Complementary Filters Shaping Using $\mathcal{H}_\infty$ Synthesis

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Outline

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Introduction

High bandwidth

- need of Sensor at low frequency $+$ sensor at high frequency
- need of merging the two
- complementary filters
- design of those filters using $\mathcal{H}_\infty$

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Sensor Fusion Architecture

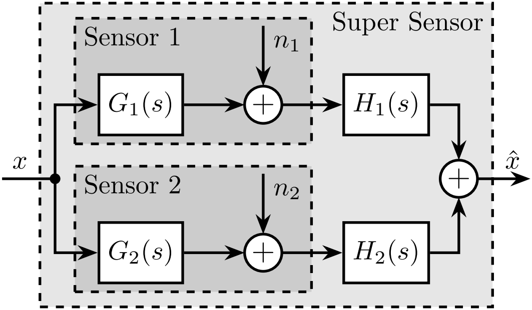

Figure: Sensor fusion architecture

$$\hat{x} = (G_1 H_1 + G_2 H_2)\, x + H_1 n_1 + H_2 n_2 \tag{1}$$

$$H_1(s) + H_2(s) = 1 \tag{2}$$

Noise Sensor Filtering

Perfect Sensor Dynamics:

$$G_1(s) = G_2(s) = 1 \tag{3}$$

Super Sensor Estimation:

$$\hat{x} = x + H_1 n_1 + H_2 n_2 \tag{4}$$

Noise in the Super Sensor:

$$\Phi_{\delta x} = |H_1|^2 \Phi_{n_1} + |H_2|^2 \Phi_{n_2} \tag{5}$$

Usually, the two sensors have high noise levels over distinct frequency regions. In order to lower the noise of the super sensor, the value of the norm $|H_1|$ has to be lowered when Φ_{n_1} is larger than Φ_{n_2} and that of $|H_2|$ lowered when Φ_{n_2} is larger than Φ_{n_1}.

Robustness of the Fusion

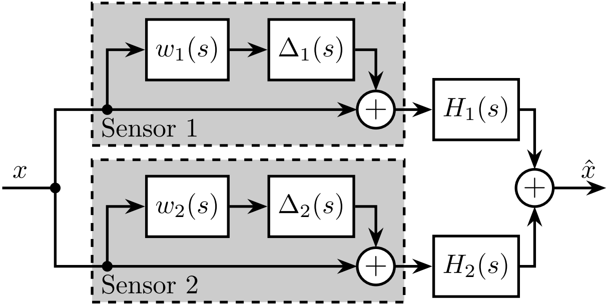

Figure: Sensor fusion architecture with sensor dynamics uncertainty

$$\frac{\hat{x}}{x} = 1 + w_1(s)H_1(s)\Delta_1(s) + w_2(s)H_2(s)\Delta_2(s) \tag{6}$$

$$\begin{aligned} & |w_1 H_1 \Delta_1| + |w_2 H_2 \Delta_2| \le \epsilon \quad \forall\omega,\ \forall\Delta_1, \forall\Delta_2 \\ \Leftrightarrow & |w_1 H_1| + |w_2 H_2| \le \epsilon \quad \forall\omega \end{aligned} \tag{7}$$

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Shaping of Complementary Filters using $\mathcal{H}_\infty$ synthesis

Synthesis Objective:

$$H_1(s) + H_2(s) = 1 \tag{8a}$$

$$|H_1(j\omega)| \le \frac{1}{|W_1(j\omega)|} \quad \forall\omega \tag{8b}$$

$$|H_2(j\omega)| \le \frac{1}{|W_2(j\omega)|} \quad \forall\omega \tag{8c}$$

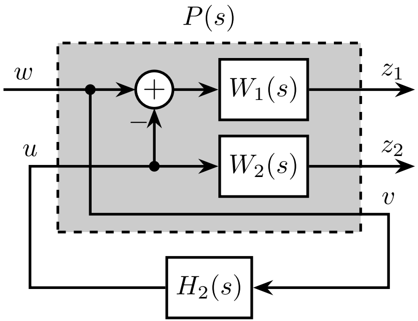


Figure: Architecture used for $\mathcal{H}_\infty$ synthesis of complementary filters

Weighting Functions Design

$$W(s) = \left(\frac{\frac{1}{\omega_0}\sqrt{\frac{1-\left(\frac{G_0}{G_c}\right)^{\frac{2}{n}}}{1-\left(\frac{G_c}{G_\infty}\right)^{\frac{2}{n}}}}\, s + \left(\frac{G_0}{G_c}\right)^{\frac{1}{n}}}{\left(\frac{1}{G_\infty}\right)^{\frac{1}{n}} \frac{1}{\omega_0}\sqrt{\frac{1-\left(\frac{G_0}{G_c}\right)^{\frac{2}{n}}}{1-\left(\frac{G_c}{G_\infty}\right)^{\frac{2}{n}}}}\, s + \left(\frac{1}{G_c}\right)^{\frac{1}{n}}} \right)^{n} \tag{11}$$

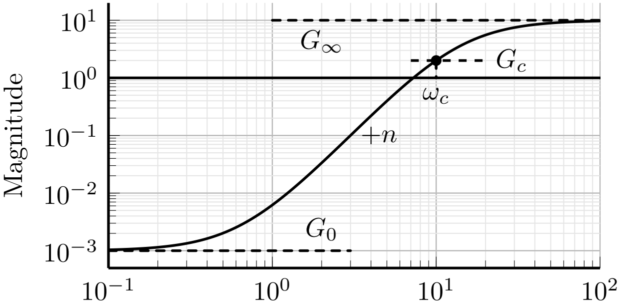


Figure: Magnitude of a weighting function generated using the formula

Validation of the proposed synthesis method

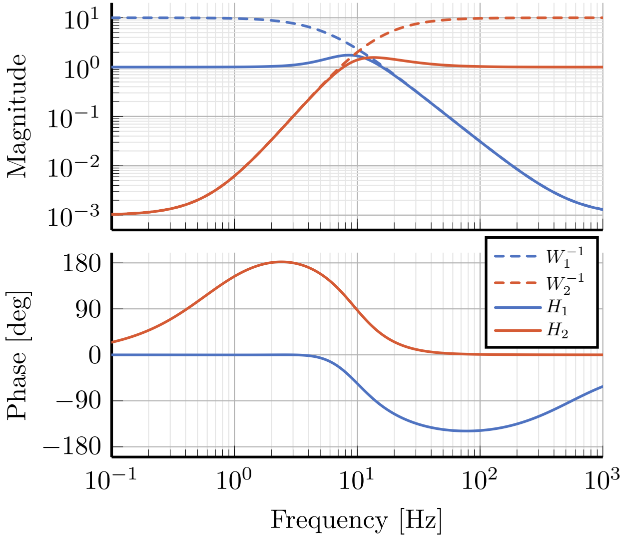


Figure: Frequency response of the weighting functions and complementary filters obtained using $\mathcal{H}_\infty$ synthesis

Synthesis of Three Complementary Filters

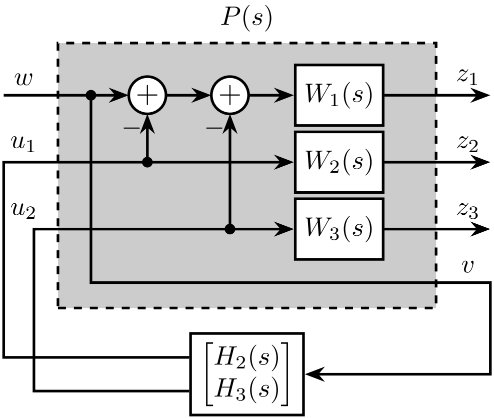
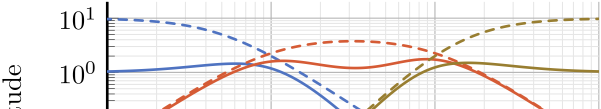


Figure: Architecture for $\mathcal{H}_\infty$ synthesis of three complementary filters



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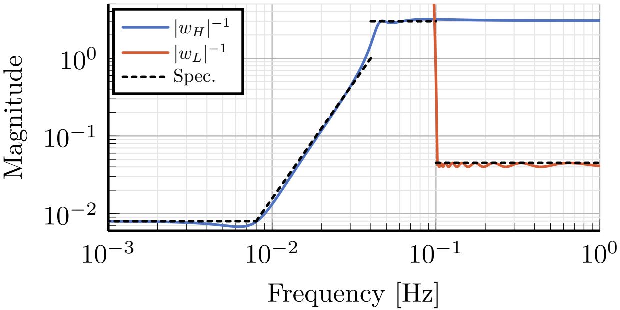
Application

Several complementary filters are used in the active isolation system at the LIGO [1, 2]. The requirements on those filters are very tight and thus their design is complex. The approach used in [1] for their design is to write the synthesis of complementary FIR filters as a convex optimization problem. The obtained FIR filters are compliant with the requirements. However they are of very high order so their implementation is quite complex.
The effectiveness of the proposed method is demonstrated by designing complementary filters with the same requirements as the one described in [1].

Complementary Filters Specifications

- From 0 to 0.008 Hz, the magnitude of the filter's transfer function should be less or equal to 8×10^{-4}
- Between 0.008 Hz to 0.04 Hz, the filter should attenuate the input signal proportional to frequency cubed
- Between 0.04 Hz to 0.1 Hz, the magnitude of the transfer function should be less than 3
- Above 0.1 Hz, the magnitude of the complementary filter should be less than 0.045



Weighting Functions Design

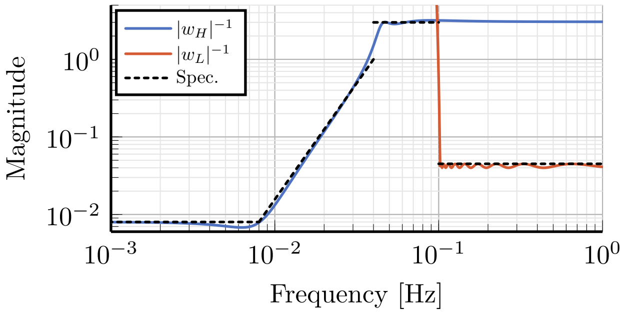


Figure: Specifications and weighting functions magnitude used for $\mathcal{H}_\infty$ synthesis

$\mathcal{H}_\infty$ Synthesis

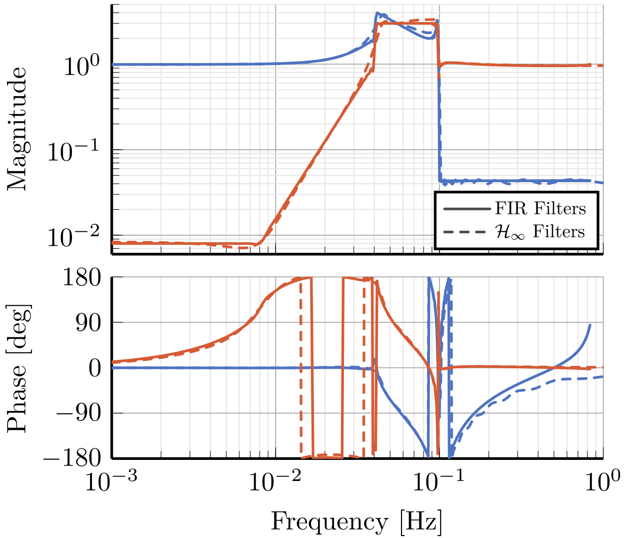


Figure: Comparison of the FIR filters (solid) designed in [1] with the filters obtained with $\mathcal{H}_\infty$ synthesis (dashed)

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Conclusion

This paper has shown how complementary filters can be used to combine multiple sensors in order to obtain a super sensor. Typical specification on the super sensor noise and on the robustness of the sensor fusion has been shown to be linked to the norm of the complementary filters. Therefore, a synthesis method that permits the shaping of the complementary filters norms has been proposed and has been successfully applied for the design of complex filters. Future work will aim at further developing this synthesis method for the robust and optimal synthesis of complementary filters used in sensor fusion.

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