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Modeling Value at Risk with Factors

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1. Introduction

Factor models are standards in investment management. For decades, Barra factor models have provided valuable risk forecasts and inputs for the portfolio construction process. Most uses of factor models have targeted longer horizons of months or years. However, we demonstrate in this paper that factor models can also provide accurate risk forecasts for shorter horizons of one to ten days. Furthermore, factor models have the advantage of explaining risk sources and providing consistency in risk management processes across all time horizons.

We present a factor model with a methodology appropriately tailored to shorter horizons. Our basic approach is to retain the same common risk factors currently used in the Barra Integrated Model (BIM) and adopt a number of techniques that exploit daily data. As we show for different asset classes, markets, and sectors, this factor model approach yields similarly accurate shorter horizon risk forecasts compared to asset-by-asset approaches.

We specifically focus on the accuracy of Value-at-Risk (VaR) estimates. We estimate VaR with volatility forecasts from two shorter horizon versions of Barra factor models: the Daily Factor Return (DFR) Model and the Scaled BIM Matrix (SBM) Model. The DFR model is a true daily factor model: it uses all the available daily data for the factor model, while the SBM model scales down the monthly estimates to the daily level. We benchmark these two models versus a method that uses an exponentially weighted moving average (EWMA) of realized portfolio returns. This method is similar to the asset-by-asset covariance approach. We show that the DFR model provides VaR estimates that are similar in accuracy to EWMA and outperforms SBM.

The paper proceeds as follows. Section 2 presents the three methods for VaR estimation: DFR, SBM, and EWMA. Section 3 provides an overview of the procedure and the test statistics we use to determine performance of the three methods. In Section 4, we describe the data and markets used in the study. In Section 5, we present the results of the performance tests for local equity market models. Section 6 covers the performance tests for local fixed income market models. Section 7 discusses the results for the integrated global model. We conclude in Section 8.

2. Methods for Estimating Volatility and VaR

We present and compare three methods for estimating the short horizon volatility of the returns required to calculate VaR: Daily Factor Return (DFR), Scaled BIM Matrix (SBM), and Exponentially Weighted Moving Average (EWMA). In conceptual terms, the DFR method consists of appropriately adjusting the factor model to the daily frequency of data, including the use of daily asset returns and a 21-day half-life for the covariance matrix. SBM uses the longer horizon factor model estimates scaled down to daily frequency. The EWMA method does not utilize factors and is closest to the asset-by-asset covariance method. In this paper, we show results for all three methods, but we focus primarily on the performance of the DFR method.[1]

2.1. The Daily Factor Return (DFR) Method

The DFR method is an adjustment of the longer horizon factor model to daily data. In the context of shorter horizon risk forecasts, analogous to the monthly factor model, we model daily asset returns in terms of one-day factor returns plus a residual:

Equation 2.1
$$\boldsymbol{r} = \boldsymbol{X}\boldsymbol{f} + \in$$

where ***r*** is the vector of asset returns, ***X*** is the matrix of asset exposures, and ***f*** is the vector of factor returns.

We can then calculate portfolio variance as:

Equation 2.2
$$\boldsymbol{\sigma^2} = \boldsymbol{h^T}(\boldsymbol{X^T F X} + \boldsymbol{\Delta})\boldsymbol{h}$$

where ***h*** is the vector of portfolio holdings, ***F*** is the covariance matrix of factor returns, and Δ is the diagonal matrix of specific return variances.[2] We estimate the covariance matrix, ***F***, with an exponentially weighted moving average:

Equation 2.3
$$\boldsymbol{F_t} = \boldsymbol{\lambda F_{t-1}} + (\boldsymbol{1} - \boldsymbol{\lambda})\boldsymbol{f_t f_t^T}$$

where $\boldsymbol{f_t}$ is the vector of factor returns on day ***t***, λ is the decay parameter calculated from the half-life ($\lambda = (\frac{1}{2})^{1/h}$, for half-life *h*). We assume that the returns are characterized by a zero mean.

We use a half-life of 21 days, which provides the best tradeoff between the responsiveness and noisiness of the risk forecast. We have found that using a longer half-life for off-diagonal elements of the covariance matrix, and updating variances with a 21-day half-life, does not improve out-of-sample forecasts.

We estimate specific return variances in the same way as for the longer horizon models.[3] To obtain one-day specific return variances, we scale the variances to a one-day horizon.

[1] BarraOne version 3.3 incorporates the DFR method.
[2] We omit the time subscripts for simplicity.
[3] For more details, see the Barra Risk Model Handbook.

2.2. The Scaled BIM Matrix (SBM) Method

We can also estimate volatility with the monthly horizon BIM matrix scaled to a one-day horizon.[4] For this method, we start with the monthly versions of Equations 2.1 and 2.2 above. We then calculate:

Equation 2.4
$$\boldsymbol{F} = \left(\frac{\mathbf{1}}{\mathbf{21}}\right)\boldsymbol{F^m}$$

where $\boldsymbol{F^m}$ is the Barra Integrated Model (BIM) monthly covariance matrix, and **21** is the average number of business days in a month.

Because updates to $\boldsymbol{F^m}$ occur only once a month, we also experimented with various other methods for updating $\boldsymbol{F^m}$ to account for intra-month changes in the volatility level. For example, we tried scaling $\boldsymbol{F^m}$ daily with the volatility of a market index. However, we found that none of these methods provided VaR backtesting performance on par with DFR models.

2.3. The Baseline (EWMA) Method

A non-factor-based alternative method is the Exponentially Weighted Moving Average of realized returns. We use EWMA as the baseline for comparing the DFR and SBM models. Using the same 21-day half-life as the daily factor return covariance in Equation 2.3, we calculate:

Equation 2.5
$$\boldsymbol{\sigma_t^2} = \boldsymbol{\lambda\sigma_{t-1}^2} + (\mathbf{1} - \boldsymbol{\lambda})\boldsymbol{r_t^2}$$

where $\boldsymbol{r_t}$ is the realized portfolio return.

We also explicitly tested the asset-by-asset covariance approach and found that it produces risk forecasts comparable in accuracy with portfolio level EWMA. Therefore, in the remainder of the paper, we present only the results of the simpler EWMA model.

If the only required risk measure is portfolio volatility, then the EWMA of the realized portfolio return history may be a good risk benchmark. However, EWMA is appropriate only for consistent (i.e., low turnover) portfolios over a short time horizon. Absent such qualifications, the forecasts are problematic, and furthermore, without factors, EWMA allows neither risk decomposition nor portfolio optimization. For these reasons, EWMA is not a practical approach for portfolio risk management. For our purposes, EWMA is nevertheless a useful benchmark, since its factorless approach reflects only trends in volatility, and we can evaluate whether our factor-based approach yields better forecasts compared to simple trending.

2.4. VaR Estimates

From these three methods, we estimate the shorter horizon volatility, σ, which we subsequently use to calculate portfolio VaR with a simple parametric assumption:

Equation 2.6
$$\boldsymbol{VaR} = \boldsymbol{\alpha} \cdot \boldsymbol{\sigma}$$

where α is a multiplier derived from the cumulative distribution for a standard Normal distribution. This approach assumes that the portfolio returns are normally distributed, which is reasonable given that the portfolios we use for the empirical study are essentially linear in the factors.

[4] Prior to the release of BarraOne version 3.3, BarraOne used SBM to calculate shorter horizon forecasts.

3. Performance Assessment of the Methods

3.1. Basic Procedure

We assess the performance of the methods for one-day horizon VaR forecasts using the following procedure. First, we start with a one-year (252 business-day) observation window. Second, using those data, we produce an out-of-sample, one day forward forecast for each of the three methods described above. Third, for each of the three methods, we determine whether the realized loss exceeds the 95% VaR. We then assess, using the appropriate test statistics, whether that fraction is less than or greater than 5%, thus determining whether the methods over- or underforecast VaR. We also employ an alternative test, based on standardized returns, to confirm the results of the VaR analysis.

3.2. Test Statistics

3.2.1. Kupiec Confidence Regions

The test statistic that we use to assess whether there are more than 5% violations of VaR is the Kupiec (1995) confidence region, which is defined by the tail points of log-likelihood ratios:

Equation 3.1
$$LR = -2\ln\left\{(1-p)^{T-N}\, p^{N}\right\} + 2\ln\left\{\left[1-\left(N/T\right)\right]^{T-N}\left(N/T\right)^{N}\right\}$$

where $\boldsymbol{T}$ defines the number of observation points in the series of volatility data, $\boldsymbol{N}$ is the number of VaR events over a period of $\boldsymbol{T}$, and $\boldsymbol{p}$ is the probability of VaR violations for a corresponding confidence level, e.g., 0.05 for 95% VaR. This test statistic is asymptotically chi-square distributed with one degree of freedom under the null hypothesis that $\boldsymbol{p}$ is the true probability. Thus, the critical value for 95% confidence level is 3.84. In other words, we would reject the null hypothesis that violations should occur 5% of the time if $LR > 3.84$ for a 95% confidence level.[5]

The critical value defines a range of acceptance for the forecasts. The idea is that in repeated sampling, 5% violations of VaR would not be exactly 5%; instead, they would range from 2.54% to 7.92% for a 252-day observation window. If the violations are less than 2.54% of the sample, then the model has most likely overforecast VaR. If the violations are more than 7.92% of the sample, then the model has most likely underforecast VaR. We use these ranges to determine whether over- or underforecasting occurs.

3.2.2. Bias Statistics

Another method commonly used to assess the risk forecast of a model is the bias statistic, defined as the standard deviation of the returns normalized by the forecast volatility. The bias statistic can be interpreted as a ratio of realized risk over forecast risk, and therefore, if forecasts are perfect, then the statistic should equal one.

More precisely, define the normalized return at time $\boldsymbol{t}$ as the return divided by the beginning-of-period risk forecast:

Equation 3.2
$$\boldsymbol{b_t = r_t / \sigma_t}$$

The bias statistic is the realized standard deviation of the normalized return over some window ending at $\boldsymbol{t=n}$:

Equation 3.3
$$\boldsymbol{B_n = \sqrt{\frac{1}{T}\sum_{t=n-T}^{n}\left(b_t - \bar{b}_n\right)^2}}$$

[5] Note the distinction between the 95% in the cumulative loss distribution for the VaR calculation and the 95% confidence level in the criteria for the acceptance/rejection of a VaR model. The two values are conceptually distinct and could be set independently.

The bias statistic focuses on the model's performance for typical returns, i.e., the center of the return distribution. In contrast, the VaR violations test focuses on the tails of the return distribution. For our tests, we calculated annual bias statistics (***T***≈252). As with the Kupiec confidence regions, the rejection regions for the bias statistics were also defined with the 95% confidence level. A portfolio is in violation if its bias statistic is outside of the interval $1 \pm \sqrt{2/T}$. Thus for ***T***=252, bias statistics within 0.91 < B < 1.09 are considered acceptable.

Note that with bias statistics, similar to the VaR case above, a value below (above) the confidence interval corresponds to an overforecast (underforecast).

4. Data

4.1. Period and Frequency

The data consist of daily asset returns and monthly exposures from the period 1 Jan 2003 to 31 May 2009. For the initial buildup of the risk forecast, we use the data from 1 Jan 2003 to 31 May 2003. For the one-day forecasts and tests, we use non-overlapping windows of one year (252 business days). This procedure results in six testing subperiods for the years ending 31 May 2004 through 2009.

4.2. Models and Markets

We use the Barra Integrated Model (BIM 207) to generate forecasts. The Barra Integrated Model (BIM) is a multiple-asset-class risk model that couples breadth of coverage (global equities, global bonds, currencies, commodities, and hedge funds) with the depth of analysis provided by our local models. The model is suitable for a wide range of investment needs, from analysis of a single country equity portfolio to a plan-wide international portfolio of equities and bonds. In-depth, accurate, local analysis requires that we choose factors that are effective in any market under study, and that we recognize that the factors developed for one market are not always appropriate for use in other markets. Thus, individual risk models are built for each market, the better to capture the behavior of the local securities.

Because it starts with individual risk models for each market, BIM retains the accuracy and detail provided by local models while forecasting risk for the global markets. For example, on any U.K. equity portfolio, the BIM risk forecast exactly matches the risk forecast from the UKE7L long-investment-horizon equity model. Consequently, the accuracy of BIM forecasts is contingent on the accuracy of local model forecasts.

We assess local and global market risk forecasting accuracy by starting local and building up to global, much as the model does. First, we assess the accuracy of shorter horizon local equity market forecasts. Second, we move on to the local fixed income market risk forecasts. Third, we assess the accuracy of global risk forecasts from BIM.

In these assessments, we focus on three representative markets: the U.S., Continental Europe, and Japan. For the local equity markets, we also examine the U.K. model, while for fixed income, some of the models include emerging market factors.

All the models we tested use the Barra specific risk model, which assumes that specific return is uncorrelated to factor return. Although our primary results in this paper are for one- and two-day forecasts, the models can produce forecasts for horizons of several days. For risk estimates over horizons greater than one day, we neglect serial correlations, and we scale covariances by the number of days in the horizon. Corrections to risk forecasts due to serial correlations tend to be small for single-country portfolios, and our empirical results support this assumption. However, the effects are not negligible in the case of global portfolios, and we later detail how we correct for serial correlation in the global context.

5. Equity Local Market Models

5.1. Equity Models

We use four longer horizon local equity market models from BIM: USE3L (U.S.), EUE2L (Continental Europe), UKE7L (U.K.), and JPE3 (Japan). For each of these models and each of the portfolios described below, we produce risk and VaR forecasts using the three methods initially set out in this paper: DFR, SBM, and EWMA.

5.2. Test Portfolios

We aimed to produce a set of test portfolios that reflect common investment styles and provide sufficient diversity for testing purposes. To that end, we constructed the test portfolios by tilting on each of the Barra fundamental factors in the equity models:

- Style factors: For each style factor, we ranked assets in the estimation universe by exposure. We then took the highest and lowest decile assets in terms of exposures, and from them we constructed two capitalization-weighted test portfolios.
- Industry factors: For each industry factor, we constructed a capitalization-weighted portfolio of all assets in the estimation universe with exposure to that industry factor.[6]
- Country factors: For each country factor, we constructed a capitalization-weighted portfolio of all assets in the estimation universe with exposure to that country factor.

We rebalance portfolios on a monthly basis in order to be consistent with model updates. In addition, in the subsequent results, we focus on total rather than active risk of the portfolios, since the results for both were found to be qualitatively similar.

5.3. Results

5.3.1. Similar Accuracy for DFR and EWMA

Table 5.1 provides a summary of the main results. For each of the four portfolio groups corresponding to the local equity market models, and for each of the three models, the table provides the percentage of times when the model over- or underforecasts 95% one-day VaR. The results are aggregated for all portfolio-years, yielding, for example, 360 distinct test scenarios for the U.S.

Table 5.2 provides an analogous summary of bias statistic tests for the portfolio's daily returns. We find that the bias statistic tests are more discriminating than the VaR backtests. In a latter part of this paper, we use this observation to compare different methods for estimating a global covariance matrix.

Both sets of results demonstrate the accuracy of DFR and EWMA over SBM. The forecasts for DFR and EWMA are relatively close in terms of over- and underforecasting. However, both are much more accurate compared to SBM, which has violations of up to approximately 50%. These results support our main conclusion that daily factor return models can produce VaR forecasts for shorter horizons that are comparable to EWMA or asset-by-asset methods.

[6] Industry and country portfolios tend to be more concentrated than style-tilted portfolios. Therefore, we apply additional filtering to eliminate "thin" portfolios. A portfolio is thin if the effective number of stocks (defined as the inverse of the Herfindahl-Hirschman Index) is less than two in the U.K. and less than three otherwise. Any portfolio that is thin at some point during a test period is excluded from the analysis. The Herfindahl-Hirschman Index (*HHI*) is a widely used measure of market concentration. In our context, $HHI = w_1^2 + w_2^2 + \ldots + w_n^2$, where n is the number of assets in a portfolio and w is the weight of each asset.

Table 5.1: Summary of 95% One-Day VaR Backtesting Results (OF=overforecast; UF=underforecast)

		EWMA		DFR		SCM	
Portfolio Group	# Port.	% OF	% UF	% OF	% UF	% OF	% UF
USE3L Factor Portfolios	**360**	**2.5%**	**17.5%**	**6.9%**	**13.3%**	**49.2%**	**32.5%**
EUE2L Factor Portfolios	**342**	**5.3%**	**9.6%**	**5.3%**	**12.6%**	**41.2%**	**29.2%**
JPE3 Factor Portfolios	**354**	**4.0%**	**3.4%**	**6.8%**	**4.0%**	**20.3%**	**31.6%**
UKE7L Factor Portfolios	**228**	**3.9%**	**7.0%**	**8.3%**	**6.6%**	**43.4%**	**28.5%**
AVERAGE		**4.2%**	**8.7%**	**6.7%**	**9.0%**	**37.8%**	**30.2%**

The tests were conducted for 60 USE3L, 57 EUE2L, 59 JPE3, and 38 UKE7L factor portfolios over 6 testing periods resulting in 360, 342, 354, and 228 independent tests respectively. A lower percentage of over- and underforecast is preferable.

Table 5.2: Summary of Bias Statistics (OF=overforecast; UF=underforecast)

		EWMA		DFR		SCM	
Portfolio Group	# Port.	% OF	% UF	% OF	% UF	% OF	% UF
USE3L Factor Portfolios	**360**	**0.3%**	**21.9%**	**8.3%**	**17.5%**	**56.4%**	**33.9%**
EUE2L Factor Portfolios	**342**	**1.2%**	**19.3%**	**5.3%**	**33.3%**	**52.3%**	**38.0%**
JPE3 Factor Portfolios	**354**	**2.0%**	**14.4%**	**7.1%**	**28.5%**	**25.1%**	**45.8%**
UKE7L Factor Portfolios	**228**	**1.8%**	**27.2%**	**17.5%**	**18.4%**	**53.9%**	**32.0%**
AVERAGE		**1.6%**	**20.2%**	**10.3%**	**25.6%**	**42.7%**	**38.7%**

5.3.2. The Value of Factor Models

The advantages of factor models can be seen by comparing and contrasting the results for two portfolios: the USE3L Large Cap portfolio (10% of all assets with the highest Size factor exposures), and the UKE7L Leisure Equipment industry portfolio (a relatively concentrated industry-tilt portfolio). Given their respective tilts, we should expect common factors, particularly Size, to be relatively more important in the USE3L Large Cap portfolio. As for the UKE7L Leisure Equipment portfolio, we expect the opposite — relatively high specific risk — since the portfolio has three effective stocks on average. Figures 5.1 and 5.2 show that these expectations are borne out. In the DFR model risk forecasts for the U.K. portfolio, the contribution of specific risk to VaR is often higher than the contribution of common factor risk.

The decomposition of risk into common and specific components, and the further breakdown of the common factors of risk, can be as valuable to managers for the shorter horizon as it has been for the longer horizons. Managers can report and help others understand the sources of risk; in the comparison outlined above, the sources of risk for the two portfolios are very different and may affect the perceived level of diversification in a portfolio. Furthermore, the explanations of risk can be used to control exposure to certain types of risk — such as those to Size or an industry in the cases described above. Understanding risk can provide valuable inputs for a variety of risk management functions, and factor models are important tools for discovering the sources of portfolio risk, whether on a shorter or longer horizon.

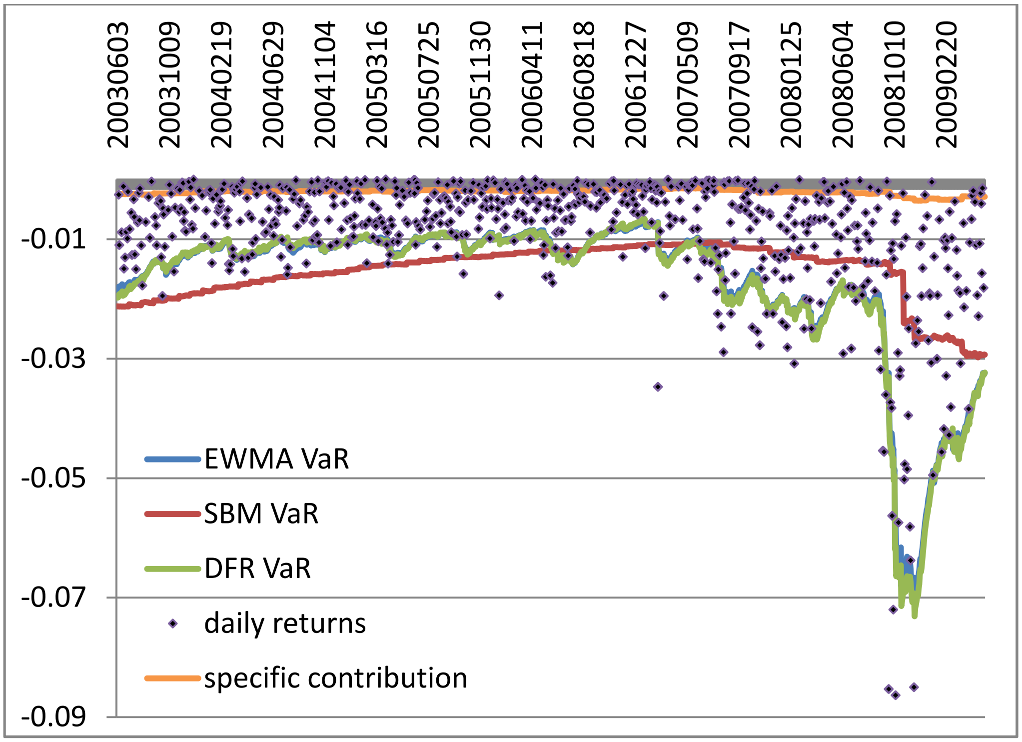

Figure 5.1: One-Day Horizon VaR Backtest, U.S. Large Cap Portfolio

Note the small contribution of specific risk to DFR and SBM VaR.

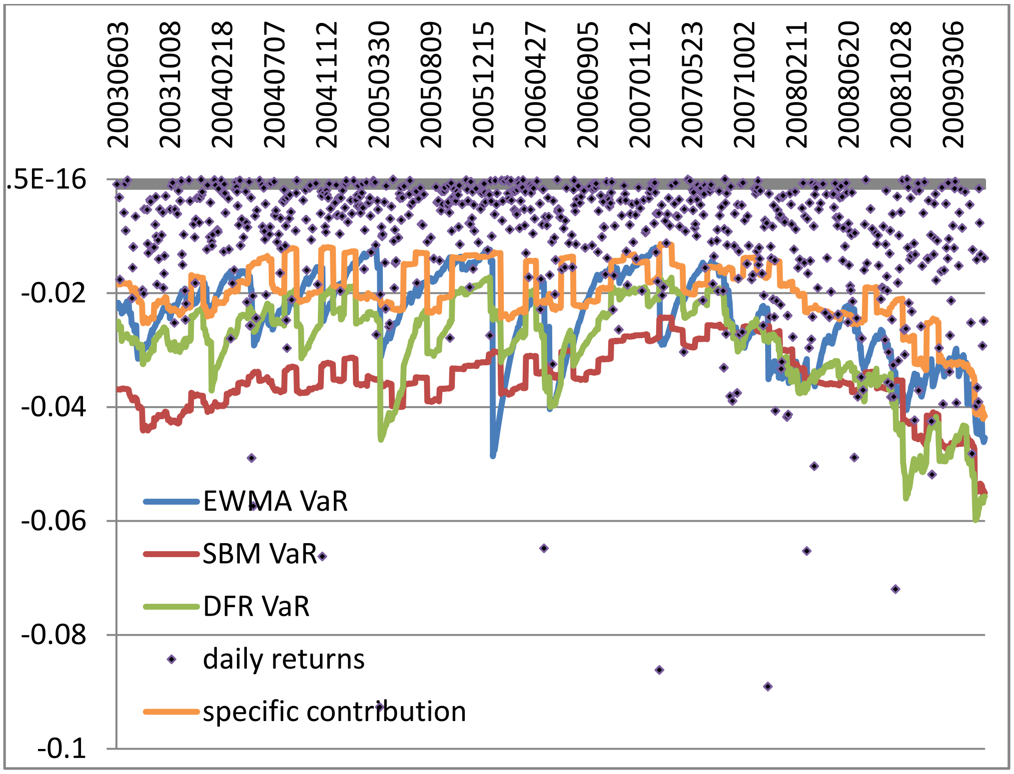

Figure 5.2: One-Day Horizon VaR Backtest, U.K. Leisure Equipment Industry Portfolio

For a concentrated portfolio, the DFR VaR model predicts higher risk than Return EWMA. This is due to the high contribution of model specific risk.

6. Fixed Income Local Market Models

6.1. Fixed Income Models

We use three longer horizon fixed income models: USB3 (U.S.), EUB2 (Euro Zone), and JPB3 (Japan). USB3 and EUB2 also incorporate an emerging market block, and we use this feature to produce forecasts for the emerging market portfolios described below. The factor model includes six types of factors[7]:

- Term Structure Shift/Twist/Butterfly
- Swap Spread
- Sector/Rating Credit Spread
- Emerging Market Spread
- Implied Volatility
- MBS Prepayment

We use the three methods, DFR, SBM, and EWMA, to estimate portfolio risk. We made two changes to the DFR factor return estimates compared to those from the longer horizon model; both changes enhanced shorter horizon forecasts. First, we allowed more outliers in the DFR model estimation; this choice improved model responsiveness during periods of high volatility and for high-yield portfolios. Second, we weighted the average Option Adjusted Spread (OAS) returns by both asset duration and amount outstanding in the DFR model, rather than just by duration as is done in the longer horizon model. Further, weighting by amount outstanding improves responsiveness, since amount outstanding is a proxy measure for asset liquidity.

6.2. Test Portfolios

We tested the three models on a wide range of fixed income portfolios:

1. Index portfolios: These portfolios consist of the Merrill Lynch index tracking portfolios in the eurozone, Japan, and the U.S.[8] They are a diverse set of portfolios including treasuries, government agencies, and a variety of corporate portfolios including High Yield (for the U.S. and the Euro Zone).
2. Treasury portfolios: These portfolios consist of treasuries for a large number of European governments, including those in the Eurozone, as well as Switzerland, the U.K., Norway, and Denmark.
3. Emerging market (EM) portfolios. These portfolios include a global emerging market bond index, the EMBI+ index, as well as portfolios made up of issuers from a single emerging market. The individual-country EM portfolios include sovereign, agency, and corporate bonds, while the EMBI+ index is made up of bonds from sovereign issuers only.
4. Random portfolios: These portfolios consist of relatively small, randomly selected bonds. We constructed 80- and 20-bond portfolios of investment-grade and high-yield bonds denominated in euros (a total of 4 different portfolio sets).[9]

6.3. Results

6.3.1. Similar Accuracy for DFR and EWMA

The results are presented in Tables 6.1 and 6.2. As with the equity market forecasts, DFR and EWMA are similar in performance, and both outperform the SBM method. The differences are

[7] Details of the fixed income risk model factor return calculations can be found, for example, in Fixed Income Risk Modeling (Breger & Cheyette, 2005).
[8] Details of the indices are provided in Appendix A1.
[9] For each type of portfolio, we constructed 50 instances by random selection at the start date and then maintained the portfolios with as little turnover as possible (replacing matured bonds by random selection from bonds with the same rating). The precise composition of the portfolios by issuer rating is fixed and described in the Appendix, while composition according to attributes such as industry and duration are allowed to vary randomly (although in practice they are usually close to market averages). Assets are weighted by amount outstanding.

particularly striking for the USA Merrill Indices and the 80-Bond Euro High Yield. Between DFR and EWMA, DFR seems to underforecast less, while EWMA has fewer overforecasts.

As observed for equity portfolios, the bias statistic appears to be a more discriminating measure of risk, and a somewhat higher proportion of portfolios fall out of the confidence bound than for VaR. However, qualitative results are similar.

Table 6.1: Summary of VaR Method Performance (OF=overforecast; UF=underforecast)

		EWMA		DFR		SBM	
Portfolio Group	# Port.	% OF	% UF	% OF	% UF	% OF	% UF
Emerging Market Bond	**13**	**5.1%**	**9.0%**	**28.2%**	**0.0%**	**64.1%**	**3.8%**
Euro Merrill Indices	**12**	**1.4%**	**13.9%**	**9.7%**	**18.1%**	**38.9%**	**22.2%**
European Treasuries	**16**	**2.1%**	**1.0%**	**1.0%**	**0.0%**	**36.5%**	**31.3%**
Japan Merrill Indices	**12**	**1.4%**	**9.7%**	**19.4%**	**8.3%**	**18.1%**	**20.8%**
USA Merrill Indices	**20**	**0.0%**	**13.3%**	**0.0%**	**23.3%**	**50.0%**	**32.5%**
20-Bond Euro Invest Grade	**50**	**8.3%**	**3.0%**	**11.3%**	**6.7%**	**41.3%**	**30.7%**
80-Bond Euro Invest Grade	**50**	**2.0%**	**0.0%**	**7.7%**	**0.3%**	**39.0%**	**33.0%**
20-Bond Euro High Yield	**50**	**7.7%**	**24.7%**	**47.0%**	**9.0%**	**75.0%**	**16.3%**
80-Bond Euro High Yield	**50**	**1.3%**	**30.0%**	**28.7%**	**7.0%**	**77.3%**	**16.3%**
AVERAGE		**3.3%**	**11.6%**	**17.0%**	**8.1%**	**48.9%**	**23.0%**

Lower values are preferable.

Table 6.2: Summary of Bias Statistics (OF=overforecast; UF=underforecast)

		EWMA		DFR		SBM	
Portfolio Group	# Port.	% OF	% UF	% OF	% UF	% OF	% UF
Emerging Market Bond	**13**	**9.0%**	**30.8%**	**51.3%**	**7.7%**	**73.1%**	**16.7%**
Euro Merrill Indices	**12**	**0.0%**	**13.9%**	**13%**	**27.8%**	**52.8%**	**29.2%**
European Treasuries	**16**	**0.0%**	**15.0%**	**0.0%**	**14.6%**	**43.8%**	**33.3%**
Japan Merrill Indices	**12**	**0.0%**	**9.7%**	**19.4%**	**8.3%**	**18.1%**	**20.8%**
USA Merrill Indices	**20**	**0.0%**	**43%**	**29.2%**	**30.6%**	**38.9%**	**36.1%**
20-Bond Euro Invest Grade	**50**	**0.7%**	**16.3%**	**3.3%**	**20.0%**	**49.7%**	**33.3%**
80-Bond Euro Invest Grade	**50**	**0.0%**	**15.3%**	**2.3%**	**13.3%**	**50.0%**	**33.0%**
20-Bond Euro High Yield	**50**	**7%**	**45%**	**60.3%**	**19.0%**	**80.0%**	**17.3%**
80-Bond Euro High Yield	**50**	**3.3%**	**42.0%**	**55.3%**	**22.7%**	**83.3%**	**16.7%**
AVERAGE		**2.2%**	**25.7%**	**26.0%**	**18.2%**	**54.4%**	**26.3%**

6.3.2. The Value of Factor Models

An examination of return and risk contributions shows the value of factor models. Figure 6.1 shows that the factor decomposition of return for the U.K. treasury portfolio is excellent; the realized portfolio return is almost identical to the portfolio common factor return. As a consequence, the factor-based DFR and the non-factor-based EWMA VaR models provide the same forecasts and accuracy. But the factor model can help to explain why this is the case. Essentially, we would expect that since treasury bonds from a single market are characterized by relative homogeneity, then changes in interest rates, an important component of the common factors, should explain returns relatively well. In fact, the close match that we observe conforms to our expectations.

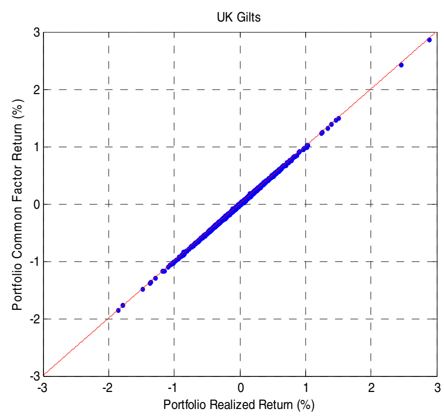

Figure 6.1: Factor Return vs. Realized Returns, U.K. Gilts (Treasury) Portfolio

The portfolio return exactly matches the model common factor return. This explains the almost exact correspondence between DFR VaR and EWMA VaR on developed market treasury portfolios (as illustrated in Table 6.1 and Table 6.2).

The relationship between common factor and realized returns is not always so tight. Figure 6.2 shows a noisier relationship between realized return and common factor return for the euro corporate bond portfolio. But even in this case, the correlation between realized and common factor returns is high (R=0.92), and we observe no significant bias. As a result, VaR forecasts from DFR and EWMA are close. The less-than-perfect match is due to the heterogeneity of corporate bonds relative to treasuries. We would expect that a specific portfolio may have corporate bond assets with exposures that are different than its proxies in the estimation universe. Nevertheless, while the results show some discrepancy between common and realized returns due to the heterogeneity, they also demonstrate the relationship remains strong.

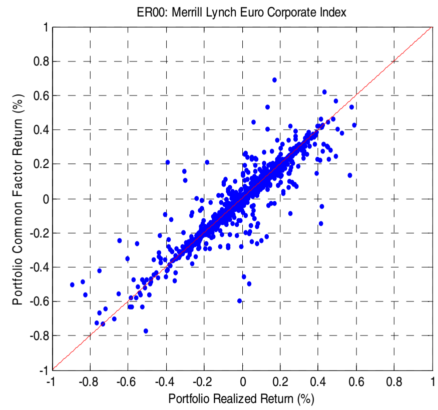

Figure 6.2: Factor Returns vs. Realized Returns, Merrill Lynch Euro Corporate Index Portfolio

For high-grade corporate bond portfolios, the match between portfolio returns and common factor returns is noisier but still exhibits a very high correlation and no significant bias.

With a factor model, we can examine the contributions to risk of common versus specific factors. For example, for small high-yield portfolios, we find that the DFR model overforecasts VaR relative to the benchmark EWMA, which is due to the large proportion of specific risk in these portfolios (Δ in Equation 2.2). Figure 6.3 shows that for a typical 20-bond high-yield portfolio, the contribution of specific risk is greater than common factor risk for many of the years we tested. For a midsize high-yield portfolio (Figure 6.4), the specific risk is less than the common factor risk but still significant. In contrast, Figure 6.5 demonstrates that a portfolio with a small allocation of investment-grade assets is characterized by negligible specific risk. This type of risk decomposition can be, and has been, useful in helping managers to diversify and control risks of certain types.

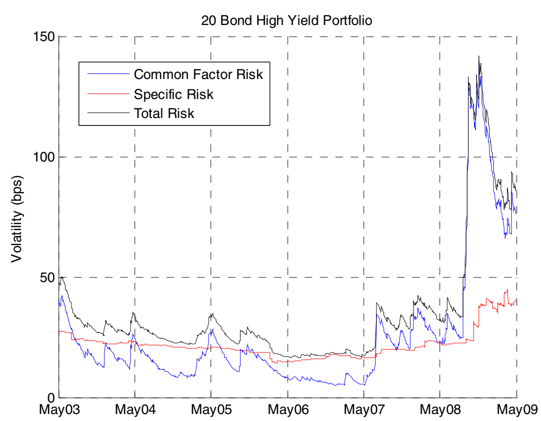

Figure 6.3: Risk Decomposition for Sample 20-Bond Euro High-Yield Portfolio

For small high yield portfolios specific risk makes a very large contribution to the total risk: usually more than the common factor risk.

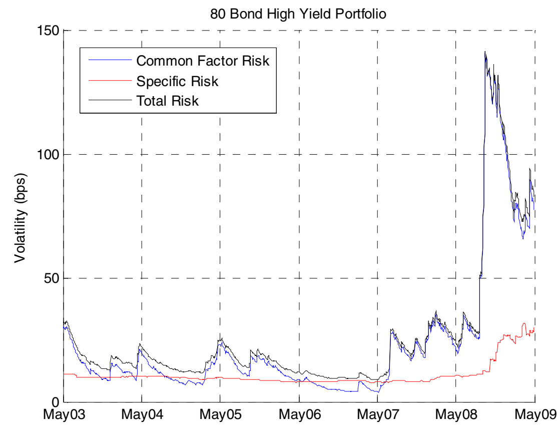

Figure 6.4: Risk Decomposition for Sample 80-Bond Euro High-Yield Portfolio

For high-yield portfolios of moderate size, specific risk contributes less to total risk than common factor risk, but the relative contributions are of similar magnitude, and specific risk occasionally dominates.

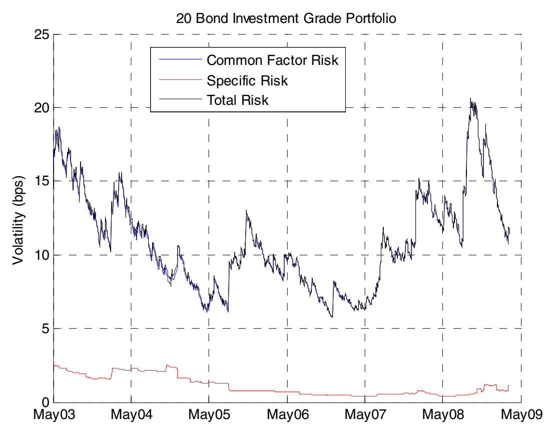

Figure 6.5: Risk Decomposition for Sample 20-Bond Euro Investment-Grade Portfolio

For investment-grade portfolios, even for portfolios with very few assets, specific risk makes a very small contribution to total risk.

7. Integrated Model

In the previous sections, we showed that the DFR model, with risk factors identical to those in the longer horizon model, provides accurate and consistent risk forecasts for shorter horizons in local markets. However, integration of the local models into a global, shorter horizon, integrated model presents its own set of challenges and decisions. This section develops alternative methods for the construction of a shorter horizon BIM, and it assesses their performance for a variety of global portfolios.

At the integrated, global level, we find that the relevance of daily data updates depends on the forecast horizon. First, for the one-day horizon, we find that the integrated model produces the best risk forecast when all DFR markets are represented in a single DFR covariance block, with all factor-by-factor covariances updated daily.[10] Second, for a horizon of two and ten days, the best solution is to update local blocks with DFR, while retaining cross-block correlations from the longer horizon BIM covariance matrix. Third, for horizons of ten days to one month, scaling the longer horizon BIM covariance matrix to the desired horizon provides adequate risk forecasts.

7.1. Building the Global Short Horizon Covariance Matrix

7.1.1. Local Market Returns

BIM builds up from the local market models, and our results for the local models suggest DFRs are the building blocks that most accurately forecast shorter horizon risk for global portfolios. Therefore, for the local markets, we use daily factor returns where available (see Table 7.1). Daily factor returns are available in eight equity markets and 13 fixed income markets. Daily factor returns are also available for most currencies.

Table 7.1: List of DFR Models and Emerging Equity Models

Models with daily factor returns available	**AUE3, BRE2, CHE2, CNE4, EUE2L, JPE3, UKE7L, USE3L;** **AUB2, CNB2, DKB1, EUB2, JPB3, NZB1, NOB1, PLB1, SAB1, SNB1, SWB2, UKB2, USB3_EMB1;** **Currency block**
Emerging equity models	**ARE1, CLE1, EGE1, HKE1, IDE1, ILE1, INE1, JOE1, KRE2, MLE1, MXE1, NZE1, OME1, PHE1, PKE1, PLE1, RUE1, SAE3, SGE1, SUE1, THE1, TRE1, TWE1**

For local markets without daily factor returns, we use one of two procedures. First, if a reliable daily index is available for the market, then we scale the volatilities of all BIM factors to reproduce the daily volatility of the market.[11] The correlations within this block are the same as those for the longer horizon BIM matrix. Second, if neither daily factor returns nor daily market index returns are available, then we divide the longer horizon variances and covariances by 21 (the average number of business days in a month) in order to derive their daily counterparts. We apply this procedure to commodity and hedge fund factors as well as some currency factors.

7.1.2. Linking Local to Global

We consider two approaches to the construction of the global covariance matrix. In the first approach, which we call the ***DFR-1B model*** ("1B" stands for one block), we use all available DFRs for equity, fixed income, and currency factors to calculate one large covariance matrix with a 21-day half-life. We then insert this covariance matrix into the original, longer horizon BIM matrix. The insertion procedure, described in Appendix A2, ensures that the resulting matrix remains positive definite by slightly modifying off-diagonal elements of the BIM matrix. As

[10] "Block" refers to a covariance matrix of a subset of the factors. This is an important unit because we build our integrated model by grouping the factors into blocks and then relating the blocks — see the BIM white paper.
[11] For example, DFRs are not available for emerging markets. See Appendix A3 for more details on the scaling procedure.

mentioned at the outset, we find that the DFR-1B model works best in terms of forecasting risk for a horizon of one day.

In the second approach, which we call the ***DFR-MB model*** (“MB” stands for multiple blocks), we compute a separate covariance matrix block for each single-country model, and we then insert each individual covariance block into the BIM matrix, again using the procedure described in Appendix A2. The resulting matrix has single-country blocks along the diagonal, with cross-market covariances reflecting the longer horizon BIM correlations. Instead of one large DFR block, there are 22 smaller DFR blocks along the diagonal, each corresponding to the local market DFR models. The DFR-MB approach works best in forecasting risk for horizons of two to ten days.

In the DFR-1B model, all factor variances and covariances within each local model, as well as the covariances between factors in different local models, are updated daily. In the DFR-MB model, the DFR block factor variances and covariances in each local model are updated daily, but cross-market covariances are updated using the rescaling method described in Appendix A2.

7.2. Test Portfolios

We chose test portfolios to represent asset allocation strategies from the point of view of the U.S. investor. The strategies vary from low to high risk as represented by the 20 portfolios shown in

Table 7.2. As one moves down the table, the corresponding asset allocation becomes riskier.

- Portfolio 1 reflects less risk in an all-bond portfolio of 80% U.S. Treasury index and 20% U.S. investment-grade corporate bond index.
- Portfolio 7 represents a somewhat riskier strategy commonly targeted to 401(k) plans of 40-year old investors: 5% U.S. Treasury securities, 35% U.S. investment grade bonds, 30% U.S. equities, 20% in other developed market equities, and the remaining 10% invested in select emerging markets with weights roughly proportional to the market capitalization.
- Portfolios 11-13 are all-equity portfolios: 20% U.S. equity, 50% developed equity, and 30% emerging equity. Each has a somewhat different tilt. Portfolio 11 is capitalization weighted in developed and emerging market equity, while portfolio 12 is heavily overweight in stocks from the Far East for both developed and emerging equity. Portfolio 13 is underweight Asian and Australian stocks.
- The weighting schemes in portfolios 14-16 and 17-19 are analogous to the weighting scheme in portfolios 11-13. However, the level of emerging market exposure, and hence, the riskiness of the portfolios, increases over the three sets. Portfolio 20 is the riskiest, with only European stocks, 45% of which is invested in Russia and 35% in Turkey.

Table 7.2 Portfolios Representing Asset Allocation Choices (FE=Far East)

Portfolio	U.S. Treasury Bonds (%)	U.S. Corporate Bonds (%)	U.S. Equities (%)	Developed Market Equities (%)	Emerging Market Equities (%)	Tilt
1	**80**	**20**				
2	**60**	**30**	**10**			
3	**40**	**40**	**15**	**5**		
4	**30**	**40**	**20**	**10**		
5	**20**	**40**	**25**	**15**		
6	**10**	**40**	**30**	**15**	**5**	
7	**5**	**35**	**30**	**20**	**10**	
8		**30**	**30**	**25**	**15**	
9		**20**	**30**	**30**	**20**	
10		**10**	**30**	**40**	**20**	
11			**20**	**50**	**30**	
12			**20**	**50**	**30**	**Over FE**
13			**20**	**50**	**30**	**Under FE**
14			**10**	**40**	**50**	
15			**10**	**40**	**50**	**Over FE**
16			**10**	**40**	**50**	**Under FE**
17				**20**	**80**	
18				**20**	**80**	**Over FE**
19				**20**	**80**	**Under FE**
20				**20**	**80**	**Europe**

Portfolios representing asset allocation choices ranging from very conservative to highly speculative strategies. Portfolios 12, 15, and 18 are heavily overweighed in stocks from the Far East (and Australia), while Portfolios 13, 16, and 19 underweight these stocks. Portfolio 20 contains only European stocks. All other portfolios are approximately capitalization weighted in equities and amount outstanding weighted in bonds.

7.3. Results

At the global level, we find that the best method depends on the horizon: (1) one-day, or (2) multiple-day. Thus, for each horizon, we first compare how DFR-1B and DFR-MB models perform based on the bias statistic test, which we have found to be more discriminating relative to the VaR tests. Then, as with the local market models, we compare the VaR backtesting of the DFR versus SBM and EWMA forecasts.

7.3.1. One-Day Horizon

We evaluate 20 portfolios for 6 testing periods, resulting in 120 distinct bias test statistics. Figure 7.1 summarizes the results in a histogram. A given model performs well when the bias statistic is between 0.91 and 1.09.

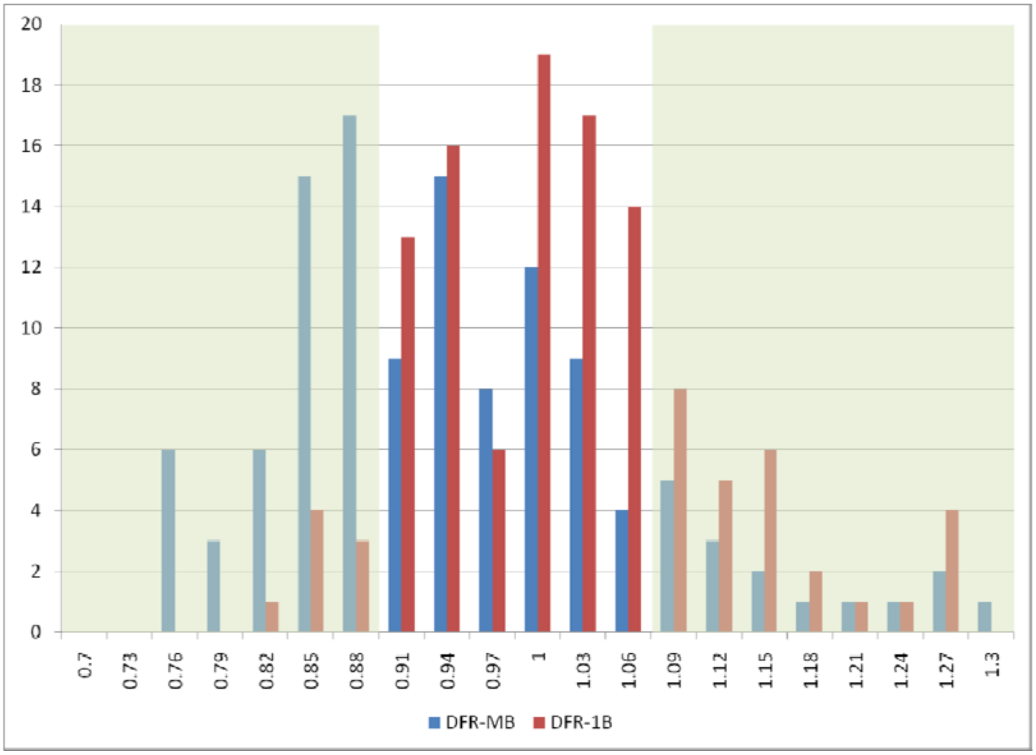

Figure 7.1: Histogram of Bias Statistic Value for Global Asset Allocation Portfolios

Histogram of bias statistic value for 20 portfolios representing different asset allocation choices for BIM, DFR-MB, and DFR-1B models. Shaded areas represent regions outside of the confidence interval. Risk is overforecast for portfolios in the left shaded region and underforecast for portfolios in the right region.

The tests verify the utility of the DFR-1B model. First, the bias tests confirm the value of a short-horizon model, since both DFR models perform better than SBM, for which 95% of the tests lie outside of the confidence interval (note that the histogram does not contain any results for SBM).

Second, the DFR-MB model performs better, but 55% of the tests also lie outside the 95% confidence interval. Since DFR-MB neglects short-term changes in cross-market correlations, we look to the DFR-1B results to show whether these cross-market correlations are relevant.

Third, the relevance of the cross-market correlations are confirmed by the performance of DFR-1B: only 30% of its test statistics lie outside the confidence interval. Furthermore, the portfolios for which the DFR-1B model fails the bias test have significant exposure to emerging markets, where DFRs are not available, and thus we use the index-scaling model. If we consider only portfolios with limited exposure to emerging market factors (Portfolios 1-10), then the difference between DFR-MB and DFR-1B models becomes even more pronounced: 52% of the portfolios are outside of the confidence interval for DFR-MB, and only 5% for DFR-1B. We find similar results for other combinations of developed market equity and fixed income index portfolios. An important conclusion is that expanding the number of local models with daily factor returns would be the most direct approach to improving the DFR-1B model.

We next compare DFR-1B to EWMA and SBM. Similar to the local market results, Figure 7.2 shows that DFR-1B and EWMA VaR have comparable results with good forecasts (in the 95% confidence interval as indicated by the solid black lines) in years 2004 to 2008. In 2005, DFR overestimates risk for Portfolios 7, 8, and 16, while EWMA accurately forecasts VaR for all 20 portfolios. In 2008, DFR underestimates risk for Portfolios 6, 15, and 18, and EWMA underforecasts risk for Portfolios 6 and 7. In 2009, both models underforecast risk for 85% of portfolios. Both outperform the SBM model, which overforecasts risk for most portfolios from 2004 to 2006, underforecasts for most portfolios in 2008 and for all portfolios in 2009.

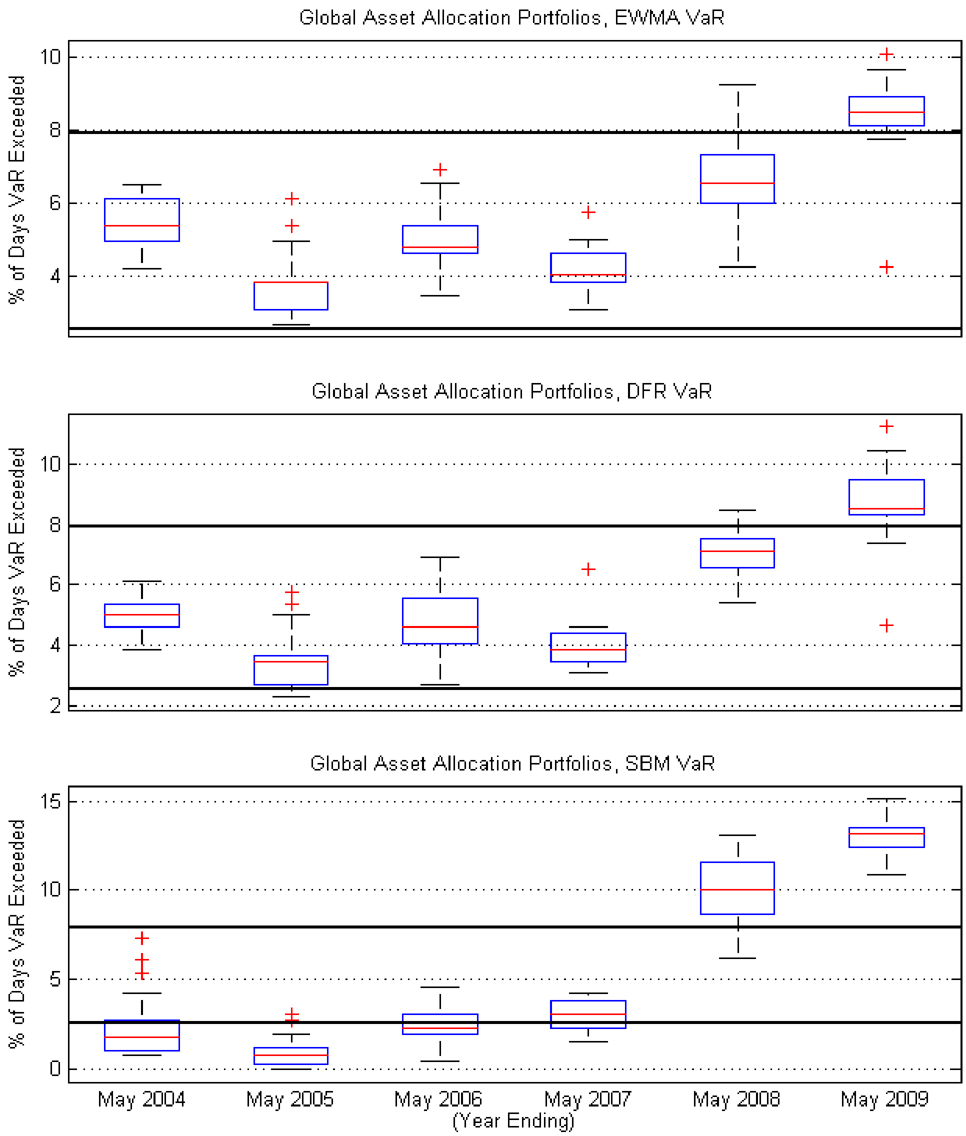

Figure 7.2: VaR Backtesting for Global Asset Allocation Portfolios

DFR-1B and EWMA VaR have similar results with good forecasts in years 2004-2008. In 2005, DFR overestimates risk for Portfolios 7, 8, and 16, while EWMA accurately forecasts VaR for all 20 portfolios. In 2009, both models underforecast risk for 85% of portfolios. The SBM model overforecasts risk for most portfolios from 2004 to 2006, and underforecasts for most portfolios in 2008 and for all portfolios in 2009.

7.3.2. Multiple-Day Horizon

As mentioned previously, the superiority of DFR-1B to DFR-MB for the one-day horizon can be attributed to the daily updating of cross-market correlations. Daily updating matters, because it matches the frequency of the return data.

With respect to a multiple-day horizon (***h***), there is a choice between: (1) scaling up, using $\sqrt{h}$ -rule, cross-block correlations from daily factor returns (DFR-1B), or (2) using cross-block correlations from the monthly factor returns (DFR-MB). For a horizon of two days, Figure 7.3 shows the results of the bias tests. These bias statistics are computed based on two-day returns with a look-back horizon of two years. The confidence interval is the same as for the one-day yearly bias tests, and there are 60 distinct portfolio tests in total. Interestingly, no overforecasting is observed for the DFR-1B model. However, the DFR-1B model underforecasts for 36 out of 60 portfolios (40% in the confidence interval), while DFR-MB model performs better, underforecasting risk for 11 and overforecasting for 2 out of 60 portfolios (78% in the confidence interval). This result suggests that the DFR-MB model is more appropriate for a two-day horizon.

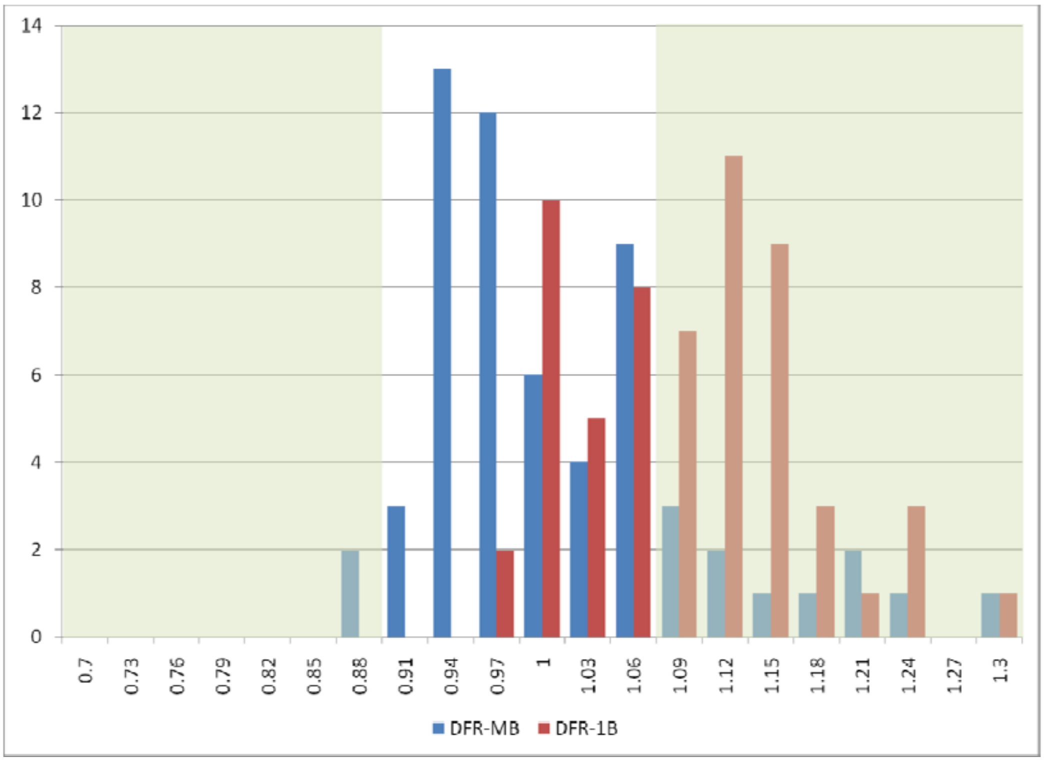

Figure 7.3: Histogram of Two-Day Bias Statistics for Global Asset Allocation Portfolios

Histogram of bias statistic value for 20 asset allocation portfolios for DFR-MB and DFR-1B models. The bias statistic is computed biannually for each portfolio, resulting in 60 distinct portfolio tests assembled in a histogram. Shaded areas represent regions outside of the confidence interval. Risk is underforecast for portfolios in the shaded region to the right.

Figure 7.4 shows the results of the VaR backtests among DFR-MB, EWMA, and SBM for the two-day horizon. EWMA based on daily portfolio returns and a 21-day half-life performs poorly when scaled to the two-day horizon, reflecting the fact that there are significant autocorrelations in the daily returns. To obtain a comparable two-day baseline, we compute EWMA based on two-day return periods with a 10.5-day half-life. In contrast to the one-day case, the DFR-MB model performs somewhat better than EWMA. In the period ending May 2009, the DFR model underforecasts VaR for 60% of the tests, while EWMA fails for 75%. Both models perform well in the two-year periods ending in 2005 and in 2007. The SBM model overforecasts risk for most portfolios in the period ending in 2005 and underforecasts for all portfolios in 2009.

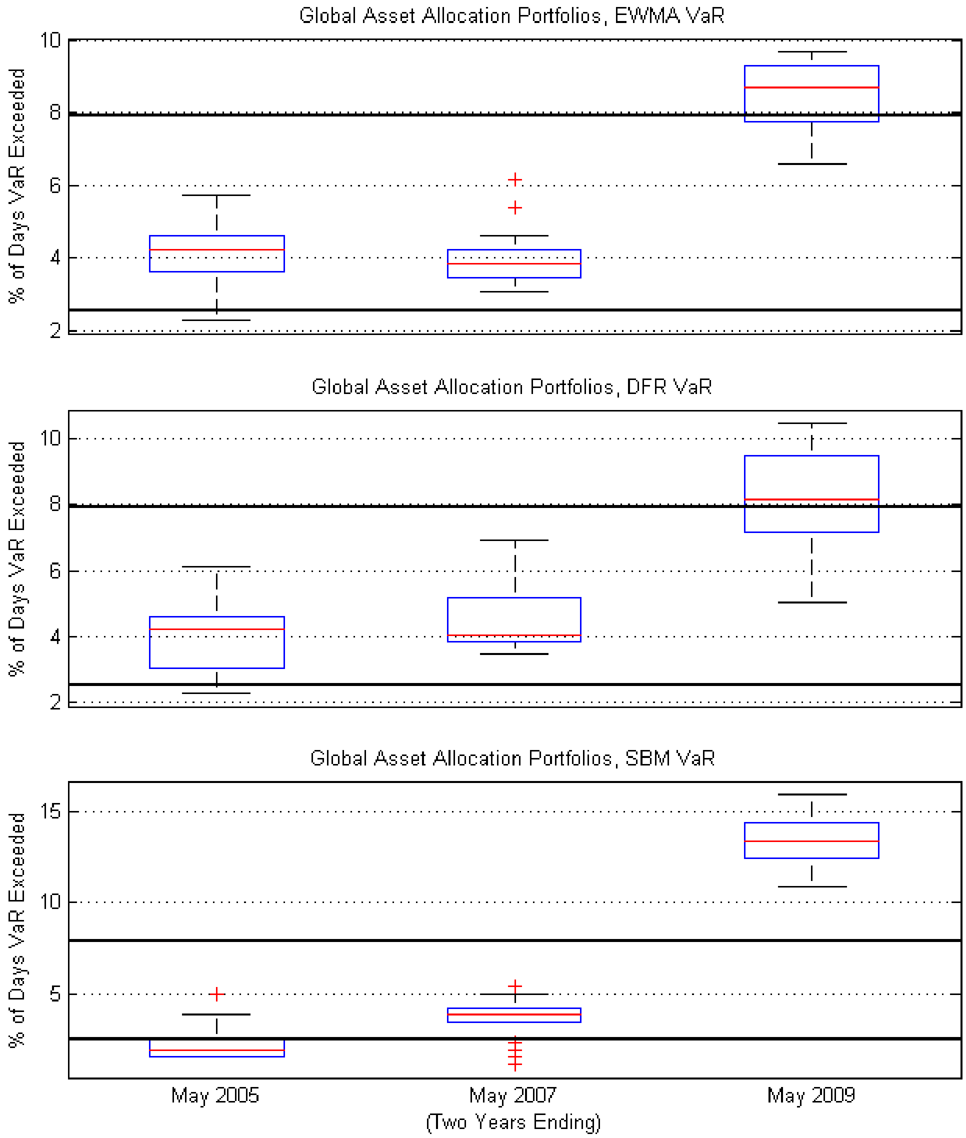

Figure 7.4: VaR Backtesting for Global Asset Allocation Portfolios on a Two-Day Horizon

DFR-MB and EWMA VaR have similar results with good forecasts in two-year periods ending in 2005 and 2007. In the period ending May 2009, the DFR model underforecasts VaR for 60% of portfolios, while EWMA fails for 75%. The SBM model overforecasts risk for most portfolios in the period ending in 2005 and underforecasts for all portfolios in 2009.

For longer horizons, we find that it becomes more difficult to differentiate among models. However, based on the two-day results and the considerations described in Appendix A4, the DFR-MB model is a better choice. For horizons longer than ten days, we find that the more responsive models based on daily factor returns and short half-life do not outperform the scaled BIM model. We therefore conclude that the rescaled monthly BIM model should be used for all horizons longer than ten days.

8. Conclusions

In this paper, we presented a model based on daily factor returns (the DFR model), which is a version of the longer horizon factor model appropriately adjusted to shorter horizons. We have shown that the DFR model is a more accurate approach than the long horizon factor model for estimating VaR and volatility for local equity markets, local fixed income markets, as well as global markets. We found that the model gives risk forecasts similar in accuracy to both EWMA and asset-by-asset covariance approaches. However, in contrast to EWMA or asset-by-asset approaches, the DFR model provides all the advantages of longer horizon factor models at shorter horizons: explanations of the sources of risk in terms of a set of intuitive, fundamental factors. With the use of Barra factor models as standards for longer horizons, the model introduced in this paper provides an extension of that consistent approach to risk management for more time horizons.

9. References

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Appendix

A1. Supplemental Tables for Fixed Income Local Models

Table A1: Merrill Lynch USA Index Portfolios

Portfolio	Description	# Bonds (mean)	Est. Univ. Coverage (mean)	Test Portfolio # (mean)
C0A0	U.S. High Grade Corporates	3405	77%	2622
C0AL	U.S. Corporates Large Cap	1293	66%	853
C0B0	U.S. Corporates AA-AAA Rated	492	69%	338
C0C0	U.S. Corporates BBB-A Rated	2910	78%	2281
C0D0	U.S. Corporates Industrials ex Telecom & Transportation	1639	81%	1321
C0J0	U.S. Corporates Finance, ex Banks	554	69%	382
C0P0	U.S. Corporates Banks	584	74%	431
C0Q0	U.S. Corporates Gas & Electric Utilities	318	79%	250
C0R0	U.S. Corporates Phones	180	72%	130
C0W0	U.S. Corporates Transportation	126	85%	107
C0Z0	U.S. Corporates All Yankees	691	65%	446
CF00	U.S. Financial Corporates	1138	72%	815
CI00	U.S. Industrial Corporates	1946	80%	1557
CU00	U.S. Utility Corporates	318	79%	250
DQG0	U.S. Agency & Quasi/Foreign Government Index	1171	55%	641
G0A0	AAA U.S. Treasury/Agency Master	1006	62%	619
G0P0	AAA U.S. Agency Master	886	57%	501
G0Q0	U.S. Treasury Master	120	99%	119
GS00	US$ Foreign Govt. and Supra National	276	54%	150
H0A0	U.S. High Yield	1911	56%	1061
	AVERAGE	1048	71%	744

Table A2: Merrill Lynch Euro Index Portfolios

Ticker	Description	# Bonds (mean)	Est. Univ. Coverage (mean)	Test Portfolio # (mean)
EB00	**Euro Financial Corporate Index**	**671**	**79%**	**528**
EBBA	**Euro Corporates Banking**	**504**	**77%**	**390**
EG00	**Euro Direct Government Index**	**247**	**100%**	**247**
EJ00	**Euro Corporates Industrials Index**	**432**	**96%**	**414**
EK00	**Euro Corporates Utilities Index**	**105**	**96%**	**101**
EMU0	**Euro Broad Market Index**	**3408**	**72%**	**2451**
EMUL	**Euro Large Cap Investment Grade Index**	**1768**	**83%**	**1470**
EQ00	**Euro Quasi-Government Index**	**441**	**51%**	**225**
ER00	**Euro Corporate Index**	**1210**	**86%**	**1043**
ER60	**Euro Corporates AAA-AA Rated**	**425**	**91%**	**388**
ERC0	**Euro Corporates BBB-A Rated**	**785**	**83%**	**655**
HE00	**Euro HighYield**	**170**	**79%**	**134**
	AVERAGE	**847**	**83%**	**670**

Table A3: Merrill Lynch Japan Index Portfolios

Ticker	Description	# Bonds (mean)	Est. Univ. Coverage (mean)	Test Portfolio # (mean)
G0Y0	**Japanese Governments**	**160**	**98%**	**157**
JC00	**Japan Corporate Index**	**1012**	**89%**	**901**
JC20	**Japan Corporate Index AA Rated**	**314**	**87%**	**272**
JC30	**Japan Corporate Index A Rated Index**	**463**	**93%**	**433**
JC40	**Japan Corporate Index BBB Rated Index**	**193**	**94%**	**182**
JF00	**Japan Financial Index**	**372**	**80%**	**297**
JFBA	**Japan Corporates Banking**	**275**	**86%**	**237**
JI00	**Japan Industrial Index**	**319**	**93%**	**296**
JP00	**Japan Broad Market Index**	**1595**	**86%**	**1365**
JPL0	**Japan Large Cap Index**	**563**	**88%**	**494**
JQ00	**Japan Quasi-Govt Index**	**423**	**73%**	**310**
JU00	**Japan Utility Index**	**311**	**99%**	**308**
	AVERAGE	**500**	**89%**	**438**

Table A4: Constituents of Random Bond Portfolios

	Number of Bonds							
Random Portfolio Type	Treasury	AAA	AA	A	BBB	BB	B	CCC
20 Bond Invest Grade	**4**	**4**	**4**	**4**	**4**	**0**	**0**	**0**
80 Bond Invest Grade	**16**	**16**	**16**	**16**	**16**	**0**	**0**	**0**
20 Bond High Yield	**0**	**0**	**0**	**0**	**0**	**8**	**8**	**4**
80 Bond High Yield	**0**	**0**	**0**	**0**	**0**	**32**	**32**	**8**

Table A5: Description of Emerging Market Portfolios

Portfolio	Country of Issue	Avg. # of Bonds	Avg. % Sov. Issues
EM_BRA	**Brazil**	**50**	**39%**
EM_CHN	**China**	**15**	**40%**
EM_COL	**Columbia**	**15**	**93%**
EM_HKG	**Hong Kong**	**20**	**6%**
EM_IND	**India**	**12**	**7%**
EM_KOR	**Korea**	**65**	**17%**
EM_MEX	**Mexico**	**48**	**53%**
EM_PAN	**Panama**	**13**	**64%**
EM_RUS	**Russia**	**17**	**10%**
EM_SIN	**Singapore**	**21**	**0%**
EM_THA	**Thailand**	**12**	**11%**
EM_TUR	**Turkey**	**14**	**100%**
EMBI+	**NA**	**73**	**100%**

A2. Covariance Matrix Scaling Procedure

In order to combine the covariance blocks of the DFR-1B or DFR-MB models with the remaining portion of the BIM covariance matrix, we use the calculation procedure described below. This procedure ensures the internal consistency of the resulting covariance matrix, i.e., it guarantees that the resulting correlation matrix is positive definite.

We start with the BIM monthly covariance matrix scaled to a short horizon. The covariance matrix is rearranged in the way shown in Figures A1 and A2. There, the covariance values that correspond to the factors in the DFR-1B or DFR-MB models are grouped along the diagonal. We can schematically represent this factor grouping of the BIM covariance matrix as follows:

Equation A1
$$\widehat{\boldsymbol{F}} = \boldsymbol{diag}\begin{pmatrix}\widehat{\boldsymbol{\sigma}}_{\boldsymbol{d}} \\ \widehat{\boldsymbol{\sigma}}_{\boldsymbol{e}}\end{pmatrix}\begin{pmatrix}\widehat{\boldsymbol{C}}_{\boldsymbol{d,d}} & \widehat{\boldsymbol{C}}_{\boldsymbol{d,e}} \\ \widehat{\boldsymbol{C}}_{\boldsymbol{e,d}} & \widehat{\boldsymbol{C}}_{\boldsymbol{e,e}}\end{pmatrix}\boldsymbol{diag}\begin{pmatrix}\widehat{\boldsymbol{\sigma}}_{\boldsymbol{d}} \\ \widehat{\boldsymbol{\sigma}}_{\boldsymbol{e}}\end{pmatrix}$$

where the caret **^** symbol indicates that the value is derived from the BIM covariance matrix, ***d*** enumerates the factors in the DFR-1B or DFR-MB models, and ***e*** enumerates factors in the emerging market block. $\widehat{\boldsymbol{\sigma}}_{\boldsymbol{d}}$ and $\widehat{\boldsymbol{\sigma}}_{\boldsymbol{e}}$ refer to the vectors of corresponding factor volatilities in BIM (scaled to the short horizon), ***diag(x)*** denotes a diagonal matrix constructed from the vector ***x***, and $\widehat{\boldsymbol{C}}_{\boldsymbol{d},\boldsymbol{d}}$, $\widehat{\boldsymbol{C}}_{\boldsymbol{e},\boldsymbol{e}}$, $\widehat{\boldsymbol{C}}_{\boldsymbol{d},\boldsymbol{e}}$ refer to the corresponding factor correlations blocks in the BIM covariance matrix.

The scaling procedure of the DFR-1B and DFR-MB covariance blocks into the regrouped BIM covariance matrix can be written as follows:

Equation A2
$$\boldsymbol{F} = \boldsymbol{diag}\begin{pmatrix} \boldsymbol{\sigma}_{\boldsymbol{d}_1} \\ \vdots \\ \boldsymbol{\sigma}_{\boldsymbol{d}_n} \\ \widehat{\boldsymbol{\sigma}}_{\boldsymbol{e}} \end{pmatrix} \begin{pmatrix} \boldsymbol{C}_{\boldsymbol{d}_1,\boldsymbol{d}_1} & \cdots & \boldsymbol{C}_{\boldsymbol{d}_1,\boldsymbol{d}_n} & \boldsymbol{C}_{\boldsymbol{d}_1,\boldsymbol{e}} \\ \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{C}_{\boldsymbol{d}_n,\boldsymbol{d}_1} & \cdots & \boldsymbol{C}_{\boldsymbol{d}_n,\boldsymbol{d}_n} & \boldsymbol{C}_{\boldsymbol{d}_n,\boldsymbol{e}} \\ \boldsymbol{C}_{\boldsymbol{e},\boldsymbol{d}_1} & \cdots & \boldsymbol{C}_{\boldsymbol{e},\boldsymbol{d}_n} & \widehat{\boldsymbol{C}}_{\boldsymbol{e},\boldsymbol{e}} \end{pmatrix} \boldsymbol{diag}\begin{pmatrix} \boldsymbol{\sigma}_{\boldsymbol{d}_1} \\ \vdots \\ \boldsymbol{\sigma}_{\boldsymbol{d}_n} \\ \widehat{\boldsymbol{\sigma}}_{\boldsymbol{e}} \end{pmatrix}$$

where ***F*** is a short-term covariance matrix, $\widehat{\boldsymbol{\sigma}}_{\boldsymbol{e}}$ and $\widehat{\boldsymbol{C}}_{\boldsymbol{e},\boldsymbol{e}}$ are defined in Equation A1, the product of $\boldsymbol{diag}(\boldsymbol{\sigma}_{\boldsymbol{d}_n})\boldsymbol{C}_{\boldsymbol{d}_n,\boldsymbol{d}_n}$ defines the DFR-1B covariance matrix (or the DFR-MB covariance matrix) to which we refer in Section 7.1, and $\boldsymbol{C}_{\boldsymbol{d}_i,\boldsymbol{d}_j}$ and $\boldsymbol{C}_{\boldsymbol{e},\boldsymbol{d}_j}$ are derived as follows:

Equation A3
$$\boldsymbol{C}_{\boldsymbol{d}_i,\boldsymbol{d}_j} = \boldsymbol{C}_{\boldsymbol{d}_i,\boldsymbol{d}_i}^{1/2} \widehat{\boldsymbol{C}}_{\boldsymbol{d}_i,\boldsymbol{d}_i}^{-1/2} \widehat{\boldsymbol{C}}_{\boldsymbol{d}_i,\boldsymbol{d}_j} \widehat{\boldsymbol{C}}_{\boldsymbol{d}_j,\boldsymbol{d}_j}^{-1/2} \boldsymbol{C}_{\boldsymbol{d}_j,\boldsymbol{d}_j}^{1/2}$$
$$\boldsymbol{C}_{\boldsymbol{e},\boldsymbol{d}_j} = \widehat{\boldsymbol{C}}_{\boldsymbol{e},\boldsymbol{d}_j} \widehat{\boldsymbol{C}}_{\boldsymbol{d}_j,\boldsymbol{d}_j}^{-1/2} \boldsymbol{C}_{\boldsymbol{d}_j,\boldsymbol{d}_j}^{1/2}$$

Note that for the calculation of the short-term covariance matrix in the context of the DFR-1B model, there is only one DFR covariance block (***n***=1 in Equation A2).

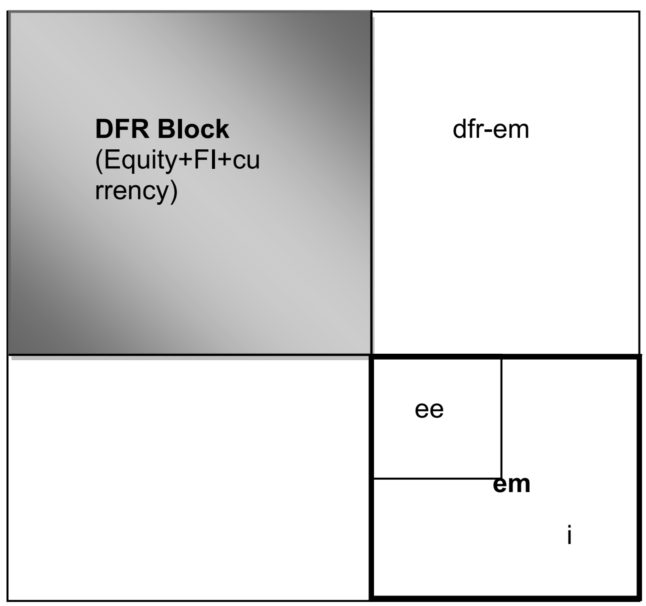

Figure A1: Structure of the Covariance Matrix

DFR block – equity, FI, and currency factors for which DFRs are available; **em** – emerging markets block, which contains **ee** and **i** sub-blocks. Sub-blocks: **ee** – emerging equity markets, where a daily index can be constructed; **i** – incomplete equity and fixed income markets, where daily data is not available. Off-diagonal, cross-covariance block is denoted **dfr-em**. DFR block is entirely constructed based on daily factor returns; it retains no information from the monthly BIM covariance matrix.

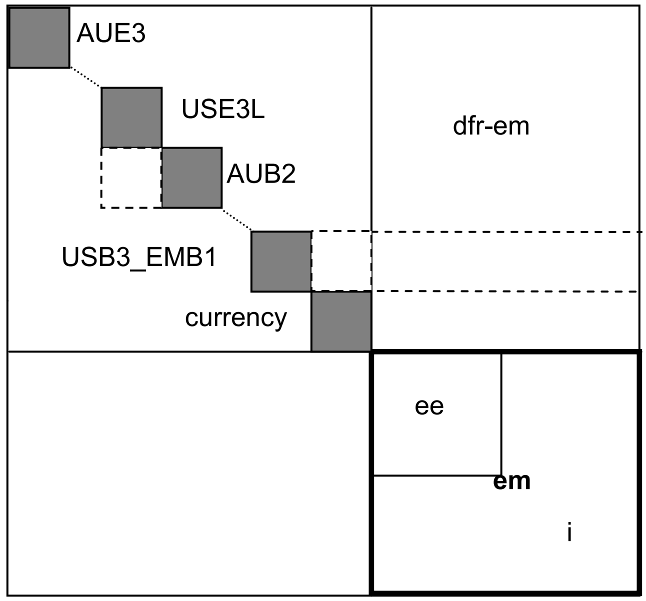

Figure A2: Structure covariance matrix for DFR-MB model

Each DFR-MB is constructed using daily factor returns for that model. All DFR-MBs are then scaled into the BIM covariance matrix separately, retaining all cross-block correlations from the investment horizon (monthly) BIM.

A3. Emerging Market Index Volatility Calculation

The quality of daily asset-level data available in emerging markets is not reliable enough to extract high-frequency correlations between the corresponding factors. We then use correlations available in the monthly BIM to construct the daily covariance matrix. To improve the model responsiveness, we rescale the factor volatilities on a daily basis using the volatility of the daily market index estimated with a 21-day half-life. This treatment is applied to emerging equity market factors in both DFR-1B and DFR-MB models.

The daily market index for any equity model is obtained by selecting from the relevant estimation universe all of the stocks with available daily returns and weighting them proportionally to their market capitalization. The EWMA estimate of the variance of the market index, σ_{index}^2, is updated daily according to Equation 2.5, while the total risk σ_{tot}^2 is estimated as in Equation 2.2 based on the BIM matrix scaled to the daily horizon according to Equation 2.4.

By scaling the variance of each factor by the ratio $\sigma_{index}^2/\sigma_{tot}^2$, we increase the responsiveness of the risk forecast to recent market events. All factor variances from a particular emerging market country model are scaled by the ratio determined for that country's market index. This amounts to multiplying all rows and columns corresponding to emerging equity market factors by $\sigma_{index}/\sigma_{tot}$. The risk forecast obtained from the resulting covariance matrix for each emerging equity market index has the desirable property of reproducing the EWMA forecast.

A4. Global Date Conventions

When daily cross-market correlations are estimated in a global model, misalignment of trading hours around the world leads to non-synchronous trading effects. These effects are seen most clearly when correlations based on daily and monthly returns are compared. Figure A3 shows the correlation between the U.S. and Japan equity indices computed over a trailing one-year window using daily and monthly returns. Correlations based on daily returns are consistently smaller, and we conclude that they do not capture all of the dependencies that exist between the U.S. and

Japan markets and that they reflect only the influence of Japan market returns on the U.S. market. In fact, the U.S. market return on a given day has a significant influence on the equity market return in Japan the following day. This can be seen by computing correlations based on the daily returns of the Japan and U.S. markets, but with the U.S. market returns lagging by one day. (This is equivalent to computing the one-day lag cross-autocorrelation between these two markets.) The lagged correlation is much larger than the correlation between returns on the same calendar day, indicating that the U.S. market influences the Japan market on the next day more than the Japan market influences the U.S. market on the same day.

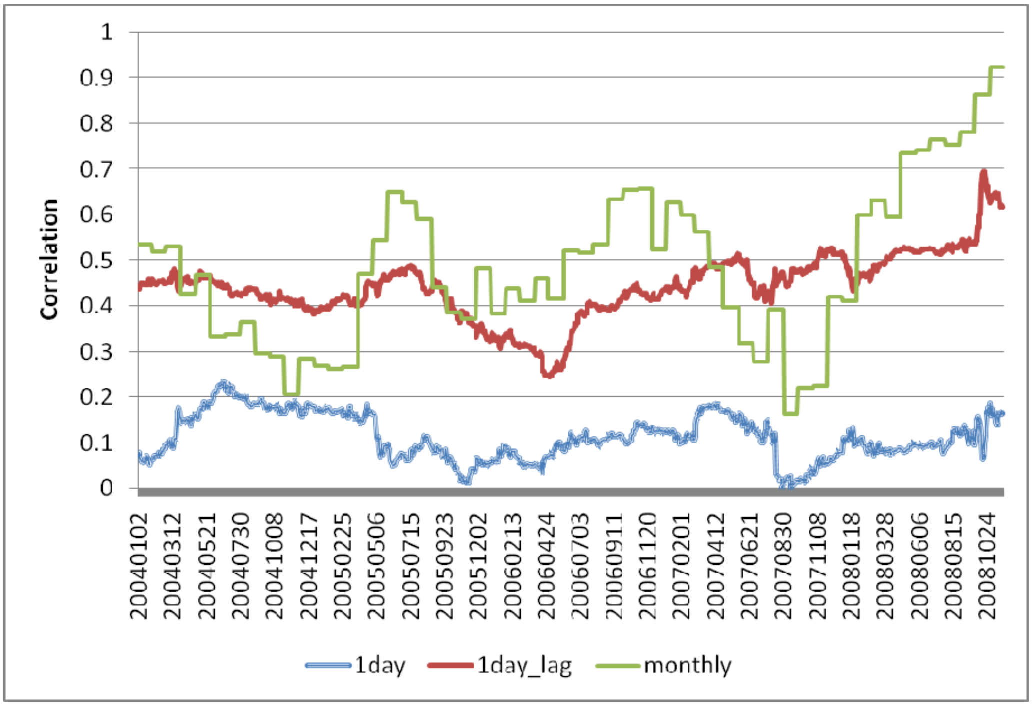

Figure A3: Correlations between the U.S. and Japan

Correlations between the U.S. and Japan capitalization-weighted estimation universe portfolios computed over a one-year trailing window. Correlations are computed using daily local return series (1day), daily local return series with a one-day lag for the U.S. returns (1day_lag), and monthly local returns (monthly).

When constructing a short-term global covariance matrix, we must define daily returns in a global setting as 24-hour returns with respect to some starting point. All possible groupings of daily returns have similar drawbacks with respect to large cross-autocorrelation coefficients between at least some markets. We use the calendar-day convention, from the beginning of trading in Japan to the market closing in the U.S. (This is probably also the only practical choice for global daily risk reporting.) As indicated above, this leads to smaller-than-expected correlations between North American and Asian portfolios, but it does not compromise the accuracy of the one-day risk forecast for a combined North American-Asian portfolio. Due to the presence of significant autocorrelation in a daily return time series, the risk forecast for a multiple-period horizon is significantly worse for North American-Asian portfolios if a simple square root of time-volatility scaling is used for time aggregation. We address this problem by constructing a different short-term covariance matrix for the multiple-day forecast that uses the DFR-MB model and retains monthly BIM correlation values for cross-market correlations.

Another difficulty in the DFR block construction is related to inconsistent holiday schedules around the world. We cannot update parts of this single DFR block in DFR-1B using Equation 2.3 while keeping other parts (corresponding to markets on holiday) unchanged, since this procedure does not necessarily preserve the positive definite property of the correlation matrix. Instead, we use a simple method that preserves the matrix sub-blocks that have a holiday. First, we update the whole DFR block using Equation 2.3, assuming zero returns for factors that correspond to the market on a holiday. This scales down volatilities of all missing factors by $\lambda^{1/2}$. At the same time, correlations between factors within a model on holiday are unchanged, while the correlations between missing factors and other factors are on average scaled down by $\lambda^{1/2}$. To retrieve pre-holiday volatility values for factors of a market on holiday, we divide all columns and rows corresponding to the missing factors by $\lambda^{1/2}$. Importantly, this multiplication preserves the positive definite property of the covariance matrix. The only resulting change is the scaling down of correlations between factors that have a holiday and other factors by $\lambda^{1/2}$. Although this introduces a small bias in the covariance matrix, we found that this bias does not materially affect the accuracy of the one-day risk forecast.

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