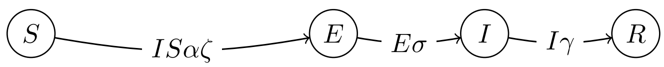
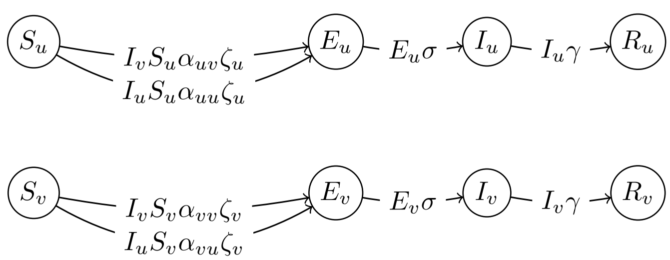
Subcritical Measles Outbreak Size

In this manuscript we model subcritical outbreaks of measles, comparing the outbreak sizes with and without heterogeneity in vaccination status.

The homogeneous model is a simple SEIR model:



The heterogeneous model is the same, but stratified into two population classes, vaccinated (v) and unvaccinated (u):



Parameter values are loosely based on known data, with consistency requirements. For the homogeneous model:

- $\alpha = 17/7$: rate of contact between individuals. Chosen so that $R_0 = 17$ would be true if ζ were at the unvaccinated level – see below.
- $\zeta = 0.0353$: probability a contact event is adequate (i.e. leads to infection if contact is between one susceptible and one infected individual), when contacts are well-mixed. Chosen to make $R_0 = \alpha\zeta/\gamma = 0.6$, given the above value of α.
- $\sigma = 1/14$: rate of transition from exposed to infectious
- $\gamma = 1/7$: rate of recovery

And for the stratified model:

- $p_v = 0.97$: proportion vaccinated.
- $\alpha_{uu} = 8$: rate of contact between unvaccinated individuals
- $\zeta_u = 1$: probability a contact with an unvaccinated susceptible individual is adequate.

The implied parameters ζ_v, α_{uv}, α_{vu}, and α_{vv} are derived from consistency conditions:

$\zeta = p_v\zeta_v + (1-p_v)\zeta_u$

$\alpha_{uv} = \alpha_{vu}$

$p_v\alpha_{uv} + (1-p_v)\alpha_{uu} = p_v\alpha_{vv} + (1-p_v)\alpha_{vu} = \alpha.$

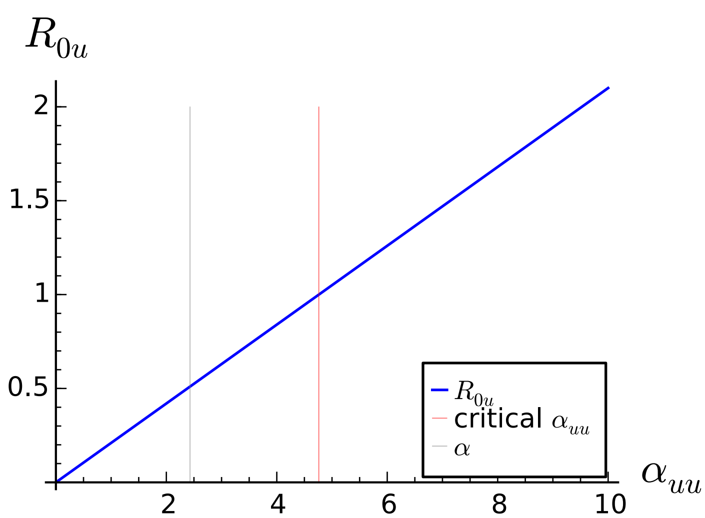
The derived values are

- $\zeta_v = 0.00546$
- $\alpha_{uv} = \alpha_{vu} = 2.26$
- $\alpha_{vv} = 2.43$

The main questions have to do with sizes of outbreaks and whether they get large when unvaccinated people cluster together, even though the population-wide R is subcritical.

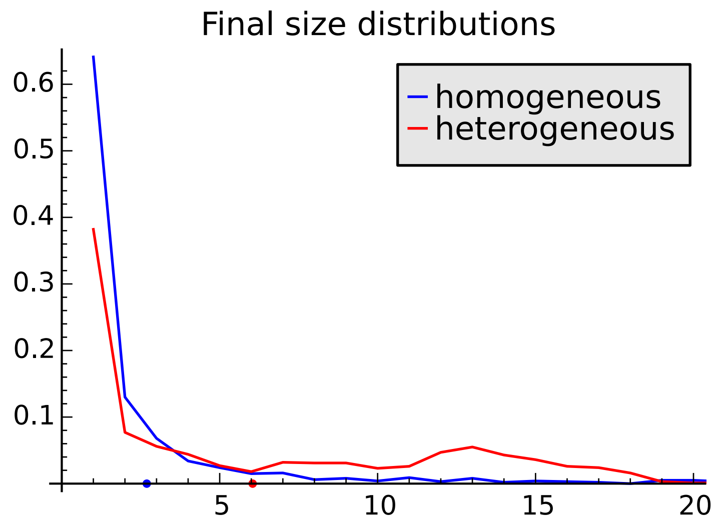
With the above parameters the initial spreading rate within the unvaccinated compartment is $R_{0u} = -\frac{\alpha_{uu}(p_v-1)\zeta_u}{\gamma} = 1.68$. I suggested in an email that this quantity should predict whether the infection can burn through that compartment.

R_{0u} is a linear function of α_{uu}:



As anticipated, sufficient contact among unvaccinated people can allow this simulated measles to become critical within that subpopulation. This critical value is attained at $\alpha_{uu} = 4.76$.

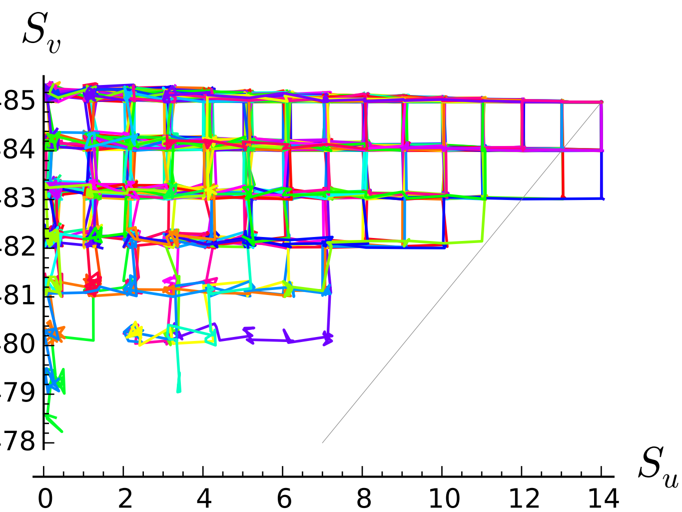
Here are the simulation results with the above values, showing the distribution of total outbreak size (final size):



These distributions are generated by 1000 simulations each, with a population of size 500 (thus with 15 unvaccinated), with initially one (unvaccinated) infectious individual and the rest susceptible. Small dots on the horizontal axis indicate the mean of each distribution.

One thing to do with this might be to vary α_{uu} from α upward and plot mean outbreak size vs. R_{0u} or something like that.

Here is a look at the time-varying dynamics of infection by compartment. The instantaneous direction of change reveals the ratio of infection rates of unvaccinated vs. vaccinated people.



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