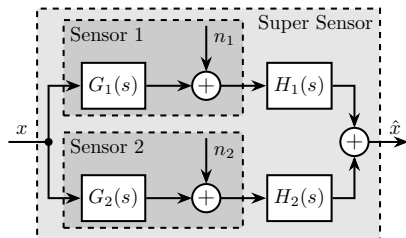# Complementary Filters Shaping Using $\mathcal{H}_\infty$ Synthesis

Control System Working Group meeting

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2019-08-28

Sensor Fusion Architecture - Noise Filtering

$$\hat{x} = (G_1 H_1 + G_2 H_2)\, x + H_1 n_1 + H_2 n_2$$

Complementary Property
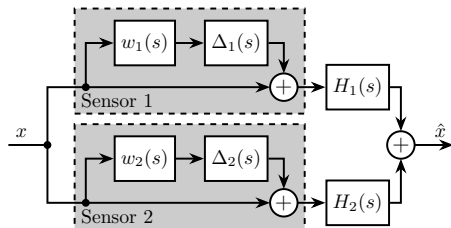
$$H_1(s) + H_2(s) = 1$$

Let's first consider **Perfectly Known Sensor Dynamics**:

$$G_1(s) = G_2(s) = 1 \implies \hat{x} = x + H_1 n_1 + H_2 n_2$$

PSD of the Super Sensor's noise

$$\Phi_{\delta x} = |H_1|^2 \, \Phi_{n_1} + |H_2|^2 \, \Phi_{n_2} \implies \text{depends on filters' norm}$$

Sensor Fusion Architecture - Robustness

Dynamic Uncertainty:

$$G_i'(s) = G_i(s)[1 + w_i(s)\Delta_i(s)],$$
$$\forall \Delta_i, \|\Delta_i\|_\infty < 1$$

Super Sensor Dynamics:

$$\frac{\hat{x}}{x} = 1 + w_1 H_1 \Delta_1 + w_2 H_2 \Delta_2$$

Limit the Super Sensor Dynamic uncertainty

Design $H_1(s)$ and $H_2(s)$ such that:

$$|w_1 H_1 \Delta_1| + |w_2 H_2 \Delta_2| \leq \epsilon \quad \forall \omega, \ \forall \Delta_1, \forall \Delta_2$$
$$\Leftrightarrow |w_1 H_1| + |w_2 H_2| \leq \epsilon \quad \forall \omega$$
$$\implies \text{depends on the filters' norm}$$

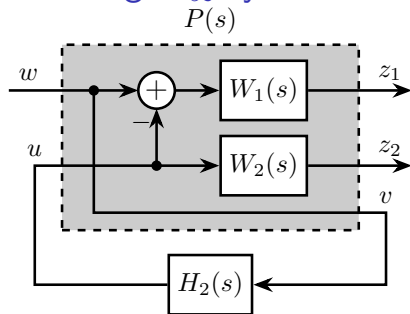# Shaping of Complementary Filters using $\mathcal{H}_\infty$ synthesis

Design Objective

$$H_1(s) + H_2(s) = 1$$

$$|H_1(j\omega)| \leq \frac{1}{|W_1(j\omega)|} \quad \forall\omega$$

$$|H_2(j\omega)| \leq \frac{1}{|W_2(j\omega)|} \quad \forall\omega$$

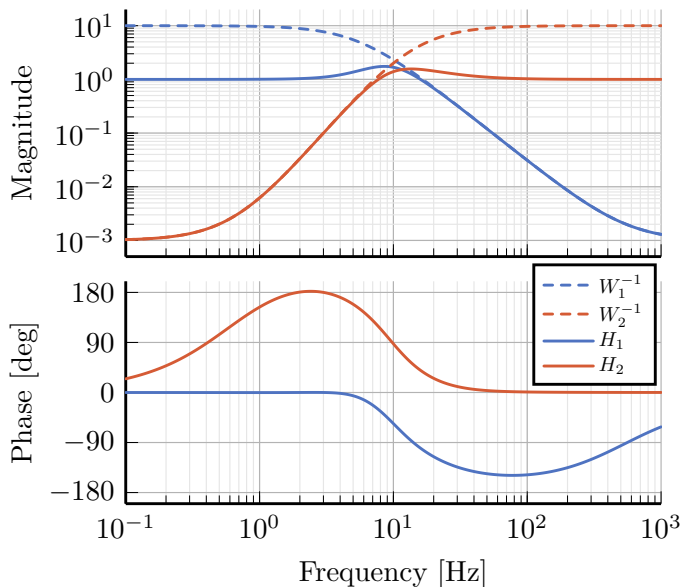$W_1(s)$ and $W_2(s)$ are proper, stable and minimum phase transfer functions

$P(s)$

$\mathcal{H}_\infty$ Synthesis

Find $H_2(s)$ such that:

$$\left\| \begin{matrix} [1 - H_2(s)]\, W_1(s) \\ H_2(s)W_2(s) \end{matrix} \right\|_\infty \leq 1$$
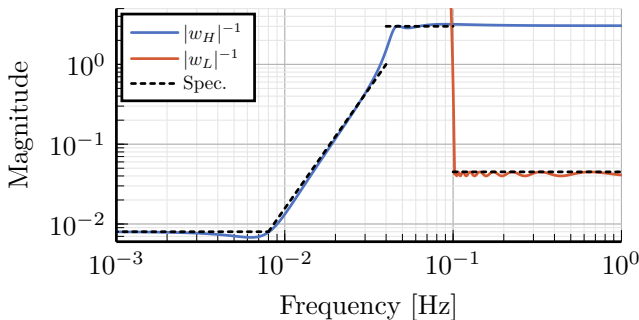
$$H_1(s) \triangleq 1 - H_2(s)$$

Validation of the proposed synthesis method

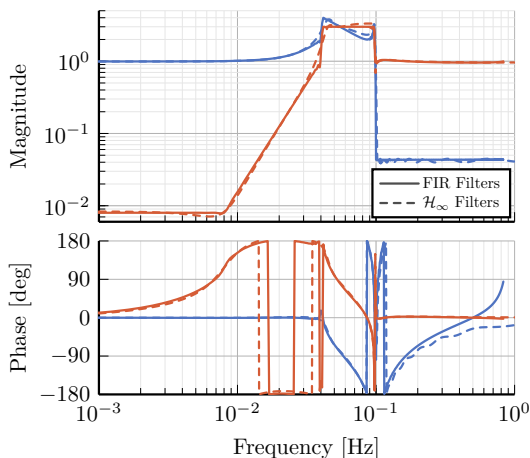# Complementary Filters Used at LIGO - Specifications

The specification are detailed in *Hua, W., Low frequency vibration isolation and alignment system for advanced LIGO* (2005)

Weighting Functions used

— Custom Designed 7th Order Transfer Function
— Type I Chebyshev Filter of Order 20

$\mathcal{H}_\infty$ Synthesis - Comparison with LIGO's FIR filters

- FIR Filters: order 512
- $\mathcal{H}_\infty$ Filters: order 27

Conclusion

Specifications: expressed as upper bounds on the filters' norm
$\mathcal{H}_\infty$ Synthesis: easily shape complex complementary filters