GooseSolid/LinearElastic

T.W.J. de Geus

*Contact: tom@geus.me – www.geus.me – https://github.com/tdegeus/GooseSolid

Abstract

Linear elasticity: a linear relation between the Cauchy stress $\boldsymbol{\sigma}$ and the linear strain $\boldsymbol{\varepsilon}$
    The model is implemented in 3-D, hence it can directly be used for either 3-D or 2-D plane strain problems.

Keywords: linear elasticity

1 Constitutive model

The stress, $\boldsymbol{\sigma}$, is set by to the elastic strain, $\boldsymbol{\varepsilon}_\mathrm{e}$, through the following linear relation:

$$\boldsymbol{\sigma} = \mathbb{C}_\mathrm{e} : \boldsymbol{\varepsilon}_\mathrm{e} \tag{1}$$

wherein $\mathbb{C}_\mathrm{e}$ is the elastic stiffness, which reads:

$$\mathbb{C}_\mathrm{e} = K\boldsymbol{I} \otimes \boldsymbol{I} + 2G(\mathbb{I}_\mathrm{s} - \tfrac{1}{3}\boldsymbol{I} \otimes \boldsymbol{I}) \tag{2}$$
$$= K\boldsymbol{I} \otimes \boldsymbol{I} + 2G\,\mathbb{I}_\mathrm{d} \tag{3}$$

with K and G the bulk and shear modulus respectively. See Appendix A for nomenclature.

A Nomenclature

- Dyadic tensor product

$$\mathbb{C} = \boldsymbol{A} \otimes \boldsymbol{B} \tag{4}$$
$$C_{ijkl} = A_{ij}\,B_{kl} \tag{5}$$

- Double tensor contraction

$$C = \boldsymbol{A} : \boldsymbol{B} \tag{6}$$
$$= A_{ij}\,B_{ji} \tag{7}$$

- Deviatoric projection tensor

$$\mathbb{I}_\mathrm{d} = \mathbb{I}_\mathrm{s} - \tfrac{1}{3}\boldsymbol{I} \otimes \boldsymbol{I} \tag{8}$$