# Assignment 4: Sobol

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## Question 1: Use the Sobel approach to generate parameter values for the 4 parameters

```
source(here("R/Catm.R"))

#windspeed in Catm function is in m/s
# generate two examples of random number from parameter distributions
# v = mean of 300 cm/s (3 m/s) and a SD of 50 cm/s (0.5 m/s)
# height = 3.5 and 5.5 m
# k_o and k_d = SD of 1% of default values


np <- 1000
k_o <- rnorm(mean = 0.1, sd = 0.1 * 0.01, n = np)
k_d <- rnorm(mean = 0.7, sd = 0.7 * 0.01, n = np)
v <- rnorm(mean = 3, sd = 0.5, n = np)
height <- runif(min = 3.5, max = 5.5, n = np)

X1 <- cbind.data.frame(k_o, k_d, v, height = height)

# repeat sampling
k_o <- rnorm(mean = 0.1, sd = 0.1 * 0.01, n = np)
k_d <- rnorm(mean = 0.7, sd = 0.7 * 0.01, n = np)
v <- rnorm(mean = 3, sd = 0.5, n = np)
height <- runif(min = 3.5, max = 5.5, n = np)

X2 <- cbind.data.frame(k_o, k_d, v, height = height)

# Use Sobel to generate parameter values for the 4 parameetes

sens_Catm_Sobol <- sobolSalt(model = NULL, X1, X2, nboot = 100)
```

## Question 2: Run the atmospheric conductance model for these parameters

```
# run atmosph conductance model for all parameter sets

parms <- as.data.frame(sens_Catm_Sobol$X)
colnames(parms) <- colnames(X1)
res <- pmap_dbl(parms, Catm)
```

```
sens_Catm_Sobol <- sensitivity::tell(sens_Catm_Sobol, res, res.names = "ga")

# main effect:  partitions variance (main effect without co-variance) - sums approximately to one
sens_Catm_Sobol$S
```

```
##      original          bias std. error   min. c.i.  max. c.i.
## X1 0.02136950 -0.0041586145 0.03090684 -0.04552517 0.08330873
## X2 0.02139135 -0.0045800677 0.03101597 -0.04545292 0.08667247
## X3 0.81116091 -0.0007803597 0.01056338  0.78922897 0.83450411
## X4 0.19709124 -0.0012452344 0.03015837  0.13147663 0.25528019
```

```
# add row names
row.names(sens_Catm_Sobol$S) <- colnames(parms)
sens_Catm_Sobol$S
```

```
##          original          bias std. error   min. c.i.  max. c.i.
## k_o    0.02136950 -0.0041586145 0.03090684 -0.04552517 0.08330873
## k_d    0.02139135 -0.0045800677 0.03101597 -0.04545292 0.08667247
## v      0.81116091 -0.0007803597 0.01056338  0.78922897 0.83450411
## height 0.19709124 -0.0012452344 0.03015837  0.13147663 0.25528019
```

```
# total effect - accounts for parameter interactions, is the T in the code
row.names(sens_Catm_Sobol$T) <- colnames(parms)
sens_Catm_Sobol$T
```

```
##           original         bias   std. error   min. c.i.   max. c.i.
## k_o    0.002967033 2.651492e-05 0.0001671285 0.002561576 0.003286085
## k_d    0.002978800 1.305424e-05 0.0002031424 0.002526283 0.003379293
## v      0.794704475 3.758261e-03 0.0303491662 0.729962507 0.851228810
## height 0.180059636 1.079031e-03 0.0100393663 0.159828341 0.197636323
```

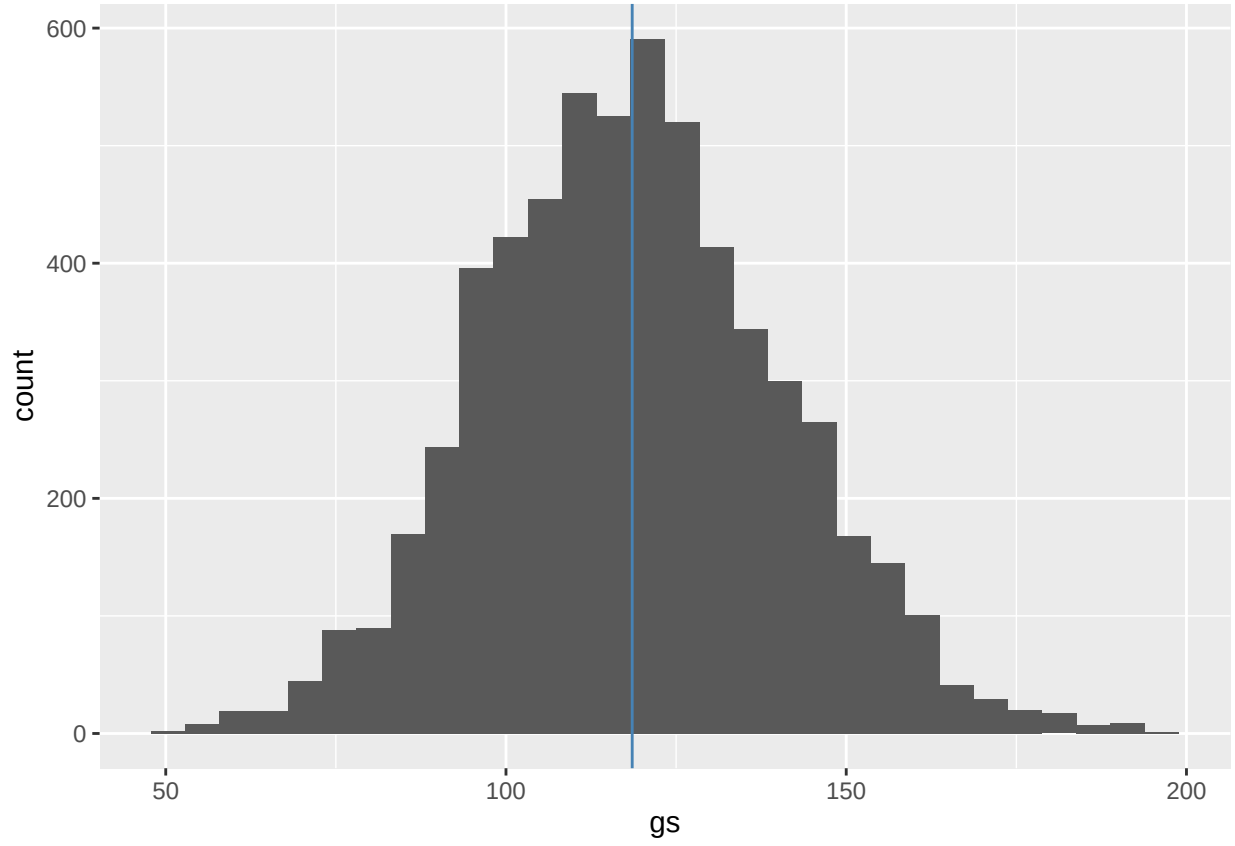
```
print(sens_Catm_Sobol)
```

```
##
## Call:
## sobolSalt(model = NULL, X1 = X1, X2 = X2, nboot = 100)
##
## Model runs: 6000
##
## Model variance: 481.6434
##
## First order indices:
##          original          bias std. error   min. c.i.  max. c.i.
## k_o    0.02136950 -0.0041586145 0.03090684 -0.04552517 0.08330873
## k_d    0.02139135 -0.0045800677 0.03101597 -0.04545292 0.08667247
## v      0.81116091 -0.0007803597 0.01056338  0.78922897 0.83450411
## height 0.19709124 -0.0012452344 0.03015837  0.13147663 0.25528019
##
## Total indices:
##           original         bias   std. error   min. c.i.   max. c.i.
## k_o    0.002967033 2.651492e-05 0.0001671285 0.002561576 0.003286085
## k_d    0.002978800 1.305424e-05 0.0002031424 0.002526283 0.003379293
## v      0.794704475 3.758261e-03 0.0303491662 0.729962507 0.851228810
## height 0.180059636 1.079031e-03 0.0100393663 0.159828341 0.197636323
```

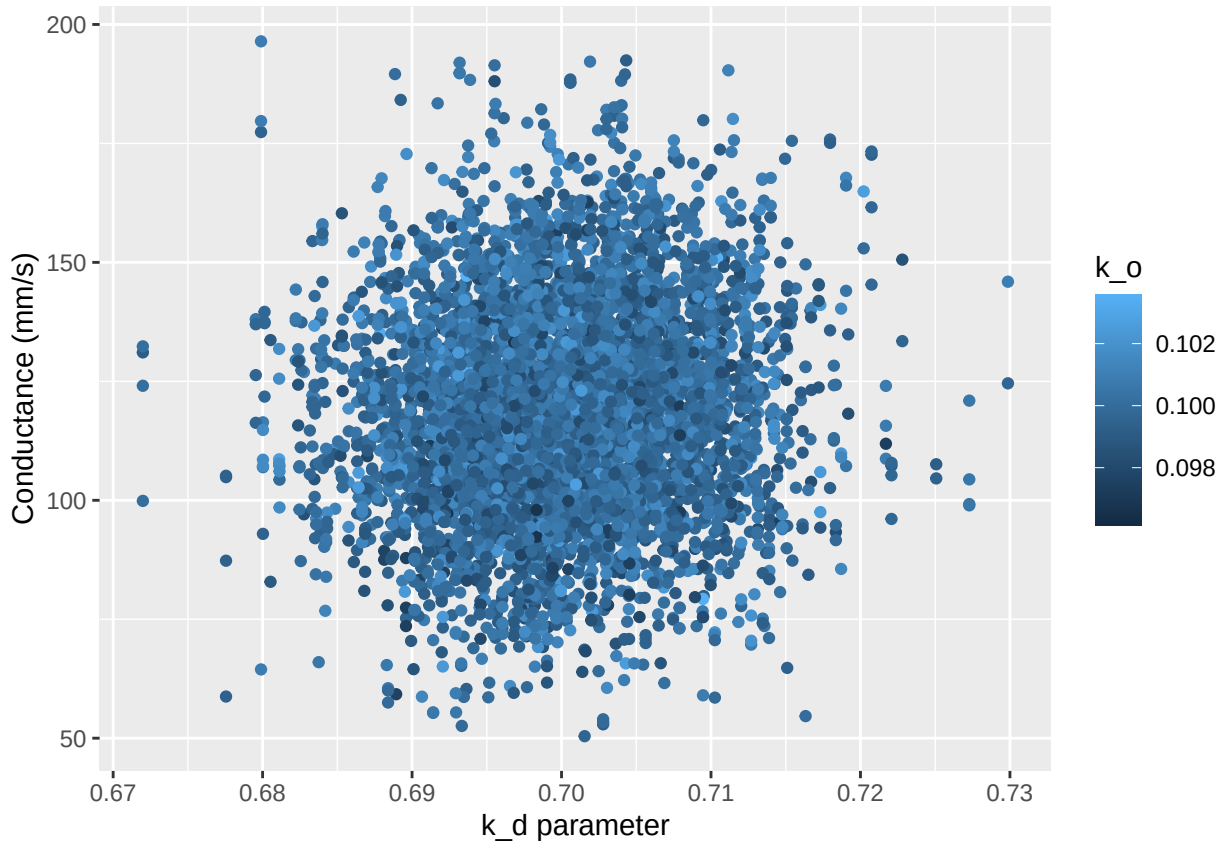
## Question 3: Plot conductance estimates in a way that accounts for parameter uncertainty

```
# graph two most sensitive parameters
sens_params <- cbind.data.frame(parms, gs = sens_Catm_Sobol$y)

# look at overall gs sensitvity to uncertainty
ggplot(sens_params, aes(x = gs)) +
  geom_histogram() +
  geom_vline(xintercept = mean(sens_params$gs), col = "steelblue")
```

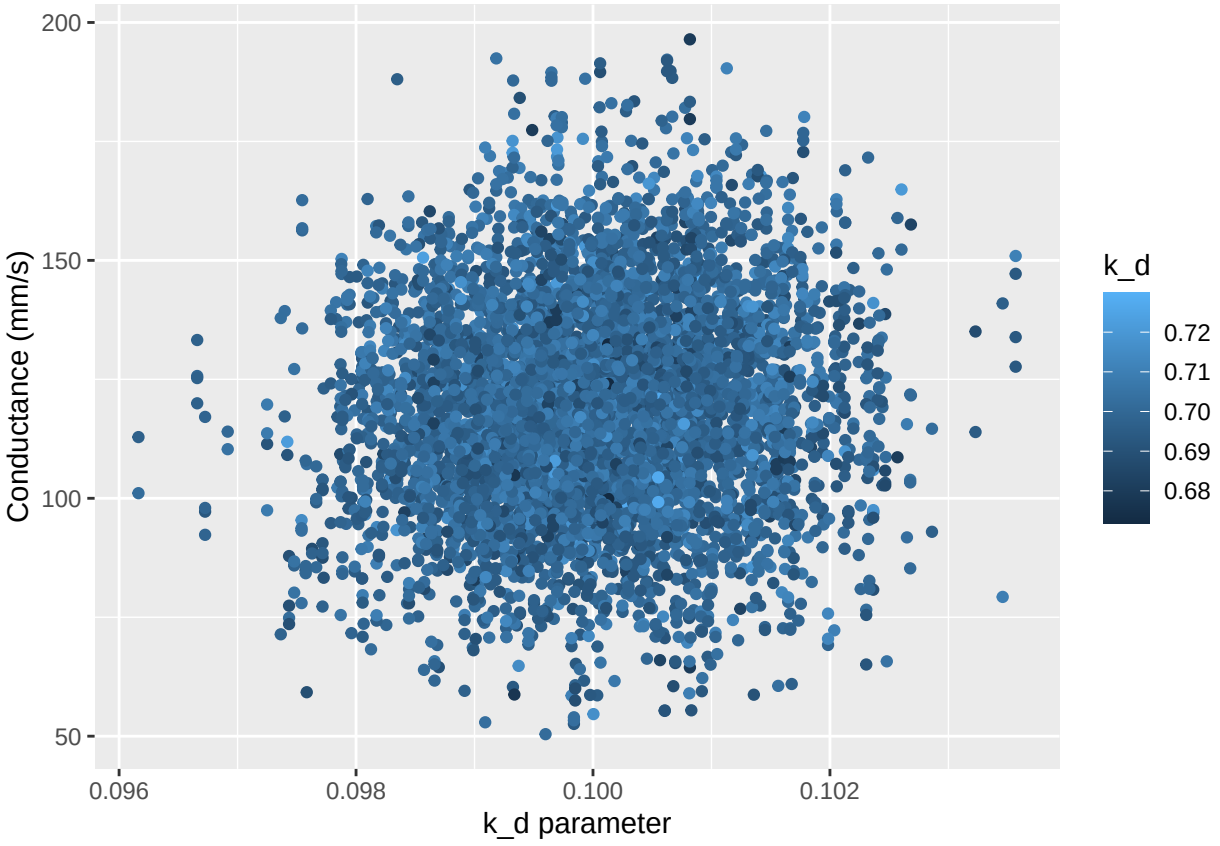
```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```
############

# (Additional) Plotting conductance estimates against k_d and k_o parameters:
# look at response of conductance to the two most important variables
ggplot(sens_params, aes(k_d, gs, col = k_o)) +
  geom_point() +
  labs(y = "Conductance (mm/s)", x = "k_d parameter")
```




```
# use second most sensitive parameter (using most important as color)
ggplot(sens_params, aes(k_o, gs, col = k_d)) +
  geom_point() +
  labs(y = "Conductance (mm/s)", x = "k_d parameter")
```




## Question 4: Plot conductance estimates against windspeed use the parameter that is 2nd in terms of total effect on response

```
#2nd paramater of total effect
T_vals <- sens_Catm_Sobol$T$original
names(T_vals) <- rownames(sens_Catm_Sobol$T)

# Sort descending
sorted_T <- sort(T_vals, decreasing = TRUE)

# View ranking
sorted_T
```

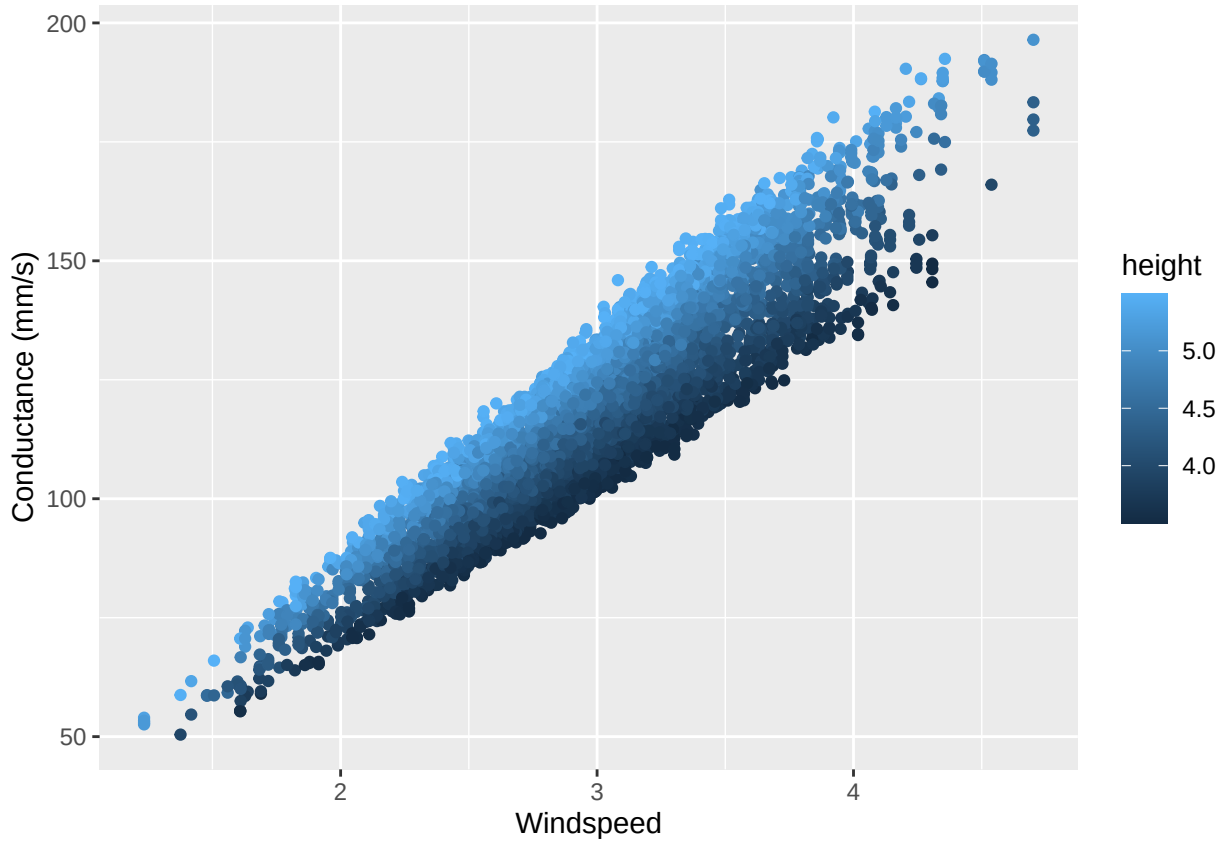
```
##           v      height         k_d         k_o
## 0.794704475 0.180059636 0.002978800 0.002967033
```

```
#2nd paramet of main effect
S_vals <- sens_Catm_Sobol$S$original
names(S_vals) <- rownames(sens_Catm_Sobol$S)

# Sort descending
sorted_S <- sort(S_vals, decreasing = TRUE)
```

```
# View ranking
sorted_S
```

```
##          v     height        k_d        k_o
## 0.81116091 0.19709124 0.02139135 0.02136950
```

```
ggplot(sens_params, aes(v, gs, col = height)) +
  geom_point() +
  labs(y = "Conductance (mm/s)", x = "Windspeed")
```




## Question 5: Estimate the Sobel Indices for your output

```
sens_Catm_Sobol2 <- sobolSalt(model = NULL, X1, X2, nboot = 100, scheme = "B")

parms2 <- as.data.frame(sens_Catm_Sobol2$X)
colnames(parms2) <- colnames(X1)
res <- pmap_dbl(parms2, Catm)


sens_Catm_Sobol2 <- sensitivity::tell(sens_Catm_Sobol2, res, res.names = "ga")
```

```r
# main effect
row.names(sens_Catm_Sobol2$S) <- colnames(parms2)
sens_Catm_Sobol2$S
```

```
##          original          bias std. error   min. c.i.  max. c.i.
## k_o    0.02324364 -0.0033836612 0.03406709 -0.03689878 0.09120967
## k_d    0.01816918 -0.0031097666 0.03386550 -0.04225528 0.08491328
## v      0.81261212  0.0008716414 0.00849372  0.79672070 0.82880952
## height 0.19917895 -0.0053534986 0.02822049  0.15216266 0.25154874
```

```r
# total effect
row.names(sens_Catm_Sobol2$T) <- colnames(parms2)
sens_Catm_Sobol2$T
```

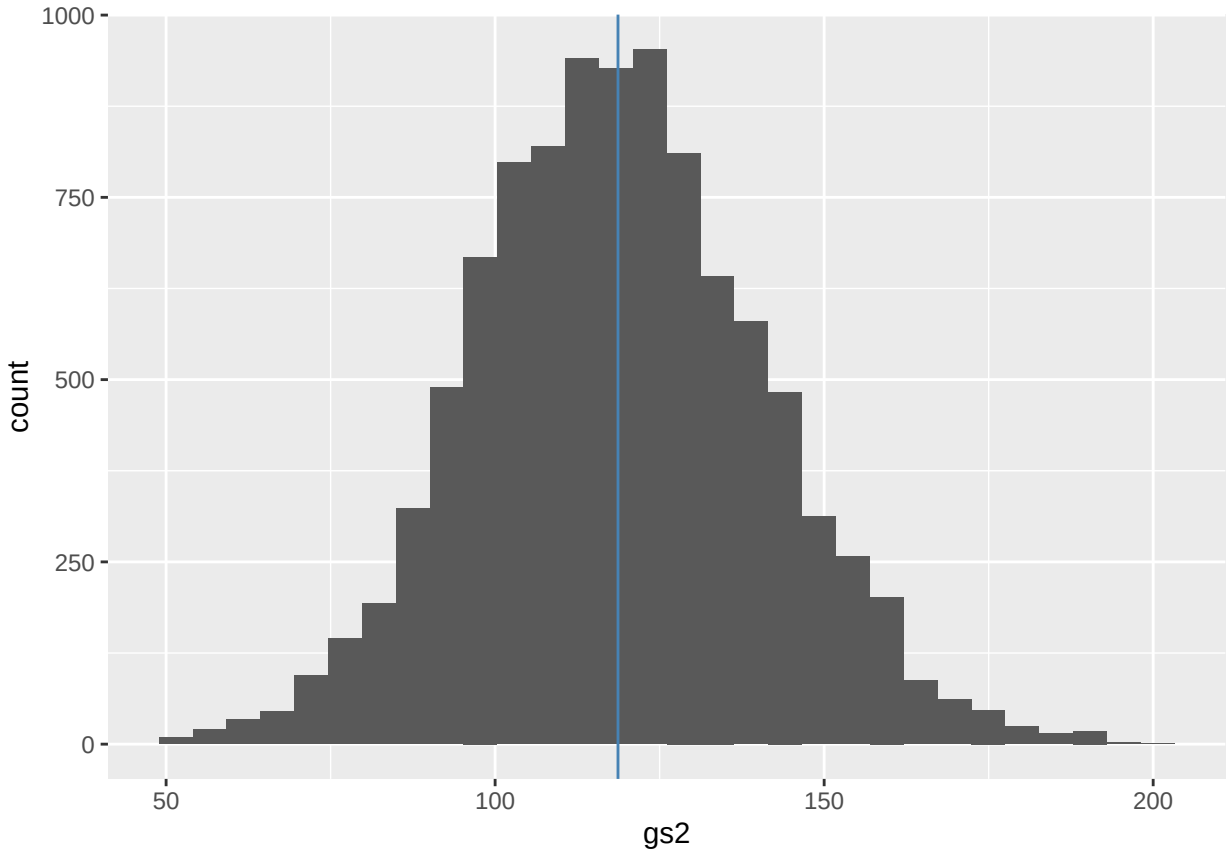
```
##           original          bias   std. error   min. c.i.   max. c.i.
## k_o    0.003009172 -1.041955e-05 0.0001409953 0.002739197 0.003333882
## k_d    0.003001199 -6.979879e-06 0.0001785948 0.002679419 0.003464260
## v      0.795668570  5.263845e-03 0.0285900542 0.748014158 0.842132592
## height 0.181860405 -7.841211e-04 0.0081939386 0.165889345 0.197263949
```

```r
# parameters are in order,
sens_Catm_Sobol2$S2
```

```
##         original        bias std. error   min. c.i.  max. c.i.
## X1X2 -0.01854181 0.003223464 0.03383767 -0.08502963 0.04132860
## X1X3 -0.02021983 0.003139467 0.03420008 -0.08856539 0.04066853
## X1X4 -0.01823945 0.003484523 0.03379816 -0.08452802 0.04103216
## X2X3 -0.01550278 0.003238606 0.03391068 -0.08149888 0.04436212
## X2X4 -0.01795988 0.003156083 0.03384481 -0.08541644 0.04247537
## X3X4 -0.01796389 0.004470998 0.02936630 -0.07792544 0.03340116
```

```r
#If you cross 0 (negative number) in the Confidence interval, it means it is not signifcant

#Plot the sobol indices

sens_params2 <- cbind.data.frame(parms2, gs2 = sens_Catm_Sobol2$y)

ggplot(sens_params2, aes(x = gs2)) +
  geom_histogram() +
  geom_vline(xintercept = mean(sens_params2$gs2), col = "steelblue")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




## Question 5: Comment on what this tells you about how atmospheric conductance and its sensitivity to variation in compared to the setting that we examined in class where wind speed was lower and less variable and vegetation was taller.

The sensitivity analysis analysis reveals that wind speed (v) is the dominant driver of variation in atmospheric conductance under this scenario. The total effect Sobol index for v is approximately 0.85, indicating that most of the variance in conductance is directly attributable to changes in wind speed alone. Vegetation height (height) also contributes meaningfully, though to a much lesser extent (~18%). In contrast, the conductance parameters k_o and k_d have negligible total effects, suggesting they do not significantly influence conductance in this scenario, either independently or through interactions.

This contrasts with the setting we examined in class, where wind speed was lower and less variable, and vegetation was taller. In that context, k_o and k_d were more influential, likely because the system was less driven by wind and more sensitive to plant-level physiological parameters. The shift in sensitivity observed here underscores how increased wind variability amplifies the importance of v in driving conductance, while the role of vegetation structure and internal resistances (k_o, k_d) becomes relatively less important.