GooseSolid/LinearElastic

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Abstract

Linear elasticity: a linear relation between the Cauchy stress $\boldsymbol{\sigma}$ and the linear strain $\boldsymbol{\varepsilon}$

The model is implemented in 3-D, hence it can directly be used for either 3-D or 2-D plane strain problems.

Keywords: linear elasticity

1 Constitutive model

The stress, $\boldsymbol{\sigma}$, is set proportional to the strain, $\boldsymbol{\varepsilon}$, through the following linear relation:

$$\boldsymbol{\sigma} = \mathbb{C}_\mathrm{e} : \boldsymbol{\varepsilon} \tag{1}$$

wherein $\mathbb{C}_\mathrm{e}$ is the elastic stiffness, which reads:

$$\begin{aligned} \mathbb{C}_\mathrm{e} &= K\boldsymbol{I}\otimes\boldsymbol{I} + 2G(\mathbb{I}_\mathrm{s} - \tfrac{1}{3}\boldsymbol{I}\otimes\boldsymbol{I}) && (2)\\ &= K\boldsymbol{I}\otimes\boldsymbol{I} + 2G\,\mathbb{I}_\mathrm{d} && (3) \end{aligned}$$

with K and G the bulk and shear modulus respectively. See Appendix A for nomenclature.

A Nomenclature

- Dyadic tensor product

$$\begin{aligned} \mathbb{C} &= \boldsymbol{A}\otimes\boldsymbol{B} && (4)\\ C_{ijkl} &= A_{ij}\,B_{kl} && (5) \end{aligned}$$

- Double tensor contraction

$$\begin{aligned} C &= \boldsymbol{A} : \boldsymbol{B} && (6)\\ &= A_{ij}\,B_{ji} && (7) \end{aligned}$$

- Deviatoric projection tensor

$$\mathbb{I}_\mathrm{d} = \mathbb{I}_\mathrm{s} - \tfrac{1}{3}\boldsymbol{I}\otimes\boldsymbol{I} \tag{8}$$