

Group meeting 16/12/2019

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Outline

Introduction

Analytical model

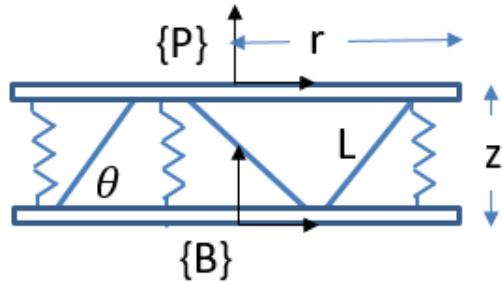
Multi-body simulation of Stewart Platform

Preliminary experiments

Future Plan

Modelling of Stewart Platform

- Geometric configuration for cubic architecture



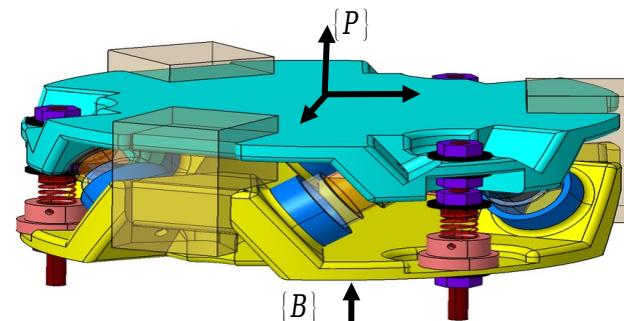
$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{2}} \right)$$

$$\frac{r}{L} = \cos \theta = \sqrt{\frac{2}{3}} \implies r = L \sqrt{\frac{2}{3}}$$

$$\frac{z}{L} = \sin \theta = \sqrt{\frac{1}{3}} \implies z = L \sqrt{\frac{1}{3}}$$

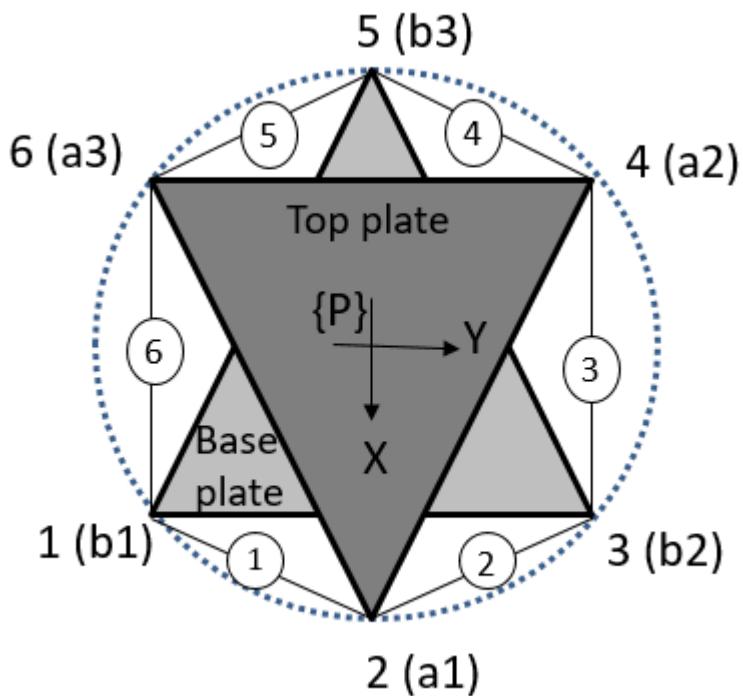
- Frame of reference

- Global frame of reference, $\{B\}$ which coincides with the centre of mass of the bottom platform.
- Payload frame of reference, $\{P\}$ which is fixed to the centre of mass of the payload plate.
- Local frame of reference which is attached to the corresponding spring and the actuator leg.



Modelling of Stewart Platform

- Location of actuators and sensors in global frame of reference



S. No.	Point	X	Y	Z
1	b1	$r/2$	$-r\sqrt{3}/2$	0
2	a1	r	0	z
3	b2	$r/2$	$r\sqrt{3}/2$	0
4	a2	$-r/2$	$r\sqrt{3}/2$	z
5	b3	$-r$	0	0
6	a3	$-r/2$	$-r\sqrt{3}/2$	z
7	B	0	0	0
8	P	0	0	z

Modelling of Stewart Platform

- Modelling of springs

Motion of payload plate

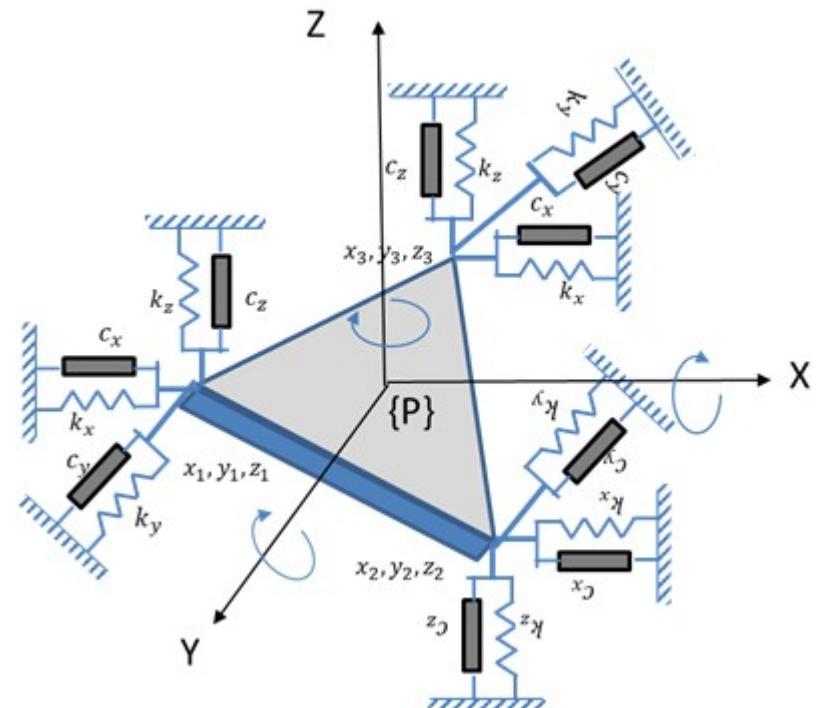
$$X = [x \ y \ z \ \theta_x \ \theta_y \ \theta_z]^T$$

Displacement of i^{th} spring

$$s_i = [s_{xi} \ s_{yi} \ s_{zi}]^T$$

External force acting on the center of mass of the payload plate

$$F_{ext} = [F \ \tau] = [F_x \ F_y \ F_z \ M_x \ M_y \ M_z]$$



Modelling of Stewart Platform

- Relation between spring motion and payload motion

Translation in X-, Y- and Z-directions

$$x = \frac{s_{x1} + s_{x2} + s_{x3}}{3} \quad y = \frac{s_{y1} + s_{y2} + s_{y3}}{3} \quad z = \frac{s_{z1} + s_{z2} + s_{z3}}{3}$$

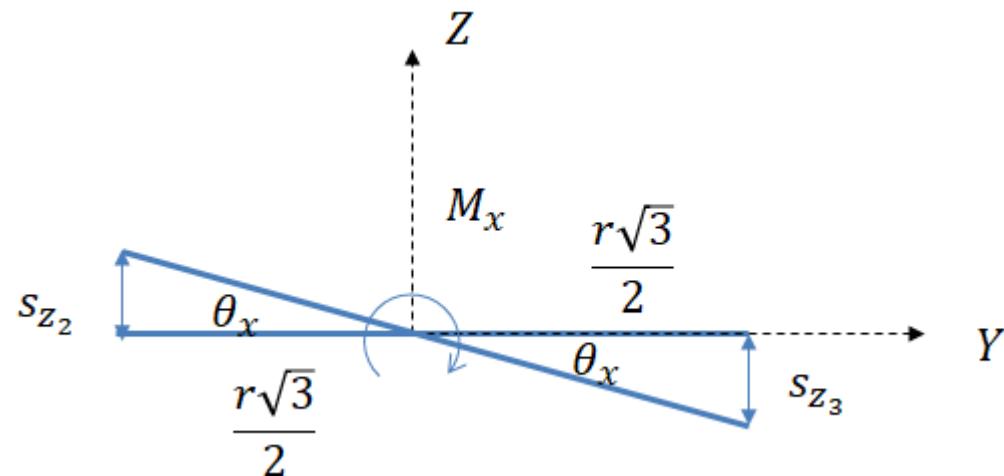
Rotation about X-axis

$$\theta_x = \frac{2s_{z2}}{r\sqrt{3}} = \frac{2s_{z3}}{r\sqrt{3}}$$

$$M_x = k_{\theta_x} \theta_x$$

$$= k_v s_{z2} \frac{r\sqrt{3}}{2} + k_v s_{z3} \frac{r\sqrt{3}}{2}$$

$$= 2k_v \left(\frac{r\sqrt{3}}{2} \theta_x \right) \frac{r\sqrt{3}}{2}$$



$$k_{\theta_x} = \frac{3k_v r^2}{2}$$

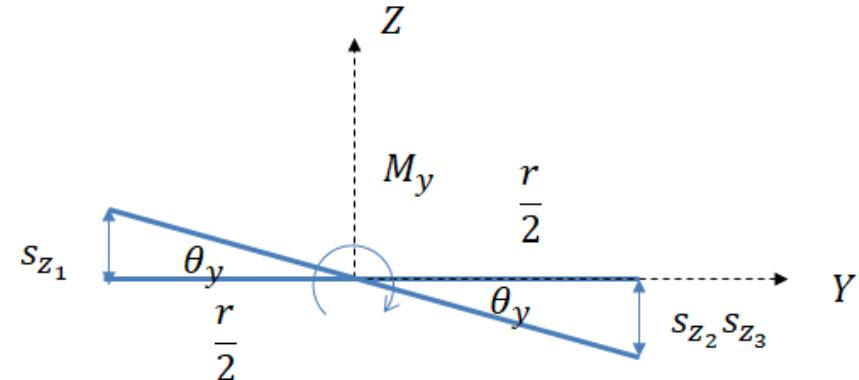
Modelling of Stewart Platform

- Relation between spring and payload displacements

Rotation about Y-axis

$$\theta_y = \frac{s_{z1}}{r} = \frac{2s_{z2}}{r} = \frac{2s_{z3}}{r}$$

$$\begin{aligned} M_y &= k_{\theta_y} \theta_y \\ &= k_v s_{z1} r + k_v s_{z2} \frac{r}{2} + k_v s_{z3} \frac{r}{2} \\ &= k_v (\theta_y r) r + k_v \left(\frac{\theta_y r}{2} \right) \frac{r}{2} + k_v \left(\frac{\theta_y r}{2} \right) \frac{r}{2} \end{aligned}$$



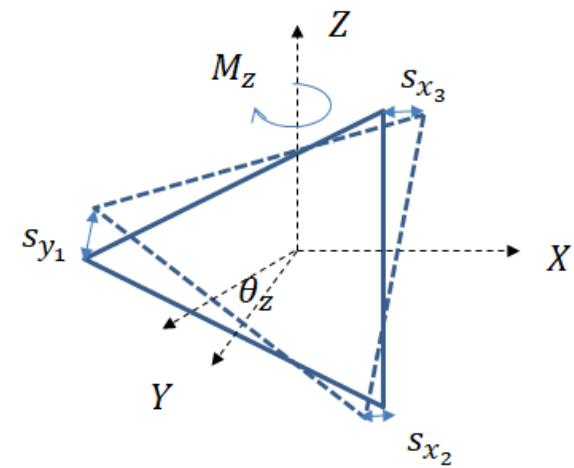
$$k_{\theta_y} = \frac{3k_v r^2}{2}$$

Rotation about Z-axis

$$\theta_z = \frac{s_{z1}}{r} = \frac{s_{z2}}{r} = \frac{s_{z3}}{r}$$

$$\begin{aligned} M_z &= k_{\theta_z} \theta_z \\ &= k_h s_{z1} r + k_h s_{z2} r + k_h s_{z3} r \\ &= k_h (3\theta_z r) r \end{aligned}$$

$$k_{\theta_z} = 3k_h r^2$$



Modelling of Stewart Platform

- Kinematic modelling of actuators

\vec{x}_o Vector connecting origin of $\{B\}$ to $\{P\}$

\vec{p}_i Vector connecting origin of $\{P\}$ to the attachment point of leg i in the payload plate

\vec{r}_i Vector connecting origin of $\{B\}$ to the attachment point of leg i in the bottom plate

\vec{q}_i Vector connecting the attachment point of leg i in the payload and bottom plates

\vec{l}_i Unit vector along the direction of leg i

$$\text{Jacobian} \quad \dot{q} = J \dot{X}$$

Absolute velocity of point A $\vec{v}_i = \vec{v} + \vec{\omega} \times \vec{p}_i$

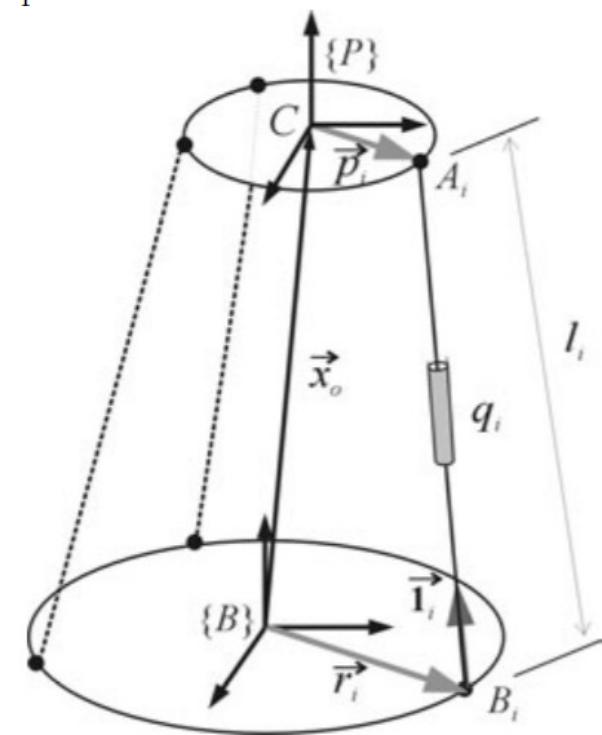
Projection along the actuator leg $\vec{q}_i = \vec{l}_i \cdot (\vec{v} + \vec{\omega} \times \vec{p}_i)$

$$= \vec{l}_i \cdot \vec{v} - \vec{l}_i \cdot \vec{p}_i \times \vec{\omega}$$

$$q_i = \vec{l}_i^T \cdot \vec{v} - \vec{l}_i^T \cdot \tilde{\vec{p}}_i \cdot \vec{\omega}$$

$$\tilde{\vec{p}} = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}$$

$$q = \begin{pmatrix} \dots & \dots \\ \vec{l}_i^T & -\vec{l}_i^T \tilde{\vec{p}}_i \\ \dots & \dots \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$



[Hanieh, 2003]

Modelling of Stewart Platform

Vectors corresponding to actuator leg and position

$$\begin{aligned}
 \vec{q}_1 &= a_1 - b_1 = (r/2, r\sqrt{3}/2, z) &= (L/\sqrt{6}, L/\sqrt{2}, L/\sqrt{3}) \\
 \vec{q}_2 &= a_2 - b_2 = (r/2, -r\sqrt{3}/2, z) &= (L/\sqrt{6}, -L/\sqrt{2}, L/\sqrt{3}) \\
 \vec{q}_3 &= a_3 - b_3 = (-r, 0, z) &= (-2L/\sqrt{6}, 0, L/\sqrt{3}) \\
 \vec{q}_4 &= a_4 - b_4 = (r/2, r\sqrt{3}/2, z) &= (L/\sqrt{6}, L/\sqrt{2}, L/\sqrt{3}) \\
 \vec{q}_5 &= a_5 - b_5 = (r/2, -r\sqrt{3}/2, z) &= (L/\sqrt{6}, -L/\sqrt{2}, L/\sqrt{3}) \\
 \vec{q}_6 &= a_6 - b_6 = (-r, 0, z) &= (-2L/\sqrt{6}, 0, L/\sqrt{3}) \\
 \vec{p}_1 &= a_1 - P = (r, 0, 0) &= (2L/\sqrt{6}, 0, 0) \\
 \vec{p}_2 &= a_1 - P = (r, 0, 0) &= (2L/\sqrt{6}, 0, 0) \\
 \vec{p}_3 &= a_2 - P = (-r/2, r\sqrt{3}/2, 0) &= (-L/\sqrt{6}, L/\sqrt{2}, 0) \\
 \vec{p}_4 &= a_2 - P = (-r/2, r\sqrt{3}/2, 0) &= (-L/\sqrt{6}, L/\sqrt{2}, 0) \\
 \vec{p}_5 &= a_3 - P = (-r/2, -r\sqrt{3}/2, 0) &= (-L/\sqrt{6}, -L/\sqrt{2}, 0) \\
 \vec{p}_6 &= a_3 - P = (-r/2, -r\sqrt{3}/2, 0) &= (-L/\sqrt{6}, -L/\sqrt{2}, 0)
 \end{aligned}$$

$$q = \begin{pmatrix} \cdots & \cdots \\ 1_i^T & -1_i^T \tilde{p}_i \\ \cdots & \cdots \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

$$J = \begin{bmatrix} 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{6} & 0 & -L\sqrt{2}/3 & L/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{6} & 0 & -L\sqrt{2}/3 & -L/\sqrt{3} \\ -2/\sqrt{6} & 0 & 1/\sqrt{6} & L/\sqrt{6} & L/\sqrt{6} & L/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{6} & L/\sqrt{6} & L/\sqrt{6} & -L/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{6} & -L/\sqrt{6} & L/\sqrt{6} & L/\sqrt{3} \\ -2/\sqrt{6} & 0 & 1/\sqrt{6} & -L/\sqrt{6} & L/\sqrt{6} & -L/\sqrt{3} \end{bmatrix}$$

$$f^T \delta q = \begin{bmatrix} F & \tau \end{bmatrix}^T \delta x$$

$$\implies \begin{bmatrix} F \\ \tau \end{bmatrix} = J^T f$$

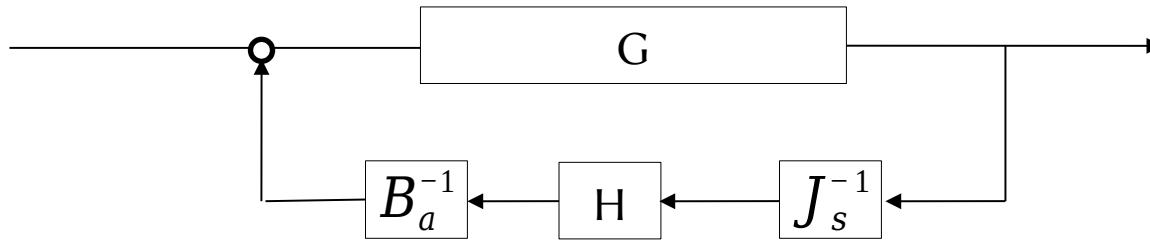
Dynamic equation of the platform

$$M \ddot{X} + K X = F_{ext} + J^T f$$

$$M = m \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_x^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_y^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_z^2 \end{bmatrix} \quad K = \begin{bmatrix} 3k_h & 0 & 0 & 0 & 0 & 0 \\ 0 & 3k_h & 0 & 0 & 0 & 0 \\ 0 & 0 & 3k_v & 0 & 0 & 0 \\ 0 & 0 & 0 & 3k_v r^2/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3k_v r^2/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3k_h r^2 \end{bmatrix}$$

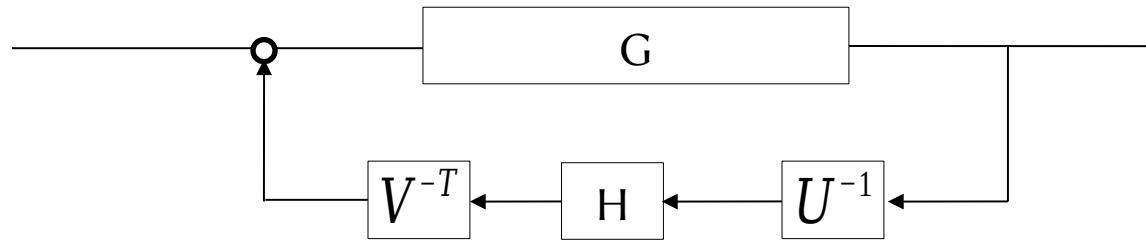
SVD controller design

Centralized controller



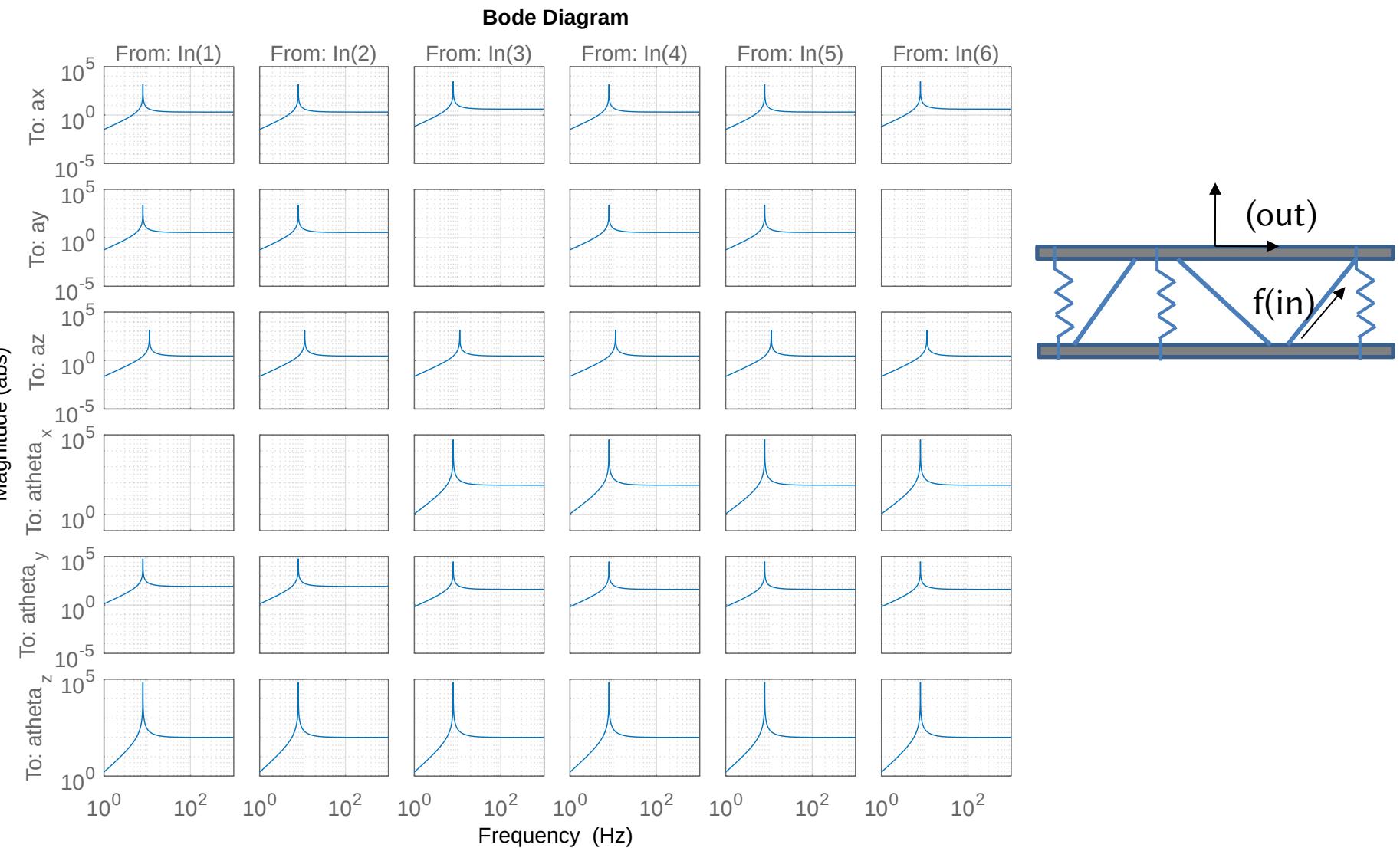
SVD controller design

$$\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^*$$



Transfer matrix is complex matrix which is dependent on the frequency.
 Select decoupling frequency and carry out SVD on the real approximation of the matrix.

Actuator to sensor

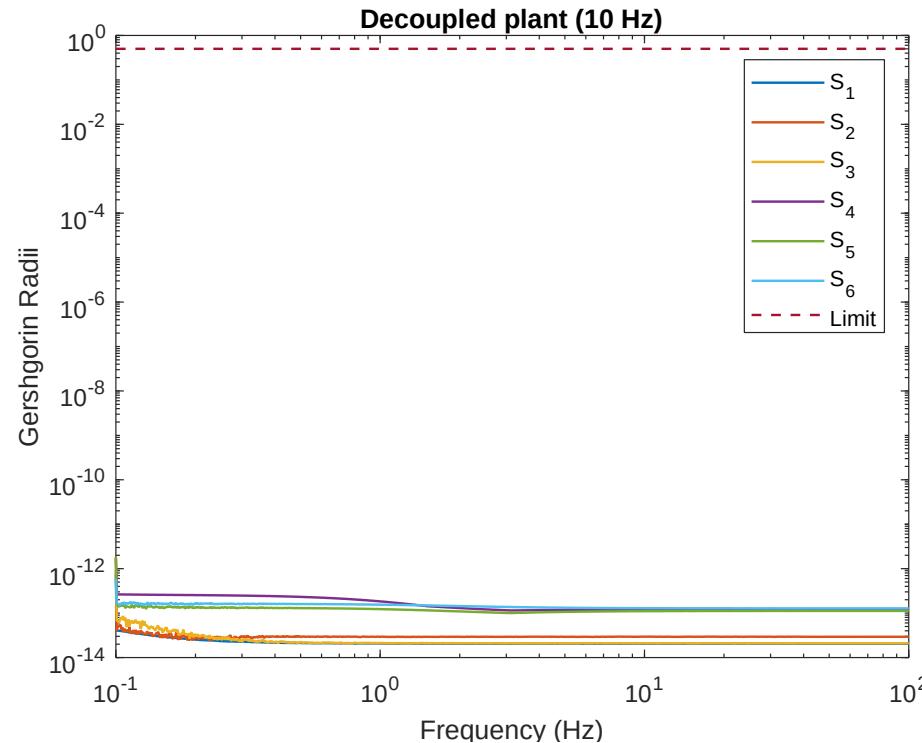
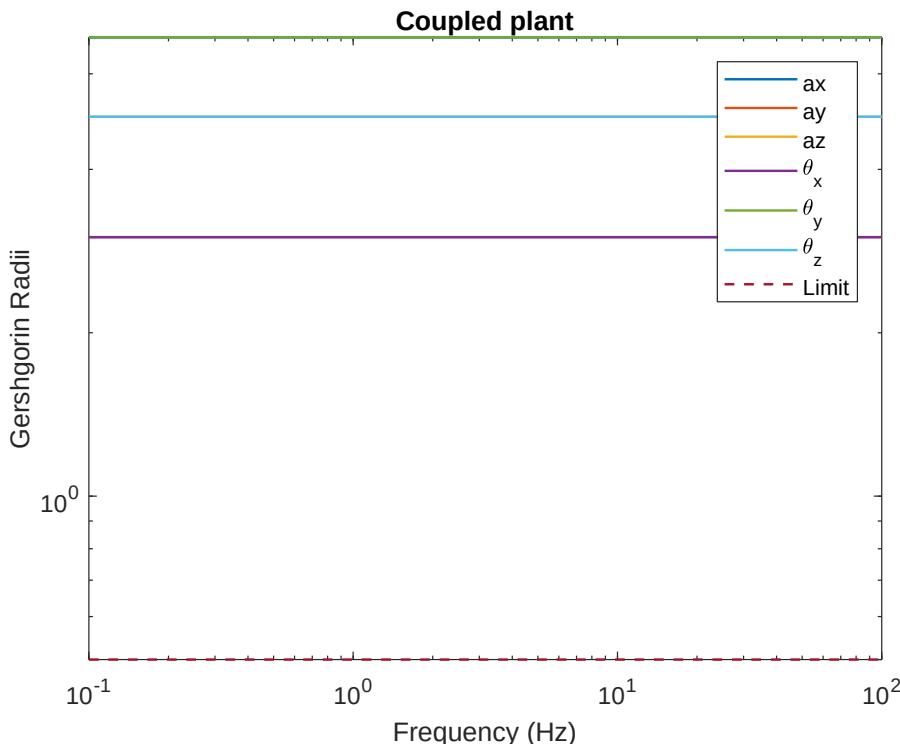


Greshgorin radii

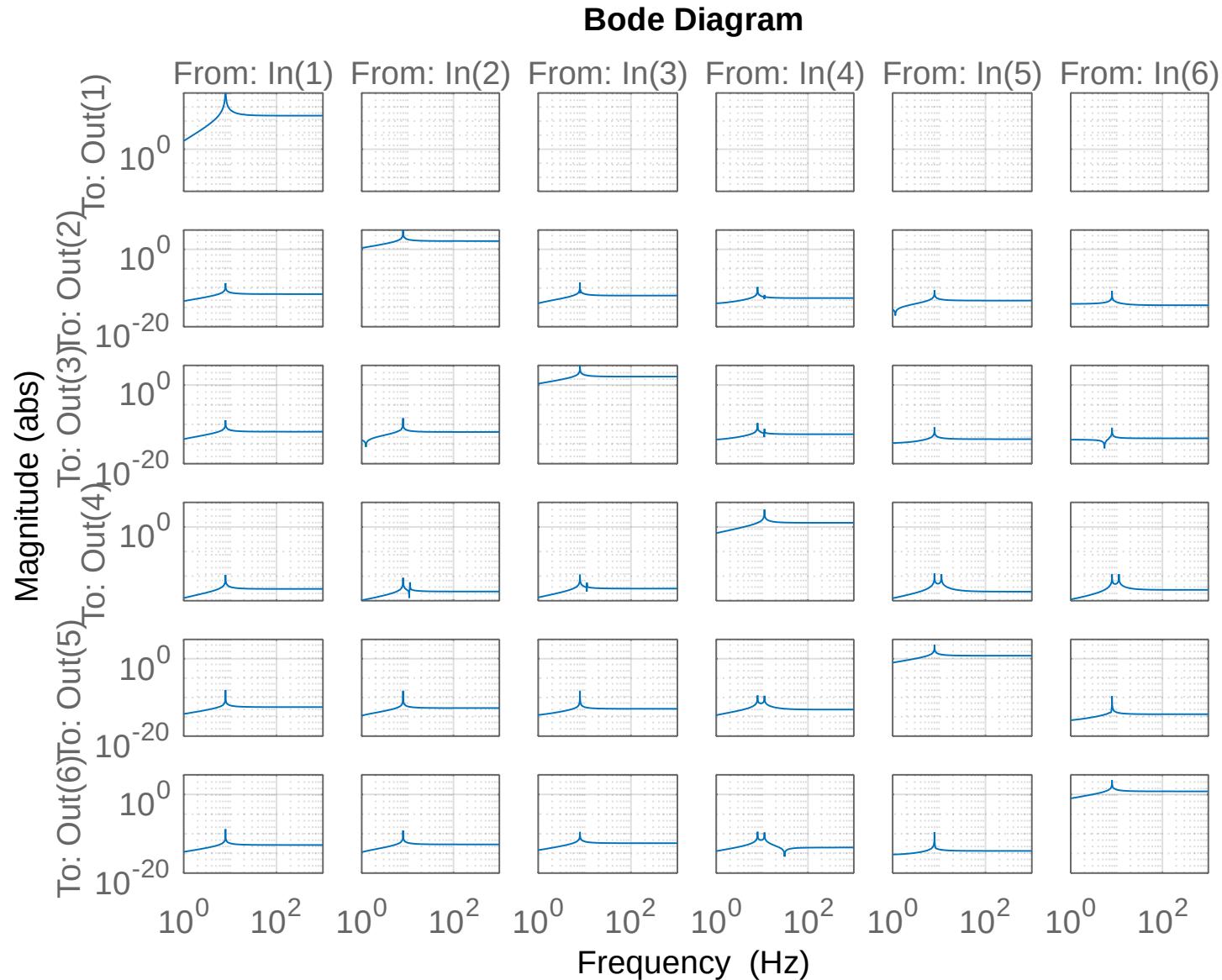
Check for diagonal dominance

$$\varrho_i(f) = \frac{\sum_{j \neq i} |H_{ij}(f)|}{|H_{ii}(f)|}$$

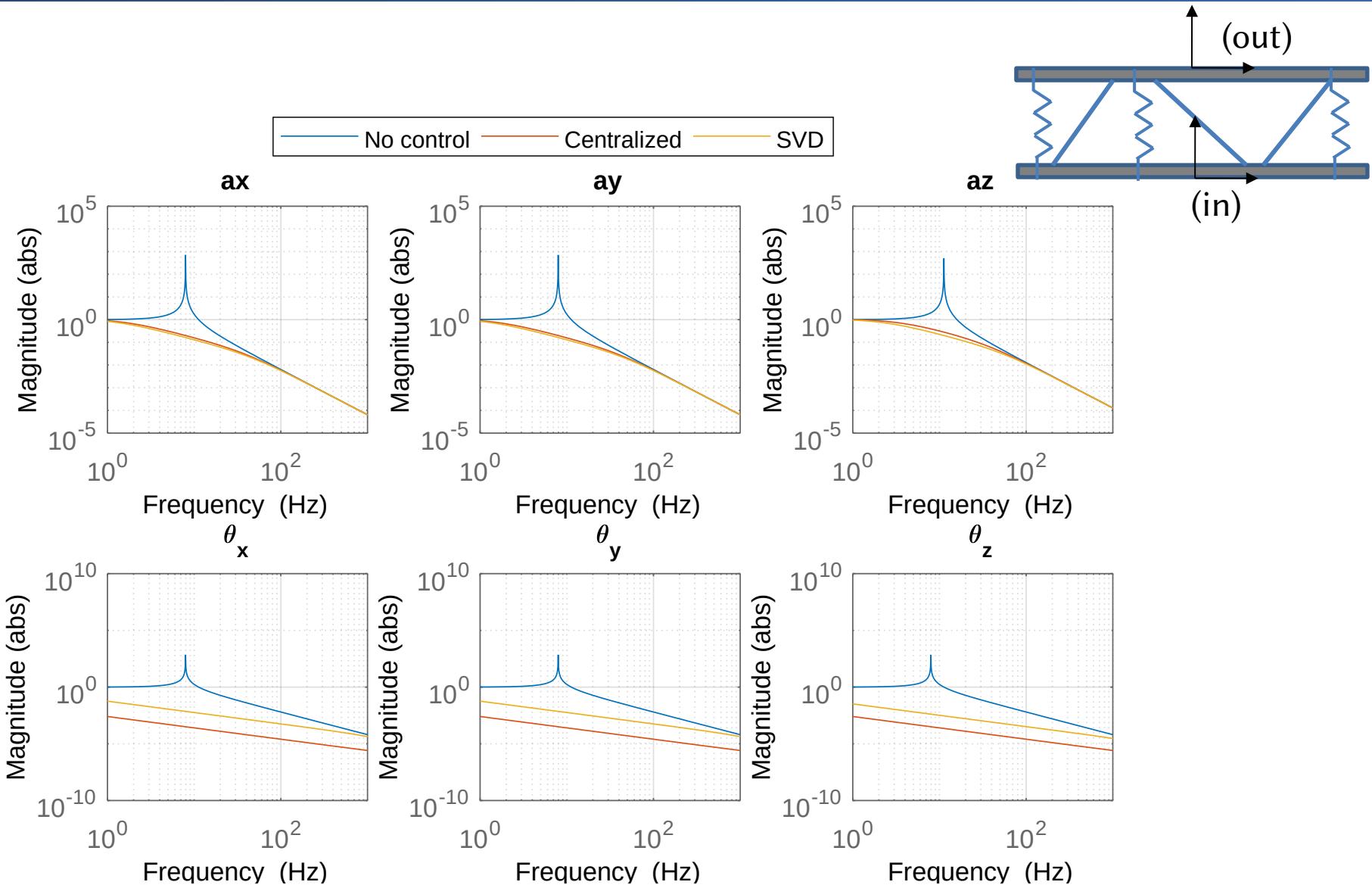
where $|H_{ij}(f)|$ denotes the magnitude of the TF from i to j at frequency f .



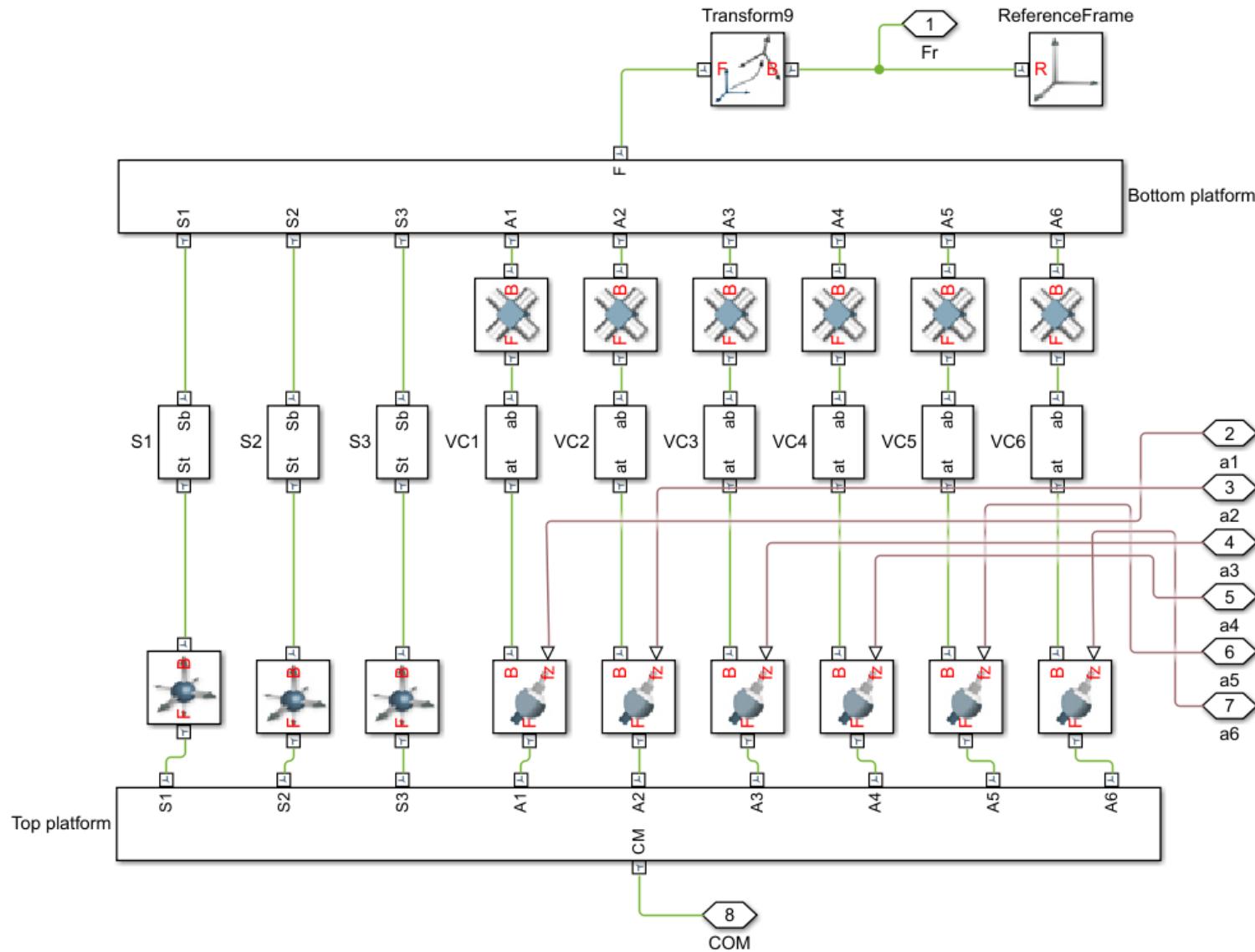
Decoupled plant



Performance



Multi-body simulation (Simscape)

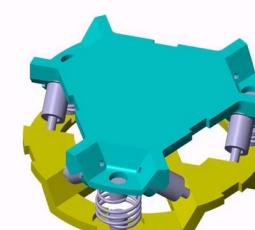
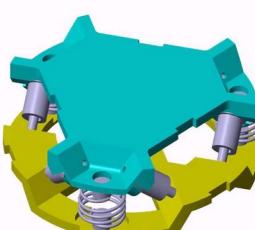
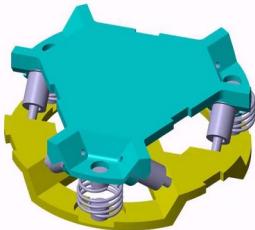


Simulation without control

F_x

F_y

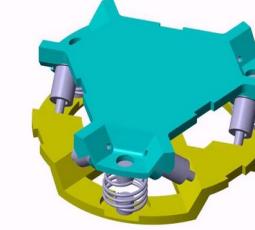
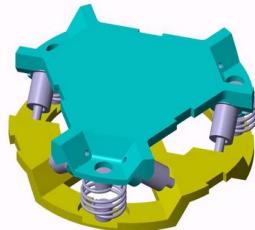
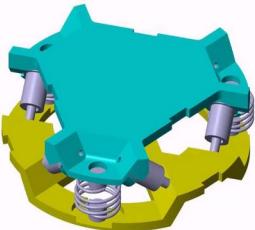
F_z



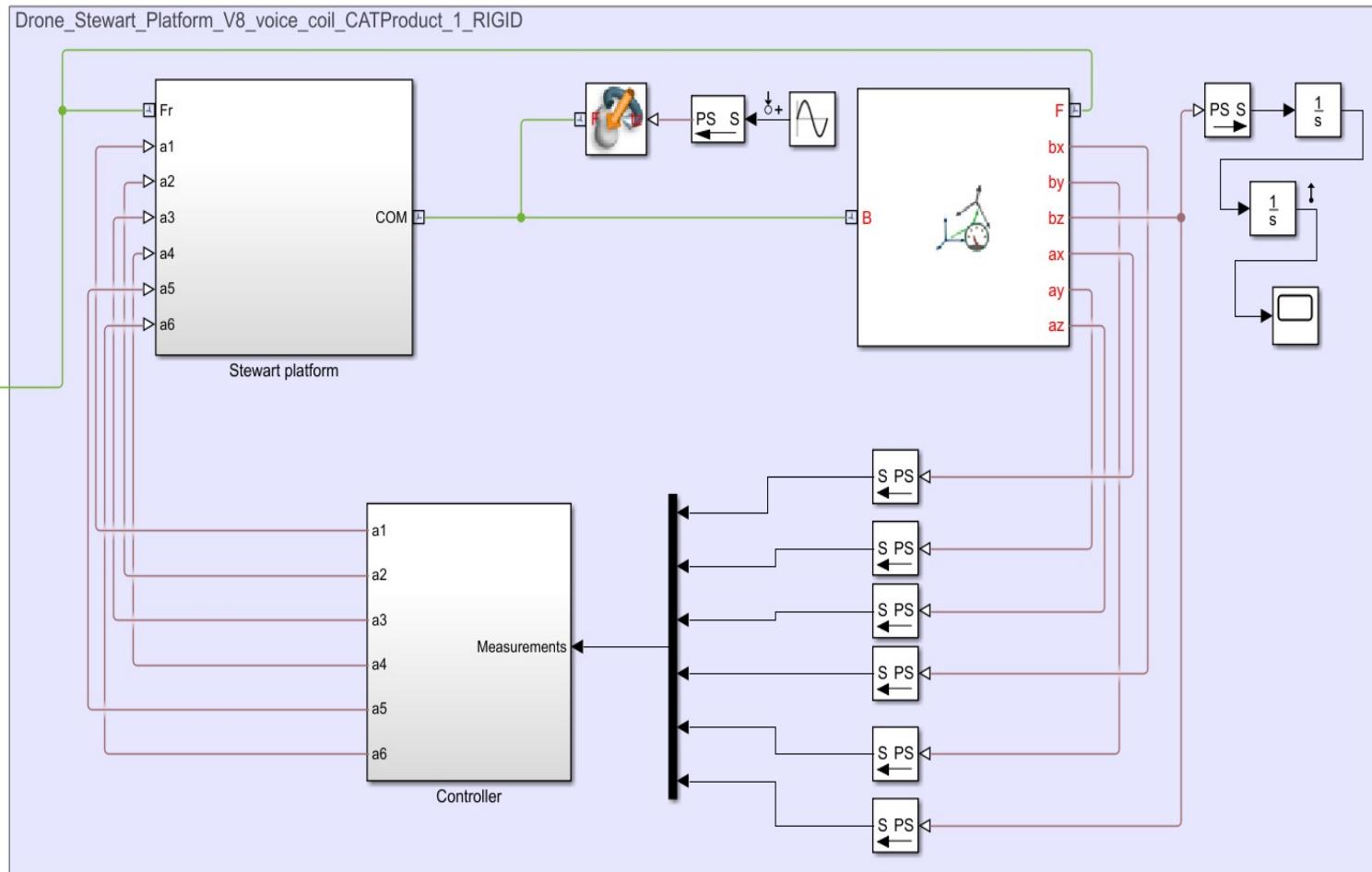
M_x

M_y

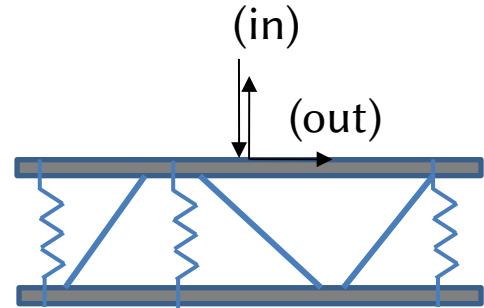
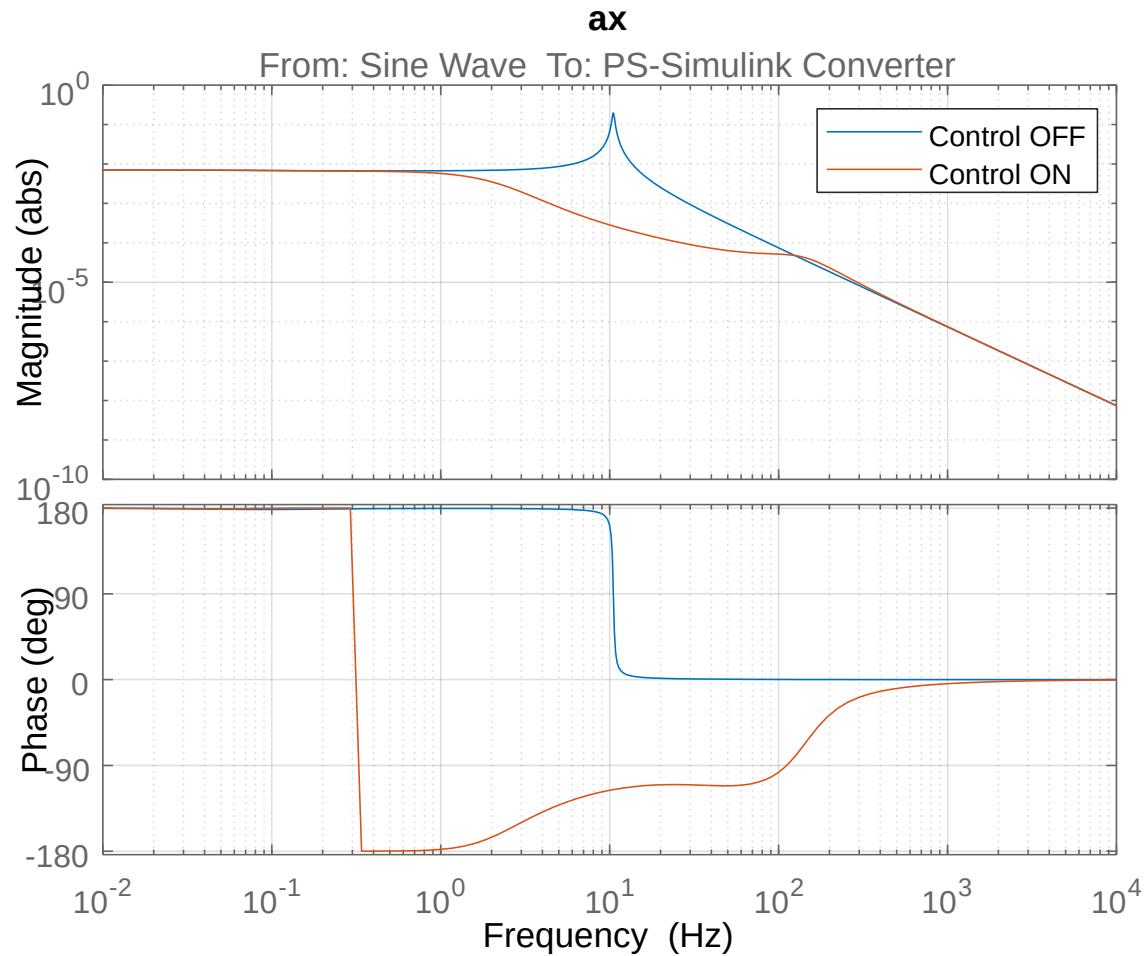
M_z



Controller implementation



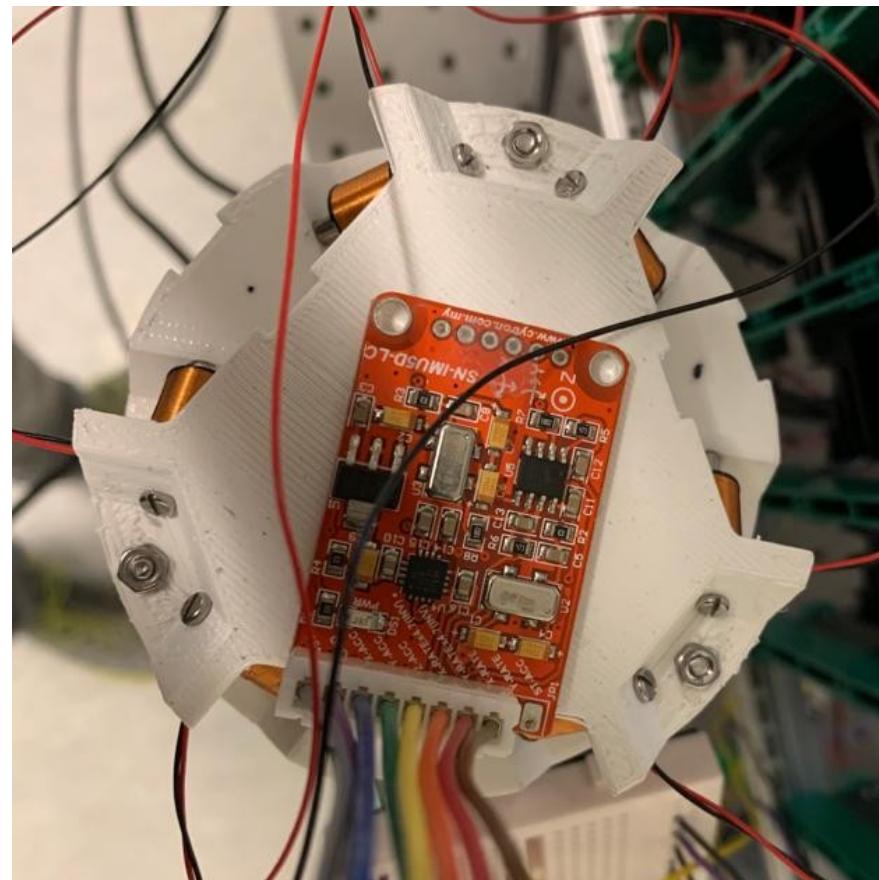
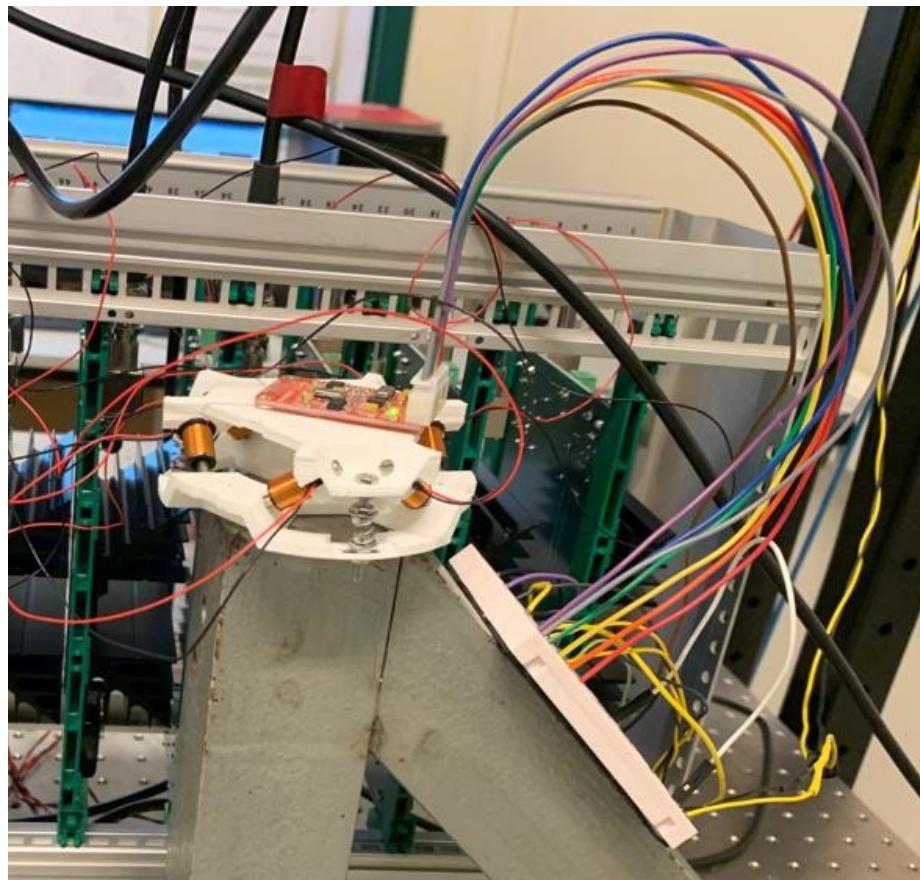
Performance



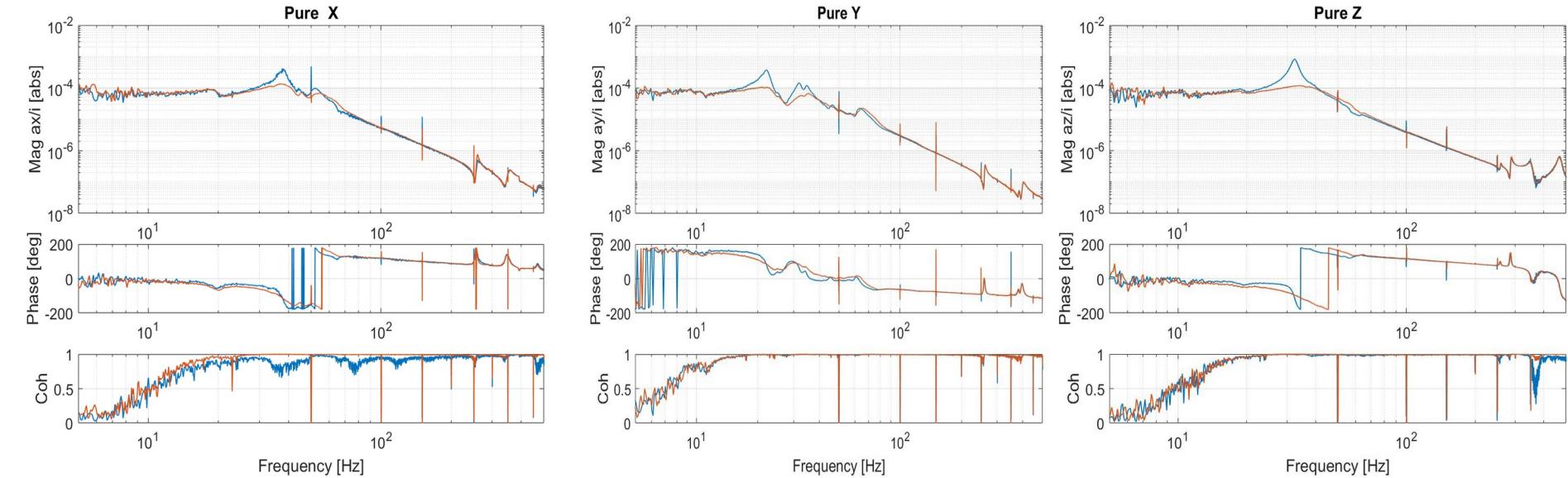
Issues:

1. Measurement of rotations at the COM
2. Initial condition of the simulation

Preliminary experiments



Results



Transfer function from the noise injected in the actuator to the acceleration in X, Y, Z

Future plans

1. Addressing issues with the Simscape model
2. Try self sensing with Vicente
3. Completion of reports

Thank you.