

Determinant formulas and cofactors

matrices q tem mto zeros longe das diagonais \rightarrow aplico cofatores p/ saber o determinante

$$\det AB = \det A \det B$$

Prop. 1 - Identidade

Prop. 2 - Changing rows

Prop. 3 - $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & 0 \\ c & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & d \end{vmatrix} =$

$\begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix} + \begin{vmatrix} a & 0 \\ c & 0 \end{vmatrix} + \begin{vmatrix} 0 & b \\ 0 & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & 0 \end{vmatrix}$

diag \uparrow \uparrow diag

$ad - bc$

Se fosse 3×3

- keep rows 2 and 3 the same, a split first row into 3 pieces $\begin{bmatrix} a & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & b & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & c \end{bmatrix}$$



$2 \rightarrow 3^3$ pieces \rightarrow a lot of them would be zero.

Os que não forem zeros \rightarrow

Survivors têm q for one entry for each row e for each column \rightarrow Se linha nula ou coluna, $\det = 0$.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & 0 & a_{23} \\ 0 & a_{32} & 0 \end{vmatrix} +$$

$(+a_{11} a_{22} a_{33})$
 $(-a_{11} a_{23} a_{32})$

$\hookrightarrow n!$ sobreviventes!

$$\begin{vmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & 0 & a_{33} \end{vmatrix} + \begin{vmatrix} 0 & a_{12} & 0 \\ 0 & 0 & a_{23} \\ a_{31} & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{21} & 0 & 0 \end{vmatrix}$$

$(-a_{21} a_{12} a_{33})$
 $(+a_{31} a_{12} a_{23})$
 $(+a_{13} a_{21} a_{32})$
 $(-a_{13} a_{22} a_{21})$

(exchange - minus)

Observe que se for 4×4 , 3 menos e 3 mais

$$\begin{vmatrix} 0 & 0 & 0 & a_{14} \\ 0 & 0 & a_{22} & 0 \\ 0 & a_{32} & 0 & 0 \\ a_{41} & 0 & 0 & 0 \end{vmatrix}$$

$+ a_{14} a_{22} a_{32} a_{41}$

Preciso de 2 exchanges

E a big formula p/ $n \times n$?

General formula - BIG FORMULA

$$\det A = \sum \pm a_{1\alpha} a_{2\beta} a_{3\gamma} \dots a_{n\omega}$$

$n!$ terms

comes from column α

metade posit.

metade negat.

$(\alpha \neq \beta \neq \gamma \dots) = \text{perm. of } (1, 2, \dots, n)$

4x4 case $\rightarrow 24$ sobreviventes - 4!

the guy in the first line can be chosen in n ways. o 2º, pode ser escolhido de $n-1$ ways.

No caso de A ser a matriz identidade, a única permutação que é $\neq 0$ é $1 \ 2 \ 3 \dots$ ($a_{11}, a_{22}, a_{33} \dots$)

ai daí 1 o produto e o determinante é 1.

Podemos provar que

$$\det AB = \det A \det B$$

$$\det A = \det A^t$$

Cofactors → connects formula for $n \times n$ determinant to determinants one smaller.

Cofactors 3×3

$$\det = a_{11} (a_{22} a_{33} - a_{23} a_{32})$$

$$+ a_{12} (\quad)$$

$$+ a_{13} (\quad)$$

$$\begin{vmatrix} a_{11} & \ominus & \ominus \\ \ominus & a_{22} & a_{23} \\ \ominus & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} \ominus & a_{12} & \ominus \\ a_{21} & \ominus & \ominus \\ \ominus & \ominus & a_{33} \end{vmatrix}$$

Cofactor of $a_{ij} = \overset{\pm}{\det} (n-1) = C_{ij}$
 matrix with row i ,
 col j } erased!

is it a plus or a minus?

$\left\{ \begin{array}{l} + \text{ if } i+j \text{ is even} \\ - \text{ if } i+j \text{ is odd} \end{array} \right.$

Cofactor formula - along row 1

$$\det A = a_{11} C_{11} + a_{12} C_{12} + \dots + a_{1n} C_{1n}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - bc$$

Example:

$$A_4 = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} =$$

$$|A_1| = 1 \quad |A_2| = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$|A_3| = -1$$

$$|A_4| = 1 \cdot \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1$$

$$\underbrace{1 \cdot 0 - 1 \cdot \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}}_{0 - 1 = -1} + 0$$

$$|A_4| = 1 \cdot |A_3| - 1 \cdot |A_2|$$

$$|A_n| = |A_{n-1}| - |A_{n-2}|$$

$$|A_1| = 1$$

$$|A_2| = 0$$

$$|A_3| = -1$$

$$|A_4| = -1$$

$$|A_5| = 0$$

$$|A_6| = 1$$

$$|A_7| = 1$$

Period
6

determinant
of $|A_6| = |A_1| = 1$

