Complementary Filters Shaping Using $\mathcal{H}_\infty$ Synthesis

Dehaeze Thomas, Verma Mohit and Collette Christophe

Abstract For many applications, large bandwidth and dynamic ranges are requiring to use several sensors, whose signals are combined using complementary filters. This paper presents a method for designing these complementary filters using $\mathcal{H}_\infty$ synthesis that allows to shape the filter norms. This method is shown to be easily applicable for the synthesis of complex complementary filters.

Introduction

A set of filters is said to be complementary if the sum of their transfer functions is equal to one at all frequencies. These filters are used when two or more sensors are measuring the same physical quantity with different noise characteristics. Unreliable frequencies of each sensor are filtered out by the complementary filters and then combined to form a super sensor giving a better estimate of the physical quantity over a wider bandwidth. This technique is called sensor fusion and is used in many applications.

In [1]–[3], various sensors (accelerometers, gyroscopes, vision sensors, etc.) are merged using complementary filters for the attitude estimation of Unmanned Aerial Vehicles (UAV). In [4], several sensor fusion configurations using different types of sensors are discussed in order to increase the control bandwidth of active vibration isolation systems. Furthermore, sensor fusion is used in the isolation systems of the Laser Interferometer Gravitational-Wave Observator (LIGO) to merge inertial sensors with relative sensors [5], [6].

As the super sensor noise characteristics largely depend on the complementary filter norms, their proper design is of primary importance for sensor fusion. In [2], [3], [7], first and second order analytical formulas of complementary filters have been presented. Higher order complementary filters have been used in [1], [4], [8]. In [7], the sensitivity and complementary sensitivity transfer functions of a feedback architecture have been proposed to be used as complementary filters. The design of such filters can then benefit from the classical control theory developments. Linear Matrix Inequalities (LMIs) are used in [9] for the synthesis of complementary filters satisfying some frequency-like performance. Finally, a synthesis method of high order Finite Impulse Response (FIR) complementary filters using convex optimization has been developed in [6], [10].

Although many design methods of complementary filters have been proposed in the literature, no simple method that allows to shape the norm of the complementary filters is available.

This paper presents a new design method of complementary filters based on $\mathcal{H}_\infty$ synthesis. This design method permits to easily shape the norms of the generated filters.

Complementary Filters Requirements

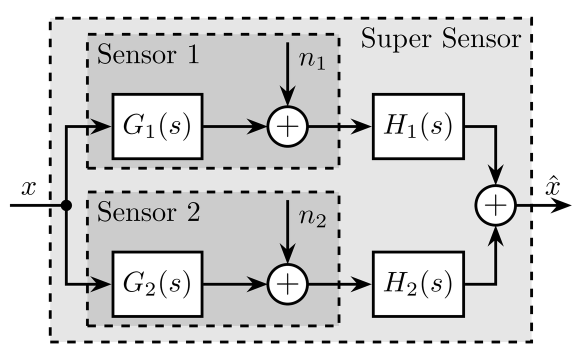

Fig. 1: Sensor fusion architecture

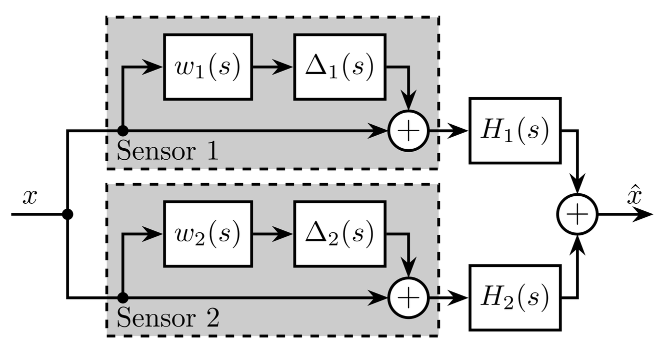

Fig. 2: Sensor fusion architecture with sensor dynamics uncertainty

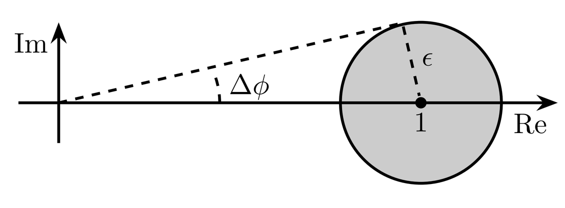

Fig. 3: Uncertainty set of the super sensor dynamics

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Synthesis Architecture

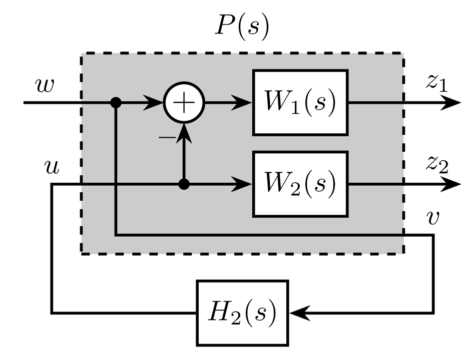

Fig. 4: Architecture used for $\mathcal{H}_\infty$ synthesis of complementary filters

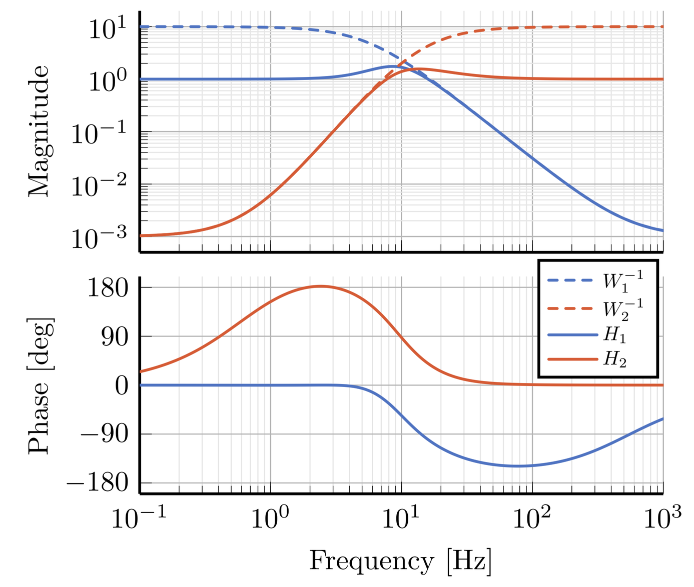

Fig. 5: Frequency response of the weighting functions and complementary filters obtained using $\mathcal{H}_\infty$ synthesis

Weighting Function Design

$$W(s) = \left(\frac{ \frac{1}{\omega_0} \sqrt{\frac{1-\left(\frac{G_0}{G_c}\right)^{\frac{2}{n}}}{1-\left(\frac{G_c}{G_\infty}\right)^{\frac{2}{n}}}} s + \left(\frac{G_0}{G_c}\right)^{\frac{1}{n}} }{ \left(\frac{1}{G_\infty}\right)^{\frac{1}{n}} \frac{1}{\omega_0} \sqrt{\frac{1-\left(\frac{G_0}{G_c}\right)^{\frac{2}{n}}}{1-\left(\frac{G_c}{G_\infty}\right)^{\frac{2}{n}}}} s + \left(\frac{1}{G_c}\right)^{\frac{1}{n}} } \right)^n \tag{1}$$

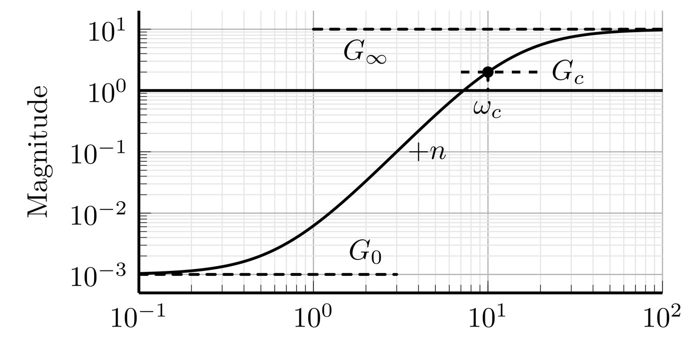

Fig. 6: Magnitude of a weighting function generated using the proposed formula eq:weight$_formula$, $G_0 = 1e^{-3}$, $G_\infty = 10$, $\omega_c = 10\,\text{Hz}$, $G_c = 2$, $n = 3$

Three Complementary Filters

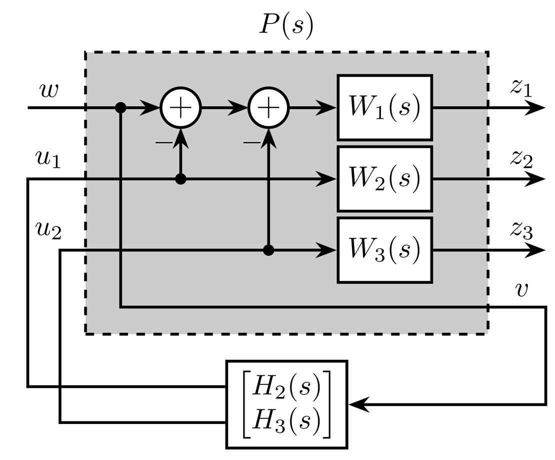

Fig. 7: Architecture for $\mathcal{H}_\infty$ synthesis of three complementary filters

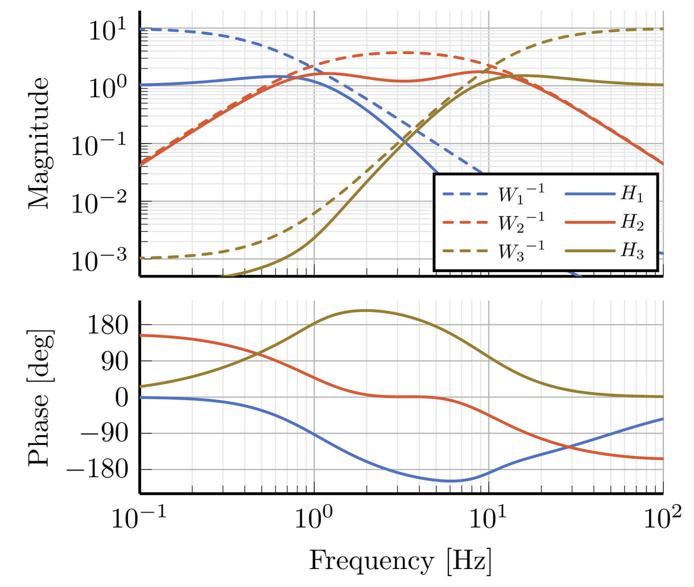

Fig. 8: Frequency response of the weighting functions and three complementary filters obtained using $\mathcal{H}_\infty$ synthesis

Application: Design of Complementary Filters used in the Active Vibration Isolation System at the LIGO

Control Configuration

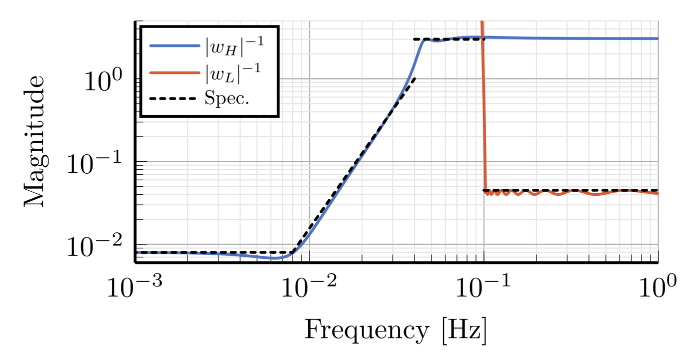

Fig. 9: Specifications and weighting functions magnitude used for $\mathcal{H}_\infty$ synthesis

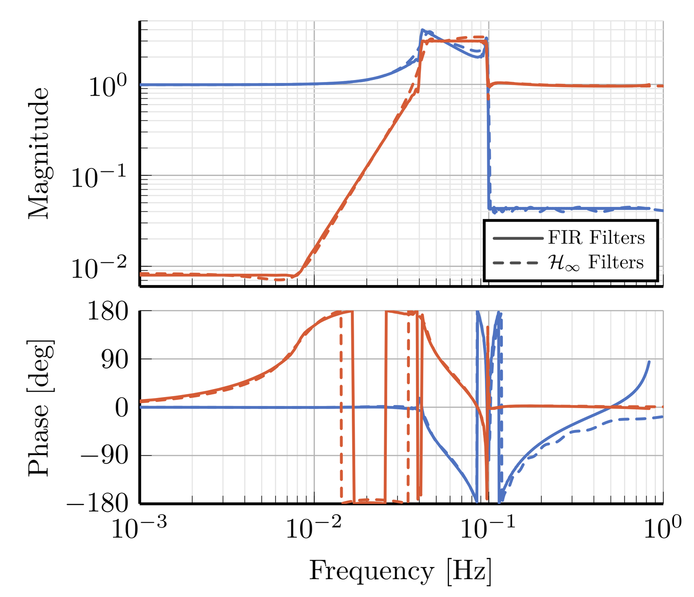

Fig. 10: Comparison of the FIR filters (solid) designed in [10] with the filters obtained with $\mathcal{H}_\infty$ synthesis (dashed)

Conclusion

This paper has shown how complementary filters can be used to combine multiple sensors in order to obtain a super sensor. Typical specification on the super sensor noise and on the robustness of the sensor fusion has been shown to be linked to the norm of the complementary filters. Therefore, a synthesis method that permits the shaping of the complementary filters norms has been proposed and has been successfully applied for the design of complex filters. Future work will aim at further developing this synthesis method for the robust and optimal synthesis of complementary filters used in sensor fusion.

Reference

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Acknowledgments This research benefited from a FRIA grant from the French Community of Belgium.