Possible collision model

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Abstract

Collisions reference: **?**

1 Series

$$G_{mn}^{(j)} = \sum_{\alpha \in S_j} B_j^{(2)}(\alpha) F_m^{(j)}(\alpha) F_n^{(j)}(\alpha). \tag{1}$$

$$
\begin{aligned}
F_n^{(j)}(\alpha) &= \frac{2}{H_1} \int_{-H}^{-\sigma_1} \varphi^{(j)}(z,\alpha) \mathscr{C}_{2n}\left(\frac{z+H}{H_1}\right) dz \\
&= 2\mathrm{sech}(\gamma H_j) \int_0^1 \cos(\kappa t) \mathscr{C}_{2n}(t) dt \\
&= \frac{(-1)^n \Gamma(\beta)}{\cosh(\gamma H_j)} \left(\frac{\kappa}{2}\right)^{-\beta} (2n+\beta) J_{2n+\beta}(\kappa),
\end{aligned} \tag{2}
$$

where $\kappa = -\mathrm{i}\gamma H_2$.

Now let $\tilde{\gamma}_{j,r} = \mathrm{i}n\pi/H_j$, and $\tilde{\kappa}_{j,r} = -\mathrm{i}\tilde{\gamma}_{j,r}H_2 = r\pi H_2/H_j = r\pi/\hat{H}_j$ ($r = 1, 2, \ldots$). Then

$$B_j^{(2)}(\tilde{\gamma}_{j,r}) = \frac{\mathrm{i}}{r\pi}, \tag{3a}$$

$$
\begin{aligned}
J_{2n+\beta}(\tilde{\kappa}_{j,r}) &= \sqrt{\frac{2}{\pi\tilde{\kappa}_{j,r}}} \cos\left(\tilde{\kappa}_{j,r} - (2n+\beta)\frac{\pi}{2} - \frac{\pi}{4}\right) \\
&= (-1)^n r^{-1/2} \sqrt{\frac{2}{\pi^2 \hat{H}_j}} \cos\left(\tilde{\kappa}_{j,r} - \beta'\right),
\end{aligned} \tag{3b}
$$

$$\beta' = \frac{\pi}{2}\left(\beta + \frac{1}{2}\right). \tag{3c}$$

Thus

$$F_n^{(j)}(\tilde{\gamma}_{j,r}) = (-1)^r r^{-(2\beta+1)/2} f_{j,n}(\beta) \cos\left(\tilde{\kappa}_{j,r} - \beta'\right), \tag{4a}$$

$$f_{j,n}(\beta) = \Gamma(\beta)(2n+\beta)\left(\frac{\pi}{2\hat{H}_j}\right)^{-\beta} \sqrt{\frac{2}{\pi^2 \hat{H}_j}}, \tag{4b}$$

and

$$\tilde{G}_{mn}^{(j)} = \sum_{r=1}^{\infty} B_j^{(2)}(\tilde{\gamma}_{j,r}) F_m^{(j)}(\tilde{\gamma}_{j,r}) F_n^{(j)}(\tilde{\gamma}_{j,r})$$

$$= \frac{i}{\pi} f_{j,n}(\beta) f_{j,m}(\beta) \sum_{r=1}^{\infty} r^{-s} \cos^2(\tilde{\kappa}_{j,r} - \beta')$$

$$= \frac{i}{2\pi} f_{j,n}(\beta) f_{j,m}(\beta) \sum_{r=1}^{\infty} r^{-s} \left(1 + \mathrm{Re}\left(e^{-2i\beta'} e^{2i\pi r/\hat{H}_j}\right)\right)$$

$$= \frac{i}{2\pi} f_{j,n}(\beta) f_{j,m}(\beta) \left(\zeta(s) + \mathrm{Re}\left(e^{-2i\beta'} \mathrm{Li}_s\left(e^{2i\pi/\hat{H}_j}\right)\right)\right), \qquad (5)$$

where $s = 2(\beta + 1)$ and

$$\mathrm{Li}_s(z) = \sum_{r=1}^{\infty} \frac{z^r}{r^s}$$

$$= \frac{z}{\Gamma(s)} \int_0^{\infty} \frac{t^{s-1} e^{-t}}{1 - z e^{-t}} dt \qquad (6)$$

is the polylogarithm function (??). This integral can be done very efficiently with Gauss-Laguerre quadrature. Also note that $\mathrm{Li}_s(1) = \zeta(s)$.