UCS405 (Discrete Mathematical Structures)

Tutorial Sheet-8

1. Which of these relations on {0, 1, 2, 3} are partial orderings?
   a) {(0, 0), (1, 1), (2, 2), (3, 3)}
   b) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}
   c) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)}
   d) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}
   e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0),(2, 2), (3, 3)}

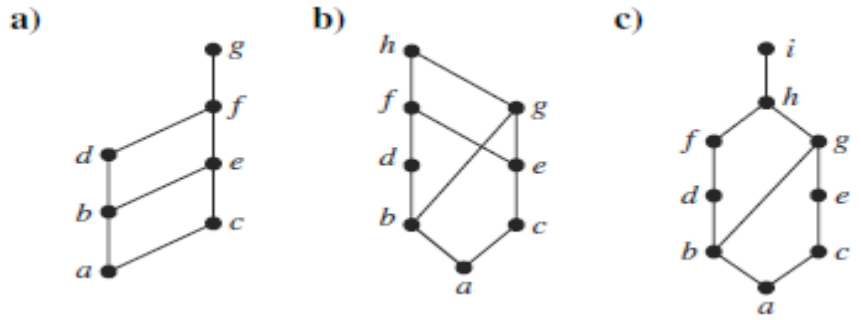
2. Draw the Hasse diagram for divisibility on the set
   a) {1, 2, 3, 4, 5, 6, 7, 8}.
   b) {1, 2, 3, 5, 7, 11, 13}.
   c) {1, 2, 3, 6, 12, 24, 36, 48}.
   d) {1, 2, 4, 8, 16, 32, 64}.

3. Answer these questions for the poset ({3, 5, 9, 15, 24, 45}, /).
   a) Find the maximal elements.
   b) Find the minimal elements.
   c) Is there a greatest element?
   d) Is there a least element?
   e) Find all upper bounds of {3, 5}.
   f) Find the least upper bound of {3, 5}, if it exists.
   g) Find all lower bounds of {15, 45}.
   h) Find the greatest lower bound of {15, 45}, if it exists.

4. Draw the Hasse diagram for inclusion on the set P(S), where S = {1, 2, 3, 4}.

5. Answer these questions for the poset ({{1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}}, ⊆).
   a) Find the maximal elements.
   b) Find the minimal elements.
   c) Is there a greatest element?
   d) Is there a least element?
   e) Find all upper bounds of {{2}, {4}}.
   f) Find the least upper bound of {{2}, {4}}, if it exists.
   g) Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.
   h) Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}}, if it exists.

6. Determine whether the posets with these Hasse diagrams are lattices



7. Schedule the tasks needed to build a house, by specifying their order, if the Hasse diagram representing these tasks is as shown in the figure.