

# Sound Automation of Magic Wands

Thibault Dardinier<sup>1</sup>, Gaurav Parthasarathy<sup>1</sup>,  
Noé Weeks<sup>1</sup>, Peter Müller<sup>1</sup>, and Alexander J. Summers<sup>2</sup>

<sup>1</sup> **ETH** zürich



# Sound Automation of Magic Wands

Binary connective in separation logic

Thibault Dardinier<sup>1</sup>, Gaurav Parthasarathy<sup>1</sup>,  
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**Goal:** Support the magic wand connective in automatic verifiers based on separation logic

# Sound Automation of Magic Wands

Binary connective in separation logic

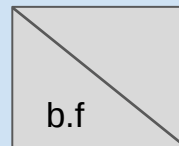
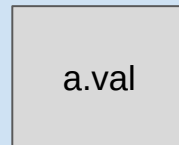
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# Separation logic: Heap-manipulating programs and ownership

## Memory



# Separation logic: Heap-manipulating programs and ownership

**Memory**

Exclusive ownership  
Permission to read and write *a.val*

a.val

b.f

# Separation logic: Heap-manipulating programs and ownership

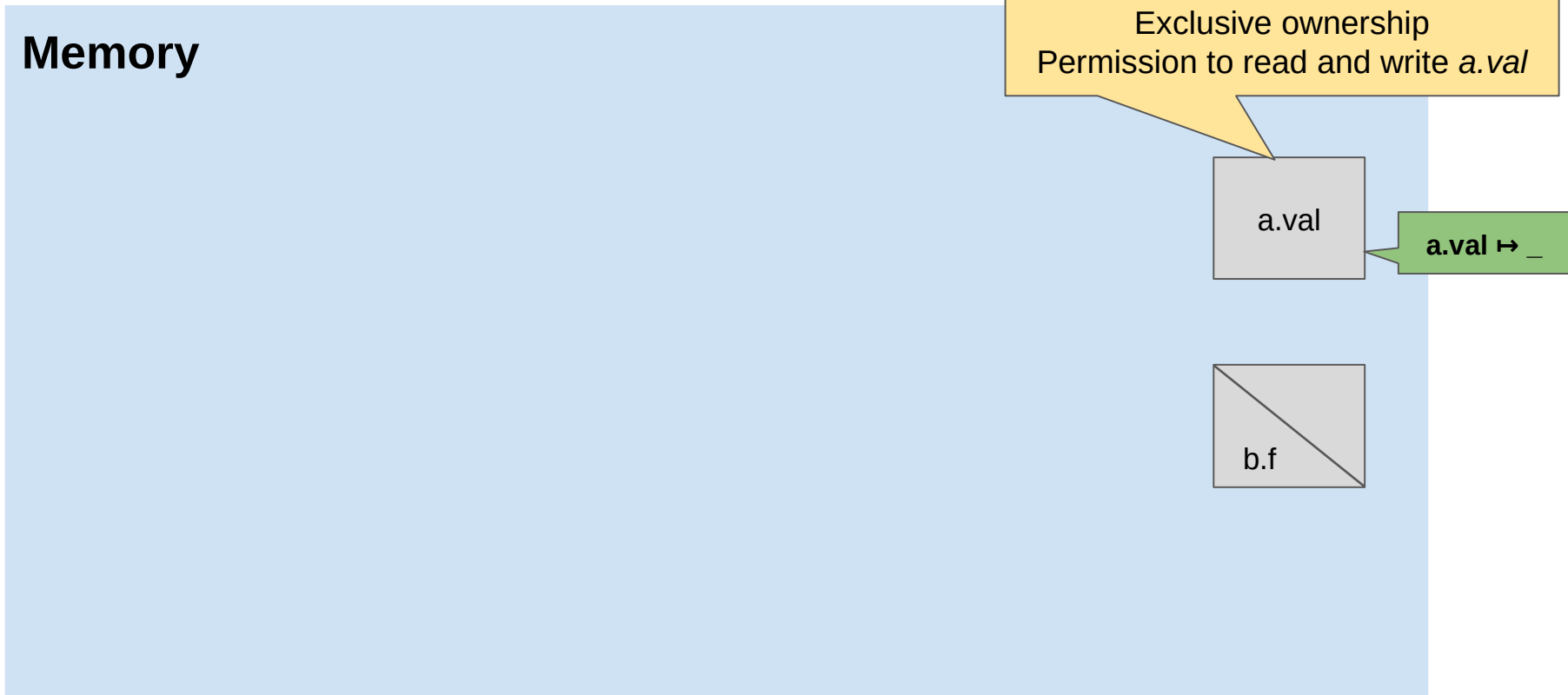
**Memory**

Exclusive ownership  
Permission to read and write *a.val*

a.val

$a.val \mapsto \_$

b.f



# Separation logic: Heap-manipulating programs and ownership

**Memory**

Exclusive ownership  
Permission to read and write  $a.val$

$a.val$

$acc(a.val)$

$a.val \mapsto \_$

$b.f$

# Separation logic: Heap-manipulating programs and ownership

**Memory**

Exclusive ownership  
Permission to read and write  $a.val$

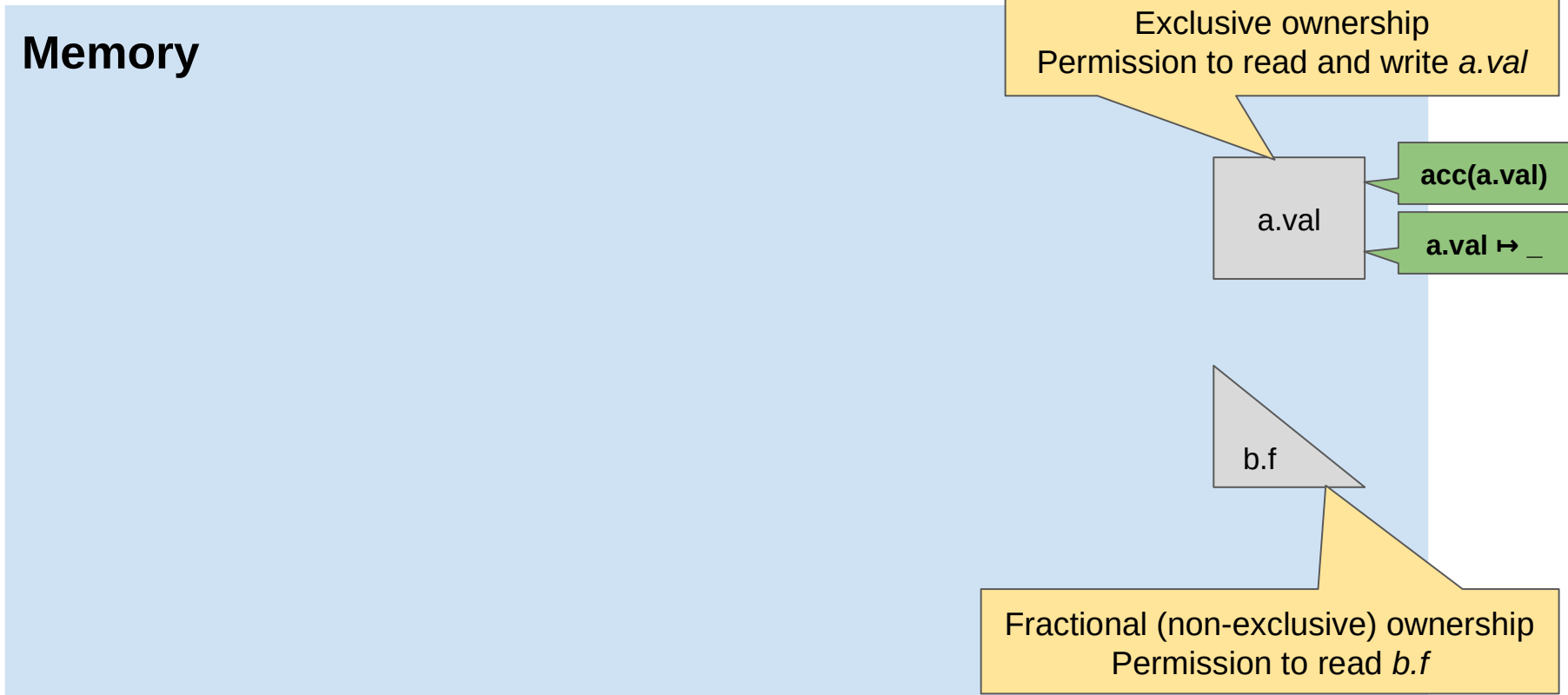
$a.val$

$acc(a.val)$

$a.val \mapsto \_$

$b.f$

Fractional (non-exclusive) ownership  
Permission to read  $b.f$



# Separation logic: Heap-manipulating programs and ownership

**Memory**

Exclusive ownership  
Permission to read and write  $a.val$

$a.val$

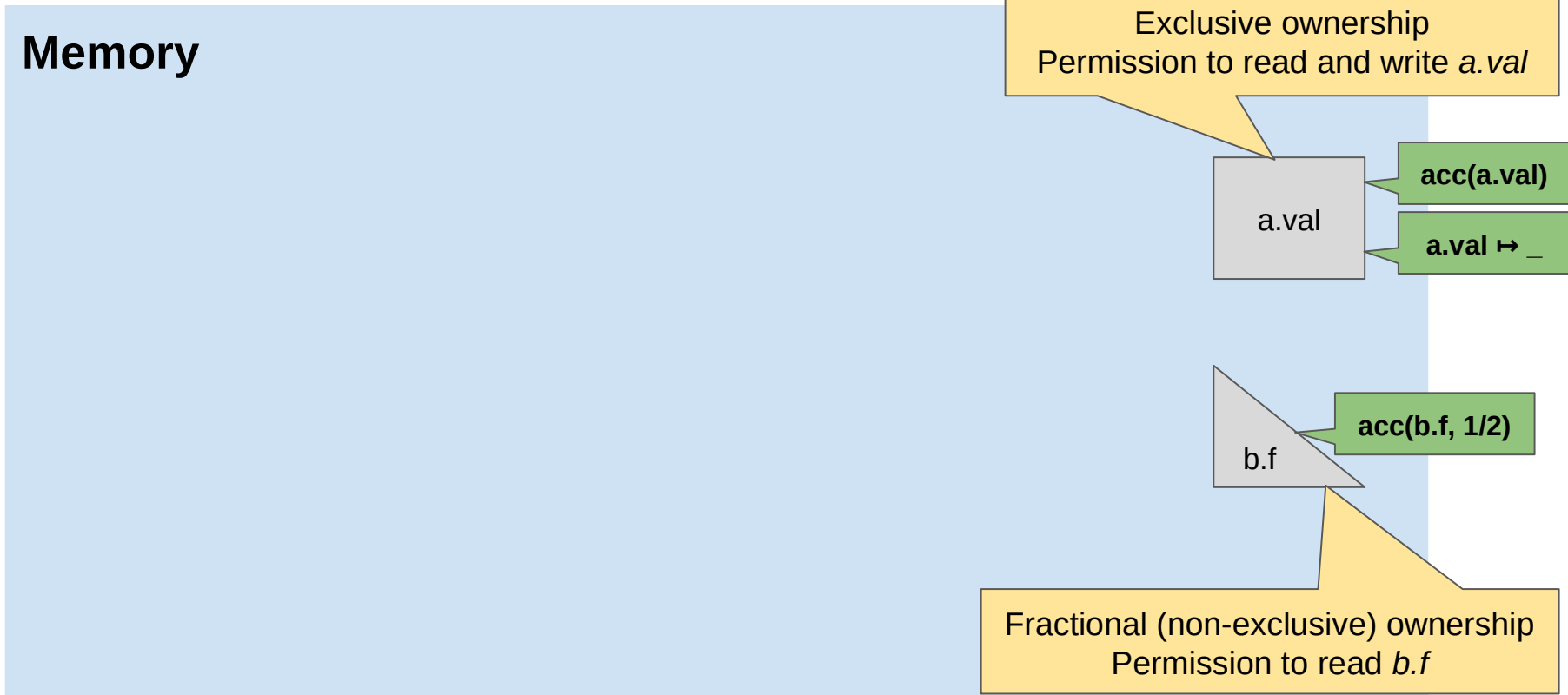
$acc(a.val)$

$a.val \mapsto \_$

$b.f$

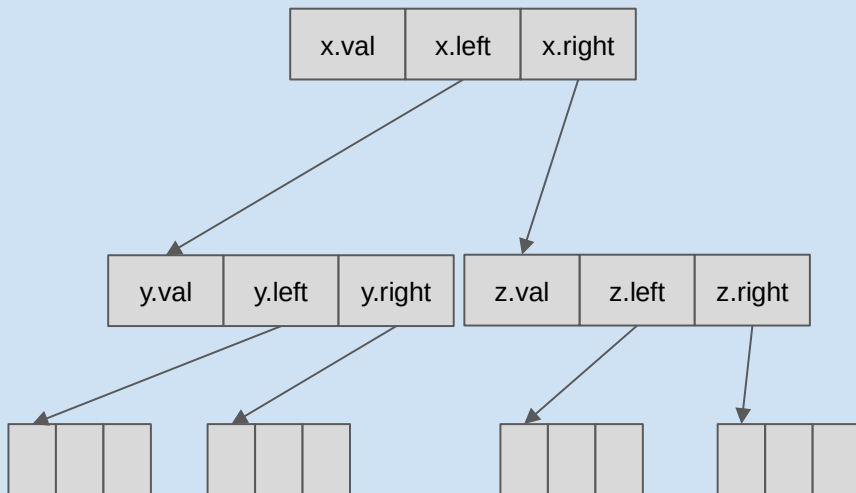
$acc(b.f, 1/2)$

Fractional (non-exclusive) ownership  
Permission to read  $b.f$

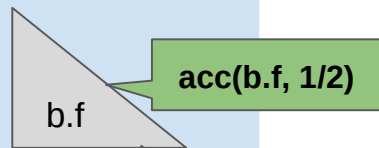
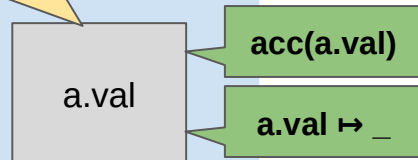


# Separation logic: Heap-manipulating programs and ownership

## Memory



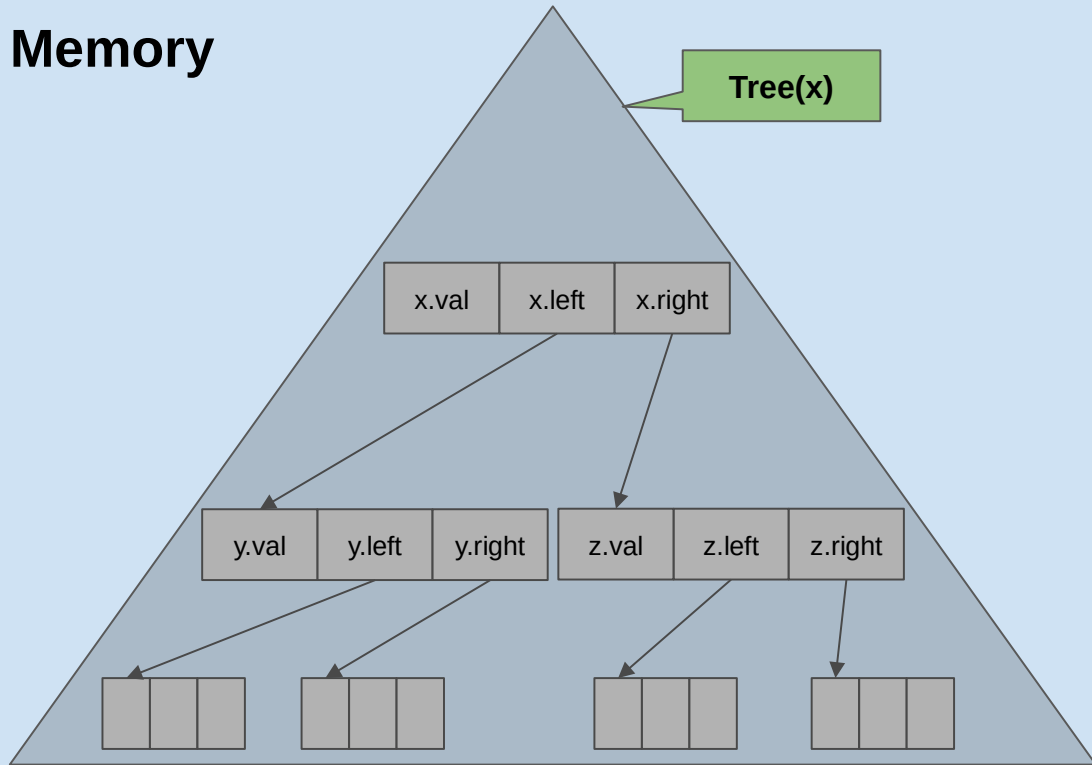
Exclusive ownership  
Permission to read and write *a.val*



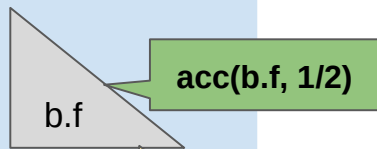
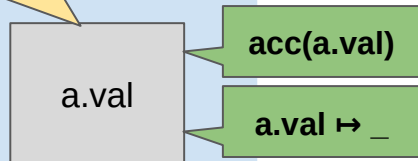
Fractional (non-exclusive) ownership  
Permission to read *b.f*

# Separation logic: Heap-manipulating programs and ownership

## Memory



Exclusive ownership  
Permission to read and write *a.val*

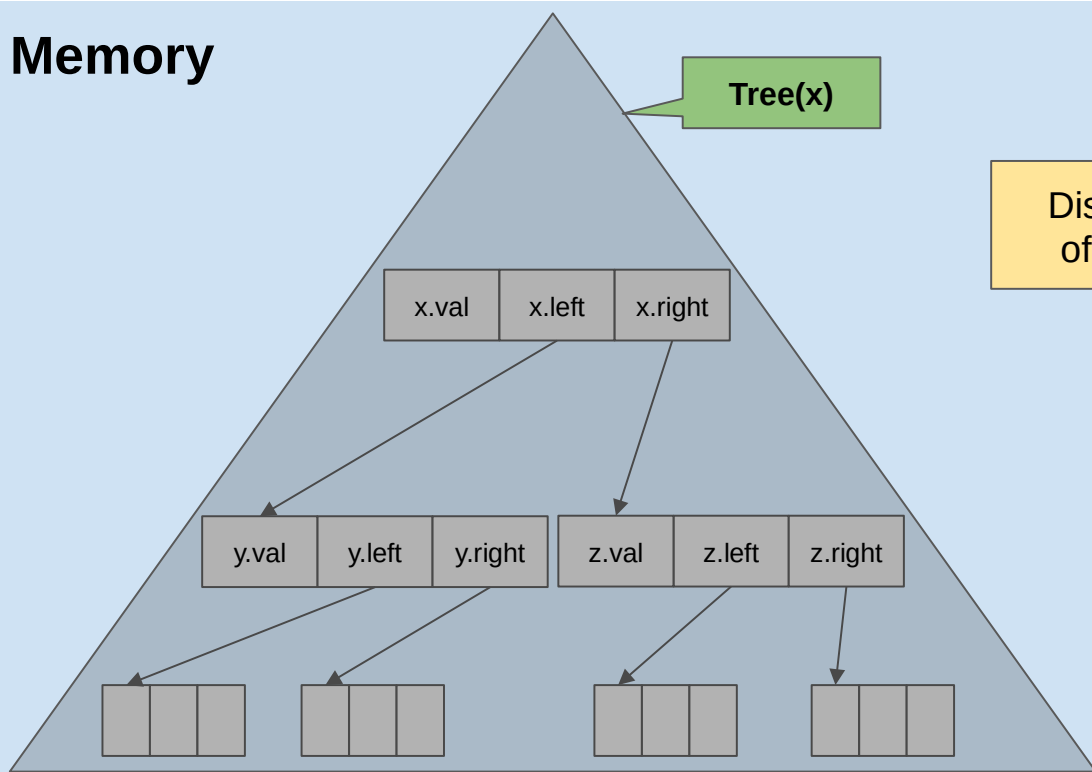


Fractional (non-exclusive) ownership  
Permission to read *b.f*

```
Tree(x: Ref)  $\triangleq$   
  acc(x.val) * acc(x.left) * acc(x.right) *  
  (x.left != null  $\Rightarrow$  Tree(x.left)) * (x.right != null  $\Rightarrow$  Tree(x.right))
```

# Separation logic: Heap-manipulating programs and ownership

## Memory



Exclusive ownership  
Permission to read and write *a.val*

Disjointness  
of memory

\*

*a.val*

**acc(a.val)**

**a.val**  $\mapsto$  \_

*b.f*

**acc(b.f, 1/2)**

Fractional (non-exclusive) ownership  
Permission to read *b.f*

**Tree(x: Ref)**  $\triangleq$

**acc(x.val)** \* **acc(x.left)** \* **acc(x.right)** \*

(**x.left**  $\neq$  **null**  $\Rightarrow$  **Tree(x.left)**) \* (**x.right**  $\neq$  **null**  $\Rightarrow$  **Tree(x.right)**)

# Using magic wands

```
method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x

  while(y.left != null)
    invariant [redacted]
    {
      y := y.left
    }
}
```

# Using magic wands

Pointer to a binary tree

Computes its leftmost leaf

```
method leftLeaf(x: Ref) : (y: Ref)  
  requires Tree(x)  
  ensures Tree(x)  
{  
  y := x  
  
  while(y.left != null)  
  invariant   
  {  
    y := y.left  
  
  }  
  
}
```

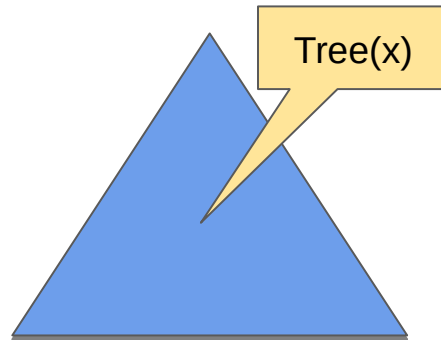
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Pointer to a binary tree

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    invariant 
    {
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    }
}
```

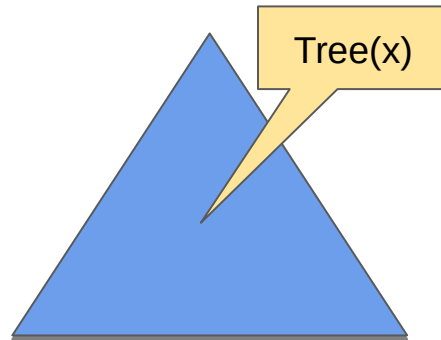


# Using magic wands

Pointer to a binary tree

Computes its leftmost leaf

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method leftLeaf(x: Ref) : (y: Ref)  
  requires Tree(x)  
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  y := x  
  
  while(y.left != null)  
    invariant   
    {  
      y := y.left  
    }  
}  
}
```

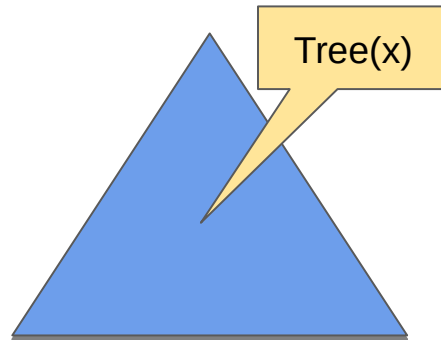


# Using magic wands

```
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  requires Tree(x)
  ensures Tree(x)
{
  y := x

  while(y.left != null)
    invariant 
    {
      y := y.left
    }
}
```

Need permission to justify read access

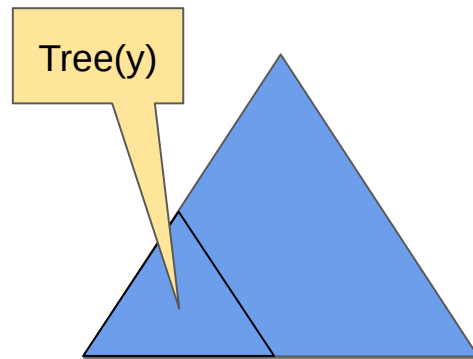


# Using magic wands

```
method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x

  while(y.left != null)
    invariant Tree(y) *
    {
      y := y.left
    }
}
```

Need permission to justify read access



# Using magic wands

```
method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x
```

How to express the remaining ownership?

```
  while(y.left != null)
```

```
  invariant Tree(y) *
```

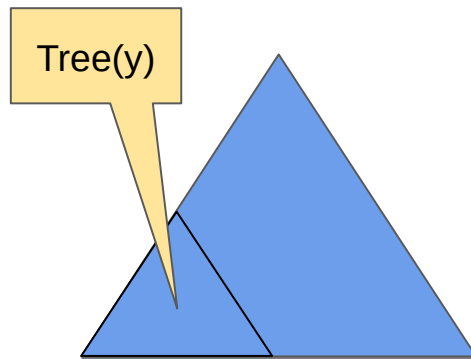
```
  {
```

```
    y := y.left
```

```
  }
```

```
}
```

Need permission to justify read access



# Using magic wands

```
method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
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How to express the remaining ownership?

```
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```

```
  invariant Tree(y) *
```

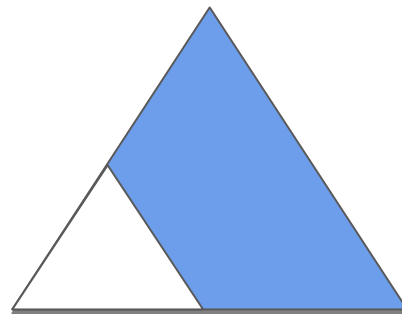
```
  {
```

```
    y := y.left
```

```
  }
```

Need permission to justify read access

```
}
```



# Using magic wands

```
method leftLeaf(x: Ref) : (y: Ref)
```

```
  requires Tree(x)
```

```
  ensures Tree(x)
```

```
{
```

```
  y := x
```

How to express the remaining ownership?

```
  while(y.left != null)
```

```
  invariant Tree(y) *
```

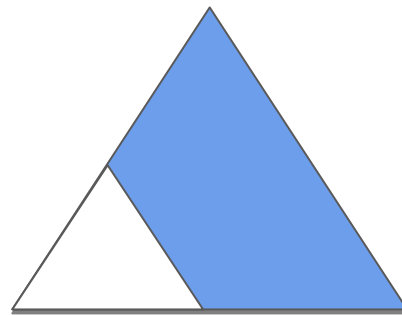
```
{
```

```
  y := y.left
```

```
}
```

Need permission to justify read access

```
}
```



$\text{Tree}(y) \multimap \text{Tree}(x)$

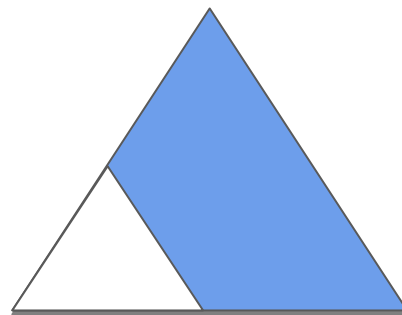
*Magic wand*

# Using magic wands

```
method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x
  How to express the remaining ownership?

  while(y.left != null)
    invariant Tree(y) * (Tree(y)  $\multimap$  Tree(x))
    {
      y := y.left
    }
  }
}
```

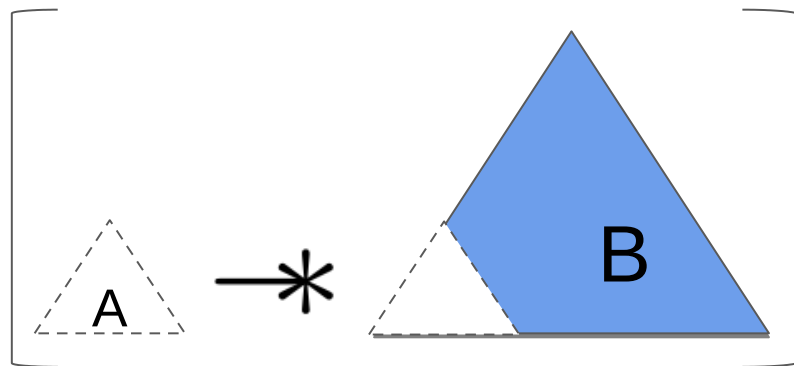
Need permission to justify read access



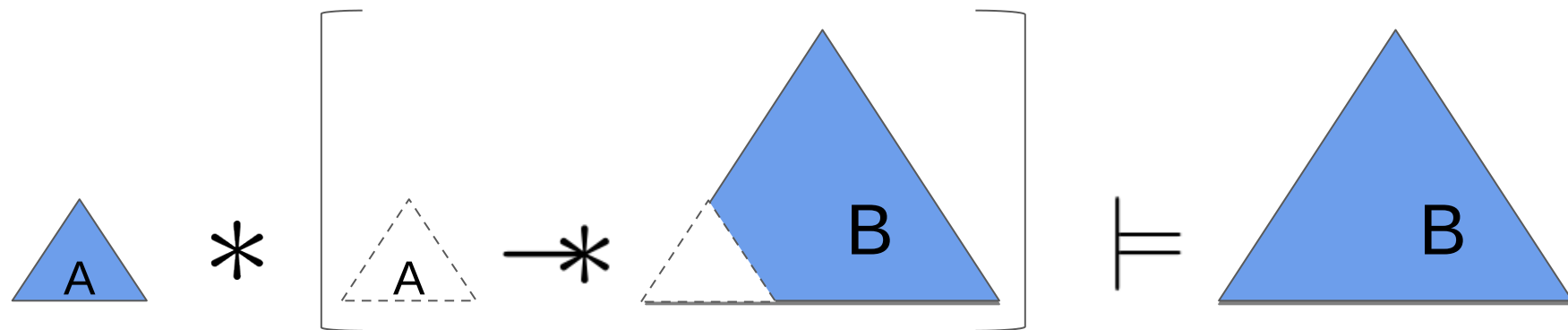
$\text{Tree}(y) \multimap \text{Tree}(x)$

***Magic wand***

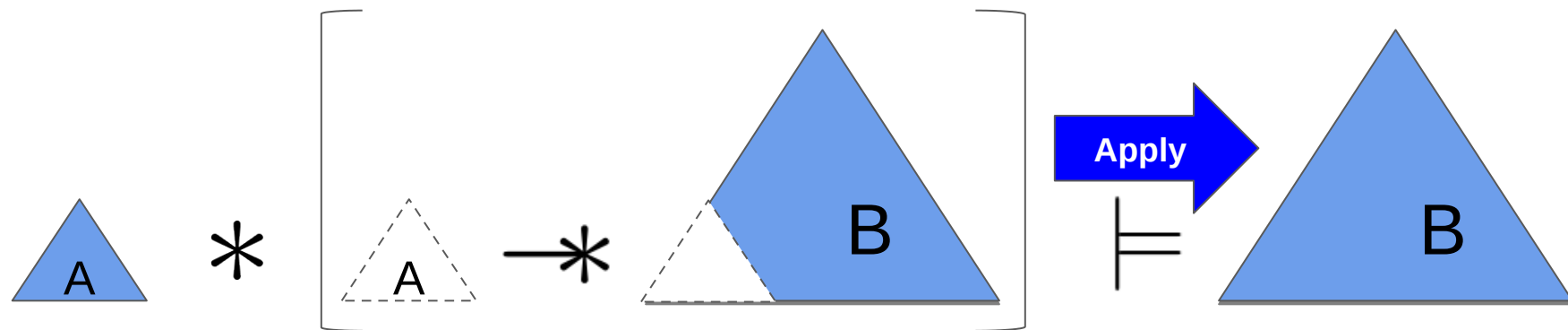
# Background: Ghost operations to manipulate magic wands



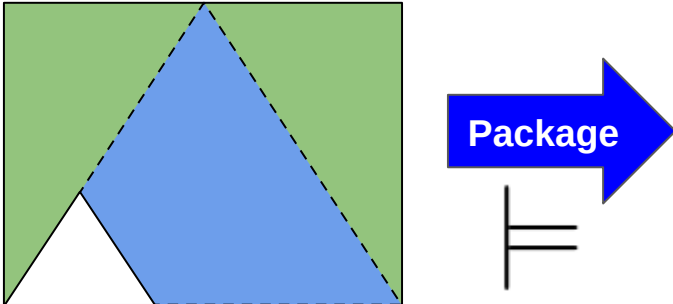
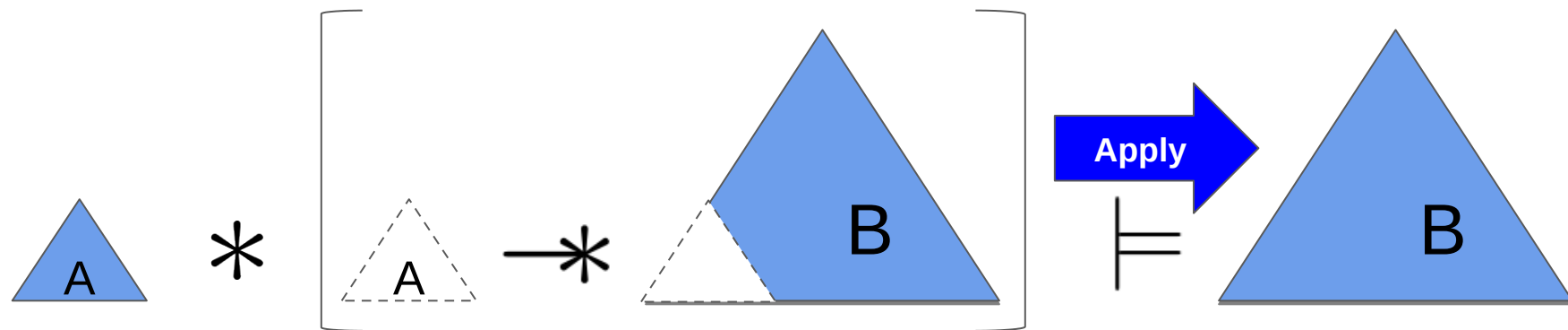
Background: Ghost operations to manipulate magic wands



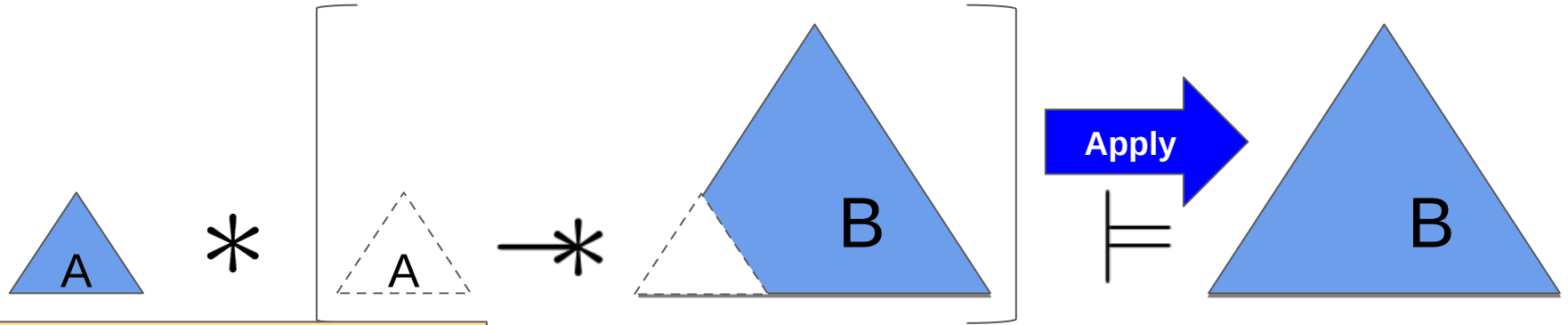
Background: Ghost operations to manipulate magic wands



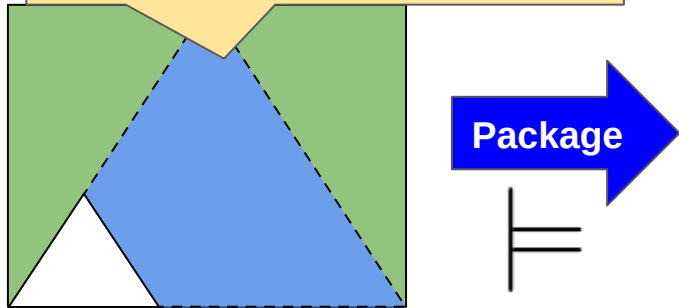
# Background: Ghost operations to manipulate magic wands



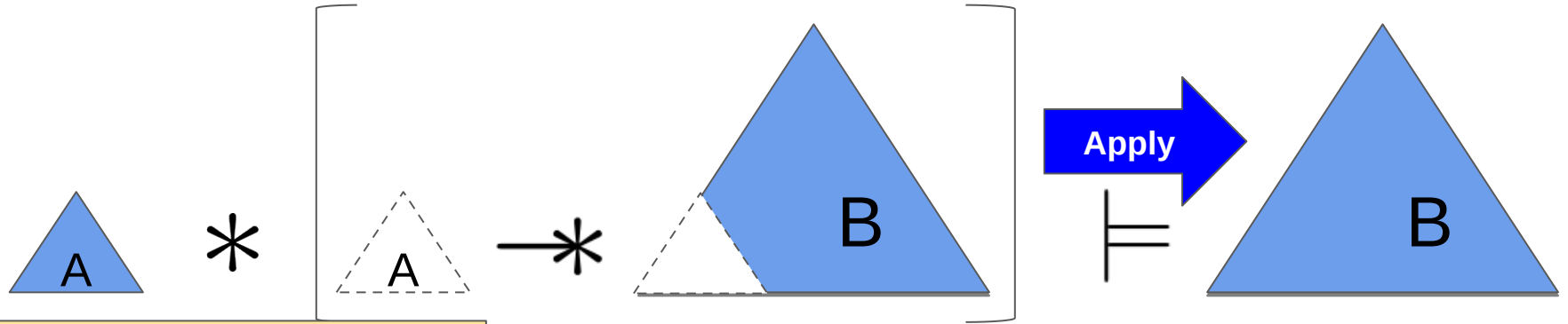
# Background: Ghost operations to manipulate magic wands



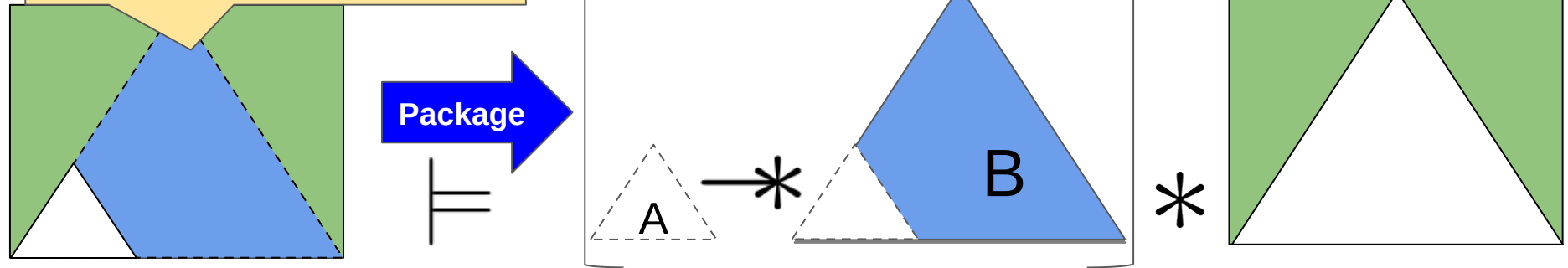
**Footprint:** Set of resources that, when combined with **A**, guarantee **B**



# Background: Ghost operations to manipulate magic wands



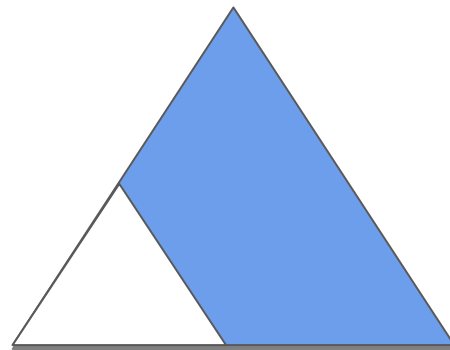
**Footprint:** Set of resources that, when combined with **A**, guarantee **B**



# Using magic wands

```
method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x

  while(y.left != null)
    invariant Tree(y) * (Tree(y) -* Tree(x))
    {
      y := y.left
    }
}
```



$\text{Tree}(y) \multimap \text{Tree}(x)$

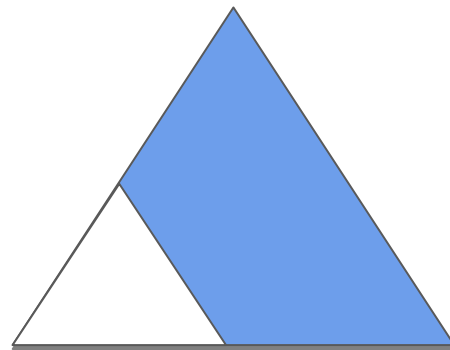
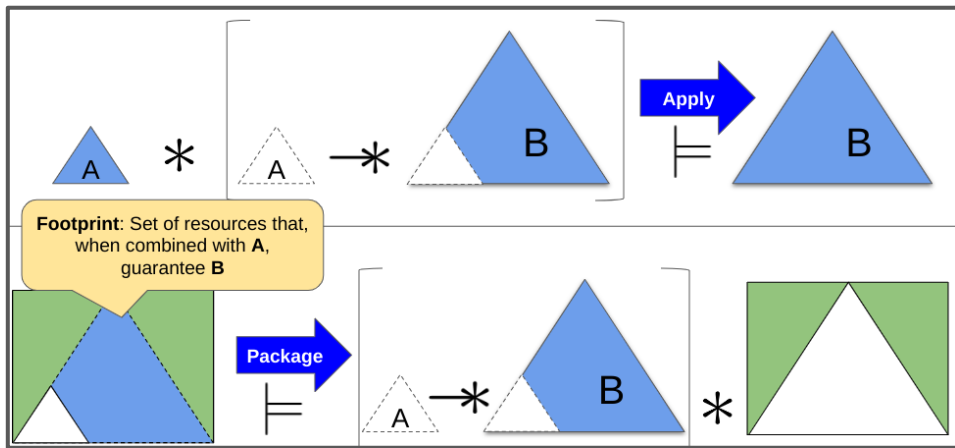
# Using magic wands

```

method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x

  while(y.left != null)
    invariant Tree(y) * (Tree(y) -* Tree(x))
    {
      y := y.left
    }
}

```



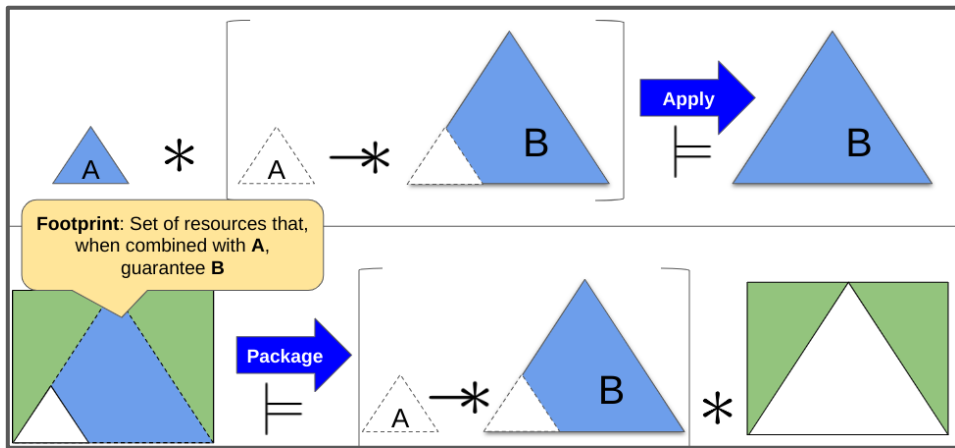
$\text{Tree}(y) \multimap \text{Tree}(x)$

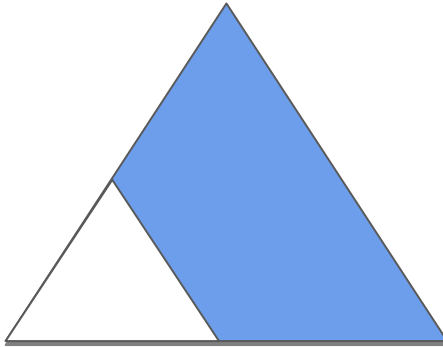
# Using magic wands

```

method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x

  while(y.left != null)
    invariant Tree(y) * (Tree(y)  $\multimap$  Tree(x))
    {
      y := y.left
    }
}
    
```



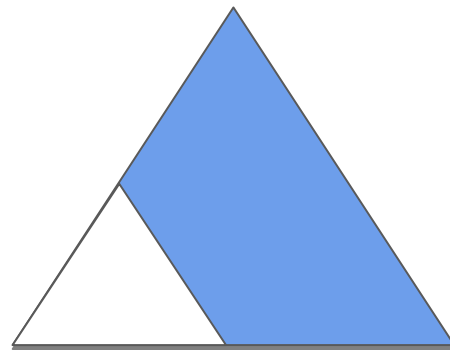
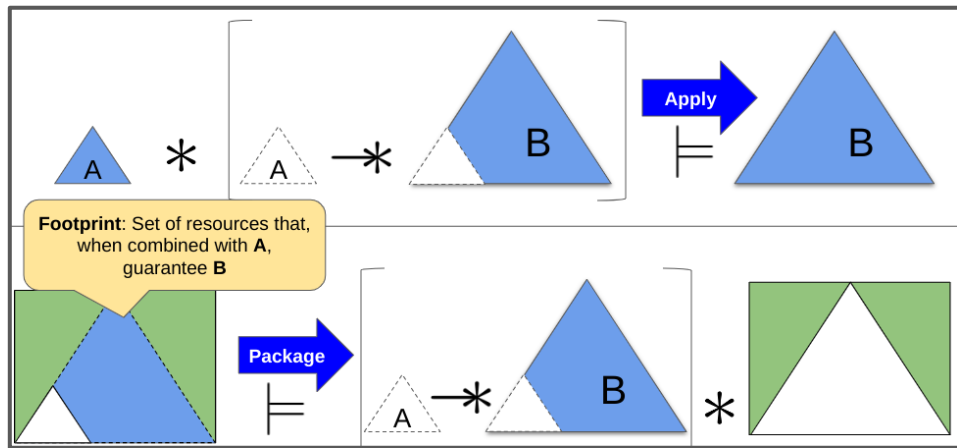

  
 $\text{Tree}(y) \multimap \text{Tree}(x)$

# Using magic wands

```

method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x
  package Tree(y) -* Tree(x)

  while(y.left != null)
    invariant Tree(y) * (Tree(y) -* Tree(x))
    {
      y := y.left
    }
}
  
```



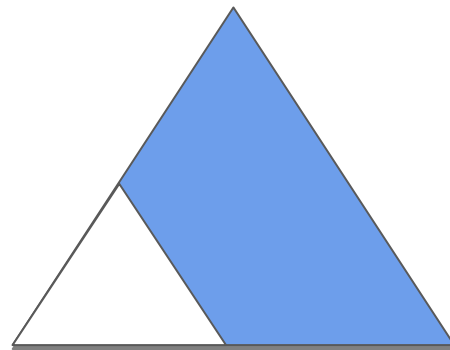
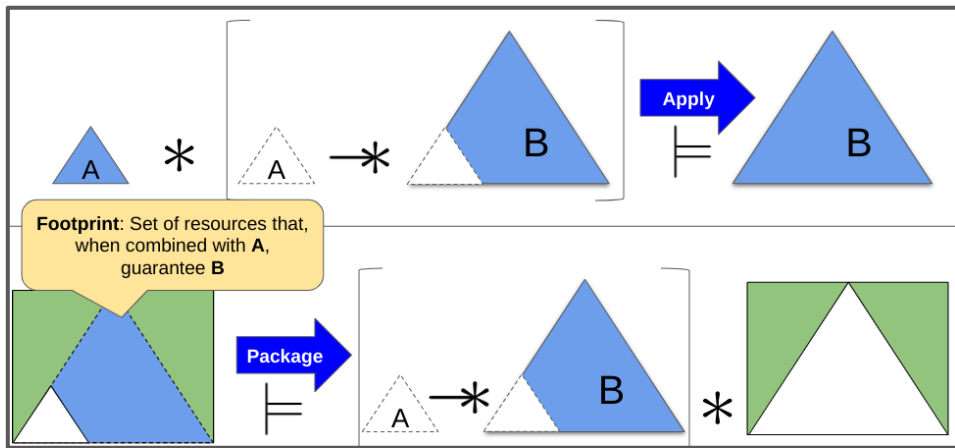
$\text{Tree}(y) \multimap \text{Tree}(x)$

# Using magic wands

```

method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x
  package Tree(y)  $\multimap$  Tree(x)

  while(y.left != null)
    invariant Tree(y) * (Tree(y)  $\multimap$  Tree(x))
    {
      y := y.left
      package Tree(y)  $\multimap$  Tree(x)
    }
}
  
```



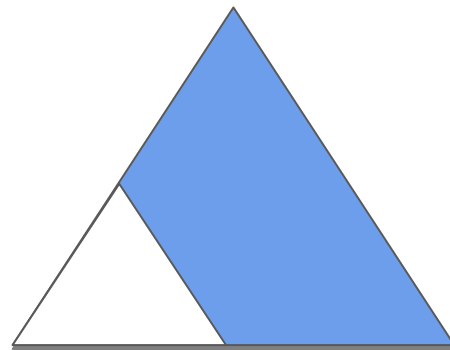
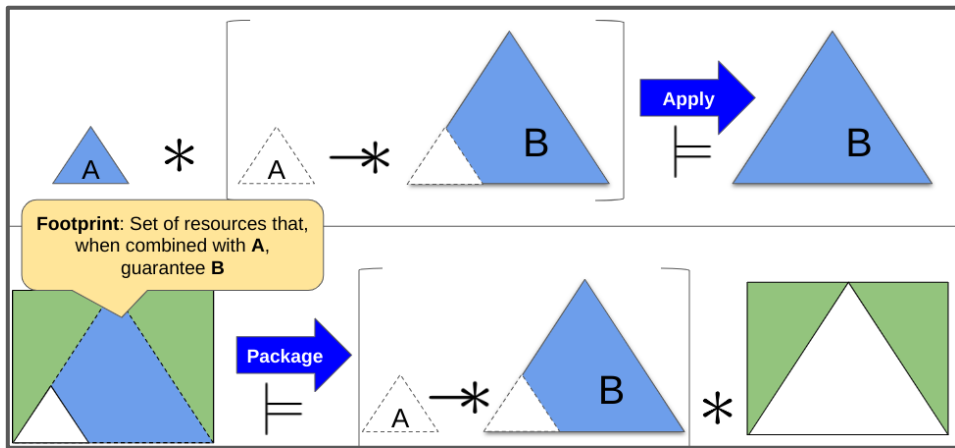
$\text{Tree}(y) \multimap \text{Tree}(x)$

# Using magic wands

```

method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x
  package Tree(y) -* Tree(x)

  while(y.left != null)
    invariant Tree(y) * (Tree(y) -* Tree(x))
    {
      y := y.left
      package Tree(y) -* Tree(x)
    }
}
    
```



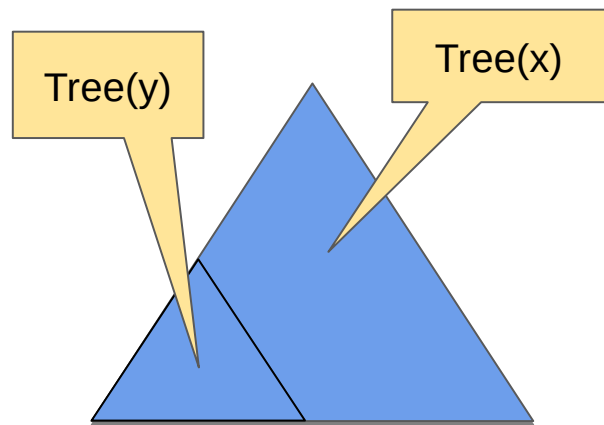
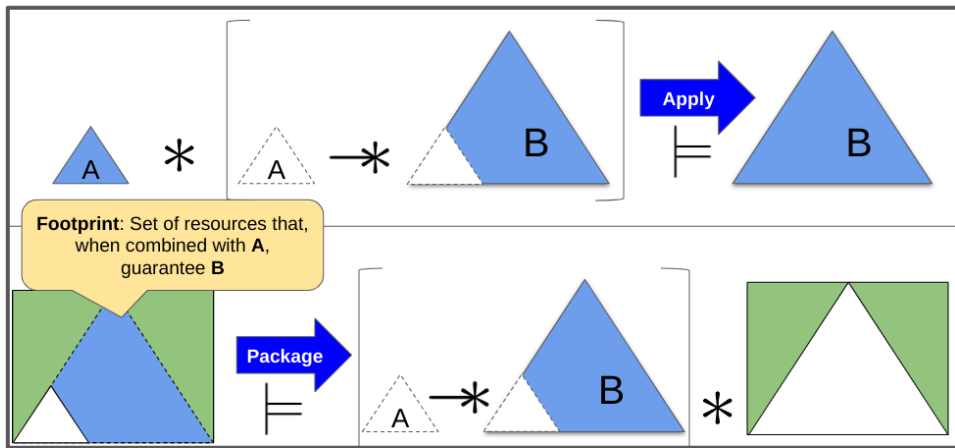
$\text{Tree}(y) \multimap \text{Tree}(x)$

# Using magic wands

```

method leftLeaf(x: Ref) : (y: Ref)
  requires Tree(x)
  ensures Tree(x)
{
  y := x
  package Tree(y) -* Tree(x)

  while(y.left != null)
    invariant Tree(y) * (Tree(y) -* Tree(x))
    {
      y := y.left
      package Tree(y) -* Tree(x)
    }
  apply Tree(y) -* Tree(x)
}
  
```



Challenge: Automatically compute a **small** and **valid** footprint

## Witnessing the elimination of magic wands

Stefan Blom · Marieke Huisman

STTT 2015

VerCors

Challenge: Automatically compute a **small** and **valid** footprint

Not automated: The user must specify the footprint

## Witnessing the elimination of magic wands

Stefan Blom · Marieke Huisman

STTT 2015

VerCors

# Challenge: Automatically compute a **small** and **valid** footprint

Not automated: The user must specify the footprint

## Witnessing the elimination of magic wands

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STTT 2015

VerCors

## Lightweight Support for Magic Wands in an Automatic Verifier

Malte Schwerhoff<sup>1</sup> and Alexander J. Summers<sup>2</sup>

ECOOP 2015

VIPER

# Challenge: Automatically compute a **small** and **valid** footprint

Not automated: The user must specify the footprint

## Witnessing the elimination of magic wands

Stefan Blom · Marieke Huisman

STTT 2015

VerCors

Automatically computes footprint

## Lightweight Support for Magic Wands in an Automatic Verifier

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ECOOP 2015

VIPER

# Challenge: Automatically compute a **small** and **valid** footprint

Not automated: The user must specify the footprint

## Witnessing the elimination of magic wands

Stefan Blom · Marieke Huisman

STTT 2015

VerCors

Automatically computes footprint

Computed footprint sometimes invalid ❌

## Lightweight Support for Magic Wands in an Automatic Verifier

Malte Schwerhoff<sup>1</sup> and Alexander J. Summers<sup>2</sup>

ECOOP 2015

VIPER

# Examples of footprints

---

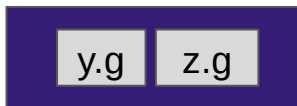
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---

# Examples of footprints

---

Current state

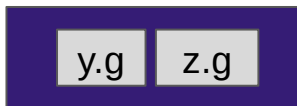


**package**  $\text{acc}(x.f) \rightarrow^* (\text{acc}(x.f) * \text{acc}(y.g))$

---

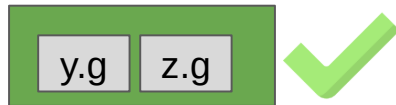
# Examples of footprints

Current state



**package**  $\text{acc}(x.f) \rightarrow^* (\text{acc}(x.f) * \text{acc}(y.g))$

Valid footprints:



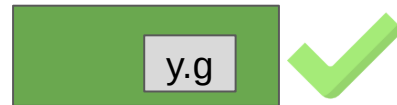
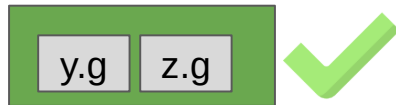
# Examples of footprints

Current state



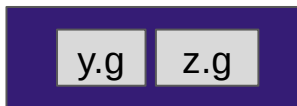
**package**  $\text{acc}(x.f) \rightarrow * (\text{acc}(x.f) * \text{acc}(y.g))$

Valid footprints:



# Examples of footprints

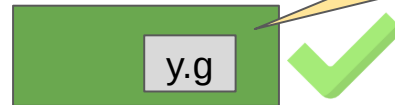
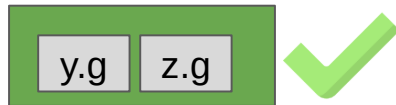
Current state



**package**  $\text{acc}(x.f) \rightarrow * (\text{acc}(x.f) * \text{acc}(y.g))$

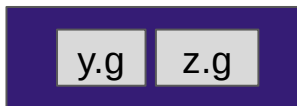
Smaller footprint

Valid footprints:



# Examples of footprints

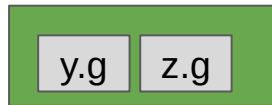
Current state



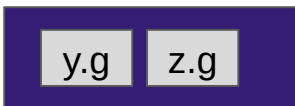
**package**  $\text{acc}(x.f) \rightarrow^* (\text{acc}(x.f) * \text{acc}(y.g))$

Smaller footprint

Valid footprints:



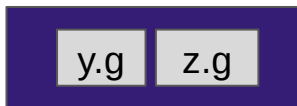
Current state



**package**  $(\text{acc}(x.f) * (x.f = y \vee x.f = z)) \rightarrow^* (\text{acc}(x.f) * \text{acc}(x.f.g))$

# Examples of footprints

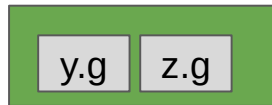
Current state



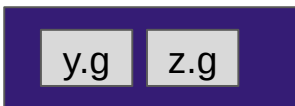
**package**  $\text{acc}(x.f) \rightarrow * (\text{acc}(x.f) * \text{acc}(y.g))$

Smaller footprint

Valid footprints:

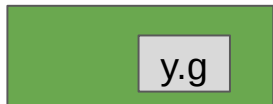


Current state



**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \rightarrow * (\text{acc}(x.f) * \text{acc}(x.f.g))$

Potential footprints:



# Examples of footprints

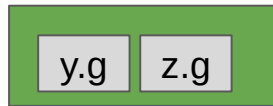
Current state



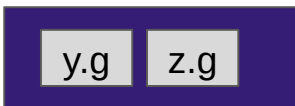
**package**  $\text{acc}(x.f) \rightarrow^* (\text{acc}(x.f) * \text{acc}(y.g))$

Smaller footprint

Valid footprints:

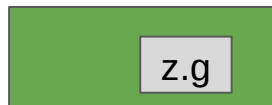
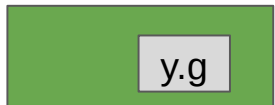


Current state



**package**  $(\text{acc}(x.f) * (x.f = y \vee x.f = z)) \rightarrow^* (\text{acc}(x.f) * \text{acc}(x.f.g))$

Potential footprints:



# Examples of footprints

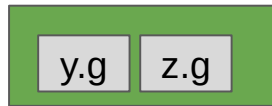
Current state



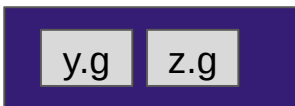
**package**  $\text{acc}(x.f) \rightarrow^* (\text{acc}(x.f) * \text{acc}(y.g))$

Smaller footprint

Valid footprints:

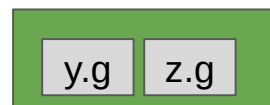
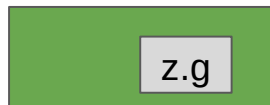
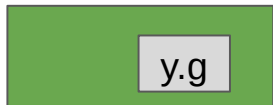


Current state



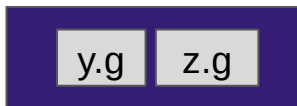
**package**  $(\text{acc}(x.f) * (x.f = y \vee x.f = z)) \rightarrow^* (\text{acc}(x.f) * \text{acc}(x.f.g))$

Potential footprints:



# Examples of footprints

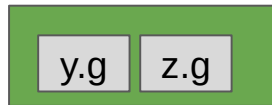
Current state



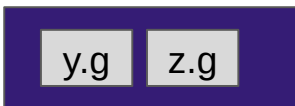
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Smaller footprint

Valid footprints:

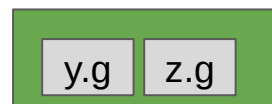
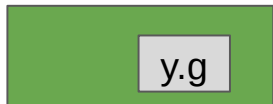


Current state

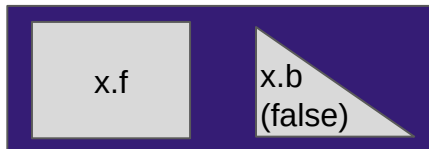


**package**  $(\text{acc}(x.f) * (x.f = y \vee x.f = z)) \rightarrow^* (\text{acc}(x.f) * \text{acc}(x.f.g))$

Potential footprints:



Current state



**package**  $\text{acc}(x.b, 1/2) \rightarrow^* (\text{acc}(x.b, 1/2) * (x.b \Rightarrow \text{acc}(x.f)))$

Valid footprints:

# Examples of footprints

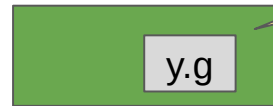
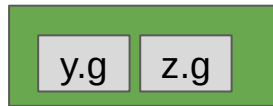
Current state



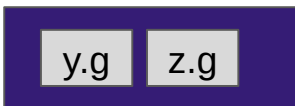
**package**  $\text{acc}(x.f) \rightarrow^* (\text{acc}(x.f) * \text{acc}(y.g))$

Smaller footprint

Valid footprints:

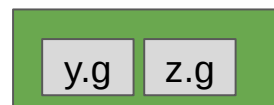
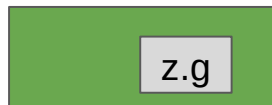
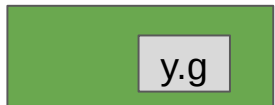


Current state

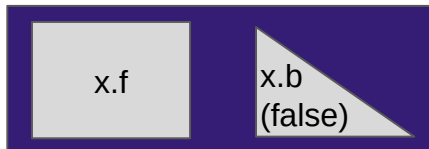


**package**  $(\text{acc}(x.f) * (x.f = y \vee x.f = z)) \rightarrow^* (\text{acc}(x.f) * \text{acc}(x.f.g))$

Potential footprints:

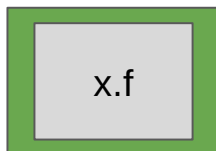


Current state



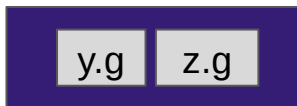
**package**  $\text{acc}(x.b, 1/2) \rightarrow^* (\text{acc}(x.b, 1/2) * (x.b \Rightarrow \text{acc}(x.f)))$

Valid footprints:



# Examples of footprints

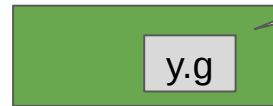
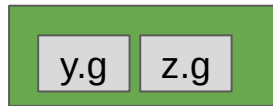
Current state



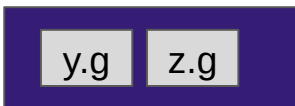
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Smaller footprint

Valid footprints:

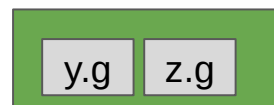
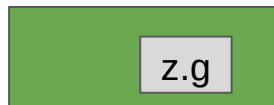
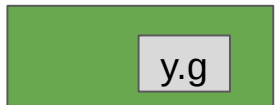


Current state

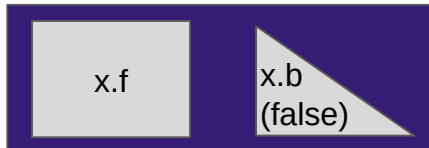


**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \rightarrow^* (\text{acc}(x.f) * \text{acc}(x.f.g))$

Potential footprints:

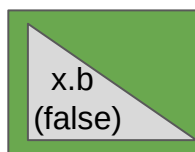
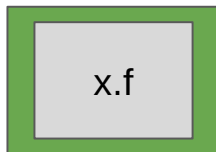


Current state



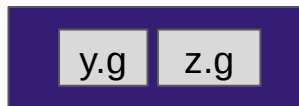
**package**  $\text{acc}(x.b, 1/2) \rightarrow^* (\text{acc}(x.b, 1/2) * (\boxed{x.b} \Rightarrow \text{acc}(x.f)))$

Valid footprints:



# Examples of footprints

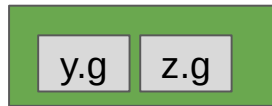
Current state



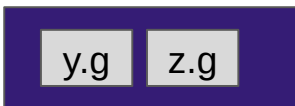
**package**  $\text{acc}(x.f) \rightarrow^* (\text{acc}(x.f) * \text{acc}(y.g))$

Smaller footprint

Valid footprints:

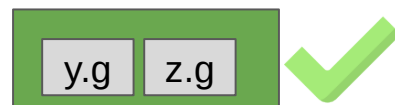
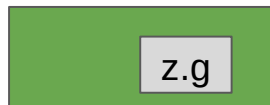
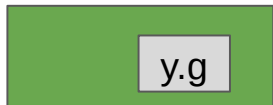


Current state

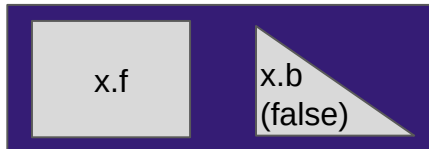


**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \rightarrow^* (\text{acc}(x.f) * \text{acc}(x.f.g))$

Potential footprints:

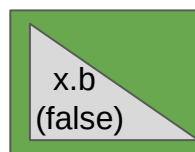
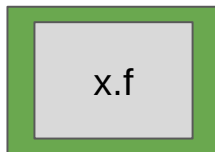


Current state



**package**  $\text{acc}(x.b, 1/2) \rightarrow^* (\text{acc}(x.b, 1/2) * (\boxed{x.b} \Rightarrow \text{acc}(x.f)))$

Valid footprints:



No minimal footprint  $\rightarrow$   
no “best” algorithm!

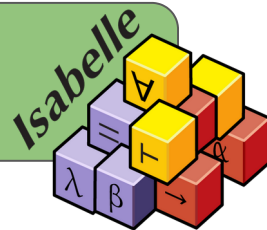
# Contributions

## **1. Package logic**

# Contributions

**Parametric** logical framework for computing footprints  
Sound and complete

## 1. Package logic



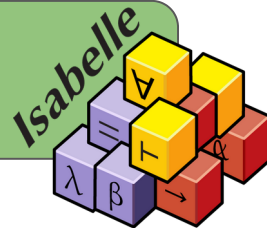
# Contributions

Standard wand



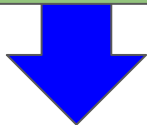
**Parametric** logical framework for computing footprints  
Sound and complete

## 1. Package logic

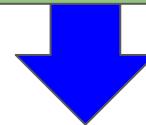
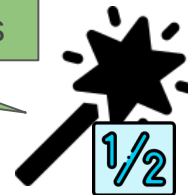


# Contributions

Standard wand

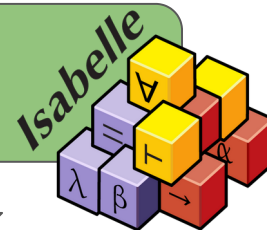


**2. Combinable wand**  
Novel definition of the magic wand, with useful properties when combined with fractional permissions



**Parametric** logical framework for computing footprints  
Sound and complete

**1. Package logic**

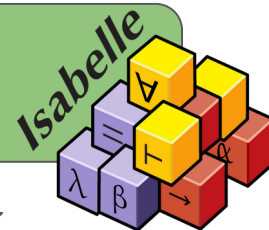
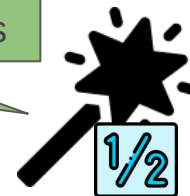


# Contributions

Standard wand

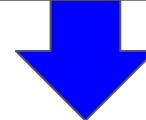
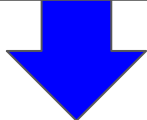


**2. Combinable wand**  
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**Parametric** logical framework for computing footprints  
Sound and complete

**1. Package logic**

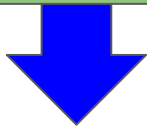


**3. Implementations and evaluation**

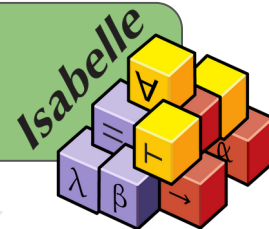


# Contributions

Standard wand



**2. Combinable wand**  
Novel definition of the magic wand, with useful properties when combined with fractional permissions



**Parametric** logical framework for computing footprints  
Sound and complete

## 1. Package logic

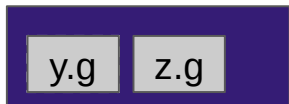
**3. Implementations and evaluation**



# Computing a footprint using the package logic (simplified)

**package**  $\text{acc}(x.f) \multimap \text{acc}(x.f) * \text{acc}(y.g)$

Current state



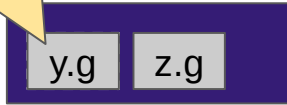
**Footprint:** Set of resources that, when combined with  $\text{acc}(x.f)$ , guarantee  $\text{acc}(x.f) * \text{acc}(y.g)$

# Computing a footprint using the package logic (simplified)

**package**  $\text{acc}(x.f) \multimap \text{acc}(x.f) * \text{acc}(y.g)$

Smallest footprint

Current state



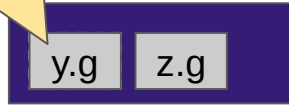
**Footprint:** Set of resources that, when combined with  $\text{acc}(x.f)$ , guarantee  $\text{acc}(x.f) * \text{acc}(y.g)$

# Computing a footprint using the package logic (simplified)

**package**  $\text{acc}(x.f) \rightarrow * \text{acc}(x.f) * \text{acc}(y.g)$

Smallest footprint

Current state



LHS



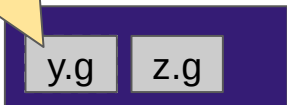
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# Computing a footprint using the package logic (simplified)

**package**  $\text{acc}(x.f) \rightarrow * \text{acc}(x.f) * \text{acc}(y.g)$

Smallest footprint

Current state



LHS



RHS

**Footprint:** Set of resources that, when combined with  $\text{acc}(x.f)$ , guarantee  $\text{acc}(x.f) * \text{acc}(y.g)$

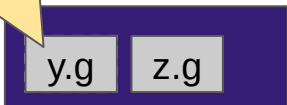
# Computing a footprint using the package logic (simplified)



**package**  $\text{acc}(x.f) \multimap \boxed{\text{acc}(x.f)} * \text{acc}(y.g)$

Smallest footprint

Current state



LHS



RHS

**Footprint:** Set of resources that, when combined with  $\text{acc}(x.f)$ , guarantee  $\text{acc}(x.f) * \text{acc}(y.g)$

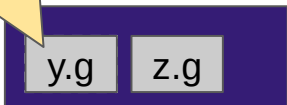
# Computing a footprint using the package logic (simplified)



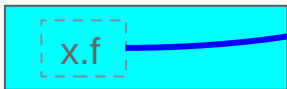
**package**  $\text{acc}(x.f) \multimap \boxed{\text{acc}(x.f)} * \text{acc}(y.g)$

Smallest footprint

Current state



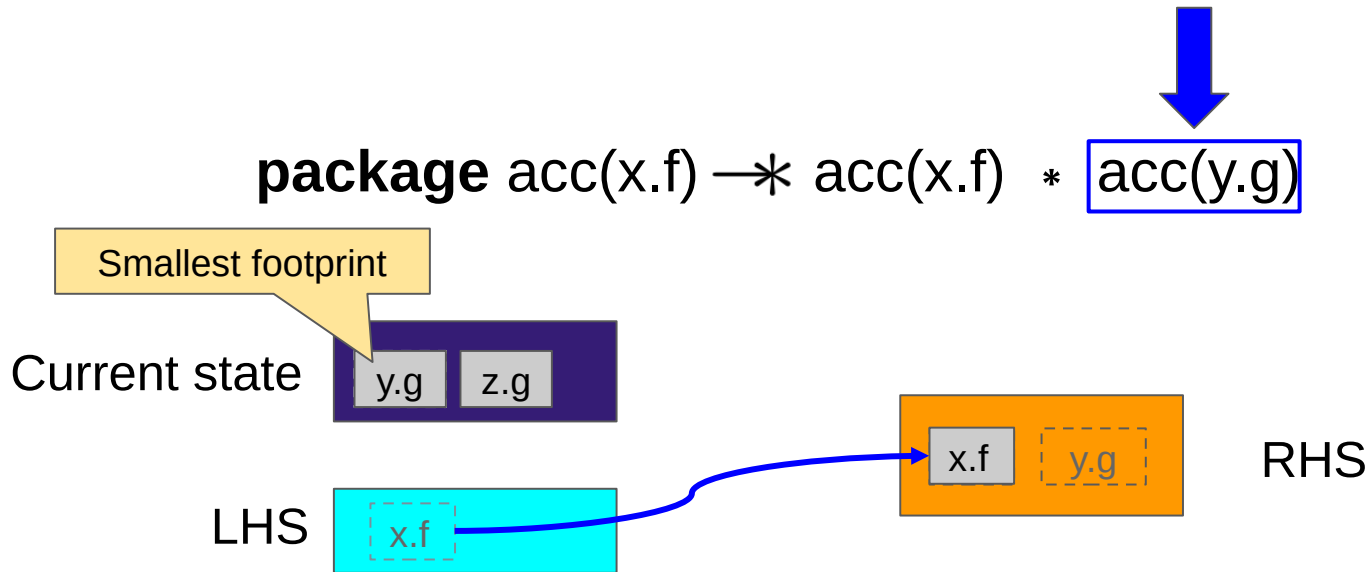
LHS



RHS

**Footprint:** Set of resources that, when combined with  $\text{acc}(x.f)$ , guarantee  $\text{acc}(x.f) * \text{acc}(y.g)$

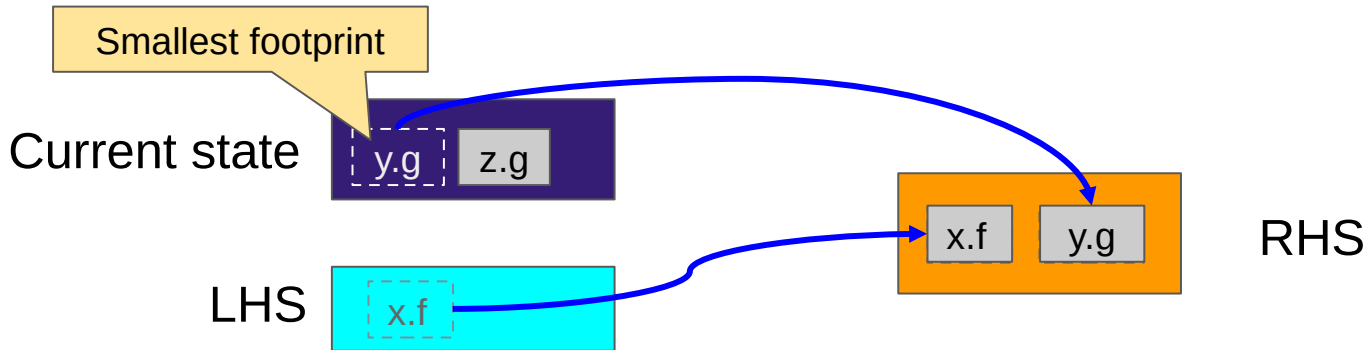
# Computing a footprint using the package logic (simplified)



**Footprint:** Set of resources that, when combined with  $\text{acc}(x.f)$ , guarantee  $\text{acc}(x.f) * \text{acc}(y.g)$

# Computing a footprint using the package logic (simplified)

**package**  $\text{acc}(x.f) \multimap \text{acc}(x.f) * \text{acc}(y.g)$



**Footprint:** Set of resources that, when combined with  $\text{acc}(x.f)$ , guarantee  $\text{acc}(x.f) * \text{acc}(y.g)$

# Using the package logic

**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \multimap \text{acc}(x.f) * \text{acc}(x.f.g)$

$h_0$ 

y.g	z.g	...
-----	-----	-----

# Using the package logic

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$h_0$ 

y.g	z.g	...
-----	-----	-----

How to represent a state satisfying a disjunction?

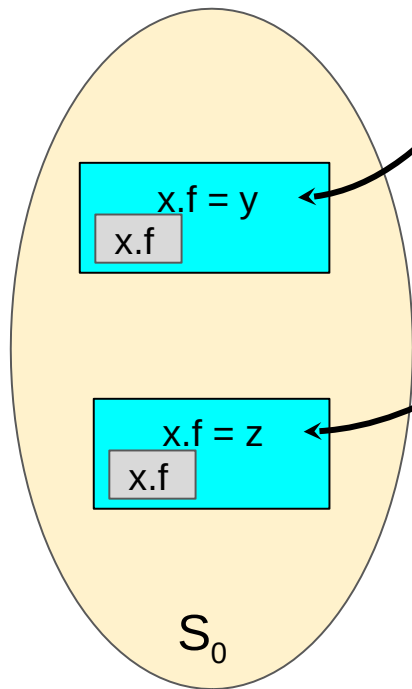
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y.g	z.g	...
-----	-----	-----

How to represent a state satisfying a disjunction?



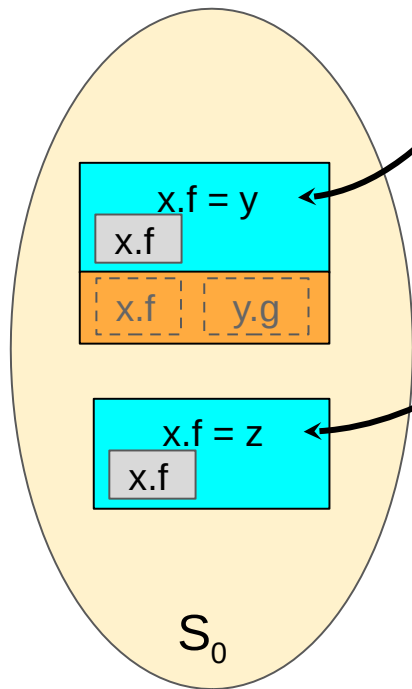
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$h_0$ 

y.g	z.g	...
-----	-----	-----

How to represent a state satisfying a disjunction?



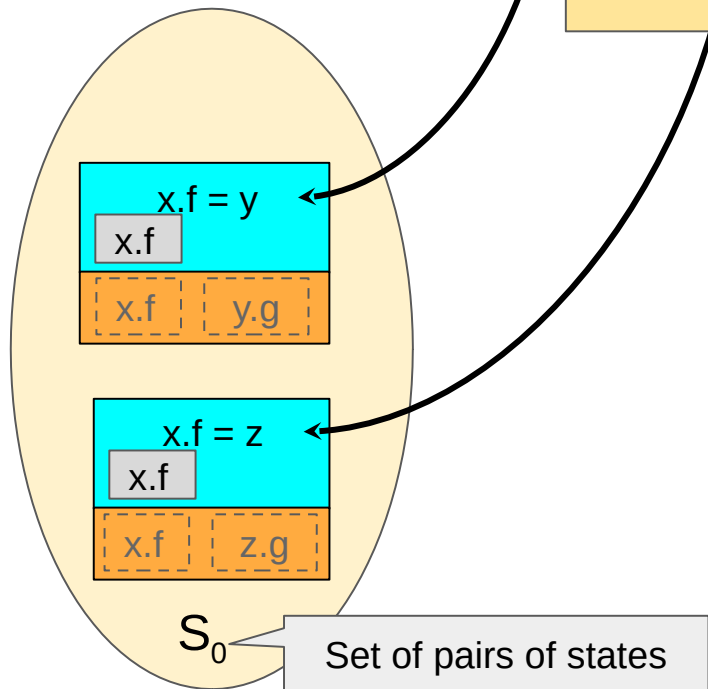
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**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \multimap \text{acc}(x.f) * \text{acc}(x.f.g)$

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y.g	z.g	...
-----	-----	-----

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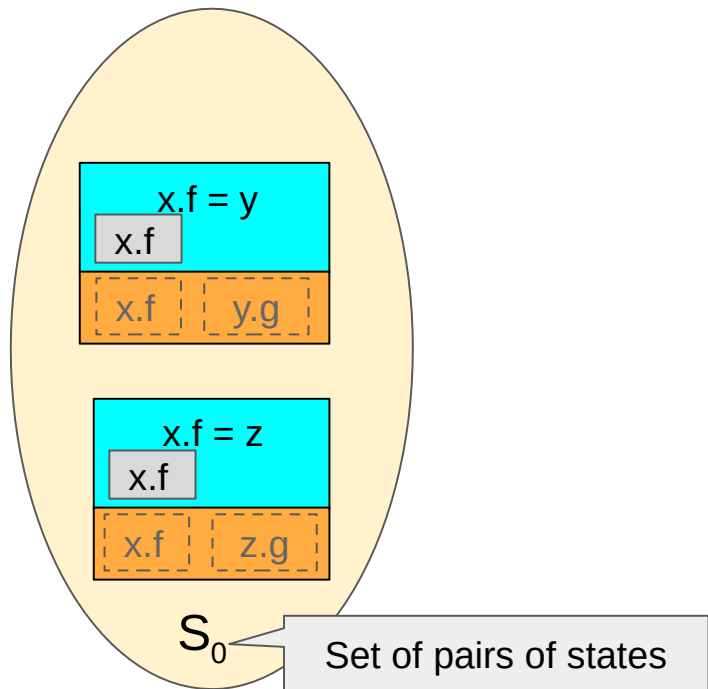
# Using the package logic



**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \multimap \boxed{\text{acc}(x.f)} * \text{acc}(x.f.g)$

$h_0$ 

y.g	z.g	...
-----	-----	-----



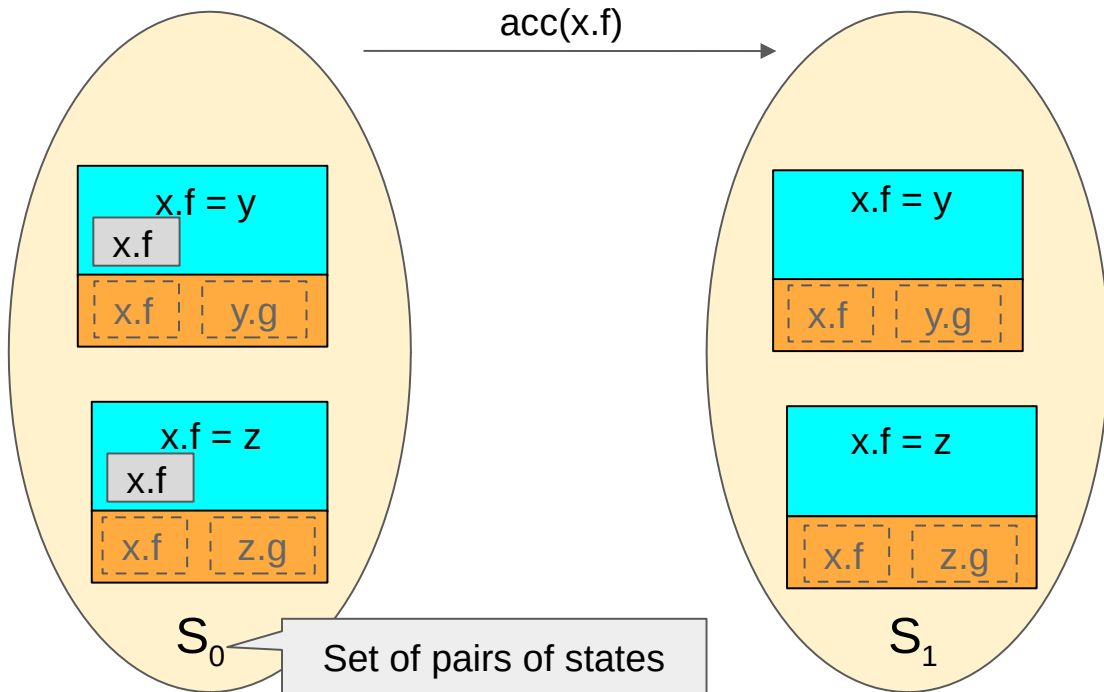
# Using the package logic



**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \multimap \boxed{\text{acc}(x.f)} * \text{acc}(x.f.g)$

$h_0$ 

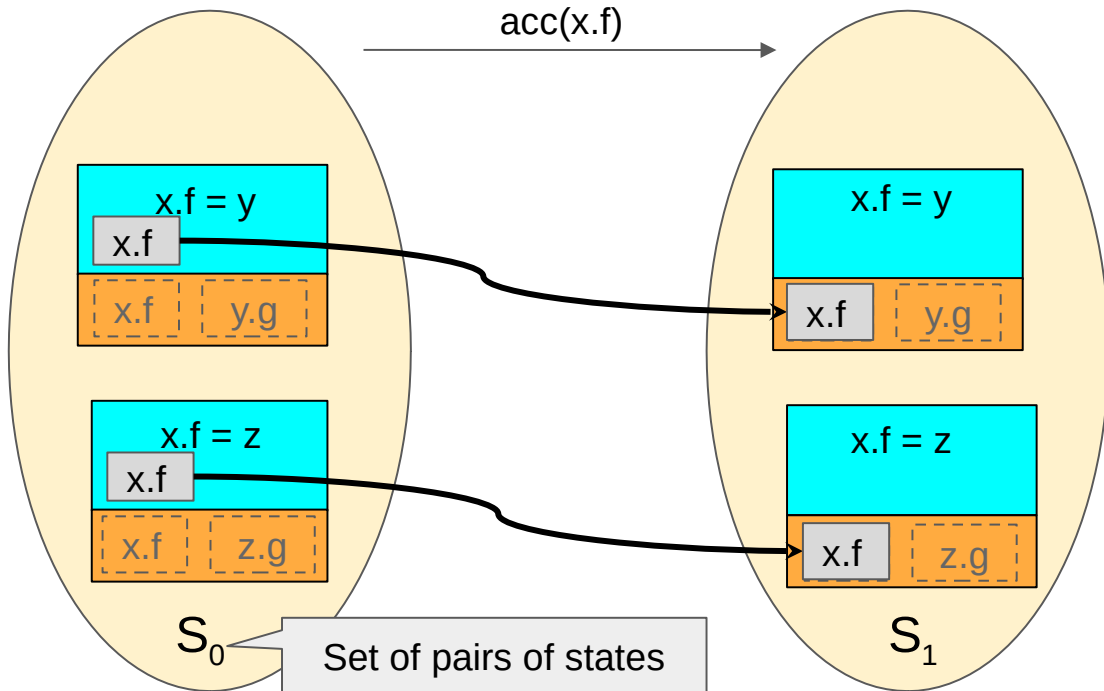
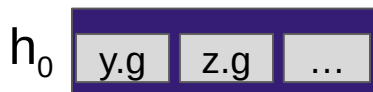
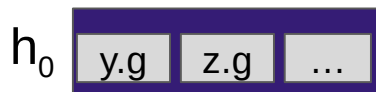
y.g	z.g	...
-----	-----	-----



# Using the package logic

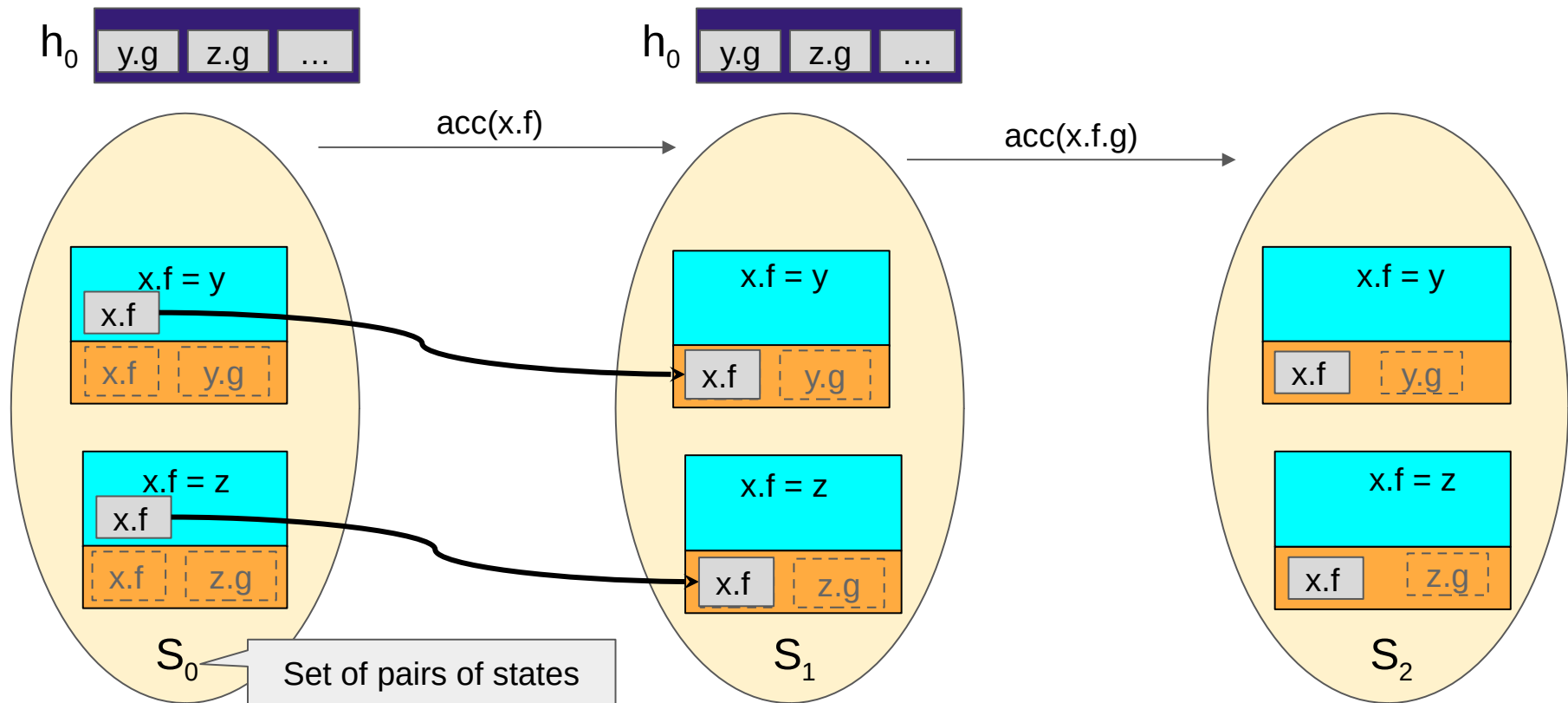


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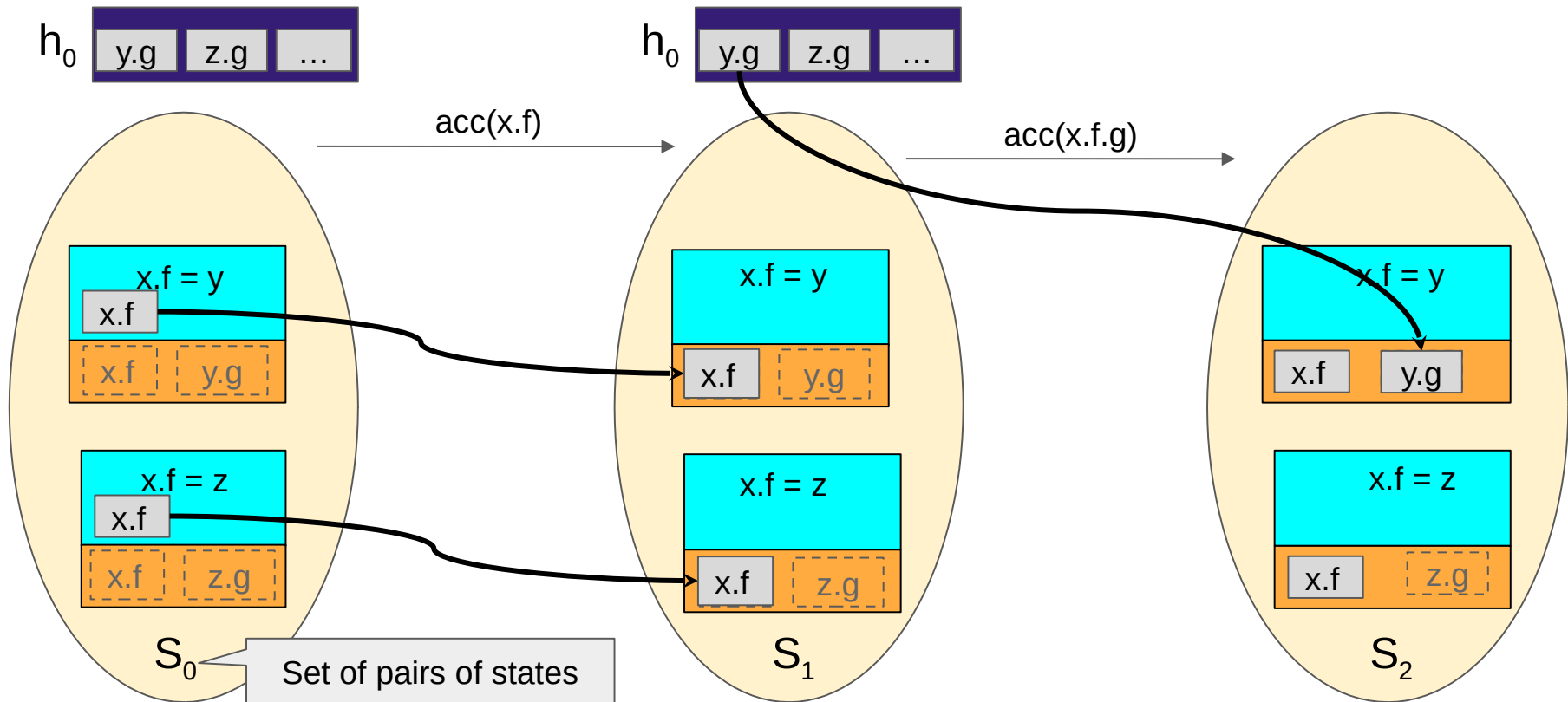
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**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \multimap \text{acc}(x.f) * \boxed{\text{acc}(x.f.g)}$



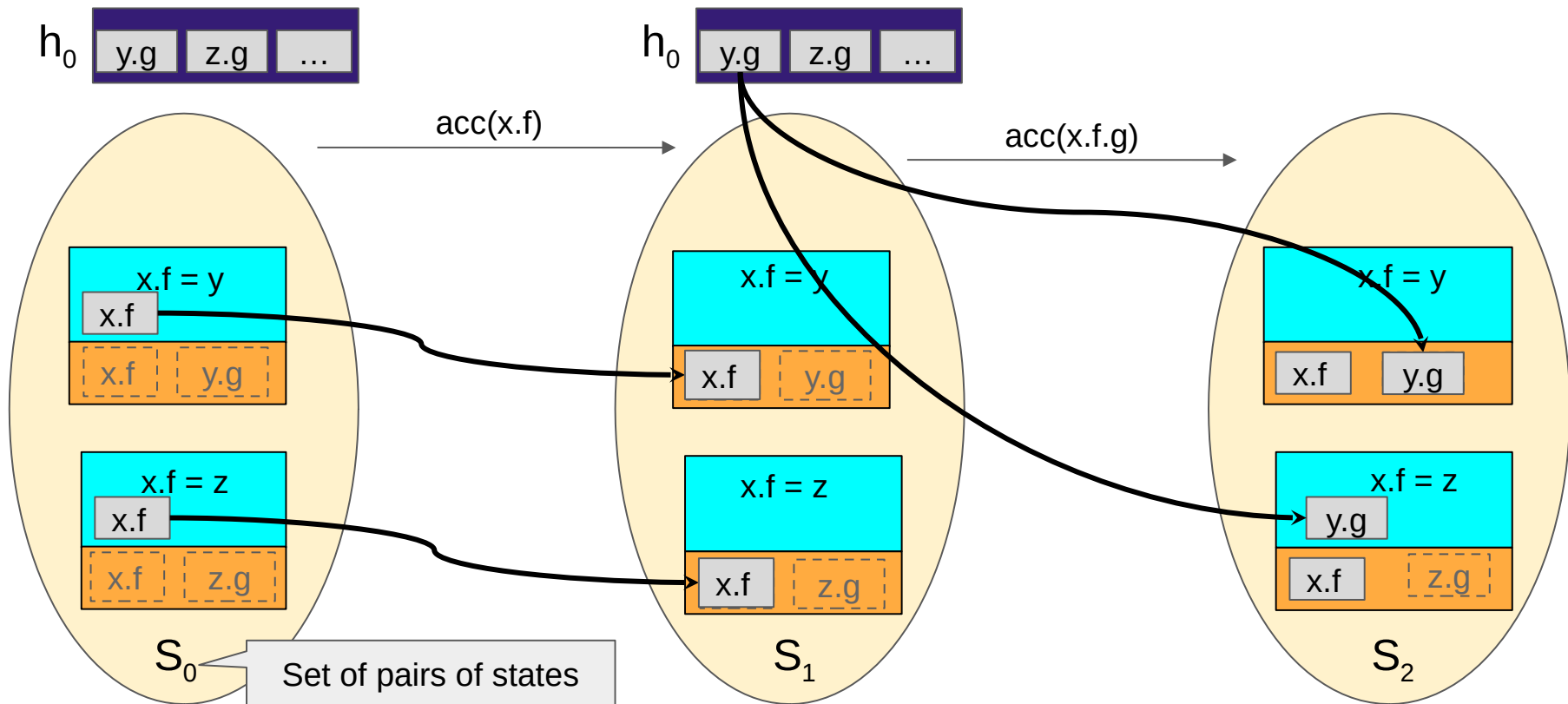
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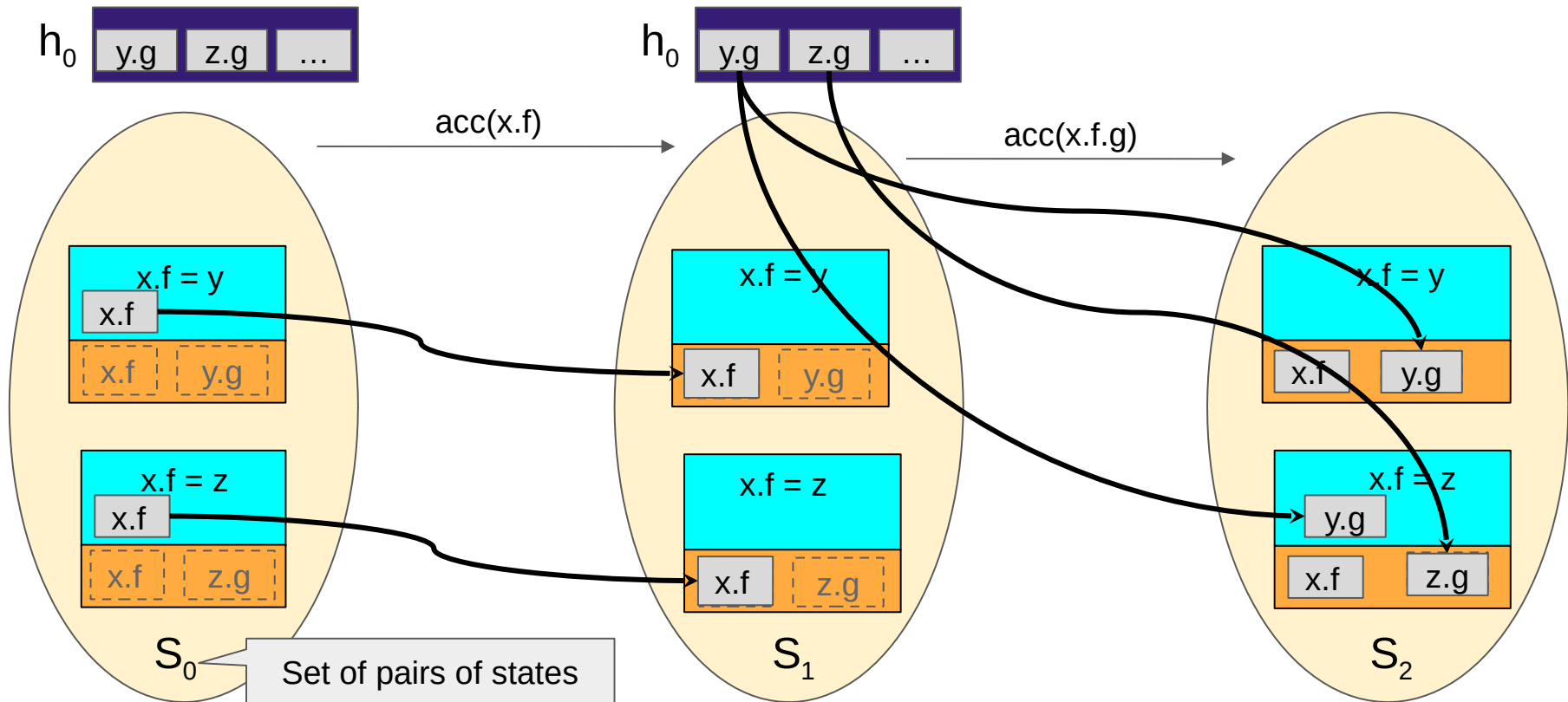
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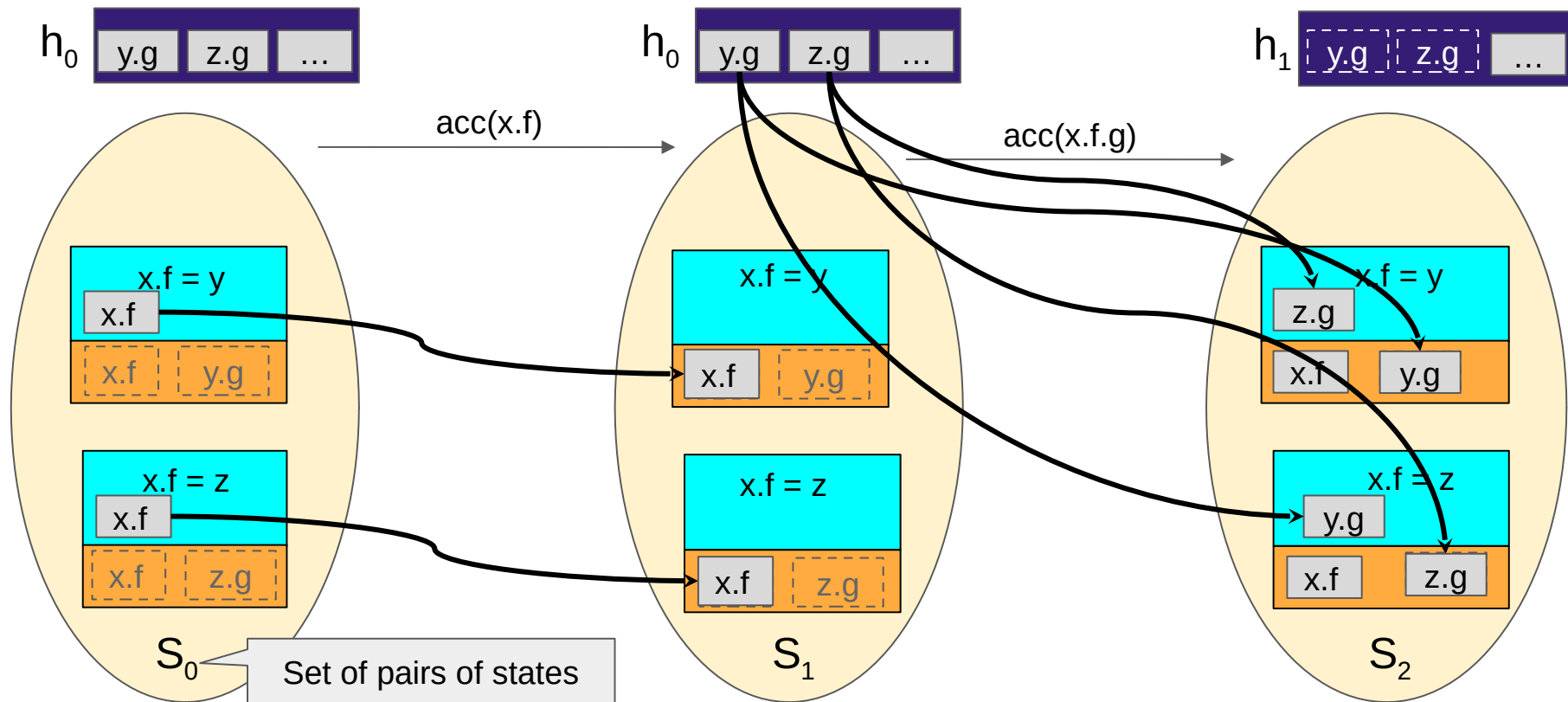
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# Using the package logic

**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \multimap \text{acc}(x.f) * \boxed{\text{acc}(x.f.g)}$

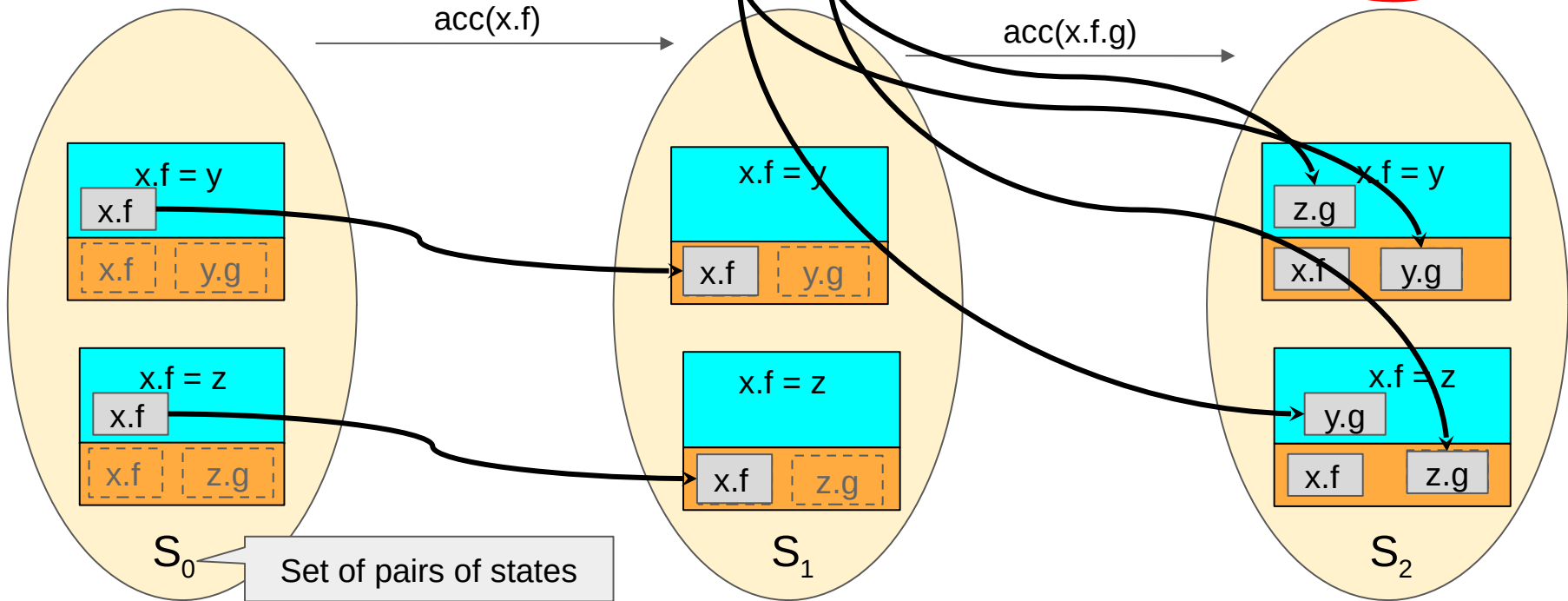
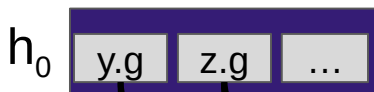
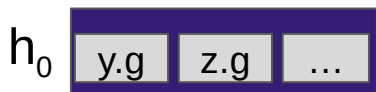


# Using the package logic

**package**  $\text{acc}(x.f) * (x.f = y \vee x.f = z) \multimap \text{acc}(x.f) * \boxed{\text{acc}(x.f.g)}$

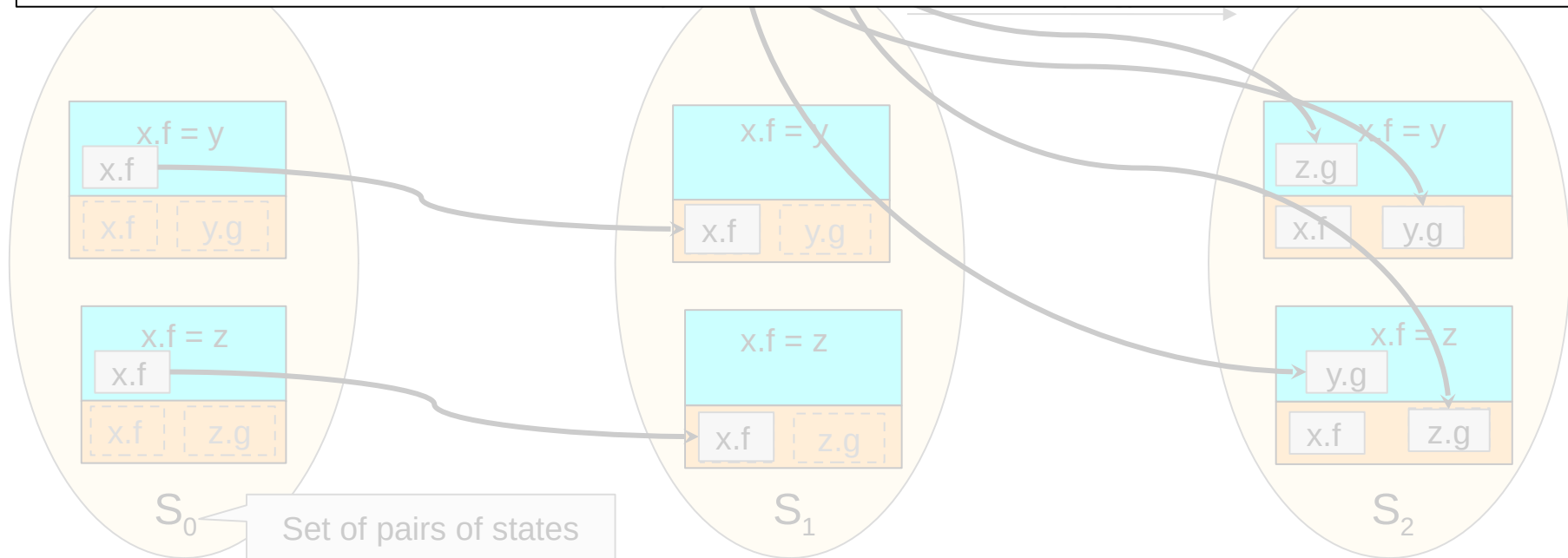


**Footprint**



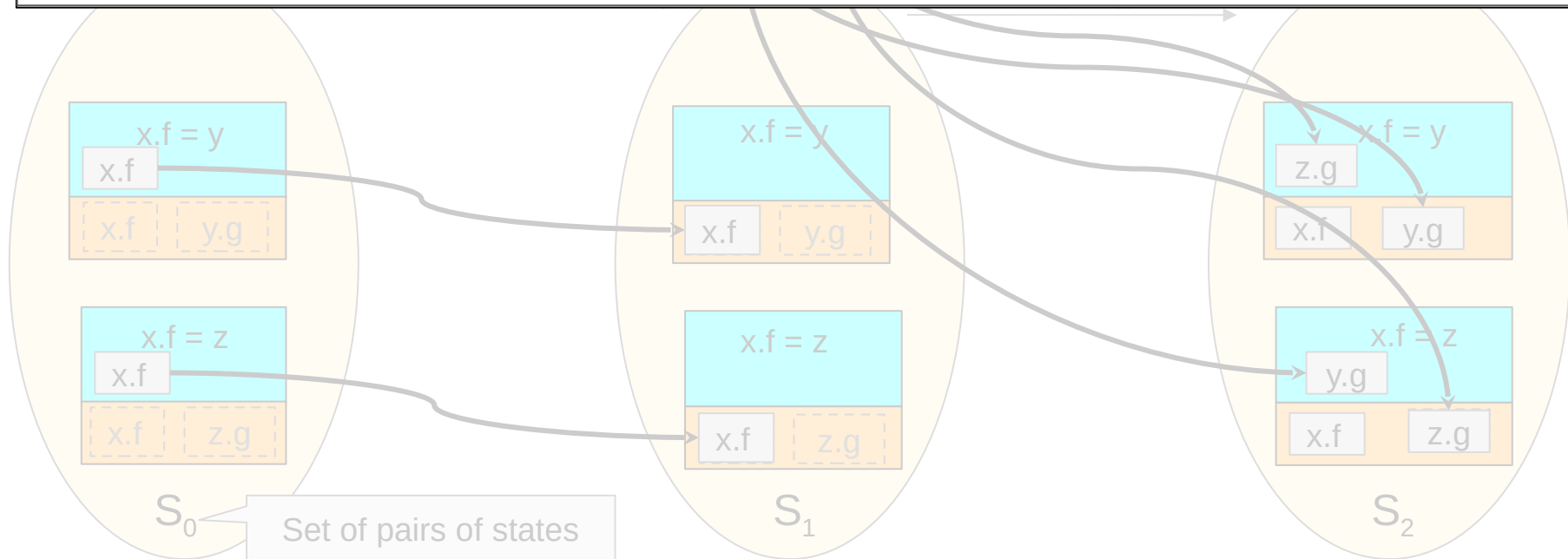
# Using the package logic

$$\begin{array}{c}
 \dots \\
 \hline
 \langle \text{acc}(\mathbf{x.f}), (h_0, S_0) \rangle \rightsquigarrow (h_0, S_1) \quad \text{Atom} \\
 \hline
 \langle \text{acc}(\mathbf{x.f}) * \text{acc}(\mathbf{x.f.g}), (h_0, S_0) \rangle \rightsquigarrow (h_1, S_2) \quad \text{Star} \\
 \hline
 \dots \\
 \hline
 \langle \text{acc}(\mathbf{x.f.g}), (h_1, S') \rangle \rightsquigarrow (h_1, S_2) \quad \text{Atom} \\
 \hline
 \langle \text{acc}(\mathbf{x.f.g}), (h_0, S_1) \rangle \rightsquigarrow (h_1, S_2) \quad \text{Extract}^\dagger
 \end{array}$$



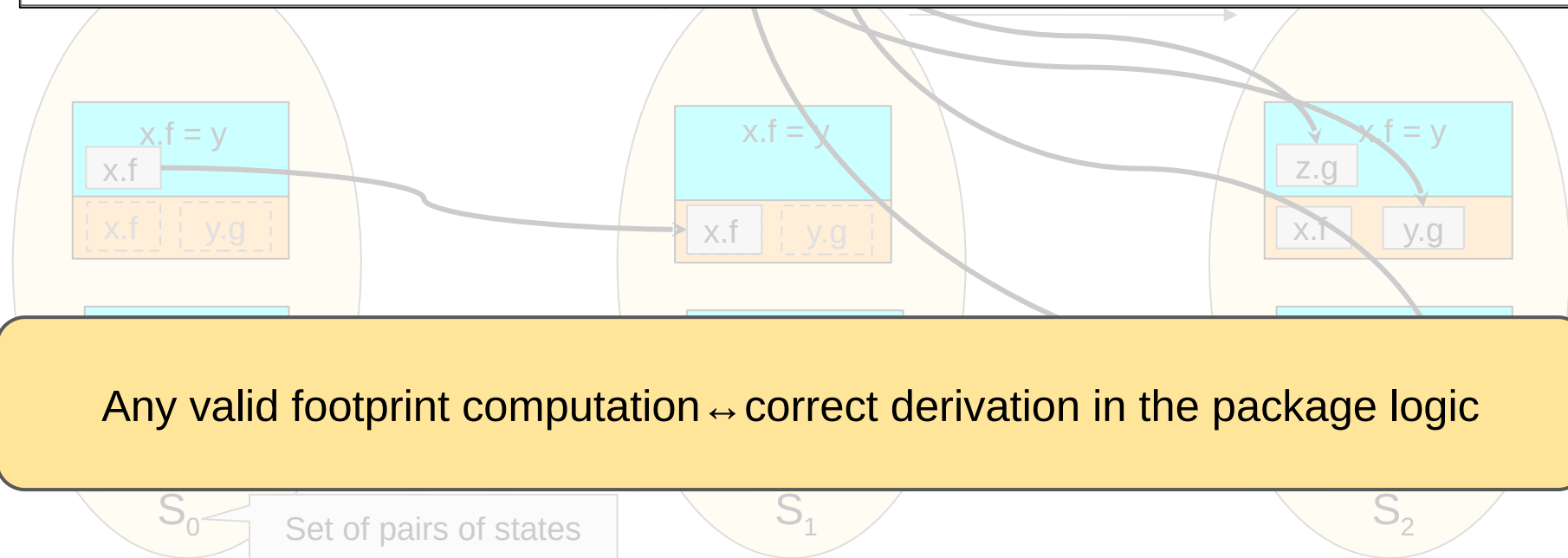
# Using the package logic

$$\begin{array}{c}
 \frac{\dots}{\langle \text{acc}(\mathbf{x.f}), (h_0, S_0) \rangle \rightsquigarrow (h_0, S_1)} \text{Atom} \qquad \frac{\dots}{\langle \text{acc}(\mathbf{x.f.g}), (h_1, S') \rangle \rightsquigarrow (h_1, S_2)} \text{Atom} \\
 \frac{\langle \text{acc}(\mathbf{x.f.g}), (h_0, S_1) \rangle \rightsquigarrow (h_1, S_2)}{\langle \text{acc}(\mathbf{x.f}) * \text{acc}(\mathbf{x.f.g}), (h_0, S_0) \rangle \rightsquigarrow (h_1, S_2)} \text{Star} \qquad \frac{\quad}{\quad} \text{Extract}^\dagger
 \end{array}$$



# Using the package logic

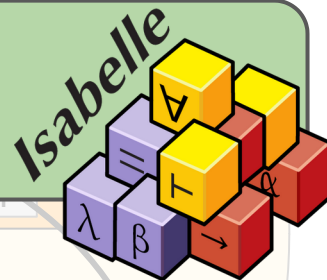
$$\begin{array}{c}
 \frac{\dots}{\langle \text{acc}(\mathbf{x.f}), (h_0, S_0) \rangle \rightsquigarrow (h_0, S_1)} \textit{Atom} \qquad \frac{\dots}{\langle \text{acc}(\mathbf{x.f.g}), (h_1, S') \rangle \rightsquigarrow (h_1, S_2)} \textit{Atom} \qquad \frac{\phantom{\dots}}{\phantom{\dots}} \dagger \\
 \frac{\phantom{\dots}}{\langle \text{acc}(\mathbf{x.f.g}), (h_0, S_1) \rangle \rightsquigarrow (h_1, S_2)} \textit{Extract} \\
 \hline
 \frac{\phantom{\dots}}{\langle \text{acc}(\mathbf{x.f}) * \text{acc}(\mathbf{x.f.g}), (h_0, S_0) \rangle \rightsquigarrow (h_1, S_2)} \textit{Star}
 \end{array}$$



# Using the package logic

$$\frac{\dots}{\langle \text{acc}(\mathbf{x.f}), (h_0, S_0) \rangle \rightsquigarrow (h_0, S_1)} \textit{Atom} \quad \frac{\dots}{\langle \text{acc}(\mathbf{x.f.g}), (h_1, S') \rangle \rightsquigarrow (h_1, S_2)} \textit{Atom} \quad \frac{\dots}{\langle \text{acc}(\mathbf{x.f.g}), (h_0, S_1) \rangle \rightsquigarrow (h_1, S_2)} \textit{Star} \quad \frac{\dots}{\langle \text{acc}(\mathbf{x.f}) * \text{acc}(\mathbf{x.f.g}), (h_0, S_0) \rangle \rightsquigarrow (h_1, S_2)} \textit{Extract}$$

The package logic is sound and complete for computing footprints



Any valid footprint computation  $\leftrightarrow$  correct derivation in the package logic

$S_0$

Set of pairs of states

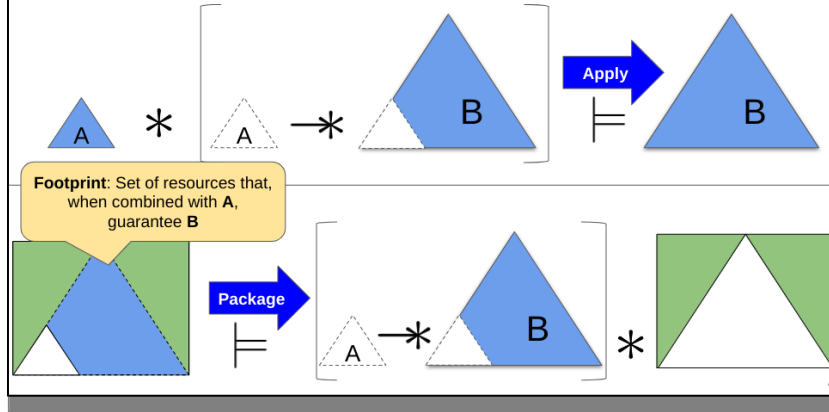
$S_1$

$S_2$

# Conclusion

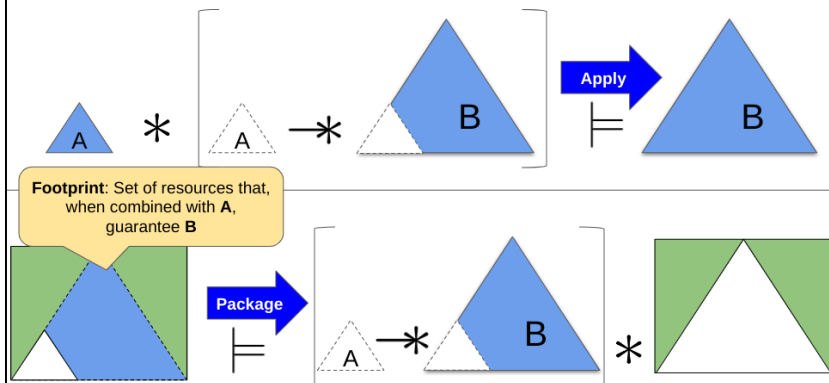
# Conclusion

Background: Ghost operations to manipulate magic wands



# Conclusion

Background: Ghost operations to manipulate magic wands



4

Challenge: Automatically compute a **small, valid** footprint

Not automated: User chooses footprint

**Witnessing the elimination of magic wands**

Stefan Blom · Marieke Huisman

2015

VerCors

Automated

Computed footprint sometimes invalid ❌

**Lightweight Support for Magic Wands in an Automatic Verifier**

Malte Schwerhoff<sup>1</sup> and Alexander J. Summers<sup>2</sup>

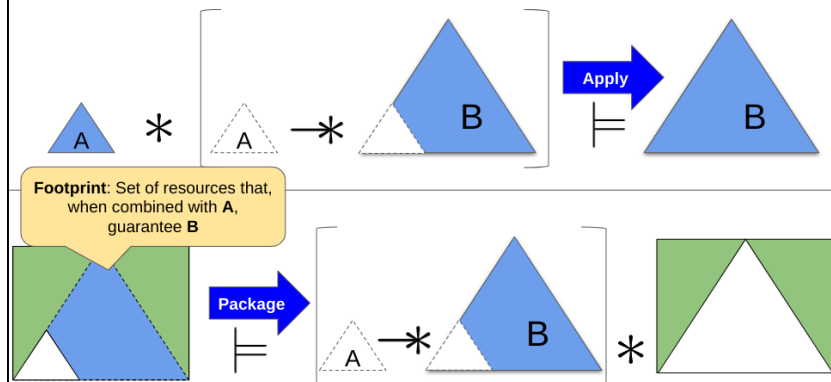
2015

VIPER

6

# Conclusion

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## Contributions

Standard wand

### 2. Combinable wand

Novel definition of the magic wand, with useful properties when combined with fractional permissions

Parametric logical framework for computing footprints  
Sound and complete

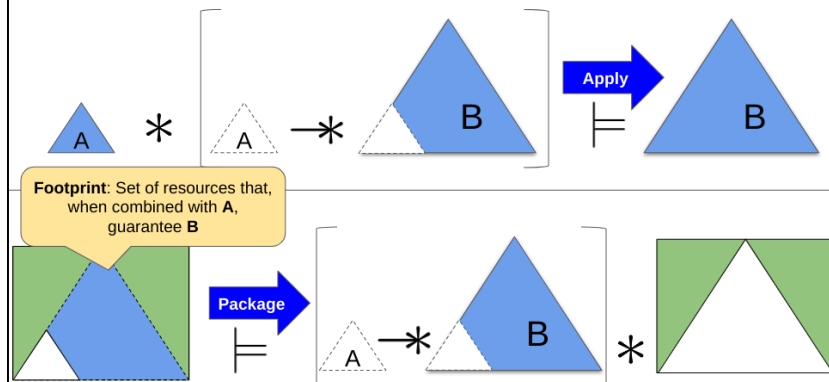
### 1. Package logic

### 3. Implementations and evaluation

8

# Conclusion

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