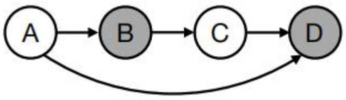
VE 492 Homework8

Due: 23:59, July. 15th

Q1. Bayes Nets: Sampling

Consider the following Bayes Net, where we have observed that B = +b and D = +d.



$P(A)$	
$+a$	0.5
$-a$	0.5

$P(B\|A)$		
$+a$	$+b$	0.8
$+a$	$-b$	0.2
$-a$	$+b$	0.4
$-a$	$-b$	0.6

$P(C\|B)$		
$+b$	$+c$	0.1
$+b$	$-c$	0.9
$-b$	$+c$	0.7
$-b$	$-c$	0.3

$P(D\|A,C)$			
$+a$	$+c$	$+d$	0.6
$+a$	$+c$	$-d$	0.4
$+a$	$-c$	$+d$	0.1
$+a$	$-c$	$-d$	0.9
$-a$	$+c$	$+d$	0.2
$-a$	$+c$	$-d$	0.8
$-a$	$-c$	$+d$	0.5
$-a$	$-c$	$-d$	0.5

a) Consider doing Gibbs sampling for this example. Assume that we have initialized all variables to the values +a, +b, +c, +d. We then unassign the variable C, such that we have A = +a, B = +b, C = ?, D = +d.
Calculate the probabilities for new values of C at this stage of the Gibbs sampling procedure.

P(C = +c at the next step of Gibbs sampling) = 0.4

P(C =- c at the next step of Gibbs sampling) = 0.6

b) Consider a sampling scheme that is a hybrid of rejection sampling and likelihood-weighted sampling. Under this scheme, we first perform rejection sampling for the variables A and B. We then take the sampled values for A and B and extend the sample to include values for variables C and D, using likelihood-weighted sampling.

(i) Below is a list of candidate samples. Mark the samples that would be rejected by the rejection sampling portion of the hybrid scheme.

A. -a, -b

B. +a, +b

C. +a, -b

D. -a, +b

(ii) To decouple from part (i), you now receive a new set of samples shown below. Fill in the weights for these samples under our hybrid scheme.

	weight
-a,+b,-c,+d	0.5
+a,+b,-c,+d	0.1
-a,+b,+c,+d	0.2
+a,+b,+c,+d	0.6

(iii) Use the weighted samples from part (ii) to calculate an estimate for P(+a| + b, +d).

The estimate of P(+a| + b, +d) =? 0.5

c) We now attempt to design an alternative hybrid sampling scheme that combines elements of likelihood-weighted and rejection sampling. For each proposed scheme, indicate whether it is valid, i.e. whether the weighted samples it produces correctly approximate the distribution P(A, C| + b, +d).

i) First collect a likelihood-weighted sample for the variables A and B. Then switch to rejection sampling for the variables C and D. In case of rejection, the values of A and B and the sample weight are thrown away. Sampling then restarts from node A.

$\boxed{\text{A. Valid}}$

B. Invalid

ii) First collect a likelihood-weighted sample for the variables A and B. Then switch to rejection sampling for the variables C and D. In case of rejection, the values of A and B and the sample weight are retained. Sampling then restarts from node C.

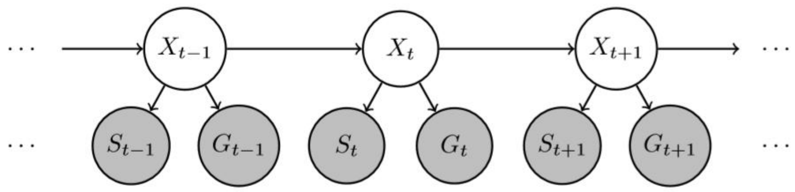
A. Valid

$\boxed{\text{B. Invalid}}$

Q2. HMM: Where is the Car?

Transportation researchers are trying to improve traffic in the city but, in order to do that, they first need to estimate the location of each of the cars in the city. They need our help to model this problem as an inference problem of an HMM. For this question, assume that only *one* car is being modeled.

a) The structure of this modified HMM is given below, which includes X, the location of the car; S, the noisy location of the car from the signal strength at a nearby cell phone tower; and G, the noisy location of the car from GPS.



We want to perform filtering with this HMM. That is, we want to compute the belief $P(x_t|s_{1:t}, g_{1:t})$, the probability of a state x_t given all past and current observations.

The **dynamics update** expression has the following form:

$P(x_t|s_{1:t-1}, g_{1:t-1}) =$ ___(i)___ ___(ii)___ ___(iii)___ $P(x_{t-1}|s_{1:t-1}, g_{1:t-1})$

Complete the expression by choosing the option that fills in each blank.

(i) A. $P(s_{1:t}, g_{1:t})$ B. $P(s_{1:t-1}, g_{1:t-1})$ C. $P(s_{1:t-1})P(g_{1:t-1})$ D. $P(s_{1:t})P(g_{1:t})$ $\boxed{\text{E. } 1}$

(ii) A. Σ_{x_t} $\boxed{\text{B. } \Sigma_{x_t-1}}$ C. $\max_{x_{t-1}}$ D. $\max_{x_t}$ E. 1

(iii) A. $P(x_{t-1}|x_{t-2})$ B. $P(x_{t-2}, x_{t-1})$ C. $P(x_{t-1}, x_t)$ $\boxed{\text{D. } P(x_t|x_{t-1})}$ E. 1

The **observation update** expression has the following form:

$P(x_t|s_{1:t}, g_{1:t}) =$ ___(iv)___ ___(v)___ ___(vi)___ $P(x_t|s_{1:t-1}, g_{1:t-1})$

Complete the expression by choosing the option that fills in each blank.

(iv) A. $P(s_{1:t-1}|s_t)P(g_{1:t-1}|g_t)$ $\boxed{\text{B. } \frac{1}{P(s_t, g_t|s_{1:t-1}, g_{1:t-1})}}$ C. $\frac{1}{P(s_{1:t-1}, g_{1:t-1}|s_t, g_t)}$

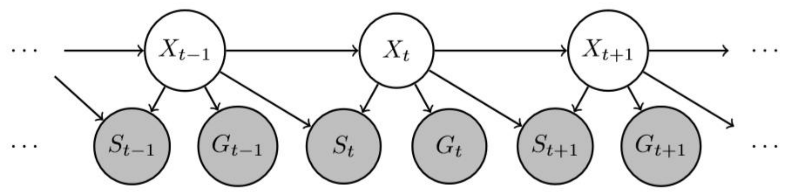
D. $P(s_t, g_t|s_{1:t-1}, g_{1:t-1})$ E. $P(s_{1:t-1}, g_{1:t-1}|s_t, g_t)$ F. $P(s_t|s_{1:t-1})P(g_t|g_{1:t-1})$

G. $\frac{1}{P(s_t|s_{1:t-1})P(g_t|g_{1:t-1})}$ H. $\frac{1}{P(s_{1:t-1}|s_t)P(g_{1:t-1}|g_t)}$ I. 1

(v) A. Σ_{x_t} B. Σ_{x_t-1} C. $\max_{x_t}$ D. $\max_{x_{t-1}}$ $\boxed{\text{E. } 1}$

(vi) A. $P(x_{t-1},s_{t-1})P(x_{t-1},g_{t-1})$ B. $P(x_{t-1},s_{t-1},g_{t-1})$ C. $P(x_t|s_t)P(x_t|g_t)$
D. $P(s_{t-1}|x_{t-1})P(g_{t-1}|x_{t-1})$ E. $P(x_t,s_t)P(x_t,g_t)$ F. $P(x_t,s_t,g_t)$
G. $P(x_{t-1}|s_{t-1})P(x_{t-1}|g_{t-1})$ $\boxed{\text{H. } P(s_t|x_t)P(g_t|x_t)}$ I. 1

b) It turns out that if the car moves too fast, the quality of the cell phone signal decreases. Thus, the signal dependent location S_t not only depends on the current state X_t but it also depends on the previous state X_{t-1}. Thus, we modify our original HMM for a new more accurate one, which is given below.



Again, we want to compute the belief $P(x_t|s_{1:t},g_{1:t})$. In this part we consider an update that combines the dynamics and observation update in a *single* update.

$P(x_t|s_{1:t},g_{1:t}) =$ __(i)__ __(ii)__ __(iii)__ __(iv)__ $P(x_{t-1}|s_{1:t-1},g_{1:t-1})$

Complete the **forward update** expression by choosing the option that fills in each blank.

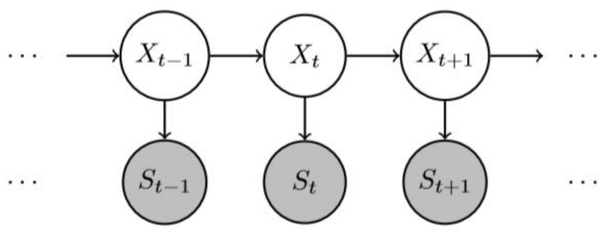
(i) A. $P(s_{1:t-1},g_{1:t-1}|s_t,g_t)$ B. $P(s_t,g_t|s_{1:t-1},g_{1:t-1})$ C. $P(s_t|s_{1:t-1})P(g_t|g_{1:t-1})$
$\boxed{\text{D. } \frac{1}{P(s_t,g_t|s_{1:t-1},g_{1:t-1})}}$ E. $\frac{1}{P(s_{1:t-1},g_{1:t-1}|s_t,g_t)}$ F. $P(s_{1:t-1}|s_t)P(g_{1:t-1}|g_t)$
G. $\frac{1}{P(s_t|s_{1:t-1})P(g_t|g_{1:t-1})}$ H. $\frac{1}{P(s_{1:t-1}|s_t)P(g_{1:t-1}|g_t)}$ I. 1

(ii) A. $\max_{x_{t-1}}$ B. $\max_{x_t}$ $\boxed{\text{C. } \Sigma_{x_{t-1}}}$ D. Σ_{x_t} E. 1

(iii) A. $P(s_{t-1}|x_{t-2},x_{t-1})P(g_{t-1}|x_{t-1})$ $\boxed{\text{B. } P(s_t|x_{t-1},x_t)P(g_t|x_t)}$ C. $P(s_t,g_t|x_t)$
D. $P(x_{t-2},x_{t-1},s_{t-1})P(x_{t-1},g_{t-1})$ E. $P(x_{t-1},x_t,s_t)P(x_t,g_t)$ F. $P(s_{t-1},g_{t-1}|x_{t-1})$
G. $P(x_{t-2},x_{t-1}|s_{t-1})P(x_{t-1}|g_{t-1})$ H. $P(x_{t-1},x_t|s_t)P(x_t|g_t)$ I. 1
J. $P(x_{t-2},x_{t-1},s_{t-1},g_{t-1})$ K. $P(x_{t-1},x_t,s_t,g_t)$

(iv) A. $P(x_{t-1},x_t)$ $\boxed{\text{B. } P(x_t|x_{t-1})}$ C. $P(x_{t-2},x_{t-1})$ D. $P(x_{t-1}|x_{t-2})$ E. 1

c) The Viterbi algorithm finds the most probable sequence of hidden states $X_{1:T}$, given a sequence of observations $s_{1:T}$, for some time $t = T$. Recall the canonical HMM structure, which is shown below.



For this canonical HMM, the Viterbi algorithm performs the following dynamic programming computations:

$$m_t[x_t] = P(s_t|x_t) \max_{x_{(t-1)}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}]$$

We consider extending the Viterbi algorithm for the modified HMM from part **(b)**. We want to find the most likely sequence of states $X_{1:T}$ given the sequence of observations $s_{1:T}$ and $g_{1:T}$. The dynamic programming update for $t > 1$ for the modified HMM has the following form:

$m_t[x_t] =$ __(i)__ __(ii)__ __(iii)__ $m_{t-1}[x_{t-1}]$

Complete the expression by choosing the option that fills in each blank.

(i) A. $\Sigma_{x_{t-1}}$ B. Σ_{x_t} C. $\max_{x_t}$ $\boxed{\text{D. } \max_{x_{t-1}}}$ E. 1

(ii) A. $P(s_{t-1}|x_{t-2},x_{t-1})P(g_{t-1}|x_{t-1})$ $\boxed{\text{B. } P(s_t|x_{t-1},x_t)P(g_t|x_t)}$ C. $P(s_t,g_t|x_t)$
D. $P(x_{t-2},x_{t-1},s_{t-1})P(x_{t-1},g_{t-1})$ E. $P(x_{t-1},x_t,s_t)P(x_t,g_t)$ F. $P(s_{t-1},g_{t-1}|x_{t-1})$
G. $P(x_{t-2},x_{t-1}|s_{t-1})P(x_{t-1}|g_{t-1})$ H. $P(x_{t-1},x_t|s_t)P(x_t|g_t)$ I. 1
J. $P(x_{t-2},x_{t-1},s_{t-1},g_{t-1})$ K. $P(x_{t-1},x_t,s_t,g_t)$

(iii) A. $P(x_{t-1},x_t)$ $\boxed{\text{B. } P(x_t|x_{t-1})}$ C. $P(x_{t-2},x_{t-1})$ D. $P(x_{t-1}|x_{t-2})$ E. 1