Initial Research

After looking through the “School Bus” article on Wikipedia, I found a link to the “Blue Bird Vision” (http://en.wikipedia.org/wiki/Blue_Bird_Vision) which is one of the most recent models of school buses that resembles what I imagine when I think of a school bus . The Wikipedia article gives a link to a spec sheet (http://www.blue-bird.com/uploadedFiles/Blue-Bird/Products/School/Vision/SB-VIS-1109.pdf

) that includes several useful dimensions and specifications:

Exterior Width: 96”

Interior Width: 90.75”

Interior Height (Headroom): 74”; 77” optional

Overall Height: 127”

Entrance Door: 27” Wide x 78” High

Tire Size: 11R22.5 G Goodyear G661HSA

Overall Length: 289” (48 Passenger) – 471” (72/77 Passenger)

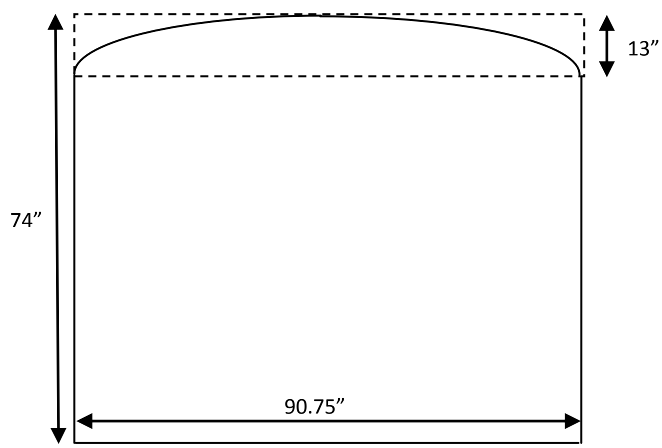
For this estimation I will use the 72 passenger bus because the 48 passenger bus sounds smaller than the ones I used to ride to school, and I’m guessing that the 77 passenger configuration has small seats crammed irregularly into extra spaces.

Estimating the Bus Interior Dimensions

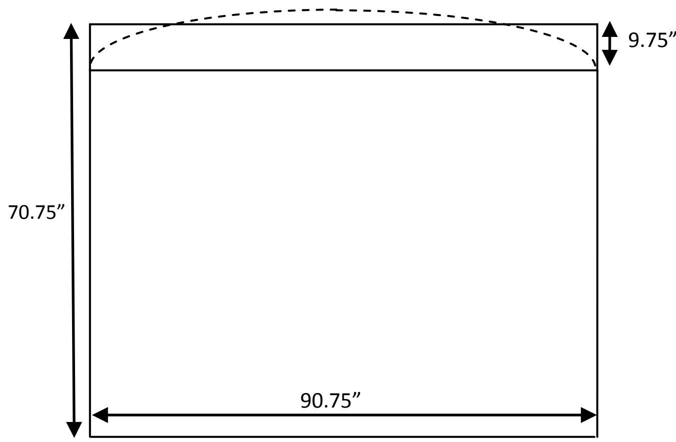
The spec sheet gave the interior dimensions for width and headroom, but not the interior length. It only gives the overall length for the entire bus, which includes the space for the engine at the front end as well as the wall thickness, so to determine interior length I’ll need to estimate how much additional length those things add. On photos of the Bluebird Vision, the length of the front end of the bus looks to be about 1.5x the diameter of the front tires.

Looking up the tires online, 11R22.5 G Goodyear G661HSA, I found that they have a diameter of 41.8" (http://www.goodyear.com/cfmx/web/truck/line.cfm?prodline=160617), so the front end adds about 41.8" x 1.5" = **62.7"** to the front of the vehicle. So the passenger compartment of the bus is about 471" – 62.7" = **408.3"** long. For the bus wall thickness I'll subtract another 5.25" (that's the difference between the exterior width, 96", and the interior width, 90.25".) So the overall estimate for the interior bus length is 403.05".

The specification for "Headroom" is 74", but this is the interior height at the highest point in the center of the bus where the aisle is; the bus has an arched roof. In photos of the Bluebird Vision, the arch on top of the school bus looks to have a height that is about 1/6 of the height of the door, which is 78" high, so I'll estimate 78" / 6 = **13"** for the arch height. The picture below shows the resulting cross-section estimate for the bus's interior space.

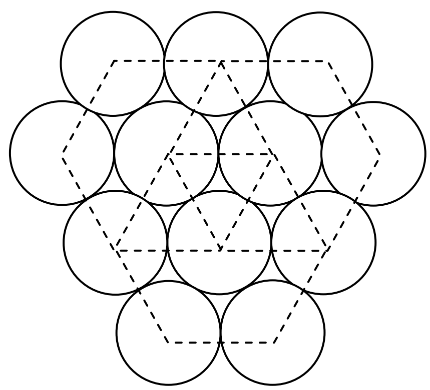


To estimate the area enclosed within the arc, I'll use 75% of the area of the bounding rectangle. A circle circumscribed within a square has $\pi/4$ (~0.785) times the area of the square, so taking 75% of the 13" x 90.75" rectangle that bounds the arch effectively approximates that area (the slight difference between 78.5% and 75% partially covers for the fact that tightly-packed spheres will fit into a rectangle better than they will into some kind of cylinder cross-section). This way, the arch can be represented by a 9.75" x 90.75" rectangle on top of the bus, and the bus's entire cross section can be simplified to a 90.75" x 70.75" rectangle. So the main interior area of the bus can be represented by a 403.05" x 90.75" x 70.75" box.



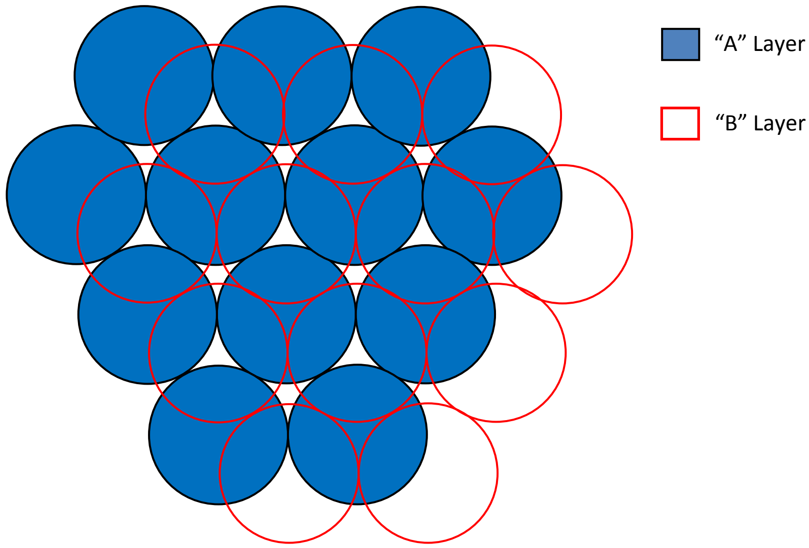
Estimating the Number of Balls that Fit into the Main Bus Area

The tightest configuration of circular objects in two-dimensional space is made up of alternating equilateral triangles, as shown below:



One of the tightest-packed configurations for spherical objects in 3D space[1] can be represented by repeating two alternating layers of spheres packed in the equilateral triangle arrangement shown above, such that the spheres in each layer rest in the spaces of the adjacent layers.

[1] The other tightest configuration for spherical objects in 3D space is a pattern of repeating three alternating layers that similarly fit into the spaces of their adjacent layers, but for simplicity the configuration with two alternating layers will be used.



In this configuration, any set of four contacting spheres forms a regular tetrahedron, so the distance between layers can be determined using the formula for the height of a regular tetrahedron:

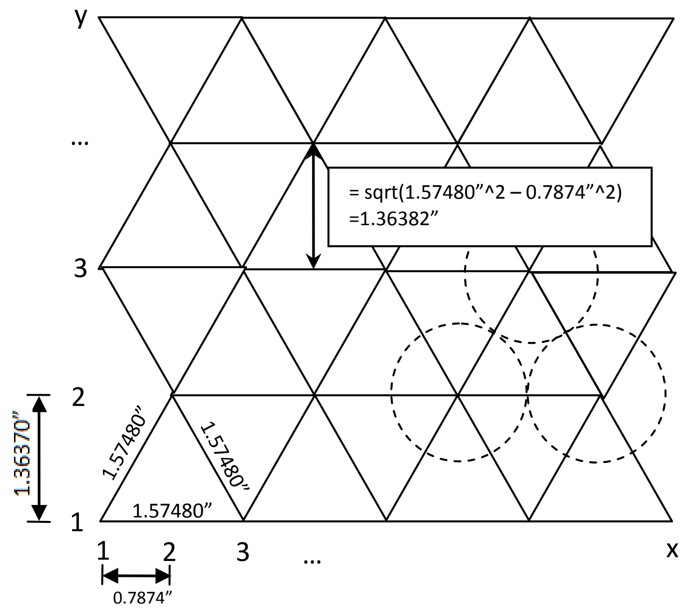
$$H = \frac{\sqrt{6}}{3}a$$

(source: http://en.wikipedia.org/wiki/Tetrahedron)

Given that a ping pong ball has a diameter of 40mm, or 1.57480", the distance between layers of ping pong balls is: √6 / 3 * 1.57480" = **1.28582".** If the layers stack in the direction of the length of the bus, then the number of layers that fill the length of the bus can be estimated as:

403.05" / 1.28582" (rounded down) = **313 layers**.

The layers make up two alternating configurations, which I will refer to as the "A" layers, and the slightly offset "B" layers. To estimate the number of balls that fit into each layer, I will use the diagram below (borrowed from last year's application), which divides a rectangular area into x columns and y rows:



The columns are separated by half of the ball diameter (0.7874"), and the rows are separated by the height of an equilateral triangle with sides equal to the ball diameter. The configuration can be broken up into two interlocking rectangular grids, one made up of the even rows and columns, and one made up of the odd rows and columns, the totals of which are determined and added together using the equation below (also borrowed from last year's application):

Number of Balls per Layer = [x / 2 (rounded up)] * [y / 2 (rounded up)] +

[x / 2 (rounded down)] * [y / 2 (rounded down)]

So to determine the number of balls that fit into the "A" layers, the dimensions of the bus cross section are plugged into these equations:

> x = [bus width (90.75") – 1.57480" (to represent the half of a ball on each edge)] / 0.7874" (rounded down) = 113
>
> y = [bus height (70.75") – 1.57480" (to represent the half of a ball on each edge)] / 1.36382" (rounded down) = 50
>
> So the number of balls that fit into an "A" layer is 57*25 + 56*25 = **2825**.

The balls in the "B" layers are simply shifted up to the centers of every other equilateral triangle in the adjacent "A" layers. The height of the center of an equilateral triangle is:

$$\frac{\sqrt{3}}{6}a$$

(source: http://en.wikipedia.org/wiki/Equilateral_triangle)

So the y values are offset by √3 / 6 * 1.57480" = 0.454606" and the x value remains the same

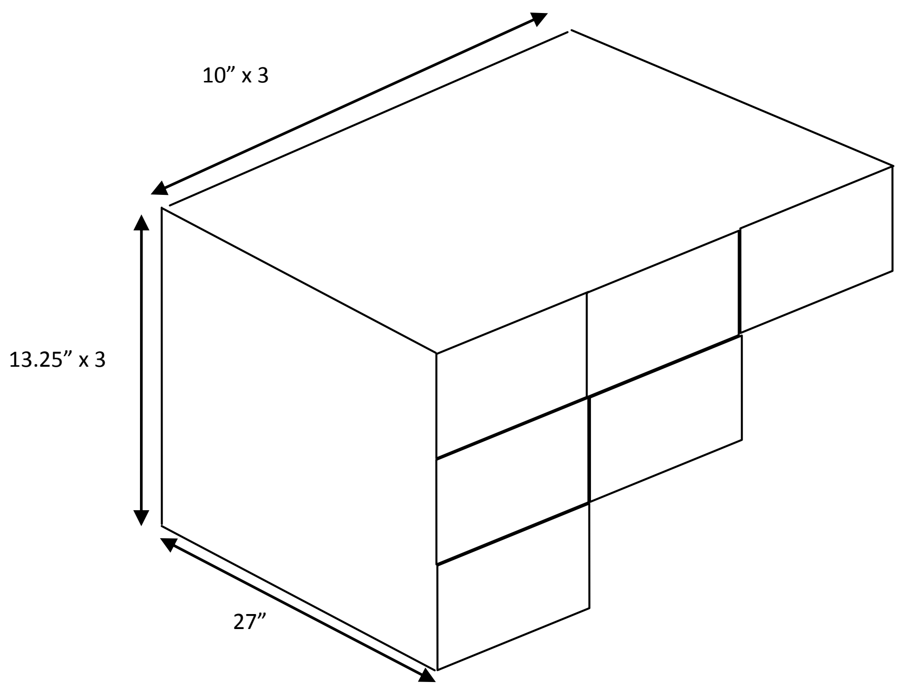
y = [bus height (70.75") – 1.57480" – 0. 454606"] / 1.36370" (rounded down) = 50

So the "B" layers also hold 2825 balls (the extra height left over when determining the y value for "A" layers is less than the 0.455" that the B layers are shifted upward.)

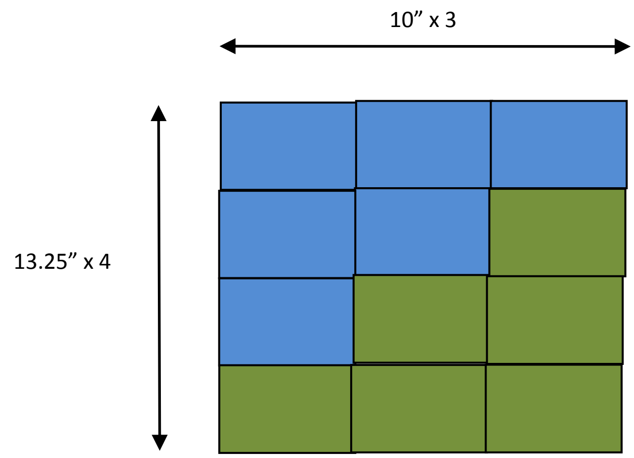
So 2825 balls/layer * 313 layers = **884225** ping pong balls fit into the main area of the school bus.

Estimating the Number of Balls that Fit into the Stairwell

There also appear to be 3 steps between the bus door and the aisle. The steps appear to be about as wide as the door (27"), and tall enough to divide the space between the aisle floor and the ground by four [overall height (127") – headroom (74")] / 4 = **13.25".** In photos the stairs appear about ¾ as long as they are tall, or about 10".



To estimate the number of balls that would fit into this space, I will simplify the calculations by representing the staircase with a box that is formed by putting two staircase volumes together, calculating the number of balls that fit into that box, and then dividing that number in half.



(13.25” x 4) x (10” x 3) x 27” = 53” x 30” x 27” box

Using the same formula as above:

Number of Layers

27” / 1.28582” (rounded down) = 20 layers

For “A” Layers:

x = (30” – 1.57480”) / 0.7874” = 36

y = (53” – 1.57480”) / 1.36370” = 37

# of balls / “A” layer = 18 * 19 + 18 * 18 = 343 balls

For “B” Layers:

x = (30” – 1. 57480”) / 0. 7874” = 36

y = (53” – 1. 57480” – 0. 454606”) / 1.36370” = 37

# of balls / “B” layer = 18 * 19 + 18 * 18 = 343 balls

20 layers * 343 balls/layer = 6860 balls in 2x stairway, **3430 balls** in stairway.

Estimating the Number of Balls Displaced by Seats

To estimate the number of balls displaced by the seats, I need to estimate the dimensions of each seat. The interior bus length is estimated to be 403”, and the front area for the staircase and driver’s seat appear to extend about 9” wider than the door (27” + 9” = 36”), so the remaining area for passengers is approximately 403” – 36” = **367”** long. To fit 72 passengers on this bus using two aisles of seats with two people per seat, creating 18 rows of seats, each seat would get approximately 20” of length, which definitely is not enough room. To fit 72 passengers with three people per seat (which you could do with smaller children), there would be 12 rows of seats (24 seats total), each getting approximately 30½” of length, which seems much more reasonable. To find the width of the seats, we can take the overall width of the bus and divide it by seven (for three people per seat and an extra person-width for the aisle in-between):

90.75” / 7 ≅ 13”

Since 13” seems like a narrow aisle for all but the smallest children, I’ll add an additional 3” to the aisle space, and split the remaining width to determine seat width:

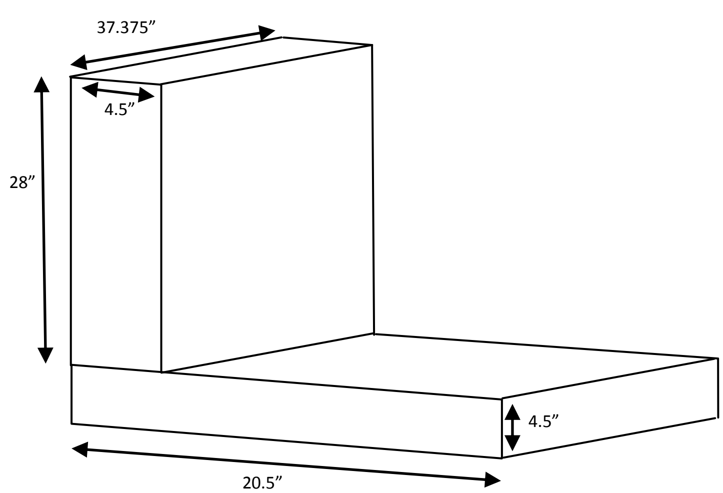
(90.75” – 16”) / 2 = 37 3/8”

This gives about 12.5” of width for each of three small children, or about 18.5” for larger people seated two per seat (which seems in line with what I remember about how much room is on a school bus.)

The Federal Motor Vehicle Safety Standard for school bus seat requirements (http://www.fmcsa.dot.gov/rules-regulations/administration/fmcsr/fmcsrruletext.aspx?reg=571.222) require that for buses made after October 2009, the seat back must extend 24” above the “seating reference point,” a point situated approximately 4” above the bottom of the seat. So we’ll assume that the seats are 28” high above the seating surface.

For the length of the seat bottom, I’ll assume that 1/3 of the seat length (~10”) is left open for leg space, and therefore the length of the seat bottom is approximately 20.5”

For seat thickness I wasn’t able to find a good reference, but from experience riding on school busses I recall that they’re about 4.5” thick.



For simplicity, I'll combine the seat back and the seat bottom to create one big (37.375" x 4.5" x 48.5") slab, and figure out the number of balls that will cover that space, rounding up for numbers of rows, columns and layers because the seats are displacing balls rather than making room for them.

4.5" / 1.28582" per layer (rounded up) = 4 layers

"A" Layers:

> x = (37.375" – 1.57480")/0.7874" (rounded up) = 46
>
> y = (48.5" – 1.57480")/1.36370 (rounded up) = 35
>
> # of balls per "A" Layer = 23 * 18 + 23 * 17 = 805

"B" Layers:

> X = 46
>
> y = (48" – 1.57480" – 0. 454606")/1.36370 (rounded up) = 34
>
> # of balls per "B" Layer = 23 * 17 + 23 * 17 = 782

So the number of balls displaced by one seat is approximately (2*805 + 2*782) = **3174**

So the 24 seats on the bus displace about 3174 * 24 = **76176 balls**.

Adding it all Up

I'll assume that the driver's seat and controls displace about the same number of balls as one seating bench (3174). And with that, the total number of balls that would fit in the bus can be estimated as:

Balls that fit in overall bus area:	884225
Balls that fit in staircase:	3430
Balls Displaced by Seats:	-76176
Ball Displaced by Driver's Seat:	-3174
Grand Total:	**808305**