Complementary Filters Shaping Using $\mathcal{H}_\infty$ Synthesis

Control System Working Group meeting

Dehaeze Thomas
Collette Christophe
Vermat Mohit

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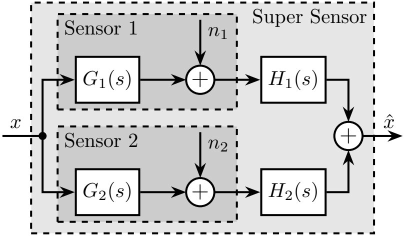
Sensor Fusion Architecture - Noise Filtering



$$\hat{x} = \left(G_1 H_1 + G_2 H_2\right) x + H_1 n_1 + H_2 n_2$$

Complementary Property

$$H_1(s) + H_2(s) = 1$$

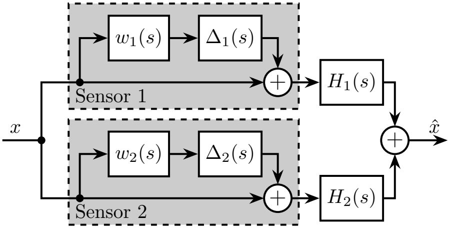
Let's first consider **Perfectly Known Sensor Dynamics**:

$$G_1(s) = G_2(s) = 1 \Longrightarrow \hat{x} = x + H_1 n_1 + H_2 n_2$$

PSD of the Super Sensor's noise

$$\Phi_{\mathsf{ss}} = \left|H_1\right|^2 \Phi_{n_1} + \left|H_2\right|^2 \Phi_{n_2} \Longrightarrow \text{depends on filters' norm}$$

Sensor Fusion Architecture - Robustness



Dynamic Uncertainty:

$$G_i'(s) = G_i(s)[1 + w_i(s)\Delta_i(s)],$$
$$\forall \Delta_i, \|\Delta_i\|_\infty < 1$$

Super Sensor Dynamics:

$$\frac{\hat{x}}{x} = 1 + w_1 H_1 \Delta_1 + w_2 H_2 \Delta_2$$

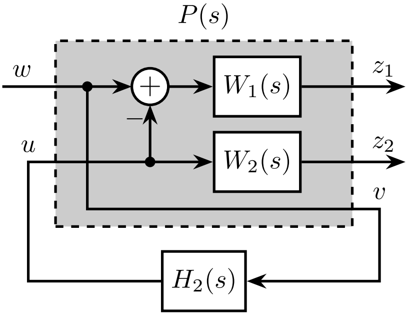
Limit the Super Sensor Dynamic uncertainty

Design $H_1(s)$ and $H_2(s)$ such that:

$$|w_1 H_1 \Delta_1| + |w_2 H_2 \Delta_2| \le \epsilon \quad \forall \omega,\ \forall \Delta_1, \forall \Delta_2$$
$$\Longleftrightarrow |w_1 H_1| + |w_2 H_2| \le \epsilon \quad \forall \omega$$

$\Longrightarrow$ depends on the filters' norm

Shaping of Complementary Filters using $\mathcal{H}_\infty$ synthesis



Design Objective

$$H_1(s) + H_2(s) = 1$$

$$|H_1(j\omega)| \le \frac{1}{|W_1(j\omega)|} \quad \forall \omega$$

$$|H_2(j\omega)| \le \frac{1}{|W_2(j\omega)|} \quad \forall \omega$$

$W_1(s)$ and $W_2(s)$ are proper, stable and minimum phase transfer functions

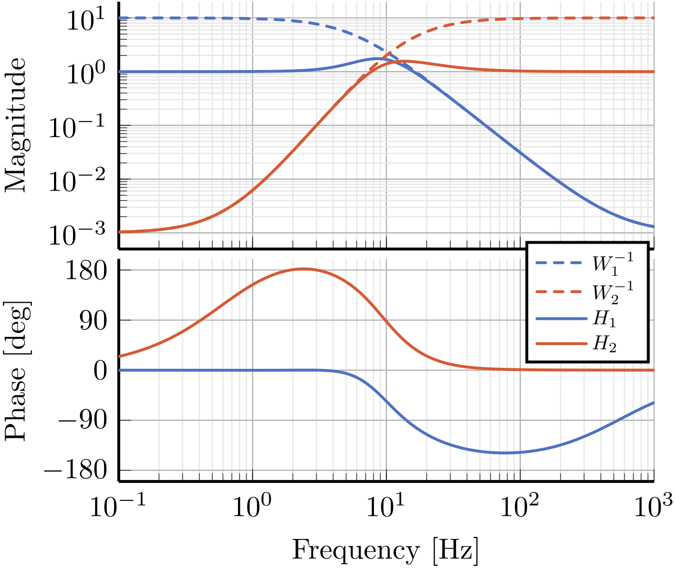
$\mathcal{H}_\infty$ Synthesis

Find $H_2(s)$ such that:

$$\left\| \begin{matrix} [1 - H_2(s)]\, W_1(s) \\ H_2(s) W_2(s) \end{matrix} \right\|_\infty \le 1$$

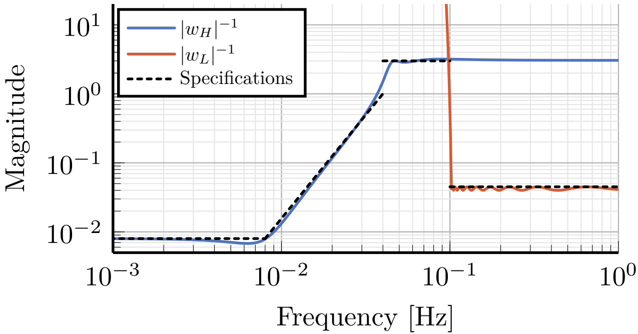
$$H_1(s) \triangleq 1 - H_2(s)$$

Validation of the proposed synthesis method



Complementary Filters Used at LIGO - Specifications

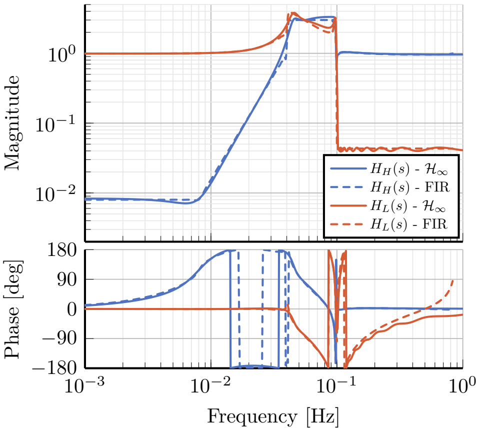
The specification are detailed in *Hua, W., Low frequency vibration isolation and alignment system for advanced LIGO (2005)*



Weighting Functions used

- Custom Designed 7^{th} Order Transfer Function
- Type I Chebyshev Filter of Order 20

$\mathcal{H}_\infty$ Synthesis - Comparison with LIGO's FIR filters



- FIR Filters: order 512
- $\mathcal{H}_\infty$ Filters: order 27

The paper and all the Matlab Scripts used for the paper are accessible here.

Conclusion

Specifications: expressed as upper bounds on the filters' norm
$\mathcal{H}_\infty$ Synthesis: easily shape complex complementary filters