

Stabilized Feedback Control of Unicycle Mobile Robots

Regular Paper

Khoukhi Amar^{1,*} and Shahab Mohamed¹

¹ Systems Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

* Corresponding author E-mail: amar@kfupm.edu.sa

Received 2 May 2012; Accepted 4 Jul 2012

DOI: 10.5772/51323

© 2013 Amar and Mohamed; licensee InTech. This is an open access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/3.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract In this paper, a stabilized feedback control is designed for a class of unicycle non-holonomic mobile robots. The approach is based on kinematic polar coordinate transformations. The suggested control scheme allows the robot to achieve stabilized near-optimal trajectories, while satisfying the hard constraints of specified initial and final postures (positions and orientations). Simulation experiments showing the effectiveness of the proposed technique are provided and discussed.

Keywords Unicycle Mobile Robots, Polar-Based Model, Posture constraints, Kinematic Feedback Control

1. Introduction

Recent advances in robotics have led to higher complexity in these systems from several perspectives, including stochastic identification, adaptivity and stabilizable feedback control design, in order address the primary challenges faced by today's engineering research community [1-5]. This paper considers the stabilizable feedback control design problem of a unicycle mobile robot. The roots of this problem date back to the pioneering work of Dubins [6] who set the problem of

characterizing the shortest path for a particle moving forward with a constant linear velocity. This simple kinematic model opened the way for many researchers into the trajectory planning problems of robotic systems. In [7] the authors considered the same problem, where backward motions were allowable. A shortest path synthesis of Dubins' car has been determined according to Pontryagin's Maximum principle in [8].

These results have been extended in [9] to a case where obstacles are present. Clothoid-like approaches have been proposed by many researchers for path preserving curvature continuity. In particular, this approach has been used as a CAD tool in [10-12]. In strongly non-linear, coupled mechanical structures, obstacle avoidance and high velocity and precision tasks, these approaches fail to give acceptable results [13, 14].

On the other hand, non-holonomic systems have velocities subject to non-holonomic constraints. Such constraints come up in locomotion applications such as wheeled mobile robots and cannot be transformed to pure configuration constraints. For an overview of modelling and control of non-holonomic systems, see [15]. Compared to holonomic ones, non-holonomic systems are much more difficult to control.

Recently, more advanced techniques have been introduced [16-19] in which several optimization methods have been implemented in the multi-objective control of wheeled mobile robots. In [16] a summary of developments in the control of non-holonomic systems is provided. The article provides an accessible presentation of the area, with various models, problem formulations, approaches and results in a proper context. This overview provides a good introduction to the subject

In this paper, we focus on a feedback control system of wheeled mobile robots involving geometric transformation from Cartesian to polar coordinates. The main objective is to achieve stable feedback control, allowing the robot to move from a given initial to a final posture including position and orientation. In Section II the system model is introduced. Section III contains the control law for transformation into polar system coordinates. Section IV provides with simulation results and discussions and finally Section V concludes this work.

2. Modelling

2.1 Kinematic and dynamic analysis

The main problem addressed in this work is posture stabilization. Consider an open 2D area, with no obstacles, as the workspace of the robot (Fig. 1). An initial and desired final configuration (posture) of the robot would be given as:

$$P_0 = \begin{bmatrix} x_0 \\ y_0 \\ \phi_0 \end{bmatrix}, P_F = \begin{bmatrix} x_F \\ y_F \\ \phi_F \end{bmatrix}$$

The non-holonomic kinematic constraints are given as:

$$A(q) = \begin{bmatrix} \sin \phi & -\cos \phi & 0 & 0 & 0 \\ \cos \phi & \sin \phi & b & -r & 0 \\ \cos \phi & \sin \phi & -b & 0 & -r \end{bmatrix} \quad (1)$$

(with r as the radius of the wheel).

Let us have a full rank matrix $S(q)$ with a rank of $(n-m)$ and in our case $n = 5$ and $m = 3$. The goal is to have:

$$S^T(q)A^T(q) = 0 \Rightarrow A(q)S(q) = 0 \quad (2)$$

The previous equation gives us the kinematic relation:

$$S(q) = \begin{bmatrix} \frac{r}{2} \cos \phi & \frac{r}{2} \cos \phi \\ \frac{r}{2} \sin \phi & \frac{r}{2} \sin \phi \\ \frac{r}{2b} & -\frac{r}{2b} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Differentiating $S(q)$ and substituting into (2) would eliminate the $A^T(q)\nu$ term and lead us to the equations considering the dynamic equation of motion as defined in [20], which will lead us to:

$$\dot{q} = S(q)\nu$$

$$\tilde{M}(q)\dot{\nu} + \tilde{V}(q, \dot{q})\nu = \tilde{B}(q)\tau$$

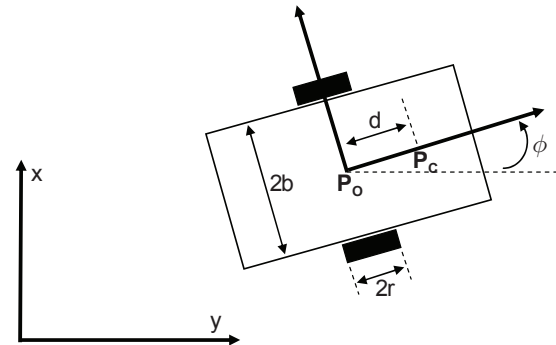


Figure 1. Two-Wheeled Unicycle Mobile Robot

with:

$$\tilde{M} = S^T M S$$

$$\tilde{V} = S^T (M\dot{S} + V S)$$

$$\tilde{B} = S^T B$$

as corresponding matrices.

One can reorganize then such that:

$$\dot{\nu} = -M^{-1}(q)V(q, \dot{q})\nu + M^{-1}(q)B(q)\tau$$

Without loss of generality, one can drop the bar notation to obtain:

$$\dot{q} = S(q)\nu$$

$$M(q)\dot{\nu} + V(q, \dot{q})\nu = B(q)\tau \quad (3)$$

The previous equations give us the following expression: (One can see also Refs. [2, 3])

$$\nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_R \\ \dot{\theta}_L \end{bmatrix} \quad (4)$$

To simplify even more, an auxiliary input u can be formed through feedback linearization as:

$$\tau = M(q)u + V(q, \dot{q})\nu$$

The input law in (4) when substituted in (3) will give us system equations in the form of [16-19]:

$$\dot{q} = S(q)\nu$$

$$\dot{\nu} = u \quad (5)$$

It is known that for posture stabilization [21-27], Brockett conditions [28] should be satisfied. So a conventional smooth feedback control scheme cannot solve for stabilization. The dynamic model of this system consists of both the kinematic relation and the decoupled "2nd-order inputs", i.e., the acceleration inputs. Furthermore, the wheel velocity inputs of the kinematic relation can be transformed into unicycle velocity inputs of linear velocity v and angular velocity ω . The transformation is a one-to-one relation that can be applied interchangeably. With this relation, one can have the same system dynamics with only a geometric change in inputs. Therefore, a complete kinematic model can be described as:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos \phi & 0 \\ \sin \phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (7)$$

One also can view the above model in this form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = g_1(p)v + g_2(p)\omega = \begin{bmatrix} \cos \phi \\ \sin \phi \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega \quad (8)$$

2.2 Polar coordinates based model

The control design with polar coordinate mapping is considered here. It is carried out to fulfil Brockett conditions and to provide natural motion solutions, as will be shown. Also, polar solution provides continuous smooth behaviour throughout the whole process except at the origin (i.e., the target). The control law of polar coordinates has a singular point in the origin [15]. However, this should not be a problem when adopting the method, as it is enough to approach only the region very near to the target.

For the posture vector of $\nu [x \ y \ \phi]^T$, the polar coordinates transformation would produce new variables in the polar space. The transformation would be executed as:

$$\begin{aligned} \rho &= \sqrt{x^2 + y^2} \\ \gamma &= \text{atan2}(y, x) - \phi + \pi \\ \delta &= \gamma + \phi \end{aligned} \quad (9)$$

We can see that ρ resembles the distance from the origin of the environment frame to the origin of the robot frame, γ is the angle that the robot's frame makes with the environment (fixed) frame and δ is the rotation angle of

the robot body inside its frame. The $\text{atan2}(\cdot, \cdot)$ function, which stands for the four-quadrant arctangent, is defined as:

$$\text{atan2}(y, x) = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right), & x > 0 \\ \pi + \tan^{-1}\left(\frac{y}{x}\right), & y \geq 0, x < 0 \\ -\pi + \tan^{-1}\left(\frac{y}{x}\right), & y < 0, x < 0 \\ \frac{\pi}{2}, & y > 0, x = 0 \\ -\frac{\pi}{2}, & y < 0, x = 0 \\ \text{undefined}, & y = 0, x = 0 \end{cases} \quad (10)$$

With the new variables in the polar space, a control law is derived to stabilize the system to the target with v, ω as the system input. First, let the error vector be:

$$\hat{e}(t) = \begin{bmatrix} x(t) - x_F \\ y(t) - y_F \\ \phi(t) - \phi_F \end{bmatrix} \quad (11)$$

Now, let the error vector be transformed into the target frame as:

$$e(t) = \begin{bmatrix} e_x(t) \\ e_y(t) \\ e_\phi(t) \end{bmatrix} = \begin{bmatrix} \cos \phi_F & \sin \phi_F & 0 \\ -\sin \phi_F & \cos \phi_F & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{e}(t) \quad (12)$$

One can have the polar-coordinate error transformation as:

$$\begin{aligned} \rho &= \sqrt{e_x^2 + e_y^2} \\ \gamma &= \text{atan2}(e_y, e_x) - e_\phi + \pi \\ \delta &= \gamma + e_\phi \end{aligned} \quad (13)$$

With the defined error variables e_y, e_x, e_ϕ , a feedback law is set to drive the system to the origin, i.e., to the target p_F . A Lyapunov-based stability analysis is utilized to come up with a feedback law for the linear and angular velocities in the form of:

$$\begin{aligned} v &= k_1 \rho \cos \gamma \\ \omega &= k_2 \gamma + k_1 \frac{\sin \gamma \cos \gamma}{\gamma} (\gamma + k_3 \sigma) \end{aligned} \quad (14)$$

With k_1, k_2, k_3 being design control gains and $k_1, k_2 > 0$. The control law stabilizes our original robot system. However, the main consequence is that the system will be

discontinuous at the target. For any initial and final postures, this control law will produce the stabilizing input trajectories along with the posture variables trajectories.

3. Control Law design

3.1 Acceleration inputs

Until now, we have only considered the kinematic solution for posture stabilization. However, the dynamic model of the robot includes inputs for linear and angular accelerations rather than velocities [29-30]. To design the acceleration inputs, the robot's linear and angular velocities must become extra system states. The complete system would then be as:

$$\begin{aligned}\dot{x} &= v \cos \phi \\ \dot{y} &= v \sin \phi \\ \dot{\phi} &= \omega \\ \dot{v} &= a_v \\ \dot{\omega} &= a_\omega\end{aligned}\quad (15)$$

To stabilize the above system, but now with acceleration inputs rather than velocities, the problem does not need a special feedback design to compensate for the new 2nd-order introduced dynamics. A solution for this is to have the new inputs computed as:

$$\begin{bmatrix} a_v \\ a_\omega \end{bmatrix} = \mathbf{k}_p \begin{bmatrix} v_d - v \\ \omega_d - \omega \end{bmatrix} + \begin{bmatrix} \dot{v}_d \\ \dot{\omega}_d \end{bmatrix}\quad (16)$$

With v, ω now as system feed-back states rather than inputs and the desired velocities v_d, ω_d computed as if they are the inputs, namely as: $v_d = k_1 \rho \cos \gamma$

$$\omega_d = k_2 \gamma + k_1 \frac{\sin \gamma \cos \gamma}{\gamma} (\gamma + k_3 \sigma)\quad (17)$$

Also, \mathbf{k}_p is a gain matrix. One can see from the above structure that the acceleration inputs try to 'track' the desired velocities with a derivative effect.

3.2 Torque inputs

The ultimate goal of this study is to stabilize robot states trajectories and stabilize torques trajectories. So, with auxiliary acceleration inputs attained, a static relation is used to find the torques. The velocities of right and left wheel v_1, v_2 are given as:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}\quad (18)$$

So the auxiliary inputs can be designed as:

$$\mathbf{u} = \mathbf{k}_p \begin{bmatrix} v_{1d} - v_1 \\ v_{2d} - v_2 \end{bmatrix} + \begin{bmatrix} \dot{v}_{1d} \\ \dot{v}_{2d} \end{bmatrix}\quad (19)$$

with the desired velocities of v_{1d}, v_{2d} computed from original velocities of v_d, ω_d .

The torques trajectories are computed by:

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\mathbf{u} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}})\boldsymbol{\nu}\quad (20)$$

Therefore, a complete control scheme for the posture stabilization of a non-holonomic unicycle mobile robot to a target of $[x_F \ y_F \ \phi_F]^T$ would be:

$$\hat{\mathbf{e}}(t) = \begin{bmatrix} x(t) - x_F \\ y(t) - y_F \\ \phi(t) - \phi_F \end{bmatrix}$$

$$\begin{bmatrix} e_x(t) \\ e_y(t) \\ e_\phi(t) \end{bmatrix} = \begin{bmatrix} \cos \phi_F & \sin \phi_F & 0 \\ -\sin \phi_F & \cos \phi_F & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{\mathbf{e}}(t)$$

$$\begin{bmatrix} \rho \\ \gamma \\ \delta \end{bmatrix} = \begin{bmatrix} \sqrt{e_x^2 + e_y^2} \\ \text{atan2}(e_y, e_x) - e_\phi + \pi \\ \gamma + e_\phi \end{bmatrix}$$

$$\begin{bmatrix} v_d \\ \omega_d \end{bmatrix} = \begin{bmatrix} k_1 \rho \cos \gamma \\ k_2 \gamma + k_1 \frac{\sin \gamma \cos \gamma}{\gamma} (\gamma + k_3 \sigma) \end{bmatrix}$$

$$\begin{bmatrix} v_{1d} \\ v_{2d} \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} v_d \\ \omega_d \end{bmatrix}$$

$$\mathbf{u} = \mathbf{k}_p \begin{bmatrix} v_{1d} - v_1 \\ v_{2d} - v_2 \end{bmatrix} + \begin{bmatrix} \dot{v}_{1d} \\ \dot{v}_{2d} \end{bmatrix}\quad (21)$$

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\mathbf{u} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}})\boldsymbol{\nu}$$

The above feedback law will allow target attainment and generates the torque commands as an output.

4. Numerical Simulations

4.1 Simulation set up

The stabilization law is tested for several initial and final positions and orientations. The overall goal of this stage is to have complete wheel torque trajectories that drive the robot system from different initial positions and orientations to different final positions and orientations. Three simulation scenarios will be presented here. Consider an open 2D environment of $(x, y) \in [-10, 10] \times [-10, 10]$, which was selected for

convenience. The study was performed using the model of the wheeled robot e-puck, shown in Fig. 2. Each simulation case shows different initial and final configurations and will also provide important information about the behaviour of the system under different situations, especially when considering the orientation of the robot. All motions in this work are expected to start from rest. The controller gains are selected as:

$$k_1 = 0.1, k_2 = 0.3, k_3 = 1.5, \mathbf{k}_p = \text{diag}(2, 2)$$

4.2 Simulated scenarios

Three scenarios were considered:

Scenario 1 $p_0 = [0 \quad -5 \quad 0]^T \rightarrow p_F = [0 \quad 5 \quad \pi/2]^T$

This case is selected as the basic test for the control law. One can see that the initial position and orientation resemble a normal setting. The final position and orientation is selected with different orientation than the initial one in order to see the manoeuvre. The first plot of Fig. 3 is the planar view of the motion, with path shown by the dotted line and orientation evolution shown by solid lines tangent to the path.

The second plot shows the error trajectories for all the posture variables of x -, y -coordinates and orientation. The third plot shows the torques variations of both wheels. The torque effect is shown for a shorter time than the motion's time, as the torque effect ends early.

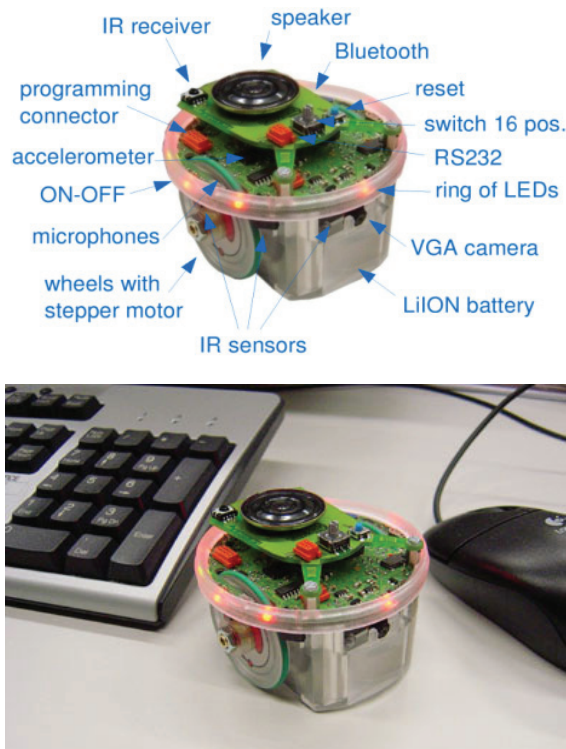


Figure 2. Wheeled mobile robot e-puck (Courtesy by <http://mobots.epfl.ch/e-puck.html>)

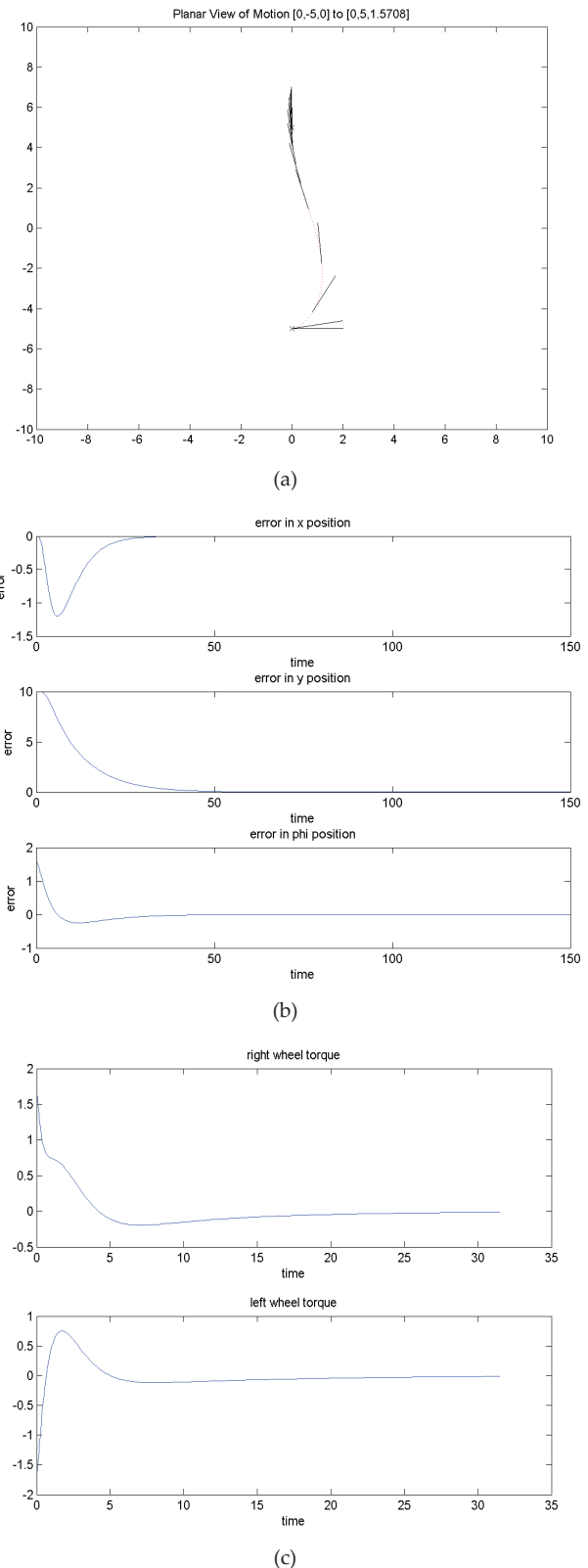
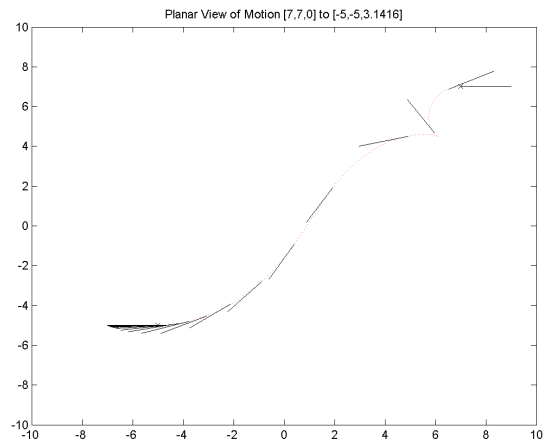


Figure 3 Simulated results for the first scenario, (a) (x, y) motion, (b) Error on (x, y, ϕ) variations, (c) torques variations

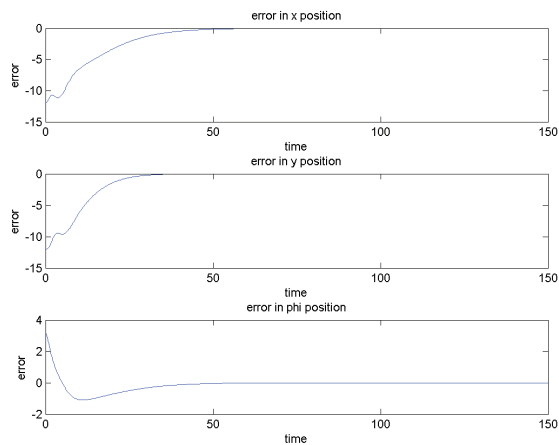
Scenario 2 $p_0 = [7 \quad 7 \quad 0]^T \rightarrow p_F = [-5 \quad -5 \quad \pi]^T$

This case includes a backward manoeuvre carried out by the robot and instructed by the control law (Fig. 4).

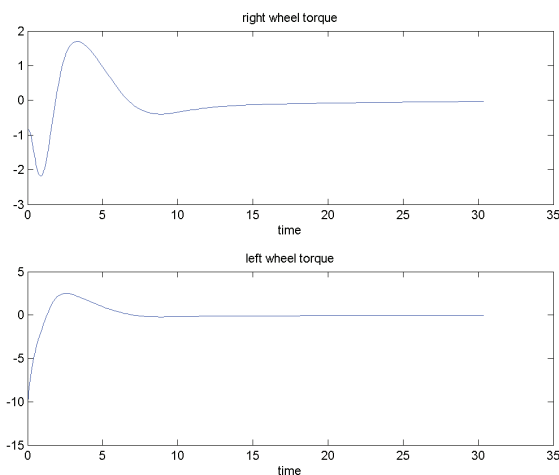
Further comments are mentioned at the end of this section.



(a)



(b)



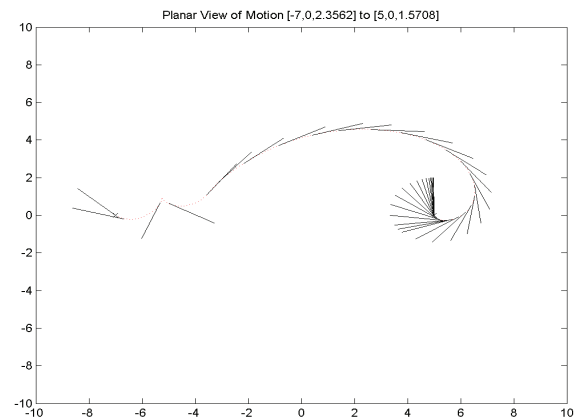
(c)

Figure 4 Simulated results for the second scenario (a) (x, y) motion, (b) Error on (x, y, ϕ) variations, (c) torques variations

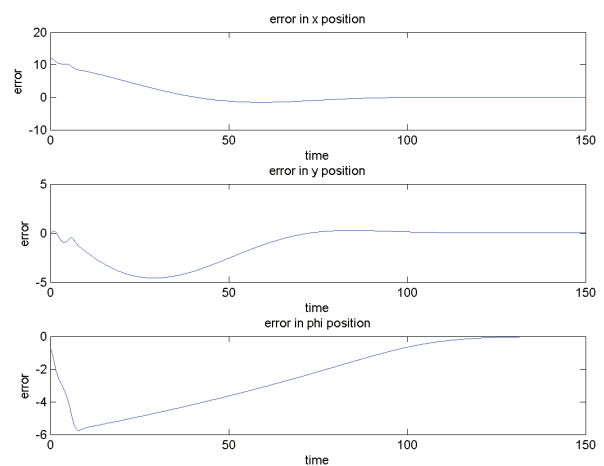
Scenario 3)

$$P_0 = \begin{bmatrix} -7 & 0 & 3\pi/4 \end{bmatrix}^T \rightarrow P_F = \begin{bmatrix} 5 & 0 & \pi/2 \end{bmatrix}^T$$

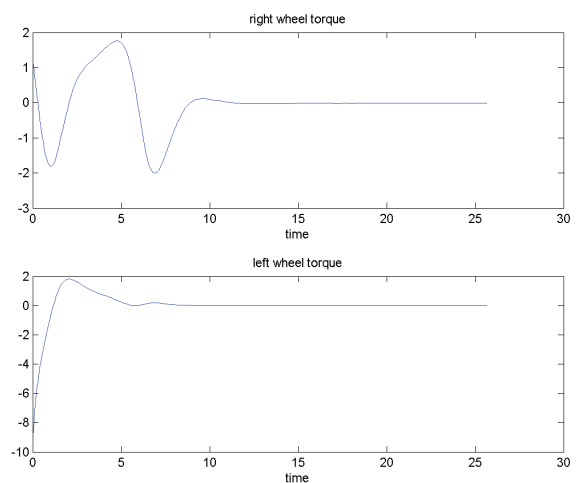
This case includes a rather harder manoeuvre (Fig. 5), as is obvious from the first plot. Also, one can observe the sharp turn carried out at the beginning of the motion.



(a)



(b)



(c)

Figure 5 Simulated results for the third scenario, (a) (x, y) motion, (b) Error on (x, y, ϕ) variations, (c) torques variations

4.3 Discussion

- The feedback controller based on polar coordinates gives good and natural results.
- The discontinuity at the origin (target) did not limit the solution performance.
- Most of the energy is spent by the wheels in a relatively small time. This should be overcome in further improvements to optimize the process and distribute the energy throughout the motion. This task is being processed as part of on-going work. Part of these problems was treated in the context of motion planning and offline programming in [31-32].

5. Conclusions

In conclusion, this paper presents a control law for a unicycle non-holonomic mobile robot. The control law stabilizes the robot with satisfactory results. The constraints on the input torques are not enforced at this stage. That is why relatively high initial torque values can be seen. This is expected as the feedback law principle dictates it. The solutions are a good initialization of the work, in terms of objective satisfaction, namely non-holonomic constraints, reaching a target, the apparent smoothness of the trajectories and the easy reproduction of any settings. The future target is to generalize this to a multi-objective system, involving energy and time minimization, limitations on input torques and obstacle avoidance.

6. Acknowledgements

The authors gratefully thank King Fahd University of Petroleum and Minerals for supporting this paper under grant #FT100029.

7. References

- [1] P. Wierach, H. P. Monner, E. Breitbach (2005): "Adaptronics - Potentials and challenges", *VDI Berichte* 1899, p 1-28.
- [2] G. S. Sukhatme, M. J. Mataric (2002): "Robots: Intelligence, Versatility, Adaptivity", *Communications of the ACM*, vol. 45, no. 3, p 30-63.
- [3] S. J. Julier, H. F. Durrant-Whyte (2003): "On the Role of Process Models in Autonomous Land Vehicle Navigation Systems", *IEEE Trans. On Rob. & Automation*, vol. 19, no. 1, p 1-14.
- [4] L. Ljung (2009): "System Identification, Theory for the User", 3rd Edition, Prentice Hall PTR.
- [5] A. Khoukhi, H. M. Khalid (2012): "Hybrid Soft Computing for Leak detection and Classification", Submitted, *Computing and Informatics*.
- [6] L. E. Dubins, (1957): "On Curves of Minimal Length with a Constraint on Average Curvature and with Prescribed Initial and Terminal Positions and Tangents", *American Journal of Mathematics*, vol. 79, p 497-516.
- [7] J. A. Reeds and L. A. Shepp (1990): "Optimal Path for a Car that Goes Both Forward and Backward", *Jou. of Pacific Mathematics*, vol. 145, no. 2, p 367-393.
- [8] P. Soureire and J. P. Laumond (1996): "Shortest Path Synthesis for a Car-Like Robot", *IEEE Trans. Automatic Control*, vol. 41, no. 5, p 672-688.
- [9] A. Bacchi, G. Casalino, C. Santilli (1996): "Planning Shortest Bounded-Curvature Paths for a Class of Non-holonomic Vehicles among Obstacles", *J. Intelligent robotic Systems*, vol. 16, p 387-405.
- [10] Y. Kanayama, B. Hartman (1997): "Smooth Local Path Planning for Autonomous Vehicles", *Int'l Jou. of Robotics Research*, vol. 16, no. 3, p 263-284.
- [11] Z.M. Bi, S.Y.T. Lang (2007): "A Framework for CAD- and Sensor-Based Robotic Coating Automation", *IEEE Trans. Industrial Electronics*, vol. 3, no. 1, p 84 - 91.
- [12] J. S. Choi and B. K. Kim (2001): "Near Minimum Time Direct Voltage Control Algorithms for Wheeled Mobile Robots with Current and Voltage Constraints", *Robotica*, vol. 19, p 29-39.
- [13] S. Kim, F. C. Park (2008): "Fast Robot Motion Generation Using Principal Components: Framework and Algorithms", *IEEE Trans. Industrial Electronics*, vol. 55, no. 6, p 2506-2516.
- [14] K. Iagnemma, S. Dubowsky (2004): "Traction Control of Wheeled Robotic Vehicles in Rough Terrain with Application to Planetary Rovers", *The Int'l Jour. of Rob. Res.*, vol. 23, no. 10-11, p 1029-1040.
- [15] T. Takei, R. Imamura, S. Yuta (2009): "Baggage Transportation and Navigation by a Wheeled Inverted Pendulum Mobile Robot", *IEEE Trans. on Ind. Electronics*, vol. 56, no. 10, p 3985-3994.
- [16] A. M. Bloc, J. Baillieul, P. Crouch, & J. E. Marsden, (2003): "Nonholonomic mechanics and control". New York, Berlin, Heidelberg: Springer. Theory, p 181-191. Boston, MA: Birkhäuser.
- [17] A. Khoukhi, L. Baron, M. Balazinski (2008): "Projected Gradient Augmented Lagrangian to Constrained Multi-Objective Trajectory Planning of Redundant Robots", *Trans. of the Canadian Society of Mech. Eng. (CSME)*, vol. 31, no. 4, p 391-405.
- [18] A. Khoukhi, L. Baron, M. Balazinski (2008) "Constrained Multi-Objective Trajectory Planning of Parallel Kinematic Machines", *Robotics and Computer Integrated Manufacturing*, vol. 25, no. 4-5, p 756-769 DOI:10.1016/j.rcim.2008.09.002.
- [19] A. Khoukhi (2010): "Data-Driven Multi-Objective Motion Planning of Parallel Kinematic Machines", *IEEE Trans. On Control System Technology*, DOI: 10.1109/TCST.2009.2036600.
- [20] O. Castillo, R. Martinez-Marroquin, P. Melin, F. Valdez, J. Soria (2012): "Comparative study of bio-inspired algorithms applied to the optimization of type-1 and type-2 fuzzy controllers for an autonomous mobile robot", *Inf. Sci.*, vol. 192, p 19-38.

- [21] O. Montel Ross, J. Camacho, R. Sepúlveda, O. Castillo (2011): "Fuzzy System to Control the Movement of a Wheeled Mobile Robot". *Sof Comp. for Intelligent Control and Mobile Robotics*, p 445-463.
- [22] R. Martinez-Soto, A. Rodríguez Díaz, O. Castillo, L. T. Aguilar (2010): "Type-2 Fuzzy Logic Controllers Optimization Using Genetic Algorithms and Particle Swarm Optimization". *GrC*, p 724-727
- [23] R. Martinez-Soto, O. Castillo, L. T. Aguilar (2009): "Optimization of interval type-2 fuzzy logic controllers for a perturbed autonomous wheeled mobile robot using genetic algorithms", *Inf. Sci.*, vol. 179, no. 13, p 2158-2174
- [24] L. Astudillo, O. Castillo, L. T. Aguilar, R. Martinez-Soto (2007): "Hybrid Control for an Autonomous Wheeled Mobile Robot Under Perturbed Torques", *IFSA*, vol. 1, p 594-603.
- [25] A. Khoukhi, M. Balazinski, L. Baron (2007): "Fuzzy Parking Manoeuvres of Vehicle Mobile Robots", *NAFIPS'2007*, San Diego, CA, USA, p 60-65, June 24-27, Best Paper Award.
- [26] A. Khoukhi (2010): "Stochastic Parameters Identification and Localization of Mobile Robots", *IEEE Int'l Workshop on Robotic and Sensors Environments ROSE10* Phoenix, Arizona, USA 15-16 October (2010).
- [27] A. Khoukhi (2010): "An Intelligent Multi-Agent System for Mobile Robots Navigation and Parking", *IEEE Int'l Workshop on Robotic and Sensors Environments ROSE10*, Phoenix, Arizona, USA 15-16 October 2010.
- [28] R. W. Brockett, (1983): "Asymptotic stability and feedback stabilization", In R. W. Brockett, R. S. Millmann, & H. J. Sussmann (Eds.), *Differential geometric control theory*, p 181-191. Boston, MA: Birkhäuser.
- [29] G. Oriolo; A. De Luca, M. Vendittelli (2002) "WMR control via dynamic feedback linearization: design, implementation and experimental validation," *Control Systems Technology, IEEE Transactions on*, vol. 10, no. 6, p 835- 852.
- [30] N. Reyhanoglu (1996): "Discontinuous feedback stabilization of the angular velocity of a rigid body with two control torques," *Decision and Control, Proceedings of the 35th IEEE* , vol. 3, p 2692-2694.
- [31] A. Khoukhi (2002): "Dynamic Modelling and Optimal Time-Energy Off-Line Programming for Mobile Robots, A Cybernetic Problem", *Kybernetes*, vol. 31, no. 5-6, p 731-733.
- [32] I. Kolmanovsky and N. H. McClamroch (1995): "Developments in nonholonomic control problems," *IEEE Control Syst. Mag.*, p 20-36.