Complementary Filters Shaping Using $\mathcal{H}_\infty$ Synthesis

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Sensor Fusion

In order to improve the estimate $\hat{x}$ of x, multiple sensors can be merged together using complementary filters.
This permits to have
High bandwidth

- need of Sensor at low frequency $+$ sensor at high frequency
- need of merging the two
- complementary filters
- design of those filters using $\mathcal{H}_\infty$

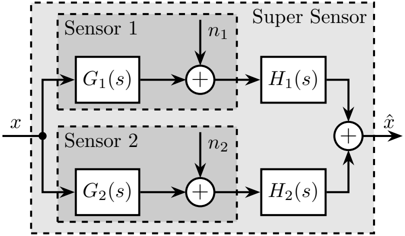
Goal:

- Higher control bandwidth
- Better estimation of some physical value

Applications:

- LIGO - Vibration isolation of precise equipment
- UAV - Angle estimation using Accelerometer and Gyroscope

Sensor Fusion Architecture - Noise Filtering



$$\hat{x} = (G_1 H_1 + G_2 H_2)\, x + H_1 n_1 + H_2 n_2$$

Complementary Property

$$H_1(s) + H_2(s) = 1$$

Let's first consider **Perfectly Known Sensor Dynamics**:

$$G_1(s) = G_2(s) = 1 \Longrightarrow \boxed{\hat{x} = x + H_1 n_1 + H_2 n_2}$$

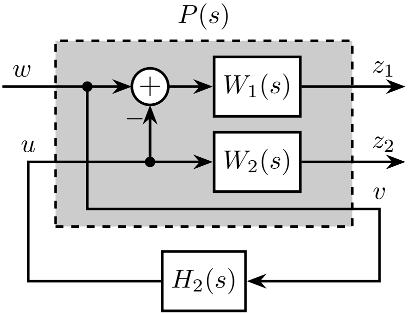
PSD of the Super Sensor's noise

$$\Phi_{\hat{x}} = |H_1|^2 \, \Phi_{n_1} + |H_2|^2 \, \Phi_{n_2} \Longrightarrow \text{depends on filters' norm}$$

Shaping of Complementary Filters using $\mathcal{H}_\infty$ synthesis

Design Objective

$$H_1(s) + H_2(s) = 1$$
$$|H_1(j\omega)| \le \frac{1}{|W_1(j\omega)|} \quad \forall\omega$$
$$|H_2(j\omega)| \le \frac{1}{|W_2(j\omega)|} \quad \forall\omega$$



$W_1(s)$ and $W_2(s)$ are proper, stable and minimum phase transfer functions

$\mathcal{H}_\infty$ Synthesis

Find $H_2(s)$ such that:

$$\left\| \begin{matrix} [1 - H_2(s)]\, W_1(s) \\ H_2(s) W_2(s) \end{matrix} \right\|_\infty \le 1$$
$$H_1(s) \triangleq 1 - H_2(s)$$

Validation of the proposed synthesis method

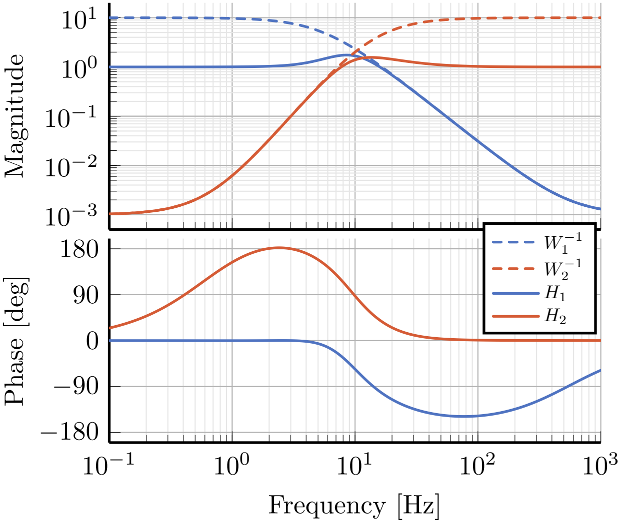


Figure: Frequency response of the weighting functions and complementary filters obtained using $\mathcal{H}_\infty$ synthesis

$\mathcal{H}_\infty$ Synthesis of Complementary filters used at LIGO

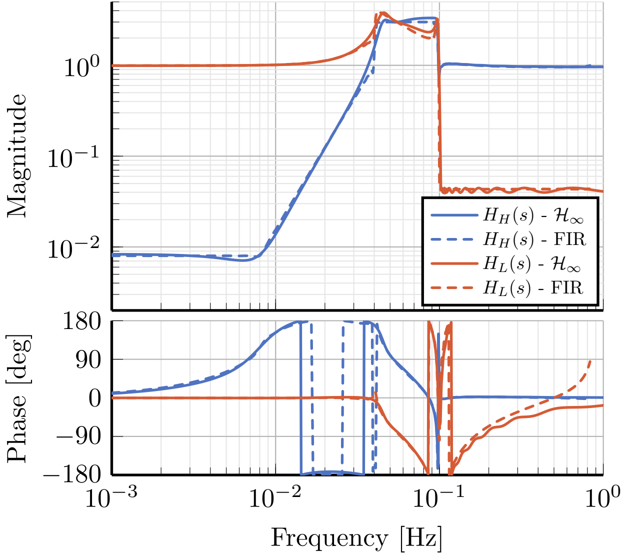


Figure: Comparison of the FIR filters (solid) designed at LIGO with the filters obtained with $\mathcal{H}_\infty$ synthesis (dashed)