Algorithm Analysis, Term I/2018-19

Assignment 5

Due: Wed, Dec 5 @ 11.59pm

Directions:

- Your solutions must be typeset. LaTeX is recommended.
- You must upload your solutions as a PDF file on Canvas before the deadline.
- You don't have to include your solutions to the programming problems in the PDF file.

Problem 1

Suppose we perform a sequence of n operations on a data structure in which the i-th operation costs i if i is an exact power of 2, and 1 otherwise. What is the amortized cost per operation?

Problem 2

Show how to implement a queue with two stacks so that the amortized cost of each ENQUEUE and each DEQUEUE operation is $O(1)$. Assume each stack operation (PUSH and POP) costs 1. (Hint: use the piggy bank technique)

Problem 3

Consider the following variants of `VERTEX-COVER` problem:

- `VERTEX-COVER`: Does there exist a vertex cover of size $\leqslant k$?
- `FIND-VERTEX-COVER`: Find a vertex cover of size $\leqslant k$.
- `OPT-VERTEX-COVER`: Find a vertex cover of minimum size.

Show that `VERTEX-COVER` $\equiv_p$ `FIND-VERTEX-COVER` $\equiv_p$ `OPT-VERTEX-COVER`. (Hint: show how a problem can be solved if a poly-time solver to another problem was given to you)

Problem 4

The `CLIQUE` problem is defined as follows. Given a graph $G = (V, E)$ and integer k, determine whether there exists a subset of vertices $C \subseteq V$ such that $|C| \geqslant k$ and for any two nodes $u, v \in C$, the edge $(u, v) \in E$. Show that `CLIQUE` is NP-complete. You may assume that `VERTEX-COVER` is NP-complete.