

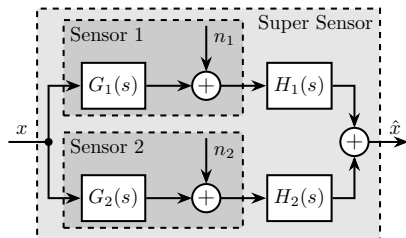
# Complementary Filters Shaping Using $\mathcal{H}_\infty$ Synthesis

Control System Working Group meeting

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2019-08-28

# Sensor Fusion Architecture - Noise Filtering



$$\hat{x} = (G_1 H_1 + G_2 H_2) x + H_1 n_1 + H_2 n_2$$

## Complementary Property

$$H_1(s) + H_2(s) = 1$$

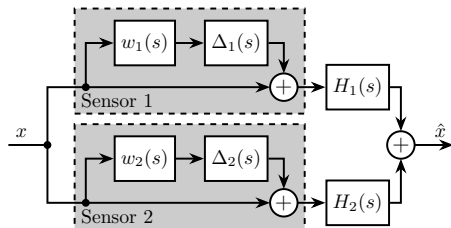
Let's first consider **Perfectly Known Sensor Dynamics**:

$$G_1(s) = G_2(s) = 1 \implies \hat{x} = x + H_1 n_1 + H_2 n_2$$

## PSD of the Super Sensor's noise

$$\Phi_{ss} = |H_1|^2 \Phi_{n_1} + |H_2|^2 \Phi_{n_2} \implies \text{depends on filters' norm}$$

# Sensor Fusion Architecture - Robustness



## Dynamic Uncertainty:

$$G'_i(s) = G_i(s)[1 + w_i(s)\Delta_i(s)],$$
$$\forall \Delta_i, \|\Delta_i\|_\infty < 1$$

## Super Sensor Dynamics:

$$\frac{\hat{x}}{x} = 1 + w_1 H_1 \Delta_1 + w_2 H_2 \Delta_2$$

## Limit the Super Sensor Dynamic uncertainty

Design  $H_1(s)$  and  $H_2(s)$  such that:

$$|w_1 H_1 \Delta_1| + |w_2 H_2 \Delta_2| \leq \epsilon \quad \forall \omega, \forall \Delta_1, \forall \Delta_2$$
$$\iff |w_1 H_1| + |w_2 H_2| \leq \epsilon \quad \forall \omega$$
$$\implies \text{depends on the filters' norm}$$

# Shaping of Complementary Filters using $\mathcal{H}_\infty$ synthesis

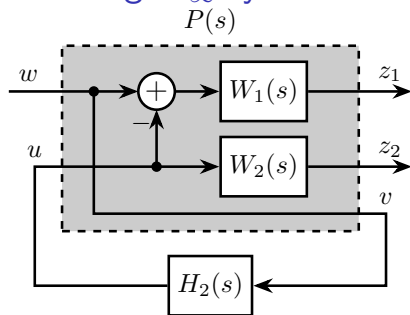
## Design Objective

$$H_1(s) + H_2(s) = 1$$

$$|H_1(j\omega)| \leq \frac{1}{|W_1(j\omega)|} \quad \forall \omega$$

$$|H_2(j\omega)| \leq \frac{1}{|W_2(j\omega)|} \quad \forall \omega$$

$W_1(s)$  and  $W_2(s)$  are proper, stable and minimum phase transfer functions



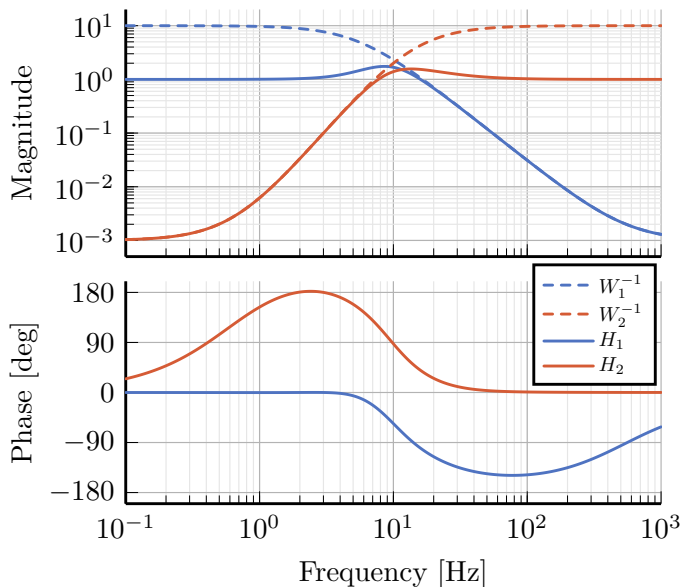
## $\mathcal{H}_\infty$ Synthesis

Find  $H_2(s)$  such that:

$$\left\| \begin{bmatrix} [1 - H_2(s)] W_1(s) \\ H_2(s) W_2(s) \end{bmatrix} \right\|_\infty \leq 1$$

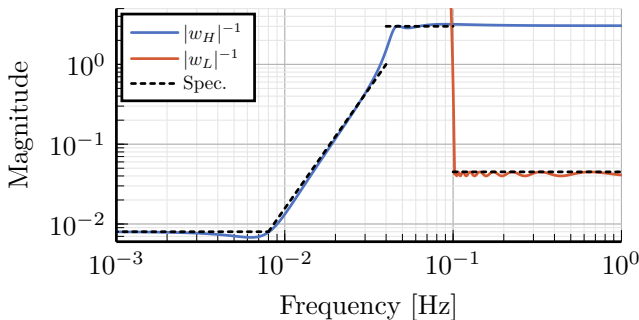
$$H_1(s) \triangleq 1 - H_2(s)$$

# Validation of the proposed synthesis method



# Complementary Filters Used at LIGO - Specifications

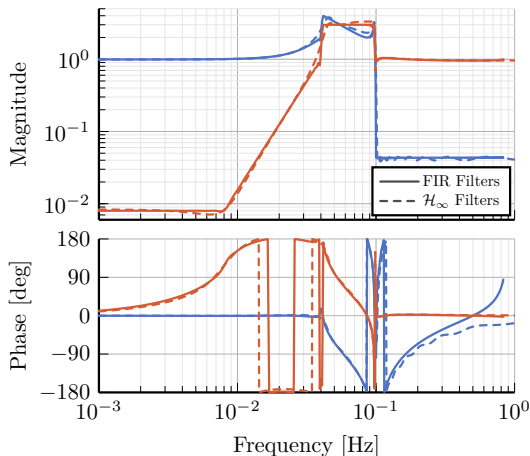
The specification are detailed in *Hua, W., Low frequency vibration isolation and alignment system for advanced LIGO (2005)*



## Weighting Functions used

- Custom Designed 7<sup>th</sup> Order Transfer Function
- Type I Chebyshev Filter of Order 20

# $\mathcal{H}_\infty$ Synthesis - Comparison with LIGO's FIR filters



- FIR Filters: order 512
- $\mathcal{H}_\infty$  Filters: order 27

The paper and all the Matlab Scripts used for the paper are accessible [here](#).

## Conclusion

Specifications: expressed as upper bounds on the filters' norm  
 $\mathcal{H}_\infty$  Synthesis: easily shape complex complementary filters