Determinant formulas and cofactors

matrizes q tem mtos zeros longe das diagonais → aplico cofatores p/ saber o determinante

$$\det AB = \det A \det B$$

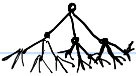
Prop. 1 - Identidade
Prop. 2 - changing rows
Prop. 3 -

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & 0 \\ c & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & d \end{vmatrix} =$$

$$\underbrace{\begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix}}_{\text{diag}} + \cancel{\begin{vmatrix} a & 0 \\ c & 0 \end{vmatrix}} + \cancel{\begin{vmatrix} 0 & b \\ 0 & d \end{vmatrix}} + \underbrace{\begin{vmatrix} 0 & b \\ c & 0 \end{vmatrix}}_{\text{diag}}$$

$$ad - bc$$

Se fosse 3×3
- keep rows 2 and 3 the same, & split first row into 3 pieces $[a\ 0\ 0]$ $[0\ b\ 0]$ $[0\ 0\ c]$



$27 \to 3^3$ pieces → a lot of them would be zero.

Os que não forem zeros →
Survivors têm a ter ~~one~~ entry for each row e for each column → Se linha nula ou col nula, det = 0.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & & 0 \\ & a_{22} & \\ 0 & & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & 0 & a_{23} \\ 0 & a_{32} & 0 \end{vmatrix} +$$

$+ a_{11} a_{22} a_{33}$ $- a_{11} a_{23} a_{32}$

└ n! sobreviventes!

$$\begin{vmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & 0 & a_{33} \end{vmatrix} + \begin{vmatrix} 0 & a_{12} & 0 \\ 0 & 0 & a_{23} \\ a_{31} & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{31} & 0 & 0 \end{vmatrix}$$

(1 exchange - minus

$- a_{21} a_{12} a_{33}$ $+ a_{31} a_{12} a_{23}$ $+ a_{13} a_{21} a_{32}$ $- a_{13} a_{22} a_{31}$

└ 3 menos e 3 mais

Observe que se for 4x4,

$$\begin{vmatrix} 0 & 0 & 0 & a_{14} \\ 0 & 0 & a_{22} & 0 \\ 0 & a_{32} & 0 & 0 \\ a_{41} & 0 & 0 & 0 \end{vmatrix}$$

Preciso de 2 exchanges

$+ a_{14} a_{22} a_{32} a_{41}$

E a big formula p/ nxn?

General formula - BIG FORMULA

$\det A = \sum \pm a_{1\alpha} a_{2\beta} a_{3\gamma} \cdots a_{n\omega}$

n! terms

comes from column α

metade posit.

metade negat.

$(\alpha \neq \beta \neq \gamma \ldots) =$ perm. of $(1, 2, \ldots, n)$

4x4 case → 24 sobreviventes - 4!

the guy in the first line can be chosen in n ways. o 2º, pode ser escolhido de n-1 ways.

no caso de A ser a matriz identidade, a única permutação que é ≠ 0 é 1 2 3 ... $(a_{11}, a_{22}, a_{33} \ldots)$

aí dá 1 o produto e o determinante é 1.

Poderei provar que

$\det AB = \det A \det B$

$\det A = \det A^t$

Cofactors → connects formula for $n \times n$ determinant to determinants one smaller.

Cofactors 3×3

$$\det = a_{11}(a_{22}a_{33} - a_{23}a_{32}) \qquad \begin{vmatrix} a_{11} & \circ & \circ \\ \circ & a_{22} & a_{23} \\ \circ & a_{32} & a_{33} \end{vmatrix}$$

$$+a_{12}(\qquad\qquad\qquad)$$

$$+a_{13}(\qquad\qquad\qquad) \qquad \begin{vmatrix} \circ & a_{12} & \circ \\ a_{21} & \circ & \circ \\ \circ & \circ & a_{33} \end{vmatrix}$$

Cofactor of a_{ij} $\overset{\pm}{=}$ det of $(n-1)$ matrix with row i, col j erased! $= C_{ij}$

is it a plus or a minus?

- $+$ if $i+j$ is even
- $-$ if $i+j$ is odd

Cofactor formula - along row 1

$$\boxed{\det A = a_{11}C_{11} + a_{12}C_{12} + \cdots + a_{1n}C_{1n}}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a.d - bc$$

Exemplo:

$$A_4 = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} =$$

$$|A_1| = 1 \quad |A_2| = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$A_{2\times 2}$

$$|A_3| = -1$$

$$|A_4| = 1 \cdot \underbrace{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}}_{1 \cdot 0 - 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + 0} - 1 \underbrace{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}}_{0} = -1$$

$$0 - 1 = -1$$

$$|A_4| = 1 \cdot |A_3| - 1 \cdot |A_2|$$

$$|A_n| = |A_{n-1}| - |A_{n-2}|$$

$|A_1| = 1$

$|A_2| = 0$

$|A_3| = -1$

$|A_4| = -1$

$|A_5| = 0$

$|A6| = 1$

$|A_7| = 1$

....

Period 6

determinant of $|A_{61}| = |A_1| = 1$