

# Homework 10 Electronic

July 31st, 2020 at 11:59pm

## 1 Propositional Logic 1

We ask a logician (who only tells the truth) about his sentimental life, and he answers the following two statements:

- I love Ann or I love Beth.
- If I love Ann, then I love Beth.

What can we conclude? Answer the following questions by "yes", "no", "unsure".

1. Does he love Ann?
2. Does he love Beth?
3. Does he love both?

**Sample Answer:**

no,no,no

## 2 Propositional Logic 2

Which of the following are correct?

- a.  $False \models True$ .
- b.  $True \models False$ .
- c.  $(A \wedge B) \models (A \Leftrightarrow B)$ .
- d.  $A \Leftrightarrow B \models A \vee B$ .
- e.  $A \Leftrightarrow B \models \neg A \vee B$ .
- f.  $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$ .
- g.  $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$ .
- h.  $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$ .
- i.  $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$ .
- j.  $(A \vee B) \wedge \neg(A \Rightarrow B)$  is satisfiable.
- k.  $(A \Leftrightarrow B) \wedge (\neg A \vee B)$  is satisfiable.
- l.  $(A \Leftrightarrow B) \Leftrightarrow C$  has the same number of models as  $A \Leftrightarrow B$  for any fixed set of proposition symbols that includes  $A, B, C$ .

**Sample Answer:**

a,b,c,d

### 3 Propositional Logic 3

We denote  $L_0$  the set of propositional logic sentences built from a set  $\mathcal{X}$  of  $n$  propositional symbols. we consider the following new formal languages, where some logical connectives are not allowed:

- $L_1$  is defined as follows:  
True and False are sentences of  $L_1$ . All symbols of  $\mathcal{X}$  are sentences of  $L_1$ . If  $s, s'$  are two sentences of  $L_1$ , then  $\neg s$ ,  $(s \wedge s')$ ,  $(s \vee s')$ , and  $(s \Rightarrow s')$  are four sentences of  $L_1$ .
- $L_2$  is defined as follows:  
True and False are sentences of  $L_2$ . All symbols of  $\mathcal{X}$  are sentences of  $L_2$ . If  $s, s'$  are two sentences of  $L_2$ , then  $\neg s$ ,  $(s \wedge s')$ , and  $(s \vee s')$  are three sentences of  $L_2$ .
- $L_3$  is defined as follows:  
True and False are sentences of  $L_3$ . All symbols of  $\mathcal{X}$  are sentences of  $L_3$ . If  $s, s'$  are two sentences of  $L_3$ , then  $\neg s$  and  $(s \wedge s')$  are two sentences of  $L_3$ .
- $L_4$  is defined as follows:  
True and False are sentences of  $L_4$ . All symbols of  $\mathcal{X}$  are sentences of  $L_4$ . If  $s$  are two sentences of  $L_4$ , then  $\neg s$  is a sentence of  $L_4$ .

We consider the following binary relation between languages:  $L \subseteq L'$  iff any sentences that can be expressed in  $L$  can equivalently be expressed in  $L'$ .

Answer "yes" or "no" the following questions.

1.  $L_1 \subseteq L_0$
2.  $L_2 \subseteq L_0$
3.  $L_3 \subseteq L_0$
4.  $L_4 \subseteq L_0$
5.  $L_0 \subseteq L_1$
6.  $L_0 \subseteq L_2$
7.  $L_0 \subseteq L_3$
8.  $L_0 \subseteq L_4$

**Sample Answer:**

no,no,no,no,no,no,no,no

### 4 First-Order Logic 1

Are the following are valid (necessarily true) sentences?

- a.  $(\exists x \ x = x) \Rightarrow (\forall y \exists z \ y = z)$ .
- b.  $\forall x \ P(x) \vee \neg P(x)$ .
- c.  $\forall x \ Smart(x) \vee (x = x)$ .

Answer "Valid" or "Invalid" the following questions.

**Sample Answer:**

Valid, Valid, Valid

## 5 First-Order Logic 2

This exercise uses the function *Map Color* and predicates *In*(*T*, *y*), *Borders*(*x*, *y*), and *Country*(*x*), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions.

- a. Paris and Marseilles are both in France.
  - (i)  $In(Paris \wedge Marseilles, France)$ .
  - (ii)  $In(Paris, France) \wedge In(Marseilles, France)$ .
  - (iii)  $In(Paris, France) \vee In(Marseilles, France)$ .
- b. There is a country that borders both Iraq and Pakistan.
  - (i)  $\exists c \text{ Country}(c) \wedge Border(c, Iraq) \wedge Border(c, Pakistan)$ .
  - (ii)  $\exists c \text{ Country}(c) \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$ .
  - (iii)  $[\exists c \text{ Country}(c)] \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$ .
  - (iv)  $\exists c Border(\text{Country}(c), Iraq \wedge Pakistan)$ .
- c. All countries that border Ecuador are in South America.
  - (i)  $\forall c \text{ Country}(c) \wedge Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$ .
  - (ii)  $\forall c \text{ Country}(c) \Rightarrow [Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)]$ .
  - (iii)  $\forall c [\text{Country}(c) \Rightarrow Border(c, Ecuador)] \Rightarrow In(c, SouthAmerica)$ .
  - (iv)  $\forall c \text{ Country}(c) \wedge Border(c, Ecuador) \wedge In(c, SouthAmerica)$ .
- d. No region in South America borders any region in Europe.
  - (i)  $\neg[\exists c, d In(c, SouthAmerica) \wedge In(d, Europe) \wedge Borders(c, d)]$ .
  - (ii)  $\forall c, d [In(c, SouthAmerica) \wedge In(d, Europe)] \Rightarrow \neg Borders(c, d)$ .
  - (iii)  $\neg \forall c In(c, SouthAmerica) \Rightarrow \exists d In(d, Europe) \wedge \neg Borders(c, d)$ .
  - (iv)  $\forall c In(c, SouthAmerica) \Rightarrow \forall d In(d, Europe) \Rightarrow \neg Borders(c, d)$ .
- e. No two adjacent countries have the same map color.
  - (i)  $\forall x, y \neg \text{Country}(x) \vee \neg \text{Country}(y) \vee \neg Borders(x, y) \vee \neg (\text{MapColor}(x) = \text{MapColor}(y))$ .
  - (ii)  $\forall x, y (\text{Country}(x) \wedge \text{Country}(y) \wedge Borders(x, y) \wedge \neg (x = y)) \Rightarrow \neg (\text{MapColor}(x) = \text{MapColor}(y))$ .
  - (iii)  $\forall x, y \text{ Country}(x) \wedge \text{Country}(y) \wedge Borders(x, y) \wedge \neg (\text{MapColor}(x) = \text{MapColor}(y))$ .
  - (iv)  $\forall x, y (\text{Country}(x) \wedge \text{Country}(y) \wedge Borders(x, y)) \Rightarrow \text{MapColor}(x \neq y)$ .

For each of the logical expressions, state whether it...

- 1 correctly expresses the English sentence;
- 2 is syntactically invalid and therefore meaningless;
- 3 is syntactically valid but does not express the meaning of the English sentence.

**Sample Answer:**

233

2333

1222

1333

3222