Capital and Labor Income Pareto Exponents across Time and Space

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Abstract

We estimate capital and labor income Pareto exponents across 428 country-year observations that span 54 countries over half a century. We document two stylized facts: (i) capital income is more unequally distributed than labor income; namely, the capital exponent (1–3) is smaller than labor (2–5), and (ii) capital and labor exponents are nearly uncorrelated. To explain these findings, we build an incomplete market model with job ladders and capital income risk that gives rise to a capital income Pareto exponent smaller than but nearly unrelated to the labor exponent. Our results suggest the importance of distinguishing income and wealth inequality.

Keywords: income fluctuation problem, inequality, power law.

JEL codes: C46, D15, D31, D52.

1 Introduction

The purpose of this paper is to estimate and document the Pareto exponents for capital and labor income separately for as many countries and years as possible. We say that a positive random variable X obeys a power law with Pareto exponent $\alpha > 0$ if the tail probability decays like a power function: $\mathrm{P}(X > x) \sim x^{-\alpha}$ for large x.[1] In the context of the income distribution, the Pareto exponent characterizes the tail heaviness of high incomes and hence top tail inequality. Our study is motivated by the following two observations. First, we are not aware of a comprehensive study that documents the capital and labor income Pareto exponents separately for many countries and years, despite their importance. Second, the Pareto exponent has desirable properties relative to other popular inequality measures such as top income shares.

Consider the first point. Conceptually, capital and labor income are very different. While the former is the return for providing capital (wealth), the latter is the return for providing labor services, and there is no particular reason

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[1] More precisely, we say that a random variable X has a Pareto upper tail with exponent $\alpha > 0$ if $\mathrm{P}(X > x) = x^{-\alpha}\ell(x)$ for some slowly varying function ℓ. A function $\ell : (0, \infty) \to \mathbb{R}$ is said to be *slowly varying (at infinity)* if it is nonzero for sufficiently large x and $\lim_{x\to\infty} \ell(tx)/\ell(x) = 1$ for each $t > 0$. See Bingham et al. (1987) for a comprehensive treatment of the theory of regular variation.

to expect a relation between the two. Although these two forms of income are conceptually distinct, it is often put together as just "income" and discussed in the context of inequality and related policies. If capital and labor income are quantitatively different, a policy design based on total income may be misleading. To give one example, consider the theory of optimal taxation (Saez, 2001), where the income Pareto exponent plays an important role. Saez and Stantcheva (2018) carefully distinguish capital and labor income and apply the theory of optimal taxation in the United States. They find that with an income elasticity of $e = 0.5$, the optimal top marginal tax rate is about 50% for labor and 60% for capital (see their Figure 5). This difference directly comes from the fact that capital and labor income Pareto exponents are distinct. Thus, distinguishing capital and labor income inequality is potentially important for policy designs.

Consider the second point about the desirable properties of Pareto exponents. In the applied literature such as Piketty (2003) and Piketty and Saez (2003), top income shares (such as the top 1% income share) are more commonly reported than the income Pareto exponent, perhaps because top shares are summary statistics that can be computed without specifying functional forms or can be understood by non-experts without special knowledge of statistics. However, Atkinson (2005) documents methodological problems regarding the cross-country comparison of top income shares, citing the differences in tax units (e.g., individual or household) and legislation (e.g., whether social security benefits are taxable). One of the reasons such issues arise is because it is not always clear how to define the population and measure small units,[2] which greatly affect the shape of the entire distribution. Using the Pareto exponents significantly alleviates these definition and measurement issues because the Pareto distribution is scale invariant (see Jessen and Mikosch, 2006 for a summary) and its exponent depends only on the tail behavior, not the entire distribution. For example, doubling the income of all households in the top 1% of the income distribution makes the top 1% income share (roughly) twice as large, but the Pareto exponent is unaffected. A similar comment applies to any inequality measure that depends on the entire distribution, such as the Gini coefficient. While we do not claim that the Pareto exponent is the only interesting inequality measure, it is certainly a robust (detail-independent) measure for top tail inequality. See Gabaix (2009, 2016) for more discussion on the robustness of the Pareto distribution.

In this paper, we use the harmonized *Luxembourg Income Study* database (hereafter LIS) to document the capital and labor income Pareto exponents across all available 428 country-year observations that span 54 countries over half a century. We document two empirical findings. First, we find that the capital income Pareto exponent is roughly in the range 1–3 and is smaller than the labor income Pareto exponent, which ranges between 2–5. This implies that capital income is more unequally distributed than labor income. This fact is unsurprising and well known for a specific country or year (see, for example, the Lorenz curve in Figure 1 of Saez and Stantcheva, 2018). However, we are not aware of a comprehensive study that systematically analyzes datasets from many countries and years, and therefore our finding suggests that capital income

[2] Imagine how to formally distinguish cities, towns, villages, and settlements; continents, islands, and islets; and inland seas, lakes, and ponds. How to define units and how to measure small units matter for the size distribution of population, land mass, and water surface area.

is generally more unequal than labor income. More specifically, we formally test the equality of capital and labor income Pareto exponents and the null is rejected in 78% of samples. In every single case of rejection, the capital exponent is smaller than the labor exponent. Second, we find that the capital income Pareto exponent is nearly unrelated to the labor exponent. In particular, the correlation between the two exponents across countries is close to zero.

To explain our empirical findings, we build a simple incomplete market model with job ladders and capital income risk. In the model, agents get randomly promoted to the next job ladder. Because individual income follows a random growth process, we obtain a Pareto-tailed labor income distribution. The agents also save assets and face idiosyncratic investment risk, which generates a Pareto-tailed wealth (hence capital income) distribution. Because the capital income Pareto exponent is mainly determined by the asset return distribution, while the labor income Pareto exponent is mainly determined by the income growth distribution, the relation between the two is weak. Furthermore, we analytically characterize the capital and labor income Pareto exponents and show that the former tends to be smaller than the latter for common parametrization. Our results suggest the importance of distinguishing income and wealth inequality.

Related literature The power law behavior of income was first recognized by Pareto (1895, 1896, 1897), who used tabulation data of tax returns in many European countries. More recent research that employs micro data include Reed (2001) for U.S., Reed (2003) for U.S., Canada, Sri Lanka, and Bohemia, Nirei and Souma (2007) for Japan, Toda (2011, 2012) for U.S., and Ibragimov and Ibragimov (2018) for Russia. These papers all concern specific countries and years. Bandourian et al. (2002) estimate eleven parametric distributions (some of which exhibit Pareto tails) using 82 household labor income datasets from *Luxembourg Income Study* (LIS) as we do, though they neither focus on the Pareto exponent nor consider capital income. Gabaix (2009) mentions "[the] tail exponent of income seems to vary between 1.5 and 3", citing Atkinson and Piketty (2007), though without providing specific details. Atkinson and Piketty (2010, Table 13A.23) document income Pareto exponents across many countries and years estimated from top income share data based on tax returns. However, these estimates are computed from total income, and since (as we document in Section 2.3) the capital income Pareto exponents tend to be smaller than labor exponents, their estimates are best understood as capital income (hence wealth) Pareto exponents. Benhabib et al. (2017) make the point that wealth is more skewed than income, citing a few Pareto exponent estimates from Badel et al. (2018) for income and Vermeulen (2018) for wealth. As mentioned in the introduction, there seems to be no comprehensive study that documents the capital and labor income Pareto exponents separately for many countries and years.

2 Pareto exponents across countries and years

In this section we estimate the capital and labor income Pareto exponents for all countries and years that are available in the LIS database described in Appendix A, which spans 54 countries over half a century (428 country-year observations in total). We then formally test for the equality of the Pareto exponents of capital and labor income.

2.1 Estimation method

For each country and year, we suppose that the (capital or labor) income observations $\{X_n\}_{n=1}^N$ are independent and identically distributed (IID) with cumulative distribution function (CDF) $F(x) = \mathrm{P}(X_n \le x)$. The assumption that the upper tail of income obeys a power law with Pareto exponent $\alpha > 0$ translates into the regular variation condition

$$1 - F(x) = x^{-\alpha}\ell(x) \tag{2.1}$$

for some slowly varying function ℓ (see Footnote 1). Note that the assumption on ℓ involves only the limit as $x \to \infty$; we are assuming a power law behavior in the upper tail without taking a stance on the shape of the entire distribution.

We are interested in estimating the Pareto exponent α for each country and year. For this purpose, we employ the Hill (1975) (maximum likelihood) estimator

$$\frac{1}{\widehat{\alpha}(k)} := \frac{1}{k}\sum_{n=1}^{k} \log\left(\frac{X_{n:N}}{X_{k:N}}\right). \tag{2.2}$$

Here $X_{n:N}$ denotes the n-th largest order statistic from the sample $\{X_n\}_{n=1}^N$ and $k \in \{1, \dots, N\}$ denotes the number of tail observations used to estimate the Pareto exponent. Results of Hall (1982) show that the standard error is $\widehat{\alpha}(k)/\sqrt{k}$ under flexible assumptions on the CDF (see (2.3) below). We use the Hill estimator because we are interested in formally testing the equality of the capital and labor income Pareto exponents using the method of Hoga (2018), for which the Hill estimator is required.[3]

When the population distribution is known to be exactly Pareto (so ℓ in (2.1) is zero below the minimum size $x_{\min}$ and constant above this threshold), it is well known that the Hill estimator for the full sample ($k = N$) is consistent, asymptotically normal, and asymptotically efficient because it is a maximum likelihood estimator. In practice, the CDF is not exactly Pareto and the researcher needs to select an appropriate value of k. For instance, if $F(x)$ satisfies

$$1 - F(x) = Cx^{-\alpha}(1 + Dx^{-\beta} + o(x^{-\beta})) \tag{2.3}$$

with some $\beta > 0$, then Hall (1982) shows that choosing $k = o(N^{2\beta/(2\beta+\alpha)})$ together with $k \to \infty$ as $N \to \infty$ is sufficient for consistency and asymptotic normality (see also Embrechts et al., 2013).[4] Notice that this choice puts a bound on the growth rate of k. The expansion (2.3) covers a wide range of distributions of interest, such as the t-distribution and the type II extreme value distribution (Daníelsson and de Vries, 1997).

Despite these asymptotic results, it is notoriously difficult to pick k optimally in finite samples (Hall, 1990; Resnick and Stărică, 1997; Danielsson et al., 2001).

[3]If we are only interested in estimating the Pareto exponents, then there are many alternative methods available. Gomes and Guillou (2015) review 13 commonly used estimators. Fedotenkov (2020) review more than 100.

[4]Recall that we write $f(x) = o(g(x))$ if for all $\epsilon > 0$, we have $|f(x)| \le \epsilon |g(x)|$ for large enough x.

In practice, researchers often plot the Hill estimator (2.2) over a range of k to find a flat region or plot the log rank $\log 1, \dots, \log N$ against the log size $\log X_{1:N}, \dots, \log X_{N:N}$ to find a region that exhibits a straight line pattern and choose a size threshold to run the log-rank regression.[5] Unfortunately, this graphical approach is not feasible in our setting because LIS does not allow researchers to download the micro data for confidentiality concerns (researchers are required to submit their execution files to conduct statistical analyses) and there is little scope for exploratory graphical data analysis. In this paper we simply use the largest 5% observations, so

$$k = \lfloor 0.05N \rfloor, \tag{2.4}$$

which is standard in the literature.[6] Unreported simulations show that our results are robust to using other thresholds such as the largest 1% and 10% observations, or using a data-driven procedure using the Kolmogorov-Smirnov distance (as in Danielsson et al., 2016).

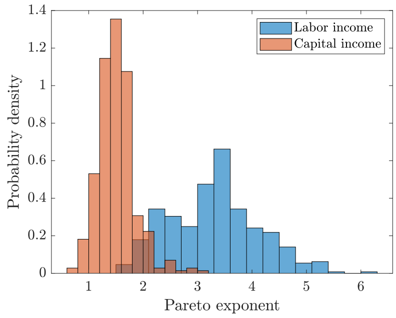
2.2 Capital and labor income Pareto exponents

We estimate the capital and labor income Pareto exponents for all countries and years available in the LIS database. The database spans 54 countries across the years 1967–2018, with a total of 428 country-year observations. The point estimates of the capital and labor income Pareto exponents for each country and year as well as their standard errors can be found in Table 2 in Appendix D. To avoid small sample issues, we restrict our analysis to countries with at least 500 positive observations for income, resulting in (all) 428 country-year pairs for labor income and 358 for capital income. For visibility, Figure 1 shows the histogram and scatter plot of the capital and labor income Pareto exponents.

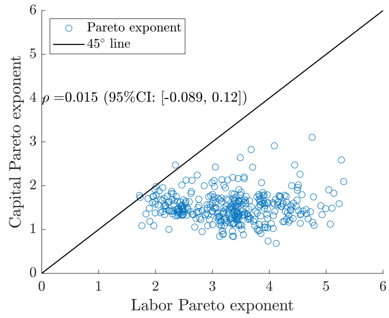
Figure 1a shows the histogram of the capital and labor income Pareto exponents pooled across all available countries and years. The capital and labor income Pareto exponents are generally in the range 1–3 and 2–5, respectively. This suggests that (i) capital income is generally more unequally distributed than labor income, but (ii) there is significant heterogeneity in both capital and labor income inequality across countries and years. Figure 1b shows the scatter plot of the Pareto exponents together with the 45° degree line. The confidence interval is computed assuming all country-year observations are independent and there is no sampling error in the point estimates of the Pareto exponents. Although it is not obvious how to account for these issues, doing so will only widen the confidence interval. Therefore the fact that the naive confidence interval contains zero suggests that the correlation between the two

[5] Gabaix and Ibragimov (2011) study the asymptotic behavior of log rank regression and show that the standard error is larger by a factor of $\sqrt{2}$ than the Hill estimator. However, they do not discuss how to select the threshold. In their empirical application, they consider the size distribution of population in U.S. metropolitan statistical areas, which are already far into the tail and hence the threshold selection is less of an issue. Ibragimov and Ibragimov (2018) apply the same methodology to Russian household income data and consider the top 5% and 10% thresholds.

[6] An alternative approach is to estimate a parametric distribution F that admits a Pareto upper tail by maximum likelihood using the entire sample. The double Pareto-lognormal distribution proposed by Reed (2003) and Reed and Jorgensen (2004) often performs best. See Toda (2012) for a horse race across several parametric distributions in the context of U.S. labor income.



(a) Histogram.



(b) Scatter plot.

Figure 1: Histogram and scatter plot of capital and labor income Pareto exponents.

Pareto exponents is weak. Furthermore, for the vast majority of countries and years, the capital income Pareto exponent is smaller than the labor exponent, again suggesting that capital income is more unequal than labor income.

How do the Pareto exponents evolve over time? Because countries appear sporadically in the LIS database and it is not obvious how to model temporal and spacial dependence of income, we only present an informal graphical analysis. To this end, we first consider only the countries (with at least 500 observations) that appear in eight or more years in the database.[7] Second, for each of these countries, we linearly interpolate the capital and labor Pareto exponents for all years from the available years. (We extrapolate by constants outside the range using the first and last observation year.) Finally, for each year we compute the median Pareto exponent across countries. Figure 2 shows the evolution of capital and labor income Pareto exponents over half a century (1967–2018). We observe that (i) the labor income Pareto exponent decreased from around 4 to 3 over the decade of 1990-2000, but (ii) the capital income Pareto exponent has been stable at around 1.5. Again, capital income appears to be more unequally distributed than the labor income.

2.3 Testing equality of capital and labor Pareto exponents

We now formally test whether the capital and labor Pareto exponents are equal. In particular our test is

$$H_0 : \alpha_{\text{lab}} = \alpha_{\text{cap}} \quad \text{against} \quad H_1 : \alpha_{\text{lab}} \neq \alpha_{\text{cap}},$$

where $\alpha_{\text{cap}}, \alpha_{\text{lab}}$ denote the capital and labor income Pareto exponents. Testing the null hypothesis H_0 is complicated by the fact that there is dependency between labor income $\{X_{\text{lab},n}\}_{n=1}^N$ and capital income $\{X_{\text{cap},n}\}_{n=1}^N$, because individuals who are rich (receive high labor income) tend to be wealthy and receive high capital income. Therefore, we apply the test recently developed by Hoga (2018), which allows for dependence in the data but assumes restrictions on

[7]There are twenty such countries, namely: Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, Germany, Ireland, Italy, Lithuania, Luxembourg, Mexico, Norway, Spain, Sweden, Switzerland, Taiwan, United Kingdom, and United States.

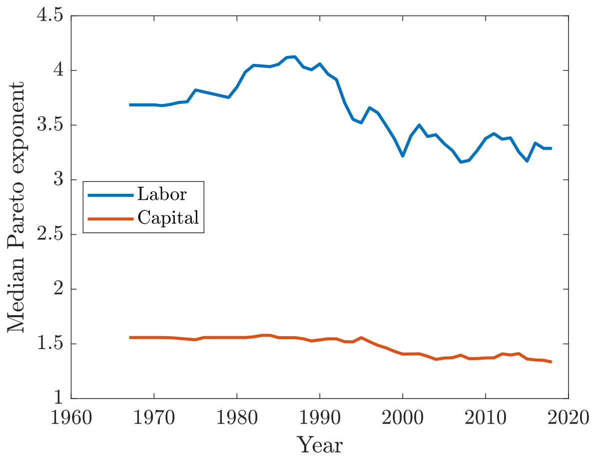


Figure 2: Time evolution of capital and labor income Pareto exponents.

the growth rate of the tail dependence (see in particular Hoga (2018, Assumption A2)). The test is based upon the inverse of the Hill estimator (2.2), which we denote by $\widehat{\gamma} \coloneqq 1/\widehat{\alpha}$. The test statistic is defined by

$$T_N = \frac{(\widehat{\gamma}_{\text{lab}}(1) - \widehat{\gamma}_{\text{cap}}(1))^2}{\int_{t_0}^1 t^2 \left[(\widehat{\gamma}_{\text{lab}}(t) - \widehat{\gamma}_{\text{cap}}(t)) - (\widehat{\gamma}_{\text{lab}}(1) - \widehat{\gamma}_{\text{cap}}(1))\right]^2 \mathrm{d}t}, \tag{2.5}$$

where $t_0 \in (0, 1)$ is a tuning parameter and $\widehat{\gamma}(t)$ is the inverse Hill estimator

$$\widehat{\gamma}(t) \coloneqq \frac{1}{\lfloor kt \rfloor} \sum_{n=1}^{\lfloor kt \rfloor} \log \left(\frac{X_{n:\lfloor Nt \rfloor}}{X_{\lfloor kt \rfloor : \lfloor Nt \rfloor}} \right). \tag{2.6}$$

Using the Hill estimator based on the subsample with only $\lfloor Nt \rfloor$ observations leads to self-normalization of the test statistic T_N and renders a test that is asymptotically pivotal. The limiting distribution is

$$T_N \xrightarrow{\mathrm{d}} \frac{W(1)^2}{\int_{t_0}^1 [W(t) - tW(1)]^2 \,\mathrm{d}t}, \tag{2.7}$$

where $W(t)$ is a standard Brownian motion. Note that since the test statistic (2.5) can be computed using only the Hill estimator and conducting numerical integration, there is no need to estimate the (potentially difficult) tail covariance. The tuning parameter t_0 affects the size of the test in finite samples: high values of t_0 make the integral in (2.5) based on too few differences of $\widehat{\gamma}$, and low values of t_0 yield volatile $\widehat{\gamma}$ in (2.6) when t is close to t_0. Both of these effects may cause size distortions. Therefore we set $t_0 = 0.2$ following the recommendation of Hoga (2018), who finds that this choice leads to favorable size properties.[8] We reject the null H_0 when the test statistic T_N is large. According to Table I

[8] The choice of t_0 in Hoga (2018) comes out using an automated selection procedure to choose k, which is different than our 5% rule. An earlier version of our paper employs the same automated procedure, which leads to very similar results.

of Hoga (2018), the 95 percentile of (2.7) for $t_0 = 0.2$ is 55.44, which we use as the critical value for testing H_0 at 5% significance level.

One issue with the test statistic (2.5) is that it requires the same number of tail observations k for both cross-sections of capital and labor income. Hence the 5% rule (2.4) discussed in Section 2.1 becomes problematic as the number of people with capital income in our data set is rather small. Many households do not hold liquid financial wealth and hence have no capital income. The resulting test is thus not feasible since k_{lab} is wildly different from k_{cap}. To overcome this issue, we only test the equality of Pareto exponents for countries that have more than 500 positive capital income observations and set $k = \lfloor 0.05 N_{\text{cap}}^{+} \rfloor$, where N_{cap}^{+} is the number of *positive* capital income observations. In practice this means that we estimate the Pareto exponent of labor income further in the tail, which is acceptable since the Pareto approximation tends to fit better for smaller k. The sample selection results in 358 country-year observations out of 428. Table 3 in Appendix D shows the test results.

We reject the null hypothesis $H_0 : \alpha_{\text{lab}} = \alpha_{\text{cap}}$ in 279 country-year observations out of 358 (78%) that meet our sample selection criterion. In every single case of rejection, we have $\widehat{\alpha}_{\text{lab}} > \widehat{\alpha}_{\text{cap}}$, and therefore we formally confirm the observation in Section 2.2 that capital income is more unequally distributed than labor income.

3 Model of capital and labor Pareto exponents

Our empirical analysis in Section 2 suggests that (i) the capital income Pareto exponent is smaller than the labor one (i.e., capital income is more unequally distributed than labor income), and (ii) the correlation between capital and labor income Pareto exponents is weak. To explain these empirical findings, we present a simple dynamic model of consumption and savings, which builds on one of the authors' prior works (Ma et al., 2020; Ma and Toda, 2020). Our model is more specialized but the characterizations are sharper.

3.1 Income fluctuation problem

Time is discrete and denoted by $t = 0, 1, 2, \dots$. Let a_t be the financial wealth of a typical agent at the beginning of period t including current income. The agent chooses consumption $c_t \geq 0$ and saves the remaining wealth $a_t - c_t$. The period utility function is $u : (0, \infty) \to \mathbb{R}$, the discount factor is $\beta > 0$, the gross return on wealth between time $t-1$ and t is $R_t > 0$, and non-financial income at time t is $Y_t > 0$. Thus the agent solves

$$\begin{aligned} &\text{maximize} && \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) && \text{(3.1a)} \\ &\text{subject to} && a_{t+1} = R_{t+1}(a_t - c_t) + Y_{t+1}, && \text{(3.1b)} \\ & && 0 \leq c_t \leq a_t, && \text{(3.1c)} \end{aligned}$$

where the initial wealth $a_0 = a > 0$ is given, (3.1b) is the budget constraint, and (3.1c) implies that the agent cannot borrow (which is without loss of generality according to the discussion in Chamberlain and Wilson, 2000). Throughout the rest of the paper we maintain the following assumptions.

Assumption 1 (CRRA utility)**.** *The utility function exhibits constant relative risk aversion (CRRA) with coefficient* $\gamma > 0$*, so* $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ *if* $\gamma \neq 1$ *and* $u(c) = \log c$ *if* $\gamma = 1$*.*

Assumption 2 (IID shocks)**.** *Let* $G_{t+1} \coloneqq Y_{t+1}/Y_t$ *be the income growth. The sequence* $\{R_{t+1}, G_{t+1}\}_{t=0}^{\infty}$ *is independent and identically distributed (*IID*).*

These assumptions are similar to Carroll (2020), except that we allow for stochastic returns on savings. Note that the asset return R_{t+1} and income growth G_{t+1} are potentially mutually dependent. Due to the IID assumption, the state variables of the income fluctuation problem (3.1) are financial wealth $a_t > 0$ and current income $Y_t > 0$. Exploiting homotheticity (Assumption 1), we can reduce the number of state variables to just one, namely the wealth-income ratio (normalized wealth) $\tilde{a}_t \coloneqq a_t/Y_t$. To see this, letting $\tilde{c}_t \coloneqq c_t/Y_t$ be the consumption-income ratio (normalized consumption), dividing the borrowing constraint (3.1c) by Y_t, we obtain $0 \leq \tilde{c}_t \leq \tilde{a}_t$. Similarly, dividing the budget constraint (3.1b) by Y_{t+1}, we obtain

$$\begin{aligned}
\tilde{a}_{t+1} = a_{t+1}/Y_{t+1} &= (R_{t+1}Y_t/Y_{t+1})(a_t/Y_t - c_t/Y_t) + 1 \\
&= (R_{t+1}/G_{t+1})(\tilde{a}_t - \tilde{c}_t) + 1 \\
&= \tilde{R}_{t+1}(\tilde{a}_t - \tilde{c}_t) + 1, \qquad (3.2)
\end{aligned}$$

where $\tilde{R}_{t+1} \coloneqq R_{t+1}/G_{t+1}$ is the asset return relative to income growth. As for the utility function, since

$$c_t = Y_t\tilde{c}_t = Y_0 \left(\prod_{s=1}^{t} G_s \right) \tilde{c}_t,$$

(here we interpret $\prod_{s=1}^{0} \bullet = 1$) assuming $Y_0 = 1$ (which is without loss of generality) and $\gamma \neq 1$, it follows from (3.1a) that

$$\begin{aligned}
\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) &= \mathrm{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=1}^{t} \beta G_s^{1-\gamma} \right) \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} \\
&= \mathrm{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=1}^{t} \tilde{\beta}_s \right) \frac{\tilde{c}_t^{1-\gamma}}{1-\gamma}, \qquad (3.3)
\end{aligned}$$

where $\tilde{\beta}_t \coloneqq \beta G_t^{1-\gamma}$. The discussion for $\gamma = 1$ is similar. Therefore the problem reduces to an income fluctuation problem with CRRA utility, random discount factors $\left\{ \tilde{\beta}_t \right\}_{t=1}^{\infty}$, stochastic returns $\left\{ \tilde{R}_t \right\}_{t=1}^{\infty}$ on wealth, and constant income ($\tilde{Y}_t \equiv 1$). The general theory of income fluctuation problems with stochastic discounting, returns, and income in a Markovian setting was developed by Ma et al. (2020). Therefore we immediately obtain the following result. In what follows, we drop the time subscript when no confusion arises.

Proposition 1. *Suppose Assumptions 1, 2 hold and*

$$\beta \,\mathrm{E}[G^{1-\gamma}] < 1 \quad \textit{and} \quad \beta \,\mathrm{E}[RG^{-\gamma}] < 1. \qquad (3.4)$$

Then the income fluctuation problem (3.1) *has a unique solution. The consumption function can be expressed as*

$$c(a, Y) = Y\tilde{c}(a/Y),$$

where $\tilde{c} : (0, \infty) \to (0, \infty)$ *is the consumption function of the detrended problem (maximizing* (3.3) *subject to* (3.2)*), which can be computed by policy function iteration.*[9]

3.2 Tail behavior of income and wealth

We now characterize the tail behavior of income and wealth in the context of the income fluctuation problem in Section 3.1.

To make the model stationary, suppose that agents survive to the next period with probability $v \in (0, 1)$ (perpetual youth model as in Yaari, 1965). Whenever agents die, they are replaced by newborn agents. For simplicity, assume that the discount factor β in (3.1a) already accounts for survival probability and that there is no market for life insurance (allowing for life insurance only changes R to R/v and is thus mathematically equivalent after reparametrization). Without loss of generality, suppose that newborn agents start with income $Y_0 = 1$. Then the income of a randomly selected agent is Y_T, where T is a geometric random variable with mean $\frac{1}{1-v}$. By the assumption on income growth, the log income of a randomly selected agent

$$\log Y_T = \log(Y_T/Y_0) = \sum_{t=1}^{T} \log G_t$$

is a geometric sum of IID random variables, for which we can characterize the tail behavior as follows.

Proposition 2 (Income Pareto exponent)**.** *Suppose that* $\mathrm{P}(G > 1) > 0$ *and* $1 < v\,\mathrm{E}[G^z] < \infty$ *for some* $z > 0$*. Then the cross-sectional income distribution has a Pareto upper tail, whose exponent is the unique positive solution* $z = \alpha_Y$ *to*

$$v\,\mathrm{E}[G^z] = 1. \tag{3.5}$$

Proof. See Beare and Toda (2017, Theorem 3.4). □

To characterize the tail behavior of wealth, we first note that the normalized consumption function $\tilde{c}$ in Proposition 1 is concave and asymptotically linear with a specific slope.

Proposition 3 (Concavity and asymptotic linearity)**.** *Let everything be as in Proposition 1. Then* $\tilde{c}$ *is concave and*

$$\lim_{a\to\infty} \frac{\tilde{c}(a)}{a} = \begin{cases} 1 - (\mathrm{E}[\beta R^{1-\gamma}])^{1/\gamma} & \text{if } \mathrm{E}[\beta R^{1-\gamma}] < 1, \\ 0 & \text{otherwise.} \end{cases} \tag{3.6}$$

[9] See Li and Stachurski (2014) and Ma et al. (2020) for details on policy function iteration.

Using Proposition 3 and setting $\rho = \min\left\{(\mathrm{E}[\beta R^{1-\gamma}])^{1/\gamma}, 1\right\}$, for high enough asset level, the detrended budget constraint (3.2) becomes approximately

$$\tilde{a}_{t+1} \approx \rho \tilde{R}_{t+1} \tilde{a}_t + 1,$$

which is a random growth process. Under specific assumptions, Ma et al. (2020, Theorem 3.3) prove that the upper tail of the stationary distribution of normalized wealth $\tilde{a}_t$ has a Pareto lower bound. Although a sharp characterization of the tail behavior is generally difficult, in our setting it is possible to obtain an exact characterization due to concavity and the IID assumption.

Proposition 4. *Let $\rho = \min\left\{(\mathrm{E}[\beta R^{1-\gamma}])^{1/\gamma}, 1\right\}$ and $H = \rho\tilde{R}$. Suppose that (i) R is thin-tailed (meaning $\mathrm{E}[R^z] < \infty$ for all $z > 0$), (ii) $\log H$ is non-lattice (not supported on an evenly spaced grid), and (iii) $\mathrm{P}(H > 1) > 0$, and $1 < v\,\mathrm{E}[H^z] < \infty$ for some $z > 0$. Then the cross-sectional normalized wealth distribution is either bounded or has a Pareto upper tail, in which case the exponent is the unique positive solution $z = \tilde{\alpha}$ of*

$$v\,\mathrm{E}[H^z] = 1. \tag{3.7}$$

The Pareto exponent for wealth and capital income is then $\alpha = \min\{\tilde{\alpha}, \alpha_Y\}$.

Proposition 4 is significant despite its simplicity. According to the model, we always have $\alpha \leq \alpha_Y$, typically with a strict inequality as we see in the numerical example below. This is in sharp contrast to canonical incomplete market general equilibrium models such as Aiyagari (1994), where agents can save using only a risk-free asset. In such models, the impossibility theorem of Stachurski and Toda (2019) implies that the tail behavior of income and wealth is the same, implying $\alpha = \alpha_Y$ in our setting.[10] Therefore, unlike canonical incomplete market models, our model can explain the empirical fact that capital income is more unequal than labor income. The key assumption leading to this conclusion is the presence of stochastic returns.

We discuss an analytically solvable example to build intuition.

Example 1. Let $\Delta > 0$ be the length of time of one period and the discount factor be $\beta = \mathrm{e}^{-\delta\Delta}$, where $\delta > 0$ is the discount rate. Suppose income grows at a constant rate $g > 0$, so $G = \mathrm{e}^{g\Delta}$. Suppose asset return is risk-free, so $R = \mathrm{e}^{r\Delta}$ with $r > 0$. Finally, let the survival probability be $v = \mathrm{e}^{-\eta\Delta}$, where η is the death rate. Then (3.5) becomes

$$1 = \mathrm{e}^{-\eta\Delta}\mathrm{e}^{zg\Delta} \iff z = \eta/g,$$

so the income Pareto exponent is $\alpha_Y = \eta/g$. (This is the classical result of Wold and Whittle (1957) in discrete-time.) Suppose in addition that $-\eta + r(1-\gamma) < 0$ so that $\beta R^{1-\gamma} < 1$. Since

$$H = (\mathrm{E}[\beta R^{1-\gamma}])^{1/\gamma}\tilde{R} = (\beta R)^{1/\gamma}/G = \mathrm{e}^{(\frac{r-\eta}{\gamma} - g)\Delta},$$

solving (3.7) the normalized wealth Pareto exponent is

$$\tilde{\alpha} = \frac{\eta\gamma}{r - \eta - g\gamma}$$

[10] The original proof in Stachurski and Toda (2019) contained an error; it has been corrected in Stachurski and Toda (2020).

assuming $r-\eta-g\gamma>0$. Therefore

$$\tilde{\alpha}<\alpha_Y \iff \frac{\eta\gamma}{r-\eta-g\gamma}<\frac{\eta}{g} \iff r>\eta+2g\gamma,$$

so the wealth (hence capital income) Pareto exponent is smaller than the labor income Pareto exponent if the return on wealth r is sufficiently large. In summary, we obtain the following result: suppose $-\eta+r(1-\gamma)<0$ and let $\alpha_{\mathrm{cap}},\alpha_{\mathrm{lab}}$ be the capital and labor income Pareto exponents. Then

$$\begin{cases}\alpha_{\mathrm{cap}}=\alpha_{\mathrm{lab}}=\frac{\eta}{g} & \text{if } r\le\eta+2g\gamma,\\ \alpha_{\mathrm{cap}}=\frac{\eta\gamma}{r-\eta-g\gamma}<\frac{\eta}{g}=\alpha_{\mathrm{lab}} & \text{if } r>\eta+2g\gamma.\end{cases} \tag{3.8}$$

Note that the labor income Pareto exponent $\alpha_{\mathrm{lab}}=\eta/g$ is highly sensitive to the income growth rate g. However, provided that $r>\eta+2g\gamma$, the capital income Pareto exponent α_{cap} is not very sensitive to the value of g because the denominator is $r-\eta-g\gamma$. This example is consistent with our result in Section 2.2 that the capital Pareto exponent is smaller than the labor Pareto exponent but the two values are only weakly related.

3.3 Numerical example

We further examine the tail behavior of income and wealth using a numerical example of the income fluctuation problem (3.1). Suppose that asset return is IID lognormal, so $\log R\sim N((\mu-\sigma^2/2)\Delta,\sigma^2\Delta)$, where $\Delta>0$ is the length of one period, μ is the expected return, and σ is volatility. Suppose every period the agent is "promoted" with some probability, so the income growth rate is

$$G_{t+1}=Y_{t+1}/Y_t=\begin{cases}1 & \text{with probability } 1-p,\\ \mathrm{e}^g & \text{with probability } p,\end{cases}$$

where $p\in(0,1)$ is the promotion probability and g is the log income growth rate conditional on promotion. We parametrize the promotion probability as $p=1-\mathrm{e}^{-\Delta/L}$, where L is the expected length of time until a promotion. Using (3.5), the labor income Pareto exponent is determined such that

$$1=v\,\mathrm{E}[G^{\alpha_Y}]=v(1-p+p\mathrm{e}^{g\alpha_Y}) \iff \alpha_Y=\frac{1}{g}\log\frac{1-v+vp}{vp}. \tag{3.9}$$

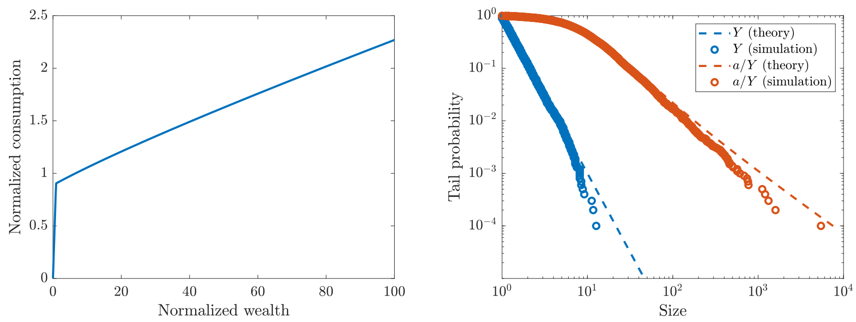
We set the parameter values as in Table 1. One unit of time corresponds to a year and one period is a quarter, so $\Delta=1/4$. The preference parameters (discount rate and risk aversion) are standard. The death rate of $\eta=0.025$ implies an average (economically active) age of $1/\eta=40$ years. The expected return and volatility roughly correspond to the stock market. We set the labor income Pareto exponent to $\alpha_Y=3$, which is roughly the median value in Figure 1. Using the survival probability $v=\mathrm{e}^{-\eta\Delta}$ and (3.9), the implied value of income growth upon promotion is $g=0.0403$. The wealth Pareto exponent determined by (3.7) is then $\alpha=1.201$.

To numerically solve the income fluctuation problem (3.1), we discretize the log asset return $\log R$ using a 7-point Gauss-Hermite quadrature and apply policy function iteration (see Appendix C). After solving the individual problem,

Table 1: Parameter values

Parameter	Symbol	Value
Length of one period	Δ	1/4
Discount rate	δ	0.04
Relative risk aversion	γ	2
Death rate	η	0.025
Expected return	μ	0.07
Volatility	σ	0.15
Expected time to promotion	L	5
Labor income Pareto exponent	α_Y	3

we apply the Pareto extrapolation algorithm developed in Gouin-Bonenfant and Toda (2018) to accurately compute the stationary (normalized) wealth distribution. Finally, we also simulate an economy with 10^4 agents. Figure 3 shows the results.



(a) Normalized consumption function.

(b) Income and wealth distributions.

Figure 3: Solution to income fluctuation problem.

Figure 3a shows the normalized consumption function $\tilde{c}(\tilde{a})$ in the range $\tilde{a} \in [0, 100]$. Consistent with Proposition 3, the consumption function is roughly linear for high asset level. Figure 3b shows the size distributions of income Y normalized wealth $\tilde{a} = a/Y$ in a log-log plot, both from the theoretical model and the simulation. The fact that the tail probability $\mathrm{P}(X > x)$ exhibits a straight line pattern in a log-log plot suggests that the size distributions have Pareto upper tails, consistent with theory. Furthermore, the slope for income is steeper than that of normalized wealth, so wealth (hence capital income) is more unequally distributed than labor income.

Finally, Figure 4 shows the income and wealth Pareto exponents when we change the income growth rate g in the range $g \in [0.02, 0.1]$, fixing other parameters. Because the income Pareto exponent is inversely proportional to income growth by (3.9), the labor income Pareto exponent is highly sensitive to income growth. On the other hand, the wealth (capital income) Pareto exponent does not depend much on income growth by the same intuition as in Example 1. Thus our model is consistent with our empirical findings in Section 2.2 that capital and labor income Pareto exponents are only weakly related.

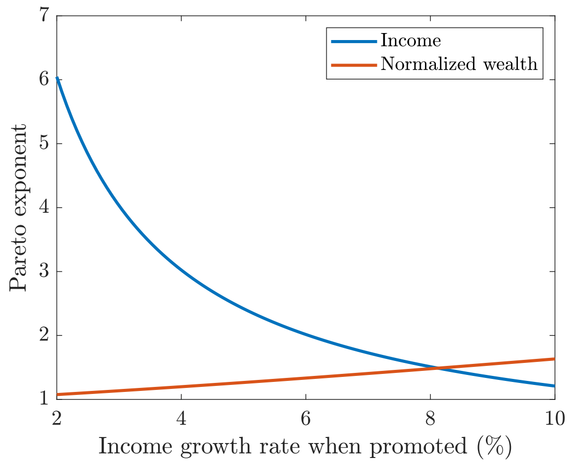


Figure 4: Dependence of income and wealth Pareto exponents on g.

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A Data

In this appendix we describe the dataset that we use and discuss its limitations.

A.1 The LIS database

We use the data from the *Luxembourg Income Study* (LIS), which is a large, harmonized database of micro-level income data that covers over 50 countries worldwide and many years since the late 1960s. In many countries, the data derive from government surveys (for example, the U.S. data is based on the *Current Population Survey*). The LIS data are available at both individual and household level. We focus on the household labor and capital income because (i) it is reasonable to assume that economic decisions such as financial planning are made at the household level, and (ii) incomes among couples are likely correlated due to assortative matching in the marriage market (Siow, 2015), which invalidates statistical estimation.[11] The LIS defines *labor income* as "cash payments and value of goods and services received from dependent employment, as well as profits/losses and value of goods from self-employment, including own consumption". *Capital income* is defined as "cash payments from property and capital (including financial and non-financial assets), including interest and dividends, rental income and royalties, and other capital income from investment in self-employment activity". Together these two categories make up total *factor income*. See the LIS 2019 USER GUIDE[12] for a detailed summary on how these data are retrieved and calculated.

A.2 Data limitations

Our analysis draws upon datasets from many different countries that are harmonized into a common framework by the LIS. However, many details about the collection of data in the different countries are omitted. For example, we find evidence of top-coding in some countries and years, as the largest income order statistic is equal to the second largest.[13] Top-coding induces an upward bias in the estimation of the Pareto exponent. This issue is not necessarily resolved if, instead, one relies on administrative tax income data, for similar biases arise such as rich households trying to understate their taxable income (Atkinson

[11] In our data, we find an average correlation of 0.22 between labor income of husband and wife, which underpins the conjectured dependency.

[12] https://www.lisdatacenter.org/wp-content/uploads/files/data-lis-guide.pdf

[13] Among all 428 country-year observations, the first and second order statistics are equal in 6 cases for labor income and 11 cases for capital income. Therefore we conjecture that the top-coding issue is not severe.

et al., 2011). Burkhauser et al. (2012) detail a method that can be used to overcome the bias due to top-coding, however at the end of their paper they show that the results are robust even if estimates are based on the top-coded series. For these reasons we treat the datasets as not being top-coded in our analysis.

Another limitation of the LIS database is that it is based on government surveys and the measurement error may be larger compared to administrative data based on tax returns. The fact that the income distribution in administrative data is often reported as tabulations, not micro data, causes no problem for estimating Pareto exponents, as Toda and Wang (2020) provide an efficient estimation method for such data. In fact, Atkinson and Piketty (2010, Table 13A.23) document income Pareto exponents across countries and years estimated from top income share data. However, their table is based on total income, and since (as we document in Section 2.3) the capital income Pareto exponents tend to be smaller than labor exponents, the estimates in Atkinson and Piketty (2010) are best understood as capital income (hence wealth) Pareto exponents. Since we are not aware of a comprehensive income database that distinguishes capital and labor income, we decided to use the LIS database.

B Proofs

Proof of Proposition 1. Applying Theorem 2.2 of Ma et al. (2020) to the IID case, a sufficient condition for the existence of a solution to the detrended problem is $\mathrm{E}[\tilde{\beta}] < 1$ and $\mathrm{E}[\tilde{\beta}\tilde{R}] < 1$, which is equivalent to (3.4). □

Proof of Proposition 3. The concavity of $\tilde{c}$ follows from Proposition 2.5 and Remark 2.1 of Ma et al. (2020). The asymptotic linearity of $\tilde{c}$ follows from Ma and Toda (2020, Theorem 2.2). Noting that

$$\mathrm{E}[\tilde{\beta}\tilde{R}^{1-\gamma}] = \mathrm{E}[\beta G^{1-\gamma}(R/G)^{1-\gamma}] = \mathrm{E}[\beta R^{1-\gamma}],$$

the limit (3.6) follows from their Example 2.2. □

Proof of Proposition 4. Since by Proposition 3 $\tilde{c}$ is concave, it is in particular Lipschitz continuous. Under the maintained assumptions, we can apply Theorem 1.8 of Mirek (2011) to deduce that the normalized wealth $\tilde{a}$ is either bounded or has a Pareto upper tail with exponent characterized as the solution to (3.7), where we have used the asymptotic linearity of $\tilde{c}$ established in Proposition 3.

By accounting, capital income (excluding capital loss) is

$$Y_{\text{cap}} := \max\{R-1, 0\}(a - c(a)) = \max\{R-1, 0\} Y(\tilde{a} - \tilde{c}(\tilde{a})).$$

Using Proposition 3, this quantity is approximately equal to $\rho \max\{R-1,0\} Y\tilde{a}$. The claim $\alpha = \min\{\tilde{\alpha}, \alpha_Y\}$ then follows because asset return R is thin-tailed and $a_t = Y_t\tilde{a}_t$ is the product of two (potentially dependent) random variables with Pareto upper tails, which inherits the smallest Pareto exponent by the result in Jessen and Mikosch (2006). □

C Solving the income fluctuation problem

In this appendix we discuss how to solve the detrended income fluctuation problem. After detrending, the problem becomes

$$\begin{aligned} &\text{maximize} && \mathrm{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=1}^{t} \beta_t \right) \frac{c_t^{1-\gamma}}{1-\gamma} \\ &\text{subject to} && a_{t+1} = R_{t+1}(a_t - c_t) + 1, \\ &&& 0 \le c_t \le a_t, \end{aligned}$$

where $\{R_t, \beta_t\}_{t=1}^{\infty}$ is IID (though R_t and β_t are generally correlated.) According to Ma et al. (2020), the Euler equation is

$$c_t^{-\gamma} = \max \left\{ \mathrm{E}_t[\beta_{t+1} R_{t+1} c_{t+1}^{-\gamma}], a_t^{-\gamma} \right\}. \tag{C.1}$$

Let $c(a)$ be the consumption function. Taking the $-1/\gamma$-th power of (C.1), we obtain

$$c(a) = \min \left\{ \left(\mathrm{E}[\beta R c(a')^{-\gamma}] \right)^{-1/\gamma}, a \right\}, \tag{C.2}$$

where $a' = R(a - c(a)) + 1$. Therefore we can compute the consumption function using the following variant of the policy function iteration algorithm:

1. Initialize the consumption function $c(a)$. For example, we can set $c(a) = \min\{a, 1 + ma\}$, where $m = \max\left\{1 - (\mathrm{E}[\beta R^{1-\gamma}])^{1/\gamma}, 0\right\}$ is the theoretical asymptotic marginal propensity to consume according to (3.6).

2. Update $c(a)$ by the right-hand side of (C.2), where $a' = R(a - c(a)) + 1$.

3. Iterate the above step until $c(a)$ converges.

While the above algorithm has no guarantee to converge unlike the "true" policy function iteration algorithm discussed in Ma et al. (2020), it has the advantage of avoiding root-finding and hence it is fast.

In Section 3, we use this algorithm on a 100-point exponential grid for normalized wealth $\tilde{a}$ that spans $[0, 10^4]$, with a median grid point of 10. The details on the exponential grid are discussed in Gouin-Bonenfant and Toda (2018).

D Tables

Table 2: Point estimates of income Pareto exponents across countries and years.

		Labor income				Capital income			
Country	Year	$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Australia	1981	5.31	0.23	528	10568	2.10	0.10	474	9488
Australia	1985	4.28	0.26	275	5492	1.41	0.09	262	5232
Australia	1989	3.97	0.17	531	10629	1.47	0.06	549	10973
Australia	1995	3.10	0.20	229	4574	1.71	0.13	168	3364
Australia	2001	3.06	0.20	226	4510	1.47	0.12	155	3105

Table 2: Point estimates of income Pareto exponents across countries and years.

		Labor income				Capital income			
Country	Year	$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Australia	2003	3.45	0.19	337	6741	1.52	0.10	224	4473
Australia	2004	3.44	0.17	395	7893	1.42	0.08	307	6145
Australia	2008	3.07	0.17	333	6666	1.28	0.08	237	4749
Australia	2010	3.39	0.15	547	10949	1.31	0.06	452	9030
Australia	2014	3.15	0.14	489	9788	1.20	0.05	478	9569
Austria	1994	3.38	0.32	109	2183	1.59	0.17	85	1693
Austria	1997	3.59	0.36	100	1998	1.76	0.19	89	1787
Austria	2000	3.17	0.36	79	1584	2.14	0.24	77	1547
Austria	2004	3.35	0.25	187	3749	0.96	0.07	203	4064
Austria	2007	3.24	0.23	205	4101	1.16	0.08	217	4334
Austria	2010	3.52	0.24	216	4323	1.00	0.06	238	4752
Austria	2013	3.37	0.24	201	4022	1.13	0.08	224	4471
Austria	2016	3.35	0.23	204	4082	1.12	0.08	191	3823
Belgium	1985	4.95	0.34	218	4357	–	–	–	–
Belgium	1988	5.36	0.48	126	2518	–	–	–	–
Belgium	1992	5.45	0.48	129	2581	–	–	–	–
Belgium	1995	3.49	0.37	90	1790	2.64	0.39	45	893
Belgium	1997	4.31	0.36	145	2906	1.62	0.11	206	4112
Belgium	2000	3.00	0.35	72	1431	1.41	0.23	36	715
Belgium	2004	3.39	0.27	156	3124	1.42	0.11	159	3185
Belgium	2007	3.36	0.24	202	4046	1.48	0.10	216	4328
Belgium	2010	4.07	0.30	186	3723	1.61	0.11	204	4072
Belgium	2013	4.22	0.31	183	3659	1.53	0.11	195	3908
Belgium	2016	3.81	0.28	183	3669	1.80	0.13	185	3703
Brazil	2006	1.99	0.03	4978	99559	1.58	0.10	239	4780
Brazil	2009	2.01	0.03	5000	100002	1.98	0.13	233	4667
Brazil	2011	1.97	0.03	4453	89066	1.76	0.15	142	2845
Brazil	2013	2.05	0.03	4583	91657	1.56	0.13	140	2804
Brazil	2016	2.06	0.03	5756	115110	1.88	0.11	315	6296
Canada	1971	3.90	0.12	1019	20386	1.66	0.07	498	9970
Canada	1975	3.89	0.12	1088	21750	1.85	0.08	536	10717
Canada	1981	4.51	0.18	617	12331	2.02	0.10	417	8342
Canada	1987	4.49	0.22	433	8667	1.90	0.12	260	5190
Canada	1991	4.39	0.16	763	15257	2.00	0.09	492	9847
Canada	1994	4.18	0.11	1408	28155	1.76	0.07	704	14077
Canada	1997	3.96	0.11	1251	25018	1.79	0.08	564	11273
Canada	1998	3.50	0.10	1198	23969	1.47	0.06	674	13486
Canada	2000	3.30	0.10	1115	22298	1.47	0.06	663	13264
Canada	2004	3.34	0.10	1080	21594	1.56	0.06	593	11861
Canada	2007	3.06	0.09	1046	20918	1.50	0.06	627	12533
Canada	2010	3.36	0.11	975	19500	1.40	0.06	517	10341
Canada	2012	3.52	0.11	953	19053	1.65	0.07	518	10370
Canada	2013	3.84	0.13	903	18069	1.64	0.07	487	9741
Canada	2014	3.41	0.11	924	18474	1.74	0.08	492	9836
Canada	2015	3.42	0.11	989	19785	1.49	0.06	528	10552

Table 2: Point estimates of income Pareto exponents across countries and years.

		Labor income				Capital income			
Country	Year	$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Canada	2016	3.47	0.11	1026	20526	1.64	0.07	532	10641
Canada	2017	3.68	0.09	1516	30313	1.52	0.05	799	15982
Chile	1990	1.95	0.06	1138	22755	1.62	0.21	62	1242
Chile	1992	1.72	0.04	1597	31934	1.78	0.19	84	1678
Chile	1994	1.99	0.04	1993	39851	1.91	0.19	104	2071
Chile	1996	2.15	0.06	1509	30177	1.53	0.12	172	3437
Chile	1998	1.99	0.04	2145	42903	1.64	0.13	167	3335
Chile	2000	1.99	0.04	2886	57716	1.38	0.09	222	4432
Chile	2003	1.89	0.03	3003	60055	1.48	0.10	232	4637
Chile	2006	2.09	0.04	3256	65111	1.38	0.08	269	5374
Chile	2009	2.18	0.04	2983	59668	1.48	0.11	181	3619
Chile	2011	2.28	0.05	2544	50887	1.82	0.14	166	3312
Chile	2013	2.21	0.04	2854	57079	1.76	0.13	182	3635
Chile	2015	2.15	0.04	3574	71477	1.71	0.11	266	5314
Chile	2017	2.18	0.04	2986	59724	1.56	0.10	241	4829
China	2002	3.16	0.11	837	16745	1.72	0.17	108	2156
China	2013	2.99	0.11	794	15878	2.06	0.09	505	10098
Colombia	2004	2.13	0.10	413	8255	1.95	0.25	60	1196
Colombia	2007	2.09	0.02	9736	194723	1.69	0.05	1189	23787
Colombia	2010	2.24	0.02	9944	198885	1.63	0.05	1247	24944
Colombia	2013	2.37	0.02	10039	200779	1.75	0.05	1283	25654
Colombia	2016	2.44	0.02	10111	202226	1.63	0.05	1199	23979
Czech Rep	1992	3.84	0.15	664	13287	–	–	–	–
Czech Rep	1996	3.38	0.10	1179	23587	1.14	0.09	154	3077
Czech Rep	2002	3.31	0.19	304	6082	1.50	0.21	49	973
Czech Rep	2004	3.19	0.26	156	3125	0.91	0.15	36	729
Czech Rep	2007	3.49	0.17	413	8254	1.11	0.12	87	1735
Czech Rep	2010	3.74	0.21	328	6551	1.23	0.15	71	1422
Czech Rep	2013	3.45	0.20	300	5996	1.59	0.21	60	1198
Czech Rep	2016	3.97	0.22	326	6526	1.25	0.17	54	1083
Denmark	1987	4.48	0.21	460	9199	1.53	0.06	579	11575
Denmark	1992	4.59	0.21	458	9164	1.42	0.06	593	11853
Denmark	1995	4.68	0.09	2865	57306	1.24	0.02	3510	70200
Denmark	2000	4.26	0.08	2977	59549	1.18	0.02	3486	69714
Denmark	2004	4.04	0.07	2991	59824	1.13	0.02	3394	67886
Denmark	2007	3.82	0.07	3063	61255	1.12	0.02	3938	78755
Denmark	2010	3.37	0.06	2974	59488	1.14	0.02	3464	69287
Denmark	2013	3.47	0.06	3044	60880	1.09	0.02	3404	68088
Denmark	2016	3.42	0.06	3098	61970	1.23	0.02	2949	58989
Dominican Rep.	2007	2.12	0.11	374	7471	1.41	0.27	27	544
Egypt	2012	1.97	0.09	505	10095	1.09	0.18	36	727
Estonia	2000	3.06	0.19	260	5196	–	–	–	–
Estonia	2004	3.23	0.26	156	3116	–	–	–	–
Estonia	2007	4.12	0.28	210	4201	0.68	0.06	112	2250
Estonia	2010	3.85	0.26	212	4241	0.91	0.09	97	1948

Table 2: Point estimates of income Pareto exponents across countries and years.

		Labor income				Capital income			
Country	Year	$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Estonia	2013	3.44	0.22	248	4951	1.22	0.14	72	1450
Finland	1987	4.37	0.18	561	11225	1.45	0.07	395	7906
Finland	1991	4.52	0.20	519	10380	1.43	0.07	417	8332
Finland	1995	4.42	0.22	393	7867	1.25	0.09	212	4236
Finland	2000	3.52	0.17	449	8977	1.10	0.06	296	5923
Finland	2004	4.07	0.19	468	9368	1.02	0.05	403	8061
Finland	2007	3.79	0.18	433	8664	1.01	0.05	425	8502
Finland	2010	3.92	0.20	377	7537	1.06	0.05	379	7571
Finland	2013	3.93	0.19	445	8899	1.04	0.05	457	9145
Finland	2016	3.72	0.18	412	8237	1.19	0.06	416	8325
France	1978	2.93	0.15	383	7664	1.75	0.21	71	1426
France	1984	1.96	0.10	394	7875	1.75	0.18	99	1974
France	1989	3.28	0.19	297	5942	1.68	0.18	84	1677
France	1994	3.13	0.16	394	7880	1.68	0.08	437	8734
France	2000	3.52	0.19	356	7112	1.58	0.08	426	8530
France	2005	3.34	0.17	399	7987	1.51	0.07	416	8330
France	2010	3.03	0.12	591	11818	1.49	0.06	583	11658
Germany	1973	3.56	0.08	1881	37614	2.16	0.05	2245	44891
Germany	1978	3.36	0.08	1787	35731	2.19	0.05	2233	44654
Germany	1981	4.66	0.46	101	2024	–	–	–	–
Germany	1983	4.54	0.11	1614	32288	2.26	0.05	2006	40130
Germany	1984	3.97	0.27	212	4240	1.04	0.07	213	4259
Germany	1987	4.24	0.31	193	3863	1.18	0.08	202	4050
Germany	1989	3.31	0.24	187	3733	1.16	0.08	197	3940
Germany	1991	4.79	0.29	272	5448	1.09	0.06	289	5789
Germany	1994	4.54	0.28	269	5384	1.34	0.08	288	5758
Germany	1995	4.66	0.29	264	5275	1.36	0.08	290	5802
Germany	1998	4.56	0.27	277	5531	1.36	0.08	307	6136
Germany	2000	3.92	0.19	439	8784	1.48	0.07	491	9829
Germany	2001	3.46	0.16	464	9273	1.26	0.06	509	10174
Germany	2002	3.57	0.17	442	8850	1.38	0.06	490	9798
Germany	2003	3.64	0.18	430	8600	1.25	0.06	494	9875
Germany	2004	3.67	0.18	415	8300	1.24	0.06	477	9544
Germany	2005	3.31	0.16	443	8869	1.24	0.05	517	10338
Germany	2006	3.29	0.16	414	8276	1.33	0.06	478	9562
Germany	2007	3.45	0.18	389	7778	1.32	0.06	453	9055
Germany	2008	3.24	0.17	365	7295	1.26	0.06	425	8497
Germany	2009	3.43	0.15	531	10611	1.36	0.06	555	11102
Germany	2010	3.45	0.14	626	12512	1.30	0.05	646	12913
Germany	2011	3.30	0.13	615	12291	1.29	0.05	633	12664
Germany	2012	3.40	0.13	696	13912	1.31	0.05	653	13051
Germany	2013	3.40	0.14	610	12199	1.32	0.05	591	11824
Germany	2014	3.85	0.16	610	12193	1.31	0.06	559	11176
Germany	2015	3.13	0.13	550	11000	1.31	0.06	531	10625
Germany	2016	3.43	0.14	611	12215	1.27	0.05	560	11203

Table 2: Point estimates of income Pareto exponents across countries and years.

		Labor income				Capital income			
Country	Year	$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Georgia	2010	2.76	0.18	243	4867	–	–	–	–
Georgia	2013	3.14	0.28	126	2512	–	–	–	–
Georgia	2016	2.93	0.27	120	2390	–	–	–	–
Greece	1995	3.68	0.28	169	3375	2.83	0.39	52	1033
Greece	2000	3.56	0.31	129	2580	2.20	0.37	36	724
Greece	2004	3.03	0.21	199	3987	2.12	0.30	50	1006
Greece	2007	2.86	0.19	227	4533	2.00	0.25	63	1253
Greece	2010	2.64	0.19	193	3858	2.12	0.28	58	1170
Greece	2013	2.73	0.17	251	5015	2.24	0.26	76	1517
Greece	2016	3.05	0.12	643	12867	2.43	0.19	165	3299
Guatemala	2006	2.01	0.08	672	13448	1.79	0.31	33	668
Guatemala	2011	2.20	0.09	631	12622	–	–	–	–
Guatemala	2014	2.28	0.10	560	11203	–	–	–	–
Hungary	1991	3.23	0.35	85	1693	–	–	–	–
Hungary	1994	3.34	0.38	77	1532	–	–	–	–
Hungary	1999	3.04	0.35	77	1531	1.98	0.35	33	666
Hungary	2005	3.41	0.41	69	1384	–	–	–	–
Hungary	2007	3.29	0.40	66	1328	–	–	–	–
Hungary	2009	3.59	0.46	62	1250	–	–	–	–
Hungary	2012	2.91	0.36	67	1343	–	–	–	–
Hungary	2015	3.18	0.33	90	1809	–	–	–	–
Iceland	2004	4.76	0.41	132	2632	1.17	0.12	92	1836
Iceland	2007	4.34	0.38	129	2583	1.17	0.11	106	2112
Iceland	2010	4.54	0.39	134	2675	1.14	0.09	150	2994
India	2004	2.35	0.05	1992	39832	2.47	0.25	94	1874
India	2011	2.12	0.05	1998	39958	1.66	0.17	101	2011
Ireland	1987	3.46	0.31	124	2482	1.33	0.13	100	1998
Ireland	1994	2.93	0.27	121	2426	1.33	0.18	56	1113
Ireland	1995	2.65	0.26	107	2146	1.34	0.19	51	1025
Ireland	2000	3.23	0.33	95	1893	1.95	0.27	51	1017
Ireland	2002	3.96	0.40	100	2000	1.61	0.26	38	752
Ireland	2003	3.03	0.23	174	3471	1.74	0.25	48	952
Ireland	2004	3.00	0.22	187	3731	1.59	0.25	40	804
Ireland	2005	2.62	0.20	176	3522	1.83	0.25	52	1049
Ireland	2006	2.49	0.19	171	3411	1.49	0.15	99	1984
Ireland	2007	2.91	0.23	157	3143	1.48	0.17	75	1500
Ireland	2008	2.73	0.22	153	3069	1.39	0.20	49	986
Ireland	2009	3.27	0.28	134	2675	2.13	0.34	39	771
Ireland	2010	3.38	0.30	126	2520	1.81	0.30	37	745
Ireland	2011	3.78	0.32	136	2720	1.74	0.28	38	768
Ireland	2012	3.35	0.28	145	2894	1.82	0.28	41	816
Ireland	2013	3.20	0.25	164	3288	2.02	0.30	46	918
Ireland	2014	3.06	0.24	164	3285	1.63	0.24	46	921
Ireland	2015	2.91	0.23	158	3159	1.57	0.23	46	914
Ireland	2016	2.76	0.22	161	3214	1.55	0.23	44	872

Table 2: Point estimates of income Pareto exponents across countries and years.

		Labor income				Capital income			
Country	Year	$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Ireland	2017	2.46	0.21	142	2835	2.02	0.33	38	753
Ireland	1996	2.95	0.29	101	2021	1.56	0.22	51	1022
Israel	1979	3.95	0.40	97	1941	–	–	–	–
Israel	1986	3.25	0.23	197	3942	–	–	–	–
Israel	1992	3.75	0.26	203	4057	–	–	–	–
Israel	1997	3.01	0.21	204	4072	–	–	–	–
Israel	2001	3.13	0.21	219	4382	1.11	0.21	28	556
Israel	2005	3.43	0.22	239	4789	1.29	0.15	74	1475
Israel	2007	3.46	0.23	234	4682	1.67	0.24	48	970
Israel	2010	2.96	0.19	237	4746	1.51	0.22	49	980
Israel	2012	3.54	0.19	343	6854	1.33	0.16	65	1305
Israel	2014	3.24	0.18	330	6607	1.69	0.21	66	1330
Israel	2016	3.45	0.18	348	6968	2.16	0.28	61	1215
Italy	1986	3.50	0.20	301	6016	1.85	0.14	174	3474
Italy	1987	3.55	0.20	302	6045	1.61	0.09	342	6844
Italy	1989	4.15	0.24	308	6154	1.57	0.09	293	5865
Italy	1991	4.43	0.26	291	5827	1.60	0.09	329	6587
Italy	1993	3.76	0.23	272	5444	1.42	0.08	336	6724
Italy	1995	3.24	0.20	274	5487	1.52	0.08	341	6811
Italy	1998	3.37	0.22	244	4876	1.49	0.08	310	6191
Italy	2000	3.10	0.19	264	5279	1.29	0.07	326	6516
Italy	2004	2.39	0.15	246	4913	1.31	0.07	332	6641
Italy	2008	3.19	0.21	239	4778	1.41	0.08	328	6552
Italy	2010	3.35	0.22	236	4719	1.33	0.07	324	6490
Italy	2014	2.76	0.19	220	4402	1.41	0.08	337	6748
Italy	2016	2.69	0.19	193	3853	1.06	0.06	310	6197
Ivory Coast	2002	1.83	0.08	491	9811	1.36	0.24	31	618
Ivory Coast	2008	1.76	0.07	580	11594	1.09	0.20	31	621
Ivory Coast	2015	1.90	0.08	565	11293	1.18	0.21	33	664
Japan	2008	3.90	0.31	163	3256	1.77	0.32	30	598
Japan	2010	4.40	0.39	127	2542	1.51	0.29	27	543
Japan	2013	4.22	0.43	95	1894	–	–	–	–
Lithuania	2009	3.42	0.24	201	4026	–	–	–	–
Lithuania	2010	4.72	0.35	184	3680	1.40	0.27	28	564
Lithuania	2011	4.36	0.31	198	3951	1.22	0.23	28	568
Lithuania	2012	3.15	0.23	187	3746	1.17	0.23	25	507
Lithuania	2013	3.61	0.26	190	3792	1.36	0.27	26	523
Lithuania	2014	2.82	0.21	176	3510	0.95	0.14	48	961
Lithuania	2015	3.13	0.24	176	3526	0.84	0.13	45	909
Lithuania	2016	3.81	0.28	184	3683	0.97	0.15	42	837
Lithuania	2017	3.25	0.24	188	3750	1.20	0.19	42	841
Luxembourg	1985	6.01	0.68	78	1552	–	–	–	–
Luxembourg	1991	5.10	0.58	77	1535	1.49	0.25	36	716
Luxembourg	1994	4.75	0.58	68	1362	3.11	0.60	27	531
Luxembourg	1997	5.23	0.54	95	1906	1.59	0.25	39	779

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income				Capital income			
		$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Luxembourg	2000	4.37	0.46	92	1840	1.55	0.23	45	892
Luxembourg	2004	4.09	0.35	138	2753	2.91	0.44	44	887
Luxembourg	2007	3.02	0.25	151	3028	1.68	0.17	103	2066
Luxembourg	2010	3.37	0.23	206	4111	1.71	0.13	175	3494
Luxembourg	2013	3.59	0.30	141	2828	1.60	0.14	127	2534
Mexico	1984	2.67	0.18	223	4461	–	–	–	–
Mexico	1989	2.25	0.10	544	10876	1.43	0.26	30	602
Mexico	1992	2.12	0.09	498	9959	–	–	–	–
Mexico	1994	2.12	0.09	603	12068	–	–	–	–
Mexico	1996	2.23	0.09	656	13128	1.62	0.32	25	505
Mexico	1998	2.23	0.10	505	10092	–	–	–	–
Mexico	2000	2.25	0.10	463	9262	–	–	–	–
Mexico	2004	2.32	0.07	1046	20928	1.58	0.22	53	1055
Mexico	2008	2.24	0.06	1359	27171	1.43	0.16	82	1635
Mexico	2010	2.33	0.07	1252	25038	1.66	0.20	66	1323
Mexico	2012	2.33	0.11	412	8247	–	–	–	–
Mexico	2014	2.38	0.08	894	17887	1.64	0.25	44	882
Mexico	2016	2.46	0.04	3227	64534	1.53	0.12	159	3186
Mexico	2018	2.52	0.04	3406	68114	1.60	0.12	169	3389
Mexico	2002	2.54	0.09	795	15905	2.13	0.40	29	587
Netherlands	1983	5.16	0.42	154	3090	–	–	–	–
Netherlands	1987	4.14	0.35	140	2792	–	–	–	–
Netherlands	1990	3.79	0.31	149	2971	1.38	0.12	128	2570
Netherlands	1993	4.30	0.32	178	3565	1.23	0.11	121	2423
Netherlands	1999	4.34	0.35	153	3061	1.32	0.13	97	1942
Netherlands	2004	3.43	0.18	358	7166	1.39	0.07	409	8171
Netherlands	2007	3.17	0.16	401	8011	0.97	0.05	457	9136
Netherlands	2010	3.65	0.18	400	8001	1.29	0.06	477	9539
Netherlands	2013	3.62	0.18	390	7808	1.30	0.06	474	9486
Norway	1979	1.73	0.08	414	8276	1.73	0.10	302	6048
Norway	1986	5.05	0.35	213	4254	1.91	0.17	129	2588
Norway	1991	3.83	0.20	368	7361	1.76	0.09	372	7448
Norway	1995	3.46	0.17	401	8027	1.12	0.05	490	9800
Norway	2000	3.19	0.13	574	11474	0.88	0.03	640	12804
Norway	2004	3.98	0.17	547	10947	0.74	0.03	649	12979
Norway	2007	3.74	0.04	8485	169708	1.09	0.01	10768	215363
Norway	2010	3.55	0.04	8861	177229	0.88	0.01	11279	225584
Norway	2013	3.35	0.04	9157	183146	0.85	0.01	11660	233192
Palestine	2017	2.82	0.22	161	3214	1.43	0.20	50	1000
Panama	2007	2.17	0.09	579	11571	1.54	0.29	28	566
Panama	2010	2.24	0.09	588	11754	1.00	0.20	26	525
Panama	2013	2.22	0.10	528	10551	–	–	–	–
Panama	2016	2.40	0.11	511	10229	–	–	–	–
Paraguay	2000	1.87	0.10	384	7685	1.71	0.32	29	585
Paraguay	2004	1.89	0.10	372	7434	–	–	–	–

Table 2: Point estimates of income Pareto exponents across countries and years.

		Labor income				Capital income			
Country	Year	$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Paraguay	2007	1.66	0.11	226	4528	–	–	–	–
Paraguay	2010	1.77	0.12	235	4697	–	–	–	–
Paraguay	2013	2.04	0.13	254	5073	–	–	–	–
Paraguay	2016	1.87	0.09	474	9470	–	–	–	–
Peru	2004	2.28	0.08	864	17289	1.87	0.18	110	2193
Peru	2007	2.29	0.07	1023	20464	1.68	0.15	129	2571
Peru	2010	2.49	0.08	986	19720	1.68	0.13	157	3137
Peru	2013	2.53	0.07	1384	27670	1.64	0.12	191	3827
Peru	2016	2.63	0.07	1633	32651	1.87	0.12	232	4649
Poland	1986	3.79	0.18	461	9213	–	–	–	–
Poland	1992	3.55	0.21	297	5944	–	–	–	–
Poland	1995	2.67	0.08	1055	21091	–	–	–	–
Poland	1999	3.31	0.10	1211	24216	–	–	–	–
Poland	2004	2.92	0.09	1178	23557	–	–	–	–
Poland	2007	2.84	0.08	1394	27872	–	–	–	–
Poland	2010	3.03	0.08	1382	27640	–	–	–	–
Poland	2013	2.91	0.08	1324	26475	–	–	–	–
Poland	2016	3.16	0.09	1299	25983	–	–	–	–
Romania	1995	3.55	0.09	1576	31519	–	–	–	–
Romania	1997	3.51	0.09	1606	32122	–	–	–	–
Russia	2000	2.39	0.21	132	2631	–	–	–	–
Russia	2004	2.91	0.25	135	2705	–	–	–	–
Russia	2007	3.69	0.30	149	2986	–	–	–	–
Russia	2010	3.21	0.20	251	5025	–	–	–	–
Russia	2011	3.40	0.16	435	8697	1.65	0.32	26	513
Russia	2013	3.38	0.08	1915	38297	1.67	0.11	218	4360
Russia	2014	3.49	0.08	1922	38433	1.79	0.12	221	4412
Russia	2015	3.65	0.07	2539	50779	1.65	0.10	288	5752
Russia	2016	3.47	0.04	6739	134781	1.91	0.07	786	15716
Russia	2017	3.71	0.07	2539	50774	2.42	0.14	284	5671
Serbia	2006	3.74	0.28	175	3491	–	–	–	–
Serbia	2010	3.84	0.30	164	3270	–	–	–	–
Serbia	2013	2.48	0.20	160	3201	–	–	–	–
Serbia	2016	3.42	0.23	228	4566	–	–	–	–
Slovakia	1992	4.31	0.18	572	11439	–	–	–	–
Slovakia	1996	4.65	0.18	659	13176	1.45	0.25	35	709
Slovakia	2004	3.14	0.22	199	3987	–	–	–	–
Slovakia	2007	4.25	0.28	227	4538	1.60	0.28	33	653
Slovakia	2010	3.83	0.26	219	4372	0.98	0.14	51	1024
Slovakia	2013	3.37	0.22	226	4527	0.84	0.10	64	1289
Slovenia	1997	5.24	0.48	119	2378	–	–	–	–
Slovenia	1999	4.91	0.37	175	3492	1.54	0.28	30	601
Slovenia	2004	5.27	0.41	164	3275	2.59	0.51	26	521
Slovenia	2007	4.78	0.38	161	3219	1.74	0.35	25	501
Slovenia	2010	4.90	0.37	173	3468	–	–	–	–

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income $\widehat{\alpha}$	s.e.	k	N	Capital income $\widehat{\alpha}$	s.e.	k	N
Slovenia	2012	4.99	0.39	163	3252	1.48	0.15	92	1841
Slovenia	2015	5.07	0.39	170	3399	1.12	0.12	84	1673
South Africa	2008	1.76	0.11	251	5011	–	–	–	–
South Africa	2010	1.89	0.14	194	3878	–	–	–	–
South Africa	2012	2.08	0.13	258	5150	–	–	–	–
South Africa	2015	2.20	0.12	336	6715	–	–	–	–
South Africa	2017	2.27	0.12	364	7281	–	–	–	–
South Korea	2006	4.11	0.15	720	14390	1.69	0.11	218	4353
South Korea	2008	3.91	0.16	627	12542	1.79	0.13	181	3616
South Korea	2010	4.15	0.17	614	12276	1.62	0.13	150	3003
South Korea	2012	4.09	0.17	596	11911	1.58	0.12	166	3329
Spain	1980	3.86	0.12	983	19651	2.07	0.24	74	1470
Spain	1985	3.76	0.32	135	2707	–	–	–	–
Spain	1990	4.23	0.15	809	16178	1.58	0.11	202	4036
Spain	1995	3.91	0.27	204	4088	1.60	0.15	112	2249
Spain	2000	3.21	0.25	169	3379	1.09	0.08	210	4206
Spain	2004	4.07	0.19	459	9175	1.32	0.09	222	4433
Spain	2007	4.37	0.20	485	9701	1.44	0.09	266	5327
Spain	2010	5.14	0.24	441	8822	1.83	0.15	152	3043
Spain	2013	3.56	0.17	441	8823	1.70	0.09	334	6681
Spain	2016	3.33	0.15	508	10165	1.52	0.08	373	7451
Sweden	1967	3.16	0.20	246	4912	1.64	0.23	49	988
Sweden	1975	3.93	0.18	477	9535	1.55	0.08	392	7845
Sweden	1981	4.45	0.21	432	8635	2.47	0.14	305	6092
Sweden	1987	4.57	0.22	433	8658	1.79	0.09	398	7953
Sweden	1992	4.50	0.19	541	10824	1.96	0.08	561	11215
Sweden	1995	3.98	0.16	583	11655	1.61	0.06	650	12993
Sweden	2000	3.62	0.16	516	10319	1.23	0.05	564	11273
Sweden	2005	3.16	0.13	598	11951	1.08	0.04	606	12120
Switzerland	1982	2.16	0.13	278	5570	1.34	0.09	212	4233
Switzerland	1992	3.56	0.24	224	4486	1.33	0.09	242	4846
Switzerland	2000	3.53	0.28	157	3140	1.41	0.12	132	2646
Switzerland	2002	3.84	0.31	154	3074	1.56	0.14	129	2578
Switzerland	2004	4.32	0.37	135	2697	1.38	0.12	132	2648
Switzerland	2006	3.11	0.19	266	5329	1.22	0.08	210	4206
Switzerland	2007	2.80	0.17	271	5427	1.23	0.08	253	5057
Switzerland	2008	3.00	0.18	291	5822	1.11	0.07	250	5004
Switzerland	2009	2.95	0.17	290	5805	1.23	0.07	270	5396
Switzerland	2010	2.92	0.17	288	5759	1.09	0.06	337	6749
Switzerland	2011	3.21	0.19	286	5714	1.24	0.07	337	6746
Switzerland	2012	3.33	0.20	274	5471	1.26	0.07	329	6586
Switzerland	2013	3.48	0.22	255	5097	1.36	0.08	310	6190
Switzerland	2014	3.12	0.19	284	5672	1.28	0.07	328	6562
Switzerland	2015	3.18	0.18	297	5947	1.36	0.07	340	6794
Switzerland	2016	3.09	0.17	316	6310	1.38	0.07	353	7068

Table 2: Point estimates of income Pareto exponents across countries and years.

		Labor income				Capital income			
Country	Year	$\widehat{\alpha}$	s.e.	k	N	$\widehat{\alpha}$	s.e.	k	N
Switzerland	2017	3.20	0.20	252	5048	1.37	0.08	278	5568
Taiwan	1981	4.20	0.15	750	14997	1.53	0.07	532	10635
Taiwan	1986	3.96	0.14	796	15911	1.35	0.05	643	12864
Taiwan	1991	4.10	0.15	788	15756	1.41	0.05	749	14978
Taiwan	1995	4.30	0.16	681	13619	1.76	0.07	709	14184
Taiwan	1997	4.27	0.17	629	12572	1.78	0.07	666	13311
Taiwan	2000	4.20	0.17	615	12301	1.75	0.07	670	13396
Taiwan	2005	4.17	0.17	595	11903	1.77	0.07	656	13127
Taiwan	2007	3.82	0.16	602	12042	1.69	0.07	664	13290
Taiwan	2010	3.85	0.15	637	12748	1.70	0.06	714	14287
Taiwan	2013	3.85	0.15	665	13300	1.56	0.06	781	15615
Taiwan	2016	3.88	0.15	682	13648	1.59	0.06	822	16449
UK	1969	3.92	0.23	283	5669	1.12	0.08	205	4102
UK	1974	3.62	0.22	259	5181	1.35	0.10	177	3547
UK	1979	4.93	0.31	247	4937	1.34	0.09	214	4280
UK	1986	4.49	0.30	226	4515	1.67	0.11	247	4942
UK	1991	3.58	0.24	222	4445	1.66	0.10	259	5174
UK	1994	3.00	0.11	788	15751	1.42	0.05	928	18563
UK	1995	3.23	0.23	205	4091	1.45	0.10	229	4584
UK	1999	2.89	0.10	760	15200	1.37	0.05	861	17223
UK	2004	2.71	0.09	851	17026	1.25	0.04	927	18531
UK	2007	2.88	0.10	764	15272	1.33	0.05	819	16371
UK	2010	2.67	0.10	762	15237	1.34	0.06	492	9839
UK	2013	2.75	0.11	613	12258	1.50	0.08	386	7714
UK	2016	2.55	0.11	588	11768	1.33	0.07	367	7340
US	1974	3.81	0.18	468	9366	1.51	0.09	278	5558
US	1979	3.64	0.07	2639	52783	1.46	0.03	2237	44749
US	1986	3.47	0.07	2283	45660	1.65	0.04	1914	38274
US	1991	3.40	0.07	2310	46195	1.54	0.04	1944	38871
US	1992	3.20	0.07	2296	45921	1.49	0.03	1907	38137
US	1993	2.57	0.05	2211	44224	1.46	0.03	1863	37252
US	1994	2.48	0.05	2213	44261	1.45	0.03	1868	37355
US	1995	2.52	0.06	1948	38958	1.48	0.04	1604	32076
US	1996	2.47	0.06	1978	39567	1.51	0.04	1558	31165
US	1997	2.40	0.05	1991	39812	1.47	0.04	1546	30913
US	1998	2.47	0.06	2013	40267	1.46	0.04	1557	31144
US	1999	2.43	0.05	2044	40886	1.44	0.04	1566	31320
US	2000	2.38	0.04	3229	64575	1.40	0.03	2348	46961
US	2001	2.35	0.04	3216	64322	1.31	0.03	2308	46168
US	2002	2.49	0.04	3200	64000	1.35	0.03	2198	43963
US	2003	2.43	0.04	3147	62939	1.37	0.03	2170	43410
US	2004	2.45	0.04	3117	62334	1.33	0.03	2118	42354
US	2005	2.31	0.04	3095	61899	1.42	0.03	2081	41624
US	2006	2.38	0.04	3085	61707	1.46	0.03	2013	40264
US	2007	2.56	0.05	3100	62002	1.49	0.03	1966	39312

Table 2: Point estimates of income Pareto exponents across countries and years.

Country	Year	Labor income $\widehat{\alpha}$	s.e.	k	N	Capital income $\widehat{\alpha}$	s.e.	k	N
US	2008	2.48	0.04	3103	62055	1.33	0.03	2026	40523
US	2009	2.48	0.04	3064	61288	1.33	0.03	1922	38446
US	2010	2.62	0.05	2997	59932	1.41	0.03	1869	37383
US	2011	2.42	0.04	2946	58921	1.40	0.03	1836	36728
US	2012	2.47	0.05	2970	59404	1.56	0.04	1843	36865
US	2013	2.50	0.06	2036	40721	1.41	0.04	1225	24494
US	2014	2.41	0.04	2905	58099	1.41	0.03	2316	46312
US	2015	2.29	0.04	2726	54510	1.36	0.03	2213	44260
US	2016	2.28	0.04	2736	54727	1.42	0.03	2270	45401
US	2017	2.39	0.05	2641	52816	1.45	0.03	2255	45102
US	2018	2.41	0.05	2658	53170	1.34	0.03	2265	45307
Uruguay	2004	2.31	0.09	684	13671	1.82	0.23	62	1241
Uruguay	2007	2.44	0.06	1855	37097	1.53	0.11	212	4236
Uruguay	2010	2.59	0.06	1771	35428	1.64	0.11	204	4085
Uruguay	2013	3.00	0.07	1763	35264	1.80	0.14	175	3497
Uruguay	2016	2.76	0.07	1674	33484	1.69	0.13	182	3632
Vietnam	2011	2.77	0.13	464	9282	1.26	0.15	68	1362
Vietnam	2013	2.97	0.14	464	9282	1.77	0.21	69	1384

Note: *$\widehat{\alpha}$: point estimate of Pareto exponent; s.e.: standard error ($\widehat{\alpha}(k)/\sqrt{k}$); k: the number of order statistics used for the Hill-estimator; N: the number of positive labor (resp. capital) income observations. "–" indicates fewer than 500 positive observations so estimates are omitted.*

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^+_{cap}	Reject H_0?
Australia	1981	5.26	0.24	2.10	0.10	9488	Yes
Australia	1985	4.22	0.26	1.40	0.09	5232	Yes
Australia	1989	3.95	0.17	1.46	0.06	10629	Yes
Australia	1995	2.91	0.22	1.70	0.13	3364	Yes
Australia	2001	3.24	0.26	1.45	0.12	3105	Yes
Australia	2003	3.25	0.22	1.51	0.10	4473	Yes
Australia	2004	3.38	0.19	1.41	0.08	6145	Yes
Australia	2008	3.30	0.21	1.27	0.08	4749	Yes
Australia	2010	3.31	0.16	1.29	0.06	9093	Yes
Australia	2014	3.14	0.14	1.19	0.05	9616	Yes
Austria	1994	3.12	0.34	1.59	0.17	1695	Yes
Austria	1997	3.55	0.37	1.78	0.19	1800	No
Austria	2000	3.10	0.35	2.10	0.24	1584	No
Austria	2004	3.34	0.24	0.99	0.07	3750	Yes
Austria	2007	3.23	0.23	1.17	0.08	4101	Yes
Austria	2010	3.51	0.24	1.05	0.07	4323	Yes
Austria	2013	3.36	0.24	1.12	0.08	4022	Yes
Austria	2016	3.31	0.24	1.11	0.08	3835	Yes
Belgium	1995	2.86	0.43	2.58	0.38	905	No

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^{+}_{cap}	Reject H_0?
Belgium	1997	4.25	0.35	1.82	0.15	2906	Yes
Belgium	2000	2.83	0.40	1.68	0.24	997	No
Belgium	2004	3.39	0.27	1.41	0.11	3185	Yes
Belgium	2007	3.35	0.24	1.48	0.10	4046	Yes
Belgium	2010	4.05	0.30	1.70	0.12	3723	Yes
Belgium	2013	4.18	0.31	1.64	0.12	3659	Yes
Belgium	2016	3.77	0.28	1.81	0.13	3669	Yes
Brazil	2006	3.32	0.21	1.53	0.10	4804	No
Brazil	2009	3.19	0.21	1.98	0.13	4724	No
Brazil	2011	2.89	0.24	1.70	0.14	2897	No
Brazil	2013	2.94	0.25	1.48	0.12	2869	Yes
Brazil	2016	3.39	0.19	1.88	0.11	6296	Yes
Canada	1971	3.56	0.16	1.66	0.07	9970	Yes
Canada	1975	4.02	0.17	1.84	0.08	10717	Yes
Canada	1981	4.59	0.22	2.01	0.10	8342	Yes
Canada	1987	4.78	0.30	1.88	0.12	5190	Yes
Canada	1991	4.34	0.20	2.00	0.09	9847	Yes
Canada	1994	4.07	0.15	1.75	0.07	14077	Yes
Canada	1997	4.01	0.17	1.79	0.08	11273	Yes
Canada	1998	3.22	0.12	1.46	0.06	13486	Yes
Canada	2000	3.17	0.12	1.47	0.06	13264	Yes
Canada	2004	3.15	0.13	1.56	0.06	11861	Yes
Canada	2007	2.85	0.11	1.49	0.06	12533	Yes
Canada	2010	3.06	0.13	1.40	0.06	10341	Yes
Canada	2012	3.35	0.15	1.65	0.07	10370	Yes
Canada	2013	3.93	0.18	1.56	0.07	9741	Yes
Canada	2014	3.23	0.15	1.71	0.08	9836	Yes
Canada	2015	3.33	0.15	1.49	0.06	10552	Yes
Canada	2016	3.38	0.15	1.64	0.07	10641	Yes
Canada	2017	3.53	0.12	1.51	0.05	15982	Yes
Chile	1990	2.24	0.28	1.62	0.21	1242	No
Chile	1992	3.42	0.37	1.76	0.19	1678	Yes
Chile	1994	2.61	0.26	1.91	0.19	2071	No
Chile	1996	2.71	0.21	1.52	0.12	3437	Yes
Chile	1998	2.43	0.19	1.64	0.13	3335	No
Chile	2000	2.28	0.15	1.38	0.09	4432	Yes
Chile	2003	2.28	0.15	1.45	0.10	4637	No
Chile	2006	2.31	0.14	1.38	0.08	5374	Yes
Chile	2009	3.02	0.22	1.48	0.11	3619	Yes
Chile	2011	3.48	0.27	1.82	0.14	3312	Yes
Chile	2013	2.87	0.21	1.76	0.13	3635	No
Chile	2015	3.27	0.20	1.71	0.11	5314	Yes
Chile	2017	2.73	0.18	1.56	0.10	4829	No
China	2002	4.24	0.41	1.72	0.17	2167	Yes
China	2013	3.10	0.14	2.06	0.09	10098	No
Colombia	2004	2.37	0.28	1.97	0.23	1411	No

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^{+}_{cap}	Reject H_0?
Colombia	2007	1.97	0.06	1.69	0.05	23787	No
Colombia	2010	2.52	0.07	1.63	0.05	24944	No
Colombia	2013	2.64	0.07	1.75	0.05	25654	Yes
Colombia	2016	2.59	0.07	1.60	0.05	23979	Yes
Czech Rep	1996	3.00	0.24	1.14	0.09	3077	Yes
Czech Rep	2002	3.34	0.48	1.50	0.21	973	No
Czech Rep	2004	2.54	0.42	0.89	0.15	729	No
Czech Rep	2007	3.62	0.39	1.11	0.12	1735	Yes
Czech Rep	2010	4.09	0.49	1.23	0.15	1422	Yes
Czech Rep	2013	3.12	0.40	1.54	0.20	1198	No
Czech Rep	2016	3.74	0.51	1.24	0.17	1083	Yes
Denmark	1987	4.46	0.21	1.64	0.08	9199	Yes
Denmark	1992	4.57	0.21	1.46	0.07	9164	Yes
Denmark	1995	4.68	0.09	1.26	0.02	57306	Yes
Denmark	2000	4.26	0.08	1.19	0.02	59549	Yes
Denmark	2004	4.03	0.07	1.17	0.02	59824	Yes
Denmark	2007	3.82	0.07	1.17	0.02	61255	Yes
Denmark	2010	3.37	0.06	1.20	0.02	59488	Yes
Denmark	2013	3.47	0.06	1.13	0.02	60880	Yes
Denmark	2016	3.41	0.06	1.23	0.02	58989	Yes
Dominican Rep.	2007	3.13	0.60	1.33	0.26	544	Yes
Egypt	2012	1.56	0.26	1.09	0.18	728	No
Estonia	2007	4.53	0.43	0.67	0.06	2250	Yes
Estonia	2010	4.68	0.48	0.91	0.09	1948	Yes
Estonia	2013	3.65	0.42	1.16	0.13	1549	Yes
Finland	1987	4.35	0.22	1.45	0.07	7906	Yes
Finland	1991	4.48	0.22	1.43	0.07	8332	Yes
Finland	1995	4.50	0.31	1.25	0.09	4236	Yes
Finland	2000	3.53	0.21	1.09	0.06	5923	Yes
Finland	2004	3.91	0.19	1.02	0.05	8061	Yes
Finland	2007	3.82	0.19	1.01	0.05	8502	Yes
Finland	2010	3.91	0.20	1.05	0.05	7537	Yes
Finland	2013	3.90	0.19	1.04	0.05	8899	Yes
Finland	2016	3.71	0.18	1.19	0.06	8237	Yes
France	1978	2.40	0.28	1.75	0.21	1426	No
France	1984	1.52	0.15	1.73	0.17	1974	No
France	1989	3.43	0.37	1.65	0.18	1677	Yes
France	1994	3.13	0.16	1.68	0.08	7880	Yes
France	2000	3.51	0.19	1.72	0.09	7112	Yes
France	2005	3.33	0.17	1.53	0.08	7987	Yes
France	2010	3.01	0.12	1.49	0.06	11658	Yes
Germany	1973	3.56	0.08	2.13	0.05	37614	No
Germany	1978	3.36	0.08	2.18	0.05	35731	No
Germany	1983	4.54	0.11	2.23	0.06	32288	Yes
Germany	1984	3.96	0.27	1.03	0.07	4240	Yes
Germany	1987	4.23	0.30	1.17	0.08	3863	Yes

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^{+}_{cap}	Reject H_0?
Germany	1989	3.29	0.24	1.18	0.09	3733	Yes
Germany	1991	4.77	0.29	1.10	0.07	5448	Yes
Germany	1994	4.54	0.28	1.32	0.08	5384	Yes
Germany	1995	4.65	0.29	1.33	0.08	5275	Yes
Germany	1998	4.56	0.27	1.31	0.08	5531	Yes
Germany	2000	3.91	0.19	1.51	0.07	8784	Yes
Germany	2001	3.46	0.16	1.31	0.06	9273	Yes
Germany	2002	3.57	0.17	1.35	0.06	8850	Yes
Germany	2003	3.64	0.18	1.28	0.06	8600	Yes
Germany	2004	3.67	0.18	1.29	0.06	8300	Yes
Germany	2005	3.29	0.16	1.27	0.06	8869	Yes
Germany	2006	3.28	0.16	1.32	0.06	8276	Yes
Germany	2007	3.44	0.17	1.35	0.07	7778	Yes
Germany	2008	3.22	0.17	1.29	0.07	7295	Yes
Germany	2009	3.43	0.15	1.37	0.06	10611	Yes
Germany	2010	3.44	0.14	1.29	0.05	12512	Yes
Germany	2011	3.30	0.13	1.28	0.05	12291	Yes
Germany	2012	3.43	0.13	1.31	0.05	13051	Yes
Germany	2013	3.40	0.14	1.32	0.05	11824	Yes
Germany	2014	3.78	0.16	1.30	0.06	11176	Yes
Germany	2015	3.09	0.13	1.31	0.06	10625	Yes
Germany	2016	3.48	0.15	1.27	0.05	11203	Yes
Greece	1995	3.70	0.51	2.83	0.39	1033	No
Greece	2000	3.80	0.63	2.13	0.35	742	No
Greece	2004	3.64	0.52	2.12	0.30	1006	No
Greece	2007	2.81	0.35	1.97	0.25	1253	No
Greece	2010	3.01	0.39	2.09	0.27	1170	No
Greece	2013	2.68	0.31	2.22	0.25	1517	No
Greece	2016	2.72	0.21	2.46	0.19	3480	No
Guatemala	2006	1.90	0.33	1.69	0.29	690	No
Hungary	1999	2.82	0.49	1.87	0.33	666	No
Iceland	2004	4.71	0.49	1.15	0.12	1840	Yes
Iceland	2007	4.40	0.43	1.16	0.11	2113	Yes
Iceland	2010	4.51	0.39	1.13	0.10	2675	Yes
India	2004	2.85	0.29	2.47	0.25	1876	No
India	2011	2.21	0.22	1.43	0.14	2027	No
Ireland	1987	3.41	0.34	1.29	0.13	1998	Yes
Ireland	1994	2.54	0.34	1.26	0.17	1114	Yes
Ireland	1995	2.31	0.32	1.30	0.18	1026	Yes
Ireland	2000	3.44	0.41	1.70	0.20	1371	Yes
Ireland	2002	4.06	0.66	1.57	0.25	752	Yes
Ireland	2003	2.26	0.33	1.74	0.25	952	No
Ireland	2004	2.12	0.34	1.47	0.23	804	No
Ireland	2005	1.94	0.27	1.80	0.25	1049	No
Ireland	2006	2.23	0.22	1.48	0.15	1984	Yes
Ireland	2007	2.61	0.30	1.47	0.17	1500	No

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^+_{cap}	Reject H_0?
Ireland	2008	3.23	0.46	1.39	0.20	986	No
Ireland	2009	2.97	0.48	2.10	0.34	771	No
Ireland	2010	3.28	0.54	1.72	0.28	745	Yes
Ireland	2011	3.07	0.50	1.72	0.28	768	Yes
Ireland	2012	3.54	0.55	1.74	0.27	816	No
Ireland	2013	2.98	0.44	1.74	0.26	918	No
Ireland	2014	3.23	0.48	1.58	0.23	921	No
Ireland	2015	3.03	0.45	1.51	0.22	914	Yes
Ireland	2016	3.18	0.48	1.55	0.23	872	No
Ireland	2017	1.99	0.32	1.92	0.31	753	No
Ireland	1996	2.68	0.38	1.56	0.22	1023	No
Israel	2001	2.27	0.43	1.07	0.20	556	Yes
Israel	2005	3.67	0.43	1.25	0.15	1475	Yes
Israel	2007	4.25	0.61	1.68	0.24	971	Yes
Israel	2010	2.30	0.33	1.50	0.21	980	No
Israel	2012	3.49	0.43	1.30	0.16	1305	Yes
Israel	2014	4.76	0.59	1.69	0.21	1330	Yes
Israel	2016	4.23	0.54	2.14	0.27	1215	Yes
Italy	1986	3.17	0.24	1.85	0.14	3474	Yes
Italy	1987	3.51	0.20	1.66	0.10	6045	Yes
Italy	1989	3.95	0.23	1.56	0.09	5865	Yes
Italy	1991	4.43	0.26	1.64	0.10	5827	Yes
Italy	1993	3.76	0.23	1.52	0.09	5444	Yes
Italy	1995	3.24	0.20	1.62	0.10	5487	Yes
Italy	1998	3.37	0.22	1.46	0.09	4876	Yes
Italy	2000	3.05	0.19	1.33	0.08	5280	Yes
Italy	2004	2.39	0.15	1.52	0.10	4913	Yes
Italy	2008	3.13	0.20	1.44	0.09	4778	Yes
Italy	2010	3.35	0.22	1.34	0.09	4719	Yes
Italy	2014	2.75	0.19	1.61	0.11	4402	Yes
Italy	2016	2.68	0.19	1.50	0.11	3853	Yes
Ivory Coast	2002	1.78	0.32	1.28	0.23	618	No
Ivory Coast	2008	1.49	0.27	1.08	0.19	621	No
Ivory Coast	2015	2.24	0.39	1.18	0.21	664	No
Japan	2008	4.09	0.75	1.60	0.29	598	No
Japan	2010	3.06	0.59	1.46	0.28	543	No
Lithuania	2010	7.07	1.34	1.39	0.26	564	Yes
Lithuania	2011	5.80	1.10	1.18	0.22	568	Yes
Lithuania	2012	4.94	0.99	1.07	0.21	507	Yes
Lithuania	2013	5.40	1.06	1.33	0.26	523	Yes
Lithuania	2014	3.23	0.47	0.94	0.14	961	No
Lithuania	2015	4.30	0.64	0.82	0.12	909	Yes
Lithuania	2016	4.34	0.67	0.96	0.15	837	Yes
Lithuania	2017	4.51	0.70	1.18	0.18	841	Yes
Luxembourg	1991	7.03	1.17	1.48	0.25	716	Yes
Luxembourg	1994	5.65	1.09	2.93	0.56	531	No

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^{+}_{cap}	Reject H_0?
Luxembourg	1997	5.21	0.83	1.58	0.25	779	Yes
Luxembourg	2000	4.01	0.60	1.55	0.23	892	Yes
Luxembourg	2004	3.83	0.58	2.85	0.43	887	No
Luxembourg	2007	3.26	0.32	1.61	0.16	2085	Yes
Luxembourg	2010	3.43	0.26	1.71	0.13	3494	Yes
Luxembourg	2013	3.68	0.33	1.59	0.14	2534	No
Mexico	1989	2.42	0.44	1.40	0.26	602	No
Mexico	1996	2.06	0.41	1.46	0.29	505	No
Mexico	2004	2.58	0.35	1.58	0.22	1055	Yes
Mexico	2008	2.21	0.24	1.43	0.16	1635	Yes
Mexico	2010	2.64	0.32	1.64	0.20	1323	Yes
Mexico	2014	2.15	0.32	1.64	0.25	882	No
Mexico	2016	1.89	0.15	1.51	0.12	3186	No
Mexico	2018	1.82	0.14	1.59	0.12	3389	No
Mexico	2002	3.67	0.68	2.06	0.38	587	Yes
Netherlands	1990	3.75	0.33	1.38	0.12	2570	Yes
Netherlands	1993	4.16	0.38	1.23	0.11	2423	Yes
Netherlands	1999	4.21	0.37	1.27	0.11	2563	Yes
Netherlands	2004	3.41	0.18	1.36	0.07	7166	Yes
Netherlands	2007	3.17	0.16	0.98	0.05	8011	Yes
Netherlands	2010	3.63	0.18	1.37	0.07	8001	Yes
Netherlands	2013	3.62	0.18	1.30	0.07	7808	Yes
Norway	1979	1.80	0.10	1.71	0.10	6048	No
Norway	1986	5.11	0.45	1.89	0.17	2588	Yes
Norway	1991	3.83	0.20	1.76	0.09	7361	Yes
Norway	1995	3.45	0.17	1.14	0.06	8027	Yes
Norway	2000	3.18	0.13	0.88	0.04	11474	Yes
Norway	2004	3.98	0.17	0.77	0.03	10947	Yes
Norway	2007	3.74	0.04	1.11	0.01	169708	Yes
Norway	2010	3.55	0.04	0.86	0.01	177229	Yes
Norway	2013	3.35	0.04	0.83	0.01	183146	Yes
Palestine	2017	2.64	0.37	1.43	0.20	1000	Yes
Panama	2007	4.01	0.76	1.54	0.29	566	Yes
Panama	2010	3.01	0.59	1.00	0.20	525	Yes
Paraguay	2000	1.75	0.33	1.67	0.31	586	No
Peru	2004	2.50	0.23	1.75	0.16	2463	Yes
Peru	2007	3.05	0.26	1.69	0.15	2667	Yes
Peru	2010	2.78	0.22	1.66	0.13	3239	Yes
Peru	2013	3.72	0.27	1.68	0.12	3930	Yes
Peru	2016	2.92	0.19	1.94	0.12	4813	Yes
Russia	2011	5.98	1.17	1.58	0.31	513	No
Russia	2013	4.25	0.29	1.67	0.11	4360	Yes
Russia	2014	5.03	0.34	1.79	0.12	4412	Yes
Russia	2015	4.89	0.29	1.65	0.10	5752	No
Russia	2016	5.19	0.19	1.91	0.07	15716	No
Russia	2017	5.54	0.33	2.42	0.14	5671	No

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^{+}_{cap}	Reject H_0?
Slovakia	1996	3.89	0.66	1.41	0.24	709	Yes
Slovakia	2007	3.31	0.58	1.60	0.28	653	No
Slovakia	2010	3.49	0.49	0.96	0.13	1024	Yes
Slovakia	2013	2.73	0.34	0.84	0.10	1289	Yes
Slovenia	1999	5.08	0.93	1.52	0.28	601	No
Slovenia	2004	5.88	1.15	2.50	0.49	521	No
Slovenia	2007	6.20	1.24	1.74	0.35	501	Yes
Slovenia	2012	6.11	0.64	1.43	0.15	1841	Yes
Slovenia	2015	5.73	0.62	1.09	0.12	1673	Yes
South Korea	2006	4.67	0.32	1.69	0.11	4353	No
South Korea	2008	4.20	0.31	1.70	0.13	3616	No
South Korea	2010	4.38	0.36	1.62	0.13	3003	No
South Korea	2012	3.79	0.29	1.58	0.12	3329	No
Spain	1980	4.21	0.49	2.07	0.24	1503	No
Spain	1990	3.84	0.27	1.58	0.11	4036	Yes
Spain	1995	3.46	0.33	1.57	0.15	2250	Yes
Spain	2000	3.19	0.24	1.09	0.08	3549	Yes
Spain	2004	4.22	0.28	1.32	0.09	4433	Yes
Spain	2007	4.30	0.26	1.41	0.09	5327	Yes
Spain	2010	6.08	0.49	1.79	0.15	3043	Yes
Spain	2013	3.59	0.20	1.69	0.09	6681	Yes
Spain	2016	3.44	0.18	1.51	0.08	7451	Yes
Sweden	1967	3.32	0.47	1.63	0.23	988	Yes
Sweden	1975	3.93	0.20	1.54	0.08	7845	Yes
Sweden	1981	4.56	0.26	2.47	0.14	6092	Yes
Sweden	1987	4.71	0.24	1.77	0.09	7953	Yes
Sweden	1992	4.49	0.19	1.98	0.08	10824	Yes
Sweden	1995	3.97	0.16	1.63	0.07	11655	Yes
Sweden	2000	3.60	0.16	1.26	0.06	10319	Yes
Sweden	2005	3.15	0.13	1.07	0.04	11951	Yes
Switzerland	1982	2.05	0.14	1.34	0.09	4233	Yes
Switzerland	1992	3.55	0.24	1.30	0.09	4486	Yes
Switzerland	2000	3.29	0.29	1.40	0.12	2646	Yes
Switzerland	2002	3.53	0.31	1.56	0.14	2578	Yes
Switzerland	2004	4.41	0.38	1.38	0.12	2648	Yes
Switzerland	2006	3.07	0.21	1.21	0.08	4206	Yes
Switzerland	2007	2.74	0.17	1.22	0.08	5057	Yes
Switzerland	2008	3.01	0.19	1.11	0.07	5004	Yes
Switzerland	2009	2.96	0.18	1.22	0.07	5396	Yes
Switzerland	2010	2.92	0.17	1.14	0.07	5759	Yes
Switzerland	2011	3.20	0.19	1.25	0.07	5714	Yes
Switzerland	2012	3.28	0.20	1.25	0.08	5471	Yes
Switzerland	2013	3.45	0.22	1.40	0.09	5097	Yes
Switzerland	2014	3.09	0.18	1.31	0.08	5672	Yes
Switzerland	2015	3.18	0.18	1.40	0.08	5947	Yes
Switzerland	2016	3.07	0.17	1.43	0.08	6310	Yes

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^{+}_{cap}	Reject H_0?
Switzerland	2017	3.19	0.20	1.46	0.09	5048	Yes
Taiwan	1981	4.50	0.19	1.53	0.07	10635	Yes
Taiwan	1986	3.88	0.15	1.34	0.05	12864	Yes
Taiwan	1991	4.11	0.15	1.41	0.05	14978	Yes
Taiwan	1995	4.29	0.16	1.77	0.07	13619	Yes
Taiwan	1997	4.27	0.17	1.83	0.07	12572	Yes
Taiwan	2000	4.19	0.17	1.76	0.07	12301	Yes
Taiwan	2005	4.17	0.17	1.80	0.07	11903	Yes
Taiwan	2007	3.82	0.16	1.75	0.07	12042	Yes
Taiwan	2010	3.85	0.15	1.74	0.07	12748	Yes
Taiwan	2013	3.85	0.15	1.55	0.06	13300	Yes
Taiwan	2016	3.88	0.15	1.66	0.06	13648	Yes
UK	1969	3.73	0.26	1.11	0.08	4102	Yes
UK	1974	3.56	0.27	1.35	0.10	3547	Yes
UK	1979	5.03	0.34	1.34	0.09	4280	Yes
UK	1986	4.48	0.30	1.79	0.12	4515	Yes
UK	1991	3.56	0.24	1.69	0.11	4445	Yes
UK	1994	3.00	0.11	1.42	0.05	15751	Yes
UK	1995	3.21	0.22	1.48	0.10	4091	Yes
UK	1999	2.89	0.10	1.40	0.05	15200	Yes
UK	2004	2.70	0.09	1.27	0.04	17314	Yes
UK	2007	2.87	0.10	1.34	0.05	15272	Yes
UK	2010	2.57	0.12	1.34	0.06	9839	Yes
UK	2013	2.79	0.14	1.50	0.08	7715	Yes
UK	2016	2.49	0.13	1.32	0.07	7340	Yes
US	1974	3.86	0.23	1.51	0.09	5558	Yes
US	1979	3.70	0.08	1.46	0.03	44749	Yes
US	1986	3.36	0.08	1.64	0.04	38274	Yes
US	1991	3.45	0.08	1.54	0.04	38871	Yes
US	1992	3.14	0.07	1.49	0.03	38137	Yes
US	1993	2.51	0.06	1.46	0.03	37252	Yes
US	1994	2.43	0.06	1.45	0.03	37355	Yes
US	1995	2.45	0.06	1.48	0.04	32076	Yes
US	1996	2.34	0.06	1.51	0.04	31165	Yes
US	1997	2.29	0.06	1.47	0.04	30913	Yes
US	1998	2.32	0.06	1.45	0.04	31144	Yes
US	1999	2.41	0.06	1.44	0.04	31320	Yes
US	2000	2.19	0.05	1.40	0.03	46961	Yes
US	2001	2.23	0.05	1.31	0.03	46168	Yes
US	2002	2.29	0.05	1.35	0.03	43963	Yes
US	2003	2.35	0.05	1.37	0.03	43410	Yes
US	2004	2.39	0.05	1.33	0.03	42354	Yes
US	2005	2.27	0.05	1.42	0.03	41624	Yes
US	2006	2.27	0.05	1.46	0.03	40264	Yes
US	2007	2.34	0.05	1.49	0.03	39312	Yes
US	2008	2.35	0.05	1.32	0.03	40523	Yes

Table 3: Hypothesis testing of $\widehat{\alpha}_{\text{lab}} = \widehat{\alpha}_{\text{cap}}$.

Country	Year	$\widehat{\alpha}_{\text{lab}}$	s.e.	$\widehat{\alpha}_{\text{cap}}$	s.e.	N^{+}_{cap}	Reject H_0?
US	2009	2.26	0.05	1.33	0.03	38446	Yes
US	2010	2.56	0.06	1.41	0.03	37383	Yes
US	2011	2.29	0.05	1.40	0.03	36728	Yes
US	2012	2.33	0.05	1.56	0.04	36865	Yes
US	2013	2.40	0.07	1.40	0.04	24494	Yes
US	2014	2.37	0.05	1.41	0.03	46312	Yes
US	2015	2.32	0.05	1.36	0.03	44260	Yes
US	2016	2.29	0.05	1.42	0.03	45401	Yes
US	2017	2.42	0.05	1.45	0.03	45102	Yes
US	2018	2.26	0.05	1.34	0.03	45307	Yes
Uruguay	2004	2.53	0.32	1.81	0.23	1241	No
Uruguay	2007	3.26	0.22	1.53	0.11	4236	Yes
Uruguay	2010	3.10	0.22	1.63	0.11	4085	Yes
Uruguay	2013	3.69	0.28	1.80	0.14	3497	Yes
Uruguay	2016	3.53	0.26	1.69	0.13	3632	Yes
Vietnam	2011	3.26	0.40	1.23	0.15	1362	Yes
Vietnam	2013	2.61	0.31	1.64	0.20	1384	No

Note: *$\widehat{\alpha}_{lab}$ (resp. $\widehat{\alpha}_{cap}$): point estimate of Pareto exponent for labor (resp. capital) income; s.e.: standard error ($\widehat{\alpha}(k)/\sqrt{k}$); N^{+}_{cap}: the number of positive capital income observations; the last column denotes if H_0 is rejected.*