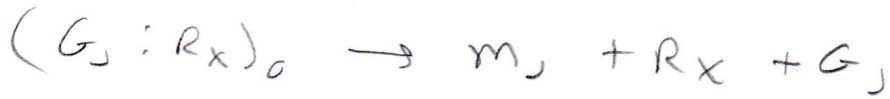


#1) Derive Expression For kinetic limit



RNAP

$$R_{x,T} = R_x^o + (G_j : R_x)_c + (G_j : R_x)_o + \sum_{i=1, j}^N \left\{ (G_i : R_x)_c + (G_i : R_x)_o \right\}$$

$$K_{x,j}^{-1} \equiv \frac{k_{+j}}{k_{-j} + k_{I,j}} \quad ; \quad \gamma_{x,j}^{-1} = \frac{k_{I,j}}{(k_{A,j} + k_{E,j})}$$

a) Balances

$$\begin{aligned} \frac{d}{dt} ((G_j : R_x)_c) &= k_{+j} G_j R_x^o - k_{-j} (G_j : R_x)_c \\ &\quad - k_{I,j} (G_j : R_x)_c \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} ((G_j : R_x)_o) &= k_{I,j} (G_j : R_x)_c - k_{A,j} (G_j : R_x)_o \\ &\quad - k_{E,j} (G_j : R_x)_o \end{aligned}$$



a) Assuming Steady State

all $\frac{d}{dt}$ Terms $\Rightarrow 0$

$$[1] \quad 0 = K_{+J}(G_J; R_x^o) - K_{-J}(G_J; R_x)_c - K_{I,J}(G_J; R_x)_c$$

$$[2] \quad 0 = K_{I,J}(G_J; R_x)_c - K_{A,J}(G_J; R_x)_o - K_{E,J}(G_J; R_x)_o$$

\Rightarrow Rearrange [1] and [2]

$$(G_J; R_x)_c = \left[\frac{K_{+J}}{K_{-J} + K_{I,J}} \right] G_J R_x^o$$

$$(G_J; R_x)_o = \left[\frac{K_{I,J}}{K_{A,J} + K_{E,J}} \right] (G_J; R_x)_c$$

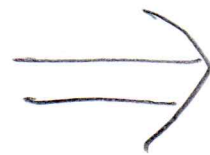
\Rightarrow Substitute in $\Sigma_{x,j,j}^{-1}, K_{x,j,j}^{-1}$

$$(G_J; R_x)_c = K_{x,j,j}^{-1} G_J R_x^o \quad \left. \begin{array}{l} \text{Substitute into [2]} \end{array} \right\}$$

$$(G_J; R_x)_o = \Sigma_{x,j,j}^{-1} (G_J; R_x)_c$$

$$\Rightarrow (G_J; R_x)_o = \Sigma_{x,j,j}^{-1} K_{x,j,j}^{-1} G_J R_x^o$$

get This value from RNAP Balance



a) RNAP

$$R_{xJT} = R_x^0 + (G_J; R_x)_c + (G_J; R_x)_o + \sum_{\sim}$$

where $\sum_{\sim} = \sum_{i=1, J}^N \{ (G_i; R_x)_c + (G_i; R_x)_o \}$

\Rightarrow Plug in eq $[\star]$ from earlier
 plug in eq $[\star\star]$ from earlier

$$R_{xJT} = R_x^0 + K_{xJJ}^{-1} G_J R_x^0 + \Sigma_{xJJ}^{-1} K_{xJJ}^{-1} G_J R_x^0 + \sum_{\sim}$$

\Rightarrow Modify \sum for R_x^0

$$\sum_{\sim} = \sum_{i=1, J}^N \{ (G_i; R_x)_c + (G_i; R_x)_o \}$$

$$\sum_{\sim}^{\star} = \sum_{i=1, J}^N \left\{ K_{xJi}^{-1} G_i \underbrace{R_x^0}_{\text{pull outside of summation}} + \Sigma_{xJi}^{-1} K_{xJi}^{-1} G_i \underbrace{R_x^0}_{\text{pull outside of summation}} \right\}$$

$$\sum_{\sim}^{\star\star} = \left[\sum_{i=1, J}^N \{ K_{xJi}^{-1} G_i + \Sigma_{xJi}^{-1} K_{xJi}^{-1} G_i \} \right] R_x^0$$

\Rightarrow So now

$$R_{xJT} = R_x^0 + K_{xJJ}^{-1} G_J R_x^0 + \Sigma_{xJJ}^{-1} K_{xJJ}^{-1} G_J R_x^0 + \sum_{\sim}^{\star\star} R_x^0$$



a) Divide Through to get just R_x^o

$$\frac{R_{xjT}}{K_{xj}^{-1} G_j + \tau_{xj}^{-1} K_{xj}^{-1} G_j + \sum_{i=j}^{**}} = R_x^o$$

\Rightarrow Multiply top and bottom by $(\tau_{xj} K_{xj})$

$$R_x^o = \frac{R_{xjT} \tau_{xj} K_{xj}}{G_j \tau_{xj} + G_j + K_{xj} \tau_{xj} + K_{xj} \tau_{xj} \sum_{i=j}^{**}}$$

looks like PSI

New Term

\Rightarrow Modify New Term

$$\Rightarrow K_{xj} \tau_{xj} \sum_{i=j}^N \left\{ K_{xj}^{-1} G_i (1 + \tau_{xj}^{-1}) \right\}$$

$$= \sum_{i=j}^N \left\{ \frac{K_{xj} \tau_{xj}}{K_{xj}} (1 + \tau_{xj}^{-1}) G_i \right\}$$

\Rightarrow Multiply Through by $\frac{\tau_{xj}^{-1}}{\tau_{xj}^{-1}} \Rightarrow$

$$\frac{\tau_{xj}^{-1}}{\tau_{xj}^{-1}} (1 + \tau_{xj}^{-1}) = \frac{1}{\tau_{xj}} (\tau_{xj} + 1) = \frac{1}{\tau_{xj}} (1 + \tau_{xj})$$

Q) § 0 Now

$$= \sum_{i=1,j}^N \left\{ \frac{k_{x,jj} \tilde{\tau}_{x,jj}}{k_{x,ji}} \left(\frac{\tilde{\tau}_{x,ji}^{-1}}{\tau_{x,ji}^{-1}} (1 + \tau_{x,ji}^{-1}) \right) G_i \right\}$$

$$= \sum_{i=1,j}^N \left\{ \frac{k_{x,jj} \tilde{\tau}_{x,jj}}{k_{x,ji} \tau_{x,ji}} (1 + \tau_{x,jj}) G_i \right\}$$

$$= \varepsilon_j$$

\Rightarrow put back into R_x^0 equation from pg 4

Now

$$R_x^0 = \frac{R_{x,jT} \tilde{\tau}_{x,jj} k_{x,jj}}{G_j (\tau_{x,jj} + 1) + \tilde{\tau}_{x,jj} k_{x,jj} + \varepsilon_j} \quad [\Phi]$$

from first order

$$\begin{aligned} \Rightarrow \Gamma_{x,jj} &= K_{E,j} (G_j; R_x)_0 \\ &= K_{E,j} \tilde{\tau}_{x,jj}^{-1} k_{x,jj}^{-1} G_j R_x^0 \end{aligned}$$

\Rightarrow Substitute in R_x^0 from $[\Phi]$



$$P_{x,j} = \frac{k_{E,j} \cancel{z_{x,j}^{-1}} \cancel{k_{x,j}^{-1}} G_j R_{x,T} \cancel{z_{x,j}} \cancel{k_{x,j}}}{G_j (1 + z_{x,j}) + z_{x,j} k_{x,j} + \epsilon_j}$$

Thus

$$P_{x,j} = k_{E,j} R_{x,T} \left(\frac{G_j}{G_j (1 + z_{x,j}) + z_{x,j} k_{x,j} + \epsilon_j} \right)$$

where

$$\epsilon_j = \sum_{i=1, i \neq j}^N \left\{ \frac{k_{x,j} z_{x,j}}{k_{x,i} z_{x,i}} (1 + z_{x,i}) G_i \right\}$$

b) Under what circumstances does a N-gene system reduce to the 1-gene system

- To reduce, ϵ_j would need to be zero, as in class there was no ϵ_j value

TO DRIVE THIS TO happen there should be an elongation limiting system

under this, $\epsilon_j \approx 0$, and $(1 + z_{x,j}) \approx 1$

So the $P_{x,j}$ in class in elongation limiters could equal the value derived here.