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CS325
HOMEWORK #7
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Problem 1:

a) Cannot be inferred: Possible for X to only be in NP
b) Cannot be inferred: It is possible to have Y in NP-hard
c) Cannot be inferred: It could be possible to have X in P
d) Can be inferred: Could be possible to have X be in P
e) Cannot be inferred: X and Y could possible be NP-complete because X reduced could be NP-complete and reduced to Y, also possibly NP-complete
f) Cannot be inferred: Possible for Y to be harder than X
g) Can be inferred: X is no harder than Y

Problem 2:

a) With the composite in NP, and subset-sum an np-complete, the statement that subset-sum can reduce composite would be false. Reducing a problem to an easier problem wouldn't make sense, subset-sum can be reduced to any other NP-complete, not an np problem. Its not known if composite is NP-complete, it is only known that it is NP.
b) True. If there is an $O(n^3)$ algorithm for subset-sum, which is NP-complete, there is then a polynomial time trial for composite which is also in NP. NP-complete is NP, so a solution for NP-complete can solve an NP problem.
c) This statement, P=NP is false. Even though a composite is NP, its not known that it is also an NP-complete.
d) If P != NP, then no problem will be able to be solved in poly. Time == FALSE.

Problem 3:

a) True: 3-SAT can be reduced to TSP. 3-Sat + TSP are both NP-complete problem. Based on the lectures, 3-SAT will get reduced to TSP but will first be reduced to dir-ham-cycle than to TSP.
b) False: If P != NP, 3-SAT will be able to be reduced to 2-SAT. Having P!=NP means that there is no poly. time algorithm for 3-SAT. We know 2-SAT is in P. If 3-SAT can be reduced to 2-SAT, 3-SAT can also be in P.
c) True: If P != NP, no NP-complete can be solved in poly. Time. NP complete problems can be solved in poly. Time, then all NP-complete can be solved in poly. Time. If P != NP, no NP-complete problems can be solved in poly time.

Problem 4:

If given the solution to HAM-PATH, we can verify it using polynomial time. We could do this by taking the answer and confirm no repeats and the first node is u and the last node to be v. Hence, HAM-PATH is in NP. Now, HAM-CYCLE is known to be NP-complete and reduces to HAM-PATH.

For HAM-CYCLE to be able to be reduced to HAM=PATH, we have the assumption that the same graph has the same number of edges. Furthermore, reducing HAM-CYCLE to HAM-PATH is trivial since if there is a HAM-CYCLE in a graph, there has to be a HAM-PATH as all nodes are connected, and there is a path in the graph where each vertex is visited exactly one time. HAM-PATH is therefore NP-complete.

Problem 5:

First, given a solution to LONG-PATH, we can verify that it is a polynomial time. We would first need to make sure the edges given are also in G, and the length of the path is at least K. With these facts, LONG-PATH should be verified as NP. Now we can think of the fact that HAM-PATH is NP-complete and reduces to LONG-PATH. Since HAM-PATH can be reduced to LONG-PATH, we are able to assume that they have the same graph, and also the same number of edges. With these facts, we can find the longest path by traversing through the vertices without repeating an edge. We are trying to find the path from u to v, that is at least k edges. From there, we can make all the edges the same weight and try to find a path from u to v, which crosses at least k number of edges. Proving LONG-PATH == NP-complete.