EXPLORING STABILITY IN DYNAMIC SYSTEMS

FIXED HARVEST

What if you wanted to take the same amount of carbon each year - what would the model look like?
Think about what you would need to do to make this realistic

FIXED HARVEST EXAMPLE

```
1  source(here("R/dharvestfixed.R"))
2
3  dharvestfixed
```

```
function (Time, biomass, parms)
{
    if (biomass < parms$mincarbon) {
        db <- parms$r * biomass * (1 - biomass/parms$K)
    }
    else {
        db <- parms$r * biomass * (1 - biomass/parms$K) - parms$harv
    }
    return(list(db))
}
```

RUN THE FOREST MODEL

- harvest of 2kgC per year (fixed rate)

- carrying capacity of 1000kg

- growth rate of 0.05

- minimum carbon before harvest is allowed 20kg

RUN FOR 100 YEARS

try it
what happens?

```
1  tm <- seq(from = 1, to = 100)
2  Cinitial <- 30
3  gps <- list(harv = 2, K = 1000, r = 0.05, mincarbon = 20)
4
5  res <- ode(Cinitial, tm, dharvestfixed, gps)
```

```
DLSODA-  At current T (=R1), MXSTEP (=I1) steps
      taken on this call before reaching TOUT
In above message, I1 = 5000

In above message, R1 = 14.2223
```

```
1  colnames(res) <- c("time", "carbon")
2  head(res)
```

```
       time    carbon
[1,]     1 30.00000
[2,]     2 29.44198
[3,]     3 28.85706
[4,]     4 28.24395
[5,]     5 27.60121
[6,]     6 26.92738
```

```
1  tail(res)
```

```
          time    carbon
[10,] 10.00000 23.88885
[11,] 11.00000 23.03457
[12,] 12.00000 22.13856
[13,] 13.00000 21.19869
```

```
[14,]  14.00000 20.21272
[15,]  14.22234 20.00000
```

NUMERICAL INTEGRATION ISSUES

‣ particularly happens if model is "stiff" (when system state reaches a threshold, behavior changes alot)

‣ our minimum carbon is a threshold

When that happens try another integration method See *ode* help for some options

EXAMPLE

```
1  # notice how it fails after a certain length of time
2  # try another method
3  res <- ode(Cinitial, tm, dharvestfixed, gps, method = "euler")
4  colnames(res) <- c("time", "carbon")
5  head(res)
```
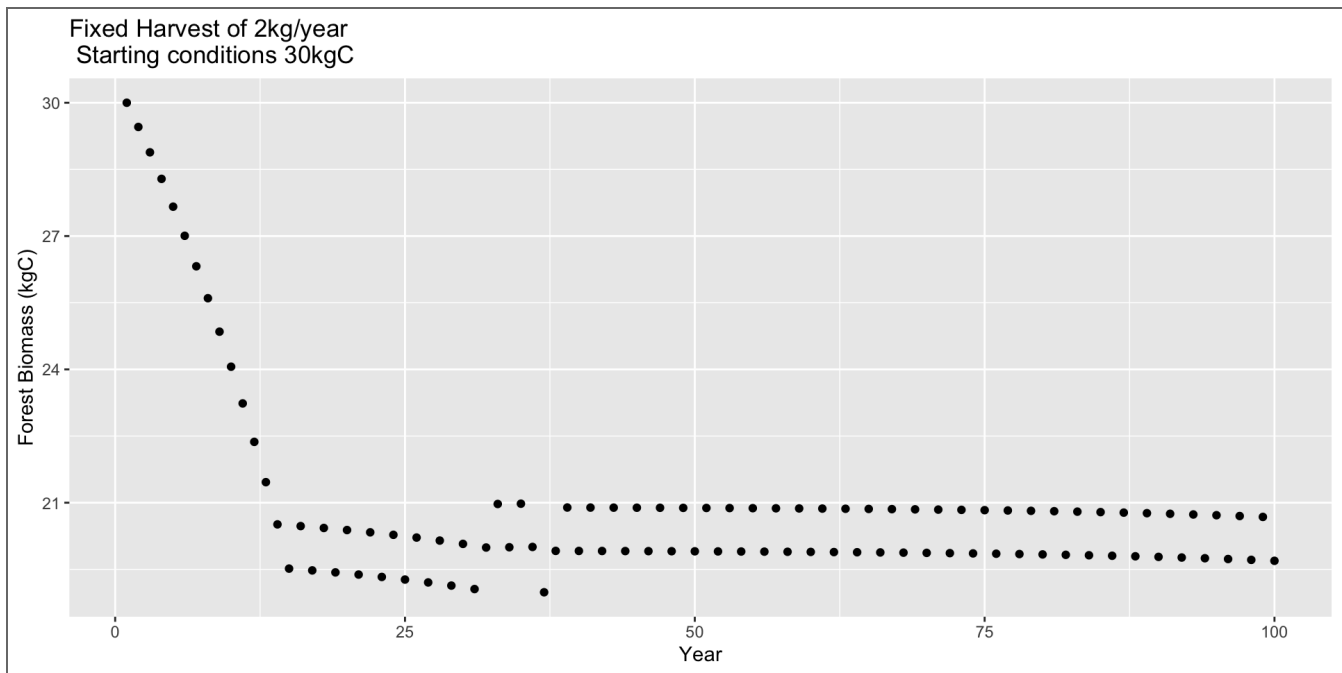
```
      time    carbon
[1,]     1 30.00000
[2,]     2 29.45500
[3,]     3 28.88437
[4,]     4 28.28687
[5,]     5 27.66121
[6,]     6 27.00601
```

```
1  tail(res)
```

```
        time    carbon
 [95,]    95 20.71847
 [96,]    96 19.73293
 [97,]    97 20.70011
 [98,]    98 19.71369
 [99,]    99 20.67994
[100,]   100 19.69256
```

```
1  ggplot(as.data.frame(res), aes(time, carbon)) +
2    geom_point() +
3    labs(y = "Forest Biomass (kgC)", x = "Year", title = "Fixed Harvest of
```

Fixed Harvest of 2kg/year
Starting conditions 30kgC

```
1  ggplot(as.data.frame(res), aes(time, carbon)) +
2    geom_line() +
3    labs(y = "Forest Biomass (kgC)", x = "Year", title = "Fixed Harvest of
```


Fixed Harvest of 2kg/year
Starting conditions 30kgC
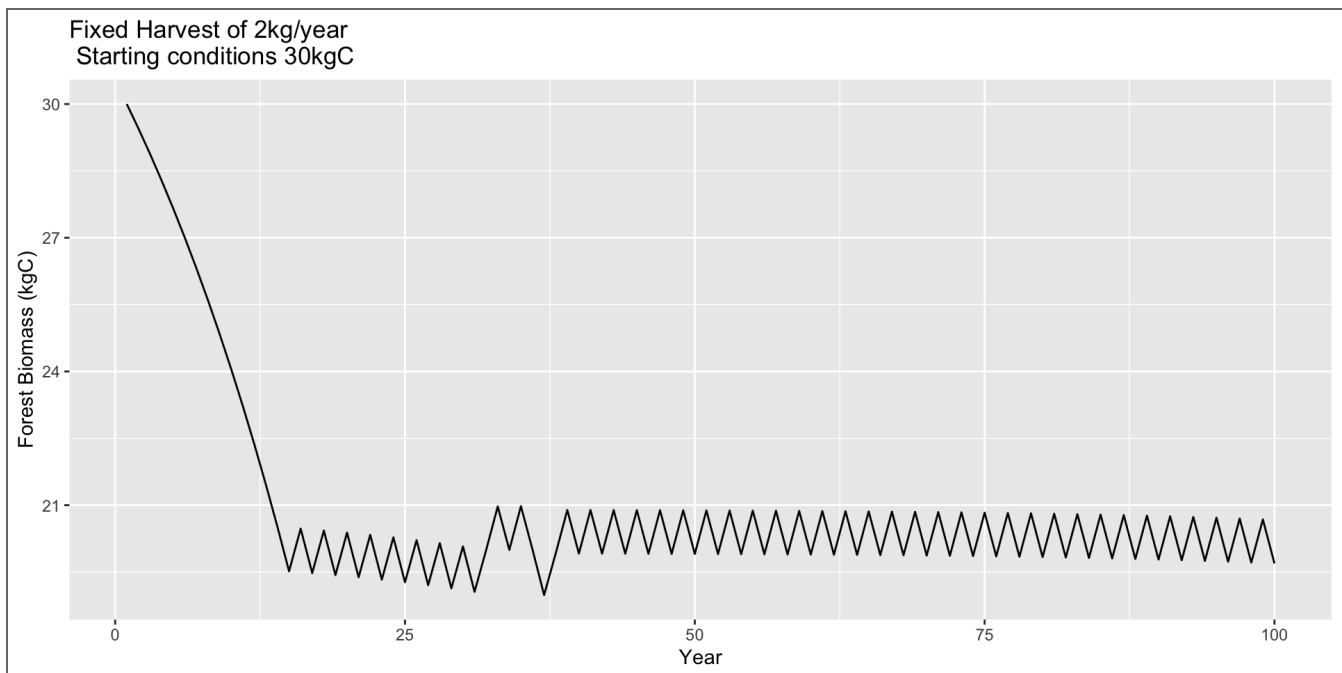
```
1  # why this pattern
```

STABILITY

Stability analysis investigates how the system state changes, particularly focusing on where the system state 'ends up' over time and what happens when there are small perturbations to the system state
Other examples?
How would you define it?

A BROADER MEASURE OF STABILITY

‣ persistence

‣ cycling doesn't necessarily mean unstable in an ecological
  sense

‣ define stability based on whether or not structure or
  function stays within expected ranges after a "press" or
  "pulse" disturbance

‣ can define metrics, similar to what we did for estimating
  performance or sensitivity

‣ multiple metrics can be useful

EXAMPLES?

Ecological systems?
Human systems?
EJ?

WHAT HARVEST RATE LEADS TO AN UNSTABLE FOREST?

using proportional harvest, what rate leads to a stable forest

```
1  source(here("R/dharvest.R"))
2  dharvest
```

```
function (Time, biomass, parms)
{
    db <- parms$r * biomass * (1 - biomass/parms$K) - parms$harv *
        biomass
    return(list(db))
}
```

```
 1  # set the time steps you want to see
 2  tm <- seq(from = 1, to = 100)
 3
 4  # inital conditions
 5  Cinitial <- 30
 6
 7  # set the parameters
 8  gps <- list(harv = 0.1, K = 1000, r = 0.05, mincarbon = 20)
 9
10  # run the model
11  res <- ode(Cinitial, tm, dharvest, gps, method = "euler")
12  colnames(res) <- c("time", "carbon")
```

GRAPH THE RESULTS

```
1  # look at the results
2  ggplot(as.data.frame(res), aes(time, carbon)) +
3    geom_point() +
4    labs(y = "Forest Biomass (kgC)", x = "Year", title = "Proportional Harv
```

```
1  # is it stable
2  # what can you change to make it stable
```


Proportional Harvest of 10% per year
Starting conditions 30kgC
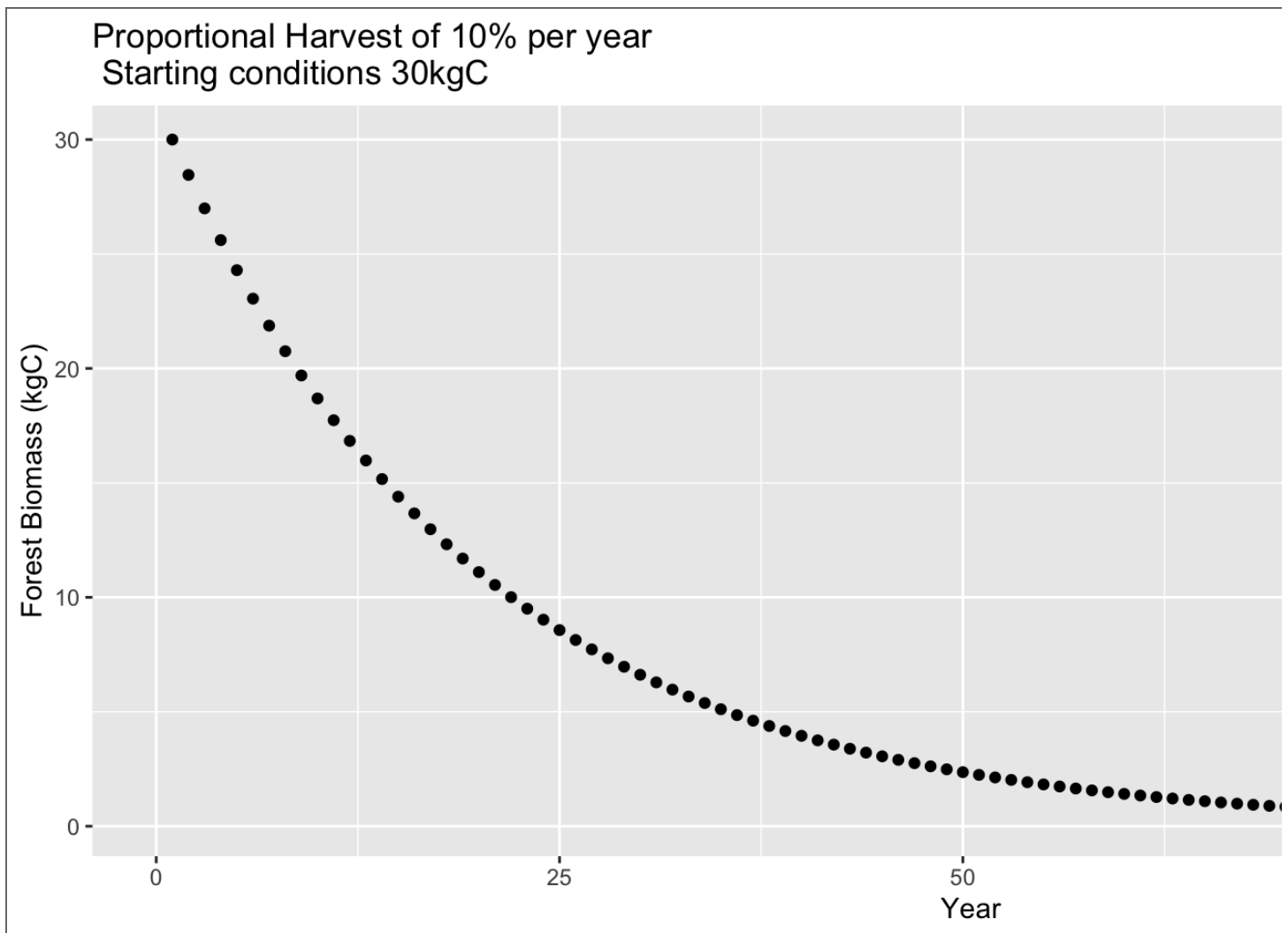
WAYS TO USE DYNAMIC MODELS

Goal: To provide insight into environmental systems, and how they respond to intervention/disturbance/change..

- sensitivity analysis of output

- output generated by solving differential equation to see system evolution in time or space

- output response to variation in inputs/parameters

- quantifying how summary measures respond to variation in inputs/parameters
  Stability focus..

- how does systems state evolution vary with inputs/parameters

- OR where does the system end up over time

MODEL VERSUS THE REAL WORLD

Models can help to develop understanding, theory and
hypothesis about stability by showing how different
assumptions (conceptual models) impact stability
The **WHY** of stability
But the real world is more complex - but we can learn by
understanding simpler models

STABILITY ANALYSIS FOR SIMPLE DIFFERENTIAL EQUATIONS

A mathematical approach..
We can look at how the derivative (rate of change) * under different system states to get a picture of when our system will be stable * (e.g where small perturbations just bring the system back to a relatively constant state)
One way we can do this WITHOUT solving the differential equation is to look at the derivatives at different values (system states)

‣ When the derivative is 0 the system is stable

‣ Positive derivatives are growth

‣ Negative are decline

We can use derivatives to see whether and when stability will happen

HARVEST AS AN EXAMPLE

graph the derivative (how the forest carbon is changing) for different forest carbons (stock/state)
Notice where derivatives are

- 0

- positive (growing)

- negative (declining)

Recall these are all for a proportional harvest rate of 4% per year and a growth rate of 0.05, etc

```
 to = 100)
0.04, K = 1000, r = 0.05)

f current values of carbon
(Ccurr = seq(from = 1, to = 300, by = 5))
t(Ccurr$Ccurr %>% map(~ dharvest(parms = gps, Time = NULL, biomass = .x)))
```

PLOT THE DERIVATIVE FOR DIFFERENCE "STARTING POINTS" (FOREST CARBON)

```
1  ggplot(Ccurr, aes(Ccurr, derv)) +
2     geom_point() +
3     geom_hline(yintercept = 0, col = "red") +
4     labs(y = "Derivative\n (Rate of change of forest carbon) \n(kg/C/year)"
```
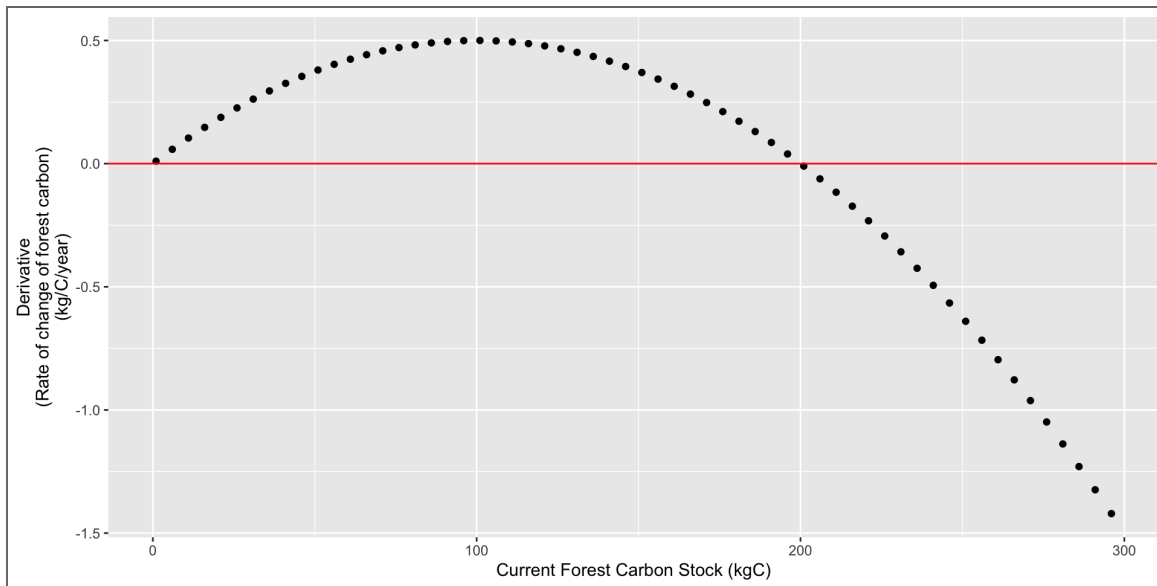
WHAT DO YOU LEARN FROM THIS?

‣ Is there a way to get a stable forest, given our harvest rate and growth rate?

‣ what size of forest would you need to *start* from

‣ try it

```
1   # rerun our model with new initial condition that we take from the plot
2   tm <- seq(from = 1, to = 100)
3   gps <- list(harv = 0.04, K = 1000, r = 0.05)
4
5   Cinitial <- 300
6   res <- ode(Cinitial, tm, dharvest, gps)
7   colnames(res) <- c("time", "carbon")
8
9   # look at the results
10  ggplot(as.data.frame(res), aes(time, carbon)) +
11    geom_point() +
12    labs(y = "Forest Biomass (kgC)", x = "Year", title = "Proportional Harv
```

USING DERIVATIVES TO ALGERBRAICALLY FIND STABILITY

We can more formally find stability by finding the system state when the derivative is zero

$$\frac{\partial C}{\partial t} = r * biomass * (1 - \frac{biomass}{K}) - harv * biomass$$

$$0 = r * biomass * (1 - \frac{biomass}{K}) - harv * biomass$$

$$biomass = K * \frac{r - h}{r}$$

$$biomass = 1000 * (0.05 - 0.04)/0.05 = 200$$

try it

# PARAMETER AND INITIAL CONDITIONS INTERACTIONS

Stability also reflects other parameters -such as harvest rate
we can also experiment with how stable harvest might
change with harvest rate
try repeating above with lower and higher harvest rates

```
1  source(here("R/dharvest.R"))
2  dharvest
```

```
function (Time, biomass, parms)
{
    db <- parms$r * biomass * (1 - biomass/parms$K) - parms$harv *
        biomass
    return(list(db))
}
```

```
 1  # given some forest characteristics - lets look at derivatives under diff
 2  lowHrate <- 0.015
 3  gps <- list(harv = lowHrate, K = 100, r = 0.05)
 4
 5  # look at the derivative over a range of forest sizes
 6
 7  findstable <- data.frame(Ccurr = seq(from = 1, to = 100, by = 5))
 8  # notice use of Time=NULL, and map to compute derivative
 9  # for different values of forest biomass
10  findstable$dervHlow <- unlist(findstable$Ccurr %>% map(~ dharvest(parms =
11
12  ggplot(findstable, aes(Ccurr, dervHlow)) +
13    geom_point() +
14    geom_hline(yintercept = 0, col = "red") +
15    labs(y = "Derivative\n (Rate of change of forest carbon) (kg/C/year)",
```
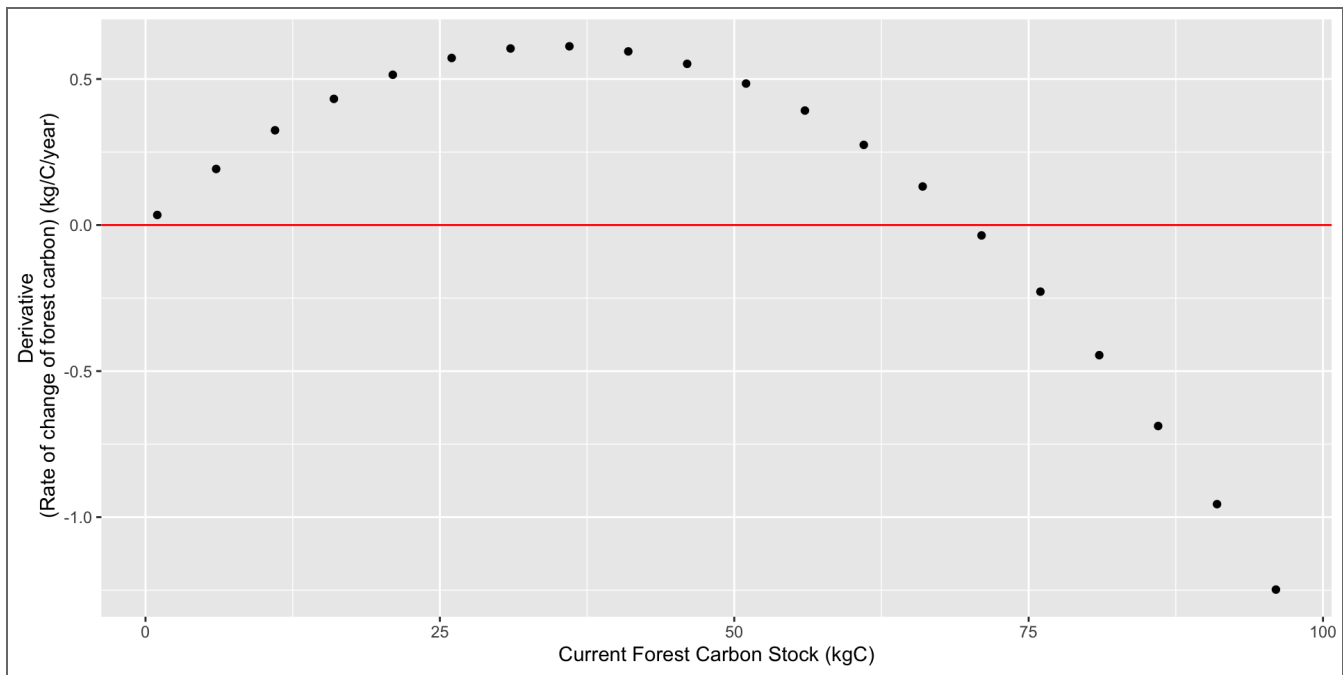
```
 1  # look at a different harvest rate
 2  midHrate <- 0.02
 3  gps <- list(harv = midHrate, K = 100, r = 0.05)
 4  findstable$dervHmid <- unlist(findstable$Ccurr %>% map(~ dharvest(parms =
 5
 6  # try high rate
 7  highHrate <- 0.05
 8  gps <- list(harv = highHrate, K = 100, r = 0.05)
 9  findstable$dervHhigh <- unlist(findstable$Ccurr %>% map(~ dharvest(parms
10
11
12  # plot them all together
13  tmp <- gather(findstable, key = "HarvestRate", value = "value", -Ccurr)
14  ggplot(tmp, aes(Ccurr, value, color = HarvestRate)) +
15    geom_point() +
16    geom_hline(yintercept = 0, col = "black") +
17    labs(x = "Forest Biomass (kgC)", y = "Forest Growth Rate (kgC/year)")
```
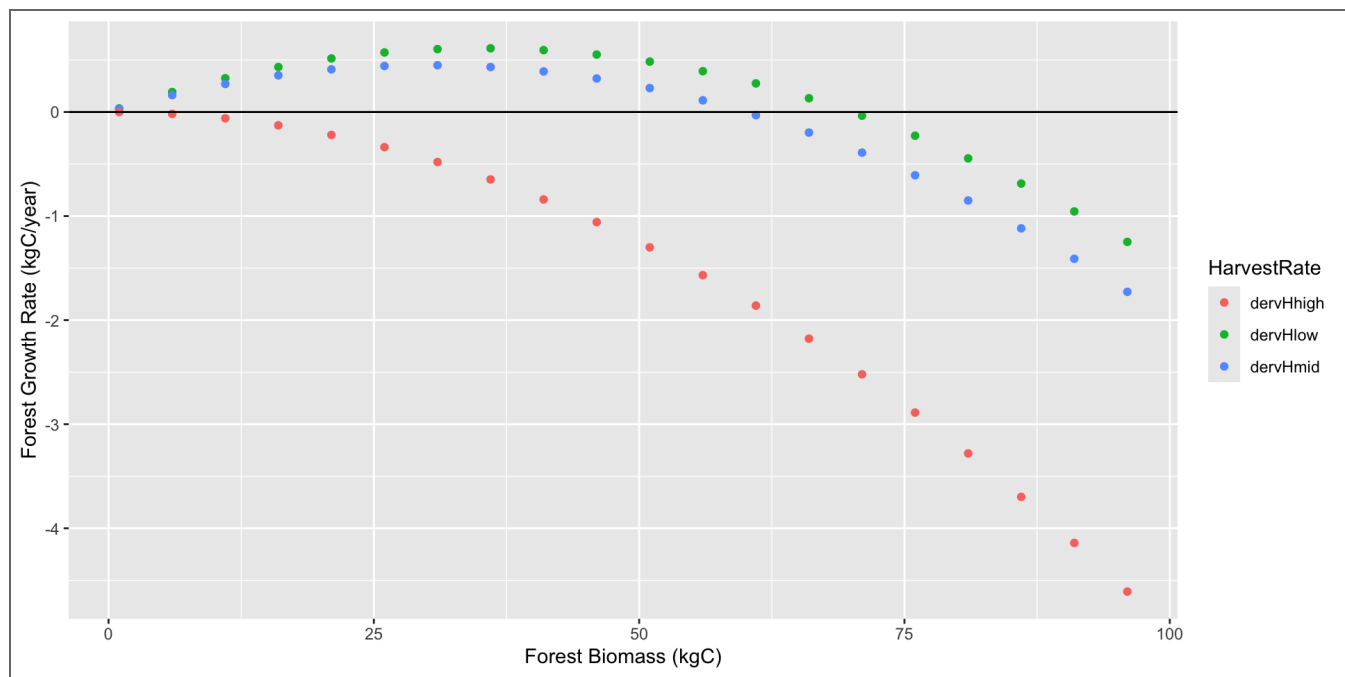
```
1  # notice how with higher harvest rates the stable population will be lowe
```

# STABILITY IN CHANGING SYSTEMS - JUST LOOKING AT DERVATIVES

Knowing the stability can help you understand the dynamics of the system

If we start with a Forest Carbon of 50 and low harvest rate what will happen

If we start with a Forest Carbon of 50 and a high harvest rate what will happen

Try it to find out if you are right

```
1  tm <- seq(from = 1, to = 500)
2  gps <- list(harv = lowHrate, K = 100, r = 0.05)
3  Pinitial <- 50
4  res <- ode(Pinitial, tm, dharvest, gps)
5  colnames(res) <- c("time", "carbon")
6  ggplot(as.data.frame(res), aes(time, carbon)) +
7    geom_point() +
8    labs(y = "Forest Biomass (kgC)", x = "Year", title = "low harvest rate"
```
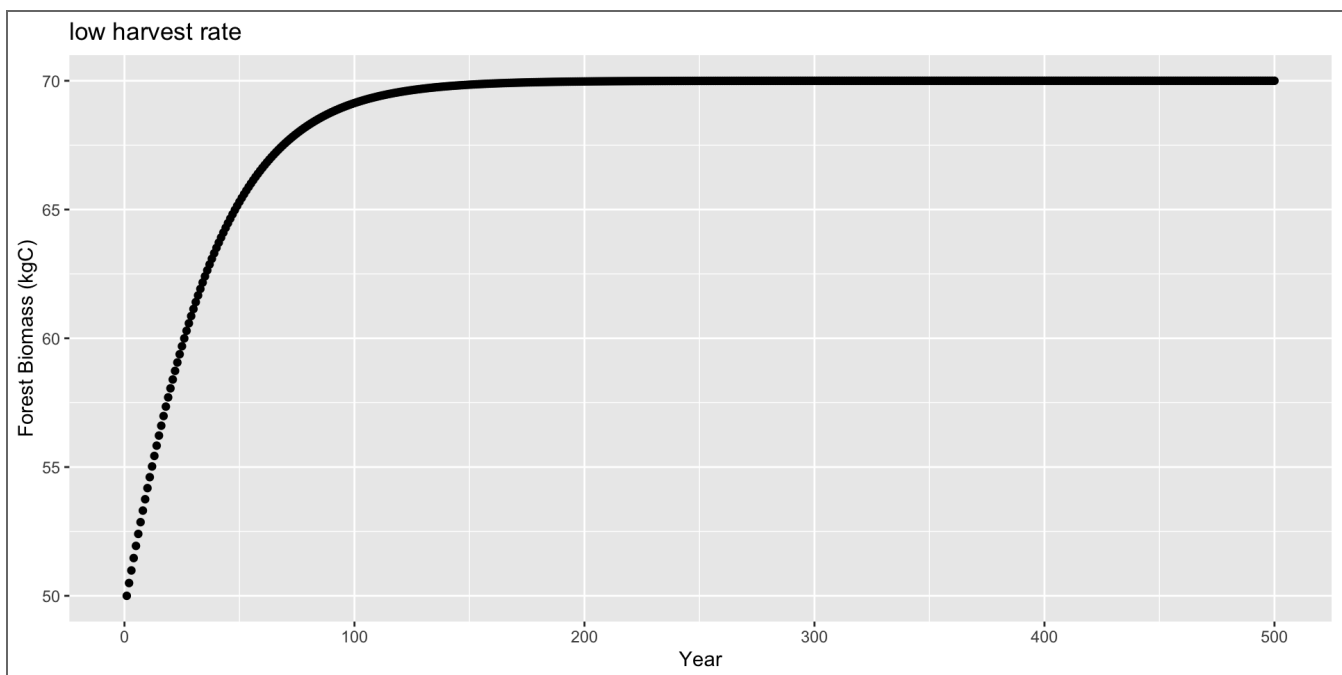


```
1  gps <- list(harv = highHrate, K = 100, r = 0.05)
2  Pinitial <- 50
```

```
3  res <- ode(Pinitial, tm, dharvest, gps)
4  colnames(res) <- c("time", "carbon")
5  ggplot(as.data.frame(res), aes(time, carbon)) +
6    geom_point() +
7    labs(y = "Forest Biomass (kgC)", x = "Year", title = "High Harvest Rate
```

High Harvest Rate
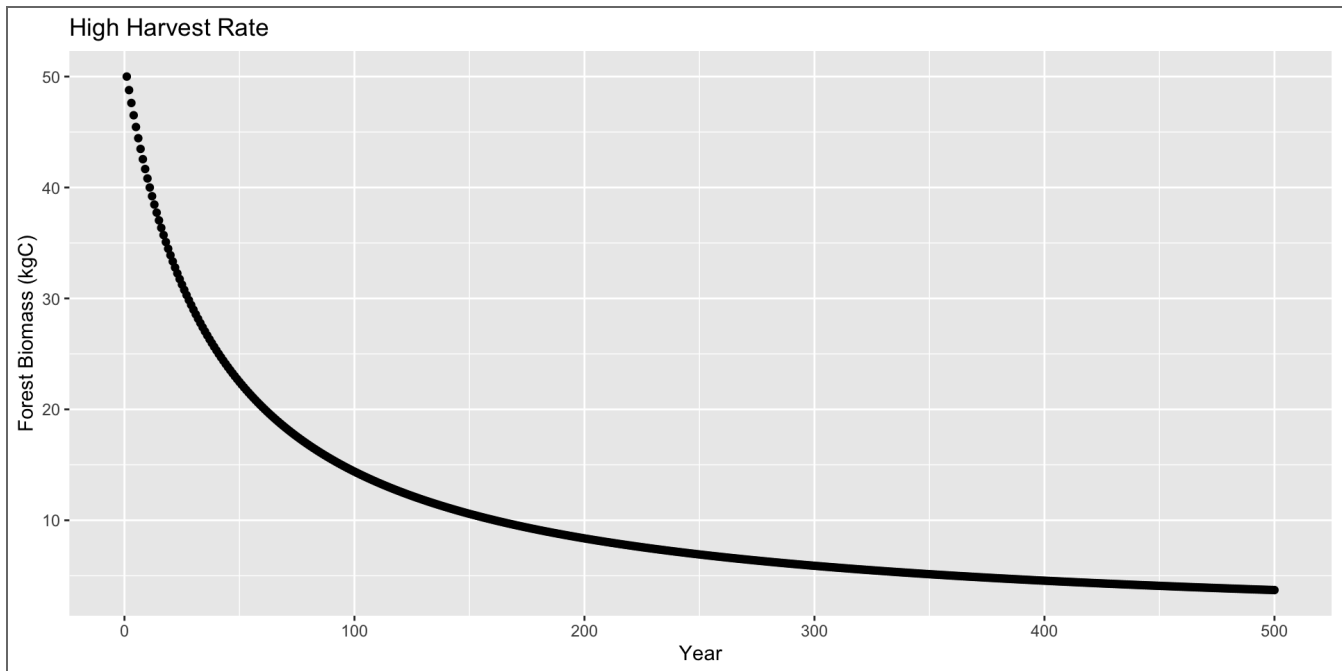
FOR NEXT CLASS

‣ Burke et al., 2021

‣ Graup et al., 2022

‣ Pick on of the papers to focus on in more detail, for this paper do the following:

‣ In 2-3 sentences, describe how parameter uncertainty was used in the paper

‣ Ask one question - something you'd like to know that wasn't covered or wasn't clear in the paper

‣ If you had the same model set up available to you, what additional scenario could you run? why?

‣ Upload to Canvas before class - and we will finalize in class

EXTRA CREDIT ASSIGNMENT

1. Add a harvesting to your forest growth model from the previous assignment You can be as simple or complex as you want (e.g you could repeat what I did above by making harvesting proportional to biomass)

2. Perform some sensitivity analysis to assess how harvesting parameters interact with forest growth parameters to influence forest carbon at the end of the simulation time period (you can choose how long this is)

3. Write 2-3 sentences to comment on what you might learn from your sensitivity analysis

Speaker notes