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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

CONTROL SYSTEMS NOTES (15EC43)

(As per Choice based Credit System (CBCS) Scheme)
IVTH SEMESTER

MODULE-1

Syllabus: Introduction to Control Systems: Types of Control Systems, Effect of Feedback Systems, Differential equation of Physical Systems – Mechanical Systems, Electrical Systems, Analogous Systems. Block diagrams and signal flow graphs: Transfer functions, Block diagram algebra and Signal Flow graphs.

Study Material Referred:

- ✓ Modern Control engineering-K Ogata.
- ✓ Control Systems Engineering- J.Nagarath and M.Gopal.
- ✓ Automatic Control Systems-Benjamin C. Kuo.
- ✓ **VTU Previous year Question papers (2010-2016)**.
- ✓ Problems and solution of controls systems -AK Jairath.
- ✓ Control Systems –KR Varamah.
- ✓ Control System Engineering-Sathyanarayan and P Ramesh Babu

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Introduction to Control System :-

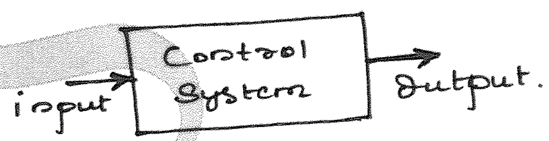
Control System :- It is an Arrangement of different physical Components such that it gives the desire Output for the given input by means of regulate or Control either direct or indirect method.

Plant :- It is defined as the portion of a system which is to be controlled or regulated. It is also called as Process.

Controller :- It is an Element of the system itself or may be external to the System. It Controls the plant or the process.

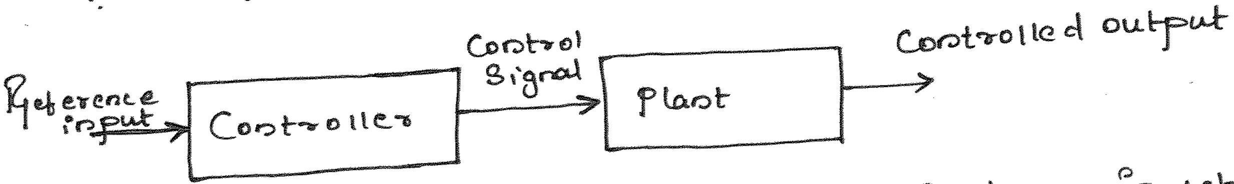
Input :- The applied Signal or Excitation Signal that is applied to a Control system to get a specified Output is Called input.

Output :- The actual response that is Obtained from a control system due to the application of the input is termed as Output



Types Of Control Systems :-

* Open - loop Control System :



* Open - loop Control system are Control systems in which the Output has no Effect upon the Control action.

* In such systems, there is no measurement of the Output and no subsequent use of that Output to generate any Control action.

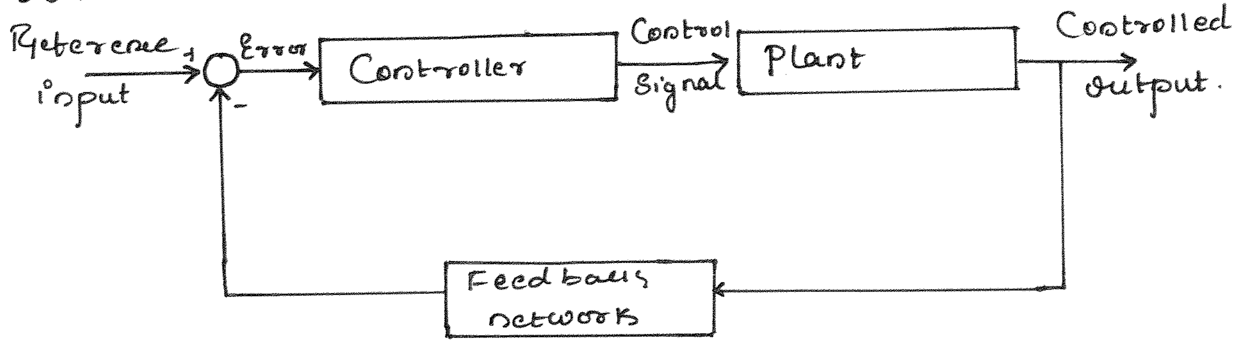
Advantages:-

* It is simple to Construct and Easy to maintain.
* It is less Expensive because of the use of minimum control devices.
* The problem of instability does not Exist.
* It is able to perform accurately once the calibration of the input is done.

Dis-advantages:-

* Disturbances, internal or External, causes drifts in the desired Output.
* Changes in Calibration Cause Errors in the system.
* Re-Calibration of the system may be necessary from time-to-time in order to maintain the required quality of the Output.

Closed-Loop Control System:-



* A closed-loop Control System is one that measures its output and adjusts its input accordingly by using a feed back signal.

* Feed back networks Consists of passive elements like R, L, C which is been used to feed back the obtained output to the input.

* The Controller subsequently produces the necessary Control Signal, which is then applied to the plant or process to reduce the error and bring the output of the system to the desired value.

* In closed loop system the functionality of the system depends on the difference between input and the feedback.

* A closed loop Control System is also called as feedback Control system.

Advantages:-

* Relatively more accurate and Expensive Components may be used to obtain an accurate Control of a given process.

* The influence of internal and External disturbances on the output can be made almost ineffective.

* Transient response of the system can be improved.

* Steady-state Error can be reduced.

Disadvantages:-

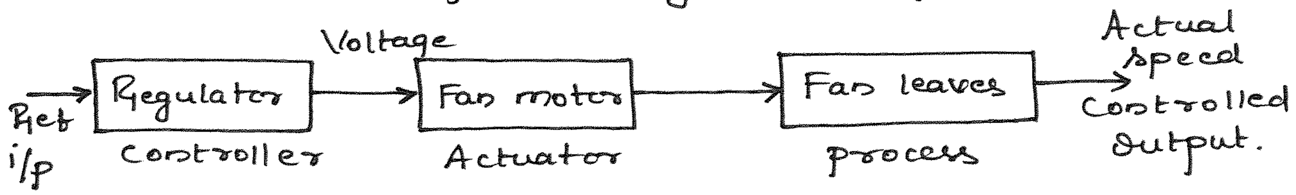
* It requires more Equipment and Components and it is costlier
* There is a tendency to Over Correct Errors, which may Create Oscillations in the System Output. This may Cause the System to drift to instability.

Comparission between Open-Loop and Closed-Loop Control System.

SL.NO	Open-Loop Control System	Closed-Loop Control System
1.	The Open-Loop System are simple to construct and cheap.	The Closed Loop System are Complicated to Construct and Costly.
2.	It Consumes less power	It Consumes more power.
3.	Any Change in the Output has no Effect On the input i.e, feed back does not Exists	Changes in the Output, affects the input which is possible by use of feed back.
4.	Highly sensitive to the disturbances	Less sensitive to the disturbances.
5.	It is inaccurate and un-reliable	Highly accurate and reliable.
6.	The Open-Loop Systems are generally stable.	More care is required to design a stable System
7.	Highly Sensitive to the Environmental Changes	Less sensitive to the Environmental Changes.

Application of an Open-Loop System :-

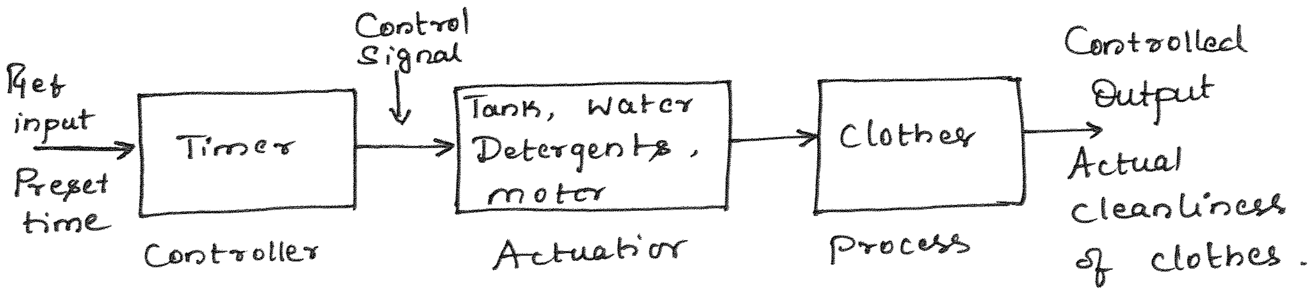
1) Control action of a celing-fan regulator :-



Block diagram of Fan Regulator

A fan regulator is a combination of a series switch and a speed regulator. For a given speed setting on the fan regulator, the fan runs at a specific speed. To obtain a different speed, the setting on the regulator is to be changed. This changes the voltage applied to the motor. The specific setting on the regulator is the reference input and the variable voltage applied to the fan motor is the control signal. The speed of the fan is the controlled output. the output speed is not measured so the control scheme is open-loop. The block diagram is shown in the figure above.

2) Working of an automatic washing machine :-

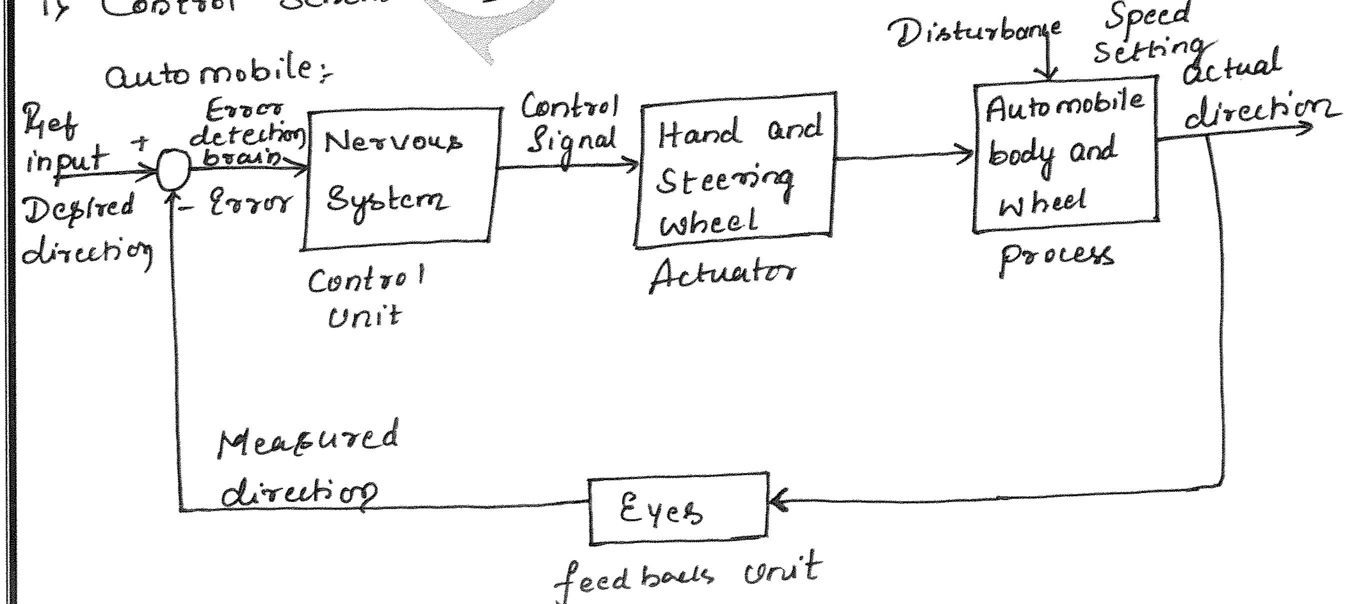


Block diagram of Automatic Washing Machine.

* An automatic washing machine is one with present wash times. The different operations like soaking, washing, rinsing, wringing and drying are all performed on a time basis.
* After the clothes to be washed are kept inside the tank, water and detergent are added in proper amounts
* The timer and the relays act as the controller. The tank, water, detergent and the motor constitute the actuator.
* The clothes that are to be washed from the process. There is no measurement of quality of wash. If cleanliness of clothes is the output parameter, there is no mechanism to judge it. The system is thus open-loop. The block diagram is shown above.

Applications of an closed loop Control System:-

1) Control Scheme in the manual steering system of an automobile:-



(Automobile Steering Control)

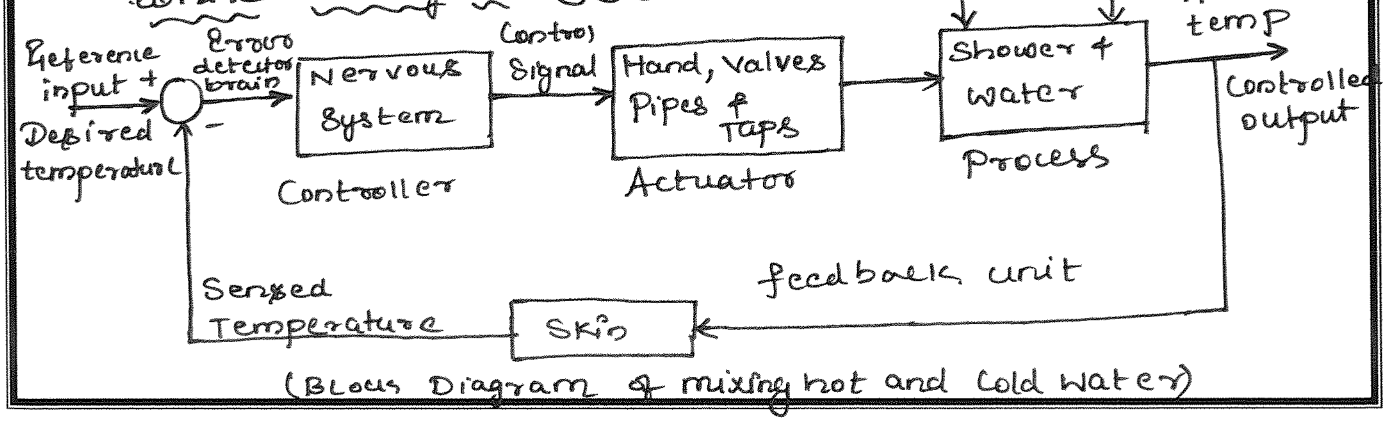
* The driver of an automobile watches the direction or heading of the vehicle with respect to a specified direction of the road. The Eye senses the deviation, if any, and the information is fed to the brain through the nervous system.

* The brain processes the signal and generates a corrective signal and it is transmitted to the hand and then to the steering wheel. The direction of the road is the reference input and heading of the automobile is the controlled input.

* The human eye acts as the Error detector. The brain and the nervous system acts as the Comparator and the Controller. The Corrective signal generated in the brain is the Control signal, which is transmitted to the hands.

* The human hands and the steering wheel form the Actuator unit, The automobile body and the wheels form the process. The scheme represents a manual feedback system and it is shown in the figure above.

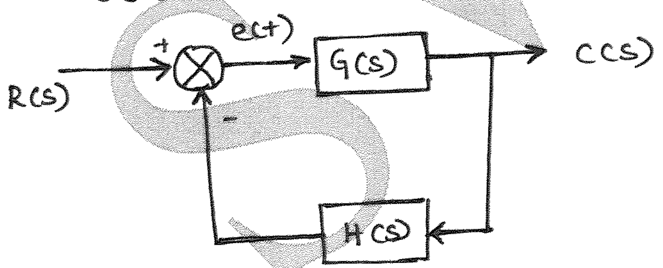
2) The Control scheme for mixing Cold and hot water while taking a shower.



(Block Diagram of mixing hot and Cold water)

* One must have some idea of the water temperature he/she wants while taking a shower. The skin acts as a temperature sensor, which measures the temperature not quantitatively but qualitatively, and conveys the information to the brain. There it is compared with the water temperature the person desires. the brain computes the differences in terms of 'too cold' or 'too hot' and activates the hand muscles to manipulate the hot and cold water valves to reduce the temperature if it is too hot or increase the temperature if it is too cold. This corrective action is reciprocal until the required water temperature is achieved. The block diagram is shown above.

Feedback System :-



* If Error signal e(t) is zero, output is controlled.
* If Error signal e(t) is not zero, output is not controlled.

For positive feedback, Error signal = r(t) + c(t)

For Negative feedback, Error signal = r(t) − c(t)

* The purpose of feedback is to reduce the error between the reference input and the system output.

⇒ +ve feed back

Unity feed back ($H(s)=1$)

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)}$$

Non-unity feed back ($H(s) \neq 1$)

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$$

⇒ −ve feed back

Unity feed back ($H(s)=1$)

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

Non-unity feed back ($H(s) \neq 1$)

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Where $G(s)$ = T.F of the forward path.
$H(s)$ = T.F of the feedback path.

* The feed back has Effects on such System performance. characteristics as stability, bandwidth, Overall gain, impedance and sensitivity.

* <u>Effect of feed back on stability</u>:-

* Stability is a notion that describes whether the System will be able to follow the input Command.
* A System is said to be unstable, if its Output is Out of Control.
* The closed loop System stability depends on loop gain. If loop gain $GH = 1$, the Output of the System becomes infinity for any finite input. and the System is said to be unstable.

* If the loop gain > 0 then system stability is improved, The feedback can improve stability or be harmful to stability if it is not properly applied

⟹ Effect of feedback on Overall gain :-

* feedback affects the gain G of a non-feedback System by a factor of $1 \pm GH$. The general effect of feedback is that it may increase or decrease the gain.

* In a practical control system, G and H are functions of frequency, so the magnitude of $1+GH$ may be >1 in on frequency range, but <1 in another. Therefore, feedback could increase the gain of the system in one frequency range but decrease it in another.

⟹ Effect of feedback on Sensitivity

* In general a good control system should insensitive to parameter variations but sensitive to input command.

* Consider G as a parameter that may vary. The sensitivity of the gain of the overall system M to the variation in G is defined as

$$S_G^M = \frac{\partial M/M}{\partial G/G}$$

* Where ∂M denotes the incremental change in M due to incremental change in G

$\partial M/M$ and $\partial G/G$ denote the percentage change in M and G respectively.

$$S_G^M = \frac{\partial M}{\partial G} \times \frac{G}{M} = \frac{1}{1+GH}$$

This relation shows that the sensitivity function can be made arbitrarily small by increasing GH, provided that the system remains stable. In an open-loop system, the gain of the system will respond in a one-to-one fashion to the variation in 'G'. In general, the sensitivity of the system gain of a feedback system to parameter variations depends on where the parameter is located.

⟹ Effects of feedback (In-Brief)

* Gain is reduced by a factor
* There is reduced of parameter variation by a factor $1+G(s)H(s)$
* There is improvement in sensitivity.
* There may be reduction of stability.

The disadvantages of reduction of gain and reduction of stability can be overcome by gain amplification and good design respectively.

* Feedback reduces the Effect of noise and disturbance on system performance.
* Bandwidth increases by the factor of $1+G(s)H(s)$
* The System becomes more accurate.

Mechanical Systems:-

Mechanical System are broadly classified into two groups.

1) Translational System.

2) Rotational system

Translational System:- In translational system, the motion of the body is along a straight line

Rotational System:- In Rotational System, the motion of the body is about it's own axis. (Circular Path).

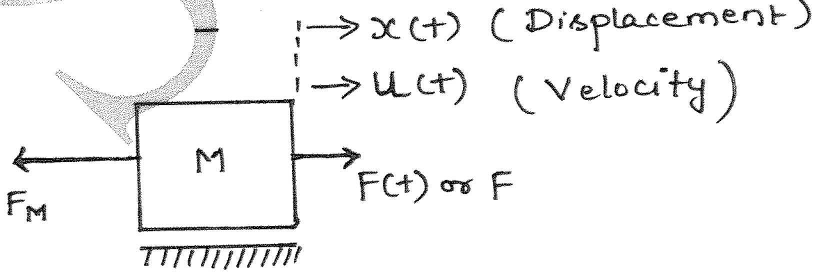
* The Basic Elements of translational System are.

① Mass

② Spring

③ Dashpot or friction.

① Mass:- Mass is the Energy (Kinetic) storage elements, where Energy Can be stored and retrieved without loss.



When a force F(t) is applied on the mass, it Produce an oppossing force F_M and it is given by

$$F_M \propto a \qquad F_M = Ma$$

$$F_M = M\frac{du(t)}{dt} = M\frac{d}{dt}\left(\frac{d\,x(t)}{dt}\right)$$

$$\boxed{F_M = M\frac{d^2x(t)}{dt^2}}$$

Where $x(t)$ and $u(t)$ are the displacement and the velocity respectively.

M is the mass, the force due to acceleration is given by

$$\boxed{F_M = M\frac{d^2x(t)}{dt^2}}$$

At Equilibrium according to network laws $F(t) = F_m$

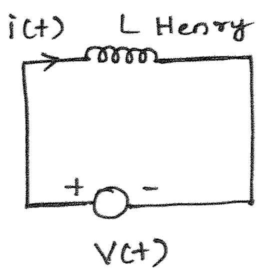
$$\therefore \boxed{F(t) = M\frac{d^2x(t)}{dt^2}}$$

W.K.T

M → Mass is an inertial Element it stores Energy in the form of Kinetic Energy is given by.

$$\boxed{K.E = W = \frac{1}{2}mu^2} \; J \quad \text{—— (1)}$$

inductance:



$$v(t) = L\frac{d\,i(t)}{dt} \quad \text{—— (2)}$$

$$v(t) = L\frac{d^2q(t)}{dt^2} \quad \text{—— (3)}$$

$$\boxed{W = \frac{1}{2}LI^2} \; J \quad \text{—— (4)}$$

Two Systems are said to be Analogous if the Mathematical Equations of the two Systems are identical

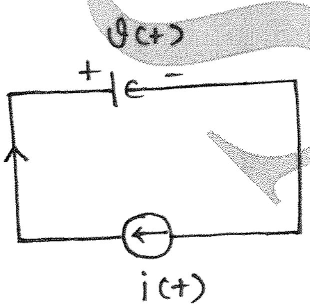
if $F(t) = V(t)$

and $M = L$ then $x(t) = q(t)$ or $u(t) = i(t)$

When force is Compared with Voltage the Corresponding Electric Circuit is said to be force -Voltage Analogous Circuit.

Mechanical System	Electrical System F-V Analogy
Force	Voltage
Velocity	Current
Displacement	Charge.
Mass	inductance.

Capacitance:-



$$i(t) = C\frac{d\,v(t)}{dt}$$

$$\text{but } v(t) = \frac{d}{dt}\phi(t),$$

$$i(t) = C\frac{d^2\phi(t)}{dt^2} \quad \text{—} \; (5)$$

$$\boxed{W = \frac{1}{2}CV^2(t)}\ J \quad \text{—} \; (6)$$

Comparing Equations (1) and (5) they are Analogous if

$F(t) = i(t)$

$M = C$

Then $x(t) = \phi(t)$

$u(t) = v(t)$

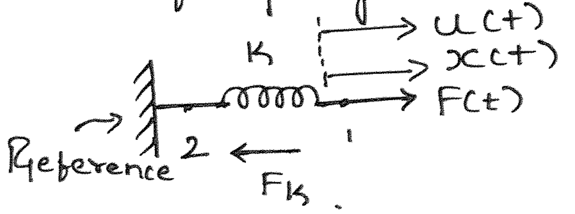
* Mass has only one displacement.

* Counter force produced by the mass is propotional to second derivative of displacement.

* When force is compared with voltage inductance is the electrical analog for mass and when force is compared with current capacitance is electrical analog for mass.

2) <u>Spring</u>:- Spring is the Energy (Potential) storage. Element where Energy can be stored and retrieved without loss.

(i) When one End of spring is connected to the reference.



For a linear spring counter force produced by the spring is propotional to net displacement of the spring.

$$F_K \propto (x(t) - 0)$$

$$F_K = K\,x(t)$$

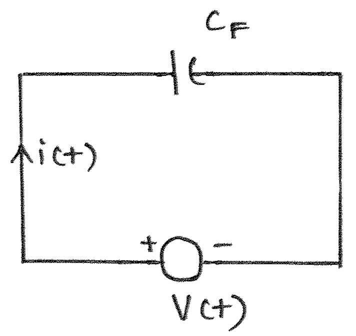
Where K is the constant of propotionality known as the spring constant.

At Equilibrium, according to Newton law.

$$F(t) = F_K$$

$$\boxed{F(t) = K\,x(t) = K\int u(t)\,dt} \quad \text{— ①}$$

<u>Capacitance</u>:-



$$i(t) = \frac{d\,q(t)}{dt}$$

$$q(t) = \int i(t)\,dt$$

$$V(t) = \frac{q(t)}{C} = \frac{1}{C}\int i(t)\,dt \quad \text{— ②}$$

Comparing Equation ① & ② They are Analogous

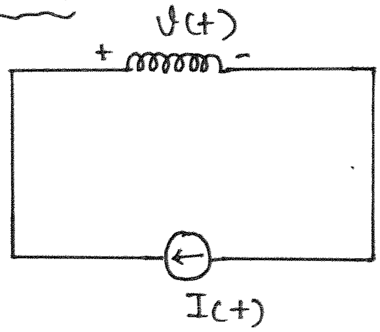
if $F(t) = V(t)$

$K = \frac{1}{C}$

$x(t) = q(t)$

$u(t) = i(t)$

Inductance:-



$$I(t) = \frac{1}{L}\int V(t)\,dt = \frac{1}{L}\phi(t) \quad \text{--- ③} \qquad \because V(t) = \frac{d}{dt}\phi(t)$$

Equations ① and ③ are Analogous if

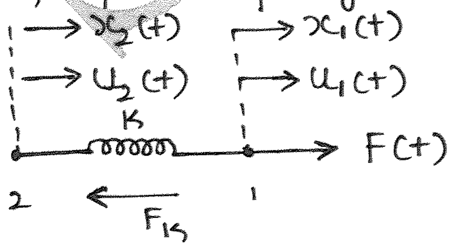
$F(t) = I(t)$

$K = \frac{1}{L}$

$x(t) = \phi(t)$

$u(t) = V(t)$

ii) When both Ends of the Spring are free to move.



Counter force produced by the Spring $F_K \propto (x_1(t) - x_2(t))$

$$F_K = K\left[x_1(t) - x_2(t)\right]$$

At Equilibrium according to Newton's law.

$$F(t) = F_K$$

$$F(t) = K\left[x_1(t) - x_2(t)\right]$$

* If One End of the Spring is connected to the reference it has one displacement and if both Ends are free to move it has two displacements.

* Counter force produced by Spring is proportional to net displacement of the Spring

* When force is Compared With Voltage, Capacitance is the Electrical Analog. When force is Compared With Current, inductance is the Electrical Analog for the Mechanical Element Spring.

3) Dashpot or Friction :-

* The friction Exists in physical Systems whenever Mechanical Surfaces are Operated in sliding contact.

* There are 3 types of Friction they are.

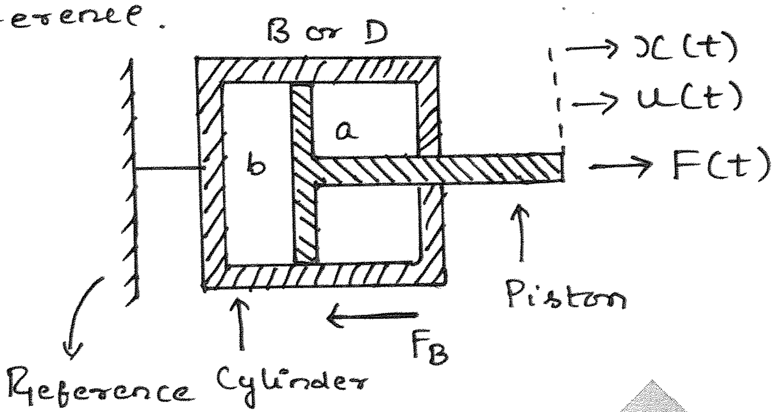
a) Coulomb friction force :- This is the force of sliding friction between dry surfaces. Coulomb friction force is substantially Constant.

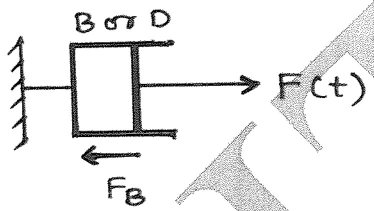
b) Viscous friction force :- It is the friction between moving surfaces separated by a Viscous fluid or between a solid body and a fluid medium. it is Proportional to the Velocity. it is predominates

c) Stiction :- This is the force required to initiate motion between two Contacting Surfaces.

Viscous friction :-

Case i> When one End of the dashpot is connected to the reference.





The Counter force produced by the Dashpot is Propotional relative velocity between piston and the cylinder.

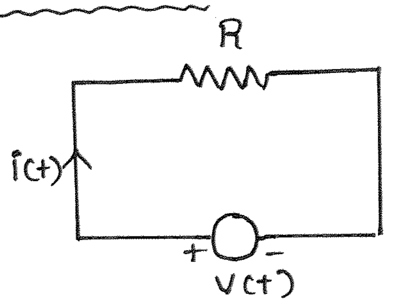
$$F_B \propto (u(t) - 0)$$

$$F_B = B\,u(t) = B\frac{dx(t)}{dt}$$

B is the constant of propotionality known as the viscous friction Co-efficient. At Equilibrium according to Newton law $\boxed{F(t) = F_B}$

$$\boxed{F(t) = B\,u(t) = B\frac{dx(t)}{dt}} \quad — (1)$$

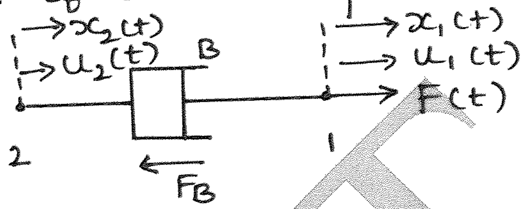
Resistance:-



$$V(t) = R \cdot i(t) = R\frac{dq(t)}{dt}$$

Case ii:

When both Ends of the dashpot are free to move.



The Counter force produced by the dashpot is

$$F_B \propto (u_1(t) - u_2(t))$$

$$F_B = B\left[u_1(t) - u_2(t)\right]$$

$$F_B = B\left[\frac{d}{dt}x_1(t) - \frac{d}{dt}x_2(t)\right]$$

At Equilibrium according to Newton's law.

$$F(t) = F_B$$

$$F(t) = B\left[u_1(t) - u_2(t)\right] = B\frac{d}{dt}\left[x_1(t) - x_2(t)\right]$$

* When One End of the dashpot is Connected to the reference it has One displacement and if both Ends are free to move it has two displacement.

* Counter force produced by the dashpot is propotional first derivative of net displacement.

* When force is Compared with Voltage resistance is the Electrical analogy for dashpot and when

force is compared with current, Conductance is the Electrical analogy for the Mechanical element dashpot.

$$F(t) = B\,u(t) = B\frac{d\,x(t)}{dt}$$

$$V(t) = R\,i(t) = R\frac{d\,q(t)}{dt}$$

$$i(t) = \frac{V(t)}{R} = G\,u(t) = G\frac{d\,\phi(t)}{dt}$$

Tabulation for converting Mechanical System to Force-Voltage and Force-Current analogy.

Mechanical System	Electrical System	
	F-V Analogy	F-I Analogy
Force [F(t)]	Voltage [V(t)]	Current [I(t)]
Velocity [u(t)]	Current [I(t)]	Voltage [V(t)]
Displacement [x(t)]	Charge [Q(t)]	Magnetic Flux [$\phi(t)$]
Mass [M]	Inductance [L]	Capacitance [C]
Spring Constant [K]	Reciprocal of Capacitance [$\frac{1}{C}$]	Reciprocal of inductance [$\frac{1}{L}$]
Compliance [$\frac{1}{K}$]	Capacitance [C]	Inductance [L]
Dashpot Constant [B]	Resistance [R]	Conductance [G]

Things to remember

F-V Analogy:-

$$\Rightarrow \frac{dq(t)}{dt} = i(t)$$

$$\Rightarrow q(t) = \int i(t)\,dt$$

F-I Analogy:-

$$\Rightarrow \frac{d\phi(t)}{dt} = V(t)$$

$$\Rightarrow \phi(t) = \int V(t)\,dt.$$

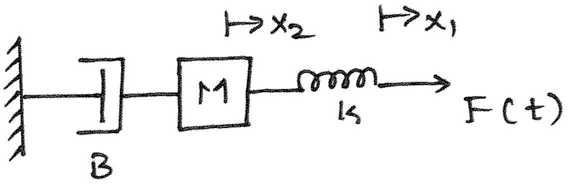
Note:-

* If the force directly acts on the mass, then the number of displacement is Equal to number of masses in the system, provided there is no direct series connected of two springs or two dashpot or dashpot and a spring.

* If the force directly acts on the spring or the dashpot then the number of displacement in the Mechanical System is Equal to number of masses +1. Provided there is no direct series Connection of two spring or two dashpot and spring and a dashpot.

Problems to be solved in class

1)



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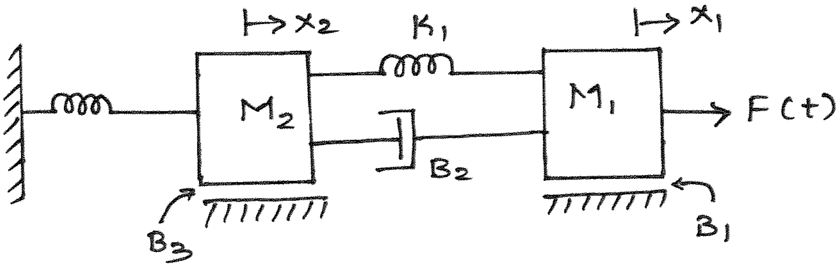

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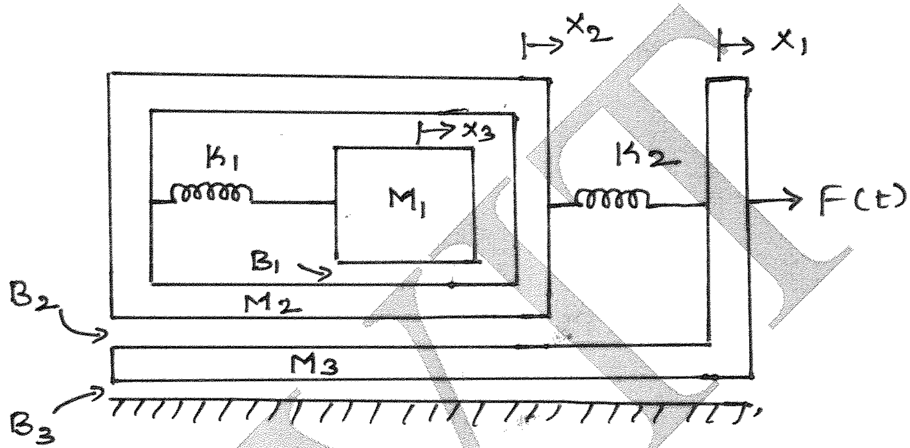

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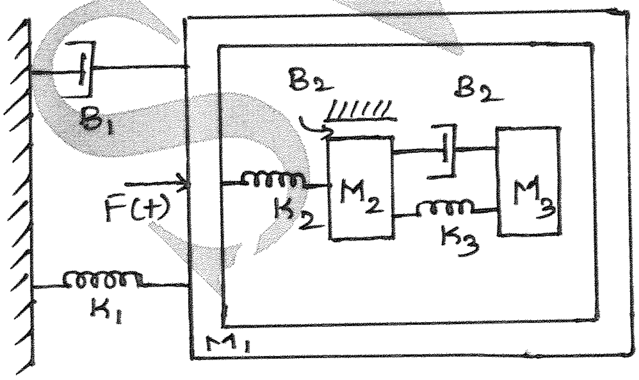

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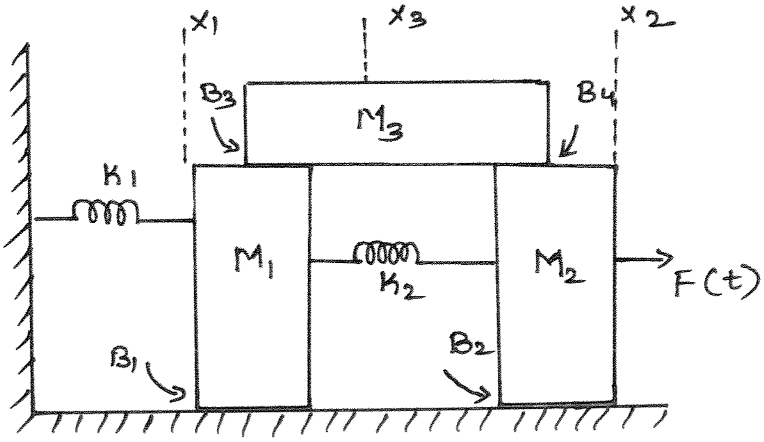
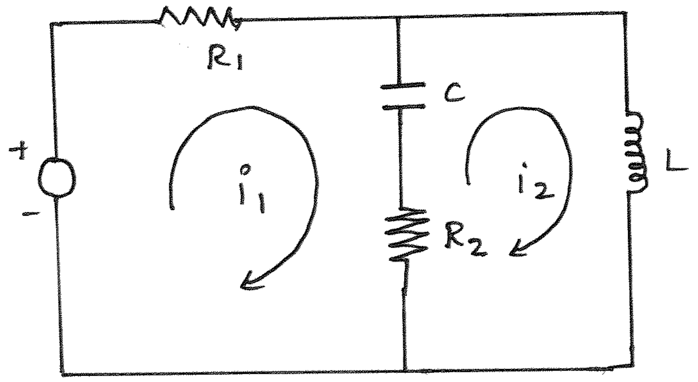
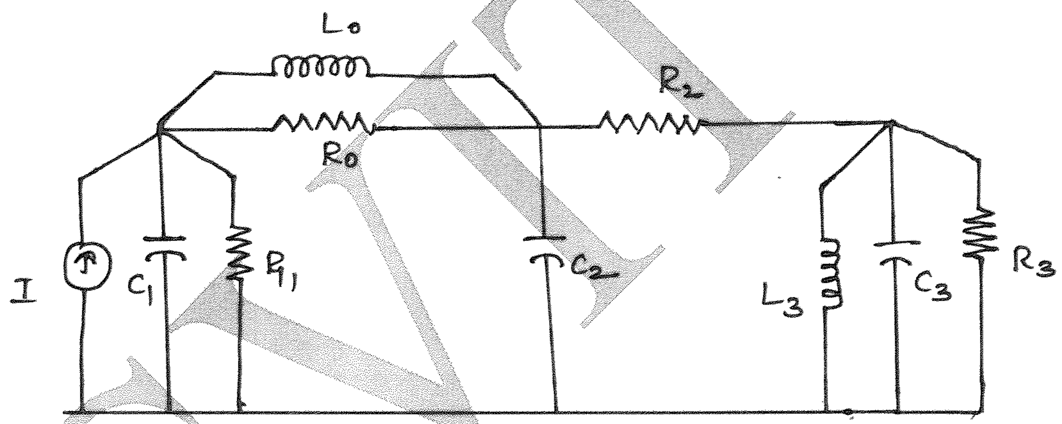


6) Draw the mechanical System for a given force Voltage. analogous Electric Circuit



7) Draw the Mechanical System for a given force Current Analogous Electric Circuit.

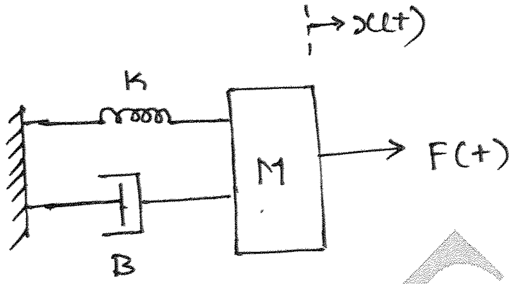


Problems on Translational Systems:-

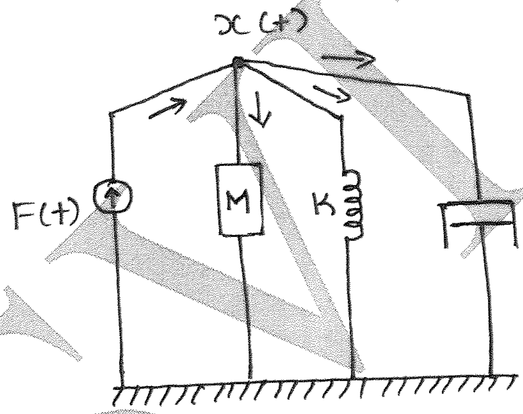
⇒ Write differential Equations for the mechanical systems shown below and also Electrical Analogous Circuit based on force Voltage analogy or force Current analogy.

Dec 2009

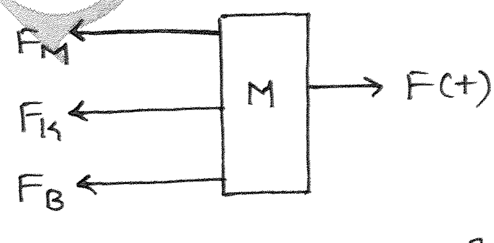
1)



Solution:- The mechanical network for a given mechanical system is as shown below.



The free body diagram is as follows



The Equilibrium Equation of a System are

$$F(t) = F_M + F_B + F_K$$

$$F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) \longrightarrow (1)$$

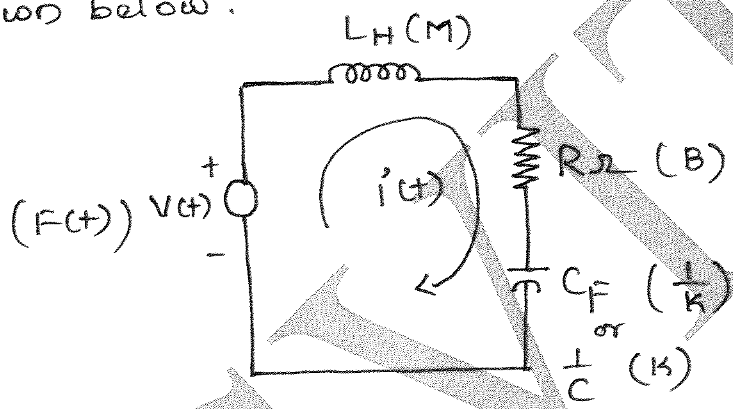
F-V Analogy:- Substituting the Electrical Analagous based on force Voltage analogy in Equation ①

$$V(t) = L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t)$$

But $i(t) = \frac{dq(t)}{dt}$ & $\int i(t)\,dt = q(t)$

$$V(t) = L\frac{d\,i(t)}{dt} + R\,i(t) + \frac{1}{C}\int i(t)\,dt \rightarrow ②$$

The Electrical Circuit Satisfying Equation ② is as shown below.



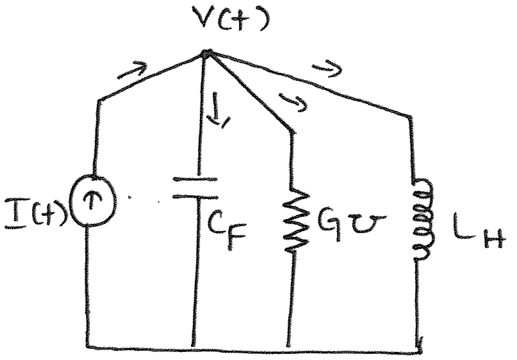
F-I Analogy:- Substituting the Electrical analogous based on ~~force~~ Current analogy in Equation ① we get.

$$I(t) = C\frac{d^2\phi(t)}{dt^2} + G\frac{d\phi(t)}{dt} + \frac{1}{L}\phi(t)$$

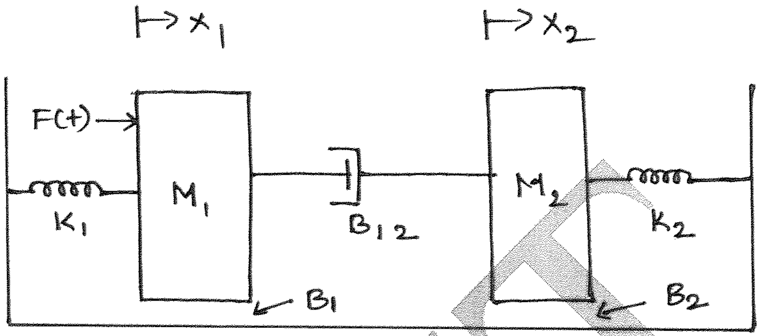
But $V(t) = \frac{d\phi(t)}{dt}$ & $\int V(t)\,dt = \phi(t)$

$$I(t) = C\frac{dV(t)}{dt} + G\,V(t) + \frac{1}{L}\int V(t)\,dt \quad — ③$$

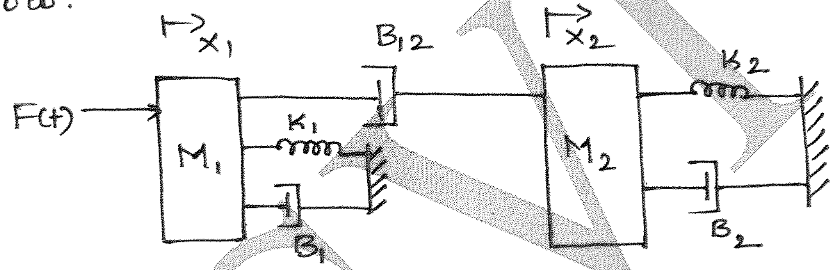
The Electrical Circuit Satisfying Equation ③ is as shown below.



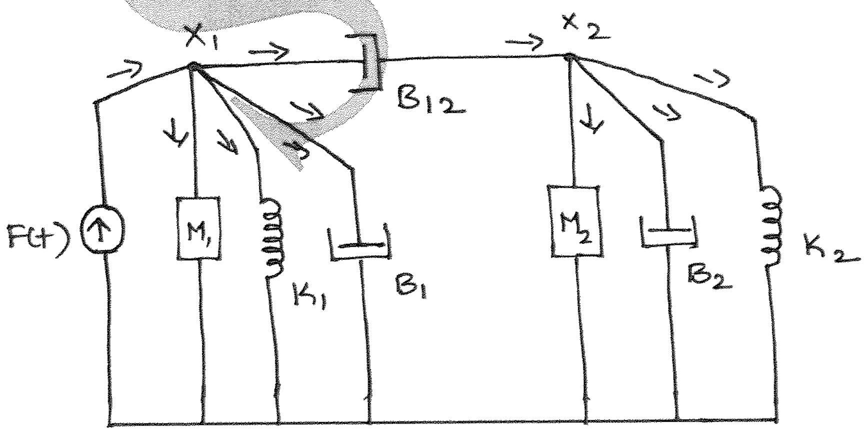
2)



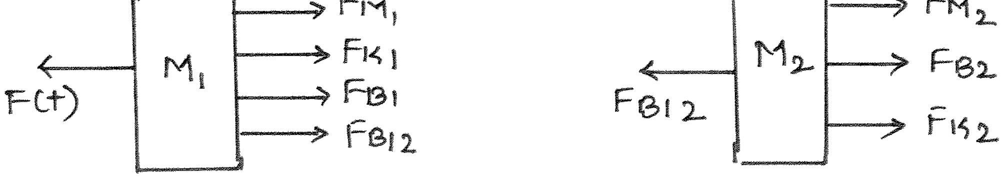
Solution:- The mechanical system is redrawn as shown below.



The Mechanical network is as shown below.



Free body diagram :-



The Equilibrium Equations are given by.

At x_1

$$F(t) = F_{M_1} + F_{K_1} + F_{B_1} + F_{B_{12}}$$

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B_1 \frac{d x_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) \longrightarrow ①$$

At x_2

$$F_{B_{12}} = F_{M_2} + F_{B_2} + F_{K_2}$$

$$B_{12} \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{d x_2}{dt} + K_2 x_2 \longrightarrow ②$$

F-V Analogy:-

Substituting Electrical analogous based on Force voltage analogy in Equation ① + ②

from ①

$$V(t) = L_1 \frac{d^2 q_1}{dt^2} + \frac{1}{C_1} q_1 + R_1 \frac{d q_1}{dt} + R_{12} \frac{d}{dt}(q_1 - q_2)$$

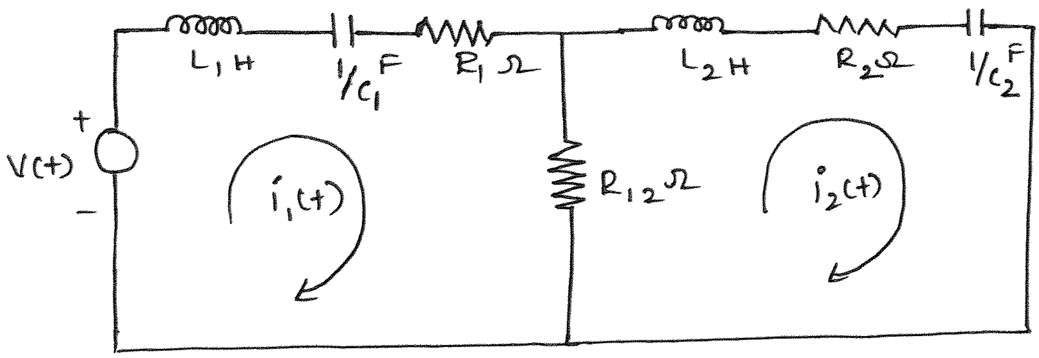
$$V(t) = L_1 \frac{d i_1}{dt} + \frac{1}{C} \int i_1 \, dt + R_1 i_1 + R_{12}(i_1 - i_2) \longrightarrow ③$$

from ②

$$R_{12} \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2 q_2}{dt^2} + R_2 \frac{d q_2}{dt} + \frac{1}{C_2} q_2$$

$$R_{12}(i_1 - i_2) = L_2 \frac{d i_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2 \, dt \longrightarrow ④$$

The Electrical Circuit Satisfying Equations ③ and ④ is as shown below.



F-I Analogy:- Substituting electrical analogous based on force current analogy in Equations ① & ② we get

from ①

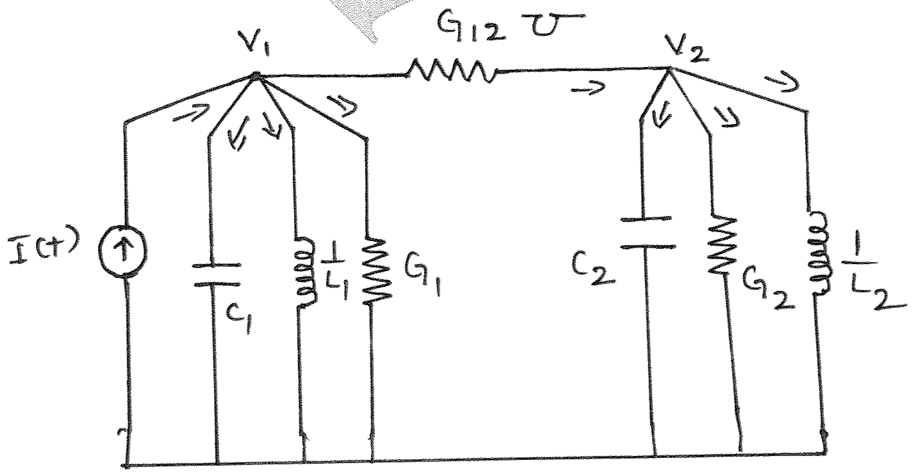
$$I(t) = C_1 \frac{d^2\phi_1}{dt^2} + \frac{1}{L_1}\phi_1 + G_1 \frac{d}{dt}\phi_1 + G_{12}\frac{d}{dt}(\phi_1 - \phi_2)$$

$$I(t) = C_1 \frac{d}{dt}V_1 + \frac{1}{L_1}\int V_1\, dt + G_1 V_1 + G_{12}(V_1 - V_2) \rightarrow (5)$$

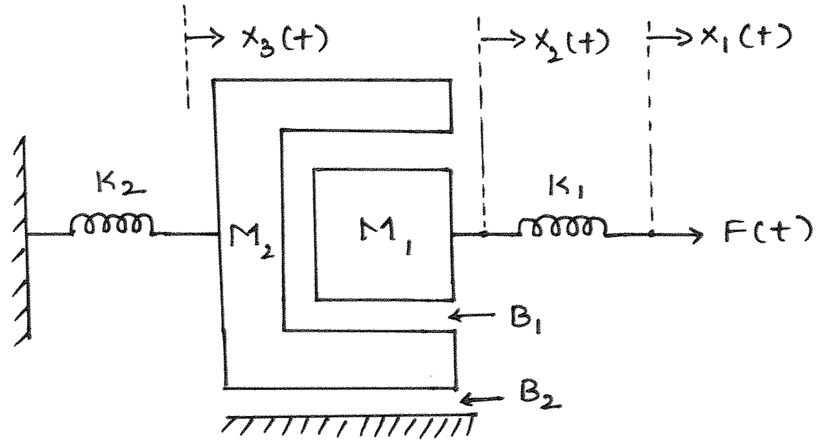
from ②

$$G_{12}\frac{d}{dt}(\phi_1 - \phi_2) = C_2 \frac{d^2}{dt^2}\phi_2 + G_2 \frac{d}{dt}\phi_2 + \frac{1}{L_2}\phi_2$$

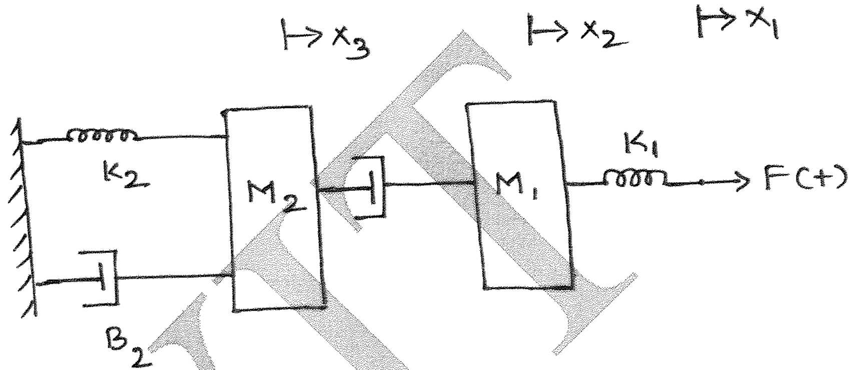
$$G_{12}(V_1 - V_2) = C_2 \frac{d}{dt}V_2 + G_2 V_2 + \frac{1}{L_2}\int V_2\, dt \rightarrow (6)$$



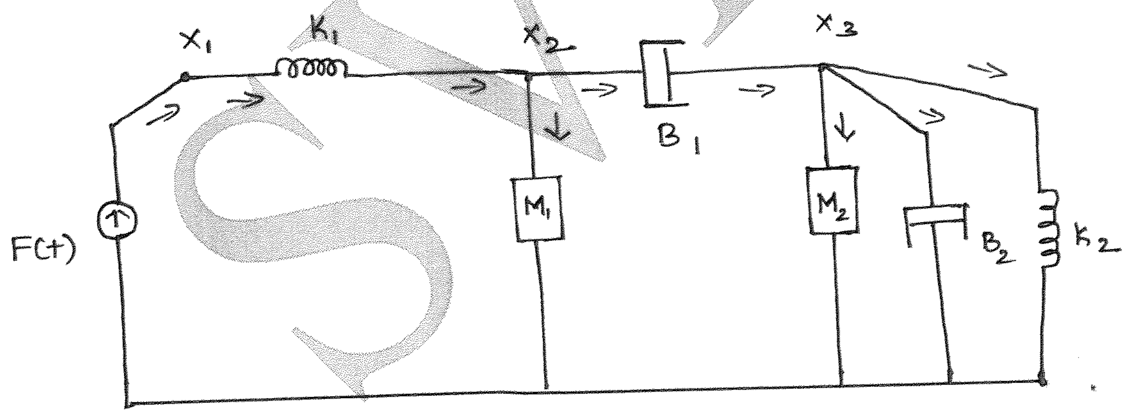
3)



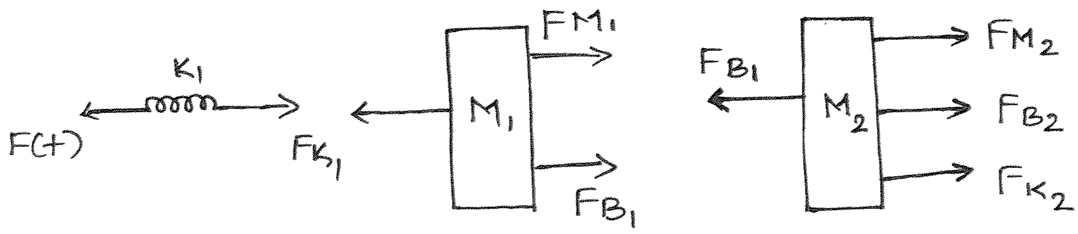
Solution :- The mechanical system is redrawn as shown below.



The Mechanical network is as shown below.



Free body diagram :-



Equilibrium Equations for a given system are.

At x_1; $F(t) = K_1(x_1 - x_2) \longrightarrow (1)$

At x_2: $K_1(x_1 - x_2) = M_1 \frac{d^2 x_2}{dt^2} + B_1 \frac{d}{dt}(x_2 - x_3) \longrightarrow (2)$

At x_3: $B_1 \frac{d}{dt}(x_2 - x_3) = M_2 \frac{d^2 x_3}{dt^2} + B_2 \frac{d x_3}{dt} + K_2 x_3 \longrightarrow (3)$

F-V Analogy:- By substituting electrical analogous based on force - Voltage analogy in equation (1), (2) & (3)

From (1)

$$V(t) = \frac{1}{C_1}(q_1 - q_2)$$

$$V(t) = \frac{1}{C_1}\int (i_1 - i_2)\,dt \longrightarrow (4)$$

from (2)

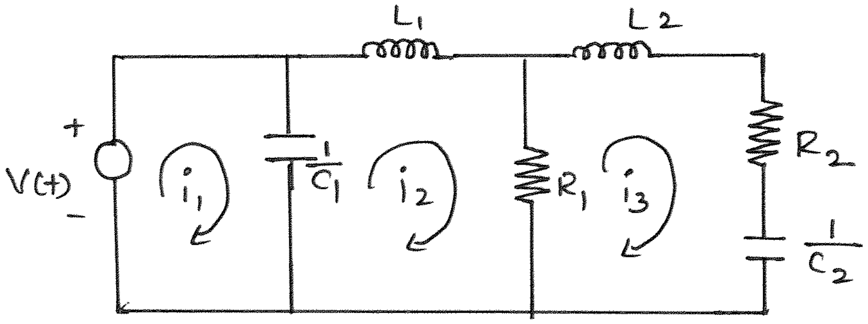
$$\frac{1}{C_1}(q_1 - q_2) = L_1 \frac{d^2 q_2}{dt^2} + R_1 \frac{d(q_2 - q_3)}{dt}$$

$$\frac{1}{C_1}\int (i_1 - i_2)\,dt = L_1 \frac{d\,i_2}{dt} + R_1(i_2 - i_2) \longrightarrow (5)$$

From (3)

$$R_1 \frac{d}{dt}(q_2 - q_3) = L_2 \frac{d^2 q_3}{dt^2} + R_2 \frac{d q_3}{dt} + \frac{1}{C_2} q_2$$

$$R_1(i_2 - i_3) = L_2 \frac{d}{dt} i_3 + R_2 i_3 + \frac{1}{C_2}\int i_3\,dt \longrightarrow (6).$$



F-I Analogy:- By substituting electrical analogous based force current analogy in equations (1), (2) and (8)

from (1)

$$I(t) = K_1(\phi_1 - \phi_2)$$

$$I(t) = \frac{1}{L_1}\int (V_1 - V_2)\, dt \quad \text{— (7)}$$

From (2)

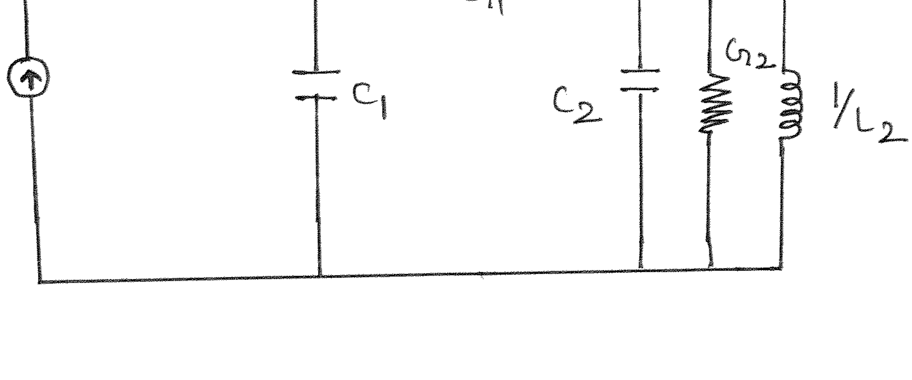
$$\frac{1}{L_1}(\phi_1 - \phi_2) = C_1 \frac{d^2\phi_2}{dt^2} + G_1 \frac{d}{dt}(\phi_2 - \phi_3)$$

$$\frac{1}{L_1}\int (V_1 - V_2)\, dt = C_1 \frac{d}{dt} V_2 + G_1 (V_2 - V_3) \quad \text{— (8)}$$

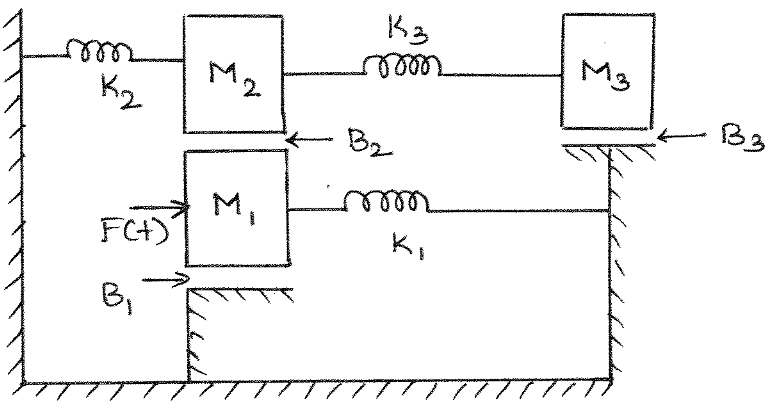
From (3)

$$G_1 \frac{d}{dt}(\phi_2 - \phi_3) = C_2 \frac{d^2}{dt^2}\phi_3 + G_2 \frac{d}{dt}\phi_3 + \frac{1}{L_2}\phi_3$$

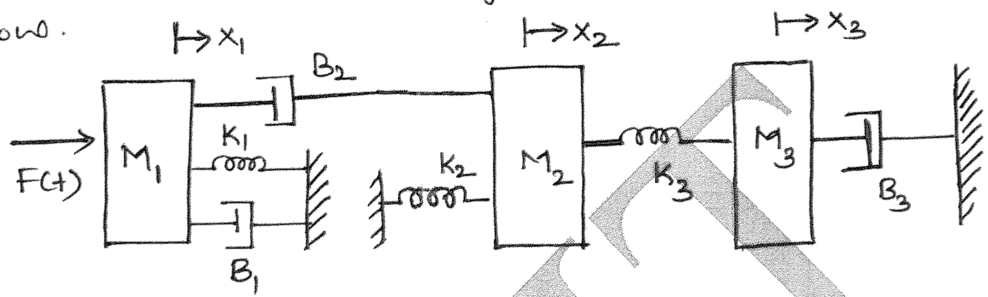
$$G_1 (V_2 - V_3) = C_2 \frac{d}{dt} V_3 + G_2 V_3 + \frac{1}{L_2}\int V_3\, dt \rightarrow (9).$$



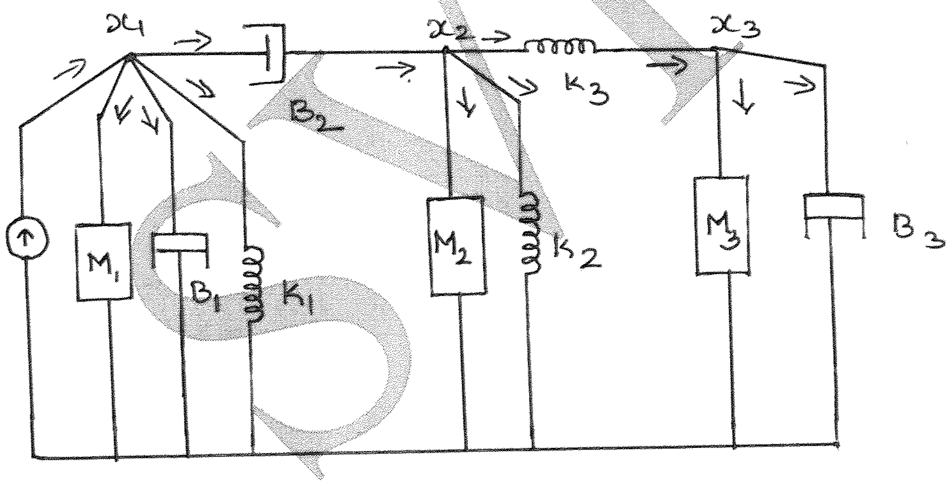
4)



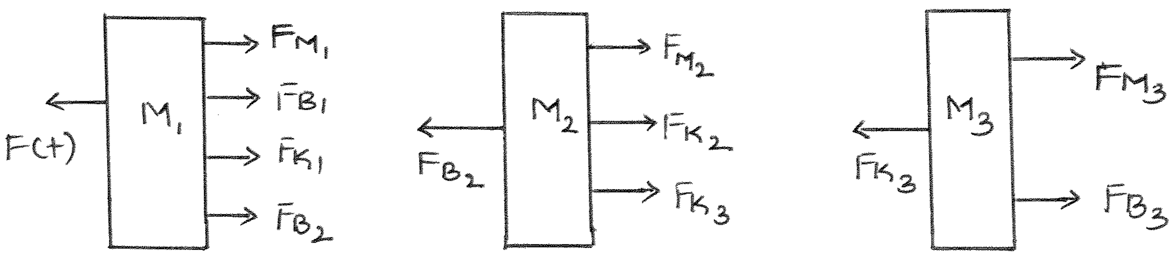
Solution: The mechanical system is redrawn as shown below.



The Mechanical network is as shown below.



Free body diagram:



Equilibrium Equations are given by.

At x_1 ; $F(t) = F_{M_1} + F_{B_1} + F_{K_1} + F_{B_2}$

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d x_1}{dt} + K_1 x_1 + B_2 \frac{d}{dt}(x_1 - x_2) \longrightarrow (1)$$

At x_2 ; $F_{B_2} = F_{M_2} + F_{K_2} + F_{K_3}$

$$B_2 \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 + K_3 (x_2 - x_3) \longrightarrow (2)$$

At x_3 ;

$$F_{K_3} = F_{m_3} + F_{B_3}$$

$$K_3 (x_2 - x_3) = M_3 \frac{d^2 x_3}{dt^2} + B_3 \frac{d x_3}{dt} \longrightarrow (3)$$

F-V Analogy:-

By Substituting Electrical Analogous based on force Voltage analogy in Equation (1), (2) and (3)

From 1:-

$$V(t) = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{d q_1}{dt} + \frac{1}{C_1} q_1 + R_2 \frac{d}{dt}(q_1 - q_2)$$

$$V(t) = L_1 \frac{d i_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 dt + R_2 (i_1 - i_2) \longrightarrow (4)$$

From 2:-

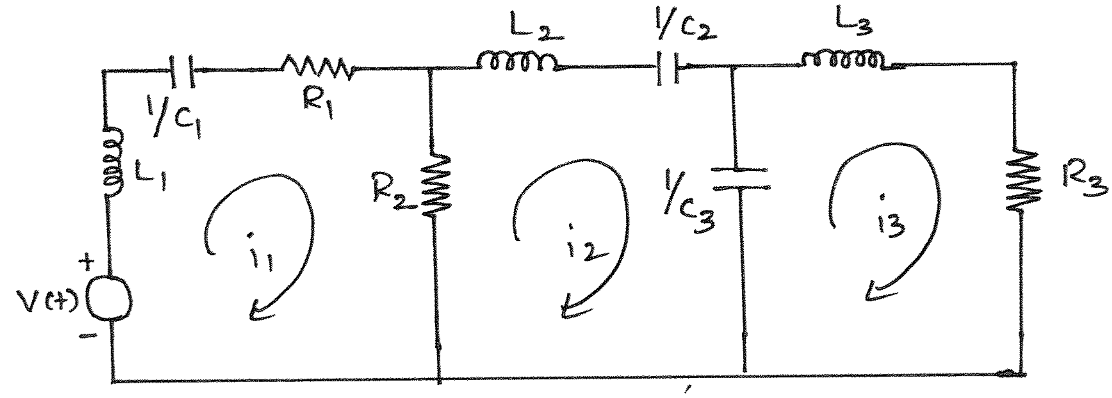
$$R_2 \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2 q_2}{dt^2} + \frac{1}{C_2} q_2 + \frac{1}{C_3}(q_2 - q_3)$$

$$R_2 (i_1 - i_2) = L_2 \frac{d i_2}{dt} + \frac{1}{C_2}\int i_2 dt + \frac{1}{C_3}\int (i_2 - i_3) dt \longrightarrow (5)$$

From 3:-

$$\frac{1}{C_3}(q_2 - q_3) = L_3 \frac{d^2 q_3}{dt^2} + R_3 \frac{d q_3}{dt}$$

$$\frac{1}{C_3}\int (i_2 - i_3) dt = L_3 \frac{d i_3}{dt} + R_3 i_3 \longrightarrow (5)$$



F-I Analogy:-

By substituting Electrical Analogous based on the force current analogy in Equations ①, ② and ③

From ①

$$I(t) = C_1 \frac{d^2 \phi_1}{dt^2} + \frac{1}{L_1}\phi_1 + G_1 \frac{d\phi_1}{dt} + G_2 \frac{d}{dt}(\phi_1 - \phi_2)$$

$$I(t) = C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int V_1\,dt + G_1 V_1 + G_2 (V_1 - V_2) \rightarrow ⑦$$

From ②

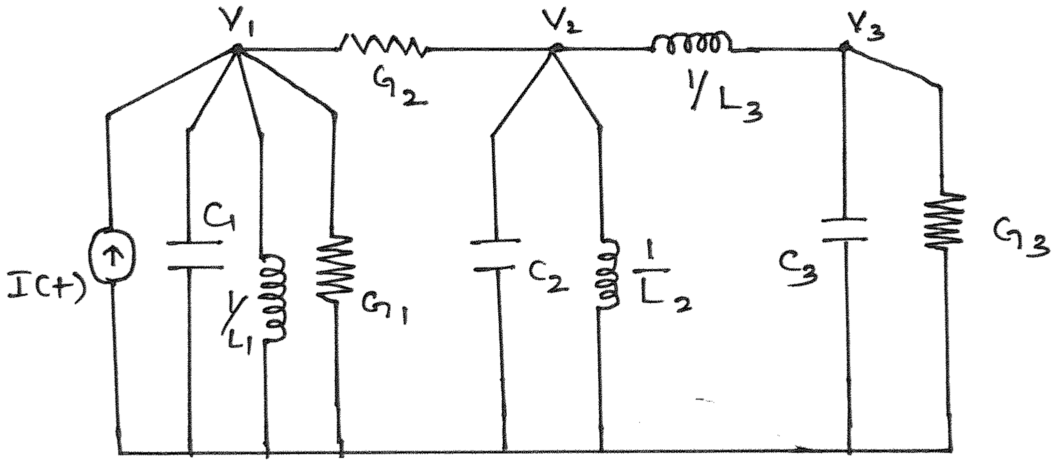
$$G_2 \frac{d}{dt}(\phi_1 - \phi_2) = C_2 \frac{d^2 \phi_2}{dt^2} + \frac{1}{L_2}\phi_2 + \frac{1}{L_3}(\phi_2 - \phi_3)$$

$$G_2 (V_1 - V_2) = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_3}\int (V_2 - V_3)\,dt \rightarrow ⑧$$

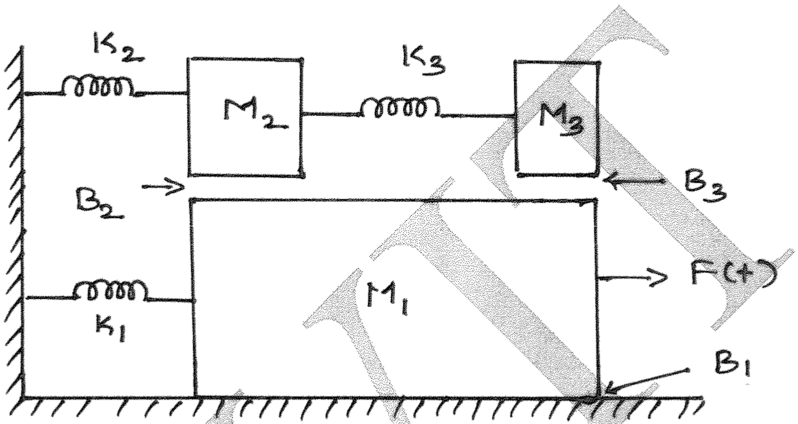
From ③

$$\frac{1}{L_3}(\phi_2 - \phi_3) = C_3 \frac{d^2 \phi_3}{dt^2} + G_3 \frac{d}{dt}\phi_3$$

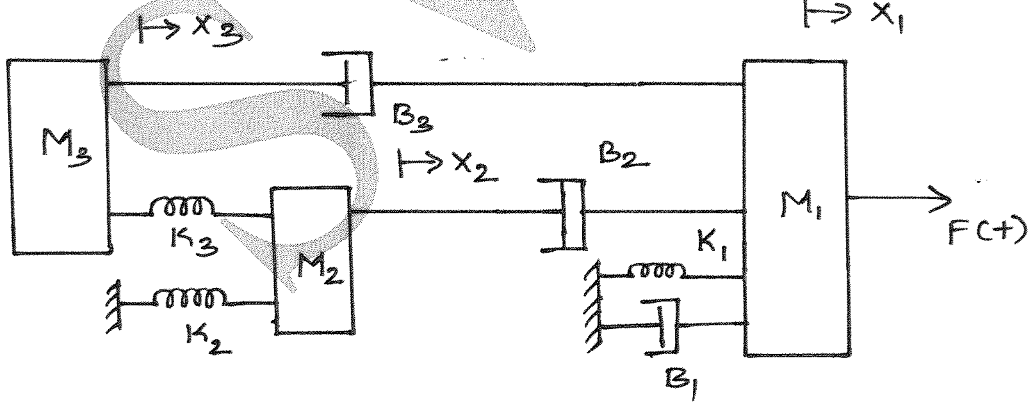
$$\frac{1}{L_3}\int (V_2 - V_3)\,dt = C_3 \frac{dV_3}{dt} + G_3 V_3 \longrightarrow ⑨$$



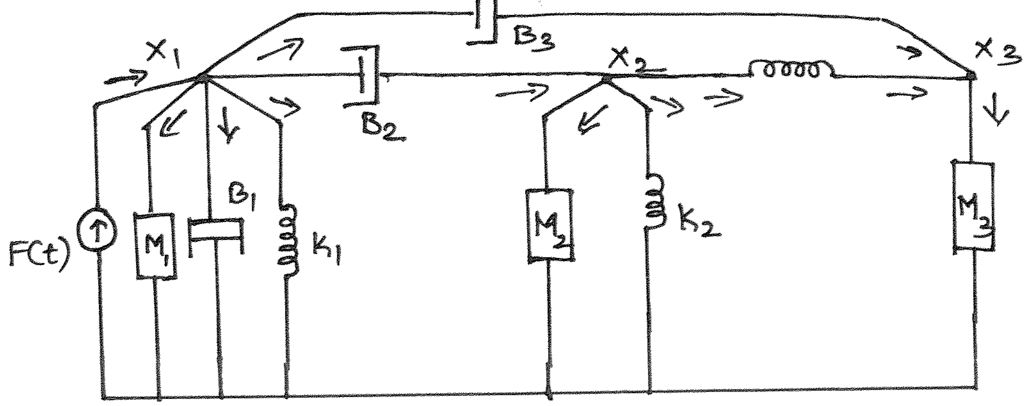
5)



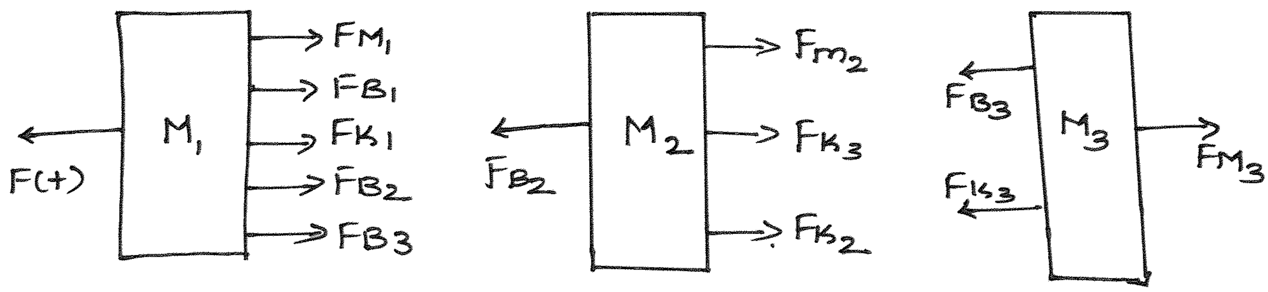
Solution:- Mechanical System is redrawn as shown below.



Mechanical network is as shown below.



Free body diagram :-



The Equilibrium Equations are given by

At x_1 ;

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B_2 \frac{d}{dt}(x_1 - x_2) + B_3 \frac{d}{dt}(x_1 - x_3) \rightarrow ①$$

At x_2 :-

$$B_2 \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 + K_3 (x_2 - x_3) \longrightarrow ②$$

At x_3 :-

$$K_3 (x_2 - x_3) + B_3 \frac{d}{dt}(x_1 - x_3) = M_3 \frac{d^2 x_3}{dt^2} \longrightarrow ③$$

F-V Analogy:-

By substituting electrical analogous based on force voltage analogy in Equation ①, ② and ③.

From ①

$$V(t) = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{1}{C_1} q_1 + R_2 \frac{d}{dt}(q_1 - q_2) + R_3 \frac{d}{dt}(q_1 - q_3)$$

$$V(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + R_2 (i_1 - i_2) + R_3 (i_1 - i_3) \rightarrow ④$$

From x_2 :-

$$R_2 \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2 q_2}{dt^2} + \frac{1}{C_2} q_2 + \frac{1}{C_3}(q_2 - q_3)$$

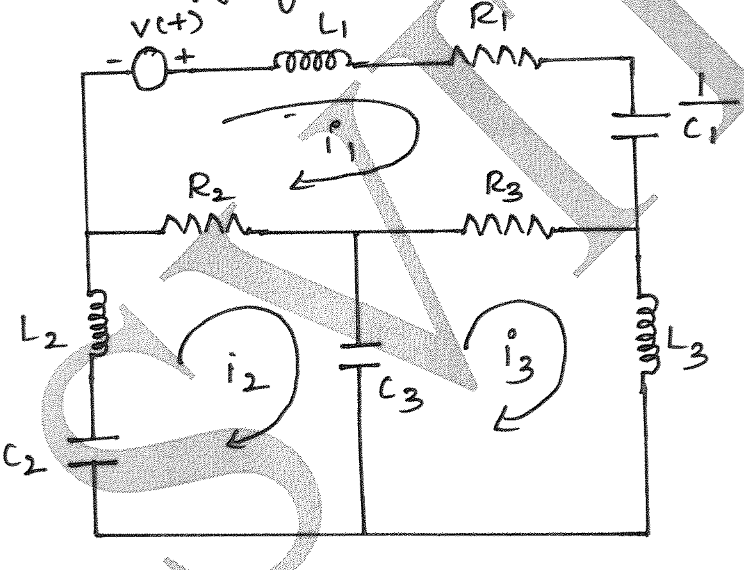
$$R_2 (i_1 - i_2) = L_2 \frac{d i_2}{dt} + \frac{1}{C_2}\int i_2\, dt + \frac{1}{C_3}\int (i_2 - i_3) dt \quad - (5)$$

From x_3 :-

$$\frac{1}{C_3}(q_2 - q_3) + R_3 \frac{d}{dt}(q_1 - q_3) = L_3 \frac{d^2}{dt^2} q_3$$

$$\frac{1}{C_3}\int (i_2 - i_3) dt + R_3 (i_1 - i_3) = L_3 \frac{d i_3}{dt} \longrightarrow (6)$$

The Circuit Satisfying Equation (4), (5) and (6)



F-I Analogy :-

By substituting electrical analogous based on F-I analogy in Equation (1), (2) and (3)

From (1)

$$I(t) = C_1 \frac{d^2 \phi_1}{dt^2} + G_1 \frac{d\phi_1}{dt} + \frac{1}{L_1}\phi_1 + G_2 \frac{d}{dt}(\phi_1 - \phi_2) + G_3 \frac{d}{dt}(\phi_1 - \phi_3)$$

$$I(t) = C_1 \frac{d V_1}{dt} + G_1 V_1 + \frac{1}{L_1}\int i_1\, dt + G_2 (V_1 - V_2) + G_3 (V_1 - V_3) \rightarrow (7)$$

From x_2 :-

$$G_2 \frac{d}{dt}(\phi_1-\phi_2) = C_2 \frac{d^2}{dt^2}\phi_2 + \frac{1}{L_2}\phi_2 + \frac{1}{L_3}(\phi_2-\phi_3)$$

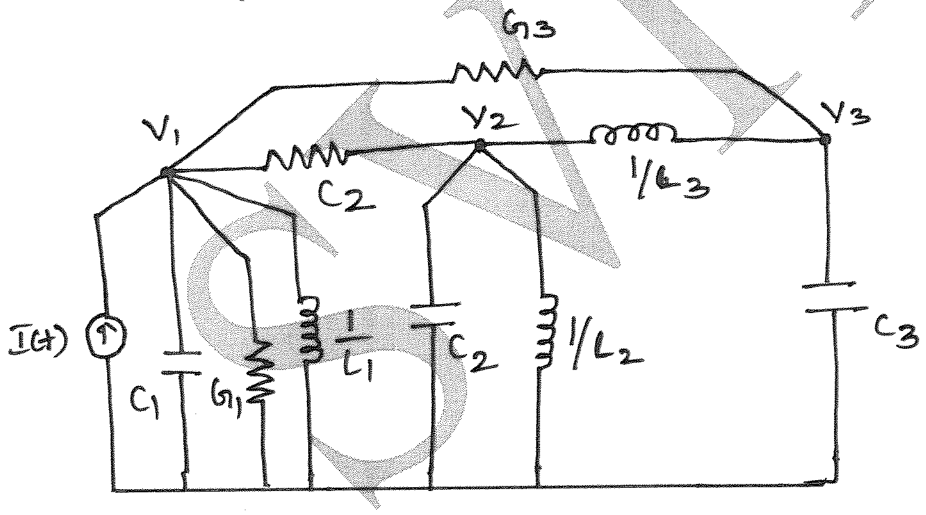
$$G_2(V_1-V_2) = C_2 \frac{d V_2}{dt} + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_3}\int (V_2-V_3)\,dt \rightarrow (8)$$

From x_3 :-

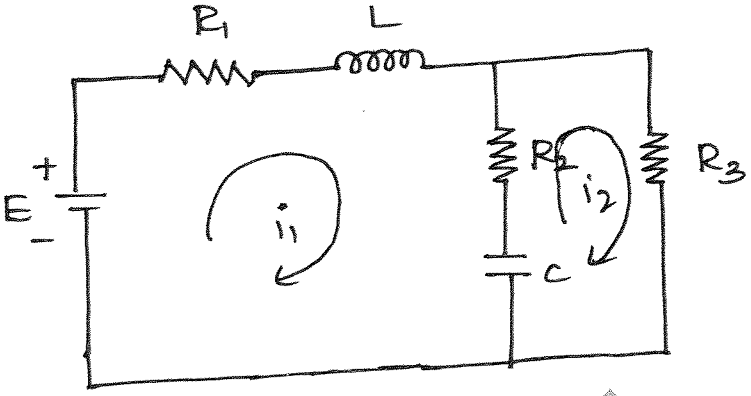
$$\frac{1}{L_3}(\phi_2-\phi_3) + G_3 \frac{d}{dt}(\phi_1-\phi_3) = C_3 \frac{d^2}{dt^2}\phi_3$$

$$\frac{1}{L_3}\int (V_2-V_3)\,dt + G_3(V_1-V_3) = C_3 \frac{d}{dt} V_3 \longrightarrow (9)$$

Circuit Satisfies Equation (7), (8) and (9)



⇒ Draw the force voltage analogous Mechanical System for the Electrical circuit shown in the figure.



Solution By applying KVL to the given circuit

from loop i_1 :-

$$E = R_1 i_1 + L\frac{di_1}{dt} + R_2(i_1 - i_2) + \frac{1}{C}\int (i_1 - i_2)\,dt$$

$$E = R_1\frac{dq_1}{dt} + L\frac{d^2 q_1}{dt^2} + R_2\frac{d}{dt}(q_1 - q_2) + \frac{1}{C}(q_1 - q_2) \rightarrow ①$$

from loop i_2 :-

$$R_3 i_2 + \frac{1}{C}\int (i_2 - i_1)\,dt + R_2(i_2 - i_1) = 0$$

$$R_3\frac{dq_2}{dt} + \frac{1}{C}(q_2 - q_1) + R_2\frac{d}{dt}(q_2 - q_1) = 0 \longrightarrow ②$$

By substituting the Mechanical Elements in Equation ① & ② based on F-V Analogy.

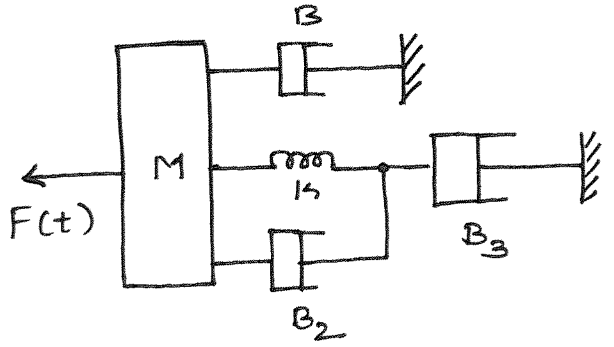
From 1 :-

$$F(t) = B_1\frac{dx_1}{dt} + M\frac{d^2 x_1}{dt^2} + B_2\frac{d(x_1 - x_2)}{dt} + K(q_1 - q_2) \rightarrow ③$$

From 2 :-

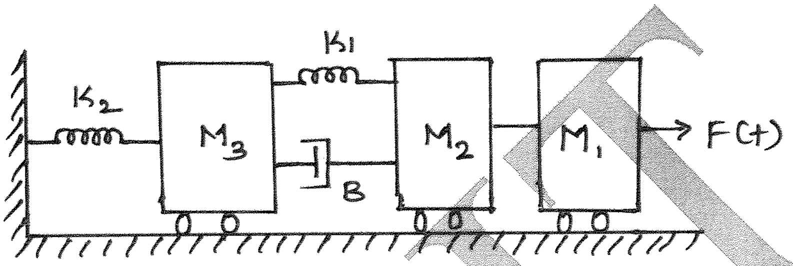
$$B_3\frac{dx_2}{dt} + K(x_2 - x_1) + B_2\frac{d}{dt}(x_2 - x_1) = 0 \rightarrow ④$$

The mechanical System satisfies Equation ③ & ④



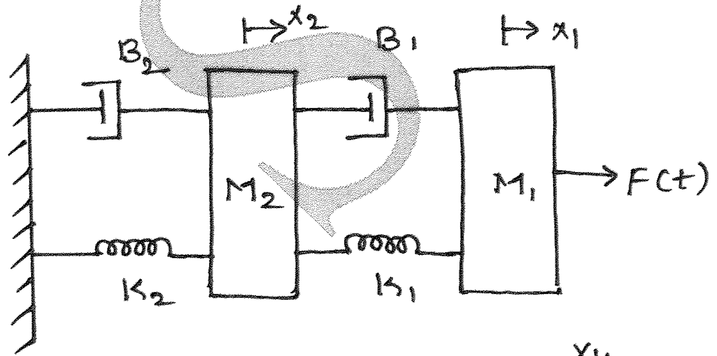
Practice Problems on Translational Systems:-

1)

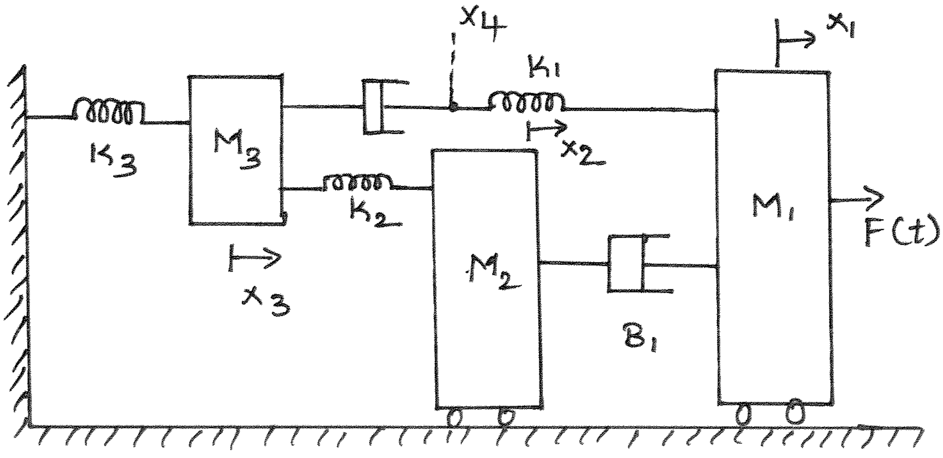


for the mechanical System shown in the figure above. write the differential Equations and draw the Electrical analogous based on Force-Voltage analogy and force-current Analogy.

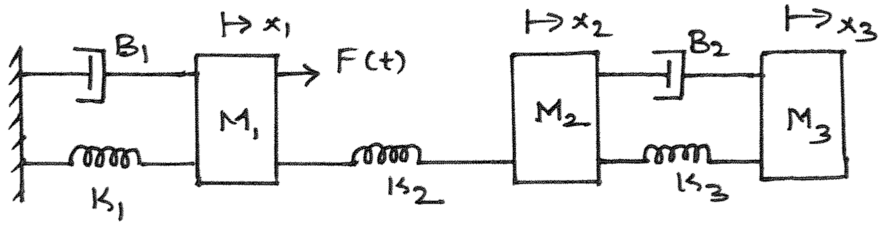
2)



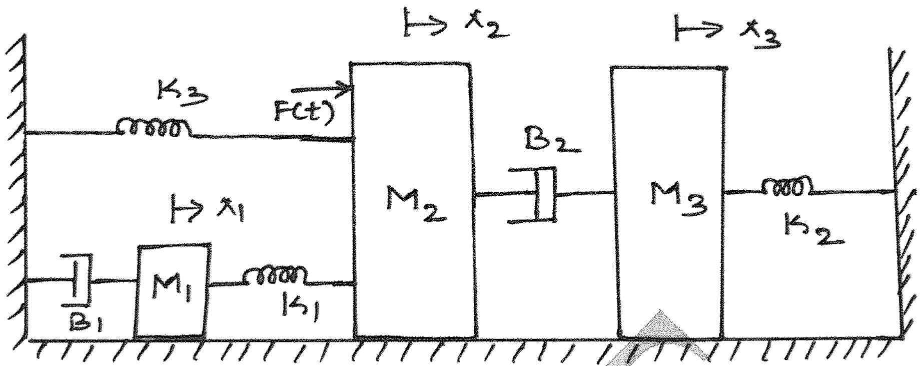
3)



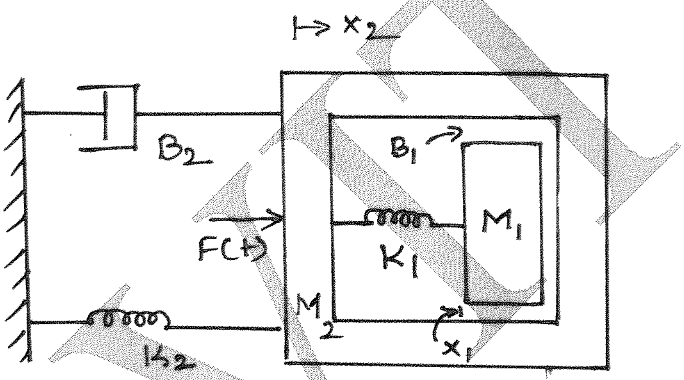
4)


x1
x2
x3
B1
F(t)
B2
M1
M2
M3
K1
K2
K3

5)


x2
x3
K3
F(t)
B2
x1
M2
M3
K2
M1
K1
B1

6)

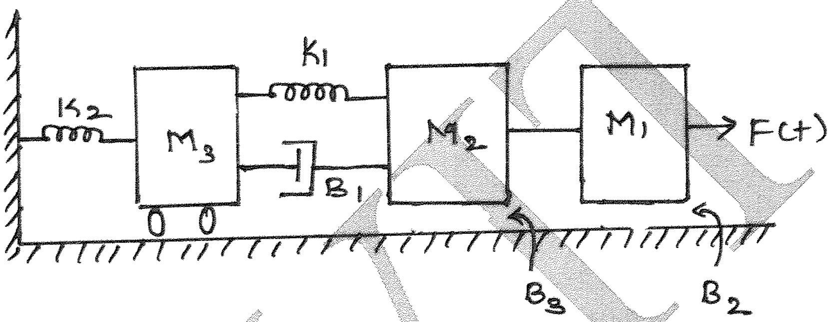

x2
B2
B1
M1
F(t)
K1
M2
x1
K2

Assignment on Translational Systems.

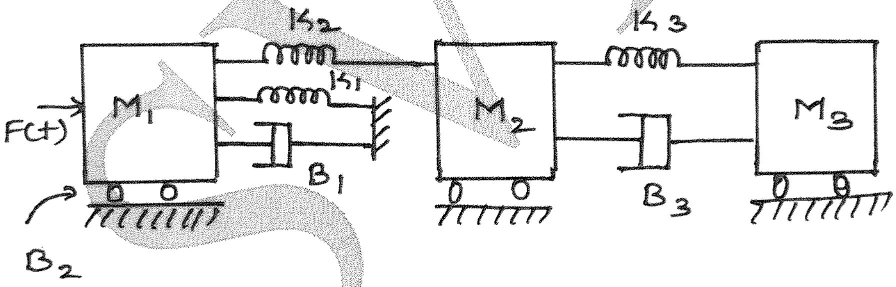
1) Define Control System. With an Example explain the types of Control System.

2) Give the comparisions between Openloop Control System and closed loop Control System.

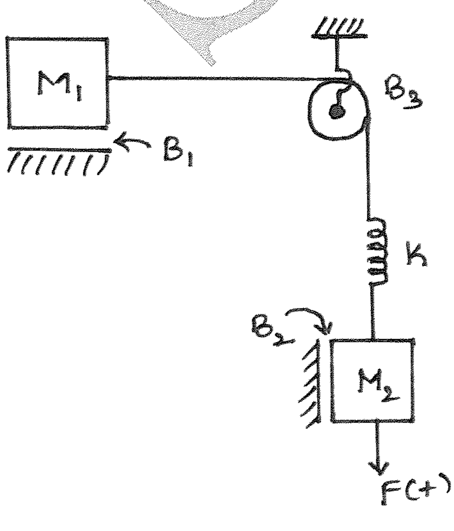
3) For a given mechanical System write the Electrical analogous circuit based force voltage and force current analogy.



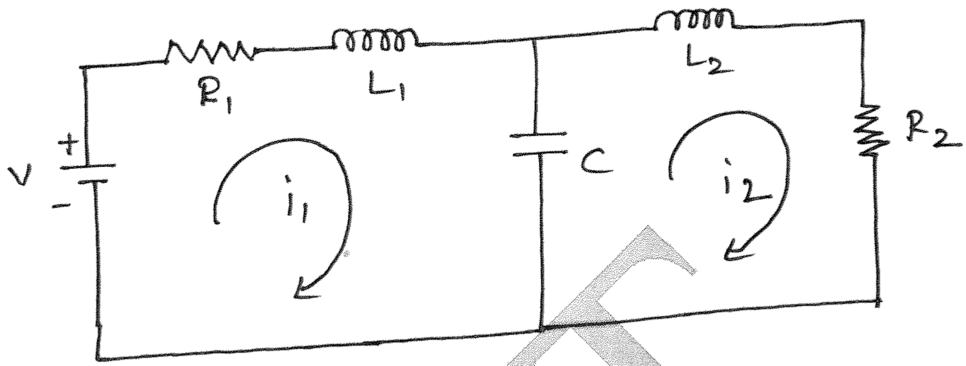
4)



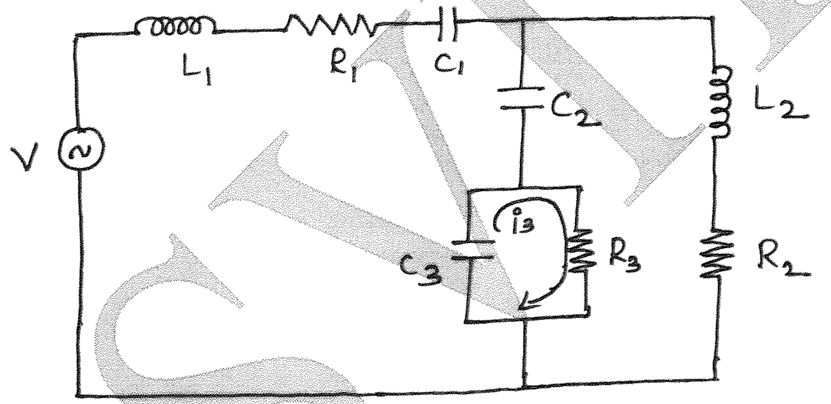
5)



6) Draw the force voltage Analogous mechanical System for the Electrical Circuit shown in figure writing the loop Equation for the Electric Circuit then transforming them to there mechanical analog



7)



Rotational Systems:-

The basic mechanical Elements of a rotational System are

a) Moment of Inertia (J)

b) Rotational Spring (K)

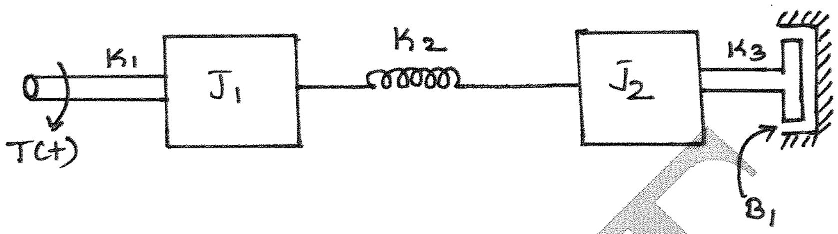
c) Rotational Dashpot (B)

* Tabulation for Converting Rotational Systems to Electrical analogous (Torque → Voltage and Torque → current)

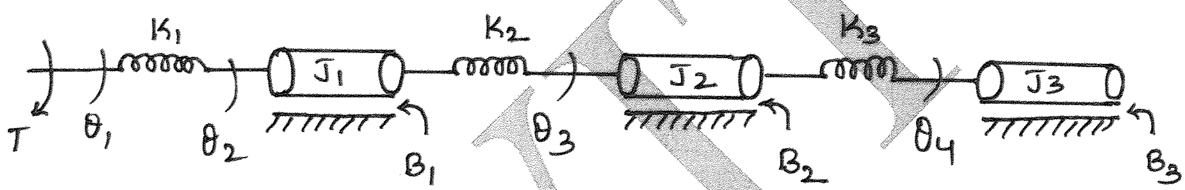
Mechanical elements	Electrical Analogous (T-V Analogy)	Electrical Analogous (T-I Analogy)
Torque [T(t)]	Voltage [V(t)]	Current [I(t)]
Angular Velocity [ω(t)]	Current [I(t)]	Voltage [V(t)]
Angular Displacement [θ(t)]	Charge [q(t)]	Magnetic Flux [ϕ(t)]
Moment of Inertia [J]	Inductance [L]	Capacitance [C]
Rotational Spring [K]	Reciprocal of capacitance [$\frac{1}{C}$]	Reciprocal of Inductance [$\frac{1}{L}$]
Rotational Dashpot [B]	Resistance [R]	Conductance [G]

Problems to be solved in the class

1) For the Mechanical System shown in the figure write the differential Equations of the system also draw the torque - voltage and torque - Current Electrical analogous.



2)



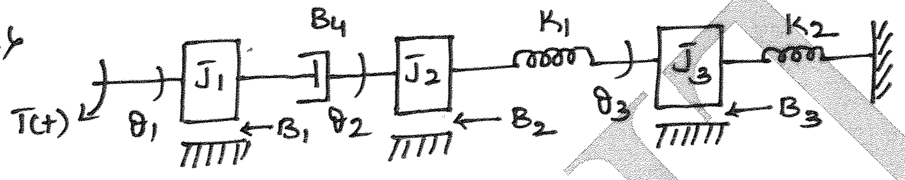
Note:- Angular Velocity $(\omega) = \frac{d}{dt}\theta(t)$

Angular Acceleration $(\alpha) = \frac{d^2\theta(t)}{dt^2}$

Problems on Rotational Systems:-

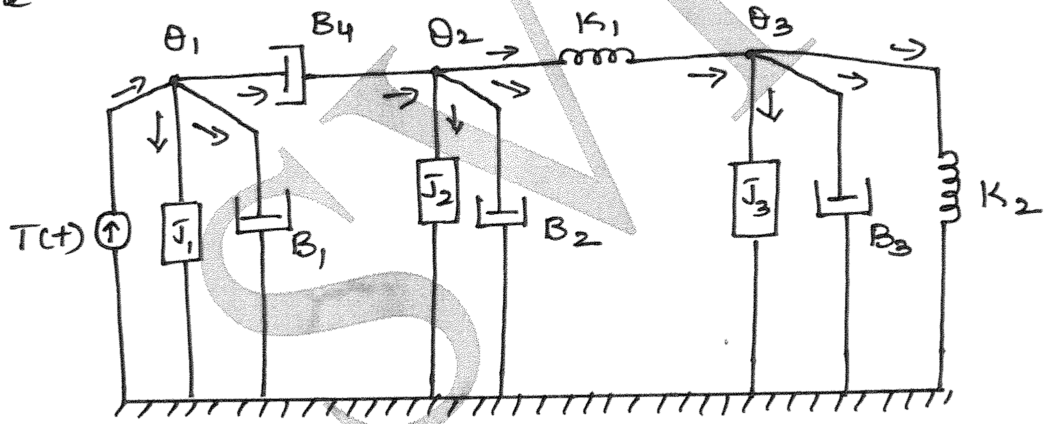
⇒ For the mechanical System shown in figure. Write the differential Equations of the System also draw the torque Voltage and torque current Electrical Analogous.

1)



Solution:-

The mechanical network for a given system



The Equilibrium Equations of the mechanical System are given by

At $\theta_1 =$ $T(t) = J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d\theta_1}{dt} + B_4\frac{d}{dt}(\theta_1-\theta_2) \rightarrow (1)$

At θ_2 ; $B_4\frac{d}{dt}(\theta_1-\theta_2) = J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} + K_1(\theta_2-\theta_3) \rightarrow (2)$

At θ_3 ; $K_1(\theta_2-\theta_3) = J_3\frac{d^2\theta_3}{dt^2} + B_3\frac{d}{dt}\theta_3 + K_2\theta_3 \rightarrow (3)$

T-V Analogy

By substituting electrical analogous based on torque voltage analogy in Equation (1), (2) and (3).

From (1),

$$v(t) = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{d}{dt} q_1 + R_4 \frac{d}{dt}(q_1 - q_2)$$

$$V(t) = L_1 \frac{d}{dt} i_1 + R_1 i_1 + R_4 (i_1 - i_2) \quad \text{—(4)}$$

From (2),

$$R_4 \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2 q_2}{dt^2} + R_2 \frac{d}{dt} q_2 + \frac{1}{C_1}(q_2 - q_3)$$

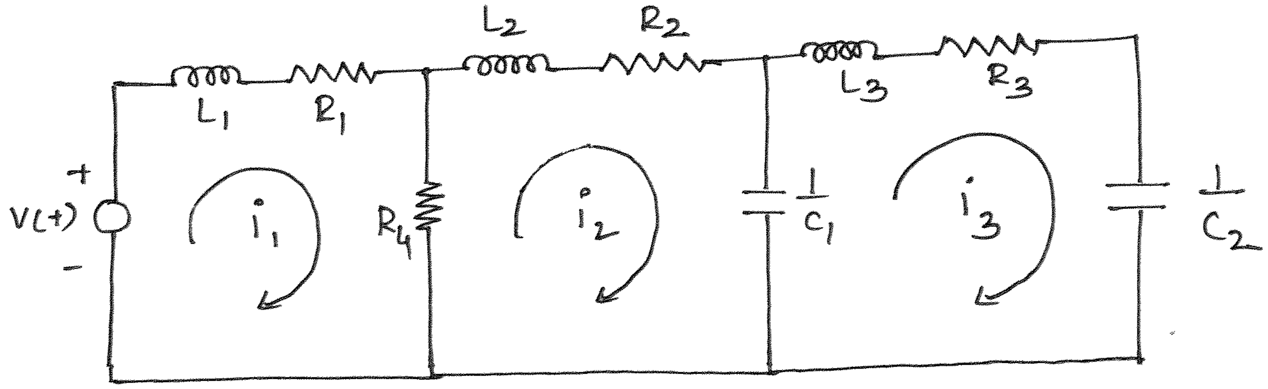
$$R_4 (i_1 - i_2) = L_2 \frac{d}{dt} i_2 + R_2 i_2 + \frac{1}{C_1} \int (i_2 - i_3)\, dt \quad \text{—(5)}$$

From (3),

$$\frac{1}{C_1}(q_2 - q_3) = L_3 \frac{d^2 q_3}{dt^2} + R_3 \frac{d q_3}{dt} + \frac{1}{C_2} q_2$$

$$\frac{1}{C_1} \int (i_2 - i_3)\, dt = L_3 \frac{d i_3}{dt} + R_3 i_3 + \frac{1}{C_2} \int i_3\, dt \quad \text{—(6)}$$

Electrical Circuit Satisfies Equation (4), (5) and (6)



T-I Analogy:-

By substituting electrical analogous based on torque. Current analogy in Equation ①, ② and ③

From ①

$$I(t) = C_1 \frac{d^2 \phi_1}{dt^2} + G_1 \frac{d \phi_1}{dt} + G_4 \frac{d}{dt}(\phi_1 - \phi_2)$$

$$I(t) = C_1 \frac{d V_1}{dt} + G_1 V_1 + G_4 (V_1 - V_2) \quad \text{—— ⑦}$$

From ②

$$G_4 \frac{d}{dt}(\phi_1 - \phi_2) = C_2 \frac{d^2 \phi_2}{dt^2} + G_2 \frac{d}{dt}\phi_2 + \frac{1}{L_1}(\phi_2 - \phi_3)$$

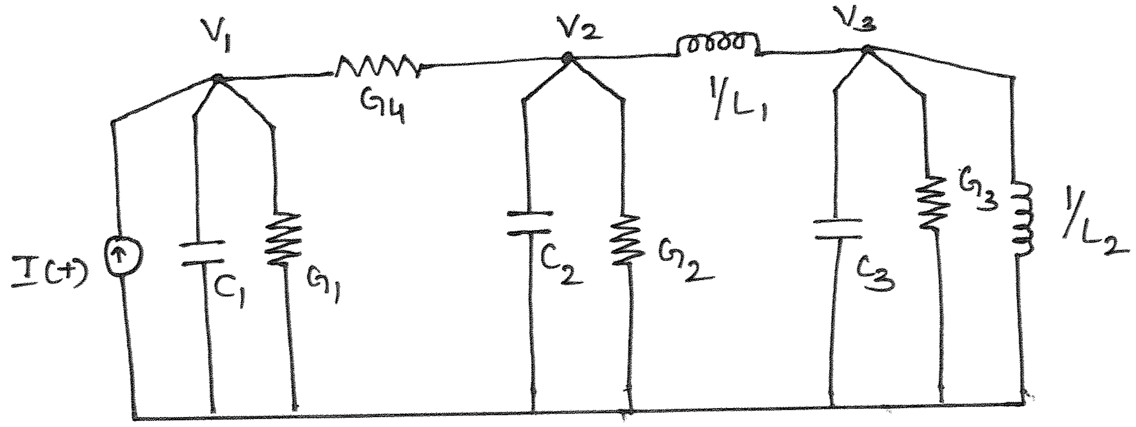
$$G_4 (V_1 - V_2) = C_2 \frac{d}{dt} V_2 + G_2 V_2 + \frac{1}{L_1}\int (V_2 - V_3)\, dt \quad \text{— ⑧}$$

From ③

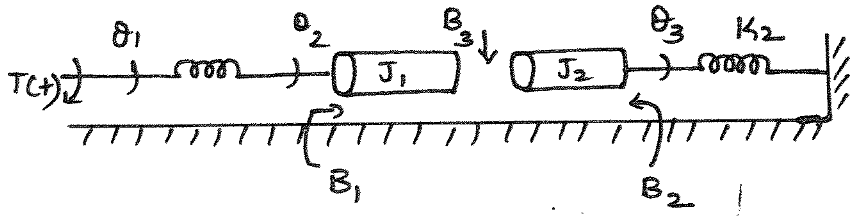
$$\frac{1}{L_1}(\phi_2 - \phi_3) = C_3 \frac{d^2 \phi_3}{dt^2} + G_3 \frac{d \phi_3}{dt} + \frac{1}{L_2}\phi_3$$

$$\frac{1}{L_1}\int (V_2 - V_3)\, dt = C_3 \frac{d V_3}{dt} + G_3 V_3 + \frac{1}{L_2}\int V_3\, dt \quad \text{— ⑨}$$

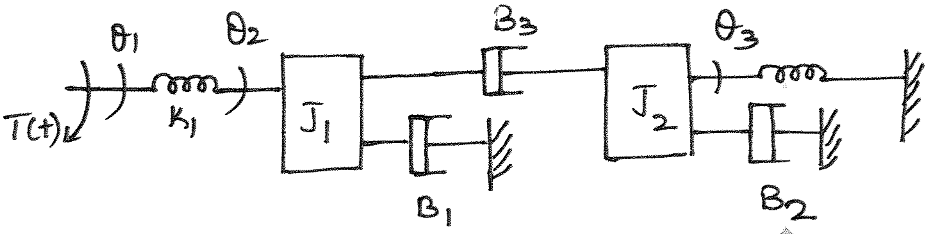
Electrical Circuit satisfying Equation 7, 8 and 9



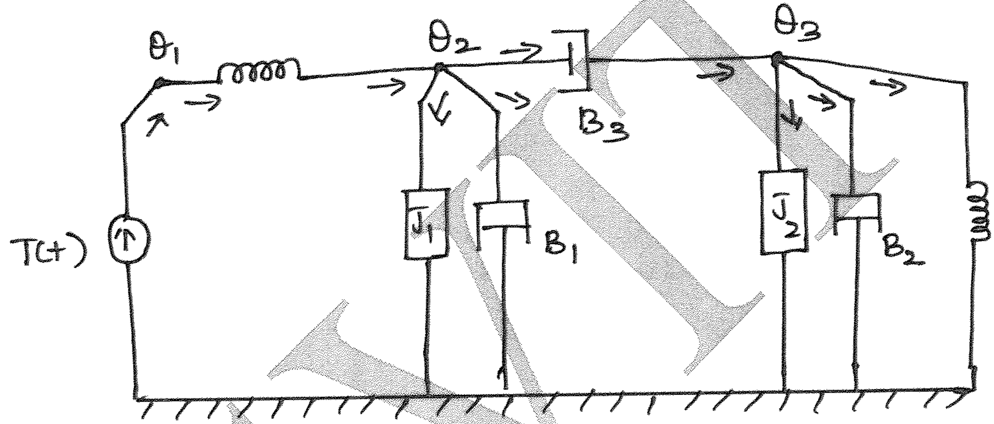
2)



Solution:- The Mechanical system is redrawn as shown below.



The Mechanical network is as drawn below.



The Equilibrium Equations are given by

At θ_1 ; $T(t) = K_1(\theta_1 - \theta_2)$ —— ①

At θ_2 ; $K_1(\theta_1 - \theta_2) = J_1 \dfrac{d^2\theta_2}{dt^2} + B_1 \dfrac{d\theta_2}{dt} + B_3 \dfrac{d}{dt}(\theta_2 - \theta_3)$ → ②

At θ_3 ; $B_3 \dfrac{d}{dt}(\theta_2 - \theta_3) = J_2 \dfrac{d^2\theta_3}{dt^2} + B_2 \dfrac{d\theta_3}{dt} + K_2\theta_3$ → ③

T-V Analogy:-

By substituting Electrical analogous based on torque. Voltage analogy in Equations ①, ② and ③.

From ①

$$V(t) = \frac{1}{C_1}(q_1 - q_2)$$

$$V(t) = \frac{1}{C_1}\int (i_1 - i_2)\,dt \quad \text{— ④}$$

From ②

$$\frac{1}{C_1}(q_1 - q_2) = L_1 \frac{d^2 q_2}{dt^2} + R_1 \frac{dq_2}{dt} + R_3 \frac{d}{dt}(q_2 - q_3)$$

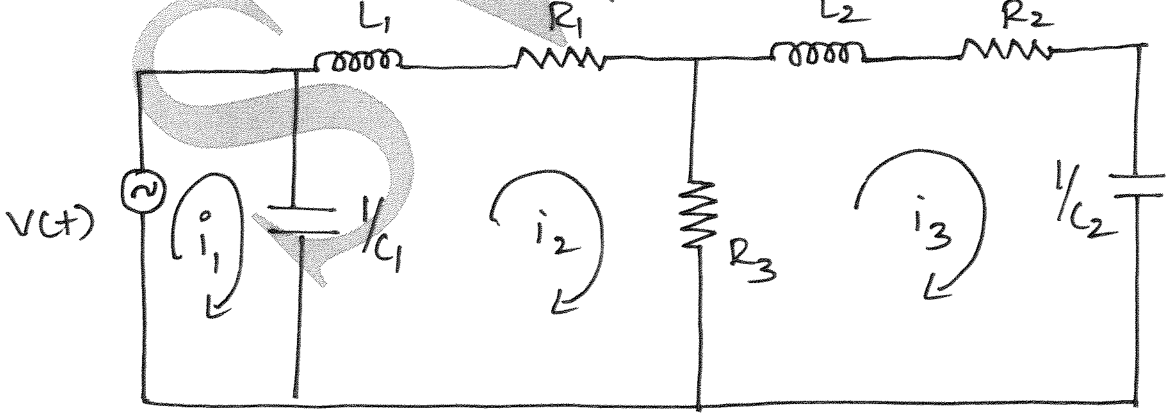
$$\frac{1}{C_1}\int (i_1 - i_2)\,dt = L_1 \frac{d i_2}{dt} + R_1 i_2 + R_3 (i_2 - i_3) \quad \text{— ⑤}$$

From ③

$$R_3 \frac{d}{dt}(q_2 - q_3) = L_1 \frac{d^2 q_3}{dt^2} + R_2 \frac{dq_3}{dt} + \frac{1}{C_2} q_3$$

$$R_3 (i_2 - i_3) = L_1 \frac{d i_3}{dt} + R_2 i_3 + \frac{1}{C_2}\int i_3\,dt \rightarrow ⑥$$

Electrical Circuit satisfying Equation ④, ⑤ and ⑥.



T-I Analogy:-

By substituting Electrical Analogous based on torque current analogy in Equation ①, ② and ③.

From ①

$$I(t) = \frac{1}{L_1}(\phi_1 - \phi_2)$$

$$I(t) = \frac{1}{L_1}\int (V_1 - V_2)\, dt \quad — ⑦$$

From ②

$$\frac{1}{L_1}(\phi_1 - \phi_2) = C_1 \frac{d^2 \phi_2}{dt^2} + G_1 \frac{d\phi_2}{dt} + G_3 \frac{d}{dt}(\phi_2 - \phi_3)$$

$$\frac{1}{L_1}\int (V_1 - V_2)\, dt = C_1 \frac{d V_2}{dt} + G_1 V_2 + G_3 (V_2 - V_3) \longrightarrow ⑧$$

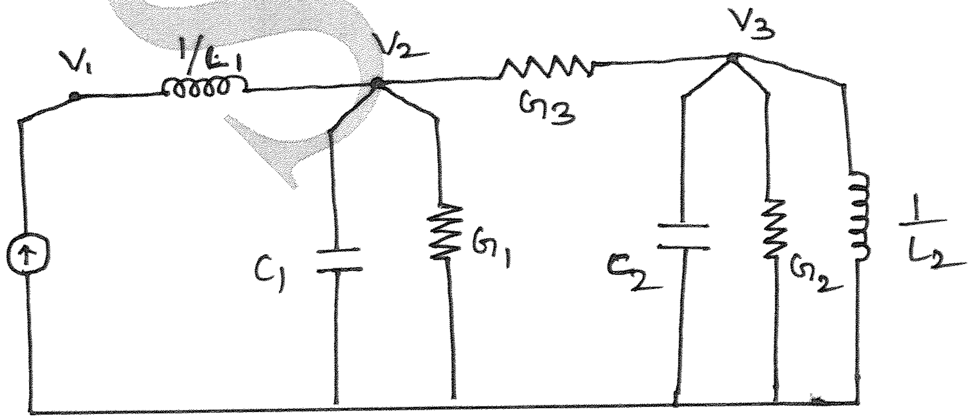
From ③

$$G_3 \frac{d}{dt}(\phi_2 - \phi_3) = C_2 \frac{d^2 \phi_3}{dt^2} + G_2 \frac{d\phi_3}{dt} + \frac{1}{L_2}\phi_3$$

or

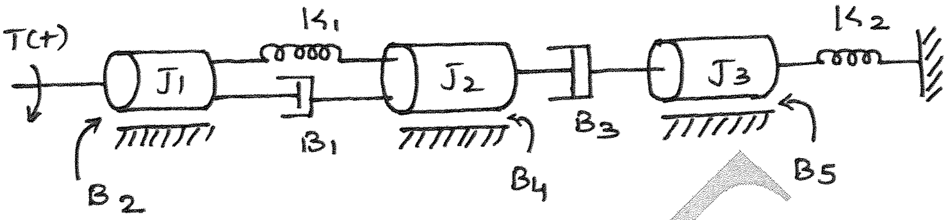
$$G_3 (V_2 - V_3) = C_2 \frac{d V_3}{dt} + G_2 V_3 + \frac{1}{L_2}\int V_3\, dt \longrightarrow ⑨$$

Electrical Circuit Satisfies Equation ⑦, ⑧ and ⑨.



Assignment on Rotational Systems.

4 Write the differential Equations describing the behavior of the mechanical system shown in the figure. Also draw an analogous Electrical Circuit. (T-V and T-I)



Transfer function :-

* Transfer function gives the mathematical Equivalent model for a System.

Defination I :-

The Transfer function is defined as the ratio of laplace Transform of the output to the Laplace transform of the input Considering all initial Condition Equals to zero;

$$r(t) \text{ (input)} \longrightarrow \boxed{\text{System}} \longrightarrow c(t) \text{ (output)}$$

For LTI Systems :-

$$\text{Transfer function} = \frac{L.T\left[\text{output}\right]}{L.T\left[\text{input}\right]} = \frac{L\left[c(t)\right]}{L\left[r(t)\right]}$$

$$T.F = \left.\frac{C(s)}{R(s)}\right|_{I.C=0}$$

LTI Systems :-

A system which has been made up of R, L, C Elements are always LTI System because R, L and C have linear characteristics and there value does not change with time.

Defination 2 :-

The Transfer function of LTI System is defined as laplace transform of Impulse response with all initial Conditions is Equal to zero.

$$\text{i.e, } T.F = \left. LT\left[\text{Impulse Response}\right]\right|_{\text{all initial Condition}=0}$$

→ Impulse response is also called as System response, natural response, free force response, Input response.

Properties of Transfer function (T.F) :-

* The transfer function of a system is the laplace transform of it's impulse response for zero initial conditions.
* The transfer function can be determined from system input - output pair by taking ratio of laplace of output to laplace of input.
* The System differential equation can be obtained from transfer function by replacing S- variable with linear differential operator D, defined by $D = \frac{d}{dt}$
* The transfer function is independent of the inputs to the system.
* The system poles/zeros can be found out from transfer function.
* Stability can be determined from the characteristic equation
* The transfer function is defined only for linear time invariant functions. it is not defined non-linear system.

Points to remember

* Laplace transforms pairs:-

SL. NO	F(t)	F(S) = L [F(t)]
1)	$\delta(t)$ impulse response at $t=0$	1
2)	$u(t)$ Unit step at $t=0$	$1/s$
3)	$u(t-T)$ Unit step at $t=T$	$\frac{1}{s}e^{-sT}$
4)	t	$1/s^2$
5)	$t^2/2$	$1/s^3$
6)	t^n	$\frac{n!}{s^{n+1}}$
7)	e^{-at}	$\frac{1}{s+a}$
8)	e^{at}	$\frac{1}{s-a}$
9)	$t \cdot e^{-at}$	$\frac{1}{(s+a)^2}$
10)	$t \cdot e^{at}$	$\frac{1}{(s-a)^2}$
11)	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
12)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
13)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
14)	$e^{-at} \sin \omega t$	$\omega/(s+\alpha)^2+\omega^2$
15)	$e^{-at} \cos \omega t$	$(s+\alpha)/(s+\alpha)^2+\omega^2$

16	$\sinh \alpha t$	$\frac{\alpha}{s^2-\alpha^2}$
17	$\cosh \alpha t$	$\frac{s}{s^2-\alpha^2}$
18	$\frac{1}{a^2}\left(at-1+e^{-at}\right)$	$\frac{1}{s^2(s+a)}$

* Laplace transform of Resistance 'R' $L_T[R] = R$
* Laplace transform of Capacitance 'C' $L_T[C] = \frac{1}{sC}$
* Laplace transform of Inductance 'L' $L_T[L] = sL$
* $L[f(t)] = F(s)$.
* $L\left[\frac{d}{dt}x(t)\right] = \left[s.X(s) - x(0)\right] = s.X(s) \quad \because x(0)=0$
* $L\left[\frac{d^2}{dt^2}x(t)\right] = \left[s^2X(s) - s\,x(0) - x'(0)\right] = s^2X(s) \quad \because x(0) = 0 = x'(0)$
* $L[t\,f(t)] = \frac{d}{ds}F(s)$.

* Laplace transform of linear Combination.

$$L\left[a f_1(t) + b f_2(t)\right] = a F_1(s) + b F_2(s)$$

Where $f_1(t)$, $f_2(t)$ are functions of time and a & b are constants.

* Scale change. $f\left[\frac{t}{a}\right] \Rightarrow a F(as)$; $a>0$

* Real translation $f(t-t_0) \Rightarrow e^{-st_0} F(s)$

* Complex translation $e^{-at} f(t) \Rightarrow F(s+a)$

* Multiplication by t $t^n f(t) \Rightarrow (-1)^n \frac{d^n F(s)}{ds^n}$

* Multiplication by $\frac{1}{t}$ $\frac{1}{t} f(t) \Rightarrow \int_s^\infty F(s)\,ds$

* Initial Value theorem $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

* Final Value theorem $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

* If the laplace transform of f(t) is F(s), then

(i) $L \frac{d}{dt} f(t) = \left[sF(s) - F(0)\right]$ $\because F(0) = 0$.

(ii) $L \frac{d^2 f(t)}{dt^2} = \left[s^2 F(s) - sF(0) - F'(0)\right]$

(iii) $L \frac{d^3 f(t)}{dt^3} = \left[s^3 F(s) - s^2 F(0) - sF'(0) - F''(0)\right]$

Where $f(0), f'(0), f''(0)$ are the values of $f(t), \frac{d}{dt} f(t), \frac{d^2 f(t)}{dt^2}$ ----- at $t=0$

* If the Laplace transform of f(t) is F(s), then

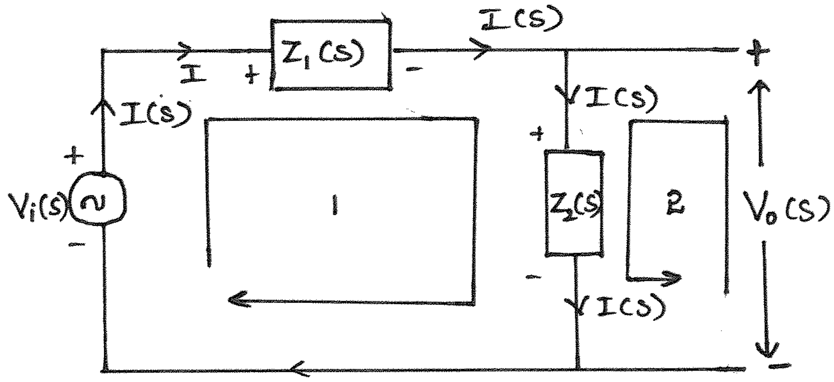
(i) $L\int f(t) = \left[\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}\right]$

(ii) $L\iint f(t) = \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0)}{s^2} + \frac{f^{2}(0)}{s}\right]$

(iii) $L\iiint f(t) = \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0)}{s^3} + \frac{f^{-2}(0)}{s^2} + \frac{f^{-3}(0)}{s}\right]$

Where $f^{-1}(0)$, $f^{-2}(0)$, $f^{-3}(0)$ are the Values of $\int f(t)$, $\iint f(t)$, $\iiint f(t)$ .... at $t = (0)$.

Transfer Function Of Electrical Networks:-



By applying KVL to loop I

$$+V_i(s) - Z_1(s)I(s) - Z_2(s)I(s) = 0$$

$$V_i(s) = Z_1(s)I(s) + Z_2(s)I(s)$$

$$V_i(s) = I(s)\left[Z_1(s) + Z_2(s)\right]$$

$$I(s) = \frac{V_i(s)}{Z_1(s) + Z_2(s)} \longrightarrow (1)$$

By applying KVL to loop 2.

$$+V_o(s) - Z_2(s)I(s) = 0$$

$$V_o(s) = Z_2(s)I(s) \longrightarrow (2)$$

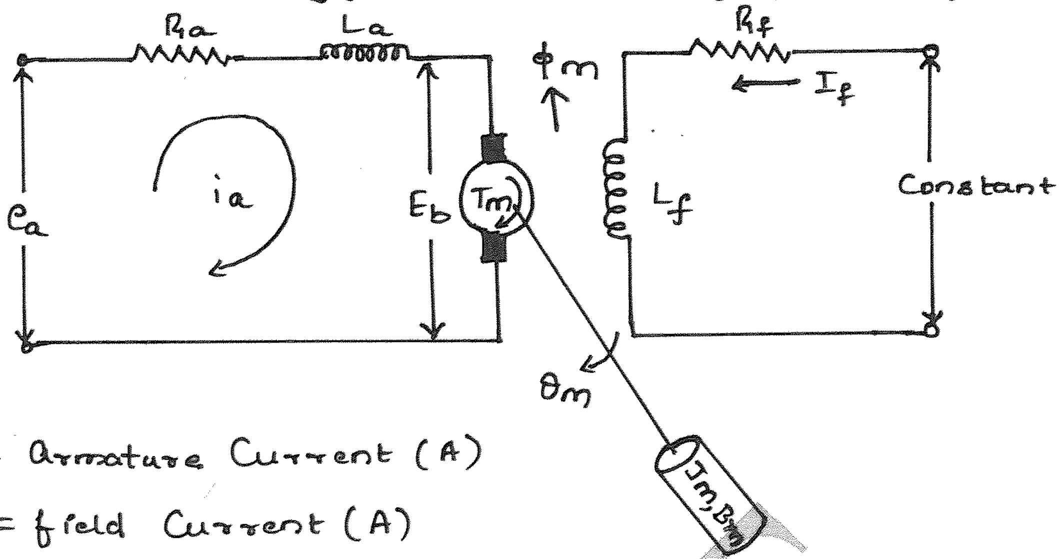
By substituting Equation (1) in (2) We get

$$V_o(s) = Z_2(s) \cdot \frac{V_i(s)}{Z_1(s) + Z_2(s)}$$

$\therefore$ Transfer function is given by

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}}$$

Transfer function of an Armature Controlled DC motor.



i_a = Armature Current (A)

i_f = field Current (A)

e_a = Applied Armature Voltage (V)

e_b = back Emf (Volts)

T_m = torque developed by motor (Nm)

θ = Angular displacement of motor-shaft (rad).

J = Equivalent moment of inertia of motor and load referred to motor shaft (Kg-m²)

B = Equivalent Viscous friction Co-Efficient of motor and load referred to motor shaft $\left(\frac{Nm}{rad/s}\right)$

* Flux is directly proportional to Current through field winding

$$\phi_m = K_f I_f = \text{Constant} \longrightarrow (1)$$

* Torque produced is proportional to product of flux and armature Current

$$T_m = K_m \phi_m I_a$$

By substituting the value of ϕ_m in (1)

$$\therefore T_m = K_m K_f I_f I_a \longrightarrow (2)$$

* Back Emf " E_b " is directly proportional to shaft Velocity " W_m ", as flux ϕ_m is constant.

* We know that velocity $W_m = \frac{d\,\theta_m(t)}{dt} = W_m(s) = s\,\theta_m(s)$

$$\text{Back Emf } E_b(s) = K_b W_m(s) = K_b s\,\theta_m(s) \rightarrow (3)$$

* By applying KVL to armature circuit we get

$$e_a = E_b + I_a(R_a) + L_a \frac{d\,i_a}{dt}$$

By taking laplace transform

$$E_a(s) = E_b(s) + I_a(s)\cdot R_a + L_a\cdot s\, I_a(s)$$

$$E_a(s) = E_b(s) + I_a(s)\left[R_a + sL_a\right]$$

$$\boxed{I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + sL_a}} \rightarrow (4)$$

* By substituting Equation (4) in (2)

$$T_m = K_m K_f I_f \left\{\frac{E_a(s) - E_b(s)}{R_a - sL_a}\right\} \quad -(5)$$

* The differential Equation is given by.

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\,\theta_m}{dt} = J_m s^2\,\theta_m(s) + B_m s\,\theta_m(s)$$

$$\therefore T_m = \left\{J_m s^2 + B_m s\right\}\theta_m(s) \quad -(6)$$

Substituting Equation (5) in (6)

$$K_m K_f I_f \left\{\frac{E_a(s) - E_b(s)}{R_a - sL_a}\right\} = \left\{J_m s^2 + B_m s\right\}\theta_m(s)$$

$$\frac{K_m K_f I_f E_a(s) - K_m K_f I_f E_b(s)}{R_a - sL_a} = \left\{ J_m s^2 + B_m s \right\} \theta_m(s) \longrightarrow (7)$$

By substituting (3) in (7)

$$\frac{K_m K_f I_f E_a(s) - K_m K_f I_f K_b s \theta_m(s)}{R_a - sL_a} = \left\{ J_m s^2 + B_m s \right\} \theta_m(s)$$

$$\frac{K_m K_f I_f E_a(s)}{R_a - sL_a} - \frac{K_m K_f I_f K_b s \theta_m(s)}{R_a - sL_a} = \left\{ J_m s^2 + B_m s \right\} \theta_m(s)$$

$$\frac{K_m K_f I_f E_a(s)}{R_a - sL_a} = \frac{K_m K_f I_f K_b s \theta_m(s)}{R_a - sL_a} + \left(J_m s^2 + B_m s \right) \theta_m(s)$$

$$\frac{K_m K_f I_f}{R_a - sL_a} E_a(s) = \left[\frac{K_m K_f I_f K_b s}{R_a - sL_a} + J_m s^2 + B_m s \right] \theta_m(s)$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m K_f I_f}{R_a + sL_a}}{\dfrac{K_m K_f I_f K_b s}{R_a + sL_a} + J_m s^2 + B_m s}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m K_f I_f}{R_a + sL_a}}{\dfrac{K_m K_f I_f K_b s + J_m s^2 (R_a + sL_a) + B_m s (R_a + sL_a)}{R_a + sL_a}}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_m K_f I_f}{K_m K_f I_f K_b s + (R_a + sL_a)(J_m s^2 + B_m s)}$$

$$= \frac{K_m K_f I_f}{(R_a + sL_a)(J_m s^2 + B_m s)\left[1 + \dfrac{K_m K_f I_f K_b s}{(R_a + sL_a)(J_m s^2 + B_m s)}\right]}$$

$$= \frac{\dfrac{K_m K_f I_f}{(R_a + sL_a)(J_m s^2 + B_m s)}}{1 + \dfrac{K_m K_f I_f K_b s}{(R_a + sL_a)(J_m s^2 + B_m s)}}$$

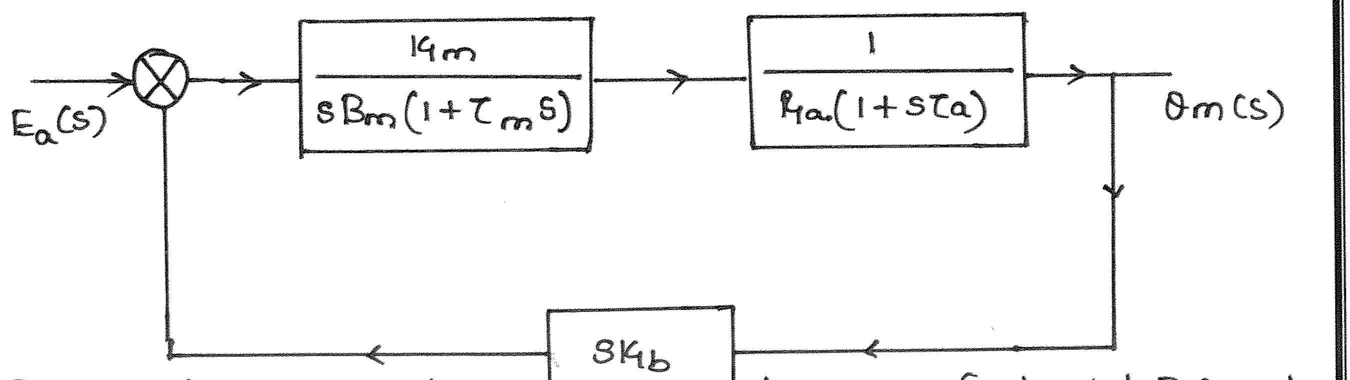
$$= \frac{\dfrac{K_m K_f I_f}{s B_m \left(s\, ^{J_m}/_{B_m} + 1\right) R_a \left(1 + ^{sL_a}/_{R_a}\right)}}{1 + \dfrac{K_m K_f I_f K_b s}{s B_m \left(s\, ^{J_m}/_{B_m} + 1\right) R_a \left(1 + ^{sL_a}/_{R_a}\right)}}$$

By substituting $K_m K_f I_f = K_m$,

$\frac{J_m}{B_m} = \tau_m$, $\frac{L_a}{R_a} = \tau_a$ we get

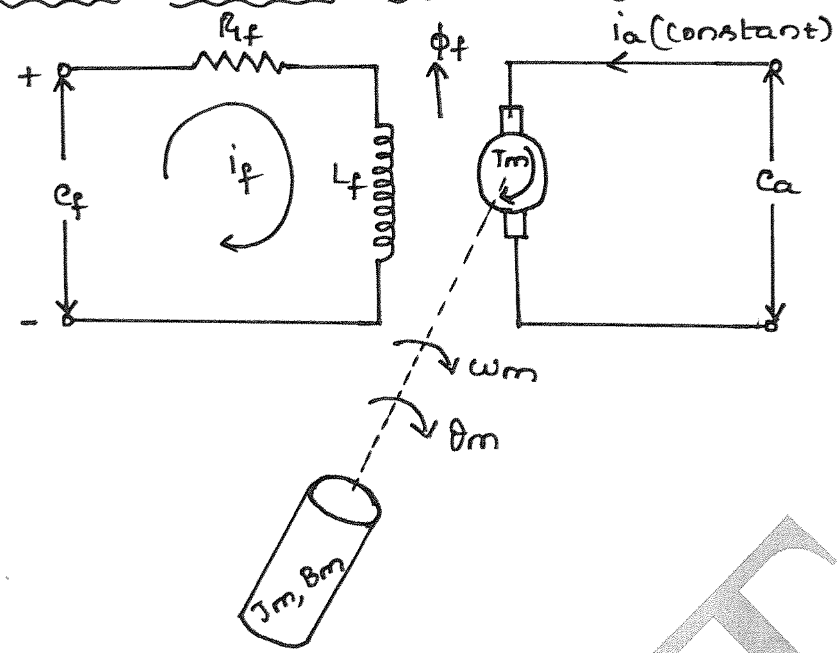
$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m}{sR_a B_m(1 + s\tau_m)(1 + s\tau_a)}}{1 + \dfrac{K_m \cdot s K_b}{sR_a B_m(1 + s\tau_m)(1 + s\tau_a)}}}$$

The block diagram below satisfies the T.F $\frac{\theta_m(s)}{E_a(s)}$



Block diagram of Armature Controlled D.C motor

Transfer function of field Controlled D.C. motor.



R_f -> field Winding resistance (Ω).

L_f -> field Winding inductance (H).

e -> field Control Voltage (V).

i_f -> field Current (A).

T_m -> torque developed by motor (Nm).

J_m -> Equivalent moment of inertia of motor (Kg-m^2).

B_m -> Equivalent Viscous friction Coefficient of motor (Nm/rad/s).

θ -> Angular displacement.

* Input to the motor is Constant Armature Current
* Magnetic flux ϕ_f is proportional to field Current I_f

$$\therefore \phi_f = K_f I_f \longrightarrow ①$$

* Torque T_m is Proportional to product of flux and armature Current

$$T_m \propto \phi_f I_a$$

$$\therefore T_m = K' \phi I_a = K' K_f I_f I_a$$

$$T_m = K_m K_f I_f \longrightarrow (2)$$

Where $K_m = K' I_a$ = Constant

By applying KVL to field current

$$L_f \frac{di_f}{dt} + R_f I_f = e_f \longrightarrow (3)$$

By taking laplace transform

$$E_f(s) = L_f(s) I_f(s) + R_f I_f(s)$$

$$E_f(s) = \left[L_f(s) + R_f\right] I_f(s)$$

$$I_f(s) = \frac{E_f(s)}{sL_f + R_f} \longrightarrow (4)$$

By applying laplace transforms for Equation (2)

$$T_m(s) = K_m K_f I_f(s)$$

By substituting the value of $I_f(s)$

$$T_m(s) = K_m K_f \frac{E_f(s)}{sL_f + R_f}$$

$$T_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)} \longrightarrow (5)$$

The differential equation is given by

$$T_m = J_m \frac{d^2 \theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} \longrightarrow (6)$$

By taking laplace transform for Equation ⑥

$$T_m(s) = J_m s^2 \theta_m(s) + B_m s\, \theta_m(s)$$

$$T_m(s) = (J_m s^2 + B_m s)\,\theta_m(s) \quad \text{—⑦}$$

By substituting Equation ⑦ in ⑤ we get

$$(s^2 J_m + sB_m)\,\theta_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)}$$

Here $E_f(s)$ is the input and $\theta_m(s)$ is the Output.

The transfer function is given by $= \dfrac{\theta_m(s)}{E_f(s)}$

$$\frac{\theta_m(s)}{E_f(s)} = \frac{K_m K_f}{(s^2 J_m + sB_m)(sL_f + R_f)}$$

$$= \frac{K_m K_f}{B_m\left[\dfrac{s^2 J_m}{B_m} + s\right] R_f\left[\dfrac{sL_f}{R_f} + 1\right]}$$

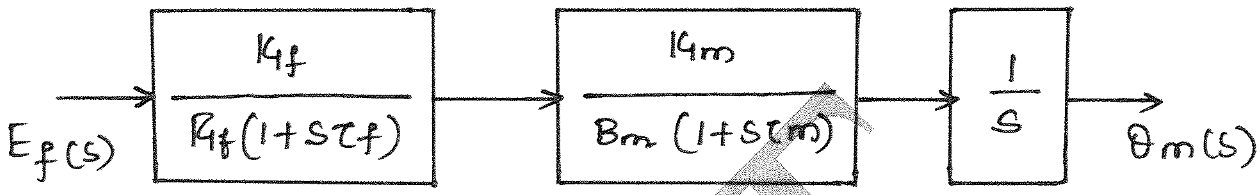
$$= \frac{K_m K_f}{sB_m R_f\left[s\dfrac{J_m}{B_m} + 1\right]\left[\dfrac{sL_f}{R_f} + 1\right]}$$

Substitute $\dfrac{J_m}{B_m} = \tau_m$ and $\dfrac{L_f}{R_f} = \tau_f$

$$\therefore \frac{\theta_m(s)}{E_f(s)} = \frac{K_m K_f}{s R_f B_m [1 + s\tau_m][1 + s\tau_f]}$$

$$T.F = \frac{\theta_m(s)}{E_f(s)} = \frac{K_f}{R_f[1+s\tau_f]} \cdot \frac{K_m}{B_m[1+s\tau_m]} \cdot \frac{1}{s}$$

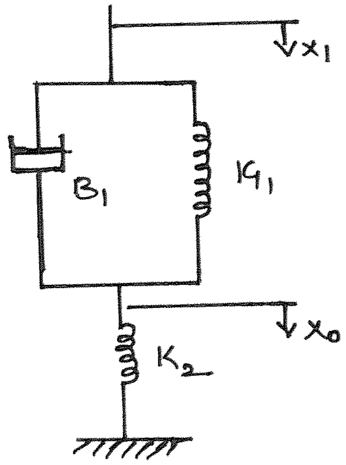
The block diagram satisfies the transfer function $\frac{\theta_m(s)}{E_f(s)}$



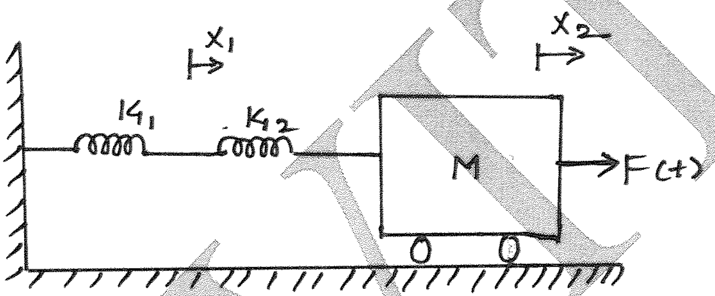
Block diagram of field controlled DC motor.

Problems to be solved in the class:

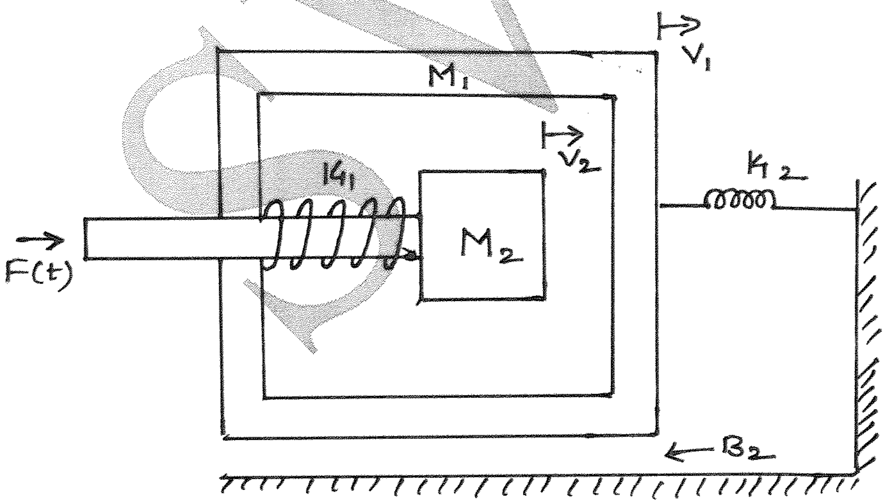
1) Obtain the transfer function for the system shown in the figure.



2)

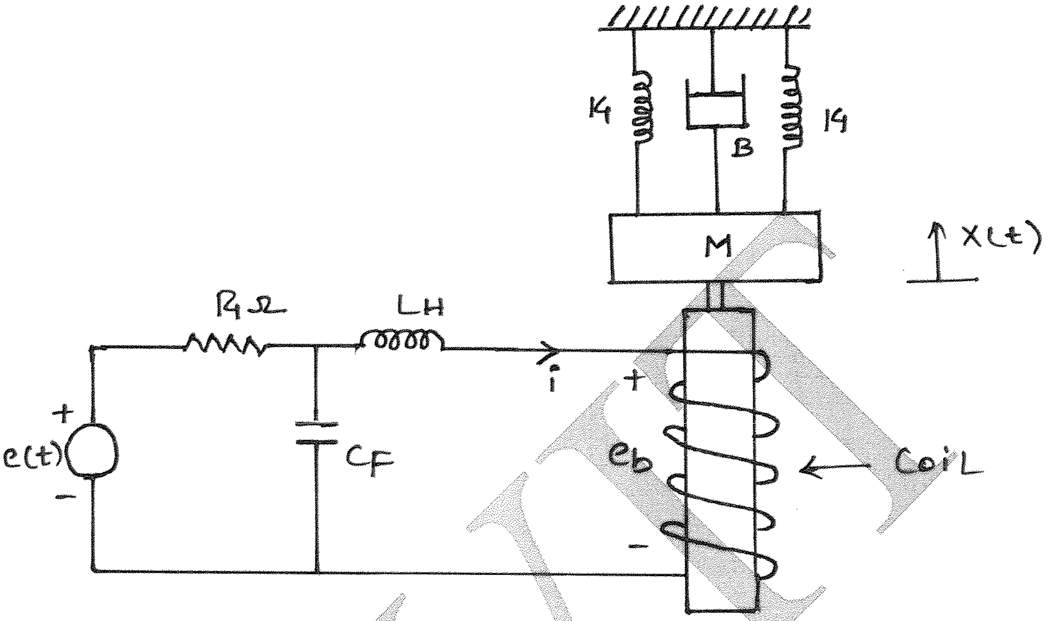


3)

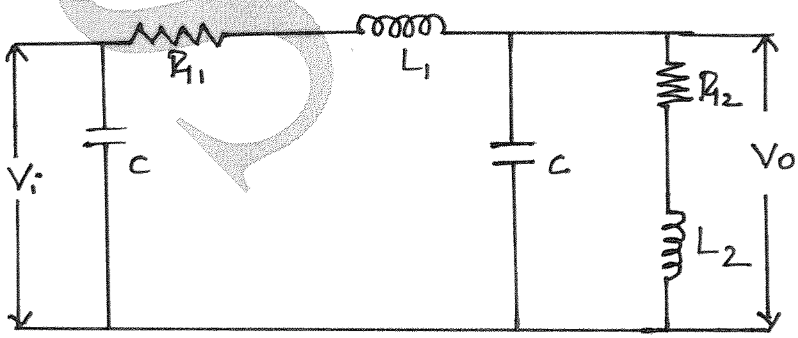


Derive the transfer function $\frac{V_2(s)}{F(s)}$ for the Mechanical System shown in the figure above.

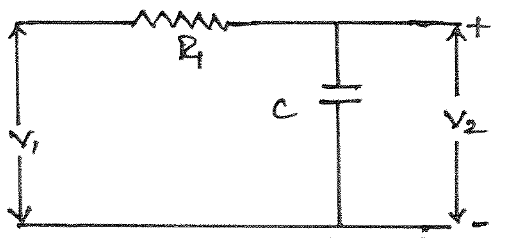
4) Find the transfer function $\frac{X(S)}{E(S)}$ for the Electromechanical System shown in figure, the coil has a back emf $e_b = K_1 \frac{dx}{dt}$ and the coil current produces a force $f_c = K_2 i$ on the mass M where K_1 & K_2 are constants.

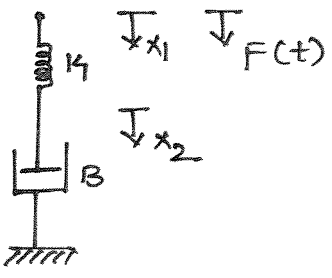


5) Determine the transfer function for the Electrical network shown in the figure Assuming zero initial condition.



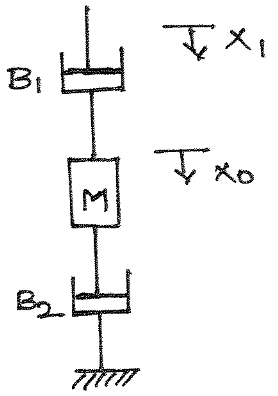
6) Show that the two systems shown in the figure ⓐ and figure ⓑ are Analogous Systems by comparing there transfer functions.





Problems On Transfer function :-

1) Obtain the transfer function for the system shown in the figure.



Solution :- The Equilibrium Equation for a given Mechanical System is given by.

$$B_1 \frac{d}{dt}(x_1 - x_0) = M \frac{d^2}{dt^2} x_0 + B_2 \frac{d}{dt} x_0$$

$$B_1 \frac{dx_1}{dt} - B_1 \frac{dx_0}{dt} = M \frac{d^2 x_0}{dt^2} + B_2 \frac{d x_0}{dt} \longrightarrow (1)$$

By Taking laplace transform for Equation (1) we get.

$$B_1 S X_1(S) - B_1 S X_0(S) = M S^2 X_0(S) + B_2 S X_0(S)$$

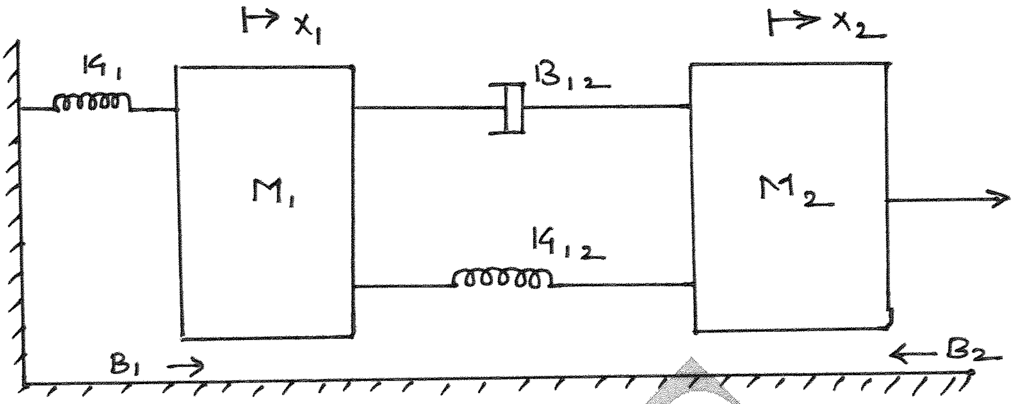
$$B_1 S X_1(S) = M S^2 X_0(S) + B_2 S X_0(S) + B_1 S X_0(S)$$

$$B_1 S X_1(S) = (M S^2 + B_2 S + B_1 S) X_0(S)$$

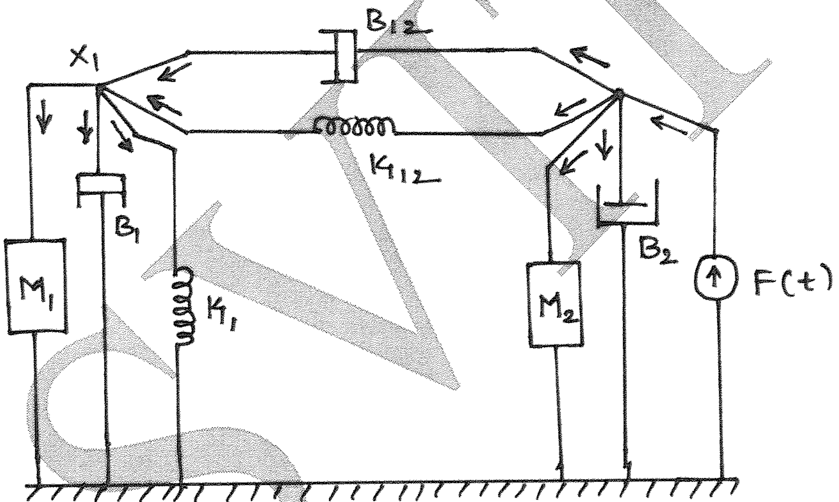
$$\frac{X_0(S)}{X_1(S)} = \frac{B_1 S}{M S^2 + B_2 S + B_1 S} = \frac{B_1}{MS + B_2 + B_1}$$

$$\boxed{\frac{X_0(S)}{X_1(S)} = \frac{B_1}{MS + B_2 + B_1}}$$

2) For a Given Mechanical System write the System Equation and Obtain Transfer function of the System.



Solution:- The Mechanical network for a given mechanical System is as shown below.



The Equilibrium Equations for a Mechanical network are.

at x_2;

$$F(t) = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{d x_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_{12}(x_2 - x_1) \rightarrow (1)$$

at x_1;

$$B_{12} \frac{d}{dt}(x_2 - x_1) + K_{12}(x_2 - x_1) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d x_1}{dt} + K_1 x_1 \rightarrow (2)$$

By taking laplace transforms for Equation (1) and (2)

For x_2 :-

$$F(S) = M_2 S^2 X_2(S) + B_2 S X_2(S) + B_{12} S (X_2(S) - X_1(S)) + K_{12}(X_2(S) - X_1(S))$$

$$F(S) = [M_2 S^2 + B_2 S] X_2(S) + [X_2(S) - X_1(S)] [S B_{12} + K_{12}]$$

$$\boxed{F(S) = (M_2 S^2 + B_2 S) X_2(S) + (S B_{12} + K_{12}) X_2(S) - (S B_{12} + K_{12}) X_1(S)} \rightarrow (3)$$

For x_1 :-

$$B_{12} S (X_2(S) - X_1(S)) + K_{12}(X_2(S) - X_1(S)) = M_1 S^2 X_1(S) + B_1 S X_1(S) + K_1 X_1(S)$$

$$X_2(S) [B_{12} S + K_{12}] = M_1 S^2 X_1(S) + B_1 S X_1(S) + K_{12} X_1(S) + K_1 X_1(S) + B_{12} S X_1(S)$$

$$\boxed{X_2(S) = \frac{X_1(S) [M_1 S^2 + B_1 S + K_1 + K_{12} + B_{12} S]}{[B_{12} S + K_{12}]}} \rightarrow (4)$$

By substituting Equation (4) in (3) for $X_2(S)$

$$F(S) = \frac{X_1(S) [M_1 S^2 + B_1 S + K_1 + K_{12} + B_{12} S]}{[B_{12} S + K_{12}]} [M_2 S^2 + B_2 S] +$$

$$\frac{X_1(S) [M_1 S^2 + B_1 S + K_1 + K_{12} + B_{12} S]}{[B_{12} S + K_{12}]} [S B_{12} + K_{12}] +$$

$$X_1(S) [S B_{12} + K_{12}]$$

By taking $X_1(S)$ common

$$F(S) = X_1(S) \left[\frac{[M_1 S^2 + B_1 S + K_1 + K_{12} + B_{12} S] [M_2 S^2 + B_2 S]}{[B_{12} S + K_{12}]} + \right.$$

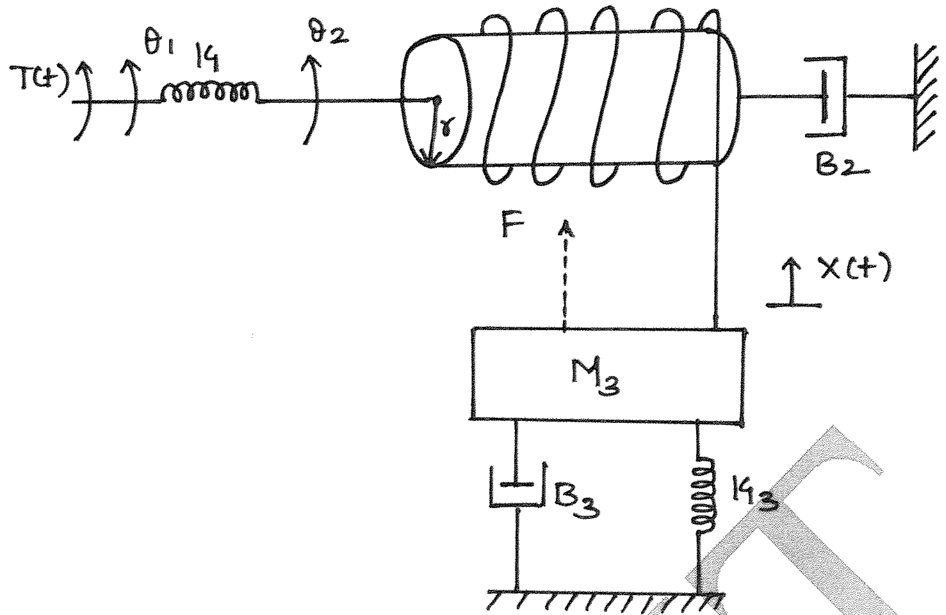
$$\left[\frac{\left[M_1S^2+SB_1+K_1+SB_{12}+K_{12}\right]\left[SB_{12}+K_{12}\right]}{\left[SB_{12}+K_{12}\right]} - \left[SB_{12}+K_{12}\right]\right]$$

$$\frac{F(S)}{X_1(S)} = \left[\frac{(M_1S^2+SB_1+K_1+SB_{12}+K_{12})\left[(SB_{12}+K_{12})+(M_2S^2+B_2S)\right]}{\left[SB_{12}+K_{12}\right]} - \left[SB_{12}+K_{12}\right]\right]$$

$$\frac{F(S)}{X_1(S)} = \frac{\left[(M_1S^2+SB_1+K_1+SB_{12}+K_{12})(SB_{12}+K_{12})+(M_2S^2+B_2S)(M_1S^2+SB_1+K_1+SB_{12}+K_{12}) - (SB_{12}+K_{12})^2\right]}{SB_{12}+K_{12}}$$

$$\boxed{\therefore \frac{X_1(S)}{F(S)} = \frac{SB_{12}+K_{12}}{\left[(M_1S^2+SB_1+K_1+SB_{12}+K_{12})(SB_{12}+K_{12})+(M_2S^2+B_2S)(M_1S^2+SB_1+K_1+SB_{12}+K_{12}) - (SB_{12}+K_{12})^2\right]}}$$

③ In the Mechanical System shown, write the differential Equation of performance for this System.



Solution: Equilibrium Equations of the Mechanical System are given by.

$$\text{At } \theta_1 = T(t) = K_1(\theta_1 - \theta_2) \quad \text{— ①}$$

$$\text{At } \theta_2 = K_1(\theta_1 - \theta_2) = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_1 \quad \text{— ②}$$

Where $T_1 = F.r \rightarrow$ ③

Where F is the force acting on the translational System at

$$x(t): F = M_3 \frac{d^2x(t)}{dt^2} + B_3 \frac{dx(t)}{dt} + K_3 x(t) \quad \text{— ④}$$

44 Find the transfer function for the system shown in the figure ① and ②. hence show that they are Analogous to Each Other.

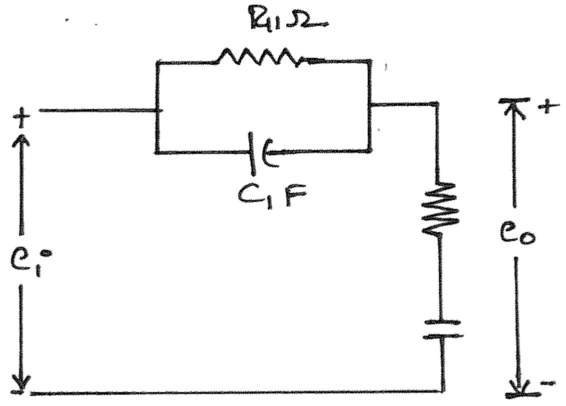


Fig ①

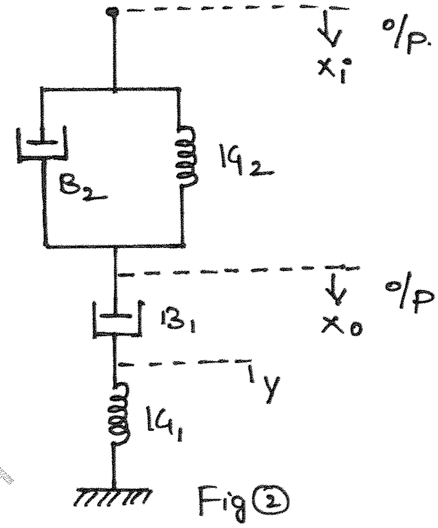


Fig ②

Solution:-

$x_i = i/p$, $x_0 = o/p$.

The Equilibrium Equation of the mechanical System is given by.

at x_0: $B_2 \frac{d}{dt}(x_i - x_0) + K_2 (x_i - x_0) = B_1 \frac{d}{dt}(x_0 - y)$ — ①

at y: $B_1 \frac{d}{dt}(x_0 - y) = K_1 y$ — ②

By taking laplace transform Assuming zero initial Condition.

From ①; $B_2 s\left[X_i(s) - X_0(s)\right] + K_2\left[X_i(s) - X_0(s)\right] = B_1 s\left[X_0(s) - Y(s)\right]$

or $(B_2 s + K_2) X_i(s) = (B_1 s + B_2 s + K_2) X_0(s) - B_1 s Y(s)$ — ③

From ②; $B_1 s\left(X_0(s) - Y(s)\right) = K_1 Y(s)$

or

$B_1 s X_0(s) = (B_1 s + K_1) Y(s)$

or $$Y(s) = \frac{B_1 s}{B_1 s + K_1} \cdot X_0(s) \quad \text{--- (4)}$$

By substituting (4) in (3)

$$(B_2 s + K_2) X_i(s) = (B_1 s + B_2 s + K_2) X_0(s) - \frac{(B_1 (s)^2 X_0(s))}{B_1 s + K_1}$$

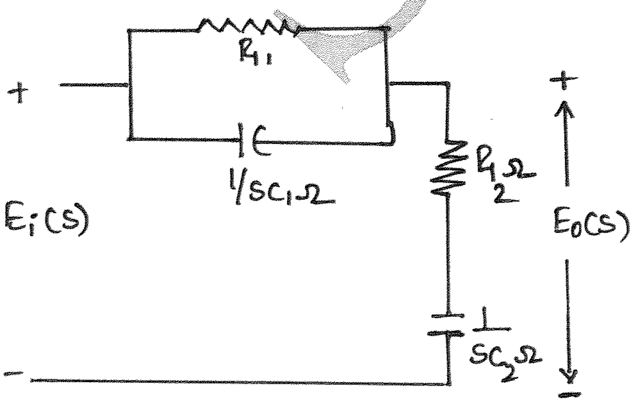
$$(B_2 s + K_2) X_i(s) = X_0(s) \left[\frac{(B_1 s + K_1)(B_1 s + B_2 s + K_2) - (B_1 s)^2}{(B_1 s + K_1)} \right]$$

$$\frac{X_0(s)}{X_i(s)} = \frac{(B_1 s + K_1)(B_2 s + K_2)}{(B_1 s + K_1)(B_1 s + B_2 s + K_2) - (B_1 s)^2}$$

$$= \frac{(B_1 s + K_1)(B_2 s + K_2)}{\cancel{(B_1 s)^2} + B_1 B_2 s^2 + B_1 s K_2 + B_1 s K_1 + B_2 s K_1 + K_1 K_2 - \cancel{(B_1 s)^2}}$$

$$\boxed{\frac{X_0(s)}{X_i(s)} = \frac{(B_1 s + K_1)(B_2 s + K_2)}{s^2 B_1 B_2 + s(B_1 K_2 + B_1 K_1 + B_2 K_1) + K_1 K_2}}$$

The transform Network for the Electrical Network is as shown below.



Note:-

$V(t) = R\, i(t)$

$V(s) = R\, i(s)$

$V(t) = L \frac{di(t)}{dt}$

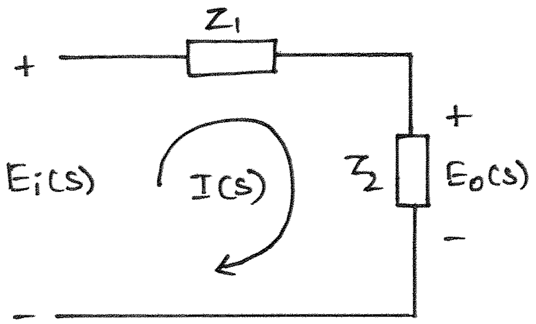
$V(s) = L\, s\, I_L(s)$

$= sL\, I_L(s)$

$V(t) = \frac{1}{C}\int i\,dt$

$V(s) = \frac{1}{C} \frac{I(s)}{s}$

$= \frac{1}{sC} I(s)$



$$Z_1 = \frac{R_1 \times \frac{1}{sC_1}}{R_1 \times \frac{1}{sC_1}} = \frac{R_1}{R_1 sC_1 + 1}$$

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{R_2 sC_2}{sC_2} + 1$$

$$I(s) = \frac{E_i(s)}{Z_1 + Z_2}$$

$$E_0(s) = Z_2 I(s)$$

$$= \frac{Z_2}{Z_1 + Z_2} \cdot E_i(s)$$

$$\frac{E_0(s)}{E_i(s)} = \frac{Z_1}{Z_1 + Z_2}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{\left(\frac{R_2 sC_2 + 1}{sC_2}\right)}{\frac{R_1}{R_1 sC_1 + 1} + \frac{R_2 sC_2 + 1}{sC_2}} = \frac{(R_1 sC_1 + 1)(R_2 sC_2 + 1)}{R_1 sC_2 + (R_1 sC_1 + 1)(R_2 sC_2 + 1)}$$

$$= \frac{C_1\left(R_1 s + \frac{1}{C_1}\right) C_2\left(R_2 s + \frac{1}{C_2}\right)}{R_1 sC_2 + R_1 R_2 s^2 C_1 C_2 + R_1 sC_1 + R_2 sC_2 + 1}$$

$$= \frac{C_1 C_2 \left(R_1 s + \frac{1}{C_1}\right)\left(R_2 s + \frac{1}{C_2}\right)}{C_1 C_2 \left[R_1 R_2 s^2 + \frac{R_1 s}{C_1} + \frac{R_1 s}{C_2} + \frac{R_2 s}{C_1} + \frac{1}{C_1 C_2}\right]}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{\left(R_1 s + \frac{1}{C_1}\right)\left(R_2 s + \frac{1}{C_2}\right)}{R_1R_2 s^2 + s\left(\frac{R_1}{C_2} + \frac{R_1}{C_1} + \frac{R_2}{C_1}\right) + \frac{1}{C_1C_2}}} \rightarrow (6)$$

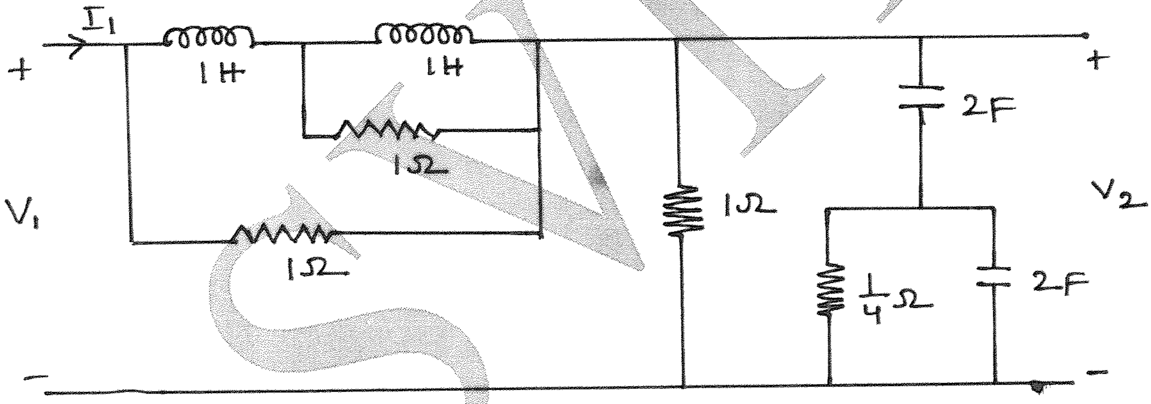
Equation (5) and (6) are mathematically simillar hence. the two System will be Analogous.

$$R_1 = B_1 \quad , \quad R_2 = B_2$$

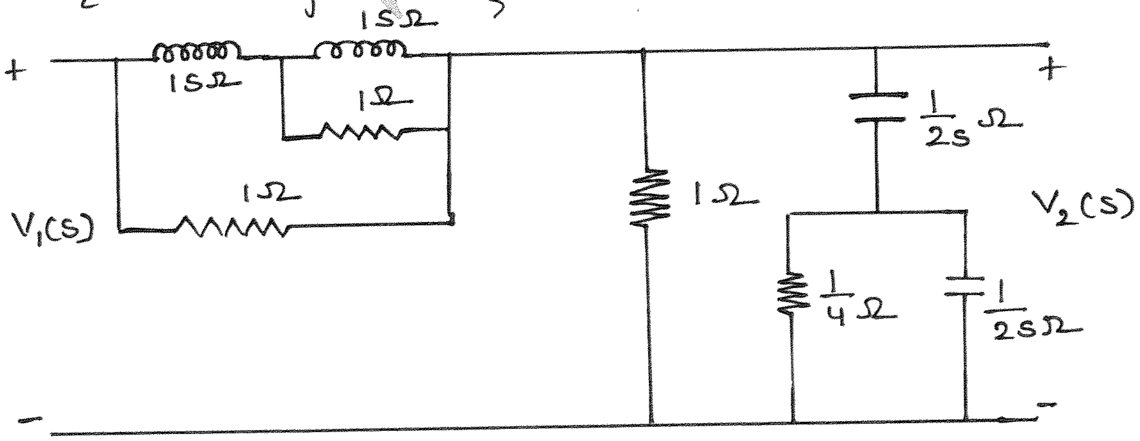
$$C_1 = \frac{1}{K_1} \quad , \quad C_2 = \frac{1}{K_2}$$

5) For the two port network shown in figure obtain the transfer function.

(i) $\frac{V_2(s)}{V_i(s)}$ (ii) $\frac{V_1(s)}{I_1(s)}$

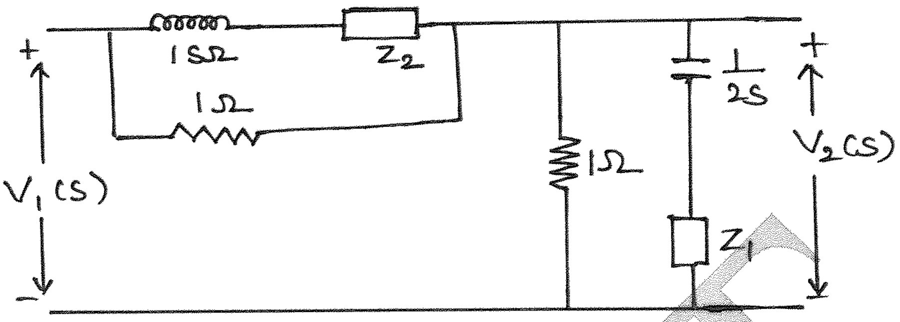


Solution :- Transformer network is as shown below.



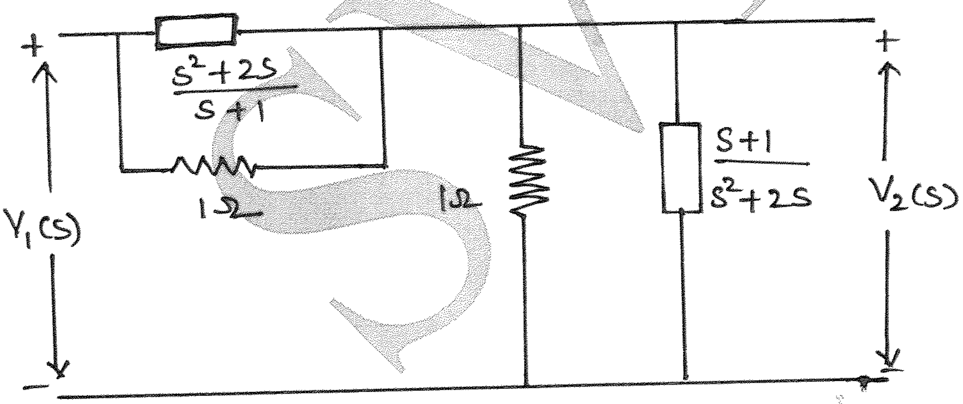
$$Z_1 = \frac{\frac{1}{4} * \frac{1}{2s}}{\frac{1}{4} * \frac{1}{2s}} = \frac{1}{2s+4}$$

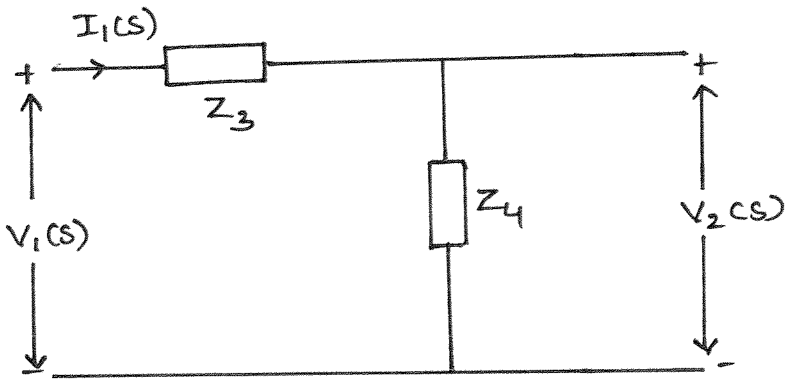
$$Z_2 = \frac{S \times 1}{S+1} = \frac{S}{S+1}$$



$$S + Z_2 = S + \frac{S}{S+1} = \frac{S^2+2S}{S+1}$$

$$Z_1 + \frac{1}{2S} = \frac{1}{2S+4} + \frac{1}{2S} = \frac{2S+2S+4}{2S(2S+4)} = \frac{4S+4}{4S^2+8S} = \frac{S+1}{S^2+2S}$$





$$Z_3 = \frac{\left(\frac{s^2+2s}{s+1}\right)\times 1}{\frac{s^2+2s}{s+1}+1} = \frac{s^2+2s}{s^2+3s+1}$$

$$Z_4 = \frac{1\times\frac{(s+1)}{s^2+2s}}{1+\frac{s+1}{s^2+2s}}$$

$$Z_4 = \frac{s+1}{s^2+2s+1+s} = \frac{s+1}{s^2+3s+1}$$

$$I_1(s) = \frac{V_1(s)}{Z_3+Z_4}$$

$$V_2(s) = I_1(s) + Z_4$$

$$V_2(s) = \frac{V_1(s)}{Z_3+Z_4}\cdot Z_4$$

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_4}{Z_3+Z_4}$$

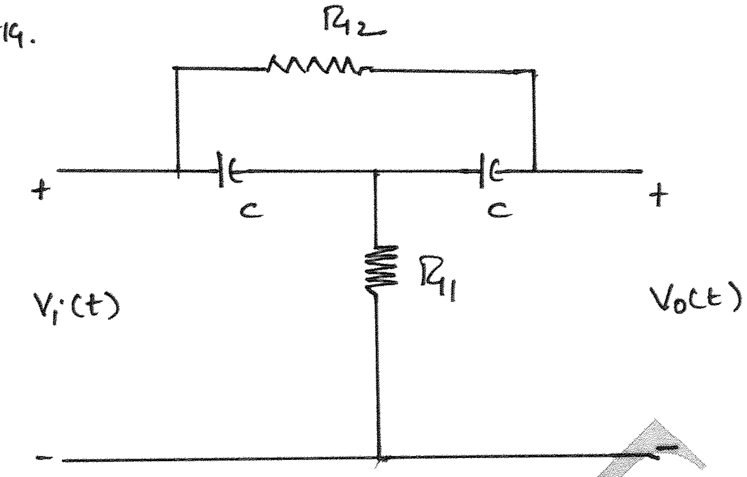
& $$\frac{V_1(s)}{I_1(s)} = Z_3+Z_4$$

$$Z_3+Z_4 = \frac{s^2+2s}{s^2+3s+1} + \frac{s+1}{s^2+3s+1}$$

$$Z_3+Z_4 = 1$$

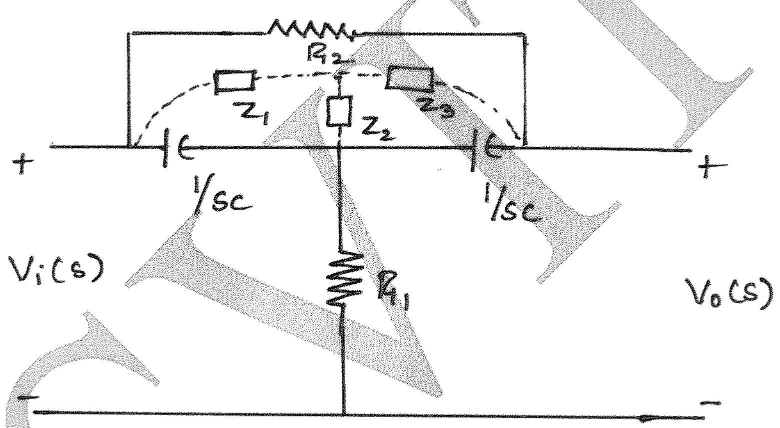
$$\therefore \boxed{\frac{V_2(s)}{V_1(s)} = \frac{Z_4}{1} = \frac{s+1}{s^2+3s+1}} \quad \& \quad \boxed{\frac{V_1(s)}{I_1(s)} = 1}$$

6) The circuit of bridge-T Network is shown in figure determine the transfer function $\frac{V_o(s)}{V_i(s)}$ of the network.

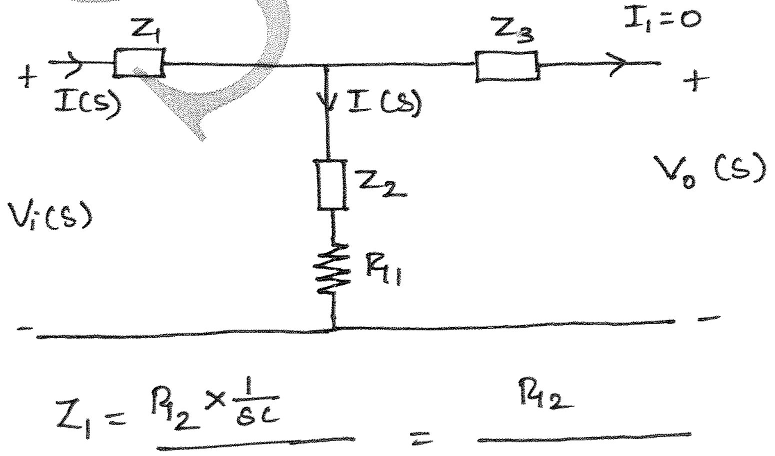


Solution :-

Transform network is as shown below.



⇓



$$Z_1 = \frac{R_2 \times \frac{1}{sC}}{R_2 + \frac{1}{sC} + \frac{1}{sC}} = \frac{R_2}{R_2 sC + 2}$$

$$Z_3 = Z_1$$

$$Z_2 = \frac{\frac{1}{sC} \times \frac{1}{sC}}{R_2 + \frac{1}{sC} + \frac{1}{sC}} = \frac{1}{sC\,(R_2 sC + 2)}$$

$$I(s) = \frac{V_i(s)}{Z_1 + Z_2 + R_1}$$

$$V_o(s) = I(s)\,(Z_2 + R_1)$$

$$V_o(s) = \frac{V_i(s)}{Z_1 + Z_2 + R_1} \cdot Z_2 + R_1$$

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2 + R_1}{Z_1 + Z_2 + R_1}$$

$$Z_1 + Z_2 = \frac{R_2}{2 + R_2 sC} + \frac{1}{sC\,(2 + R_2 sC)}$$

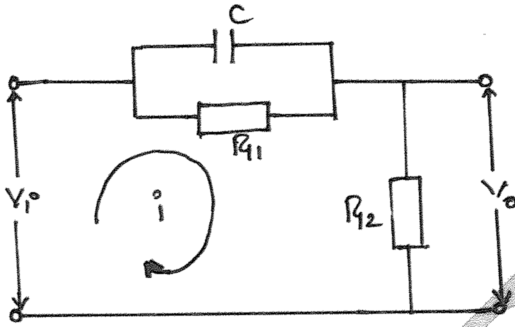
$$= \frac{R_2 sC + 1}{sC\,(2 + R_2 sC)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC(2 + R_2 sC)} + R_1}{\frac{(R_2 sC + 1)}{sC\,(2 + R_2 sC)} + R_1}$$

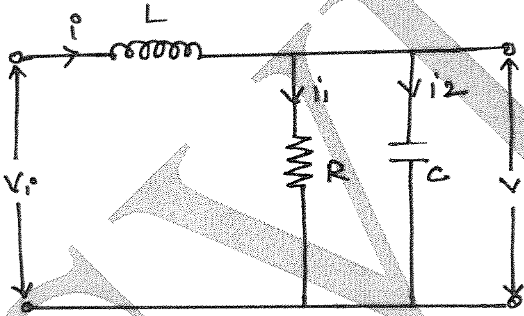
Transfer Function: $\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1 + R_1 sC\,(2 + R_2 sC)}{R_2 sC + 1 + R_1 sC\,(2 + R_2 sC)}}$

Practice Problems on Transfer Function

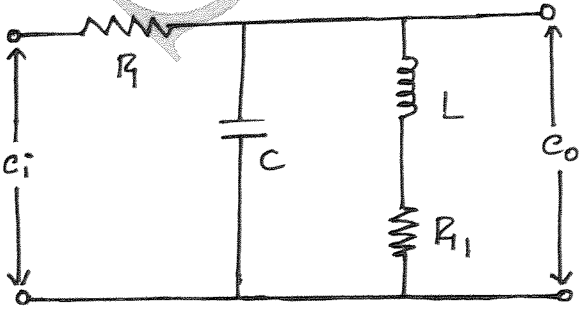
① Derive the transfer function $\frac{V_o(S)}{V_i(S)}$ for the circuit shown in the figure.



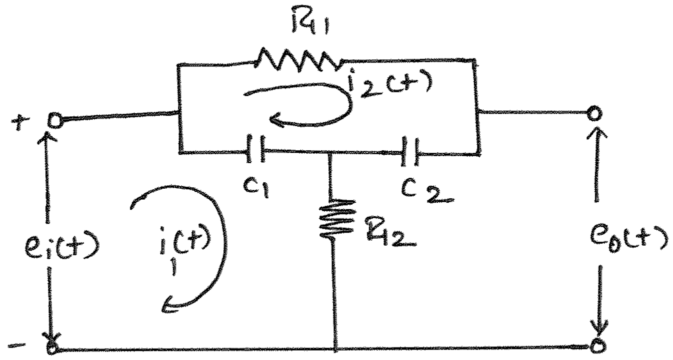
② Find the transfer function of the Electrical network shown in the figure.



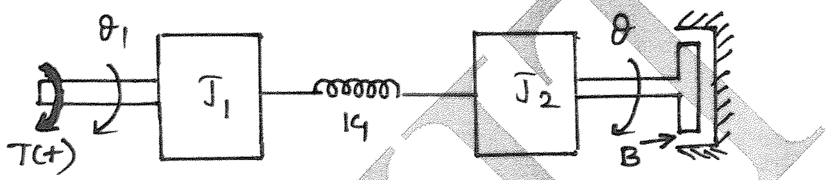
③ Derive the transfer function of the network shown in the figure



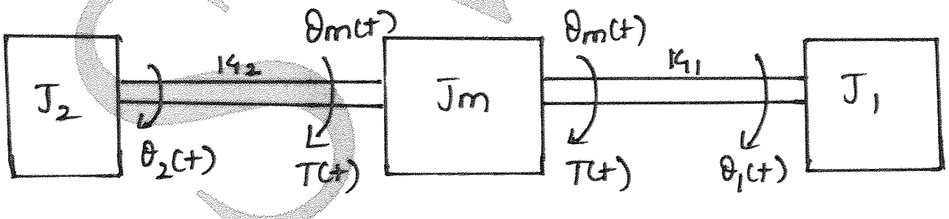
4) Find the transfer function of the Electrical network shown in the figure



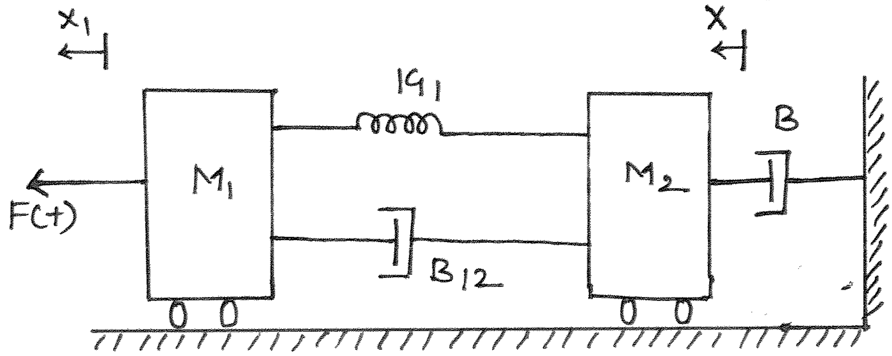
5) Obtain the transfer function of the mechanical. System shown in the figure.



6) Find the transfer function $\frac{\theta_1(s)}{T(s)}$ for the system shown in the figure.

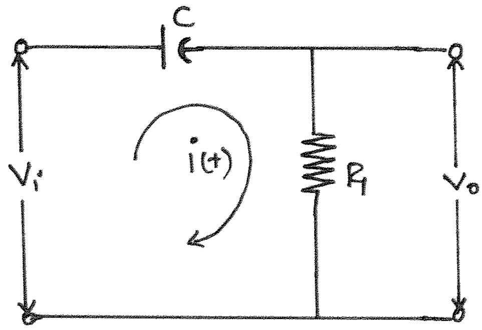


7) Write the differential Equations governing the Mechanical translational system shown in the figure. and determine the transfer function $\frac{X(S)}{F(S)}$

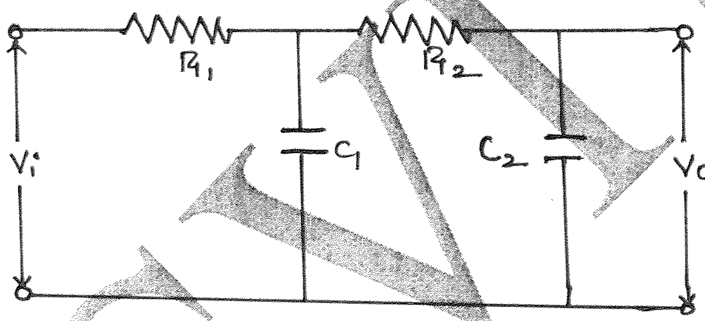


Assignment Problems On Transfer Function

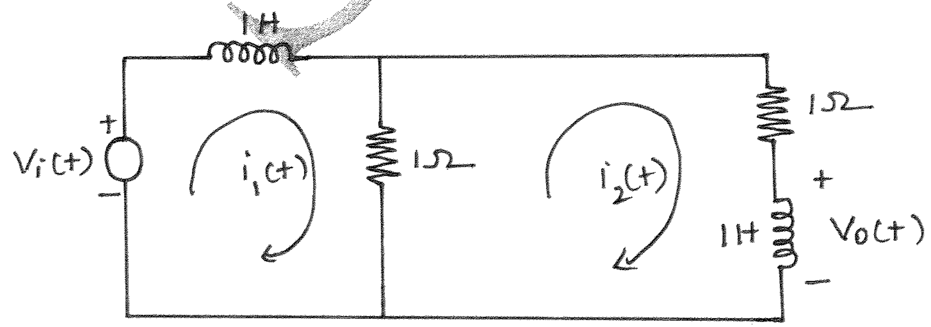
① Derive the transfer function $\frac{V_o(s)}{V_i(s)}$ for the Circuit shown in the figure.



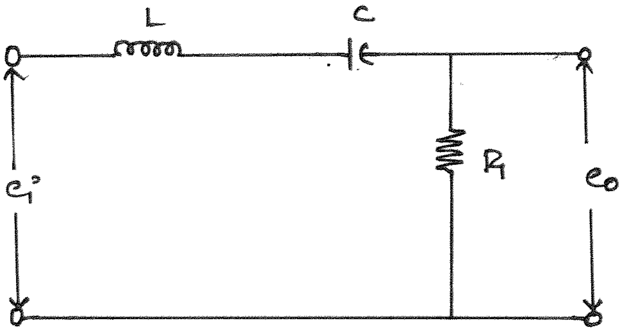
② Find the transfer function $\frac{V_o(s)}{V_i(s)}$ for the network shown in the figure.



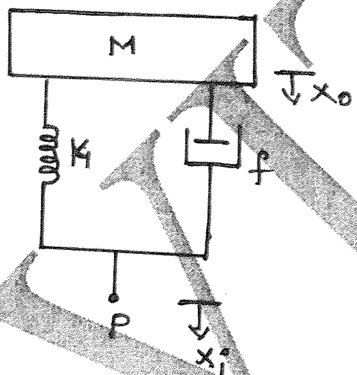
③ Find the input - Output relationship of the Electrical network shown in the figure



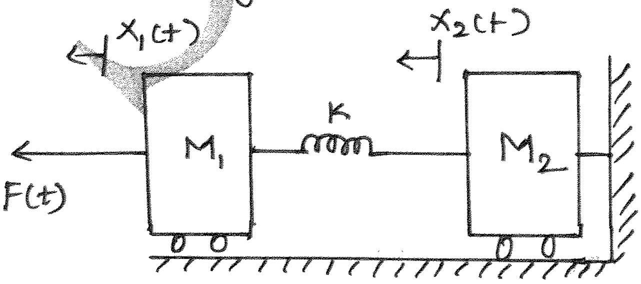
4) Derive the transfer function of the network shown in the figure.



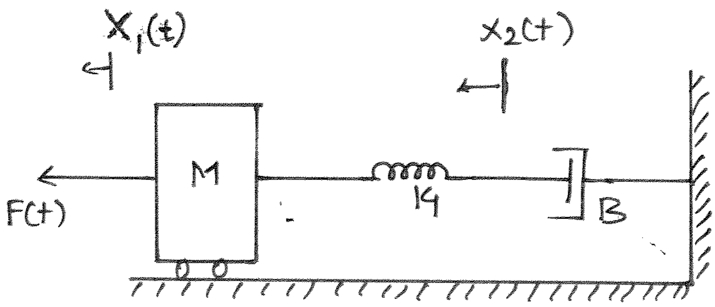
5) Find the transfer function $\frac{X_o(s)}{X_i(s)}$ for the system shown in the figure



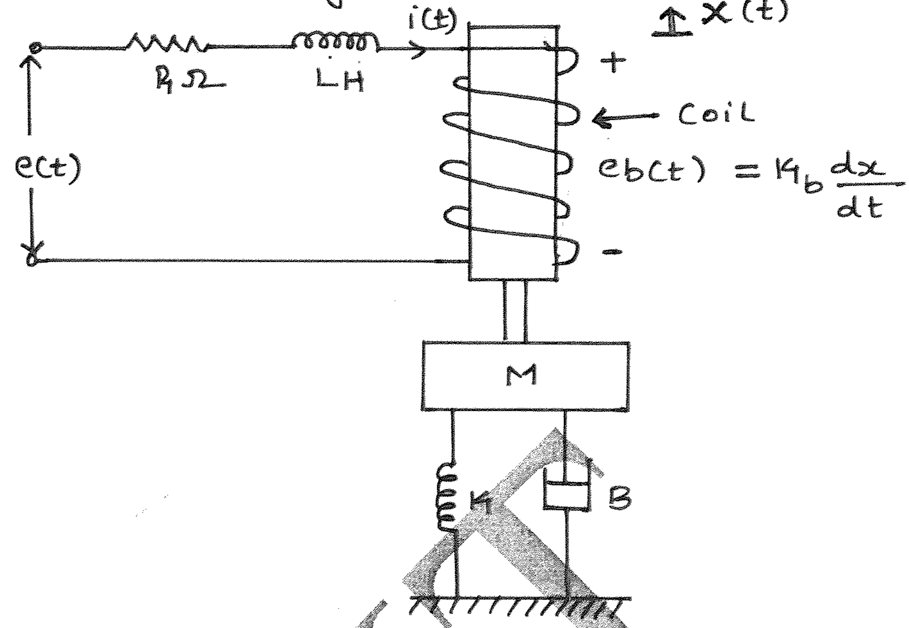
6) Find the transfer function $\frac{X_1(s)}{F(s)}$ for the system shown in the figure.



7)

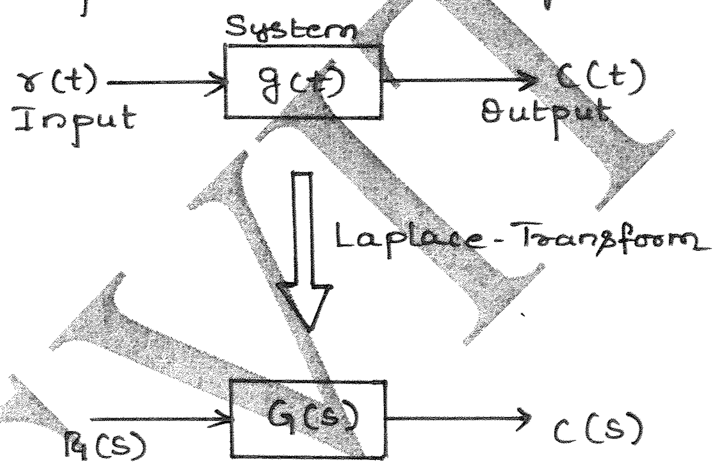


8) Find the transfer function $\frac{X(s)}{E(s)}$ for the Electromechanical System shown in the figure.



Block Diagram Algebra

- * In this section we discuss development of block diagram for the systems.
- * A system consists of number of components, the function of each component is represented by a block.
- * All the blocks are inter connected by lines with arrows indicating the flow of signals from the output of one block to the input of another.
- * Such a block diagram gives an overall idea of the inter-relationships that exist among various components.

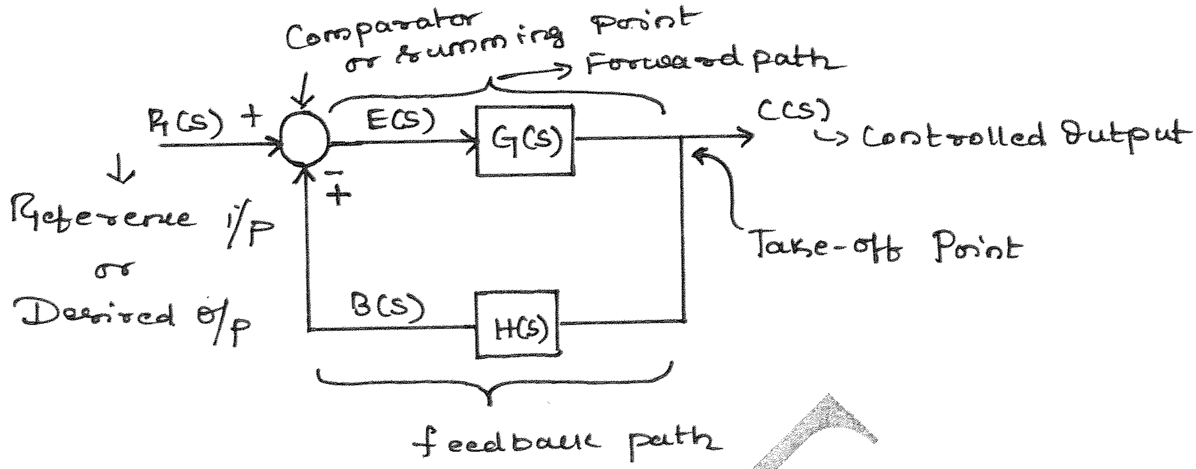


- * Let us consider a system with transfer function G(S).
- * The system can be represented by a block as shown in the figure above.
- * The input signal in to the block is R(S) which is the Laplace transform of input signal r(t)
- * The output signal of the block is C(S) which is the Laplace transform of the output signal c(t).
- * The flow of signal is unidirectional from the input to the output. The output C(S) is equal to the convolution of the input signal and transfer function G(S)

  i.e, $\boxed{C(S) = G(S)\,R(S)}$

⇒ Block Diagram Transformation.

1) Feedback Control System (Eliminating feedback loop).



* The Error signal E(s) is given by

$$E(s) = R(s) - B(s) \quad \text{——} \ (1)$$

W.K.T $\quad G(s) = \dfrac{C(s)}{E(s)}$

or $\quad E(s) = \dfrac{C(s)}{G(s)}$

&

$$H(s) = \frac{B(s)}{C(s)}$$

or

$$B(s) = H(s) \cdot C(s)$$

By substituting the value of E(s) & B(s) in (1)

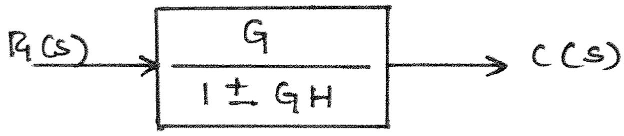
$$\frac{C(s)}{G(s)} = R(s) - H(s) \cdot C(s)$$

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

$$C(s) + G(s)H(s)C(s) = G(s)R(s)$$

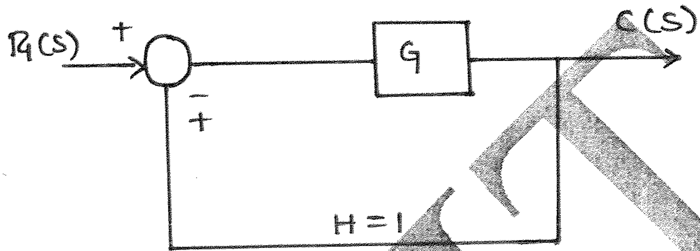
$$C(s)\left[1 + G(s)H(s)\right] = G(s)R(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{G}{1+GH}}$$



$\dfrac{C(s)}{R(s)}$ is known as the overall transfer function or closed loop transfer function.

If H = 1, then it is said to be Unity feed back control system, shown in the figure.

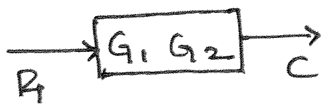


$$\boxed{\frac{C(s)}{R(s)} = \frac{C}{R} = \frac{G}{1 \pm G}}$$

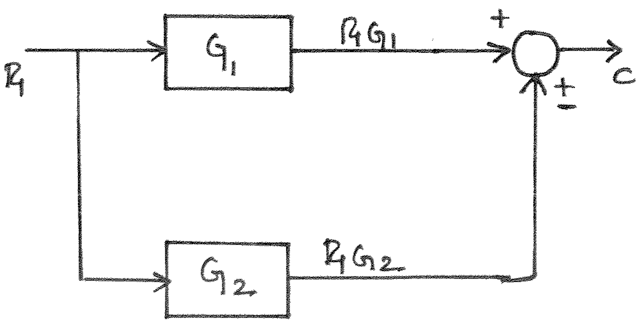
2) Blocks in cascade.

$C = R\,G_1 G_2$

$\dfrac{C}{R} = G_1 G_2$

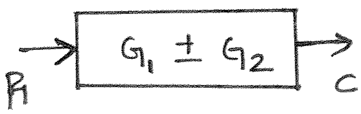


3) Blocks in parallel.

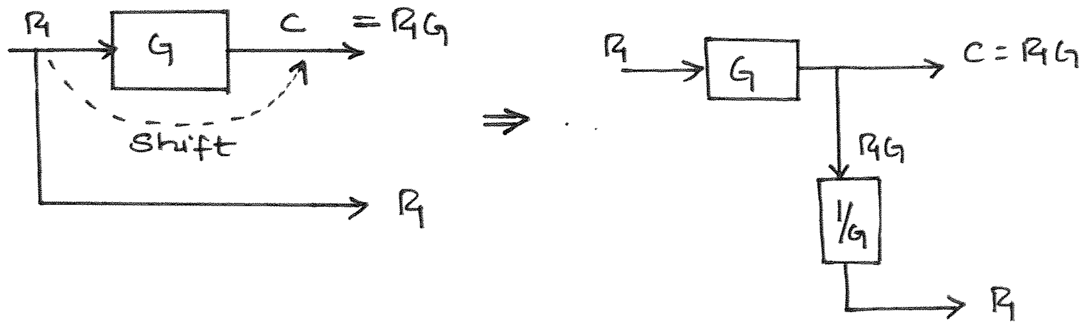


$C = RG_1 \pm RG_2 = R(G_1 \pm G_2)$

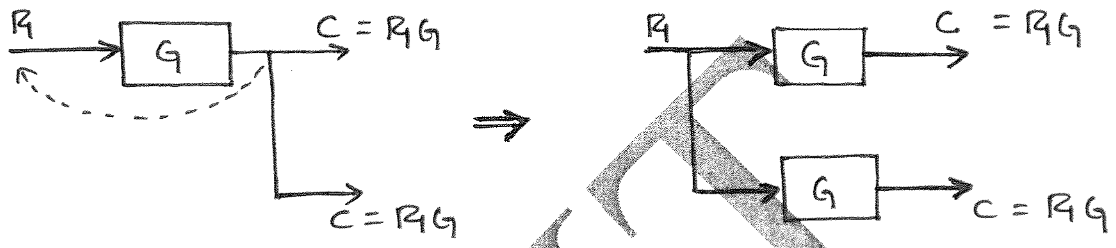
$\dfrac{C}{R} = G_1 \pm G_2$



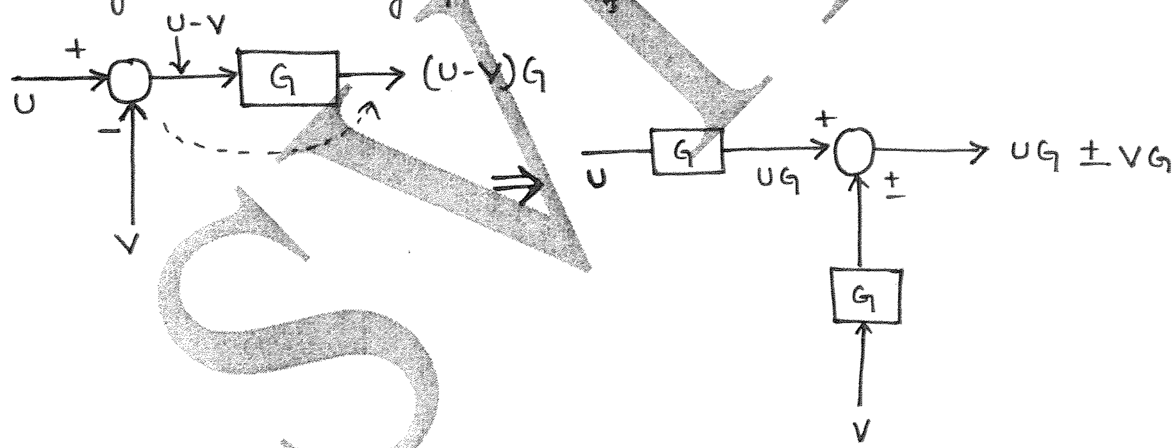
4) Shifting a take off point after a block.



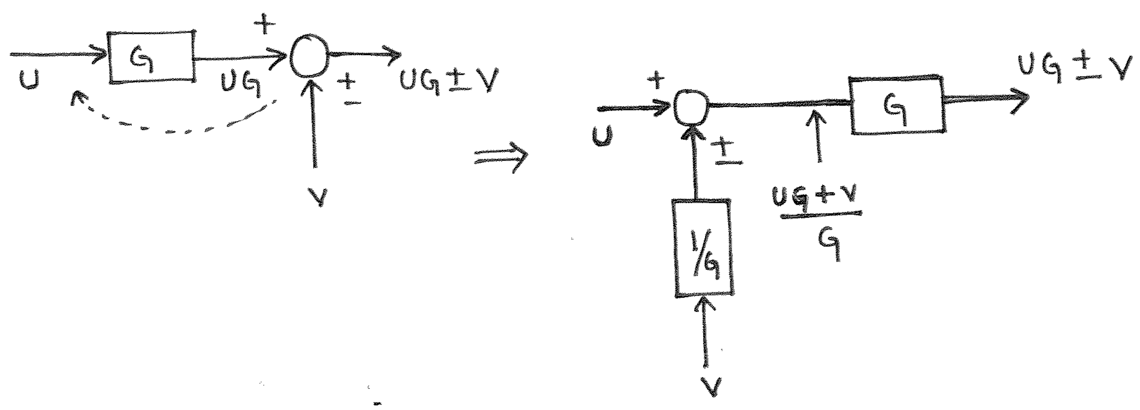
5) Shifting a take-off point before the block.



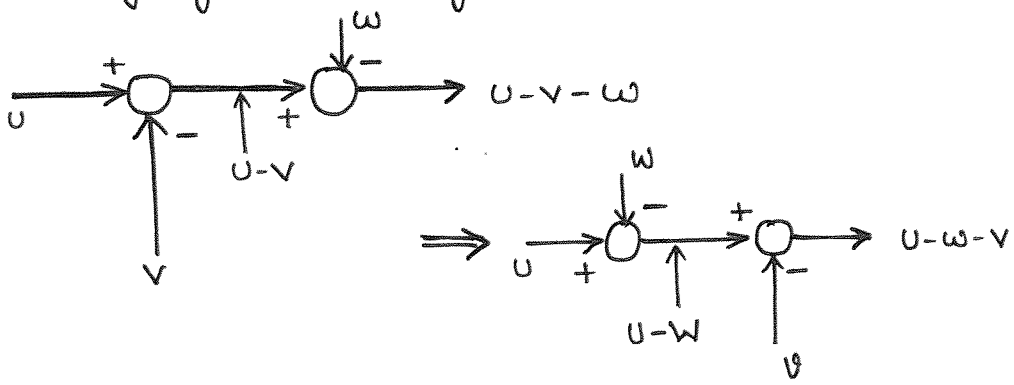
6) Moving a Summing point after a block.



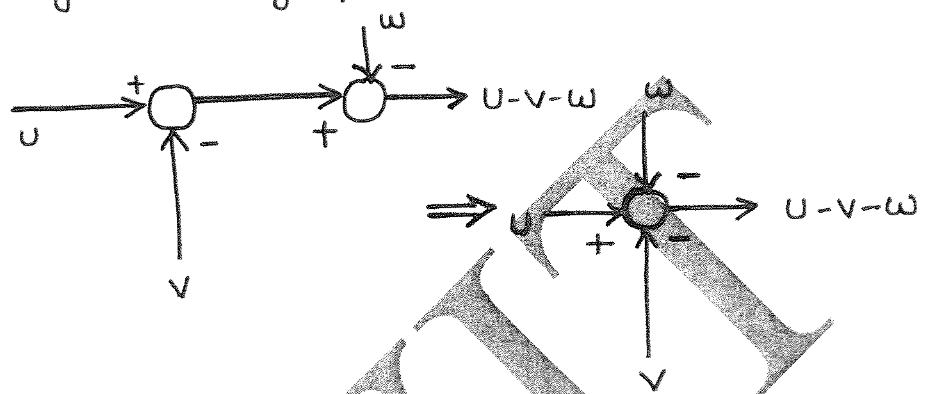
7) Moving a Summing point before the block.



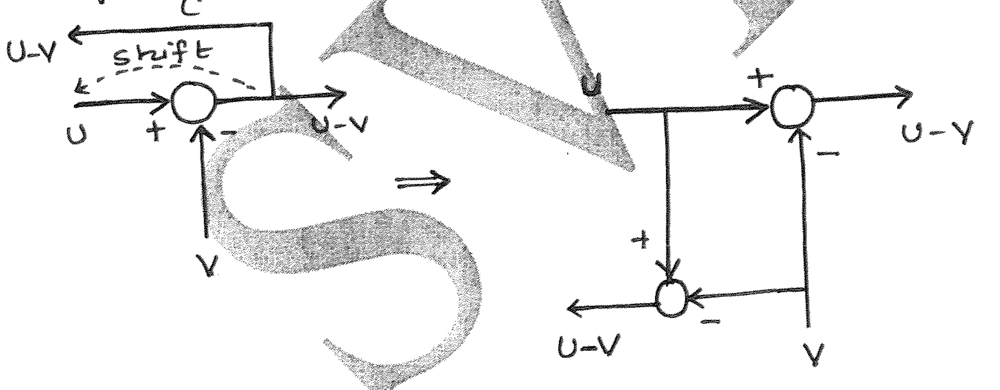
8) Rearranging Summing Points.



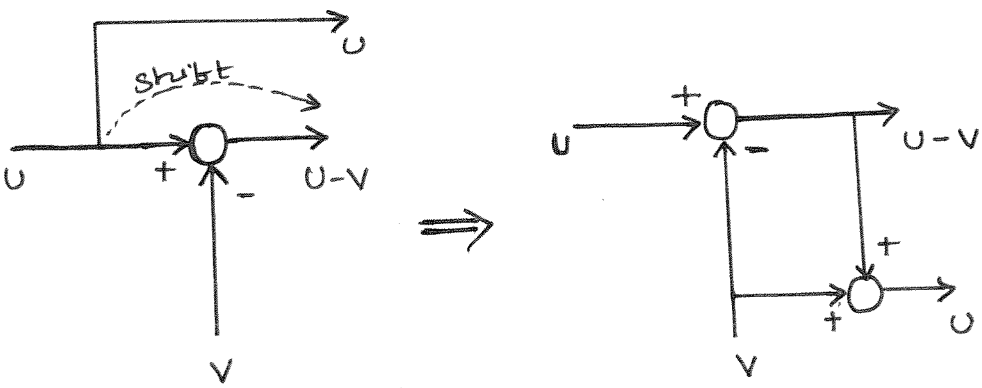
9) Adding Summing points.



10) Shifting a take-off point before a summing Point



11) Shifting take off point after the summing point.



* **Steps for reduction of Complicated Block diagram:**

step1:- Combine all Cascade blocks.

step2:- Combine all parallel blocks.

step3:- Eliminate all minor feed back loops.

step4:- Shift Summing points to the left and take off Points to the right of the major loop.

step5:- Repeat steps 1 to 4 until the Canonical form has been achieved for a particular input.

step6:- Repeat steps 1 to 5 for each input as required

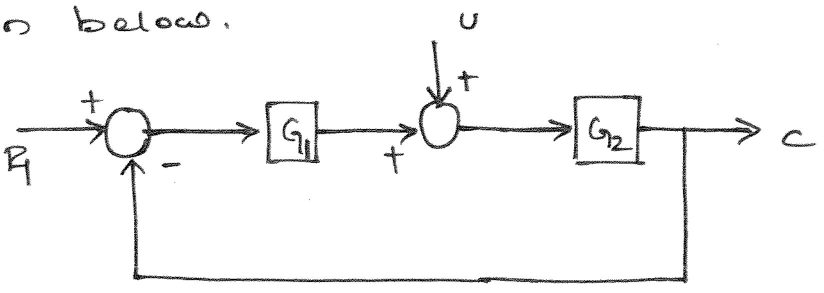
* **Superposition of Multiple Inputs:-**

→ Some time it is necessary to Evaluate system Performance when several inputs are simultaneously applied at different points of the System.

→ When multiple inputs are present in a linear System, Each input is treated independently of the others.

For Example:-

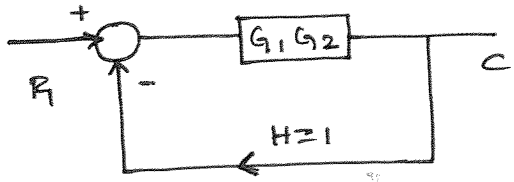
Determine the transfer function for the blockdiagram shown below.



Step 1:- Consider any one input at a time, (U)

Put U = 0,

Step 2:- Block diagram is redrawn as shown below.



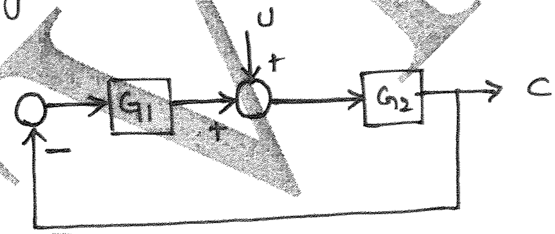
Step 3:- The Output Equation $\frac{C}{R}$ is given by

$$\boxed{\frac{C}{R} = \frac{G_1 G_2}{1+G_1G_2}}$$

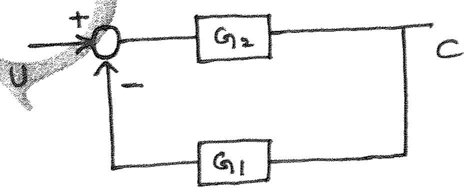
Step 4:- By considering second input (R)

Put R = 0

Step 5:- Block diagram is redrawn as shown below.



⇓ Similar.



The output transfer function is given by

$$\frac{C}{U} = \frac{G_2}{1+G_2G_1}$$

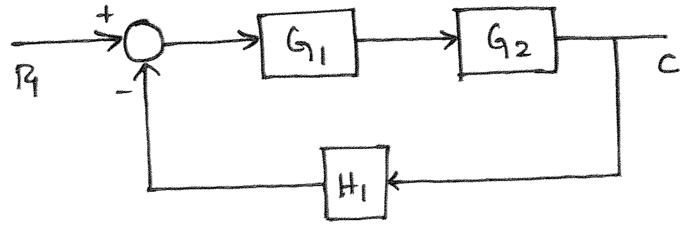
The total output is given by

$$C = \left[\frac{G_1 \; G_2}{1+G_1G_2}\right] R + \left[\frac{G_2}{1+G_2G_1}\right] U$$

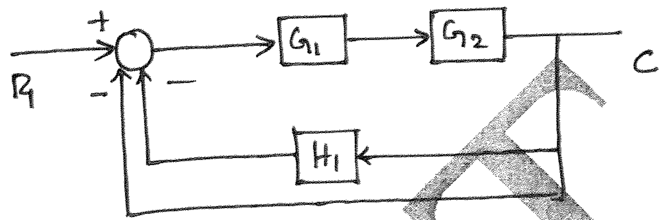
Problems to be solved in Class

⇒ For the block diagram shown in the figure find the Overall transfer function.

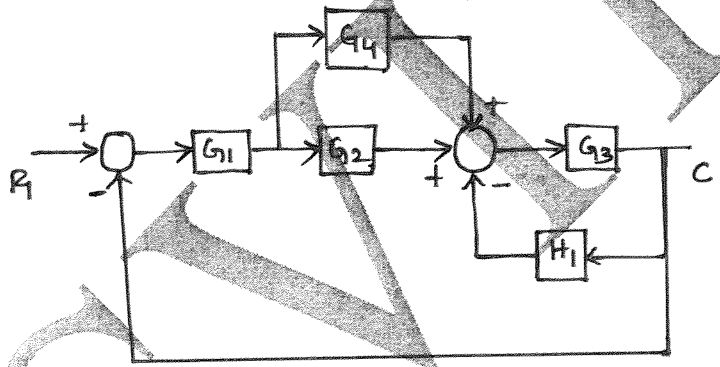
1)



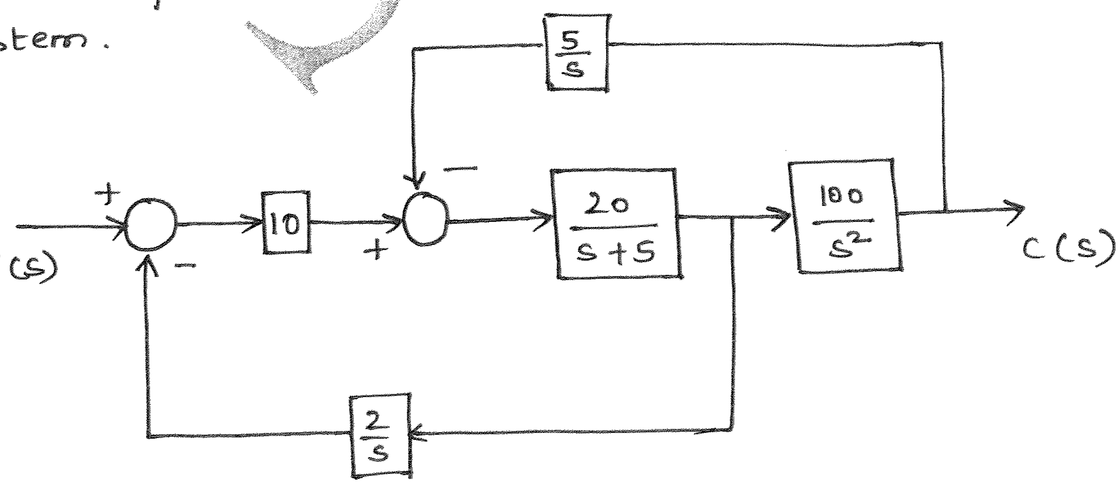
2)



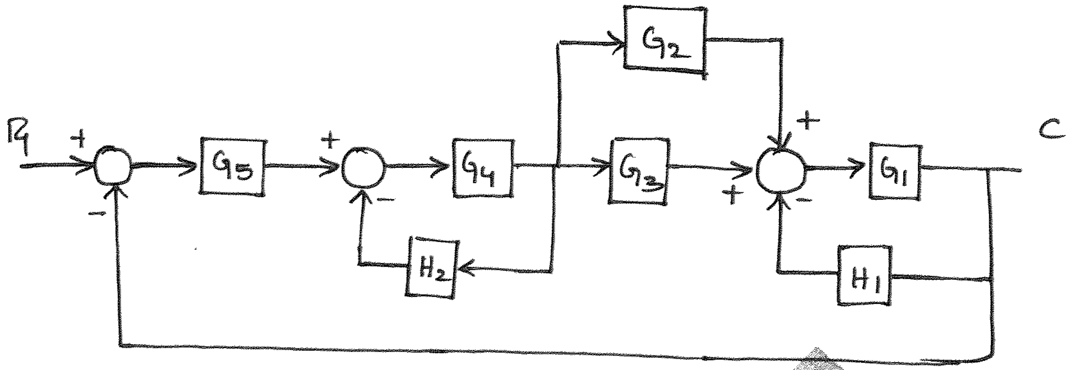
3)



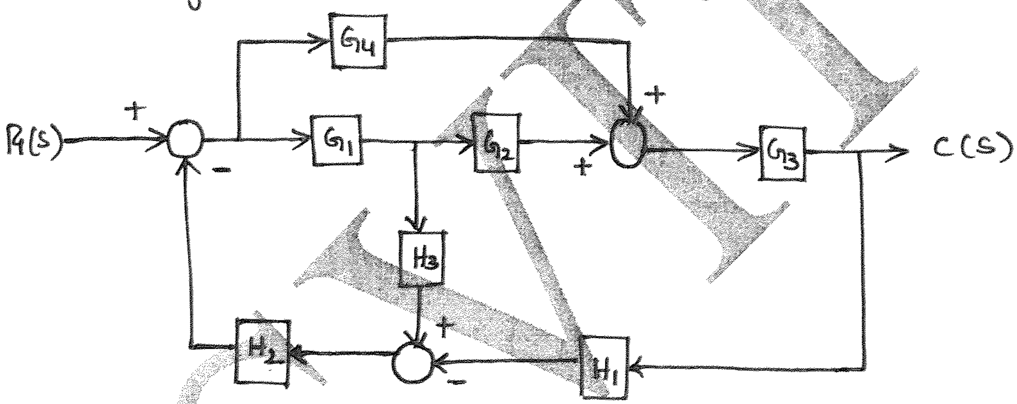
⇒ Reduce the block diagram shown in the figure to its simplest form and Determine the order of the system.



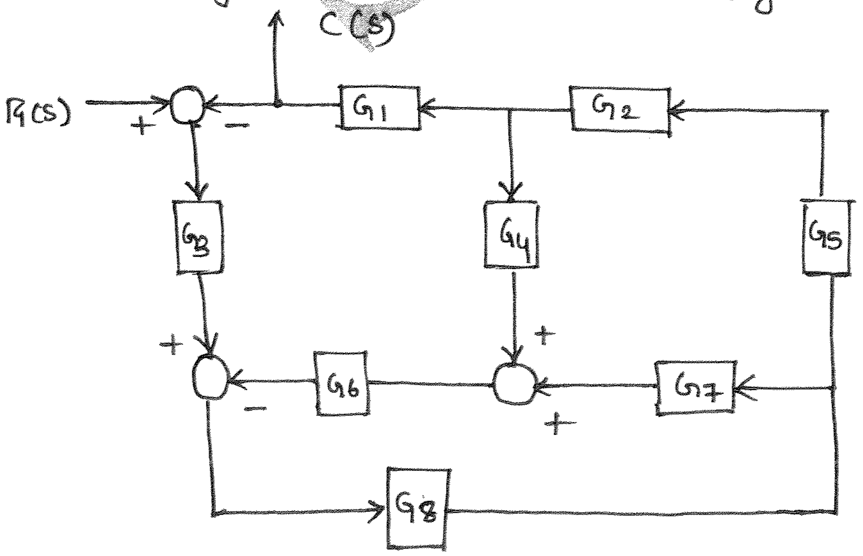
⟹ Find the Overall transfer function for the block diagram shown below. using block diagram reduction technique.



⟹ Determine the Overall transfer function for the block diagram shown in the figure.



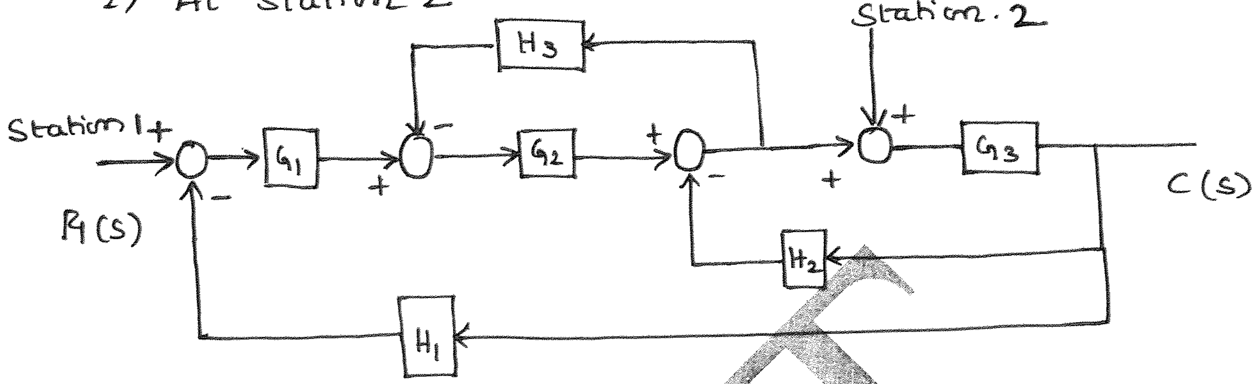
⟹ Find the Overall transfer function $\frac{C(S)}{R(S)}$ for the Block diagram shown in the figure.



=> For the system represented by the block diagram shown in the figure. Evaluate the closed loop transfer function when the input are is

1> At station 1

2> At station 2



=> The performance equations of a controlled system are given by the following set a linear algebraic. equation draw the block diagram and determine. the overall transfer function $\frac{C(S)}{R(S)}$ by reducing the block diagram in steps.

$$E_1(S) = R(S) - H_3(S)\, C(S)$$

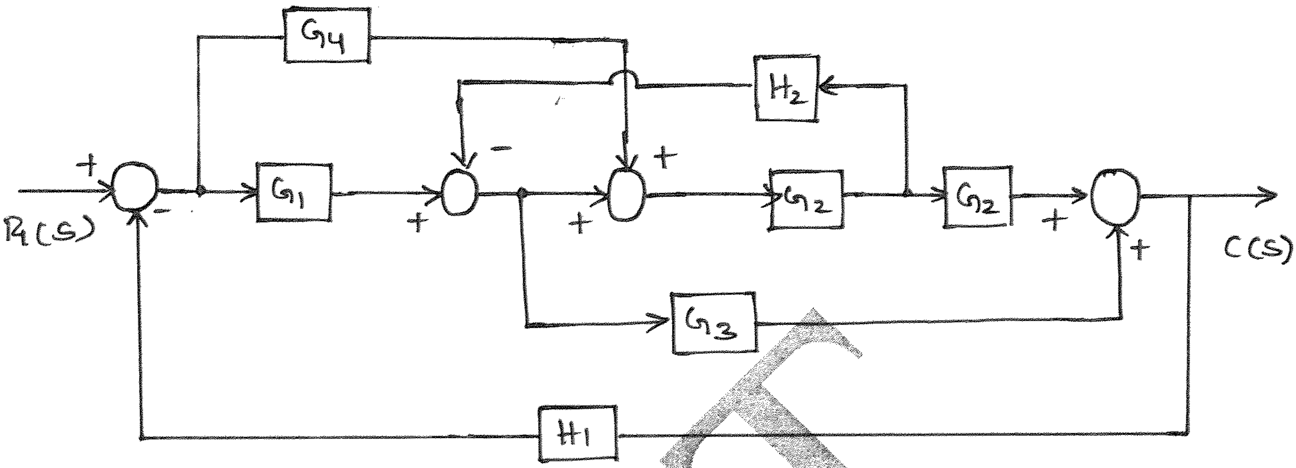
$$E_2(S) = E_1(S) - H_1(S)\, E_4(S)$$

$$E_3(S) = G_1(S)\, E_2(S) - H_2(S)\, C(S)$$

$$E_4(S) = E_3(S)\, G_2(S)$$

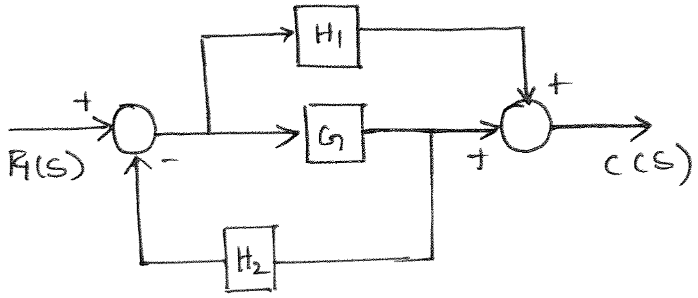
$$C(S) = G_3(S)\, E_4(S)$$

=> Determine the overall transfer function for the system represented by the block diagram shown in the figure using block diagram reduction technique.



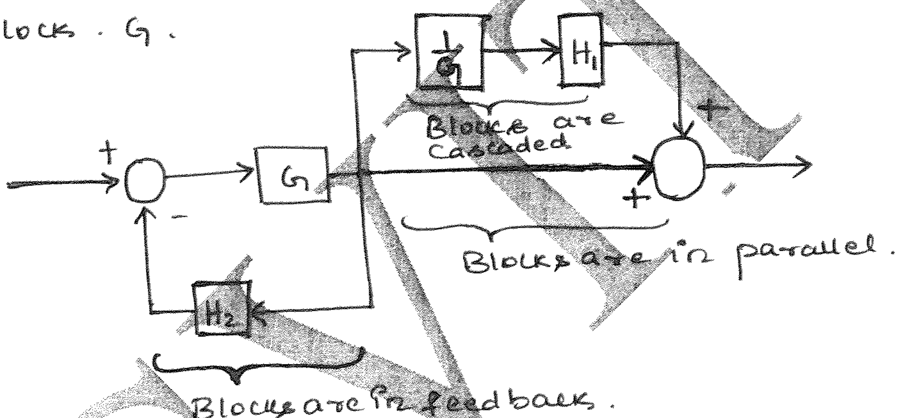
Problems on Block diagram Algebra:-

1) Simplify the block diagram shown in the figure and obtain the overall transfer function $\frac{C(S)}{R(S)}$

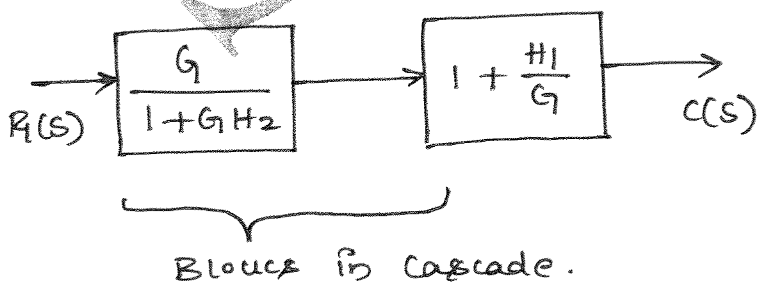


Solution:-

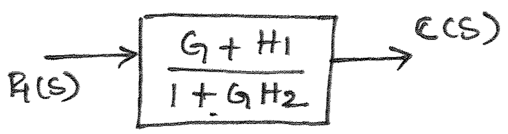
Step 1:- By shifting the take off point after the block G.



Step 2:- Eliminating the feedback loop and combining the blocks in parallel and cascaded.



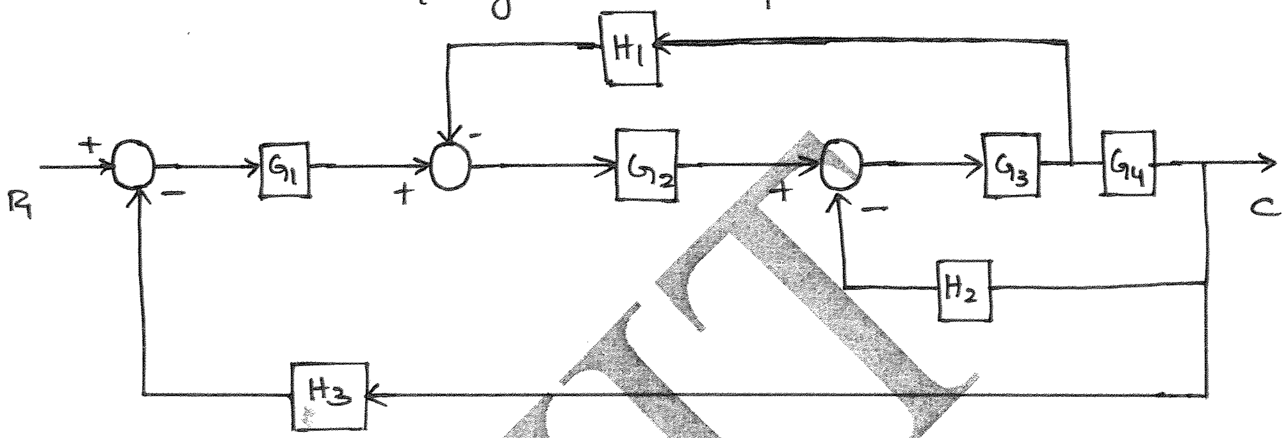
Step 3:- Combining blocks in cascade.



Hence the Overall trasfer function is given by

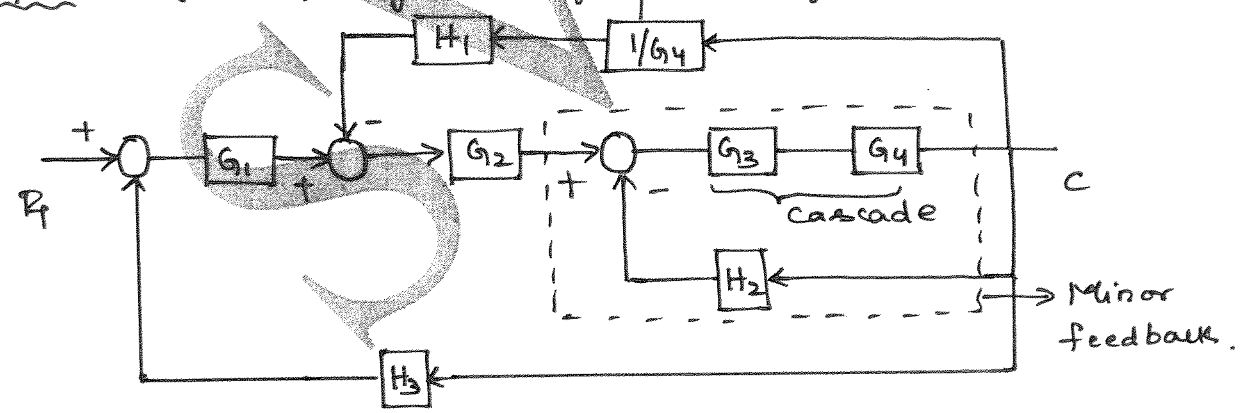
$$\boxed{\frac{C(s)}{R(s)} = \frac{G + H_1}{1 + GH_2}}$$

2) Find the ratio $\frac{C(s)}{R(s)}$ for the block diagram shown below using block diagram reduction technique.

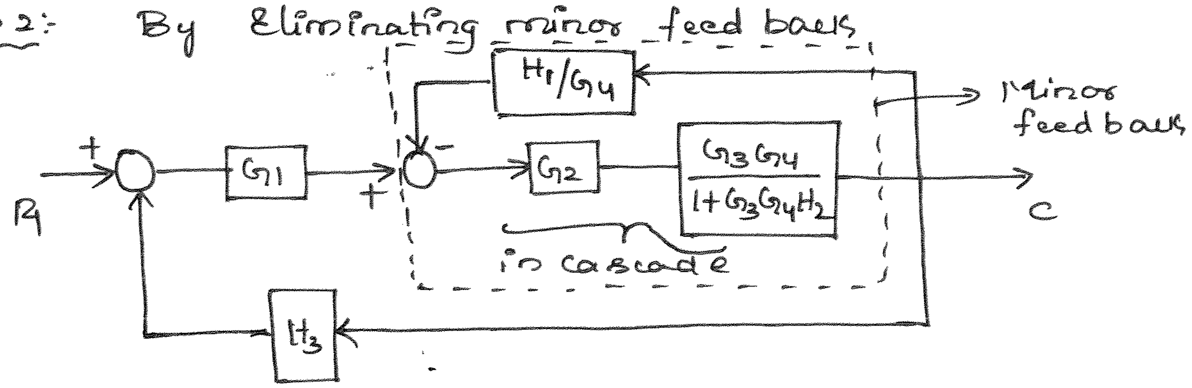


Solution:-

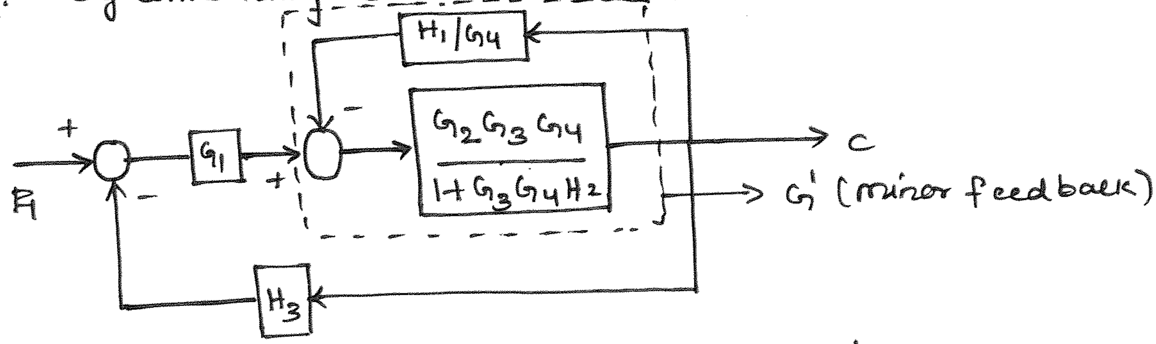
Step1:- By shifting take off point after block G_3.



Step 2:- By Eliminating minor feed back



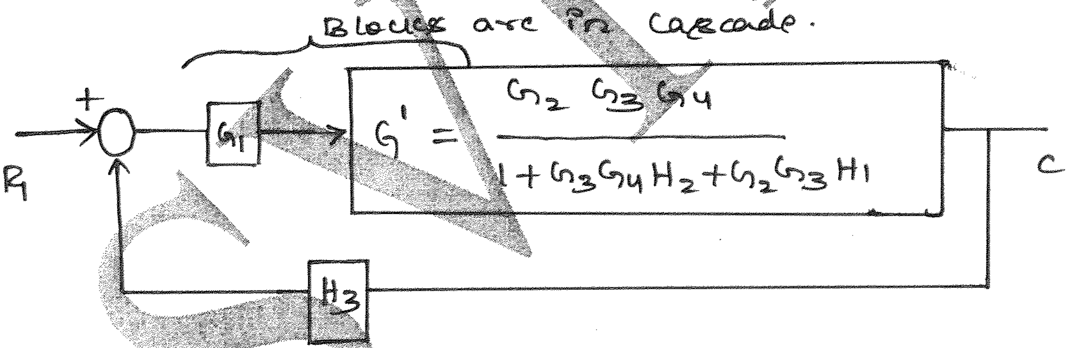
Step 3:- By Combining cascaded blocks

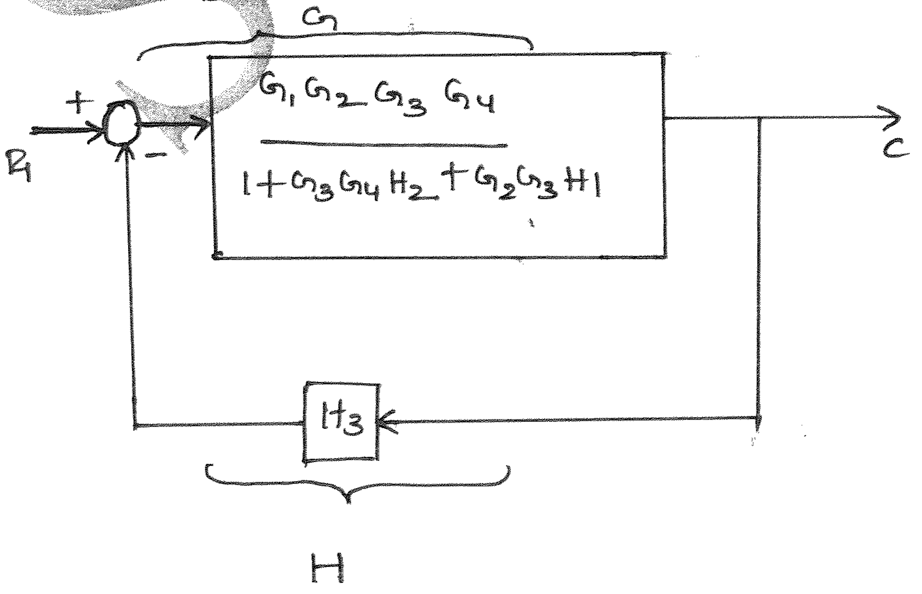


Step 4:- By Eliminating minor feed back G'

$$G' = \frac{\dfrac{G_2 G_3 G_4}{1+G_3 G_4 H_2}}{1 + \dfrac{G_2 G_3 G_4}{1+G_3 G_4 H_2} \cdot \dfrac{H_1}{G_4}}$$

$$\boxed{G' = \frac{G_2 G_3 G_4}{1 + G_3 G_4 H_2 + G_2 G_3 H_1}}$$



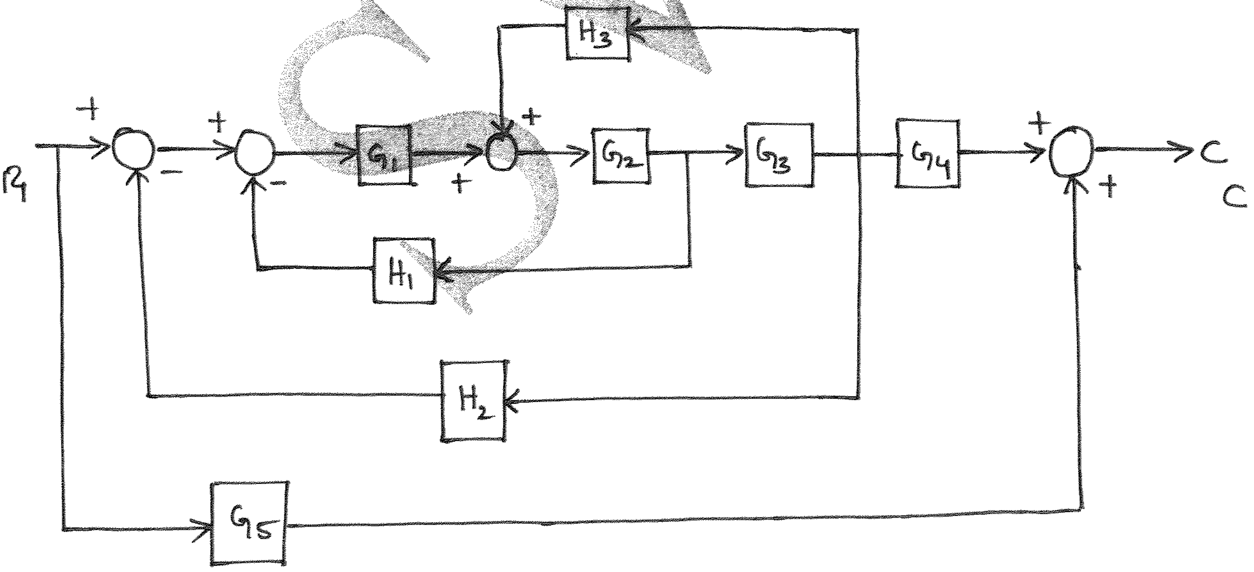


$$\frac{C}{R} = \frac{G}{1+GH} = \frac{\dfrac{G_1 G_2 G_3 G_4}{1+G_3 G_4 H_2 + G_2 G_3 H_1}}{1 + \dfrac{G_1 G_2 G_3 G_4}{1+G_3 G_4 H_2 + G_3 G_2 H_1} \cdot H_3}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3 G_4}{1+G_3 G_4 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 G_4 H_3}$$

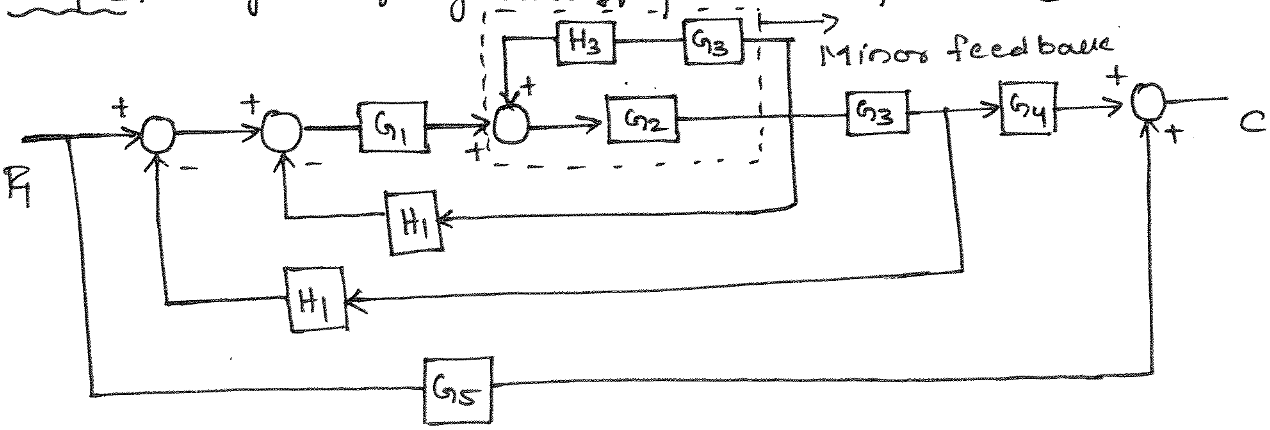
$$R \longrightarrow \boxed{\dfrac{G_1 G_2 G_3 G_4}{1+G_3 G_4 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 G_4 H_3}} \longrightarrow C$$

3) For the block diagram shown in the figure determine the over all transfer function.

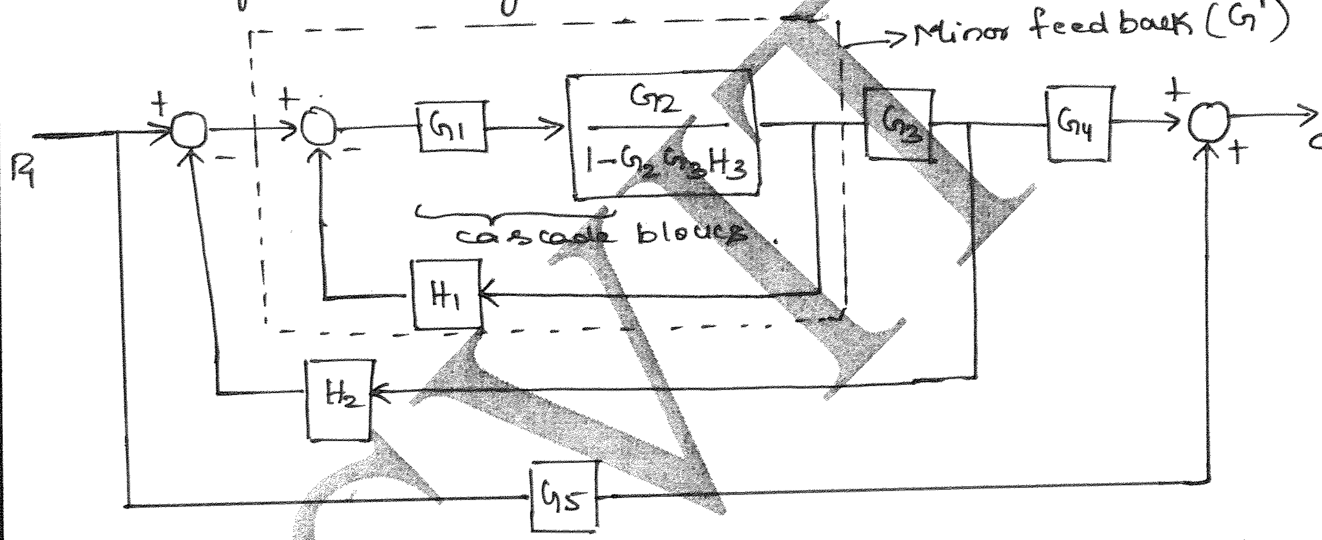


Solution :-

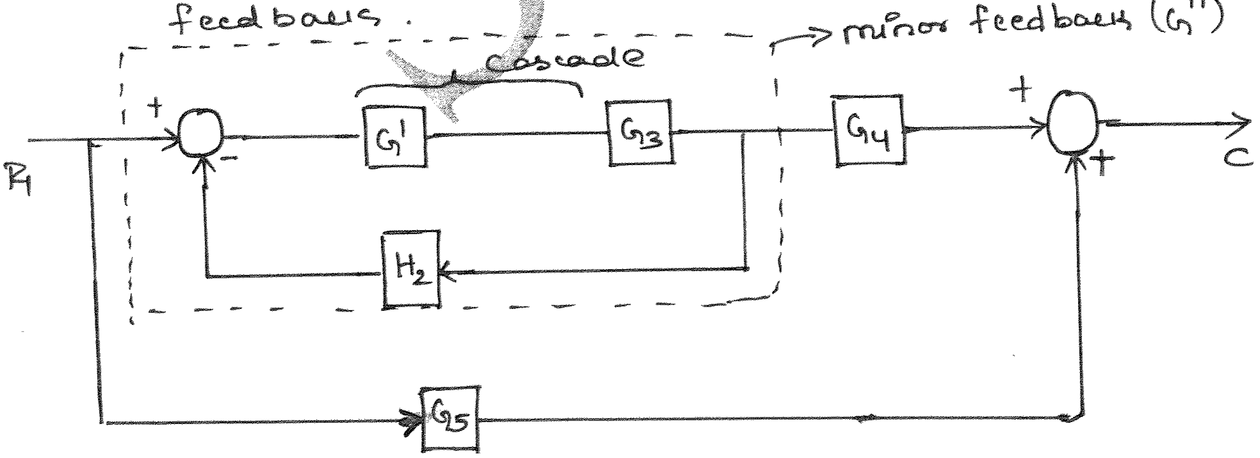
Step 1 :- By shifting takeoff point before G_3



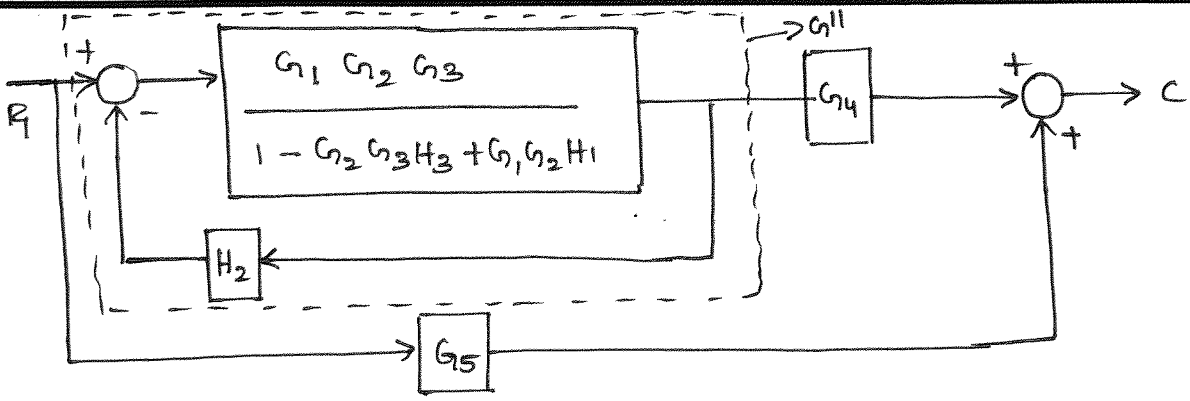
Step 2 :- By Eliminating minor feed back.



Step 3 :- By combining cascaded blocks and Eliminating minor feedbacks.



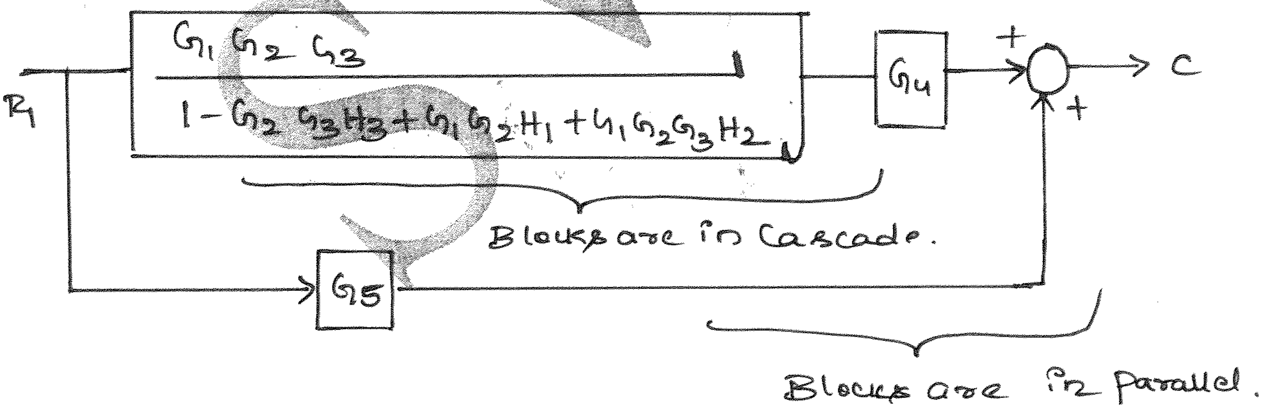
$$G' = \frac{\dfrac{G_1 G_2}{1 - G_2 G_3 H_3}}{1 + \dfrac{G_1 G_2}{1 - G_2 G_3 H_3} \cdot H_1} = \frac{G_1 \; G_2}{1 - G_2 G_3 H_3 + G_1 G_2 H_1}$$



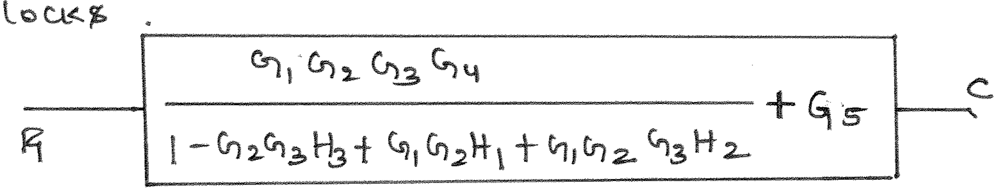
Step 4: By Eliminating minor feedback G''

$$G'' = \frac{\dfrac{G_1 G_2 G_3}{1 - G_2 G_3 H_3 + G_1 G_2 H_1}}{1 - \dfrac{G_1 G_2 G_3}{1 - G_2 G_3 H_3 + G_1 G_2 H_1} \cdot H_2}$$

$$\boxed{G'' = \frac{G_1 G_2 G_3}{1 - G_2 G_3 H_3 + G_1 G_2 H_1 + G_1 G_2 G_3 H_2}}$$

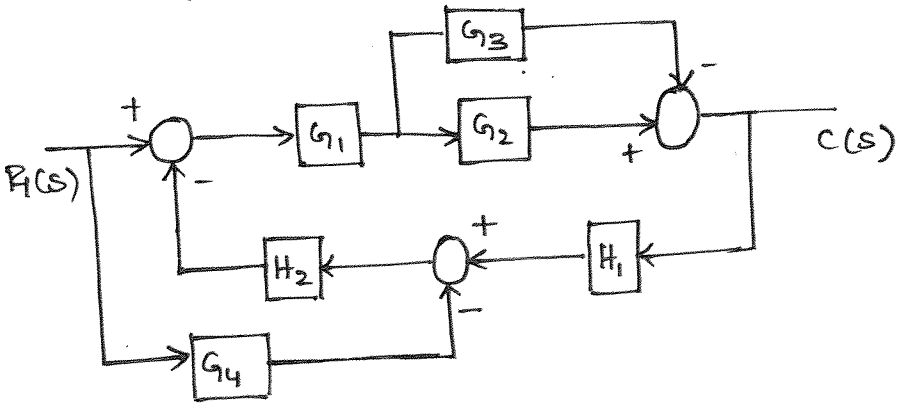


Step 5: By Combining cascade blocks and parallel Blocks.



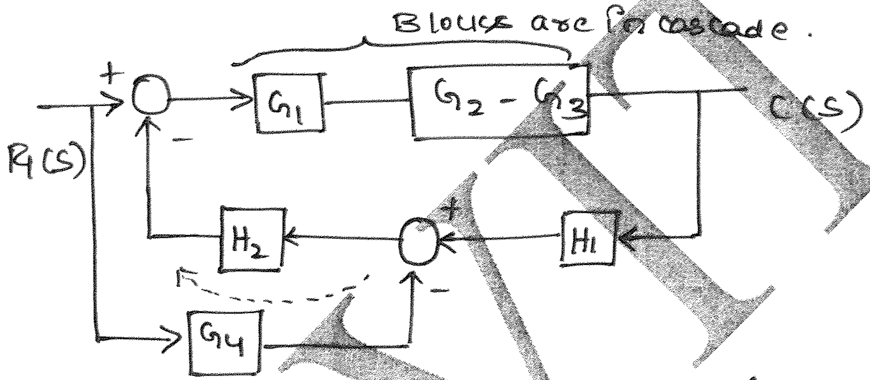
$$\frac{G_1 G_2 G_3 G_4}{1 - G_2 G_3 H_3 + G_1 G_2 H_1 + G_1 G_2 G_3 H_2} + G_5$$

4) Determine the overall transfer function for the block diagram shown in the figure.

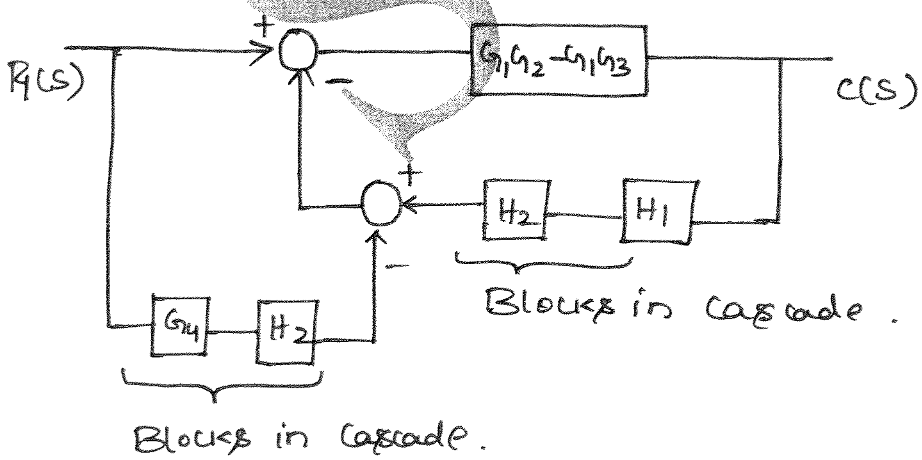


Solution:-

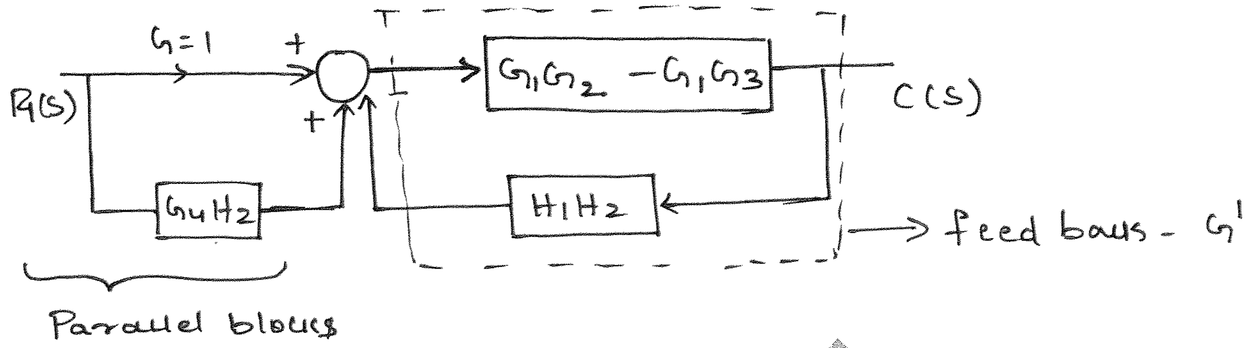
Step1:- By combining parallel blocks $(G_2 + G_3)$



Step2:- By combining cascaded blocks $(G_1, (G_2 - G_3))$ and By shifting summing point after Block (H_2)

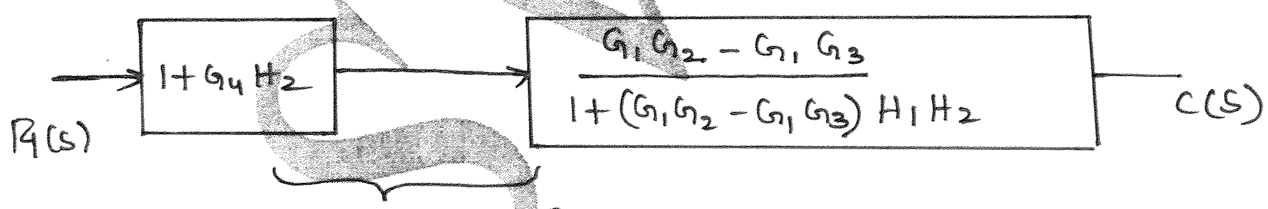


Step 3:- By Combining cascaded blocks and
By combining summing blocks (adding)



Step 4:- By Combining parallel blocks and by Eliminating feed back G'

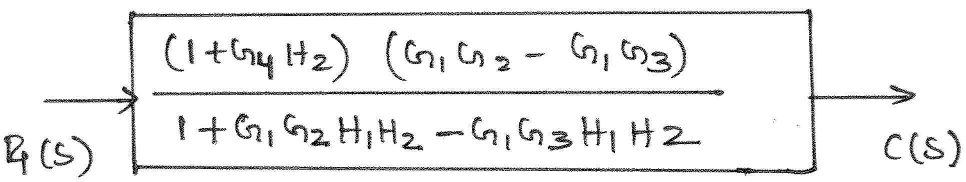
$$G' = \frac{G}{1+GH} = \frac{G_1G_2 - G_1G_3}{1+(G_1G_2 - G_1G_3)H_1H_2}$$



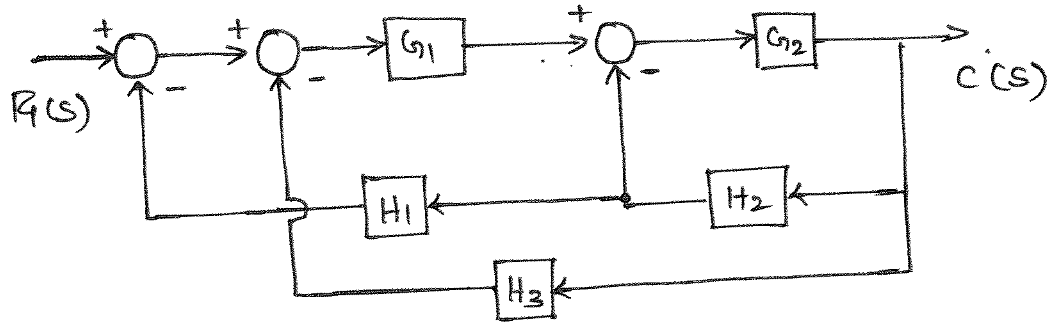
Blocks are in Cascade.

Step 5:- By Combining cascaded blocks.

$$\frac{C(S)}{R(S)} = \frac{(1+G_4H_2)(G_1G_2 - G_1G_3)}{1+(G_1G_2 - G_1G_3)H_1H_2}$$

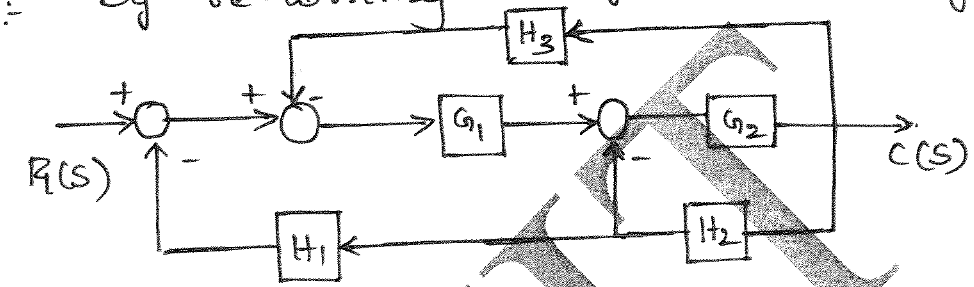


5) Determine the transfer function for the block diagram shown in the figure.

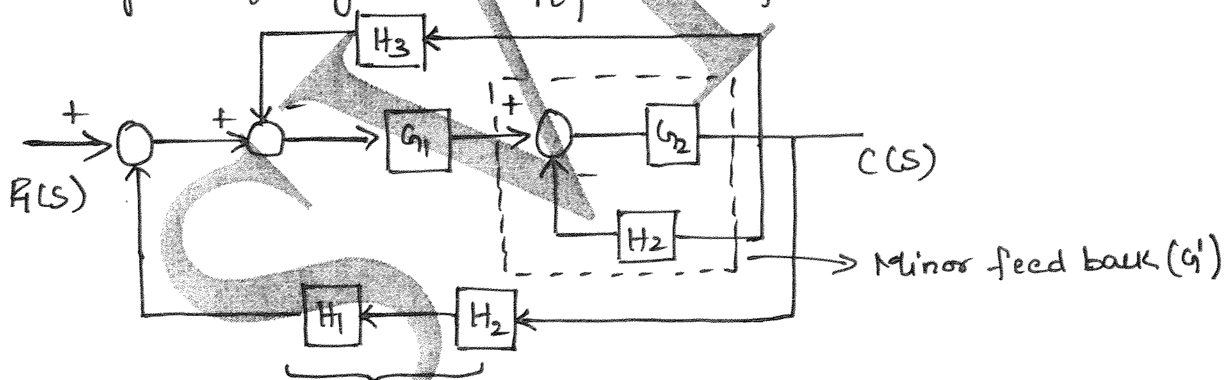


Solution:-

Step1:- By re-writing the given block diagram.

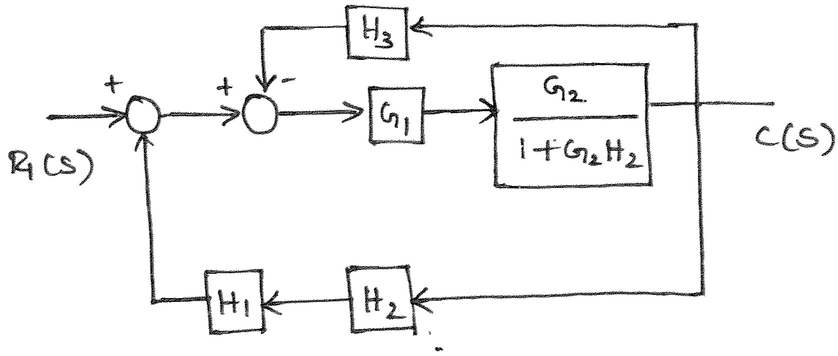


Step2:- By shifting take-off point befor H_2

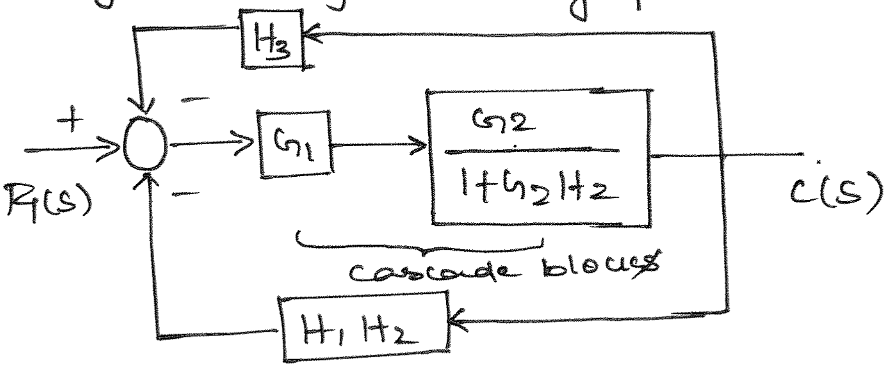


Blocks are in cascade.

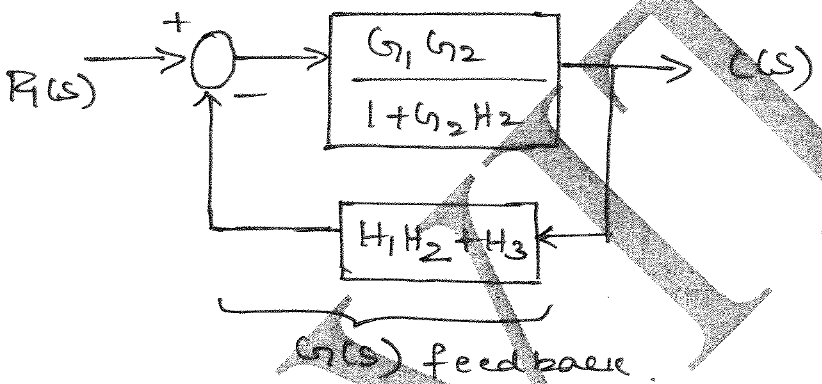
$$G' = \frac{G}{1+GH} = \frac{G_2}{1+G_2H_2}$$



Step 3:- By combining summing point



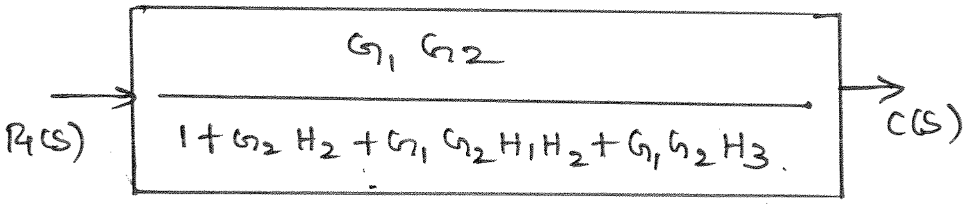
Step 4:- By combining cascaded blocks and parallel blocks (H_3, $H_1 H_2$)



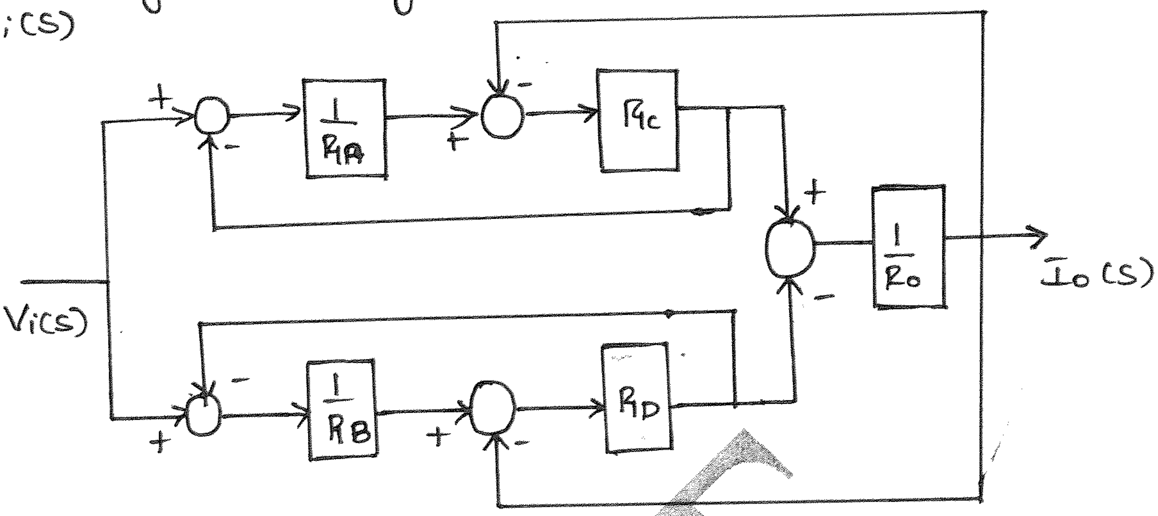
Step 5:- By eliminating the feed back.

$$G(s) = \frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\dfrac{G_1 G_2}{1+G_2 H_2}}{1 + \dfrac{G_1 G_2}{1+G_2 H_2} \cdot (H_1 H_2 + H_3)}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_1 H_2 + G_1 G_2 H_3}$$

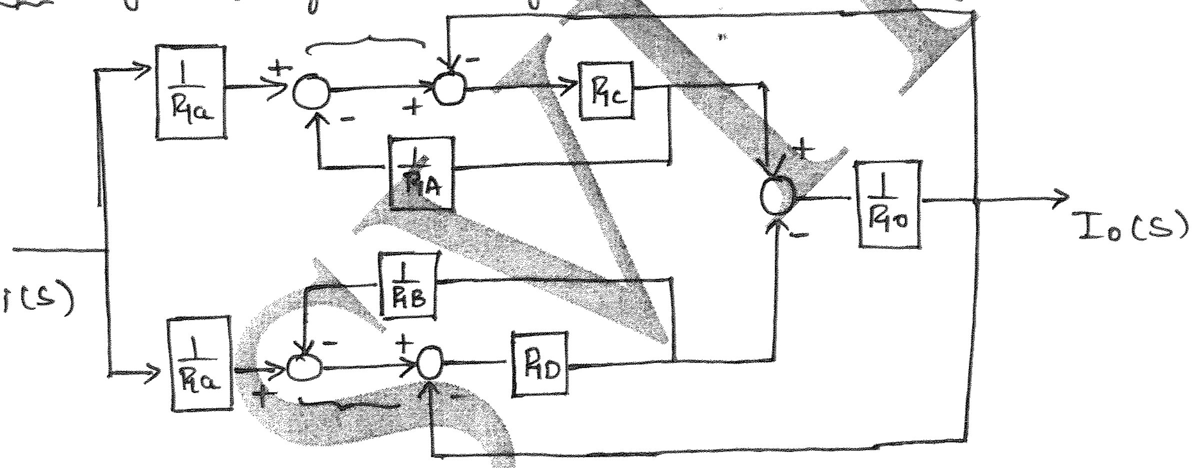


6) For the system shown in the figure determine $\frac{I_o(s)}{V_i(s)}$ by blockdiagram reduction technique.

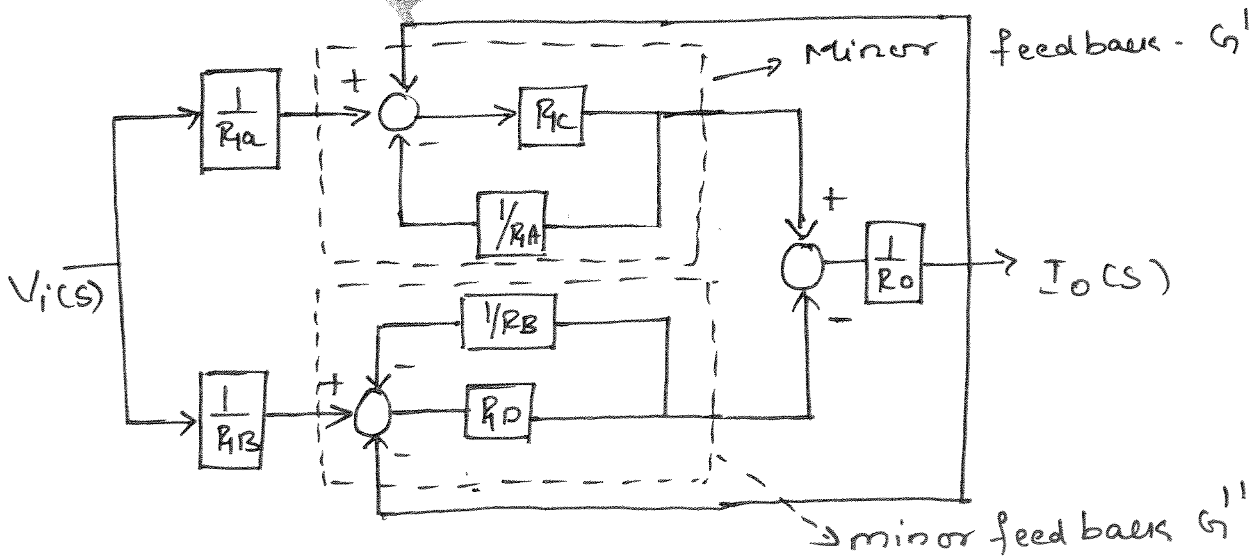


Solution :-

Step 1. By shifting summing points after block $\left(\frac{1}{R_A} + \frac{1}{R_B}\right)$



Step 2: By Combining the summing point



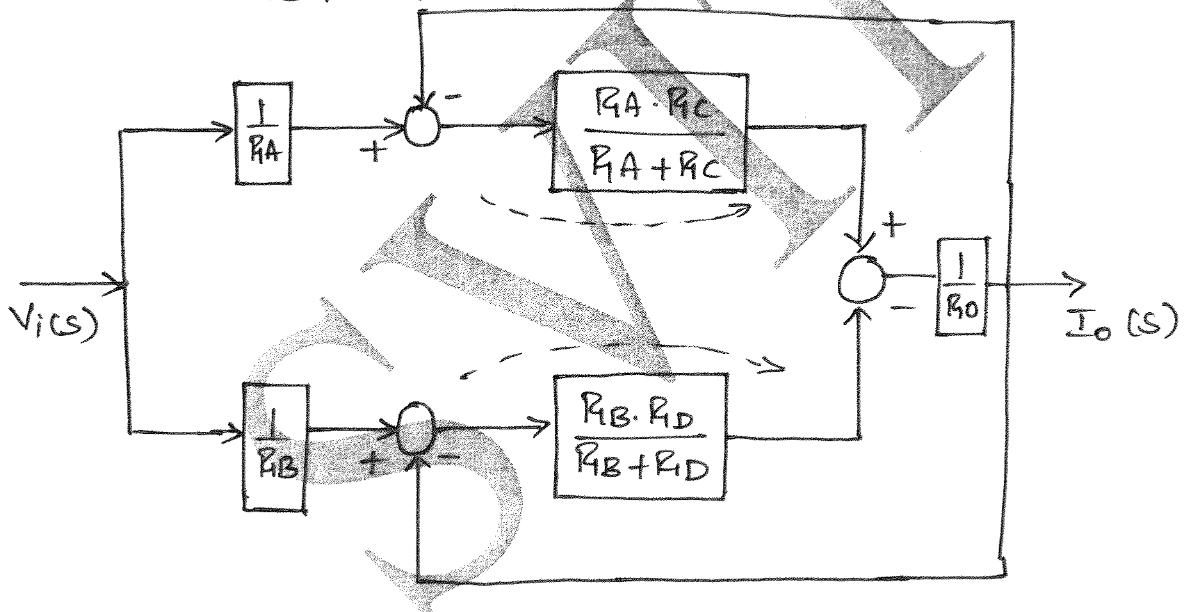
Step 3 :- By Eliminating the feed back G' and G''

$$G' = \frac{G}{1+GH} = \frac{R_C}{1+R_C \cdot \frac{1}{R_A}} = \frac{R_C}{\frac{R_A+R_C}{R_A}} = \frac{R_A R_C}{R_A+R_C}$$

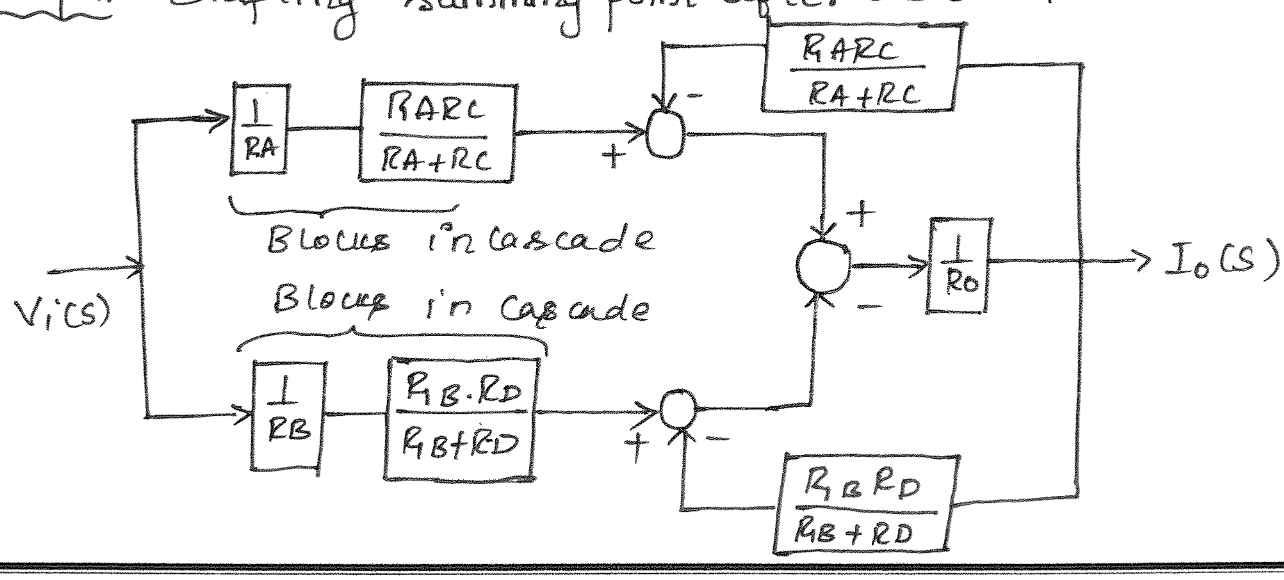
$$G' = \frac{R_A R_C}{R_A+R_C}$$

$$G'' = \frac{G}{1+GH} = \frac{R_D}{1+R_D \cdot \frac{1}{R_B}} = \frac{R_D}{\frac{R_B+R_D}{R_B}} = \frac{R_B \cdot R_D}{R_B+R_D}$$

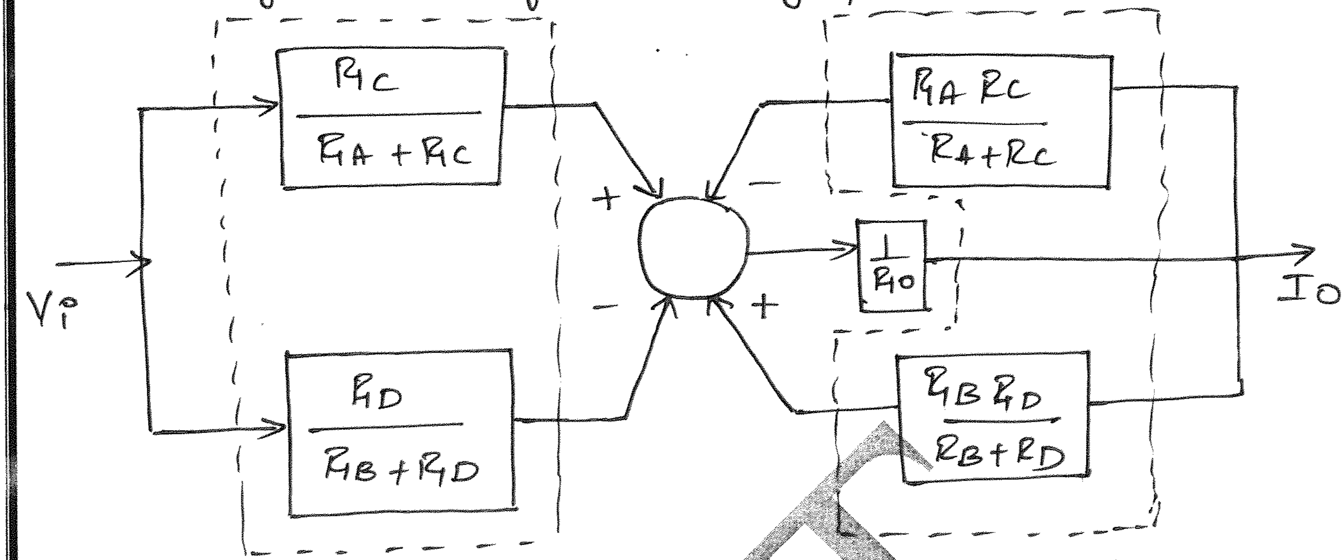
$$G'' = \frac{R_B \cdot R_D}{R_B+R_D}$$



Step 4:- Shifting summing point after the blocks.

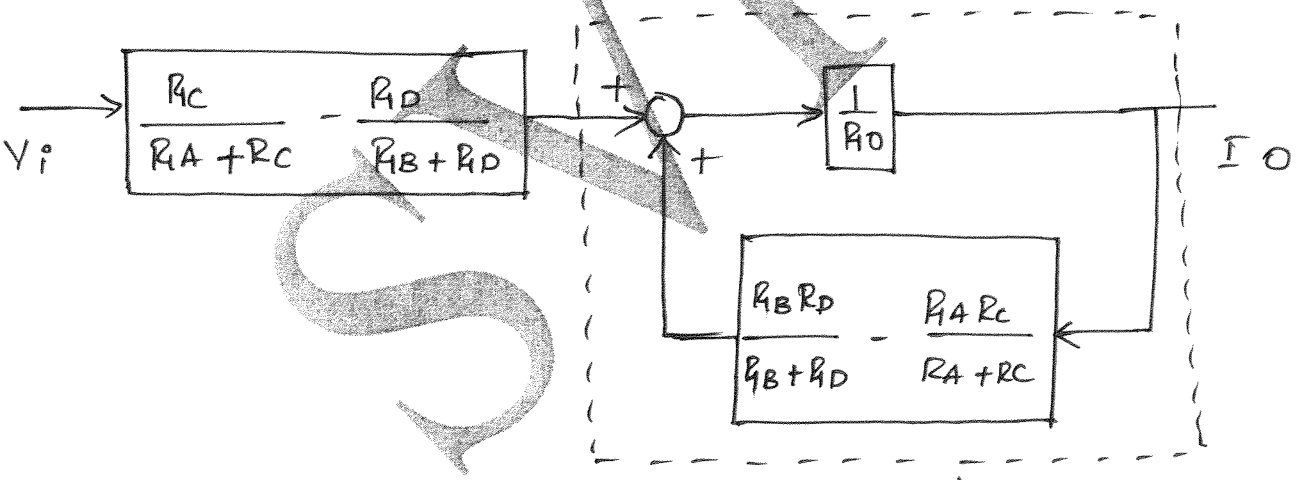


Step 5:- By Combining Cascaded blocks and By Combining Summing points.



Parallel Blocks Parallel Blocks

Step 6:- By Combining Parallel Blocks



feedback - G'

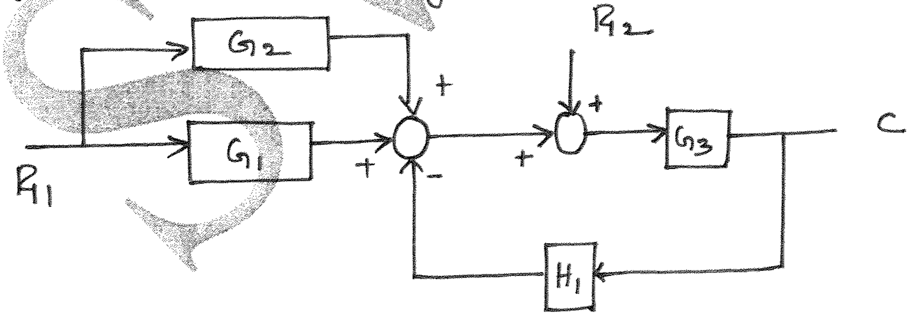
$$G' = \frac{G}{1+GH} = \frac{G}{1-GH} = \frac{1/R_O}{1-\frac{1}{R_O}\left[\frac{R_BR_D}{R_B+R_D}-\frac{R_AR_C}{R_A+R_C}\right]}$$

$$V_i \rightarrow \boxed{\frac{R_C(R_B+R_D)-R_D(R_A+R_C)}{(R_A+R_C)(R_B+R_D)}} \rightarrow \boxed{\frac{1/R_O}{1-\frac{1}{R_O}\left[\frac{R_BR_D(R_A+R_C)-R_AR_C(R_B+R_D)}{(R_A+R_C)(R_B+R_D)}\right]}} \rightarrow I_o$$

$$\frac{I_o(s)}{V_i(s)} = \frac{\left[R_C(R_B+R_D)-R_D(R_A+R_C)\right]}{(R_A+R_C)(R_B+R_D)} * \frac{(R_A+R_C)(R_B+R_D)}{R_O(R_A+R_C)(R_B+R_D)-R_BR_D(R_A+R_C)+R_AR_C(R_B+R_D)}$$

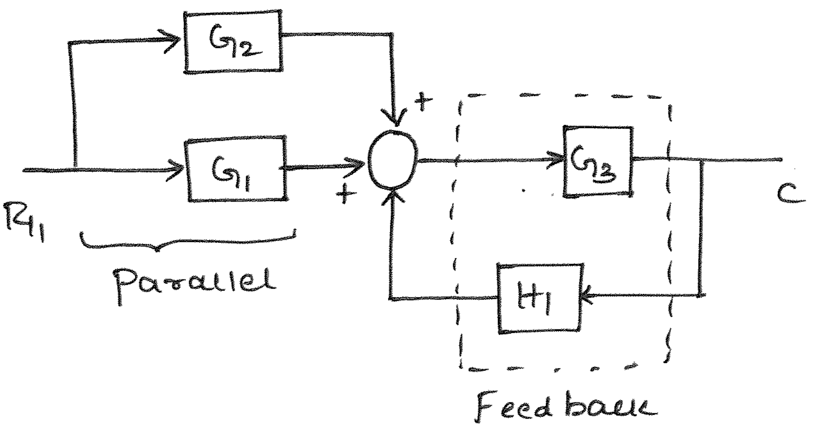
$$\boxed{\frac{I_o(s)}{V_i(s)} = \frac{R_C(R_B+R_D)-R_D(R_A+R_C)}{R_O(R_A+R_C)(R_B+R_D)-R_BR_D(R_A+R_C)+R_AR_C(R_B+R_D)}}$$

7) Determine the transfer function $\frac{C}{R_1}$ and $\frac{C}{R_2}$ for the Block diagram shown in figure.



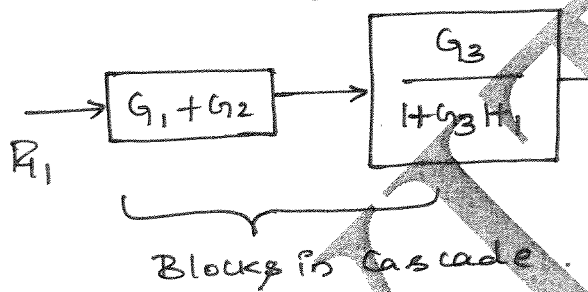
Solution:-

Case 1:- By applying superposition principle i.e considering 1 input at a time, consider R_1 with R_2 to be 0. Block diagram as shown below.



Step1:- By combining parallel blocks and

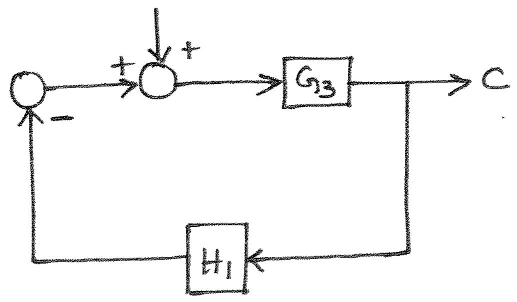
By Eliminating feedback blocks.



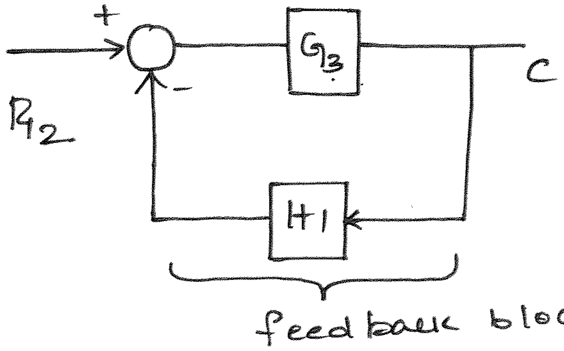
Step2:- By Eliminating cascaded blocks the Transfer function $\frac{C}{R_1}$ is given by

$$\boxed{\frac{C}{R_1} = \frac{(G_1+G_2)\,G_3}{1+G_3H_1}}$$

Case ii: Considering the input R_2 with $R_1=0$ the block diagram is as shown below



Step 1:- By Combining Summing points.

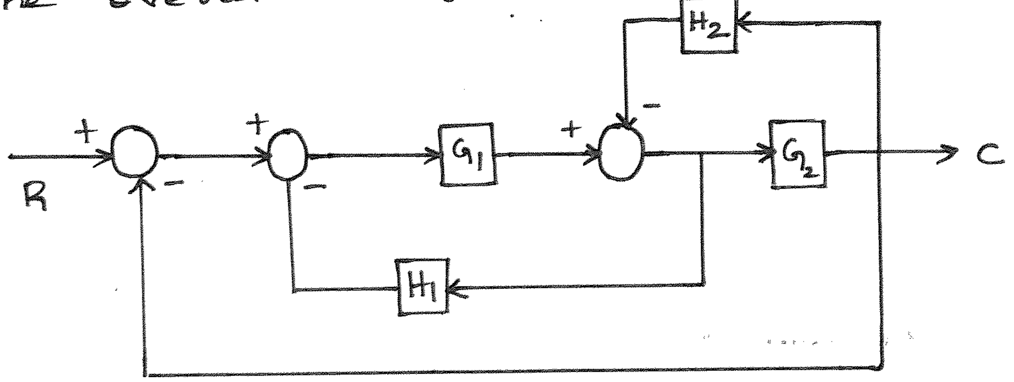


The Overall transfer function $\frac{C}{R_2}$ is given by

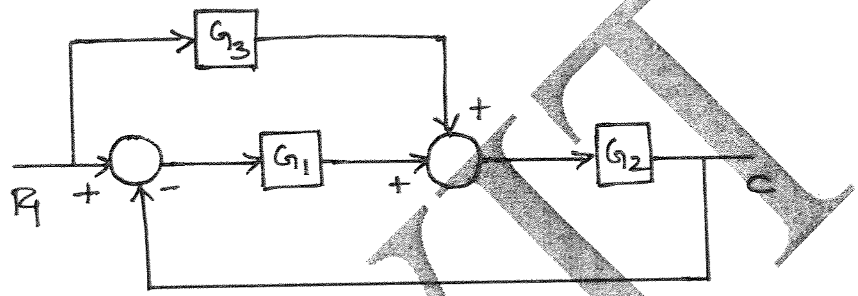
$$\boxed{\frac{C}{R_2} = \frac{G_3}{1+G_3H_1}}$$

Practice Problems:-

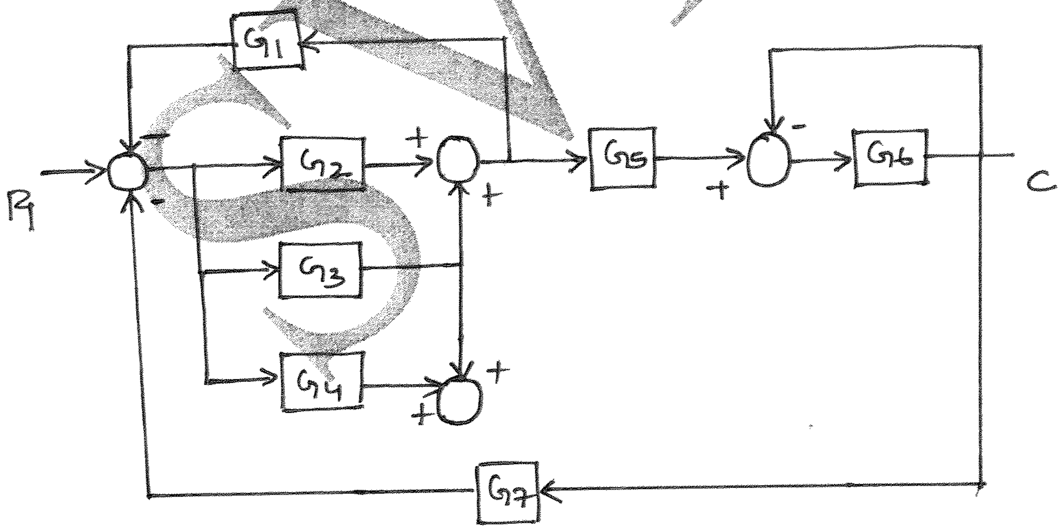
1) using block diagram reduction techniques, Obtain the overall transfer function



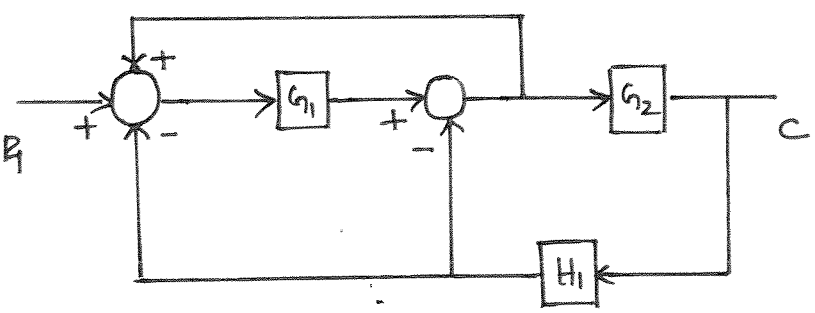
2)



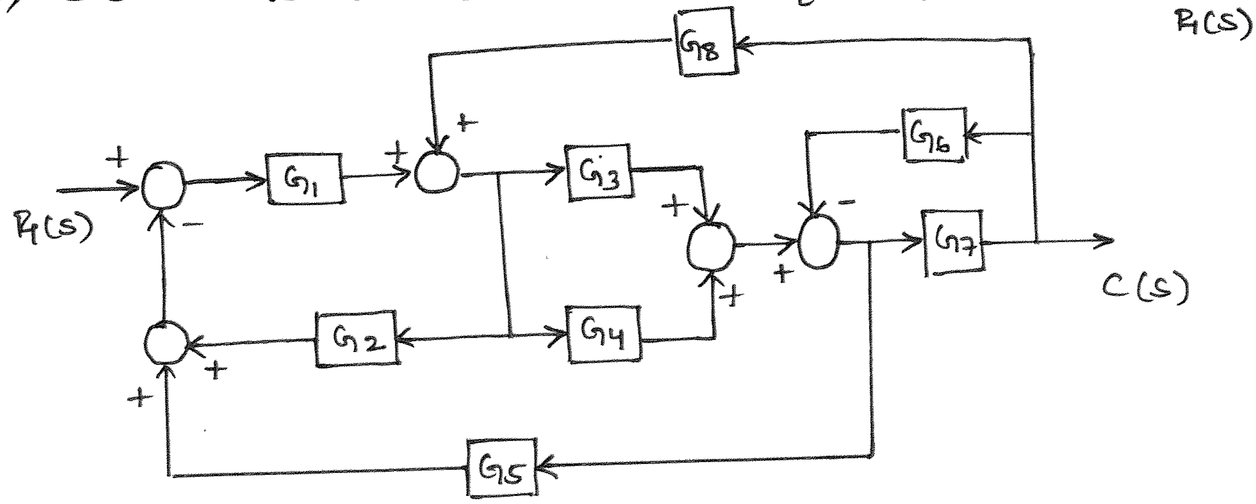
3)



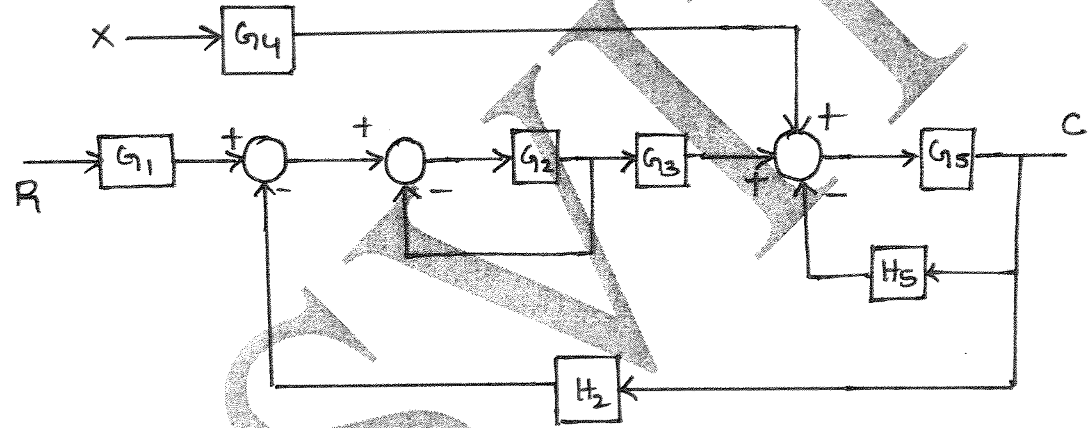
4)



5) Determine the overall transfer function $\frac{C(s)}{R(s)}$

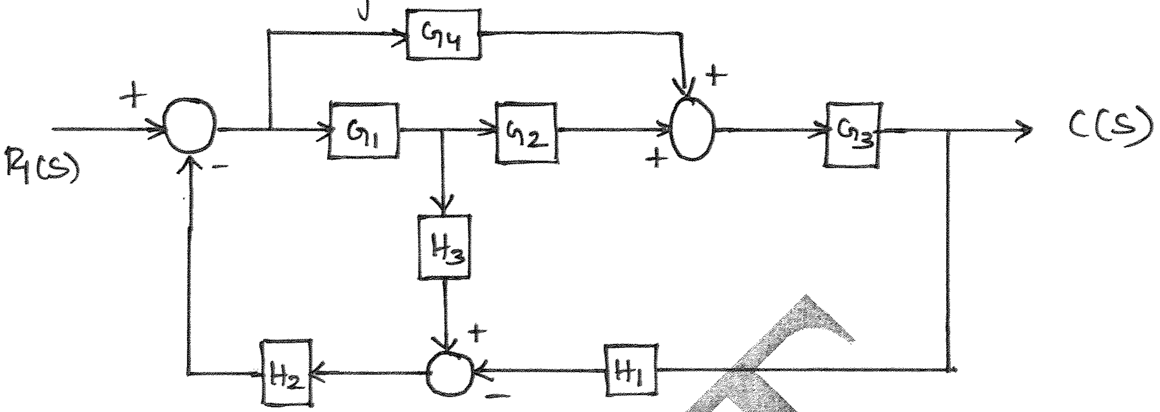


6) using the block diagram reduction technique, find the transfer function for each input

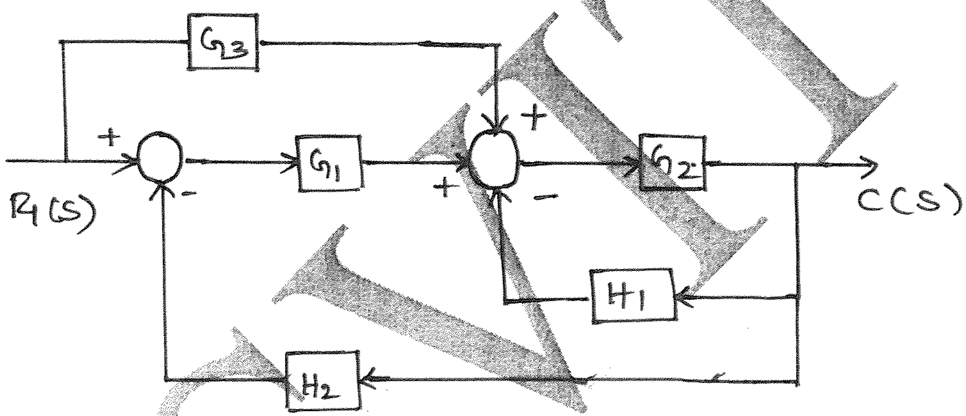


Assignment Problems On Block Diagram Algebra

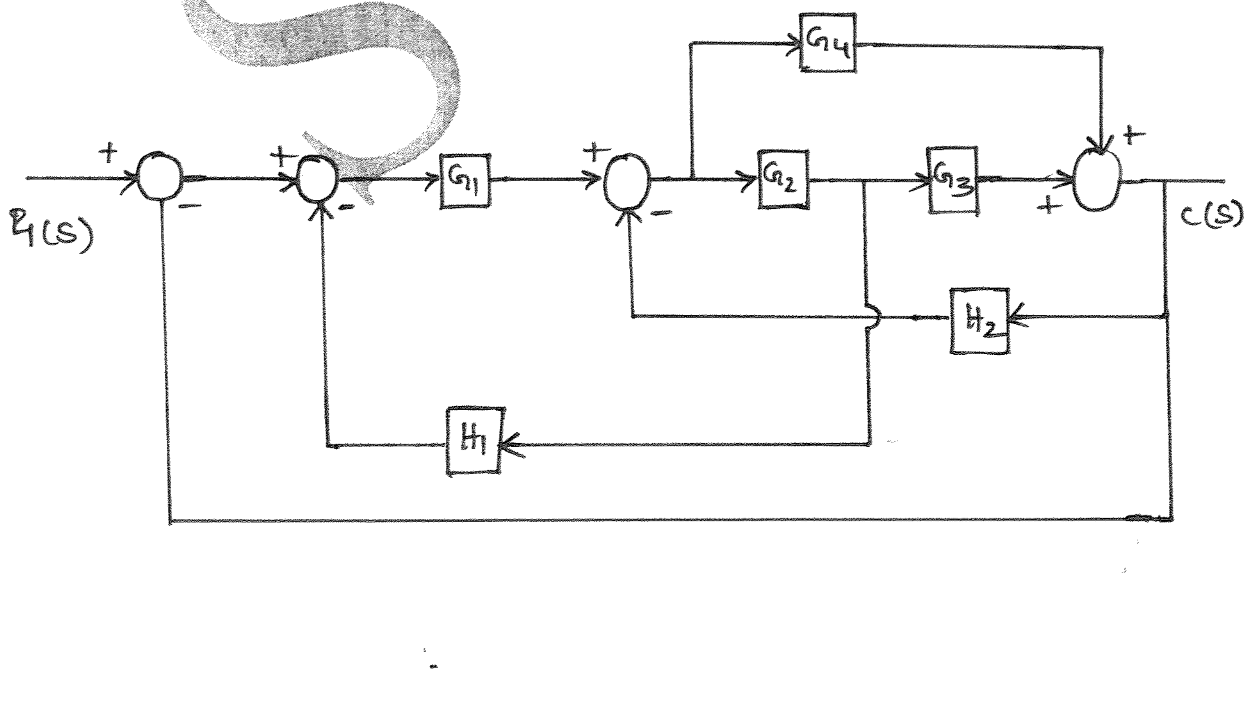
1) Determine the Overall transfer function for the block diagram shown in the figure.



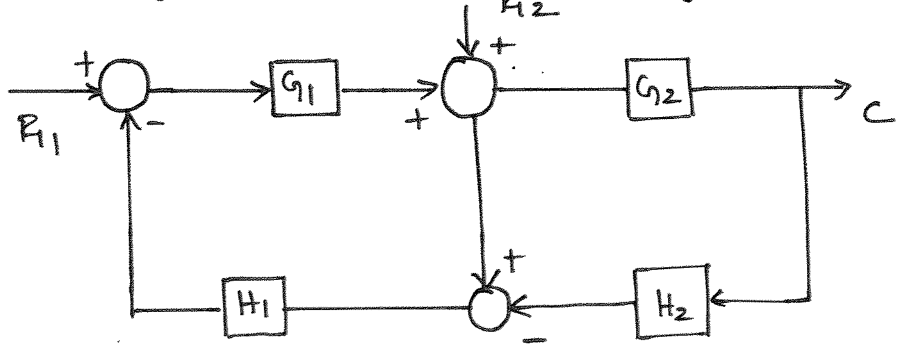
2)



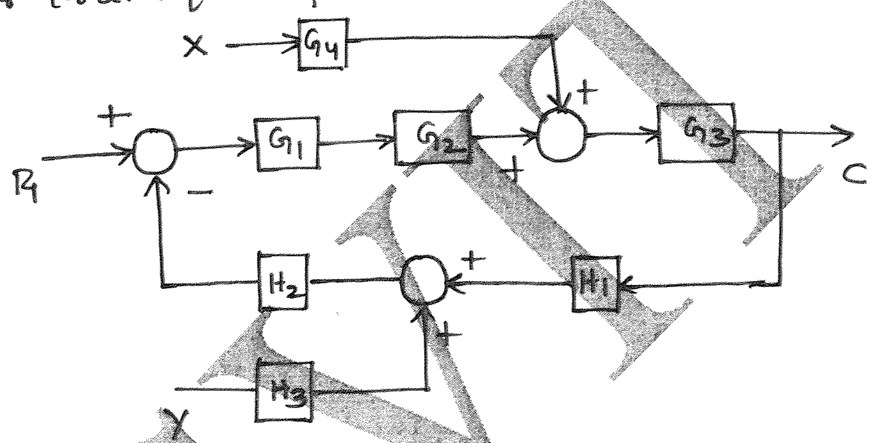
3)



4) Derive an Expression for the total Output for the system represented by the block diagram.



5) Using Block diagram reduction Technique find the over transfer function.



Signal Flow Graph

* It is defined as the graphical representation of a set of simultaneous Equations or transfer function.

* The Overall transfer function for a signal flow graph can be determined by using Mason's gain formula.
  it is given by.

$$\boxed{M(s) = \sum_{K=1}^{N} \frac{P_K \Delta_K}{\Delta}}$$

Where. $\Delta = 1 - \Sigma P_{m1} + \Sigma P_{m2} - \Sigma P_{m3} + \cdots\cdots$

P_K is the forward path gain of the K^{th} forward path.

Δ_K is the Co-factor.

* Forward path:- It is the Journey from input to Output node in the direction of arrows without touching in between more than once.

  N → no of forward path present in signal flow graph.

* Loop:- A path is said to be a loop if we start from a node and come back to the same node in the direction of arrows without touching any nodes in between more than Once.

* Two-non-touching Loops:- Two loops are said to be non-touching, if they don't have a Common node or Variable between them.

Δ is the determinant of the signal flow graph.

ΣP_{m1} is the sum of loop gains of all possible Combinations of single loop.

$\Sigma P m_2$ is sum of product of loop gain of all possible combinations of 2 non-touching loops.

$\Sigma P m_3$ is the sum of product of the loop gain of all combinations of 3 non-touching loops.

Δ_K is the co-factor of the graph. The expression for Δ_K is similar to Δ, but it must be applied to that part of the graph not touching the K^{th} forward path.

If a forward path touches all single loop present in the graph then the corresponding $\Delta K = 1$

⇒ Procedure to Construct Signal flow graph from linear Equations:

* let us Consider a system described by a set of linear Equations.

$$x_2 = a_{12}x_1 + a_{32}x_3 + a_{42}x_4$$

$$x_3 = a_{23}x_2$$

$$x_4 = a_{24}x_2 + a_{34}x_3 + a_{44}x_4$$

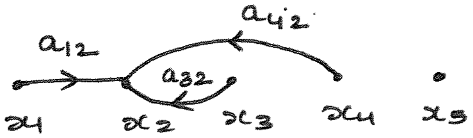
$$x_5 = a_{25}x_2 + a_{45}x_4$$

* Where x_1 is the input Variable and x_5 is the Output Variable.

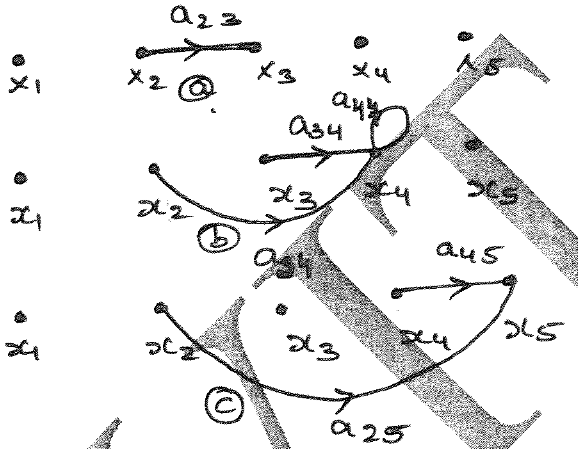
* When Constructing signal flowgraph, the nodes are used to represent Variables.

* Therefore locate the nodes x_1, x_2, x_3, x_4, x_5 as shown in the figure below.

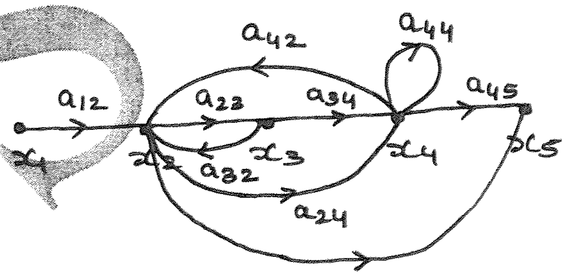
* The first Equation states that x_2 is Equal to the sum of three incoming signals and it's signal flow graph is shown in the figure.



* Similarly the signal flow graph for the remaining three Equations are shown in the figure.



* The Complete signal flow graph is a Combination of all the four parts as shown in the figure.



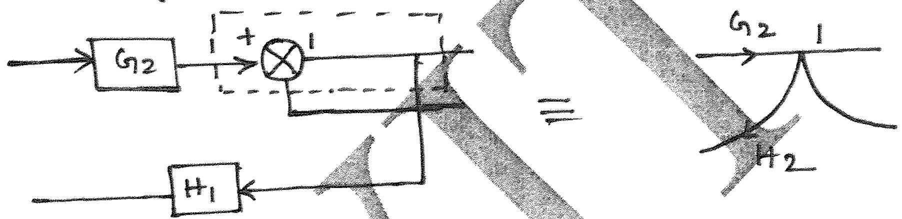
⇒ Procedure to draw signal flow graph from Block diagram.

Step1:- Replace the input signal and Output signal by nodes.

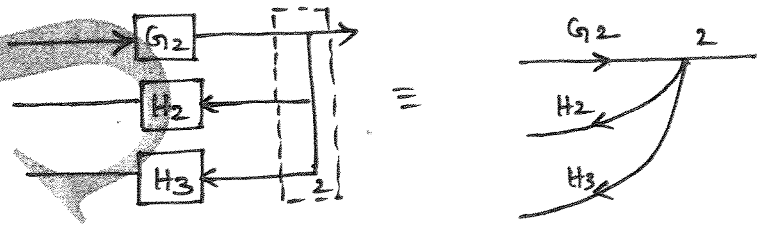
step2:- Replace all the summing points by nodes.

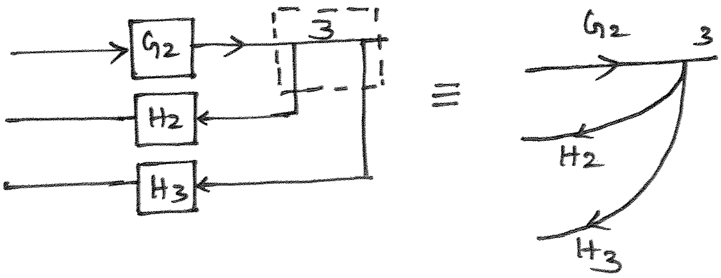
step3:- Replace all the take off points by nodes.

Step4:- If the branch Connecting a summing point and take off point has unity gain, then the summing Point and take off point can be combined and represented by a single node.

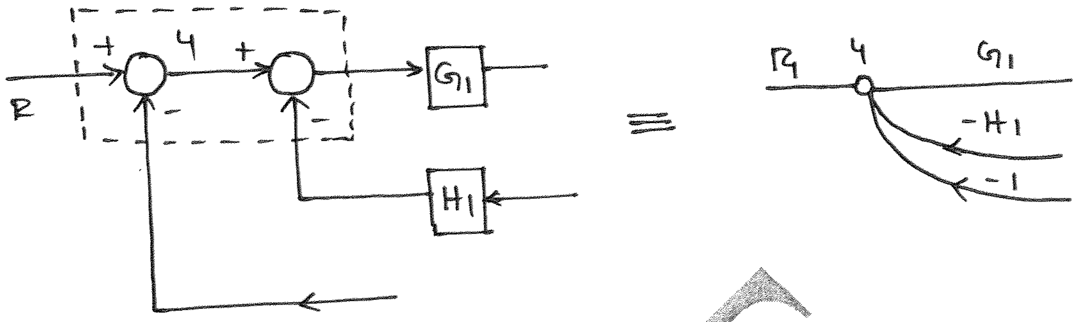


Step5:- If there are more take off points from the same signal then all the take off points can be combined and represented by a single node.





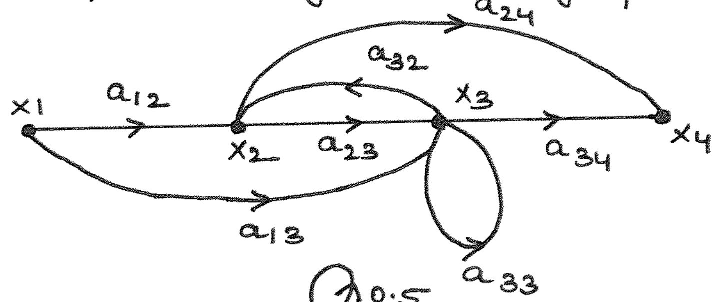
Step 6:- If the gain of the link connecting two summing points is one, then the two summing points can be combined and can be replaced by a single node as shown in the figure.



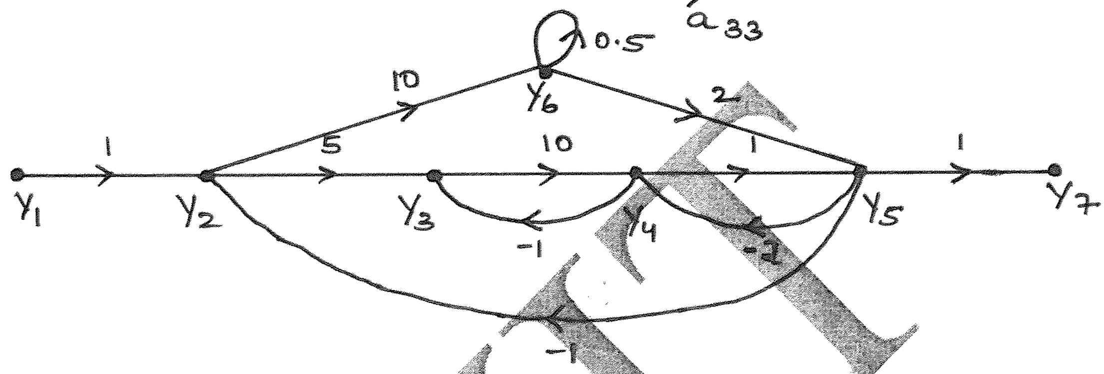
Step 7:- In summing point, subtract a signal instead of adding and then multiply the transmittance by -1 while representing in signal flow graph as shown in the figure above.

Problems to be solved in the class.

1) using mason's gain formula find the Overall transfer function. $\frac{x_4}{x_1}$ for the Signal flow graph shown below.

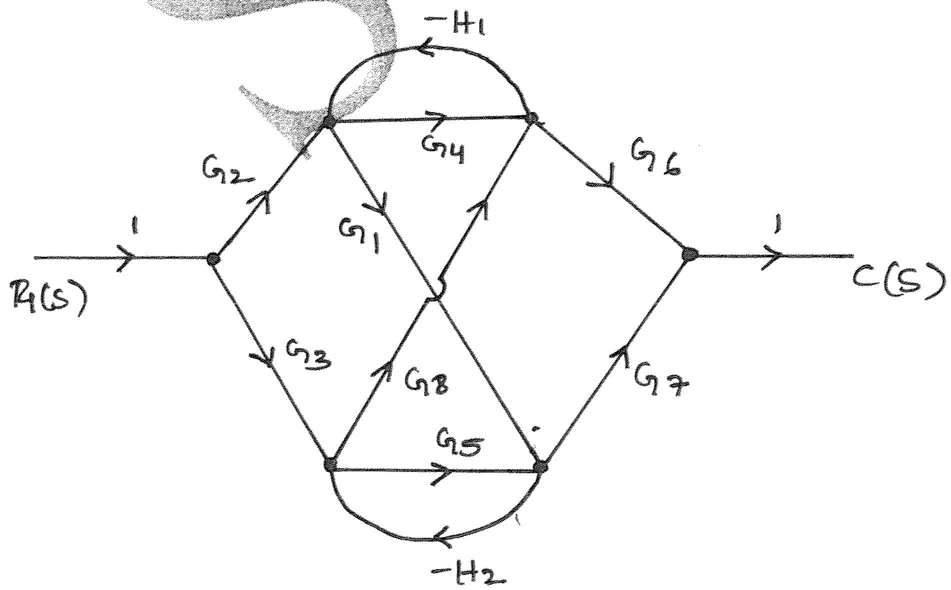


2)



using mason's gain formula determine $\frac{Y_7}{Y_1}$ and $\frac{Y_3}{Y_1}$ for the signal flow graph shown in the figure.

3) Determine the Overall transfer function of system whose signal flow graph as shown below.



4) Draw the signal flow graph for the system of equations given below and obtain the transfer function using mason's gains formula.

$$X_2 = G_1 X_1 - H_1 X_2 - H_2 X_3 - H_6 X_6$$

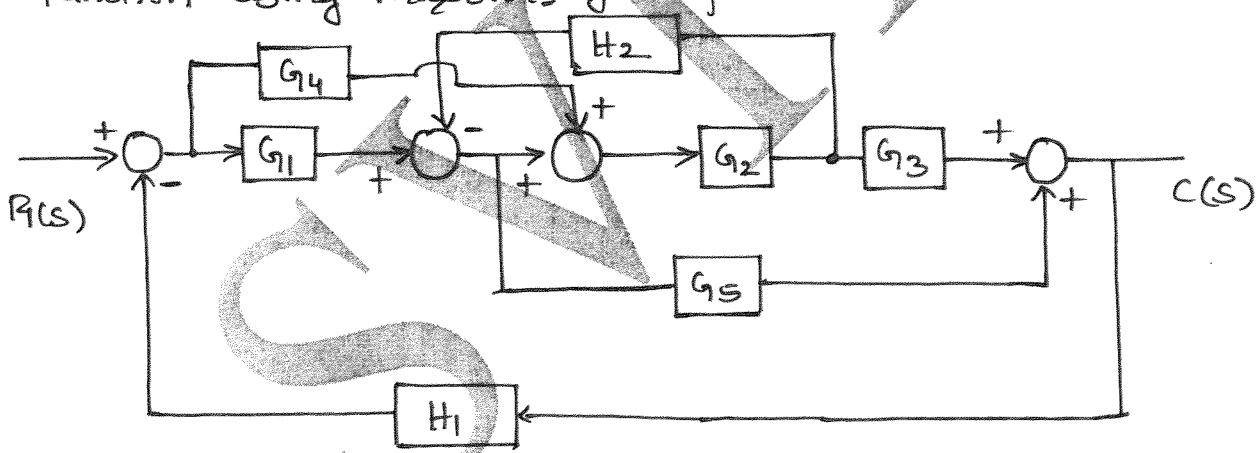
$$X_3 = G_1 X_1 + G_2 X_2 - H_3 X_3$$

$$X_4 = G_2 X_2 + G_3 X_3 - H_4 X_5$$

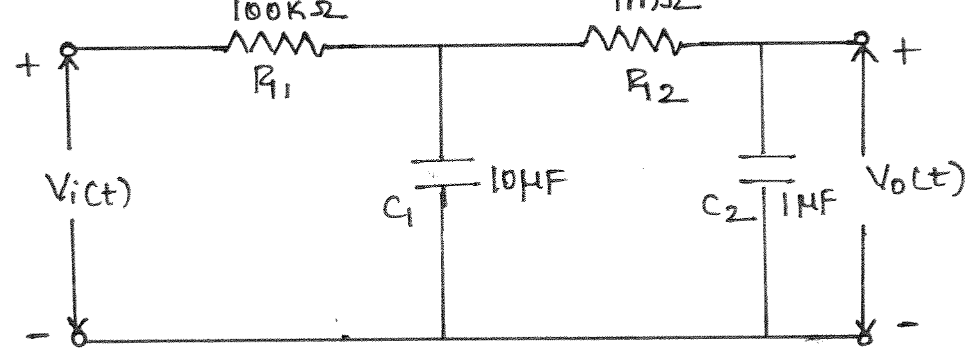
$$X_5 = G_5 X_4 - H_5 X_6$$

$$X_6 = G_5 X_5$$

5) Draw the Signal flow graph for the block diagram shown in the figure. Determine the overall transfer function using mason's gain formula.

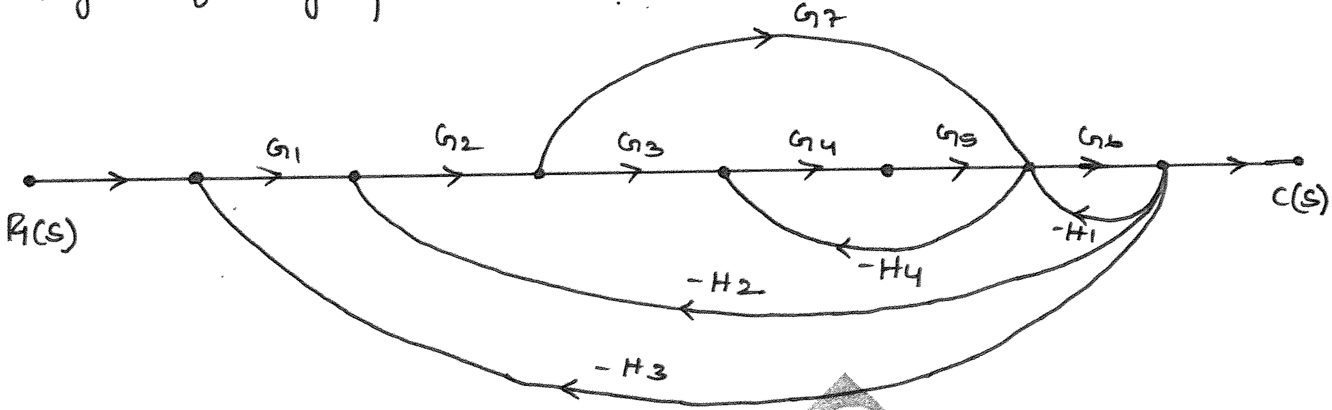


6) For the Electrical Circuit shown in the figure, find the overall transfer function using Mason's gain formula.



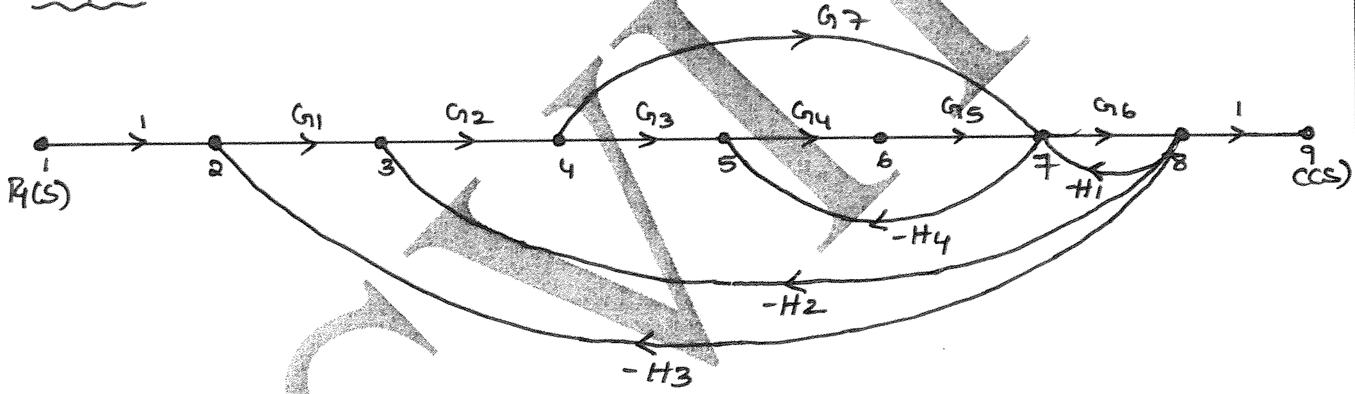
Problems on Signal flow graph

1) Obtain the transfer function of the system for the Signal flow graph shown below



Solution:-

Step 1:- Name the nodes by using numbers.



Step 2:- Forward path gains.

$P_1 = G_1 G_2 G_3 G_4 G_5 G_6 \quad (1, 2, 3, 4, 5, 6, 7, 8, 9)$

$P_2 = G_1 G_2 G_7 G_6 \quad (1, 2, 3, 4, 7, 8, 9)$

Number of forward path $n = 2$,

Step 3:- Single loop gains

$P_{11} = -G_4 G_5 H_4 \quad (5, 6, 7)$

$P_{21} = -G_6 H_1 \quad (7, 8)$

$P_{31} = -G_1 G_2 G_3 G_4 G_5 G_6 H_3 \quad (2, 3, 4, 5, 6, 7, 8)$

$P_{41} = -G_2 G_3 G_4 G_5 G_6 H_2 \quad (3, 4, 5, 6, 7, 8)$

$P_{51} = -G_2 G_7 G_6 H_2 \quad (3,4,7,8)$

$P_{61} = -G_1 G_2 G_7 G_6 H_3 \quad (2,3,4,7,8)$

Step 4:- Two non-touching loop gains.

* Two non touching loops and higher order is absent.

$\therefore \sum P_{m_2}$ & Onwards is zero.

Step 5:- To find ΔK.

Note:- * Number of forward path is equal to number of Co-factor (ΔK)

* By comparing forward path gains with single loop gains.

for Ex:- $\Delta_1 = 1 - \underbrace{P_{11} + P_{31}}_{\text{Sequence which is not Common}} - P_{51}$

$\Delta_1 = 1 - (0) = 1$

$\Delta_2 = 1 - (0) = 1$

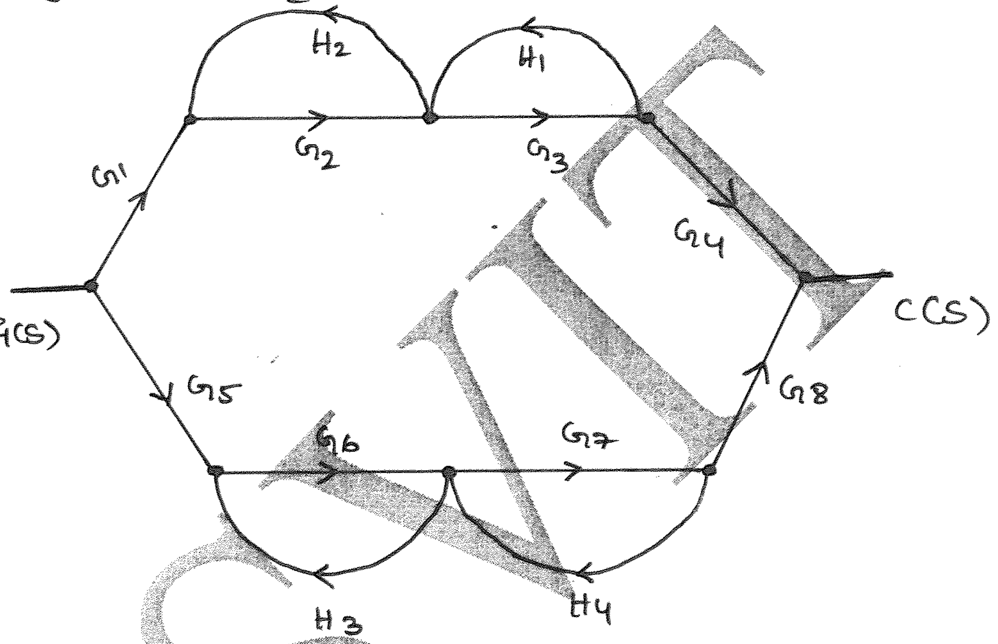
Step 6:- Find the Overall transfer function using mason's gain formula.

$$\frac{C(s)}{R(s)} = \frac{\sum_{K=1}^{n=2} P_K \Delta_K}{1 - \sum_{m=1}^{6} P_{m_1} + 0}$$

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51} + P_{61})}$$

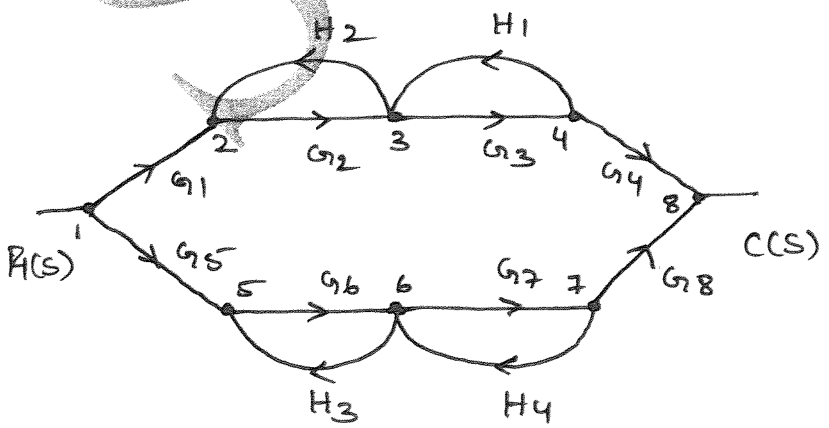
$$\frac{C(s)}{R(s)} = \frac{(G_1G_2G_3G_4G_5G_6)(1) + (G_1G_2G_7G_6)(1)}{1-\left((-G_4G_5H_4)+(-G_6H_1)+(-G_1G_2G_3G_4G_5G_6H_3) + (-G_2G_3G_4G_5G_6H_2)+(-G_2G_7G_6H_2) + (-G_1G_2G_7G_6H_3)\right)}$$

2) Obtain the overall transfer for the system shown in figure using mason's rule.



Solution:

⇒



⇒ Forward path gains.

$P_1 = G_1\,G_2\,G_3\,G_4 \quad (1,2,3,4,8)$

$P_2 = G_5\,G_6\,G_7\,G_8 \quad (1,5,6,7,8)$

Number of forward path, $n = 2$.

⇒ Single loop gains:-

$P_{11} = G_2 H_2 \; (2,3)$

$P_{21} = G_3 \; H_1 \; (3,4)$

$P_{31} = G_6 \; H_3 \; (5,6)$

$P_{41} = G_7 \; H_4 \; (6,7)$

⇒ Two non-touching loop gains.

$P_{12} = P_{11} \; P_{31} = G_2 H_2 G_6 H_3 \; (2,3,5,6)$

$P_{22} = P_{11} \; P_{41} = G_2 H_2 G_7 H_4 \; (2,3,6,7)$

$P_{32} = P_{21} \; P_{31} = G_3 H_1 G_6 H_3 \; (3,4,5,6)$

$P_{42} = P_{21} \; P_{41} = G_3 H_1 G_7 H_4 \; (3,4,6,7)$

ΣP_{m3} + onwards is zero.

⇒ Co-factor of graph.

$\Delta_1 = 1 - (P_{31} + P_{41}) + 0 = 1 - (G_6 H_3 + G_7 H_4)$

$\Delta_2 = 1 - (P_{11} + P_{21}) + 0 = 1 - (G_2 H_2 + G_3 H_1)$

⇒ Overall transfer function

$$M(S) = \sum_{K=1}^{N} \frac{P_K \Delta_K}{\Delta}$$

where Δ is $1 - \Sigma P_{m1} + \Sigma P_{m2} - \Sigma P_{m3} + - + \cdots$

$$M(S) = \frac{\sum_{K=1}^{2} P_K \Delta_K}{1 - \sum_{m=1}^{4} P_{m1} + \sum_{m=1}^{4} P_{m2} - 0} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{1 - (P_{11} + P_{21} + P_{31} + P_{41}) + (P_{12} + P_{22} + P_{32} + P_{42})}$$

$$\frac{C(S)}{R(S)} = \frac{(G_1G_2G_3G_4)(1-G_6H_3-G_7H_4) + (G_5G_6G_7G_8)(1-G_2H_2+G_3H_1)}{1-[G_2H_2+G_3H_1+G_6H_3+G_7H_4] + [G_2H_2G_6H_3 + G_2H_2G_7H_4 + G_3H_1G_6H_3 + G_3H_1G_7H_4]}$$

34) Draw the signal flow graph for the system of equations given below and obtain the overall transfer function using mason's rule.

$$x_2 = x_1 - H_3x_8$$

$$x_3 = G_1x_2 - H_2x_8$$

$$x_4 = G_2x_3$$

$$x_5 = G_3x_4 - H_4x_6$$

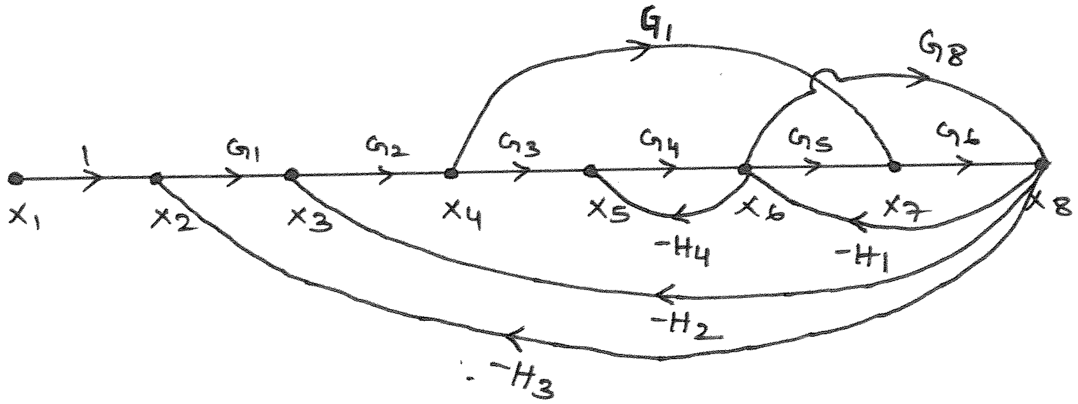
$$x_6 = G_4x_5 - H_1x_8$$

$$x_7 = G_7x_4 + G_5x_6$$

$$x_8 = G_8x_6 + G_6x_7$$

Solution:-

* The signal flow graph satisfying the above equation is as shown below.



⇒ Forward path gains.

$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$ (1, 2, 3, 4, 5, 6, 7, 8)

$P_2 = G_1 G_2 G_7 G_6$ (1, 2, 3, 4, 7, 8)

$P_3 = G_1 G_2 G_3 G_4 G_8$ (1, 2, 3, 4, 5, 6, 7, 8)

Number of forward path $n = 3$.

⇒ Single loop gains.

$P_{11} = -G_1 G_2 G_3 G_4 G_5 G_6 H_3$ (2, 3, 4, 5, 6, 7, 8)

$P_{21} = -G_1 G_2 G_7 G_6 H_3$ (2, 3, 4, 7, 8)

$P_{31} = -G_1 G_2 G_3 G_4 G_8 H_3$ (2, 3, 4, 5, 6, 8)

$P_{41} = -G_2 G_3 G_4 G_5 G_6 H_2$ (3, 4, 5, 6, 7, 8)

$P_{51} = -G_2 G_7 G_6 H_2$ (3, 4, 7, 8)

$P_{61} = -G_2 G_3 G_4 G_8 H_2$ (3, 4, 5, 6, 8)

$P_{71} = -G_4 H_4$ (5, 6)

$P_{81} = -G_5 G_6 H_1$ (6, 7, 8)

$P_{91} = -G_8 H_1$ (6, 8)

⇒ Two-non touching loop gains

$P_{12} = P_{21} P_{71} = G_1 G_2 G_7 G_6 H_3 G_4 H_4$ (2, 3, 4, 5, 6, 7, 8)

$P_{22} = P_{51} P_{71} = G_2 H_7 G_6 H_2 G_4 H_4$ (3, 4, 5, 6, 7, 8)

ΣP_{m3} and Onwards is zero.

⇒ Co-factor of Graph.

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - P_{71} + 0 = 1 + G_4 H_4$

$\Delta_3 = 1$

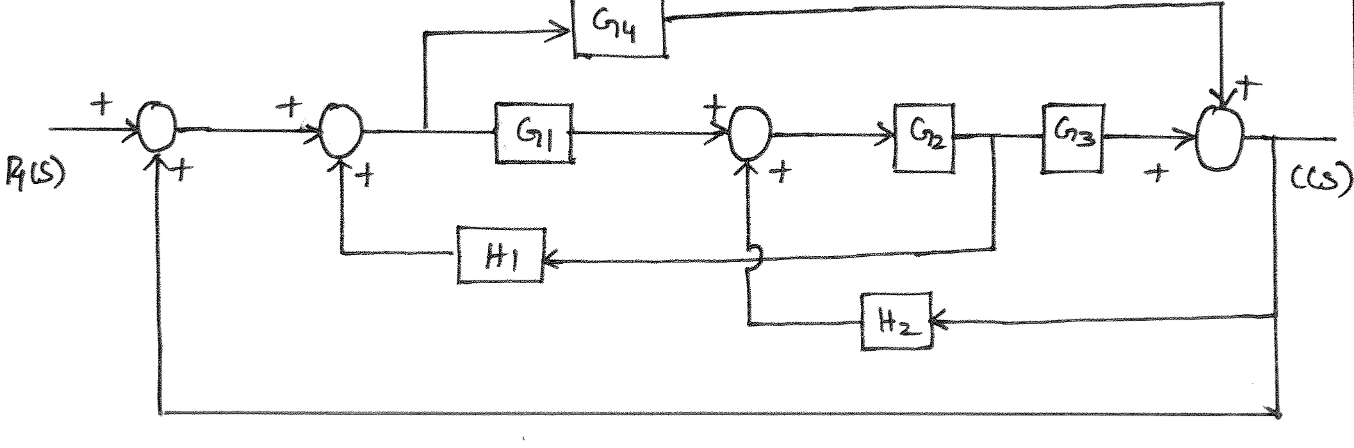
$\Rightarrow$ Overall transfer function.

$$M(s) = \frac{\sum_{K=1}^{N} P_K \Delta_K}{\Delta}$$

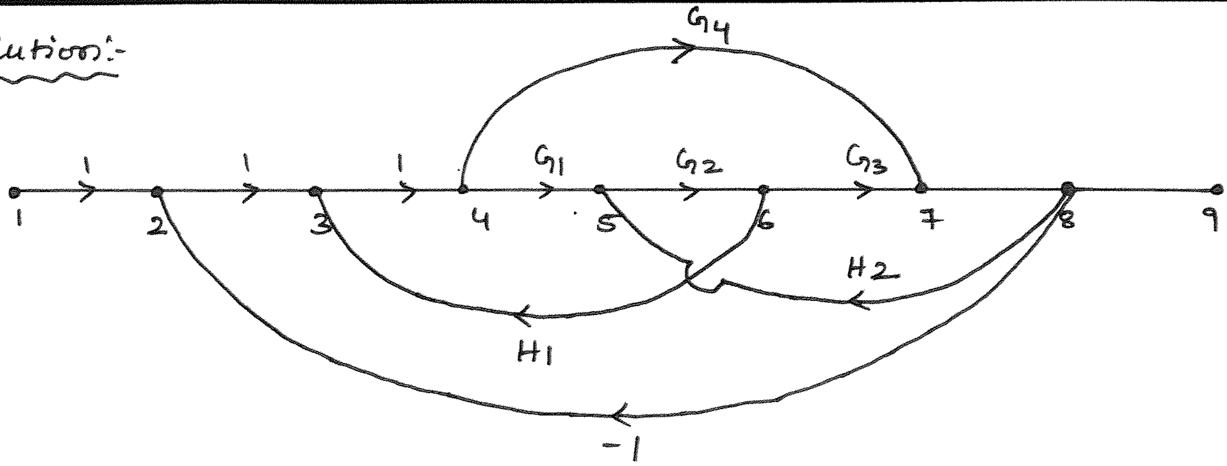
$$\frac{X_8}{X_1} = \frac{\sum_{K=1}^{3} P_K \Delta_K}{1 - \sum_{m=1}^{9} P_{m1} + \sum_{m=1}^{2} P_{m2} - 0} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{1 - [P_{11}+P_{21}+P_{31}+P_{41}+P_{51}+P_{61}+P_{71}+P_{81}+P_{91}] + [P_{12}+P_{22}]}$$

$$\frac{X_8}{X_1} = \frac{G_1G_2G_3G_4G_8 + (G_1G_2G_7G_6)(1+G_4H_4) + G_1G_3G_3G_4G_5G_6}{1 - [-G_1G_2G_3G_4G_5G_6H_3 - G_1G_2G_7G_6H_3 - G_1G_2G_3G_4G_8H_3 - G_2G_3G_4G_5G_6H_2 - G_2G_7G_6H_2 - G_2G_3G_4G_8H_2 - G_4H_4 - G_8H_1 + [G_1G_2G_7G_6H_3G_4H_4 + G_2G_7G_6H_2G_4H_4]]}$$

4) Draw the Signal Flow graph and determine the overall trasfer function for the block diagram shown in the figure.



Solution:-



The Signal flow graph for the given block diagram is as shown above.

⇒ Forward path gains.

$P_1 = G_1 G_2 G_3$ (1, 2, 3, 4, 5, 6, 7, 8, 9)

$P_2 = G_4$ (1, 2, 3, 4, 7, 8, 9)

⇒ Single loop gains

$P_{11} = G_1 G_2 G_3$ (2, 3, 4, 5, 6, 7, 8)

$P_{21} = G_1 G_2 H_1$ (3, 4, 5, 6)

$P_{31} = G_2 G_3 H_2$ (5, 6, 7, 8)

$P_{41} = G_4$ (2, 3, 4, 7, 8)

$P_{51} = G_4 H_2 G_2 H_1$ (3, 4, 7, 8, 5, 6)

ΣP_{m2} 4 onwards is zero.

⇒ To find Δ_K

$\Delta_1 = 1 - (0) = 1$

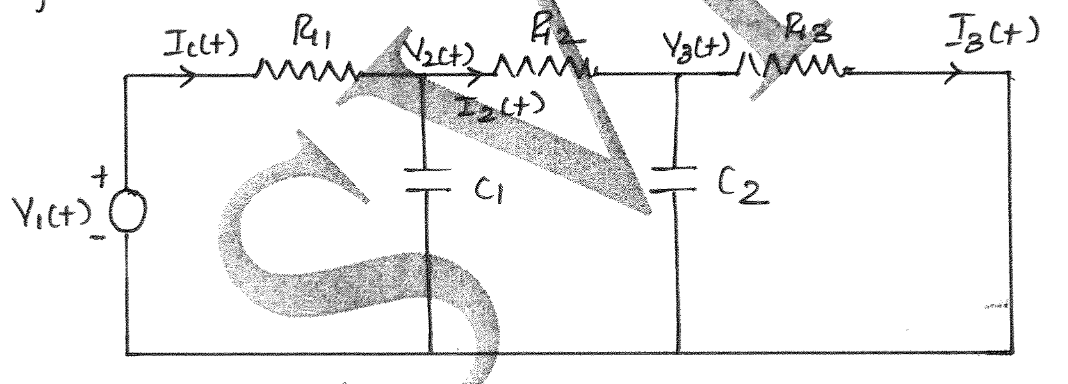
$\Delta_2 = 1 - (0) = 1$

Overall transfer Function

$$\frac{C(s)}{R(s)} = \frac{\sum_{K=1}^{n=2} P_K \Delta_K}{1-\sum_{m=1}^{5} P_{m1} + 0} = \frac{P_1\Delta_1 + P_2\Delta_2}{1-\left[P_{11}+P_{21}+P_{31}+P_{41}+P_{51}\right]+0}$$

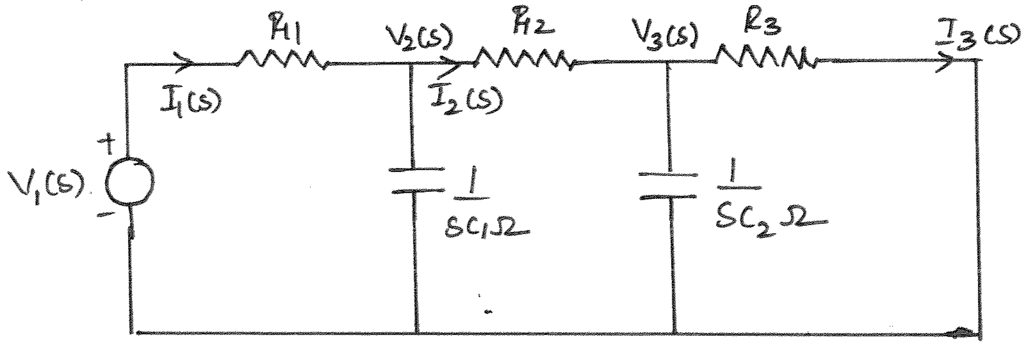
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_4}{1-\left[G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_4 + G_4H_2G_2H_1\right]}$$

54 For the Circuit shown in the Figure write the Performance Equation Considering the Voltage & Current Variable as indicated. draw the Corresponding signal flow graph and determine $\frac{I_3(s)}{V_1(s)}$ using mason's gain formula.



Solution:-

By taking laplace transform, the network is as shown below.



$$I_1(s) = \frac{V_1(s) - V_2(s)}{R_1}$$

$$I_1(s) = \frac{1}{R_1} V_1(s) - \frac{1}{R_1} V_2(s) \longrightarrow (1)$$

$$I_2(s) = \frac{V_2(s) - V_3(s)}{R_2}$$

$$= \frac{1}{R_2} V_2(s) - \frac{1}{R_2} V_3(s) \longrightarrow (2)$$

$$I_3(s) = \frac{V_3(s)}{R_3} \longrightarrow (3)$$

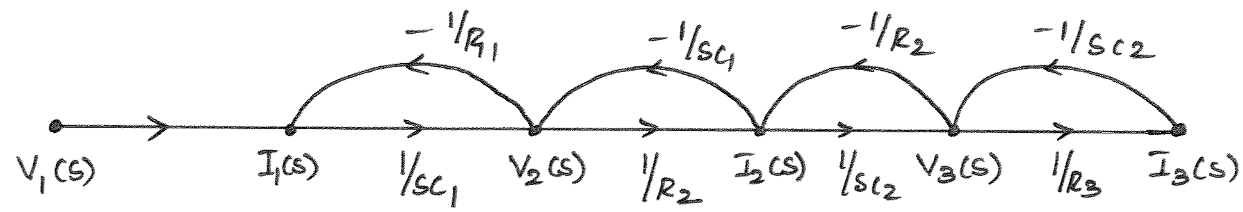
$$V_2(s) = \frac{1}{sC_1} \left(I_1(s) - I_2(s) \right)$$

$$V_2(s) = \frac{1}{sC_1} I_1(s) - \frac{1}{sC_1} I_2(s) \longrightarrow (4)$$

$$V_3(s) = \frac{1}{sC_2} \left(I_2(s) - I_3(s) \right)$$

$$V_3(s) = \frac{1}{sC_2} I_2(s) - \frac{1}{sC_2} I_3(s) \text{ --- } (5)$$

=> Signal flow graph is drawn is as shown below.



⇒ Forward path gains

$$P_1 = \frac{1}{R_1} \cdot \frac{1}{sC_1} \cdot \frac{1}{R_2} \cdot \frac{1}{sC_2} \cdot \frac{1}{R_3} = \frac{1}{s^2 C_1 C_2 R_1 R_2 R_3}$$

Number of forward path gain, $n = 1$

⇒ Single Loop gains

$$P_{11} = -\frac{1}{sC_1 R_1}$$

$$P_{21} = -\frac{1}{sC_1 R_2}$$

$$P_{31} = -\frac{1}{sC_2 R_2}$$

$$P_{41} = -\frac{1}{sC_2 R_3}$$

⇒ Two -non-touching Loop gains

$$P_{12} = P_{11} P_{31} = \frac{1}{s^2 C_1 C_2 R_1 R_2}$$

$$P_{22} = P_{11} P_{41} = \frac{1}{s^2 C_1 C_2 R_1 R_3}$$

$$P_{32} = P_{21} P_{41} = \frac{1}{s^2 C_1 C_2 R_2 R_3}$$

ΣP_{m3} and Onwards is zero.

⇒ To find Δ_K

$\Delta_1 = 1 - 0 = 1$

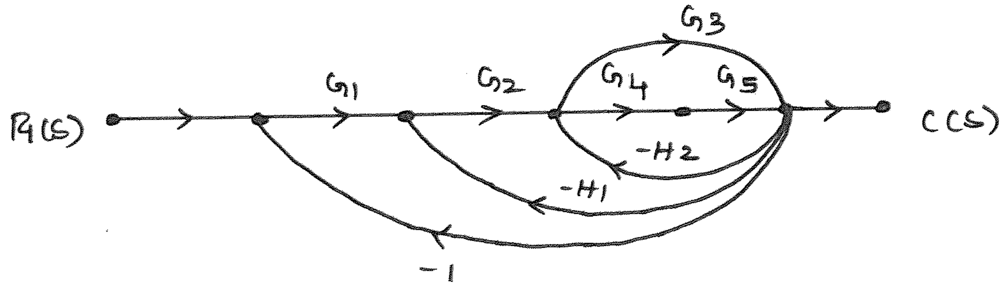
⇒ The Overall transfer function is given by

$$T.F = \frac{I_3(s)}{V_1(s)} = \frac{\sum_{K=1}^{n=1} P_K \Delta_K}{1 - \sum_{m=1}^{4} P_{m1} + \sum_{m=1}^{3} P_{m2} - 0.}$$

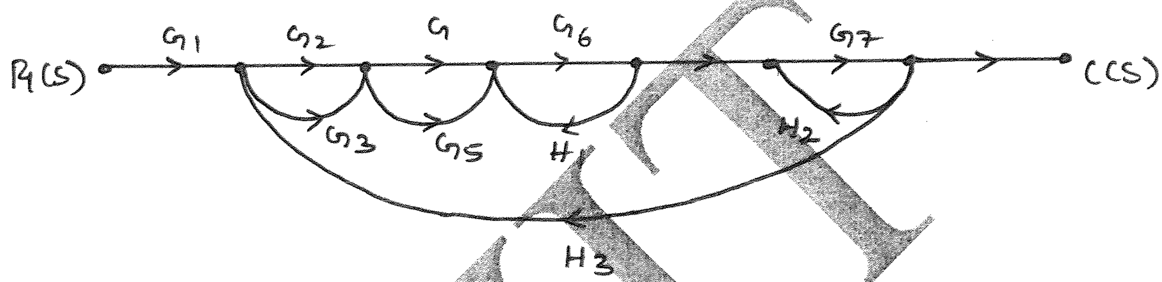
$$\frac{I_3(s)}{V_1(s)} = \frac{P_1 \Delta_1}{1 - (P_{11} + P_{21} + P_{31} + P_{41}) + (P_{12} + P_{22} + P_{32})}$$

Practice Problems on Signal flow graph

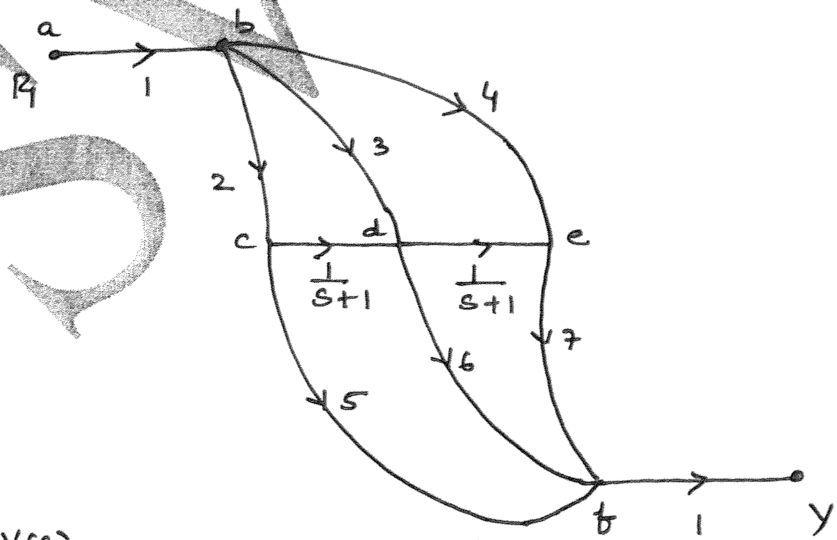
1) Determine the Overall transfer function for the system shown in the figure.



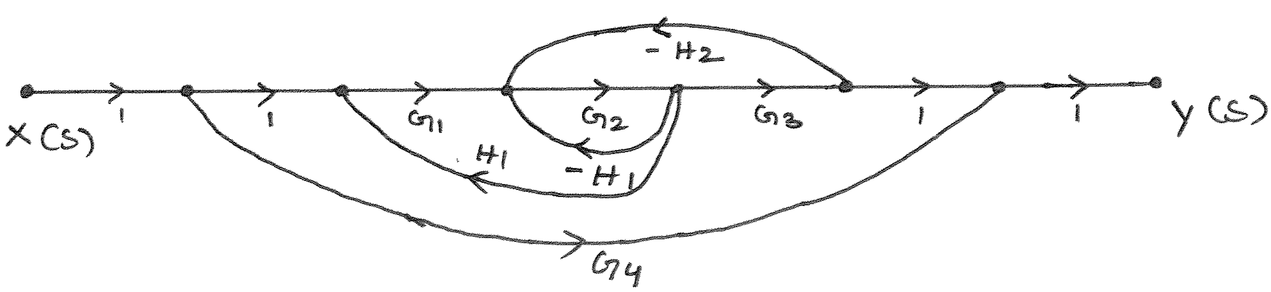
2)



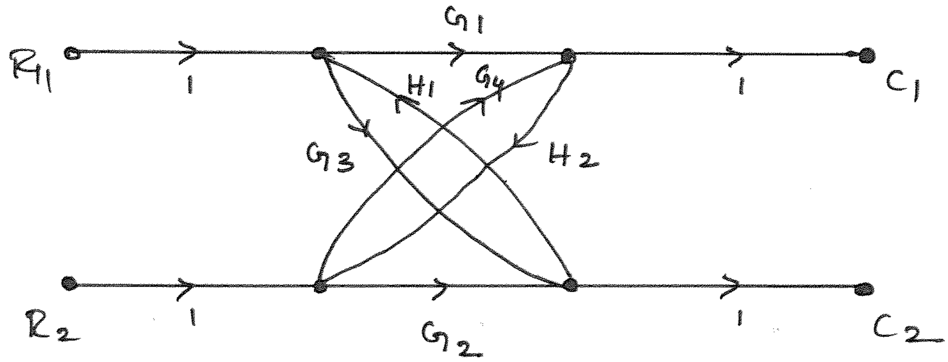
3) Determine the transmittance of the signal flow graph.



4) Determine $\frac{Y(S)}{X(S)}$ using mason's rule.

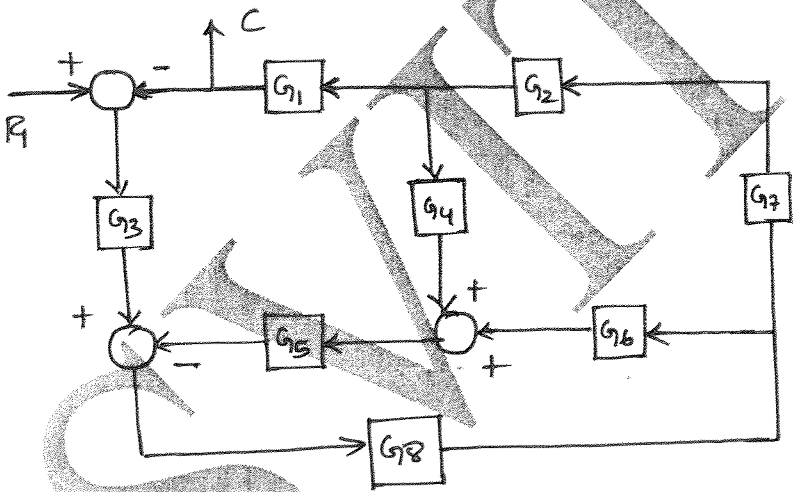


5)



Determine the overall transfer function using mason's rule.

6) Draw a signal flow graph for the block diagram shown in the figure.



Assignment Problems On Signal flow graph

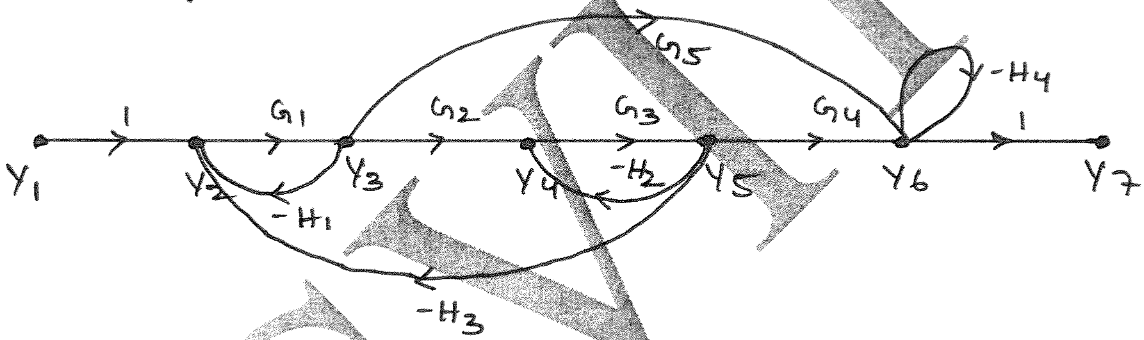
1) Draw Signal flow graph from the following Equations and Obtain the Overall transfer function.

$$x_2 = a_{21} x_1 + a_{23} x_3$$

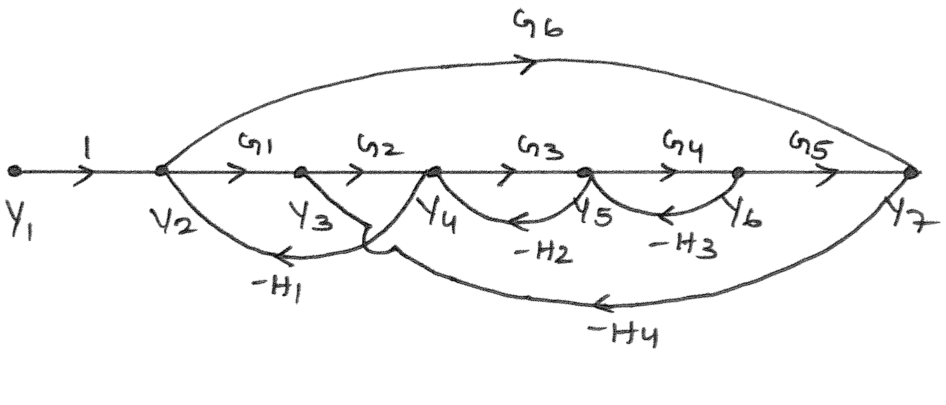
$$x_3 = a_{31} x_1 + a_{32} x_2 + a_{33} x_3$$

$$x_4 = a_{42} x_2 + a_{43} x_3$$

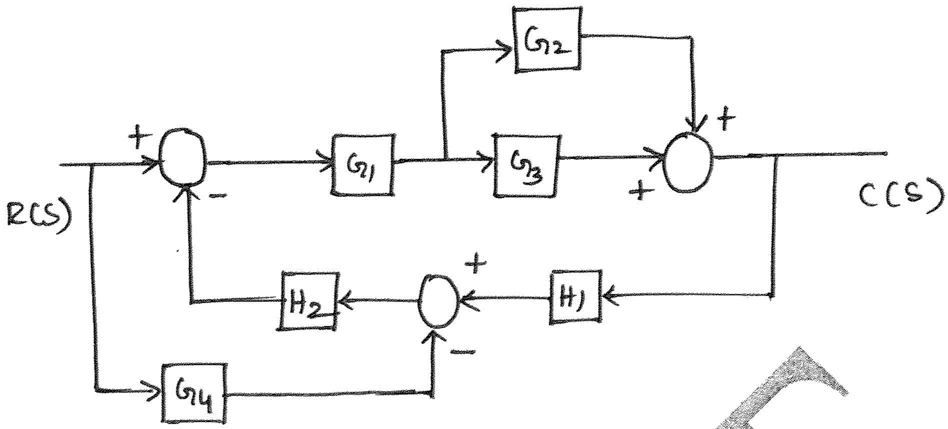
2) Determine the Overall transfer function $\frac{Y_7}{Y_4}$ of the Signal flow graph shown in the figure. using mason's gain formula.



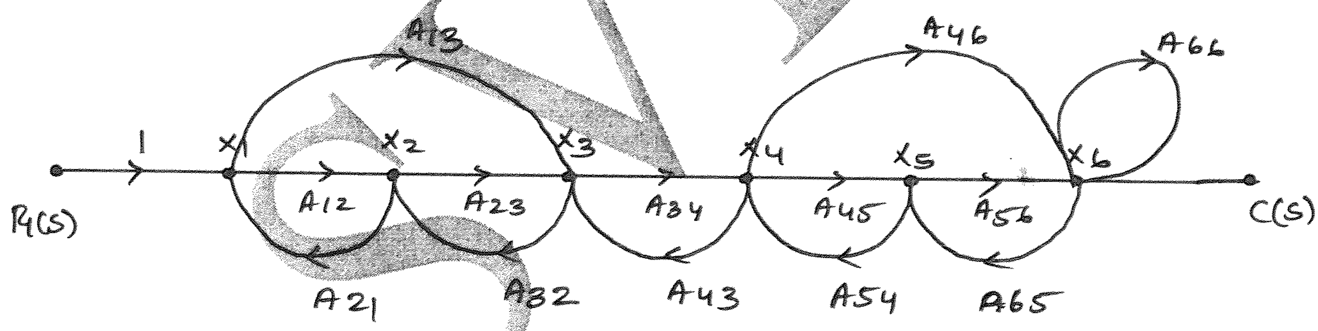
3) Find the transfer function $\frac{Y_7}{Y_1}$ using mason's gain formula for the Signal flow graph shown in figure.



4) Draw the signal flow graph for the block diagram shown in the figure and Evaluate $\frac{C(s)}{R(s)}$ using mason's formula.



5) Find the Overall transfer function $\frac{C(s)}{R(s)}$ using mason's rule for the system shown in the figure.



Solution for Practice Problems :-

Translational System :-

Problem -1

* → F-V Analogy :-

Equation 1 → $V(t) = L_1 \frac{d}{dt} i_1;$

Equation 2 → $0 = L_2 \frac{d}{dt} i_2 + \frac{1}{C_1}\int (i_2 - i_3)\,dt + R_1(i_2 - i_3);$

Equation 3 → $\frac{1}{C_1}\int (i_2 - i_3)\,dt + R(i_2 - i_3) = L_2 \frac{d}{dt} i_3 + \frac{1}{C_2}\int i_3\,dt;$

* → F-I Analogy :-

Equation 1 → $I(t) = C_1 \frac{d}{dt} V_1;$

Equation 2 → $0 = C_2 \frac{d}{dt} V_2 + \frac{1}{L_1}\int (V_2 - V_3)\,dt + G(V_2 - V_3);$

Equation 3 → $\frac{1}{L_1}\int (V_2 - V_3)\,dt + G(V_2 - V_3) = C_2 \frac{d}{dt} V_3 + \frac{1}{L_2}\int V_3\,dt;$

Problem 2 :-

* → F-V Analogy :-

Equation 1 :- $V(t) = L_1 \frac{d}{dt} i_1 + R_1(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt;$

Equation 2 :- $R_1(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt = L_2 \frac{d}{dt} i_2 + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt;$

* → F-I Analogy :-

Equation 1 : $I(t) = C_1 \frac{d}{dt} V_1 + G_1(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_2)\,dt;$

Equation 2 :- $G_1(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_2)\,dt = C_2 \frac{d}{dt} V_2 + G_2 V_2 + \frac{1}{L_2}\int V_2\,dt;$

Problem 3:-

* → F-V Analogy:-

Equation 1:- $V(t) = L_1 \frac{d}{dt} i_1 + \frac{1}{C_1}\int (i_1 - i_4)\,dt + R_1(i_1 - i_2)$;

Equation 2:- $R_1(i_1 - i_2) = L_2 \frac{d}{dt} i_2 + \frac{1}{C_2}\int (i_2 - i_3)\,dt$;

Equation 3:- $\frac{1}{C_1}\int (i_1 - i_4)\,dt = R_2(i_4 - i_3)$;

Equation 4:- $R_2(i_4 - i_3) + \frac{1}{C_2}\int (i_2 - i_3)\,dt = L_3 \frac{d}{dt} i_3 + R_3 i_3 + \frac{1}{C_3}\int i_3\,dt$;

* → F-I Analogy:-

Equation 1: $I(t) = C_1 \frac{d}{dt} V_1 + \frac{1}{L_1}\int (V_1 - V_4)\,dt + G_1(V_1 - V_2)$;

Equation 2:- $G_1(V_1 - V_2) = C_2 \frac{d}{dt} V_2 + \frac{1}{L_2}\int (V_2 - V_3)\,dt$;

Equation 3:- $\frac{1}{L_1}\int (V_1 - V_4)\,dt = G_2(V_4 - V_3)$;

Equation 4:- $G_2(V_4 - V_3) + \frac{1}{L_2}\int (V_2 - V_3)\,dt = C_3 \frac{d}{dt} V_3 + G_3 V_3 + \frac{1}{L_3}\int V_3\,dt$;

Problem 4:-

* → F-V Analogy:-

Equation 1:- $V(t) = L_1 \frac{d}{dt} i_1 + R_1 i_1 + \frac{1}{C_1}\int i_1\,dt + \frac{1}{C_2}\int (i_1 - i_2)\,dt$;

Equation 2:- $\frac{1}{C_2}\int (i_1 - i_2)\,dt = L_1 \frac{d}{dt} i_2 + \frac{1}{C_3}\int (i_2 - i_3)\,dt + R_2(i_2 - i_3)$;

Equation 3:- $\frac{1}{C_3}\int (i_2 - i_3)\,dt + R_2(i_2 - i_3) = L_2 \frac{d}{dt} i_3$;

$* \rightarrow$ F-I Analogy :-

Equation 1 :- $I(t) = C_1 \frac{d}{dt} V_1 + G_1 V_1 + \frac{1}{L_1}\int V_1 dt + \frac{1}{L_2}\int (V_1 - V_2) dt$;

Equation 2 : $\frac{1}{L_2}\int (V_2 - V_3) dt = C_1 \frac{d V_2}{dt} + \frac{1}{L_3}\int (V_2 - V_3) dt + G_2 (V_2 - V_3)$;

Equation 3 :- $\frac{1}{L_3}\int (V_2 - V_3) dt + G_2 (V_2 - V_3) = C_2 \frac{d V_3}{dt}$;

Problem 5 :

$* \rightarrow$ F-V Analogy :-

Equation 1 : $V(t) = L_2 \frac{d}{dt} i_2 + \frac{1}{C_3}\int i_2 dt + \frac{1}{C_1}\int (i_2 - i_1) dt + R_2 (i_2 - i_3)$;

Equation 2 : $\frac{1}{C_1}\int (i_2 - i_1) dt = L_1 \frac{d}{dt} i_1 + R_1 i_1$;

Equation 3 : $R_2 (i_2 - i_3) = L_3 \frac{d i_3}{dt} + \frac{1}{C_2}\int i_3 dt$;

$* \rightarrow$ F-I Analogy :-

Equation 1 : $I(t) = C_2 \frac{d V_2}{dt} + \frac{1}{L_3}\int V_2 dt + \frac{1}{L_1}\int (V_2 - V_1) dt + G_2 (V_2 - V_3)$;

Equation 2 : $\frac{1}{L_1}\int (V_2 - V_1) dt = C_1 \frac{d V_1}{dt} + G_1 V_1$;

Equation 3 : $G_2 (V_2 - V_3) = C_3 \frac{d}{dt} V_3 + \frac{1}{L_2}\int V_3 dt$;

Problem 6 :-

$* \rightarrow$ F-V Analogy :-

Equation 1 : $V(t) = L_2 \frac{d}{dt} i_2 + R_2 i_2 + \frac{1}{C_2}\int i_2 dt + \frac{1}{C_1}\int ({}_2 - i_1) dt + R_1 (i_2 - i_1)$;

Equation 2 : $\frac{1}{C_1}\int (i_2 - i_1) dt + R_1 (i_2 - i_1) = L_1 \frac{d}{dt} i_1$;

* → F - I Analogy:-

Equation 1 :- $I(t) = C_2 \frac{dV_2}{dt} + G_2 V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_1}\int (V_2 - V_1)\,dt + G_1 (V_2 - V_1);$

Equation 2 : $\frac{1}{L_1}\int (V_2 - V_1)\,dt + G_1 (V_2 - V_1) = C_1 \frac{d}{dt} V_1 ;$

Transfer Function:

Problem 1 :-

$$\frac{V_o(S)}{V_i(S)} = \frac{R_2}{R_1 + R_2}\left[\frac{1 + SCR_1}{1 + \frac{SCR_1R_2}{R_1+R_2}}\right]$$

Problem 2 :-

$$\frac{V_o(S)}{V_i(S)} = \frac{R}{S^2 LCR + SL + R}$$

Problem 3 :-

$$\frac{E_o(S)}{E_i(S)} = \frac{R_1 + SL}{R + R_1 + S(L + RR_1C) + S^2 RLC}$$

Problem 4 :-

$$\frac{E_o(S)}{E_i(S)} = \frac{1 + S^2 R_1 R_2 C_1 C_2 + SR_2 (C_1 + C_2)}{1 + S^2 R_1 R_2 C_1 C_2 + S(R_1 C_2 + R_2 C_1 + R_2 C_2)}$$

Problem 5 :-

$$\frac{\theta(S)}{T(S)} = \frac{K}{(J_1 J_2 S^4 + J_1 B S^3 + (KJ_1 + KJ_2) S^2 + KBS)}$$

Problem 6 :-

$$\frac{\theta_1(s)}{T(s)} = \frac{(J_2 s^2 + K_2)K_1}{J_1 J_2 J_m s^6 + (K_2 J_m J_1 + J_2 J_1 K_2 + J_2 J_1 K_1 + J_2 J_m K_1)s^4 + (J_2 K_2 K_1 + J_1 K_1 K_2 + J_m K_1 K_2)s^2}$$

Problem 7 :-

$$\frac{X(s)}{F(s)} = \frac{B_{12} s + K_1}{s(M_1 M_1 s^3 + (M_2 B_{12} + M_1 B + M_1 B_{12})s^2 + (m_1 K_1 + M_2 K_1 + B B_{12})s + K_1 B)}$$

Block Diagram Algebra

Problem 1 :-

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 + G_1 H_1}$$

Problem 2 :-

$$\frac{C(s)}{R(s)} = \frac{G_2 (G_1 + G_3)}{1 + G_1 G_2}$$

Problem 3 :-

$$\frac{C(s)}{R(s)} = \frac{G_3 G_6 + G_4 G_6 + G_2 G_5 G_6 + G_3 G_5 G_6}{1 + G_1 G_2 + G_1 G_3 + G_6 + G_1 G_2 G_6 + G_1 G_3 G_6 + G_3 G_6 G_7 + G_4 G_6 G_7 + G_2 G_5 G_6 G_7 + G_3 G_5 G_6 G_7}$$

Problem 4 :-

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 H_1 + G_2 H_1 - G_1}$$

Problem 5:-

$$\frac{C(s)}{R(s)} = \frac{G_1 G_7 (G_3 + G_4)}{1 + G_6 G_7 - G_3 G_7 G_8 - G_4 G_7 G_8 + G_1 G_3 G_5 + G_1 G_4 G_5 + G_1 G_2 + G_1 G_2 G_6 G_7}$$

Problem 6:-

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_5}{1 + G_2 + G_5 H_5 + G_2 G_5 H_5 + G_2 G_3 G_5 H_2}$$

Signal Flow Graph

Problem 1:-

$$M(s) = \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_4 G_5 + G_1 G_2 G_3}{1 + H_2 G_4 G_5 + G_2 G_4 G_5 H_1 + G_1 G_2 G_4 G_5 + G_1 G_2 G_3 + G_2 G_3 H_1 + G_3 H_2}$$

Problem 2:-

$$M(s) = \frac{C(s)}{R(s)} = \frac{G_1 G_6 G_7 (G_2 + G_3)(G_4 + G_5)}{1 - G_6 G_7 H_3 (G_2 + G_3)(G_4 + G_5) - G_6 H_1 - G_7 H_2 + G_6 G_7 H_1 H_2}$$

Problem 3:-

$$M(s) = \frac{C(s)}{R(s)} = \frac{56(s+1)^2 + 33(s+1) + 14}{(s+1)^2}$$

Problem 4:-

$$M(s) = \frac{Y(s)}{X(s)} = \frac{G_1 G_2 G_3 + G_4 (1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2)}{1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2}$$

Problem 5:-

$$M(s) = \frac{C_2(s)}{R_1(s)} = \frac{G_3(1-G_4H_2) + G_1G_2H_2}{(1-G_4H_2) + H_1\left(G_3(G_4H_2-1) - G_1G_2H_2\right)}$$

Problem 6:-

$$M(s) = \frac{C(s)}{R(s)} = \frac{(1 + G_5G_6G_8 + G_2G_4G_5G_7G_8)}{1+G_5G_6G_8 + G_2G_4G_5G_7G_8 + G_1G_2G_3G_7G_8}$$