- Lec 1 - Geometry of linear equations
- Lec 2 - Elimination with matrices
- Lec 3 - Matrix operations and inverses
- Lec 4 - LU / LDU Factorization
- Lec 5 - Transposes and Permutations

$\}$ Problem Set 1.

Lec 2

Elimination with matrices

Todo software package resolve equações por eliminação.

Normally it succeeds (se A é a good matrix)

1st Pivot

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 12 \\ 2 \end{bmatrix}$$

Step 1

① Accept the first equation. Multiply by sth that knocks out $\underline{x}$ of equation 2. Eliminate $\underline{\underline{x}}$.

$$\begin{array}{rrrr} & -3 & -6 & -3 \\ + & 3 & 8 & 1 \\ \hline & 0 & 2 & -2 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} \rightarrow \text{Idêntica!}$$

Step 2: Limpei (2,1) position. Seria limpar (3,1) position mas já tenho!

Second pivot! Agora tenho + 2 eq. c/ y e z.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

2nd pivot - I want to wipe out (3,2) position.

$$\begin{array}{rrr} 0 & -4 & 4 \\ + \; 0 & 4 & 1 \\ \hline 0 & 0 & 5 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

$\underbrace{\text{PIVOTS!}}$

U → Upper triangular matrix

Pivots can't be zero!

this matrix is great! Gave me 3 pivots, none of them zero!

Elimination - quer transform A em U

Se eu quisesse saber o determinante dela, era só multiplicar os pivots.

How could this have failure?

- Se tiver um 0 no $\underline{\underline{x}}$ na 1ª equação, mude a posição das linhas. Exchange rows.
- Se aparecer zeros nos lugares do pivot, a matriz ñ é invertível.

↳ That's how we identify failure.

Back Substitution - volta a analisar ← →

Augmented matrix - you've tacked sthon!

Tenho q fazer a msm coisa dos 2 lados!

$$\underbrace{\begin{matrix} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{matrix}}_{\text{augmented matrix!}} \rightarrow \begin{matrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 5 & -10 \end{matrix}$$

$$Ax = b \qquad Ux = c$$

Because the system is triangular!

Back substi.

$z = -2$

$y = 1$

$x = 2$

row × matrix = row

$$\underset{1\times 3}{\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}} \times \underset{3\times 3}{\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}} = \begin{array}{l} 1\times \text{row1} \\ +2\times \text{row2} \\ +7\times \text{row3} \end{array}$$

lin. comb. of rows.

$1\times[\text{row1}] + 2[\text{row2}] + 7\times[\text{row3}]$

matrix × column

$$\underbrace{\underset{3\times 3}{\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}}}_{3\times 1} = \begin{array}{l} 3\times \text{col1} \\ + \\ 4\times \text{col2} \\ + \\ 5\times \text{col3} \end{array}$$

$$\underset{1\times 3}{\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}} \times \underset{3\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}} = \begin{bmatrix} 1\times 1 + 2\times 3 + 7\times 1 & 1\times 2 + 2\times 2 + 7\times 1 & 3\times 1 + 2\times 1 + 7\times 0 \end{bmatrix}$$

Quero uma matriz q subtraia 3 x row 1 from row 2

row 1 is not changing.   25:07

changing row 2.

$$\begin{bmatrix} 1 & 0 & 0 \\ & & \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

aqui

aqui

remains unchanged

Elimination steps - express as matrices

Elimination matrices

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

Subtraia

row2 - 3×row1

$$\begin{array}{rrrr} & 3 & 6 & 3 \\ & 3 & 8 & 1 \\ - & 3 & 6 & 3 \end{array}$$

pega 1× a row1 e nenhuma as outras rows

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

→ pega apenas a row3 e none of the other rows

Matriz identidade ñ transforma nada!

→ E_{21} → matrix that fixes (2,1) position - vira zero.

$\underbrace{\phantom{E_{21}}}$ elementary matrix

Step 2 ← row 3 − 2. row 2

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}}_{E_{32}} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

E_{32} → elim. matrix

1st is not involved!

$$\underbrace{\overset{E_{32}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}} \cdot \overset{E_{21}}{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 6 & -2 & 1 \end{bmatrix}$$

seria e_{31}? ?

Put steps together!

associative law

$$E_{32}(E_{21}A) = U$$

Que matriz é esta?

There's a good way to do this!

→ a melhor forma ñ é pensar em como ir de A p/ U, mas sim a melhor forma de ir de U p/ A.

- Reversing steps!
- Inverse! Todas as matrizes aqui vistas têm inversas!

Permutation matrix → multiplica pela identidade com as linhas q ve quer inverter, invertidas

Poderia multiplicar à esquerda p/ exchange columns?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\text{exchange 2 columns of identity.}} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

↳ NO! If I multiply on the left, I'm doing row operations.

$AB \neq BA$!

Inverses

$$A \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{quero achar a matriz q desfaz este passo.}} = I$$

A^{-1}

quero achar a matriz q desfaz este passo.

subtrair row2 - 3row1

inverse step: 3.row1 + row2

mudei row 2 subtraindo dela 3.row1
p/ inverter, somo 3.row1 à row2.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$