05-0: **Parsing**

- LL(1) – Left-to-right, Leftmost derivation, 1-symbol lookahead parsers
  - Need to guess which rule to apply after looking at only the first element in the rule
- LR parsers – Left-to-right, Rightmost derivation parsers
  - Look at the entire right-hand side of the rule before deciding which rule to apply

05-1: **LR Parsing**

- Maintain a stack
- Shift terminals from the input stream to the stack, until the top of the stack is the same as the right-hand side of a rule
- When the top of the stack is the same as the right-hand side of a rule *reduce* by that rule – replace the right-hand side of the rule on the stack with the left-hand side of the rule.
- Continue shifting elements and reducing by rules, until the input has been consumed and the stack contains only the initial symbol

05-2: **LR Parsing Example**

(0) $E' \rightarrow E\$$
(1) $E \rightarrow E + T$
(2) $E \rightarrow T$
(3) $T \rightarrow T*$ num
(4) $T \rightarrow$ num

3 + 4 * 5$
3 * 4 + 5$

05-3: **LR Parsing**

- How do we know when to shift, and when to reduce?
- Use a Deterministic Finite Automaton
  - Combination of DFA and a stack is called a Push-down automaton
- We will put both states and symbols on the stack
- When the end-of-file marker is shifted, accept the string

05-4: **LR Parsing Example**

(0) $E' \rightarrow E\$$
(1) $E \rightarrow E + T$
(2) $E \rightarrow T$
(3) $T \rightarrow T*$ num
(4) $T \rightarrow$ num

	num	+	*	$	E	T
1	s2				g3	g4
2		r(4)	r(4)	r(4)		
3		s7		a		
4		r(2)	s5	r(2)		
5	s6					
6		r(3)	r(3)	r(3)		
7	s2					g8
8		r(1)	s(5)	r(1)		

05-5: **LR Parsing**

- LR(0) Parsers. Reduce as soon as the top of the stack is the same as the left-hand side of a rule
- SLR(1) Parsers. More powerful than LR(0) – adds some lookahead information
- LR(1) Parsers. More powerful than SLR(1) – adds more sophisticated lookahead information

- LALR Parsers. *Almost* as powerful as LR(1), but uses much less memory (smaller table sizes)

05-6: **LR(0) Parsing**

- Reads the input file Left-to-Right LR(0)
- Creates a Rightmost derivation LR(0)
- No Lookahead (0-symbol lookahead) LR(0)

LR(0) parsers are the simplest of the LR parsers

05-7: **LR Parsing Example**

(0) $S' \rightarrow S\$$
(1) $S \rightarrow AA$
(2) $S \rightarrow$ bc
(3) $A \rightarrow$ ba$A$
(4) $A \rightarrow$ c

05-8: **LR Parsing Example**

$S' \rightarrow S\$$
$S \rightarrow AA$
$S \rightarrow$ bc
$A \rightarrow$ ba$A$
$A \rightarrow$ c

	a	b	c
S'		$S' \rightarrow S\$$	$S' \rightarrow S\$$
S		$S \rightarrow$ bc $S \rightarrow AA$	$S \rightarrow AA$
A		$A \rightarrow$ ba$A$	$A \rightarrow$ c

Not LL(1)!

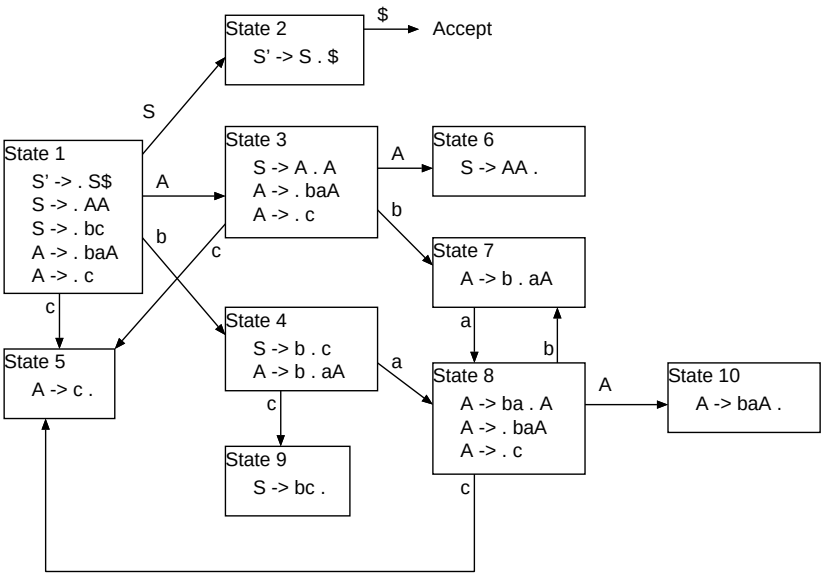
05-9: **LR(0) Items**

- An LR(0) item consists of
  - A rule from the CFG
  - A "." in the rule, which indicates where we currently are in the rule
- $S \rightarrow$ ab . c
  - Trying to parse the rule $S \rightarrow$ abc
  - Already seen "ab", looking for a "c"

05-10: **LR(0) States & Transitions**

(0) $S' \rightarrow S\$$
(1) $S \rightarrow AA$
(2) $S \rightarrow$ bc
(3) $A \rightarrow$ ba$A$
(4) $A \rightarrow$ c

05-11: **LR(0) States & Transitions**



05-12: **LR(0) Parse Table**

	a	b	c	$	S	A
1		s4	s5		g2	g3
2				accept		
3		s7	s5			g6
4	s8		s9			
5	r(4)	r(4)	r(4)	r(4)		
6	r(1)	r(1)	r(1)	r(1)		
7	s8					
8		s7	s5			g10
9	r(2)	r(2)	r(2)	r(2)		
10	r(3)	r(3)	r(3)	r(3)		

05-13: **Closure & Transitions**

- Two basic operations for creating LR(0) parsers:
  - Finding the *closure* of a state
  - Finding the transitions out of a state

05-14: **Closure**

1. For each item in the state of the form $S \rightarrow \alpha \ . \ S_1\beta$, where α and β are (possibly empty) strings of terminals and non-terminals, and S_1 is a non-terminal:
   - For each rule of the form $S_1 \rightarrow \gamma$ add the item $S_1 \rightarrow \ . \ \gamma$ if it is not already there
2. If any items were added in step 1, go back to step 1 and repeat

05-15: **Closure**

- If a "." appears right before the non-terminal S in an item
  - Add items for all S rules to the state, with the "." at the beginning of the rule
- Repeat until no more items can be added

05-16: **Finding Transitions**

1. If the end-of-file terminal $ appears before the "." in some item in the original state, create a transition from the original state to an "accept" state, transitioning on $.
2. For each terminal `a` (other than $) that appears before the "." in some item in the original state:
   - Create a new empty state.
   - For each item in the original state of the form $S \rightarrow \alpha$. `a` γ, where α and γ are (possibly empty) strings of terminals an non-terminals, add the item $S \rightarrow \alpha$`a` . γ to the new state.
   - Find the closure of the new state.
   - Add a transition from the original state to the new state, labeled with `a`.
3. For each non-terminal `S` that appears before the "." in some item in the original state:
   - Create a new empty state.
   - For each items in the original state of the form $S_1 \rightarrow \alpha$. $S\gamma$, where α and γ are (possibly empty) strings of terminals an non-terminals, add the item $S_1 \rightarrow \alpha S$. γ to the new state.
   - Find the closure of the new state.
   - Add a transition from the original state to the new state, labeled with S.

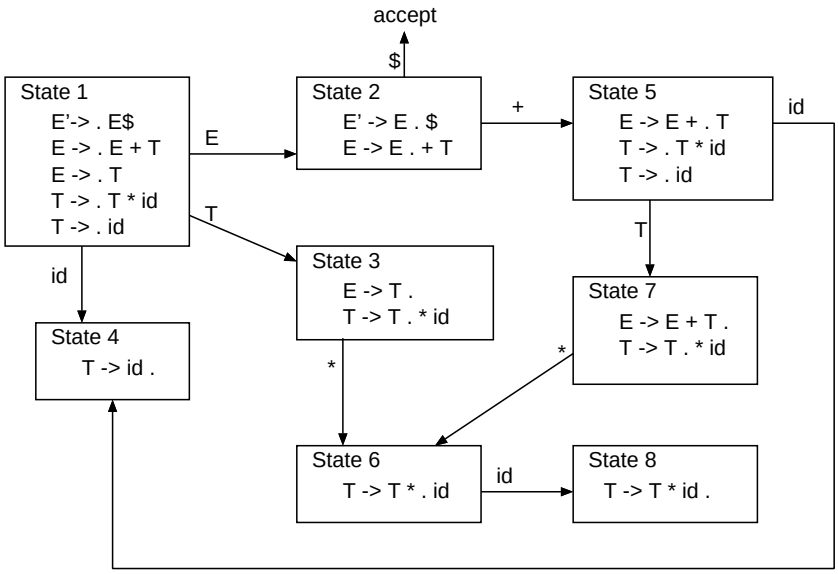
05-17: **Finding Transitions**

- If a "." appears just before a terminal `a` in at least one item:
  - Create a new state
  - Add all items where the "." is just before an `a`
  - Move the "." past the `a` in the new state
  - Find the closure of the new state
  - Add a transition to the new state, labeled with an `a`.

05-18: **Another LR(0) Example**

(0) $E' \rightarrow E\$$
(1) $E \rightarrow E + T$
(2) $E \rightarrow T$
(3) $T \rightarrow T$ * id
(4) $T \rightarrow$ id

05-19: **LR(0) States & Transitions**



05-20: **LR(0) Parse Table**

	id	+	*	$	E	T
1	s4				g2	g3
2		s5		accept		
3	r(2)	r(2)	r(2),s6	r(2)		
4	r(4)	r(4)	r(4)	r(4)		
5	s4					g7
6	s8					
7	r(1)	r(1)	r(1),s6	r(1)		
8	r(3)	r(3)	r(3)	r(3)		

05-21: **Shift-Reduce Conflict**

- In state 3, on a *, should we shift, or reduce? Why?

05-22: **Shift-Reduce Conflict**

- In state 3, on a *, should we shift, or reduce? Why?
  - If we reduce, then we're stuck – since the top of the stack will contain E, the next symbol in the input stream is *, and * cannot follow E in any partial derivation!
- If a state contains the item:

  $S \rightarrow \gamma$.

  we should only reduce if *the next terminal can follow* S

05-23: **SLR(1)**

- Add simple lookahead (the *S* in SLR(1) is for *simple*
- In LR(0) parsers, if state k contains the item "$S \rightarrow \gamma$." (where $S \rightarrow \gamma$ is rule (n))
  - Put r(n) in state k, in all columns

- In SLR(0) parsers, if state k contains the item "$S \rightarrow \gamma$." (where $S \rightarrow \gamma$ is rule (n))
  - Put r(n) in state k, in all columns *in the follow set of* S

05-24: **SLR(1) Parse Table**

	id	+	*	$	E	T
1	s4				g2	g3
2		s5		accept		
3		r(2)	s6	r(2)		
4		r(4)	r(4)	r(4)		
5	s4					g7
6	s8					
7		r(1)	s6	r(1)		
8		r(3)	r(3)	r(3)		

x + y * z w + x + y * z
x * y + z w * x * y + z

05-25: **YALR(0)E**

- Yet another LR(0) Example

(0) $S' \rightarrow S\$$
(1) $S \rightarrow AB$
(2) $A \rightarrow A$a
(3) $A \rightarrow$ a
(4) $B \rightarrow B$b
(4) $B \rightarrow$ b

05-26: **YALR(0)E**

05-27: **YASLR(1)E**

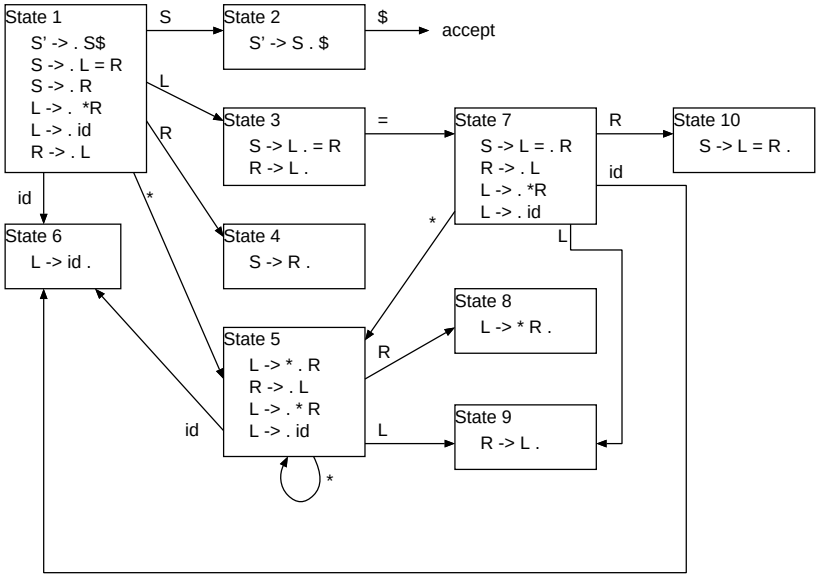
- Yet another SLR(1) Example

(0) $E' \rightarrow E\$$
(1) $E \rightarrow V$
(2) $E \rightarrow$ num
(3) $V \rightarrow$ id
(4) $V \rightarrow V[E]$

05-28: **Yet Another Example**

(0) $S' \rightarrow S\$$
(1) $S \rightarrow L = R$
(2) $S \rightarrow R$
(3) $L \rightarrow {*}R$
(4) $L \rightarrow$ id
(5) $R \rightarrow L$

05-29: **LR(0) States & Transitions**



05-30: **SLR(1) Parse Table**

	id	=	*	$	S	L	R
1	s6		s5		g2	g3	g4
2				accept			
3		r(5),s7		r(5)			
4				r(2)			
5	s6		s5			g9	g8
6		r(4)		r(4)			
7	s6		s5			g9	g10
8		r(3)		r(3)			
9		r(5)		r(5)			
10				r(1)			

05-31: **Why SLR(1) Fails**

$S \rightarrow L$. = R

$R \rightarrow L$.

- In this state, on a =, should be shift or reduce?
- An = can follow an R – only if the R is preceded by a *
- We need a more sophisticated lookahead scheme to disambiguate this situation

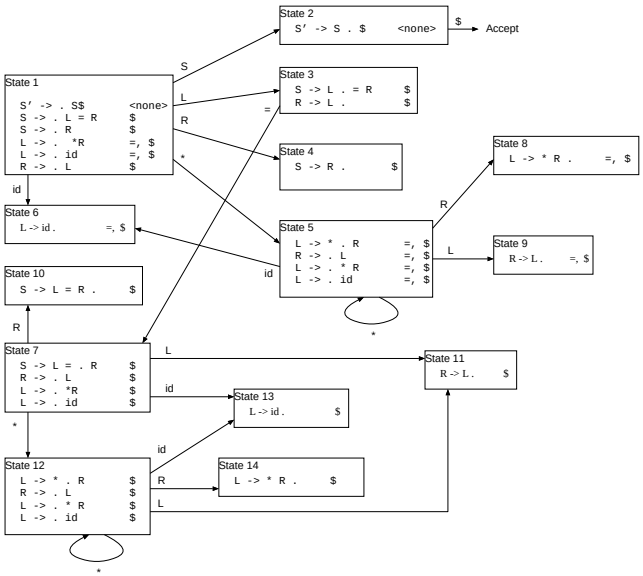
05-32: **LR(1) Items**

- Like LR(0) items, contain a rule with a "."
- Also contain lookahead information – the terminals that could follow this rule, *in the current derivation*
  - More sophisticated than SLR(1), which only look at what terminals could follow the LHS of the rule in *any* derivation

05-33: **LR(1) Example**

(0) $S' \rightarrow S\$$
(1) $S \rightarrow L = R$
(2) $S \rightarrow R$
(3) $L \rightarrow *R$
(4) $L \rightarrow \text{id}$
(5) $R \rightarrow L$

05-34: **LR(1) States and Transitions**



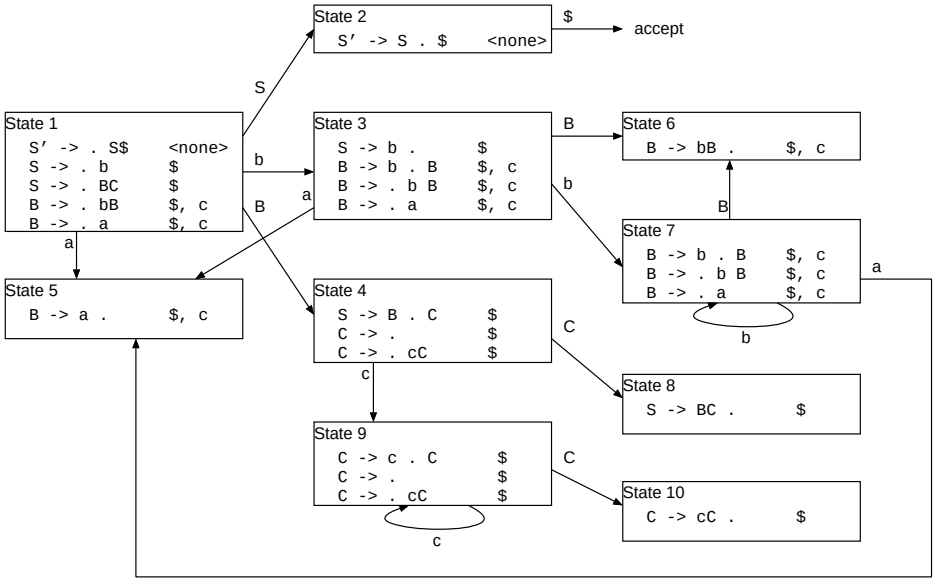
05-35: **LR(1) Parse Table**

	id	=	*	$	S	L	R
1	s6		s5		g2	g3	g4
2				accept			
3		s7		r(5)			
4				r(2)			
5	s6		s5			g9	g8
6		r(4)		r(4)			
7	s13		s12			g11	g10
8		r(3)		r(3)			
9		r(5)		r(5)			
10				r(1)			
11				r(5)			
12	s13		s12			g11	g14
13				r(4)			
14		r(3)		r(3)			

05-36: **More LR(1) Examples**

(0) $S' \rightarrow S\$$
(1) $S \rightarrow BC$
(2) $S \rightarrow \text{b}$
(3) $B \rightarrow \text{b}B$
(4) $B \rightarrow \text{a}$
(5) $C \rightarrow \epsilon$
(6) $C \rightarrow \text{c}C$

05-37: **LR(1) States & Transitions**



05-38: **LR(1) Parse Table**

	a	b	c	$	S	B	C
1	s5	s3			g2	g4	
2				accept			
3	s5	s7		r(2)		g6	
4			s9	r(5)			g8
5			r(4)	r(4)			
6			r(3)	r(3)			
7	s5	s7				g6	
8				r(1)			
9			s9				g10
10				r(6)			

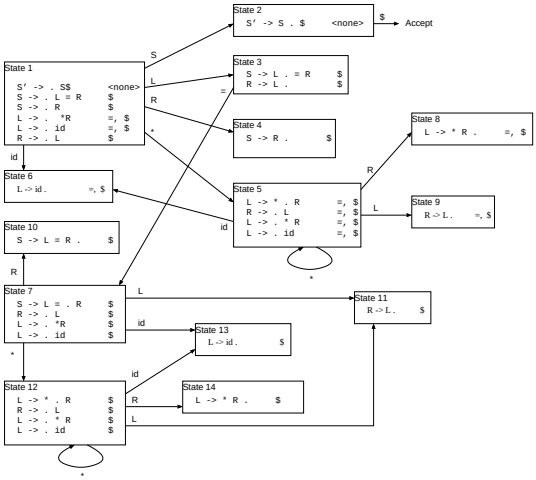
05-39: **LALR Parsers**

- LR(1) Parsers are more powerful than LR(0) or SLR(1) parsers
- LR(1) Parsers can have many more states than LR(0) or SLR(1) parsers
  - My simpleJava implementation has 139 LR(0) states, and *thousands* of LR(1) states
- We'd like *nearly* the power of LR(1), with the memory requirements of LR(0)

05-40: **LALR Parsers**

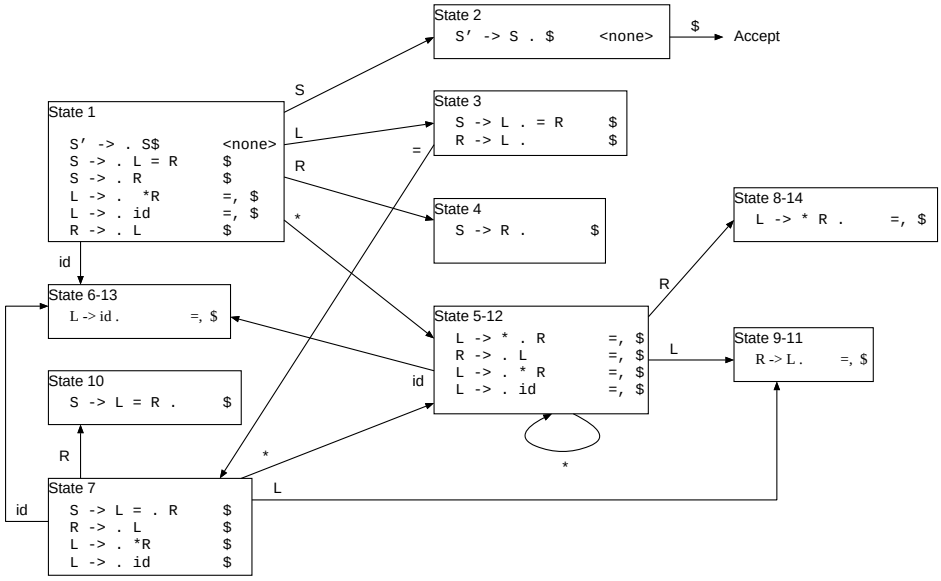
- LR(1) parsers can have large numbers of states
- Many of the states will be nearly the same – they will differ only in Lookahead
- IDEA – Combine states that differ only in lookahead values
  - Set lookahead of combined state to union of lookahead values from combining states

05-41: **LALR Parser Example**



Can combine 5 & 12, 6 & 13, 8 & 14, 9 & 11

05-42: **LALR Parser Example**



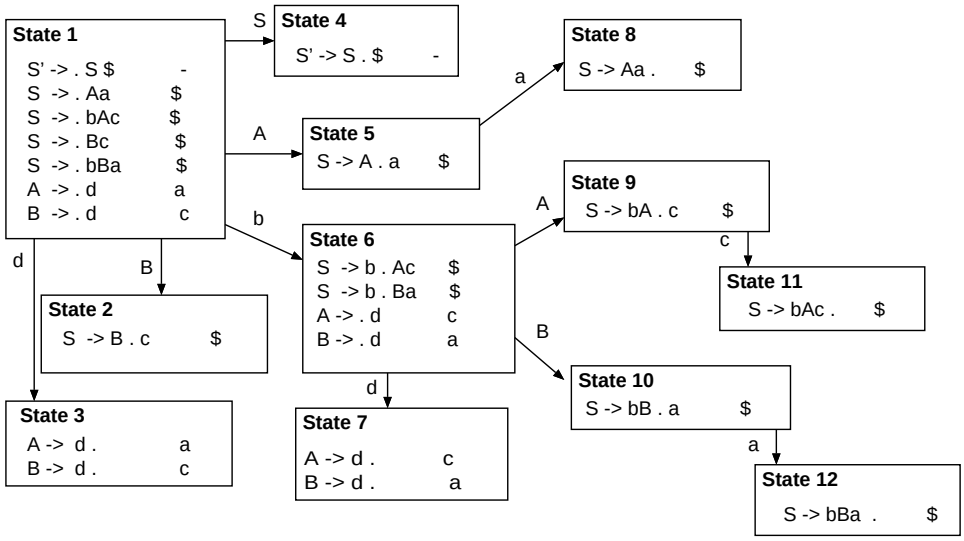
05-43: **LALR Parser Example**

	id	=	*	$	S	L	R
1	s6-13		s5-12		g2	g3	g4
2				accept			
3		s7		r(5)			
4				r(2)			
5-12	s6-13		s5-12			g9-11	g8-14
6-13		r(4)		r(4)			
7	s6-13		s5-12			g9-11	g10
8-14		r(3)		r(3)			
9-11		r(5)		r(5)			
10				r(1)			

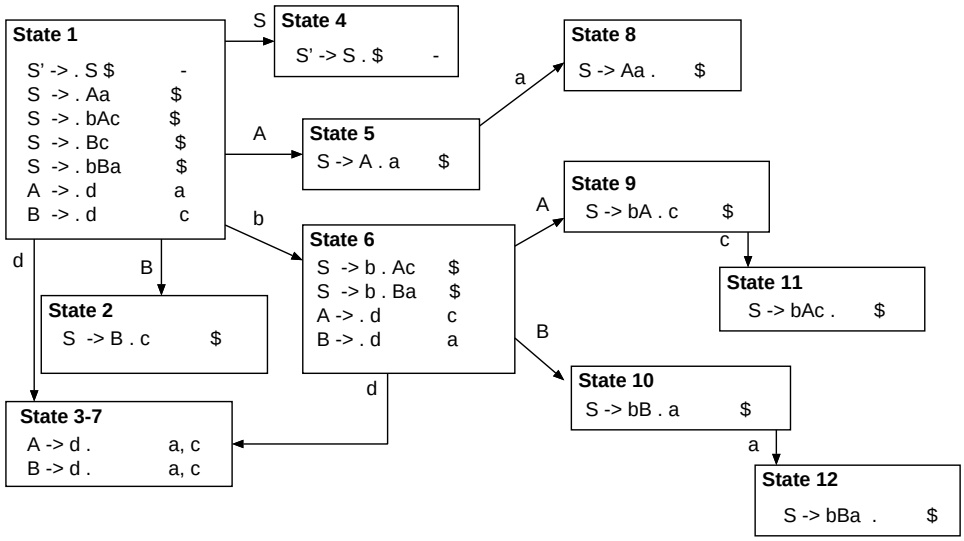
05-44: **More LALR Examples**

(0) $S' \rightarrow S\$$
(1) $S \rightarrow A\mathrm{a}$
(2) $S \rightarrow \mathrm{b}A\mathrm{c}$
(3) $S \rightarrow B\mathrm{c}$
(4) $S \rightarrow \mathrm{b}B\mathrm{a}$
(5) $A \rightarrow \mathrm{d}$
(6) $B \rightarrow \mathrm{d}$

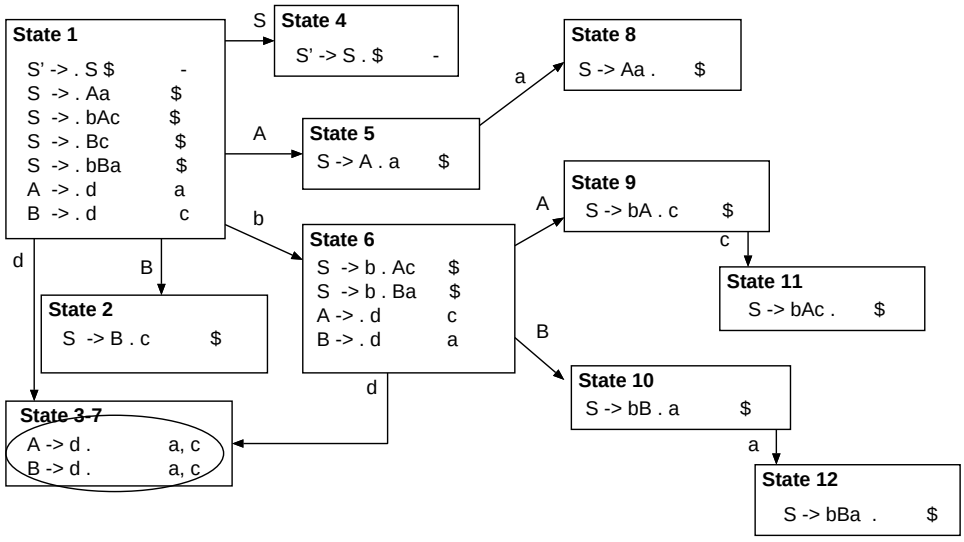
05-45: **More LALR Examples**



05-46: **More LALR Examples**



05-47: **More LALR Examples**


State 1
S' -> . S $ -
S -> . Aa $
S -> . bAc $
S -> . Bc $
S -> . bBa $
A -> . d a
B -> . d c
S
State 4
S' -> S . $ -
A
State 5
S -> A . a $
a
State 8
S -> Aa . $
b
State 6
S -> b . Ac $
S -> b . Ba $
A -> . d c
B -> . d a
A
State 9
S -> bA . c $
c
State 11
S -> bAc . $
B
State 10
S -> bB . a $
a
State 12
S -> bBa . $
d
B
State 2
S -> B . c $
d
State 3-7
A -> d . a, c
B -> d . a, c