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Article *in* European Journal of Finance · September 2009

DOI: 10.1080/13518470902853566 · Source: RePEc

Gordon J. Alexander
University of Minnesota Twin Cities

The European Journal of Finance,
Vol. 15, Nos. 5–6, July–September 2009, 451–461

From Markowitz to modern risk management

Gordon J. Alexander*

Department of Finance, University of Minnesota, Minneapolis, MN 55455, USA

Nobel Laureate Harry Markowitz is often referred to as the 'founder of Modern portfolio theory' and deservedly so given his enormous influence on the money management industry.[1] However, it is my contention that he should also be referred to as the 'founder of Modern Risk Management' since his contributions to portfolio theory formed the basis for how risk is currently viewed and managed. More specifically, Markowitz argued that a portfolio of securities should be viewed through the lens of statistics where the probability distribution of its rate of return is evaluated in terms of its expected value and standard deviation. Since the ultimate selection of a portfolio involves the evaluation and management of risk as measured by standard deviation, it is clear that Markowitz's process of portfolio selection represents the birth of modern risk management whereby risk is quantified and controlled. In this paper, I will first, introduce *value-at-risk* as a measure of risk and how it relates to standard deviation, the risk measure at the heart of the model of Markowitz. Second, I will similarly introduce *conditional value-at-risk* (also known as *expected shortfall*) as a measure of risk and compare it with VaR. Third, I will briefly introduce *stress testing* as a supplemental means of controlling risk and will then present my conclusions.[2]

Keywords: risk management; value-at-risk; conditional value-at-risk; stress testing

1. Value-at-risk

In 1989, the CEO of J.P. Morgan Bank ('Morgan', now JPMorgan Chase) asked for a daily 4:15 pm report that detailed the market risk of the bank's trading portfolio. Morgan subsequently ended up using value-at-risk (VaR) as a measure of market risk, and set up a subsidiary in 1994 known as RiskMetrics that not only educated the global marketplace about VaR but also freely provided data via a website to assist institutions in the estimation of their own VaR. In 1998, the success of RiskMetrics Group led to its spinoff by Morgan and listing on the New York Stock Exchange under the ticker *RMG*. VaR has subsequently risen to such a high level of prominence as a measure of risk that it is now 'widely used by corporate treasurers and fund managers as well as financial institutions' (Hull 2007, 195).

Just what is VaR? Hull (2007, 477) defines VaR as 'a loss that will not be exceeded at some specified confidence level'. Since the user must specify a confidence level t and a time horizon, a portfolio p's VaR, denoted VaR_p, can be stated more formally as:

$$\mathrm{VaR}_p = zS_p - E_p, \tag{1}$$

where z denotes the $(1 - t)$ quantile of the portfolio's return distribution, and S_p and E_p denote the portfolio's standard deviation and expected return, respectively, as measured over the specified time horizon.[3] Hence, a hypothetical portfolio whose annual return is normally distributed with

*Email: gjalex@umn.edu

ISSN 1351-847X print/ISSN 1466-4364 online

DOI: 10.1080/13518470902853566
http://www.informaworld.com

a mean of 10% and a standard deviation of 20% would have an annual VaR based on a 99% confidence level of

$$2.33 \times 20\% - 10\% = 36.6\%. \tag{2}$$

For example, if the portfolio has a current market value of €1,000,000,000, then it has a 1% chance of losing at least €366,000,000 by the end of the year.

When evaluating any portfolio on the basis of the mean and standard deviation of its return distribution, it is common to plot the location of the portfolio in a two-dimensional diagram as indicated by, for example, point p in Figure 1. However, it is also quite easy to indicate any portfolio's VaR in the diagram by rewriting Equation (1) as:

$$E_p = zS_p - \mathrm{VaR}_p. \tag{3}$$

Note that this equation corresponds to a straight line with a slope of z and vertical intercept of $-\mathrm{VaR}_p$. Thus, the VaR of our hypothetical portfolio can be found by extending a line with a slope of 2.33 toward the vertical axis and noting that the intercept occurs at the value of -36.6. Since $-36.6\% = -\mathrm{VaR}_p$, it can be seen that the portfolio has a VaR of 36.6%.

According to the Markowitz model, an investor seeks to identify his or her optimal portfolio from the set of all possible portfolios that can be formed from an arbitrary set of n securities. The process of identifying the optimal portfolio involves identifying the *mean–variance boundary*. Specifically, a portfolio p belongs to the mean–variance boundary if and only if, for some expected return E^*, p solves the following problem:

$$\text{Minimize } V \tag{4}$$

$$\text{Subject to : } \boldsymbol{E}'\boldsymbol{W} = E^* \tag{5}$$

$$\boldsymbol{W}'\boldsymbol{1} = 1, \tag{6}$$

where $\boldsymbol{W}$ is an $n \times 1$ vector of weights representing the proportion of the investor's wealth that is to be invested in each one of the n securities; $V = \boldsymbol{W}'\boldsymbol{C}\boldsymbol{W}$ is the variance of the portfolio with weight vector $\boldsymbol{W}$; $\boldsymbol{C}$ is the $n \times n$ variance-covariance matrix; $\boldsymbol{E}$ is an $n \times 1$ vector of the expected returns of the n securities; and $\mathbf{1}$ is an $n \times 1$ unit vector. Black (1972) and Merton (1972) have shown that the portfolios solving this minimization problem for varying values of E^* lie on a hyperbola in mean–standard deviation space. The upper half of this boundary, beginning with the *minimum variance portfolio* ('MVP') is known as the *mean–variance efficient frontier* (or simply 'efficient frontier'). The punch line of the Markowitz model is that an expected utility maximizing

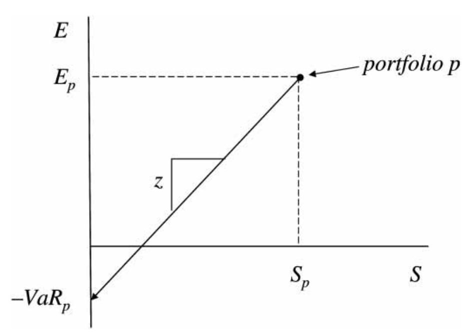

Figure 1. Representing a portfolio's VaR in mean–standard deviation space.

portfolio manager will select for investment a portfolio on the efficient frontier with the highest *certainty equivalent return* that is identified by the use of indifference curves.

Similarly, a portfolio p belongs to the *mean–VaR boundary* if and only if, for some expected return E^*, p solves the following problem:

$$\text{Minimize VaR} \tag{7}$$

$$\text{Subject to : } \boldsymbol{E}'\boldsymbol{W} = E^* \tag{8}$$

$$\boldsymbol{W}'\boldsymbol{1} = 1, \tag{9}$$

where VaR is defined in Equation (1). Note that the mean–variance and mean–VaR boundaries are identical since minimizing VaR is equivalent to minimizing V for a given level of expected return since z and E^* are constants in the context of objective function (7).

This observation leads to the following two theorems.

THEOREM 1 *If the minimum VaR portfolio* (*'MVaRP'*) *exists, then it is mean–variance efficient.*

This can be seen in Figure 2 by noting what happens when lines with slope z are extended from points on the mean–variance boundary to the vertical axis, beginning with the point $p1$ corresponding to MVP. In particular, at first these lines have higher vertical intercepts but then starting at point MVaRP ($p2$) the lines have continually lower intercepts. Remembering what was shown in Figure 1, it follows that point MVaRP corresponds to the portfolio with minimum VaR.[4] Accordingly, the *mean–VaR efficient frontier* is the upper part of the mean–VaR boundary, beginning at MVaRP. Theorem 2 follows directly:

THEOREM 2 *MVP is mean–VaR inefficient for any* $t < 1$.

Figure 3 shows why this is so. Specifically, MVP has a lower expected return than MVaRP as shown in Figure 2. This observation means that there are portfolios that have both higher expected returns and lower VaRs than MVP. Hence, it follows that MVP is mean–VaR inefficient and that, accordingly, the mean–VaR efficient frontier is a proper subset of the mean–variance efficient frontier.

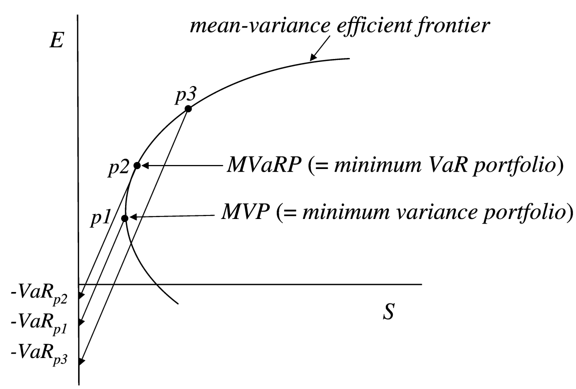

Figure 2. Comparing the minimum variance and minimum VaR portfolios.

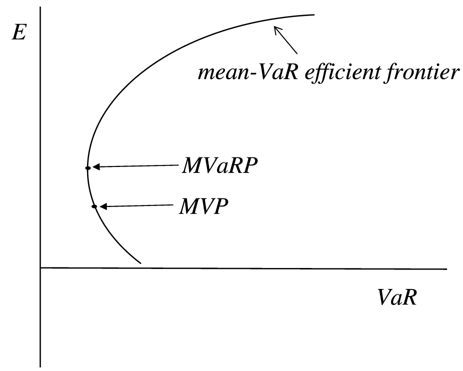

Figure 3. The mean–VaR efficient frontier.

What bearing do these observations have on the portfolio manager who uses the Markowitz mean–variance model for portfolio selection? Consider a situation at a bank such as JP Morgan Chase. To control the 'tail risk' of the bank's trading portfolio, it is reasonable to assume that the CEO has some notion of a maximum level of VaR that he or she believes is acceptable.[5] It follows that the portfolio manager must identify the *VaR-constrained mean–variance boundary.* Specifically, a portfolio p belongs to this boundary if and only if, for some expected return E^*, p solves the following problem:

$$\text{Minimize } V \tag{10}$$

$$\text{Subject to : } \boldsymbol{E}'\boldsymbol{W} = E^* \tag{11}$$

$$\boldsymbol{W}'\boldsymbol{1} = 1 \tag{12}$$

$$zS - E^* \leq B, \tag{13}$$

where B is the maximum level of VaR that is acceptable to the CEO. Note that the last constraint can be rewritten as $E^* \geq zS - B$. Using different values of E^*, the constraint forces the portfolio manager to select a portfolio on or above a line in mean–standard deviation space with an intercept of B and a slope of z.

Figure 4 indicates what the CEO hopes will happen with this constraint. In this 'good' scenario, the CEO is preventing the portfolio manager from selecting a portfolio with a relatively high standard deviation since such portfolios, plotting below the constraint, are now infeasible. However, Figure 5 shows a possible 'bad' scenario. Here the CEO is also preventing the portfolio manager from selecting a portfolio with a relatively small standard deviation (such as MVP) as they are infeasible. Hence, it is possible that the portfolio manager might be forced to select a portfolio with a larger standard deviation than he or she would select if unconstrained.

At this point it is tempting to say 'so what?' Figure 6 shows why this is an undesirable outcome. Consider efficient portfolios S and L, where S has a relatively small expected return and standard deviation and L has a relatively large expected return and standard deviation. It follows that their cumulative probability distributions intersect at only one point. This point corresponds to a confidence level of t_i. Imagine that the CEO is using a confidence level of t, which means that the VaRs of S and L are equal to VaR_S and VaR_L, respectively, where $\text{VaR}_S > \text{VaR}_L$. While the portfolio manager would like to select S, he or she will be forced to select portfolio L when

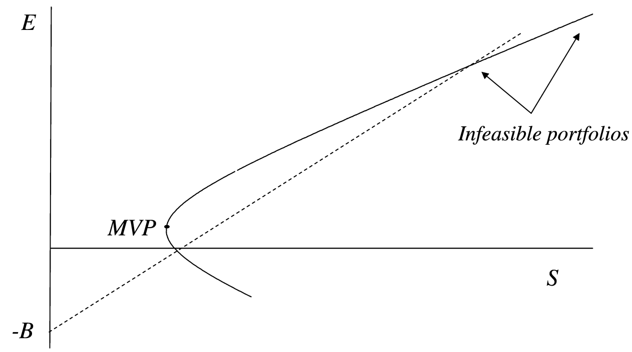

Figure 4. What the CEO hopes will happen with a VaR constraint.

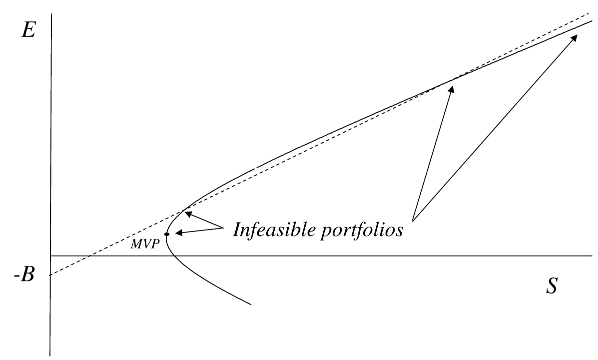

Figure 5. What can happen with a VaR constraint.

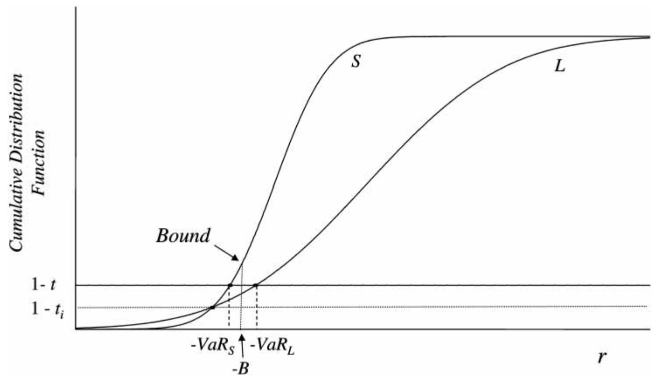

Figure 6. Using a VaR constraint can lead to a perverse outcome.

a bound of B is imposed where $\mathrm{VaR}_S > B > \mathrm{VaR}_L$. However, a visual inspection of the figure indicates that L has a higher probability of 'extremely bad outcomes', where 'extremely bad outcomes' are those that occur at a point below where the two distributions intersect. Thus, a VaR constraint can have the perverse effect of forcing the portfolio manager to select a portfolio with a higher probability of incurring an 'extremely bad outcome'. A remedy that has been proposed to ameliorate this problem is discussed next.

2. Conditional value-at-risk

The conditional value-at-risk (CVaR) of a portfolio is defined as its expected return, conditioned on the return being less than or equal to its VaR.[6] Using a 99% confidence level as an example, VaR involves measuring the upper end of the bottom 1% tail of the distribution, whereas CVaR measures the expected value of the 1% tail. More precisely, to calculate a portfolio's CVaR, one must first solve the following equation for k:

$$-\left[\int_{-\infty}^{-z} x\phi(x)\,\mathrm{d}x\right] \Big/ (1-t) = k, \tag{14}$$

where $\phi(x)$ is the standard normal probability function. Having solved Equation (14) for k, portfolio p's CVaR is

$$\mathrm{CVaR}_p = kS_p - E_p. \tag{15}$$

Note that $\mathrm{CVaR}_p > \mathrm{VaR}_p$ since $k > z$.[7]

Continuing with the earlier example involving a hypothetical portfolio whose annual return is normally distributed with a mean of 10% and a standard deviation of 20%, it can be determined from Equation (14) that based on a 99% confidence level, $k = 2.67$. Hence, the portfolio's annual CVaR is equal to $2.67 \times 20\% - 10\% = 43.4\%$. Accordingly, the portfolio with a current market value of €1,000,000,000 is expected to suffer a loss of €434,000,000 by the end of the year if a 'bad event' occurs, where a bad event is one that causes the portfolio to lose an amount equal to or greater than its VaR of €366,000,000.

Similar to VaR, it is quite easy to indicate a portfolio's CVaR in a mean–standard deviation diagram. Note that Equation (15) can be written as:

$$E_p = kS_p - \mathrm{CVaR}_p. \tag{16}$$

Like Equation (3), this equation corresponds to a straight line with a slope of k and vertical intercept of $-\mathrm{CVaR}_p$. As illustrated in Figure 7, our hypothetical portfolio's CVaR can be found by extending a line with a slope of 2.67 toward the vertical axis and noting that the intercept occurs at the value of -43.4. Since $-43.4\% = -\mathrm{CVaR}_p$, it can be seen that the portfolio has a CVaR of 43.4%. Furthermore, since VaR and CVaR are found by extending lines from the point where the portfolio plots in the figure and that the line for CVaR has a slope greater than the slope of the line for VaR,the intercept for CVaR lies below the intercept for VaR.

Continuing with the VaR analogy, portfolio p belongs to the *mean–CVaR boundary* if and only if, for some expected return E^*, p solves the following problem:

$$\text{Minimize CVaR} \tag{17}$$

$$\text{Subject to}: \boldsymbol{E}'\boldsymbol{W} = E^* \tag{18}$$

$$\boldsymbol{W}'\boldsymbol{1} = 1, \tag{19}$$

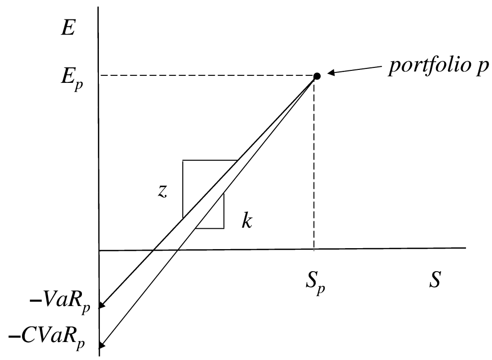

Figure 7. Representing a portfolio's CVaR in mean–standard deviation space.

where CVaR is defined in Equation (15). Furthermore, the mean–variance, mean–VaR, and mean–CVaR boundaries are identical since minimizing CVaR and VaR are equivalent to minimizing V for a given level of expected return. Theorem 3 follows.

THEOREM 3 *If the minimum CVaR portfolio* ('*MCVaRP*') *exists, then it is mean–variance efficient.*

This can be seen in Figure 8 by noting what happens when lines with slope k are extended from points on the mean–variance boundary to the vertical axis, beginning with the point $p1$ corresponding to MVP. As with VaR, at first these lines have higher vertical intercepts but then starting at point MCVaRP ($p2$) the lines have continually lower intercepts. Thus, point MCVaRP corresponds to the portfolio with minimum CVaR.[8] Accordingly, the *mean–CVaR efficient frontier* is the upper part of the mean–CVaR boundary, beginning at MCVaRP. Theorem 4 follows directly.

THEOREM 4 *MVP is mean–CVaR inefficient for any* $t < 1$.

Figure 9 shows that since MVP has a lower expected return that MCVaRP, it follows that MVP in *mean–CVaR* inefficient.

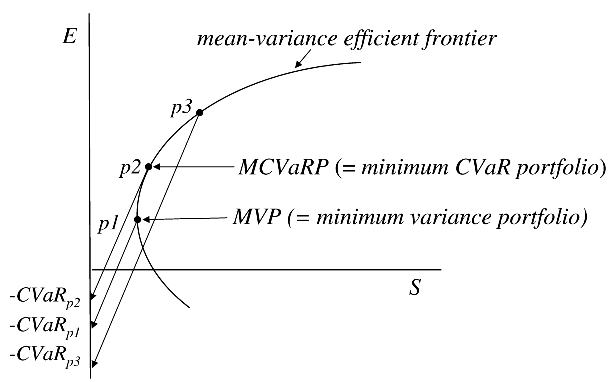

Figure 8. Comparing the minimum variance and minimum CVaR portfolios.

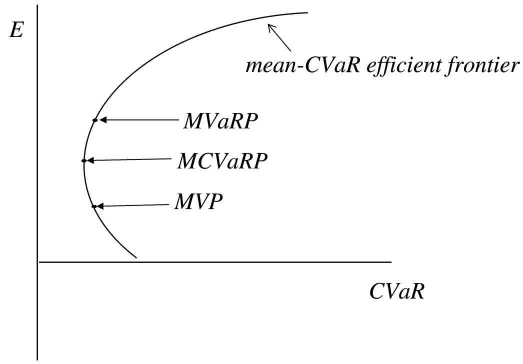

Figure 9. The mean–CVaR efficient frontier.

Note that the lines drawn in Figure 9 to determine the location of MCVaRP have a steeper slope (k) than the slope (z) of the lines drawn in Figure 3 to determine the location of MVaRP. This means that the expected return of MVaRP is greater than the expected return of MCVaRP. Theorem 5, illustrated in Figure 9, follows:

THEOREM 5 *MVaRP is mean–CVaR efficient for any* $t < 1$.

It follows that MCVaRP is mean–VaR inefficient since MCVaRP lies below MVaRP.

Similar to what was shown before with VaR, consider a portfolio manager at a bank who uses the Markowitz mean–variance model for portfolio selection but has a CEO who wishes to control the 'tail risk' of the bank's trading portfolio by setting a bound on the acceptable level of CVaR. It follows that the portfolio manager must identify the *CVaR-constrained mean–variance boundary.* Specifically, a portfolio p belongs to this boundary if and only if, for some expected return E^*, p solves the following problem:

$$\text{Minimize } V \tag{20}$$

$$\text{Subject to}: \boldsymbol{E}'\boldsymbol{W} = E^* \tag{21}$$

$$\boldsymbol{W}'\boldsymbol{1} = 1 \tag{22}$$

$$kS - E^* \leq B. \tag{23}$$

Note that the last constraint can be rewritten as $E^* \geq kS - B$, indicating that the portfolio manager must select a portfolio on or above a line in mean–standard deviation space with an intercept of B and a slope of k, but now $k > z$.

Figure 10 expands upon Figure 5 to show that a 'good' scenario is still possible when the same bound B is used. As with a VaR constraint, the CVaR constraint prevents the portfolio manager from selecting a portfolio with a relatively high standard deviation since such portfolios, plotting below the constraint, are now infeasible. Note that the CVaR constraint is 'tighter' than the VaR constraint in the sense that some previously feasible high-risk portfolios are now infeasible if the same bound B is utilized for both constraints since the slope of the CVaR constraint (k) is steeper than the slope of the VaR constraint (z). However, when examining Figure 5 it is also straightforward to see that a 'bad' scenario is more acute with CVaR relative to VaR in that more low-risk portfolios are precluded from consideration given the steeper slope when the same bound

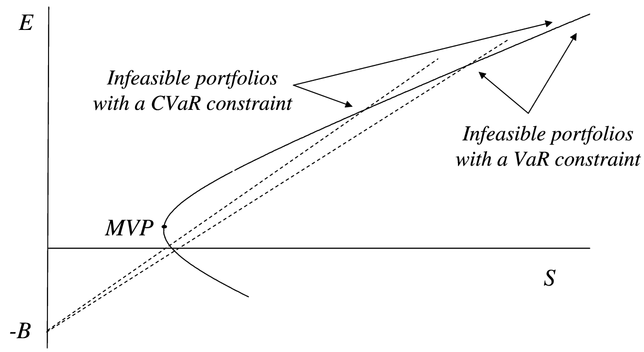

Figure 10. Comparing VaR and CVaR constraints with the same bound.

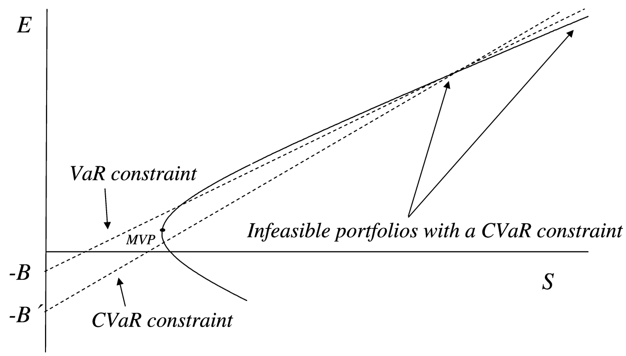

Figure 11. Comparing VaR and CVaR constraints with different bounds.

B is used. However, Figure 11 shows that it is possible to use a larger bound B' so that it precludes the same high-risk portfolios as the VaR constraint but does not preclude the low-risk portfolios from selection, unlike the VaR constraint.

3. Stress testing

A portfolio that passes stress testing and is thus permissible for selection is one whose returns under certain scenarios are equal to or greater than some bound. These scenarios can be historical ones, such as the crash in the US stock market in October 1987 and the terrorist attacks in New York City on 11 September 2001, or hypothetical ones, such as a major earthquake or a sudden large drop in the value of the US dollar.[9] Portfolios whose estimated return, conditioned on each scenario i happening, must constraints of the following form satisfy in order to be feasible:

$$\boldsymbol{W}'\boldsymbol{R}_i \geq T, \quad i = 1, 2, \ldots, m, \tag{24}$$

where $\boldsymbol{R}_i$ is an $n \times 1$ vector of estimated returns on the n securities conditioned on scenario i occurring and T is the bound; there are m scenarios.[10]

Note how a portfolio can have its 'tail risk' limited by using stress testing in conjunction with either VaR or CVaR by adding Equation (24) to the optimization problems given in

Equations (10)–(13) and (20)–(23), respectively. Furthermore, Bloomberg machines can be used to conduct stress tests of assorted historical events as well as calculate the VaR of a portfolio using several confidence levels.

4. Conclusion

Modern risk management has it roots in the mean–variance model of Markowitz, but focuses on the bottom tail of the return distribution in evaluating and controlling the risk of a portfolio. The most commonly used measure of tail risk, VaR, focuses on the upper end of the tail distribution in that it represents the return of the $(1 - t)$ quantile of the return distribution for a confidence level of t. However, it has been criticized by ignoring the shape of the tail distribution below this level. In response, CVaR has been advocated by, for example, Artzner et al. (1999), since it measures the expected value of the tail distribution. It can be viewed as the expected return of the portfolio, conditioned on a 'bad event' occurring. In this address, I show how VaR and CVaR of a portfolio can be represented in the mean–standard deviation diagram of Markowitz. I also show how the mean–variance, mean–VaR, and mean–CVaR boundaries are equivalent, but that the mean–VaR efficient frontier is a proper subset of the mean–CVaR efficient frontier, which in turn is a proper subset of the mean–variance efficient frontier.

Continuing, I show that portfolio managers who are limited to selecting a portfolio whose VaR does not exceed some bound could be forced to choose a portfolio that has not only a larger standard deviation but also a larger CVaR. Thus, a risk management system based on VaR could lead to the perverse result of a riskier portfolio being selected. However, the use of CVaR, with properly chosen bounds, avoids such a result.

International banks are currently required to use risk management systems based on VaR, with an amount of capital required for their trading portfolio being based on its magnitude. In addition, banks are required to have a rigorous stress testing system in place. This leaves open the question of whether a risk management system based on VaR and stress testing is as effective as one based on CVaR. In a recently completed working paper, Baptista, Yan, and I have used a simple model based on historical simulation to examine this question (see Alexander, Baptista, and Yan 2008). While the combination of VaR and stress testing seems to work well when short selling is not allowed, the combination can perform poorly when it is allowed. Since short selling is typically involved in trading portfolios, this raises serious concerns about the use of such risk management systems. When coupled with various implementation problems such as non-stationary distributions, increased liquidity problems during times of stress, acute estimation risk in modeling the tail distribution of a portfolio, and Black Swans ('unknown unknowns') of Taleb (2007), it is hardly surprising that Greenspan (2008) would comment that the current financial crisis 'will leave many casualties. Particularly hard hit will be much of today's financial risk-valuation system, significant parts of which failed under stress. … The problems, at least in the early stages of this crisis, were most pronounced among banks whose regulatory oversight has been elaborate for years'. While much work has been done to improve such systems, it is clear that much work remains to be done.

Acknowledgements

This paper is based on the Keynote Address given at the Asset Management and International Capital Markets Conference that was held in Frankfurt on 29 May 2008. The author is indebted to Alexandre M. Baptista for comments and many stimulating discussions on the topic.

Notes

1. Wikipedia, for example, makes such a reference to Markowitz: http://en.wikipedia.org/wiki/People_known_as_the_father_or_mother_of_something#Economics. Also see the press release announcing his being awarded the 1990 Alfred Nobel Memorial Prize in Economic Sciences: http://nobelprize.org/nobel_prizes/economics/laureates/1990/press.html.
2. My comments draw largely from Alexander and Baptista (2002, 2004, 2006). An excellent book on VaR, CVaR, and stress testing is Jorion (2007). Also see Crouhy, Galai, and Mark (2006) and Hull (2007).
3. Technically VaR is equal to $E_p - zS_p$. However, this typically produces a negative number. The desire to express VaR as a positive number results in Equation (1). It is also important to point out that Equation (1) and the subsequent analysis is based on the assumption that the portfolio's returns have a normal distribution (it is straightforward to show that similar results are obtained with a t-distribution).
4. Note that MVaRP might have an infinite expected return depending on the shape of the hyperbola and confidence level. I assume it has a finite expected return in all of my examples.
5. Motives for controlling the tail risk of the bank's trading portfolio are two-fold. First, it is a means of limiting the probability of default. Second, the bank's capital requirement is based in part on the VaR of its trading portfolio. For example, see Crouhy, Galai, and Mark (2006, 154–61) and the Basle Committee on Banking Supervision (2006), respectively. Also see Ball and Fang (2006) for a survey of the literature on VaR and bank regulation.
6. Due to the practice of expressing VaR and CVaR as positive numbers, technically −CVaR is the portfolio's expected return conditioned on the return being less than or equal to −VaR.
7. It is possible for a portfolio's VaR and CVaR to be equal when the lower mass of the probability density function is concentrated at the portfolio's VaR. That is, VaR = CVaR when the probability of observing a return less than the portfolio's VaR is zero. Since such distributions are highly unlikely, for expository purposes it is assumed that $\text{CVaR}_p > \text{VaR}_p$.
8. Note that MCVaRP might have an infinite expected return depending on the shape of the hyperbola and confidence level. I assume it has a finite expected return in all of my examples.
9. Banks are currently required to stress test their portfolios; see Basle Committee on Banking Supervision (2006) and Committee on the Global Financial System (2005).
10. The bounds can be of different values for the various scenarios; a constant bound is used here simply for expository purposes.

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