Complementary Filters Shaping Using $\mathcal{H}_\infty$ Synthesis

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Abstract For many applications, large bandwidth and dynamic ranges are requiring to use several sensors, whose signals are combined using complementary filters. This paper presents a method for designing these complementary filters using $\mathcal{H}_\infty$ synthesis that allows to shape the filter norms. This method is shown to be easily applicable for the synthesis of complex complementary filters.

Sensor Fusion Architecture

Let's consider two sensors measuring the same physical quantity x with dynamics $G_1(s)$ and $G_2(s)$, and with uncorrelated noise characteristics n_1 and n_2.

The signals from both sensors are fed into two **complementary filters** $H_1(s)$ and $H_2(s)$ and then combined to yield an estimate $\hat{x}$ of x as shown in Fig. 1.

$$\hat{x} = (G_1 H_1 + G_2 H_2)\, x + H_1 n_1 + H_2 n_2$$

The complementary property of $H_1(s)$ and $H_2(s)$ implies that their transfer function sum is equal to one at all frequencies:

$$\boxed{H_1(s) + H_2(s) = 1}$$

The combined sensors forms a so called **super sensor**. Let's determine the noise property and dynamical uncertainty of such super sensor.

Complementary Filters Requirements

Noise Property

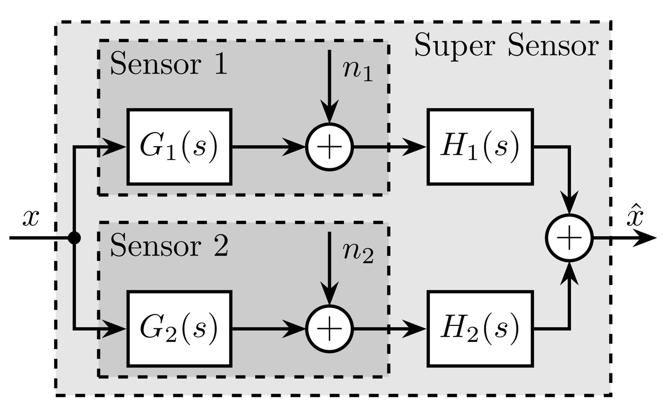

Fig. 1: Sensor fusion architecture

First suppose **known sensor dynamics**, such that the dynamics can be inverted:

$$G_1(s) = G_2(s) = 1$$

The estimate $\hat{x}$ is then:

$$\boxed{\hat{x} = x + H_1 n_1 + H_2 n_2}$$

The signal x is kept **undistorted** while the noises n_1 and n_2 are **filtered out by the complementary filters**.

Estimate error δx:

$$\delta x \triangleq \hat{x} - x = H_1 n_1 + H_2 n_2$$

PSD of the super sensor noise:

$$\boxed{\Phi_{\delta x} = |H_1|^2 \Phi_{n_1} + |H_2|^2 \Phi_{n_2}}$$

Dynamical Uncertainty

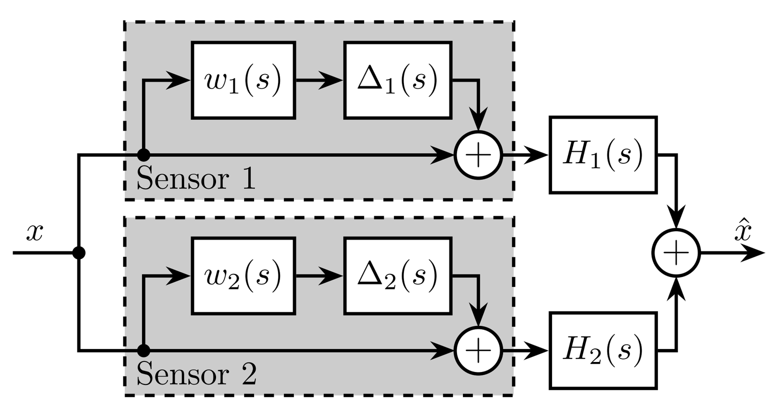

Fig. 2: Fusion of sensors with dynamics uncertainty

Sensor dynamic uncertainty is represented by **multiplicative input uncertainty**:

$$G_i'(s) = G_i(s)[1 + w_i(s)\Delta_i(s)], \quad \forall \Delta_i, \|\Delta_i\|_\infty < 1$$

The super sensor dynamics is:

$$\boxed{\frac{\hat{x}}{x} = 1 + w_1 H_1 \Delta_1 + w_2 H_2 \Delta_2}$$

The super sensor dynamic uncertainty is represented in the complex plane in Fig. 3.

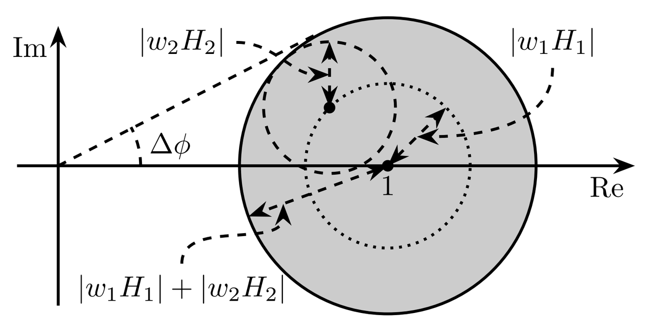

Fig. 3: Uncertainty set of the super sensor dynamics

As shown in the analysis above, **the performance and robustness of the sensor fusion architecture depends on the complementary filters norms**. Therefore, the development of a synthesis method of complementary filters that allows the shaping of their norm is necessary.

Complementary Filters Shaping using $\mathcal{H}_\infty$ Synthesis

The synthesis objective is to shape the norm of two filters $H_1(s)$ and $H_2(s)$ while ensuring their complementary property. This is equivalent as to finding stable transfer functions $H_1(s)$ and $H_2(s)$ such that the following conditions are satisfied.

$$\begin{cases} H_1(s) + H_2(s) = 1 & \\ |H_1(j\omega)| \leq \frac{1}{|W_1(j\omega)|} & \forall \omega \\ |H_2(j\omega)| \leq \frac{1}{|W_2(j\omega)|} & \forall \omega \end{cases}$$

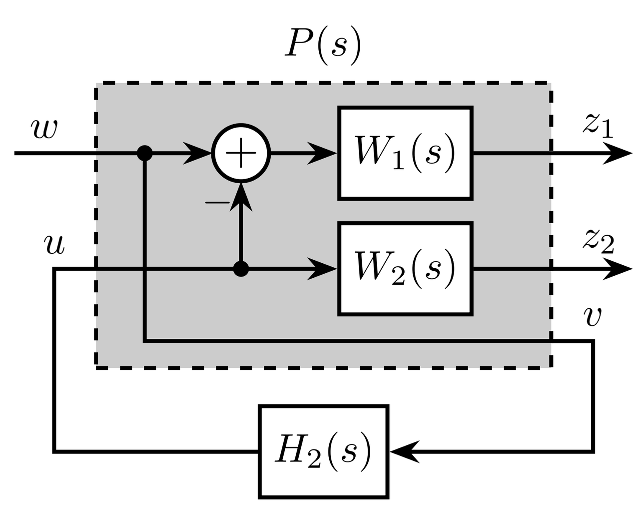

Fig. 4: Architecture used for $\mathcal{H}_\infty$ synthesis

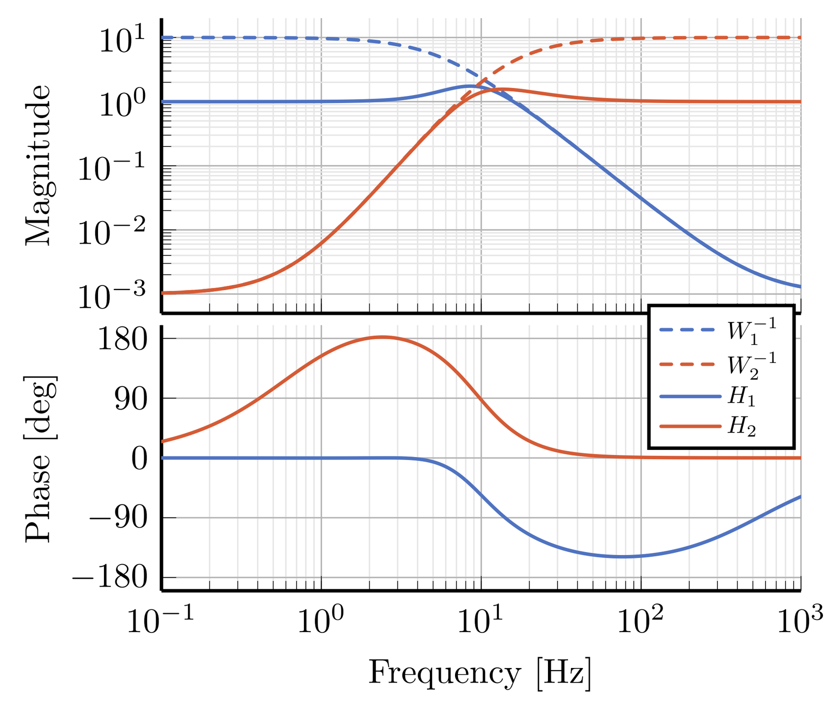

Fig. 5: Frequency response of the weighting functions and complementary filters obtained using $\mathcal{H}_\infty$ synthesis

Weighting Function Design

$$W(s) = \left(\frac{ \frac{1}{\omega_0} \sqrt{ \frac{1 - \left(\frac{G_0}{G_c}\right)^{\frac{2}{n}}}{1 - \left(\frac{G_c}{G_\infty}\right)^{\frac{2}{n}}} } s + \left(\frac{G_0}{G_c}\right)^{\frac{1}{n}} }{ \left(\frac{1}{G_\infty}\right)^{\frac{1}{n}} \frac{1}{\omega_0} \sqrt{ \frac{1 - \left(\frac{G_0}{G_c}\right)^{\frac{2}{n}}}{1 - \left(\frac{G_c}{G_\infty}\right)^{\frac{2}{n}}} } s + \left(\frac{1}{G_c}\right)^{\frac{1}{n}} } \right)^n$$

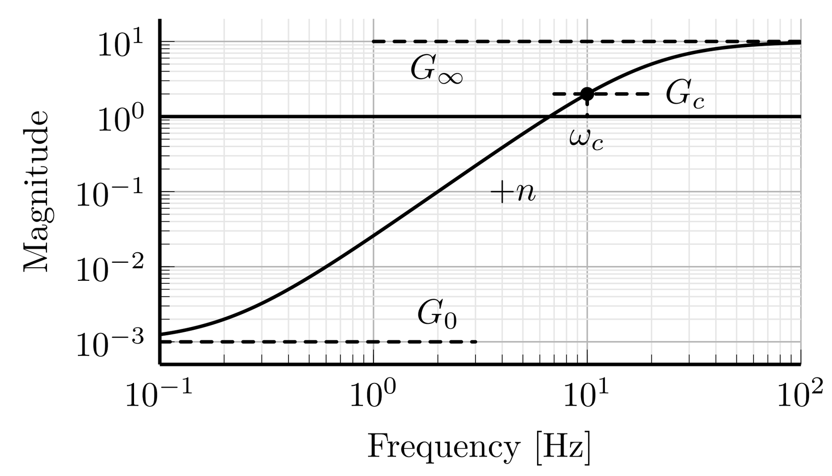

Fig. 6: Magnitude of a weighting function generated using the proposed formula (??), $G_0 = 1e^{-3}$, $G_\infty = 10$, $\omega_c = 10\,\text{Hz}$, $G_c = 2$, $n = 3$

Design of Complementary Filters used in the Active Vibration Isolation System at the LIGO

Control Configuration

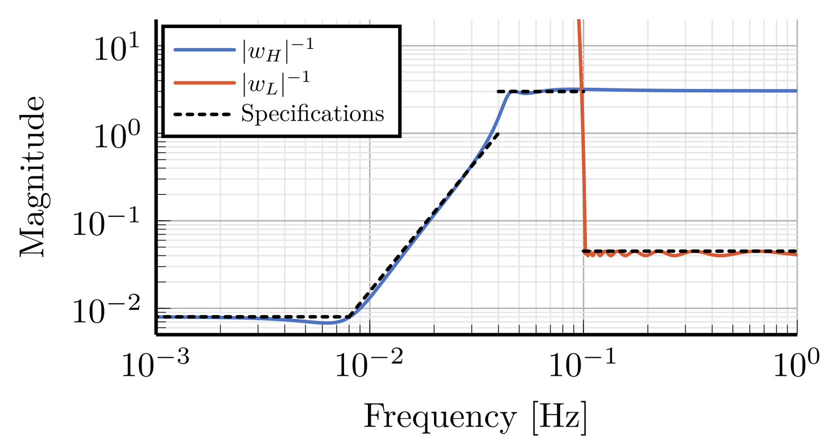

Fig. 7: Specifications and weighting functions magnitude used for $\mathcal{H}_\infty$ synthesis

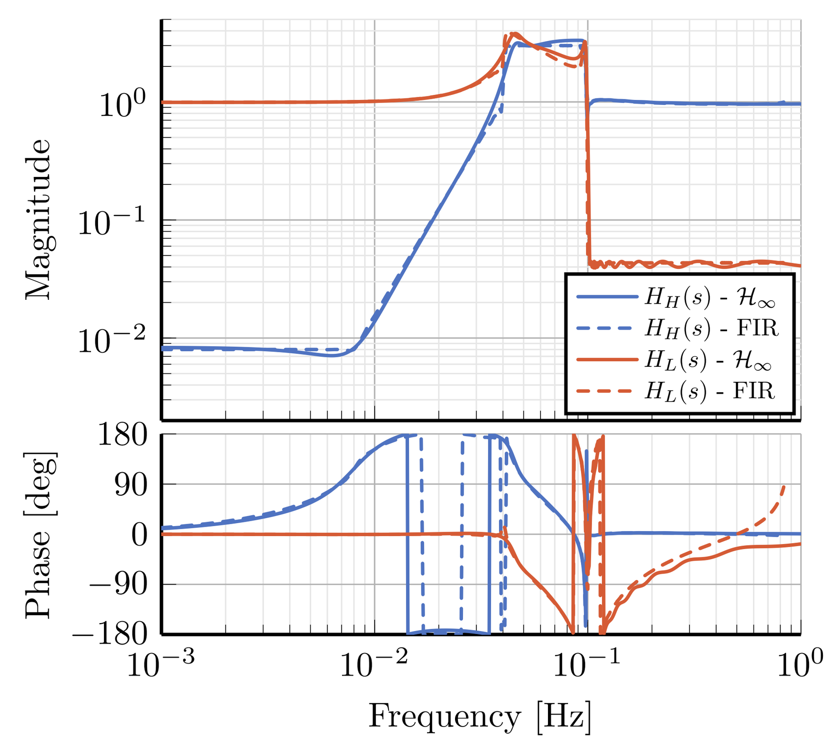

Fig. 8: Comparison of the FIR filters (solid) designed in [1] with the filters obtained with $\mathcal{H}_\infty$ synthesis (dashed)

Conclusion

This paper has shown how complementary filters can be used to combine multiple sensors in order to obtain a super sensor. Typical specification on the super sensor noise and on the robustness of the sensor fusion has been shown to be linked to the norm of the complementary filters. Therefore, a synthesis method that permits the shaping of the complementary filters norms has been proposed and has been successfully applied for the design of complex filters. Future work will aim at further developing this synthesis method for the robust and optimal synthesis of complementary filters used in sensor fusion.