Homework 10 Written

July 31st, 2020 at 11:59pm

1 Propositional Logic 1

A logician tells to his son: "If you don't finish your dinner, you will not play video games afterwards." After the son finishes his meal, he is sent to bed right away.

Which mistake did he make by thinking that he would be able to play video games after dinner?

Solution: Denote the event 'finish dinner' as D, and 'play video games' as G. The logician's words could then be expressed as $\neg D \Rightarrow \neg G$, whose contrapositive is $G \Rightarrow D$. Being able to play video games after dinner is $D \Rightarrow G$ in logic language. The son treats $G \Rightarrow D$ and $D \Rightarrow G$ as a logical equivalence, which is incorrect.

2 Propositional Logic 2

Write the following sentences in CNF form.

a. $\neg(p \vee (q \wedge r))$

b. $(\neg p \Rightarrow q) \vee \neg(q \wedge r)$

c. $(p \Rightarrow \neg q) \Leftrightarrow ((q \wedge \neg r) \Rightarrow (\neg p))$

Solution:

a. $\neg(p \vee (q \wedge r)) \equiv \neg p \wedge \neg(q \wedge r) \equiv \neg p \wedge (\neg q \vee \neg r)$

b. $(\neg p \Rightarrow q) \vee \neg(q \wedge r) \equiv p \vee q \vee \neg q \vee \neg r$

c. $(p \Rightarrow \neg q) \Leftrightarrow ((q \wedge \neg r) \Rightarrow (\neg p)) \equiv (\neg p \vee \neg q) \Leftrightarrow (\neg(q \wedge \neg r) \vee \neg p) \equiv (\neg p \vee \neg q) \Leftrightarrow (\neg q \vee r \vee \neg p)$
$\equiv ((\neg p \vee \neg q) \Rightarrow (\neg q \vee r \vee \neg p)) \wedge ((\neg q \vee r \vee \neg p) \Rightarrow (\neg p \vee \neg q))$
$\equiv (\neg(\neg p \vee \neg q) \vee (\neg q \vee r \vee \neg p)) \wedge (\neg(\neg q \vee r \vee \neg p) \vee (\neg p \vee \neg q))$
$\equiv ((p \wedge q) \vee (\neg q \vee r \vee \neg p)) \wedge ((q \wedge \neg r \wedge p) \vee (\neg p \vee \neg q))$
$\equiv (p \vee \neg q \vee r \vee \neg p) \wedge (q \vee \neg q \vee r \vee \neg p) \wedge (q \vee \neg p \vee \neg q) \wedge (\neg r \vee \neg p \vee \neg q) \wedge (p \vee \neg p \vee \neg q)$

3 Propositional Logic 3

Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \vee C$.

b. $\neg A \vee \neg B \vee \neg C \vee \neg D$.

c. $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

Solution:

a. 12

b. 15

c. 0

For a, b, The truth table is shown below

A	B	C	D	$B \vee C$	$\neg A \vee \neg B \vee \neg C \vee \neg D$
T	T	T	T	T	F
T	T	T	F	T	T
T	T	F	T	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	T	F	T	T
T	F	F	T	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	T	F	T	T
F	T	F	T	T	T
F	T	F	F	T	T
F	F	T	T	T	T
F	F	T	F	T	T
F	F	F	T	F	T
F	F	F	F	F	T

For c, $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D \equiv (\neg A \vee B) \wedge A \wedge \neg B \wedge C \wedge D \equiv False$, so 0 model.

4 Propositional Logic 4

We have defined four binary logical connectives.

a. Are there any others that might be useful?

b. How many binary connectives can there be?

c. Why are some of them not very useful?

Solution:

a. Yes. For instance, xor ($\oplus$).

b. 16. For two propositions A, B, there could be $2^4 = 16$ binary combinations of $A, B, \neg A, \neg B$.

c. Some of them might not be useful, such as the combinations representing $A, \neg A, B, \neg B, True$, $False$. These binary combinations do not represent new logical relationships between the two propositions other than unary combinations, and are actually not influnced by one of the two propositions or both of the propositions.

5 Propositional Logic 5

The inference rule *Modus Tollens* is written as follows:

$$\frac{\neg q, p \Rightarrow q}{\neg p}$$

Prove that Modus Tollens is equivalent to Modus Ponens, i.e., the latter can be proved from the former, and the other around.

Solution:

1. Modus Tollens $\Rightarrow$ Modus Ponens:
   Modus Tollens is equivalent to $(\neg q \wedge (p \Rightarrow q)) \Rightarrow \neg p \equiv (\neg q \wedge (\neg p \vee q)) \Rightarrow \neg p$. Subsititute $a = \neg q,\ b = \neg p$ into the sentence, we then have $(a \wedge (b \vee \neg a)) \Rightarrow b \equiv (a \wedge (a \Rightarrow b)) \Rightarrow b$, which is just Modus Ponens.
2. Modus Ponens $\Rightarrow$ Modus Tollens:
   Modus Ponens, which is written as follows:

$$\frac{p, p \Rightarrow q}{q}$$

   is equivalent to $(p \wedge (p \Rightarrow q)) \Rightarrow q \equiv (p \wedge (\neg p \vee q)) \Rightarrow q$. Subsititute $a = \neg q,\ b = \neg p$ into the sentence, we then have $(\neg b \wedge (b \vee \neg a)) \Rightarrow \neg a \equiv (\neg b \wedge (a \Rightarrow b)) \Rightarrow \neg a$, which is just Modus Tollens.

6 First-Order Logic 1

Translate the following sentences in first-order logic:

a. Alice likes everything that Bob dislikes.
b. Bob doesn't like everything Alice likes.
c. Charles doesn't like anything Alice likes.
d. David likes anything everybody else dislikes.
e. I like writing sentences in first-order logic.
f. A parent of my sibling is my parent.
g. A child of my parent, who is not me, is my sibling.

Try to use a minimum number of predicates, functions, and constants.

Solution:

Meaning of functions:

- $Likes(x, y)$: Person x likes something y
- $Person(x)$: x is a person
- $FirstOrderLogicSentence(x)$: x is a sentence in first-order logic
- $ILikeWrite(x)$: x is a sentence I like writing
- $ParentAndChild(x, y)$: x is a parent of y, or equivalently, y is a child of x

a. $\forall x\ \neg Likes(Bob, x) \Rightarrow Likes(Alice, x)$

b. $\exists x\ Likes(Alice, x) \Rightarrow \neg Likes(Bob, x)$

c. $\forall x\ Likes(Alice, x) \Rightarrow \neg Likes(Charles, x)$

d. $\forall x, y\ Person(y) \wedge \neg(y = David) \wedge (\neg Likes(y, x)) \Rightarrow Likes(David, x)$

e. $\forall s\ FirstOrderLogicSentence(s) \Rightarrow ILikeWrite(s)$

f. $\forall x\ ParentAndChild(x, my\ sibling) \wedge ParentAndChild(x, I)$

g. $\forall c\ ParentAndChild(my\ parent, c) \wedge \neg(c = me) \Rightarrow c = my\ sibling$

7 First-Order Logic 2

Translate into good, natural English (no xs or ys):

$$\forall x, y, l\ SpeaksLanguage(x, l) \wedge SpeaksLanguage(y, l) \Rightarrow Understands(x, y).$$

Solution: All the people who speak the same language could understand each other.

8 First-Order Logic 3

Consider a first-order logical knowledge base that describes worlds containing people, songs, albums (e.g., ”Meet the Beatles”) and disks (i.e., particular physical instances of CDs), The vocabulary contains the following symbols:

- $CopyOf(d, a)$: Predicate. Disk d is a copy of album a.
- $Owns(p, d)$: Predicate. Person p owns disk d.
- $Sings(p, s, a)$: Album a includes a recording of song s sung by person p.
- $Wrote(p, s)$: Person p wrote song s.
- $McCartney$, $Gersharin$, $BHoliday$, Joe, $EleanorRigby$, $TheManILove$, $Revolver$: Constants with the obvious meanings.

Express the following statements in first-order logic:

a. Gershwin wrote ”The Man I Love.”

b. Gershwin did not write ”Eleanor Rigby.”

c. Either Gershwin or McCartney wrote ”The Man I Love.”

d. Joe has written at least one song.

e. Joe owns a copy of Revolver.

f. Every song that McCartney sings on Revolver was written by McCartney.

g. Gershwin did not write any of the songs on *Revolver*.

h. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)

i. There is a single album that contains every song that Joe has written.

j. Joe owns a copy of an album that has Billie Holiday singing ”The Man I Love.”

k. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)

l. Joe owns a copy of every album on which all the songs are sung by Billie Holiday.

Solution:

a. $Wrote(Gershwin, TheManILove)$
b. $\neg Wrote(Gershwin, EleanorRigby)$
c. $Wrote(Gershwin, TheManILove) \lor Wrote(McCartney, TheManILove)$
d. $\exists s\ Wrote(Joe, s)$
e. $\exists d\ CopyOf(d, Revolver) \land Owns(Joe, d)$
f. $\forall s\ Sings(McCartney, s, Revolver) \land Wrote(McCartney, s)$
g. $\forall s, p\ Sings(p, s, Revolver) \land \neg Wrote(Gershwin, s)$
h. $\forall s\ Wrote(Gershwin, s) \Rightarrow \exists a\ \forall p\ Sings(p, s, a)$
i. $\exists a\ \forall p, s\ Wrote(Joe, s) \land Sings(p, s, a)$
j. $\exists a, d\ CopyOf(d, a) \land Owns(Joe, d) \land Sings(BillieHoliday, TheManILove, a)$
k. $\forall a \exists s\ Sings(McCartney, s, a) \Rightarrow \exists d\ CopyOf(d, a) \land Owns(Joe, d)$
l. $\forall a \forall s\ Sings(BillieHoliday, s, a) \Rightarrow \exists d\ CopyOf(d, a) \land Owns(Joe, d)$