Mathematics of Gerrymandering

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Motivation

This research focuses on the mathematics of gerrymandering. Gerrymandering refers to how political parties draw district boundaries to give them better odds at receiving a majority during congressional elections. Every 10 years, local district boundaries in the United States are redrawn for congressional elections by the current majority party for each state. With such change in political cartography, redrawing district boundaries not only affects who wins but also the people who live within them. With such loosely defined criteria for redistricting, we seek to quantify the process of redrawing district lines to determine when this partisan process becomes an act of manipulation and exclusion.

The Metropolis-Hastings Algorithm

We will be using the The Metropolis Hastings (M.H.) algorithm to produce a random walk that has as its unique stationary distribution the ideal probability distribution, p, of all possible districting plans for a given U.S state. We begin with an arbitrary sample in our distribution, x_0. We then choose at random a different sample that is maximally similar to x_0, x^*. We then choose $x_1 = x^*$ with probability $\min(1, \frac{p(x^*)}{p(x_0)})$ and otherwise choose $x_1 = x_0$. We repeat this process for several iterations.

Iowa

By collecting congressional maps and election results, we implemented the M.H. algorithm in python to produce a sample space for Iowa. The random walk model considers local and federal redistricting requirements using self-defined parameters which measure: compactness of districts and population division.

District Maps

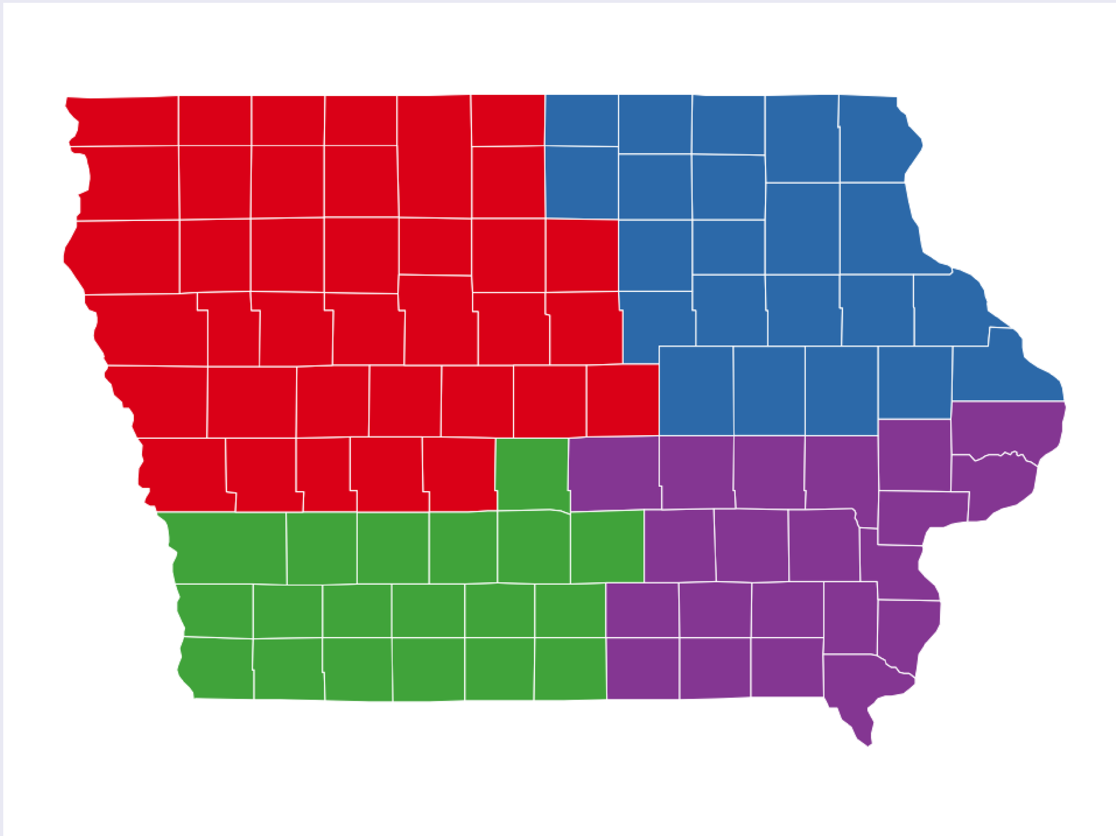

Figure: Above is a sample redistricting plan generated by our algorithm.

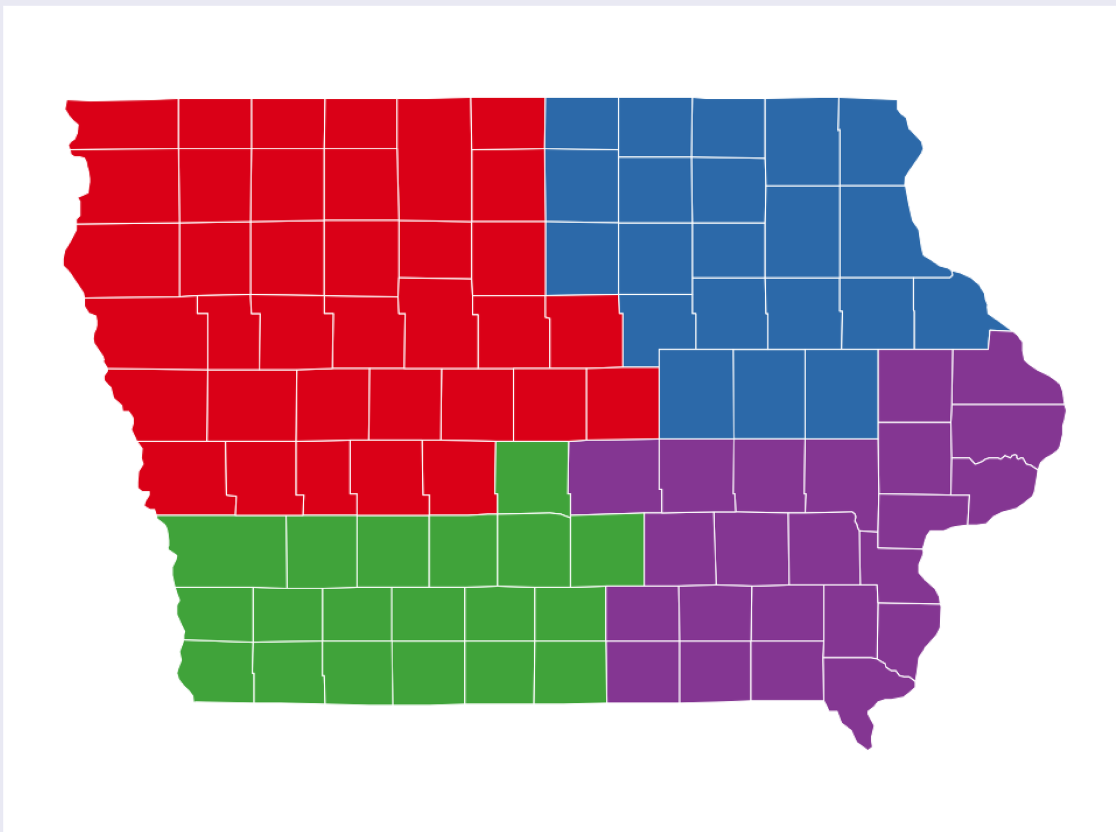

Figure: Above is a sample redistricting plan generated by our algorithm.

Election Simulations

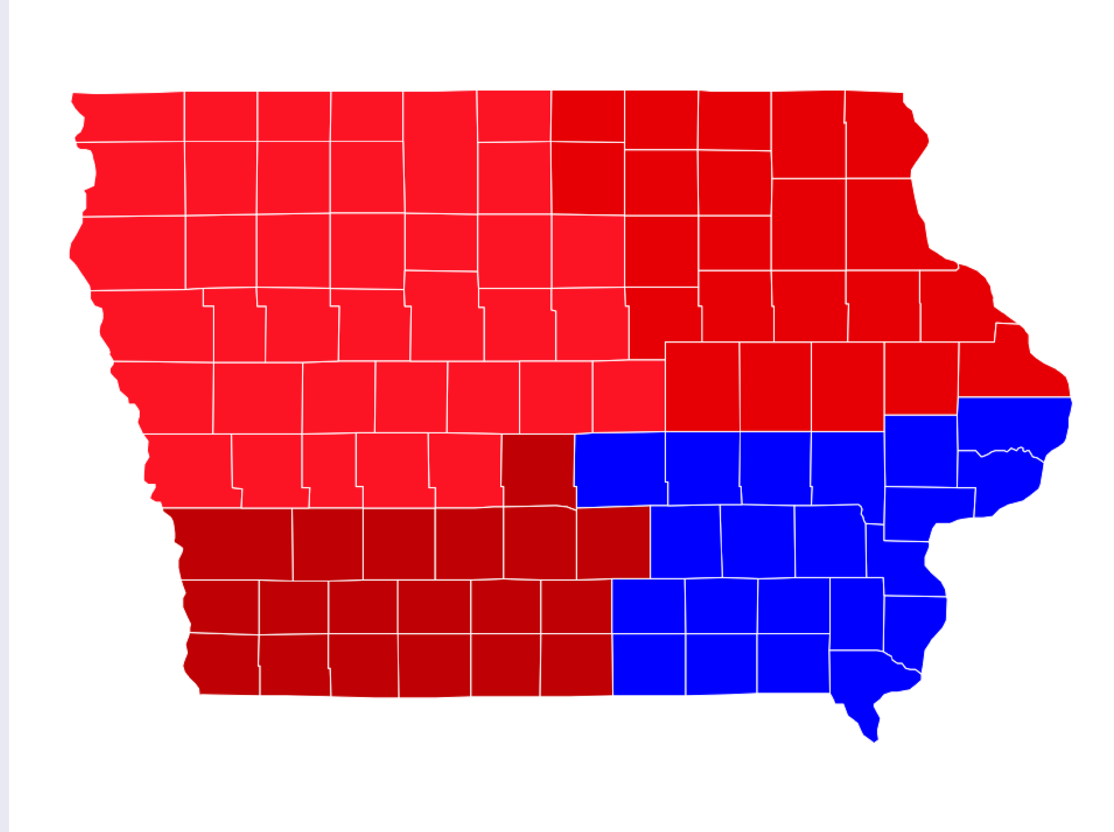

Figure: Above is a result of election simulation corresponding to the left redistricting plan by using 2016 presidential election data. Republicans won in the three red districts and Democratics only won in the one blue district.

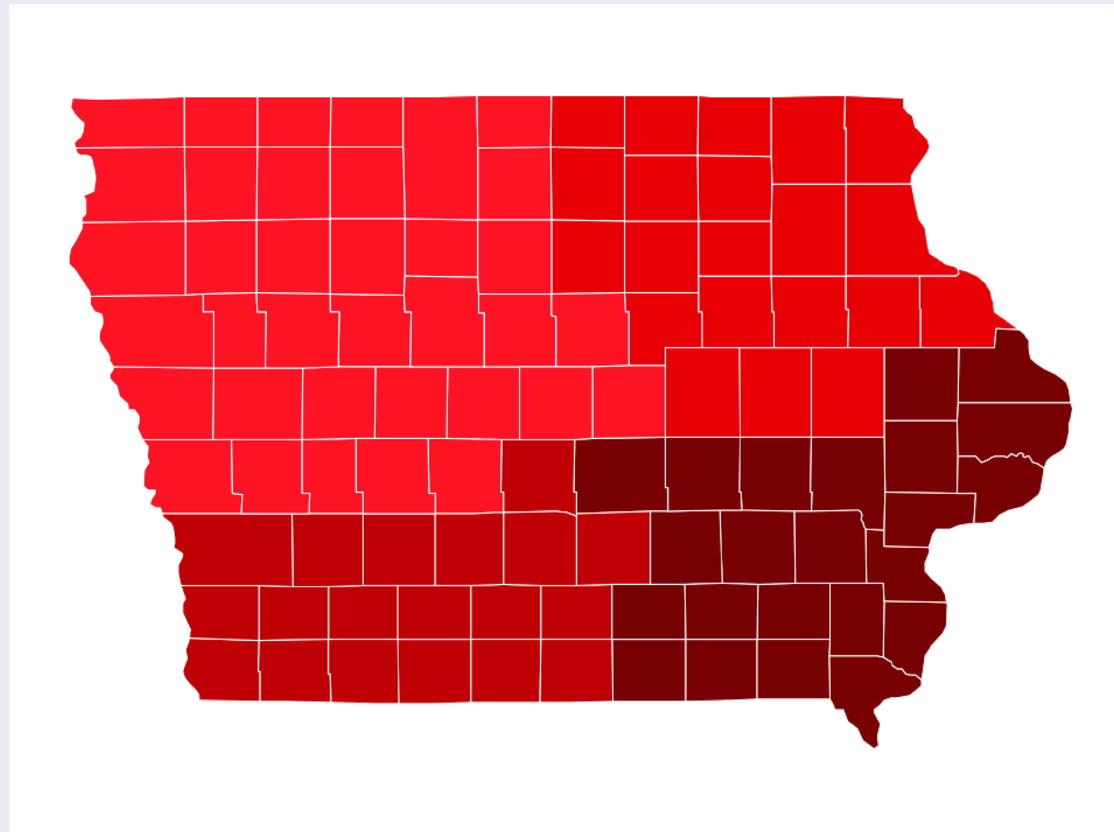

Figure: Above is a result of election simulation corresponding to the left redistricting plan by using 2016 presidential election data. Republicans won in every district.

Energies for Iowa

Population energy:

$$\sum_{\text{Districts}} \left(\text{District Pop.} - \frac{\text{State pop.}}{\text{Number of districts}} \right)^2$$

Compactness energy:

$$\sum_{\text{Districts}} \frac{(\text{District perimeter})^2}{\text{District area}}$$

Accept candidate with probability:

$$\min\left(1, \frac{\exp(\text{weighted sum of current energies})}{\exp(\text{weighted sum of candidate energies})}\right)$$

Future Work

We will define majority-minority energy equation in a way that ensures minorities have an equal opportunity to elect representatives of their choice. We will also define the counties-split energy equation where division of boundaries of cities and counties should be minimized. Ultimately, our goal is to simulate election results with respect to our sample districts and compare the simulated outcomes with actual district maps and their election results.

This study will help to determine if the current redrawn maps legitimately reflect the people they purport to represent. The next state we will be applying our model to will be the state of Washington.

References

Bangia, S., Dou, B., Guo, S., Mattingly, J., & Vaughn, C. (n.d.). Quantifying Gerrymandering.