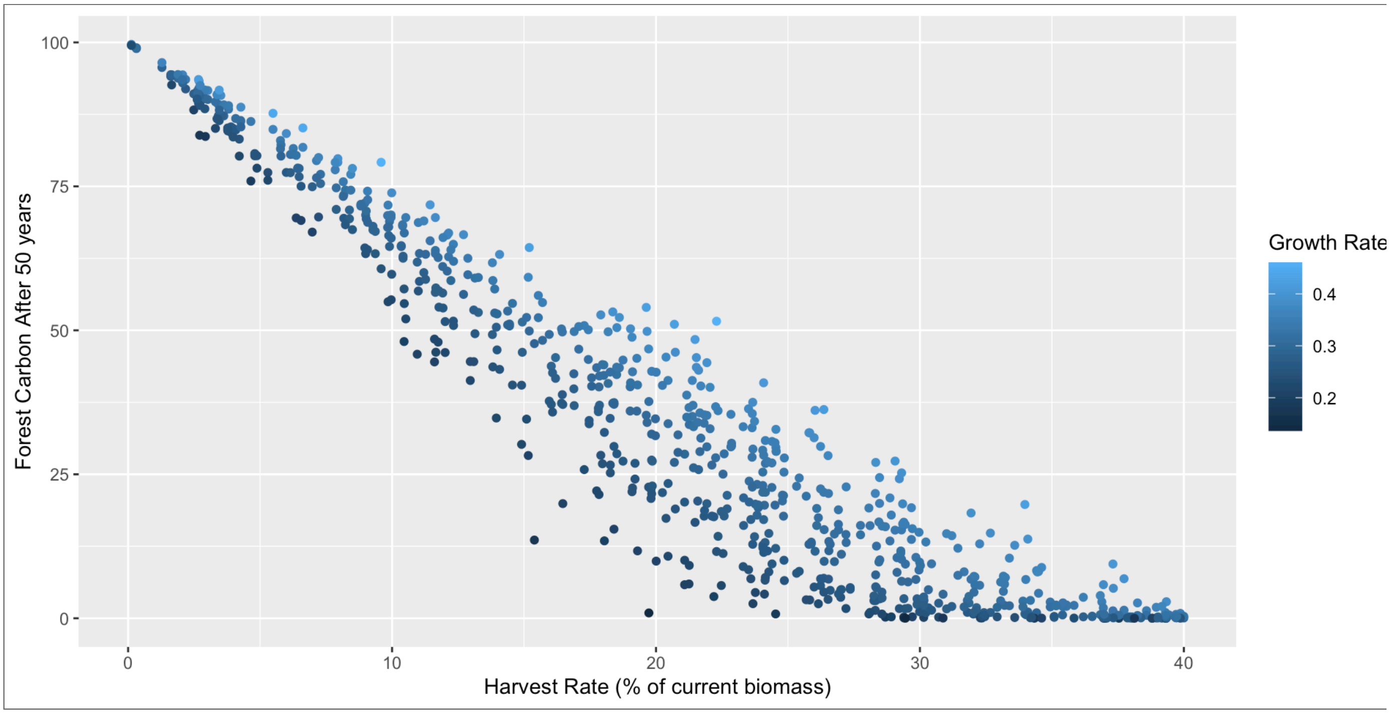
EXPLORE MORE

- graph response by parameter
- sobol indices

notice how we can see the impact of harvesting - and how growth rates reduce the sustainable harvest

GRAPH THE RESULTS

```
# link with parameters to see how parameters impact results
# report harvest as a percent for easy of interpretation
allresp <- cbind.data.frame(sens_forest$X, allres)
ggplot(allresp, aes(harv * 100, Clong, col = r)) +
  geom_point() +
  labs(y = "Forest Carbon After 50 years", x = "Harvest Rate (% of
```



COMPUTE SOBOL INDICES

```r
# sobol indices
sens_forest_C50 <- sensitivity::tell(sens_forest, allres$Clong)
rownames(sens_forest_C50$T) <- c("r", "harv")
sens_forest_C50$T
```

```
       original         bias  std. error  min. c.i.  max. c.i.
r    0.06250169 0.0007946407 0.009101695 0.04337397 0.07853703
harv 0.91548093 0.0065125614 0.069835985 0.76994324 1.04761962
```

```r
rownames(sens_forest_C50$S) <- c("r", "harv")
sens_forest_C50$S
```

```
       original         bias std. error   min. c.i. max. c.i.
r    0.04671916 -0.006810670 0.07317682 -0.08733211 0.1987345
harv 0.91787042 -0.001064374 0.01373252  0.89556040 0.9468316
```

FIXED HARVEST

What if you wanted to take the same amount of carbon each year - what would the model look like?
Think about what you would need to do to make this realistic

FIXED HARVEST EXAMPLE

```
source(here("R/dharvestfixed.R"))

dharvestfixed
```

```
function (Time, biomass, parms)
{
    if (biomass < parms$mincarbon) {
        db <- parms$r * biomass * (1 - biomass/parms$K)
    }
    else {
        db <- parms$r * biomass * (1 - biomass/parms$K) - parms$harv
    }
    return(list(db))
}
```

```
# try it out with different initial conditions to watch how the sys
tm <- seq(from = 1, to = 100)
Cinitial <- 30
gps <- list(harv = 2, K = 1000, r = 0.05, mincarbon = 20)

res <- ode(Cinitial, tm, dharvestfixed, gps)
```

```
DLSODA-  At current T (=R1), MXSTEP (=I1) steps
      taken on this call before reaching TOUT
In above message, I1 = 5000

In above message, R1 = 14.2223
```

```
colnames(res) <- c("time", "carbon")
head(res)
```

```
     time   carbon
[1,]    1 30.00000
[2,]    2 29.44198
[3,]    3 28.85706
[4,]    4 28.24395
[5,]    5 27.60121
[6,]    6 26.92738
```

```
tail(res)
```

```
          time   carbon
[10,] 10.00000 23.88885
[11,] 11.00000 23.03457
[12,] 12.00000 22.13856
```

```
[13,] 13.00000 21.19869
[14,] 14.00000 20.21272
[15,] 14.22234 20.00000
```

```
# notice how it fails after a certain length of time
# try another method
res <- ode(Cinitial, tm, dharvestfixed, gps, method = "euler")
colnames(res) <- c("time", "carbon")
head(res)
```

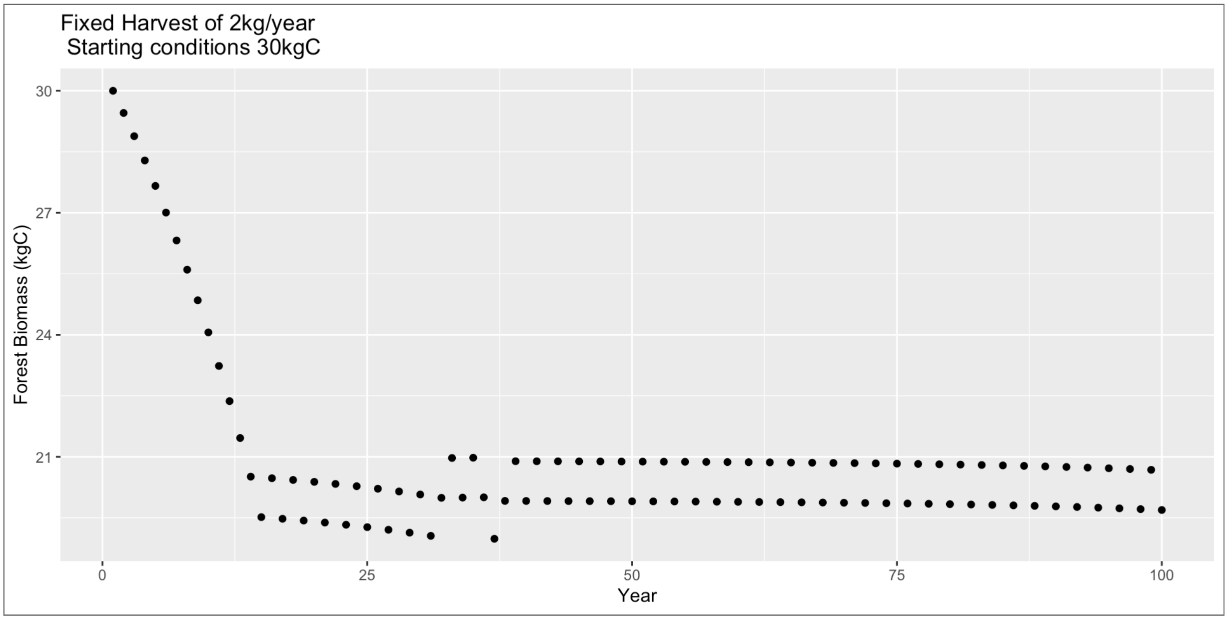
```
     time   carbon
[1,]    1 30.00000
[2,]    2 29.45500
[3,]    3 28.88437
[4,]    4 28.28687
[5,]    5 27.66121
[6,]    6 27.00601
```

```
tail(res)
```

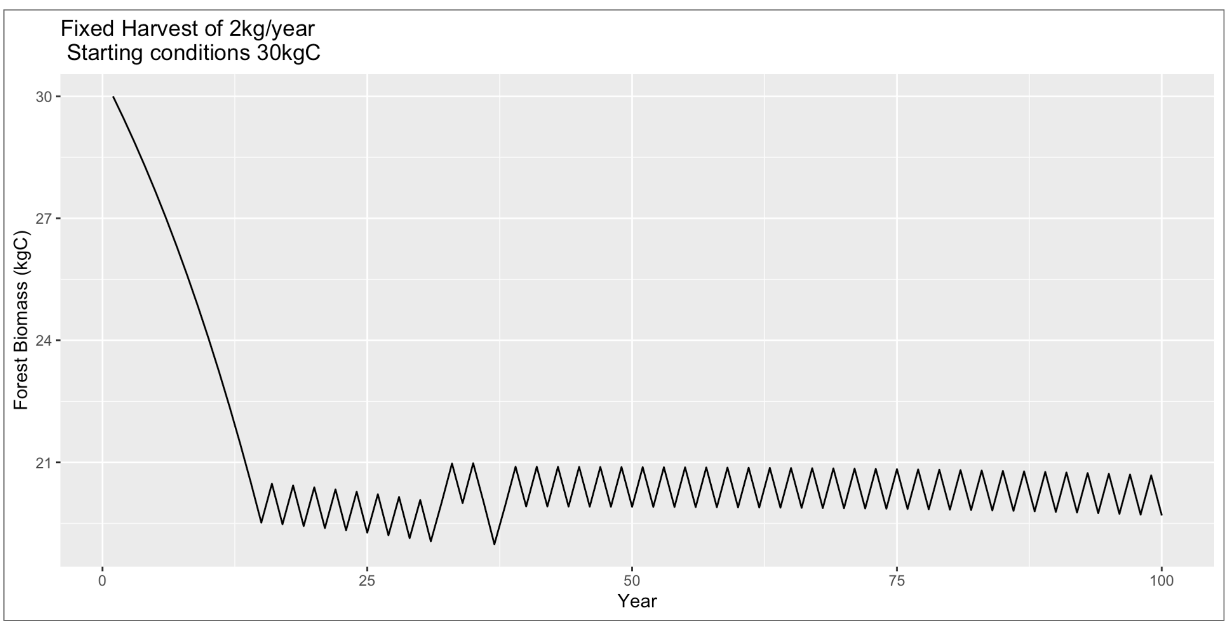
```
       time   carbon
 [95,]   95 20.71847
 [96,]   96 19.73293
 [97,]   97 20.70011
 [98,]   98 19.71369
 [99,]   99 20.67994
[100,]  100 19.69256
```

PLOT THE RESULTS

```
ggplot(as.data.frame(res), aes(time, carbon)) +
  geom_point() +
  labs(y = "Forest Biomass (kgC)", x = "Year", title = "Fixed Harve
```




```
ggplot(as.data.frame(res), aes(time, carbon)) +
  geom_line() +
  labs(y = "Forest Biomass (kgC)", x = "Year", title = "Fixed Harve
```




```
# why this pattern
```