Mathematics of Gerrymandering

WXML Winter 2018

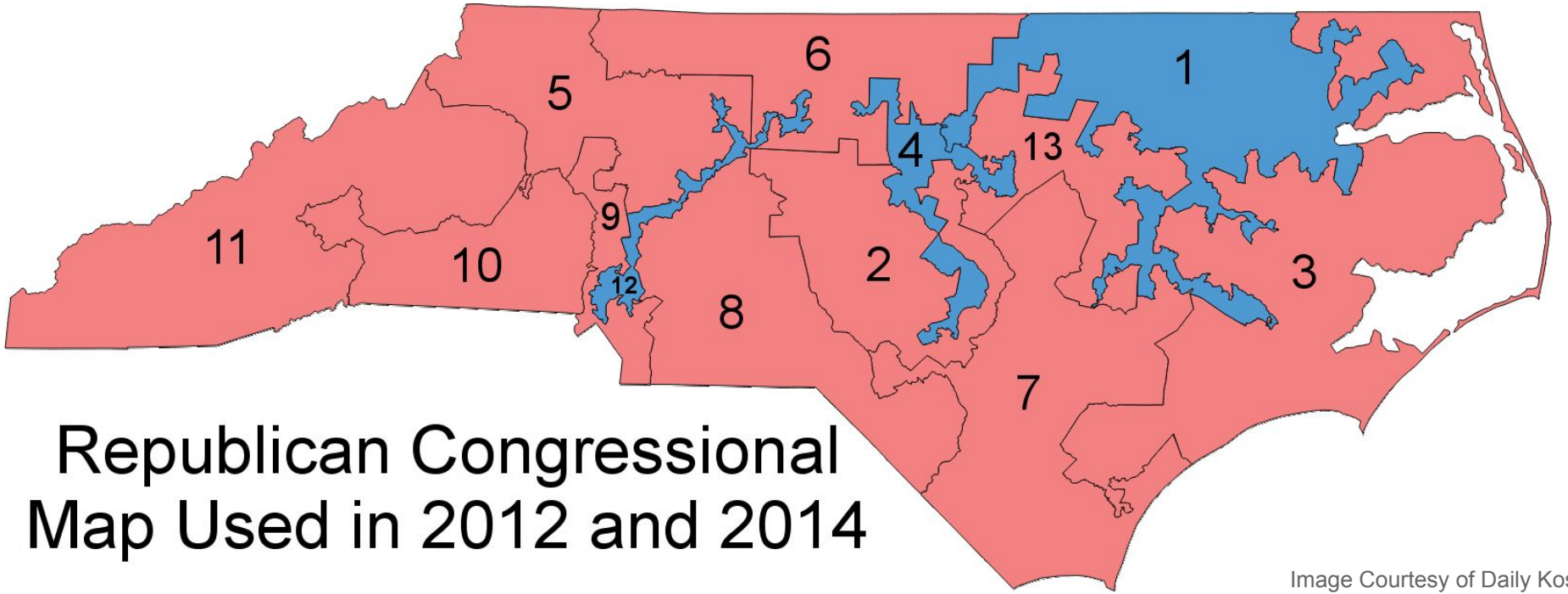
What is Gerrymandering ?

“Packing & Cracking “

North Carolina’s 12th Legislative district

Image Courtesy of WPSU

North Carolina



Republican Congressional Map Used in 2012 and 2014

Image Courtesy of Daily Kos

Metropolis-Hastings Algorithm

How to sample from large space?

Random walk along graph allows sampling from distribution. Over time the probabilities converge to actual distribution probabilities.

Calculating the ratio of probabilities locally generally easier than calculating entire distribution.

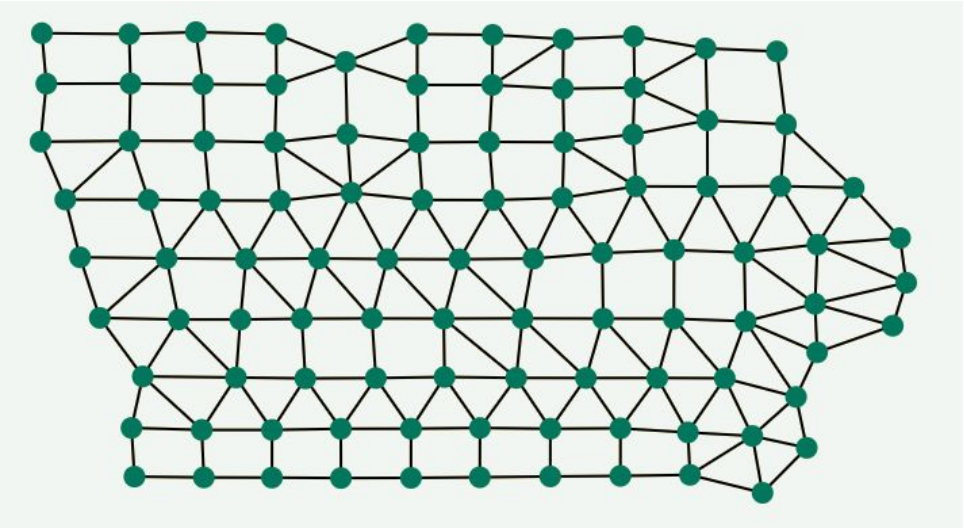
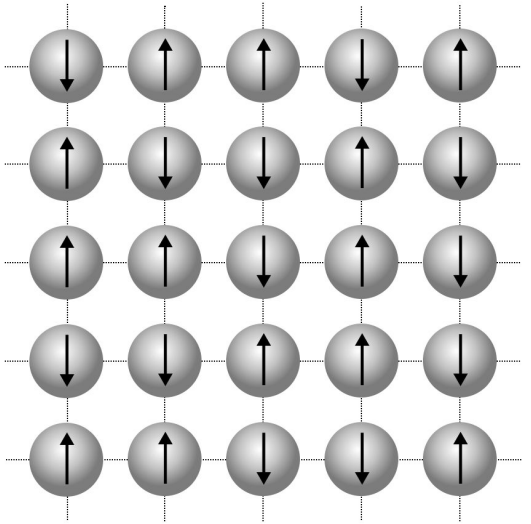


Image courtesy of Duke University

Ising Model

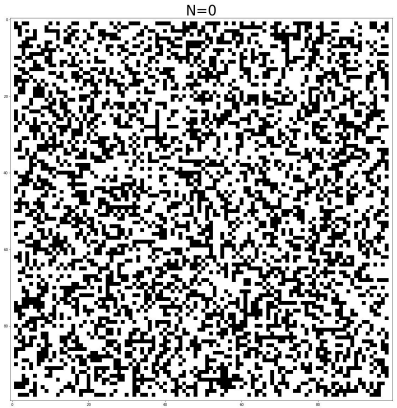
As an exercise of Metropolis-Hastings Algorithm, we reproduced Ising model, which is a mathematical model of magnetism. This is much simpler so we chose this to practice.

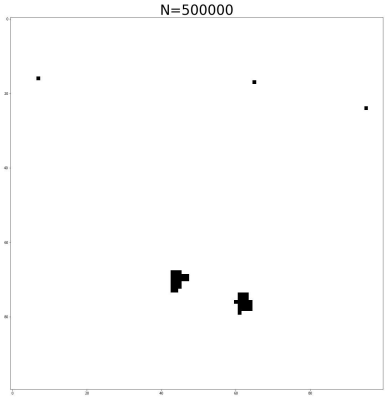
This model is similar to planning congressional district. Atoms represent precincts, spins are congressional districts and edges mean that precincts share a border. Also, the probability function is already given from physics, which we need to find out for redistricting distribution.

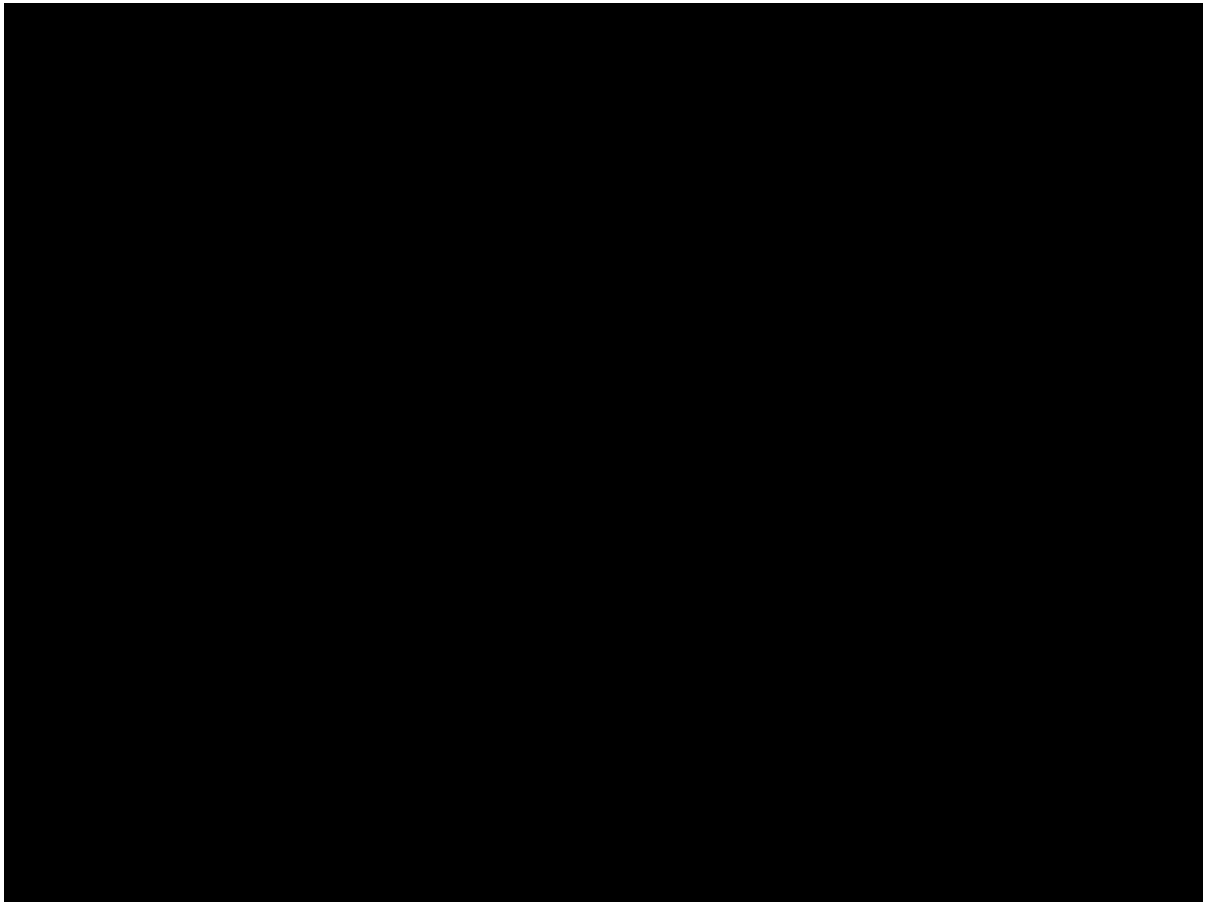


Implementing the Metropolis & Ising Model

- Starting at random assignments of 1 & -1's on the grid
- At each step of random walk, choose another assignment which only differ by 1 cell with current sample
- Decide whether to accept/reject candidate by probability vector calculated with Ising Model







https://drive.google.com/open?id=1h16FBfJHfXRPQYotQetGoWiLCgNmNTuB

Thank you.

Weifan Jiang
Namyoung Kim
Alex Robkin
Leo Segovia