Elasto-viso-plasticity

$$\varepsilon = \varepsilon_e + \varepsilon_p$$

$$\sigma = {}^4\mathbb{C} : \varepsilon_e = {}^4\mathbb{C} : (\varepsilon - \varepsilon_p) \qquad \text{with } {}^4\mathbb{C} = K\,\mathbb{I}\otimes\mathbb{I} + 2G\,{}^4\mathbb{I}_d$$

yield function

$$f(\sigma, \varepsilon_p) = \sigma_{eq} - \sigma_y(\varepsilon_p) \qquad \sigma_{eq} = \sqrt{\tfrac{3}{2}\,\sigma_d : \sigma_d}$$

normality:

$$\dot{\varepsilon}_p = \dot{\gamma}\,\frac{\partial f}{\partial \sigma} \qquad \dot{\varepsilon}_p = \sqrt{\tfrac{2}{3}\,\dot{\varepsilon}_p : \dot{\varepsilon}_p} = \dot{\gamma} \quad , \quad \varepsilon_p = \int_0^T \dot{\varepsilon}_p \, dt$$

evolution of plasticity

$$\dot{\gamma} = \dot{\gamma}_0 \left(\frac{\sigma_{eq}}{\sigma_y} \right)^{1/m}$$

The deviatoric response can be written as (ε_p is strictly deviatoric)

$$\sigma_d = 2G\,(\varepsilon_d - \varepsilon_p)$$

or, in rate form:

$$\dot{\sigma}_d = 2G\,(\dot{\varepsilon}_d - \dot{\varepsilon}_p)$$

$$\dot{\sigma}_d = 2G\left[\dot{\varepsilon}_d - \dot{\gamma}_0\left(\frac{\sigma_{eq}}{\sigma_y}\right)^{1/m} \frac{3}{2}\,\frac{\sigma_d}{\sigma_{eq}}\right] \qquad \left(\frac{\partial f}{\partial \sigma} \equiv \frac{3}{2}\,\frac{\sigma_d}{\sigma_{eq}}\right)$$

Simple shear: $\dot{\varepsilon} = \begin{bmatrix} 0 & \dot{\gamma} & 0 \\ \dot{\gamma} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \sigma = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $m = 1$; $\dfrac{\sigma_y}{\dot{\gamma}_0} = \dfrac{3G}{\eta}$

$$\dot{\tau} = 2G\,\dot{\gamma} - \eta\tau$$