Market consistent valuation of deferred taxes*

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Abstract

This paper develops a continuous time framework to value deferred taxes using Black and Scholes (1973) type option pricing techniques. The valuation renders a market consistent pricing procedure, meaning that the final price of tax deferrals depends on parameters observed in the market. The framework is flexible enough to value deferred taxes like loss carryforward, loss carryback or liabilities arising from temporary differences. In addition, our approach unravels the influence of leverage on deferred tax values and proposes an alternative to the Modigliani-Miller theorem, as we can model the tax shield as a financial option. A simulation study over multiple time horizons shows that carryforward value is negatively influenced by leverage, whereas carryback and latent tax liability values increase. An empirical application serves to illustrate the practical use of our model: the loss absorbing capacity of deferred taxes (LAC DT) for European insurers.

Keywords: Deferred tax valuation, Option pricing, Loss absorbing capacity of deferred taxes

JEL Codes: G32 (Value of Firms), H32 (Fiscal Policies and Behavior of Firms), H25 (Business Taxes)

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1 Introduction

In this paper we describe the market consistent valuation of deferred taxes. Deferred taxes are balance sheet items of firms with a certain history of fiscal profits and losses. They reflect the advantage or disadvantage of such firms to pay less or more taxes compared to a hypothetical, similar firm without any history of fiscal profits and losses.

Deferred taxes are commonly valued according to accounting standards, which recognize and value them using a single deterministic scenario. This, however, does not reflect the contingent characteristics of deferred taxes that their payoff, i.e. the tax advantage or disadvantage, is a non-linear function of the future fiscal profits and losses of the firm. In this paper we construct a model to take account of this non-linear payoff using basic option pricing techniques. Thereby, the model yields pricing formulas that depend on market observable parameters, resulting in a market consistent valuation of deferred taxes.

The market consistent valuation of deferred taxes differs from extant accounting valuation methods in the following ways:

- The market consistent valuation of *deferred tax assets* (DTA) and *deferred tax liabilities* (DTL) is typically lower than those obtained with current accounting methods. This is because a market consistent valuation reflects scenarios in which DTA's/DTL's do not materialize, while a single deterministic scenario is more or less all or nothing.

- Profits in previous years and the possibility in a tax regime to carry back future losses to previous years also results in a DTA under the market consistent valuation principles. However, this does not result in a DTA under conventional accounting standards. The reason for this comes from the deterministic scenario, which only considers a profit situation in which the carry back potential is not being realized. On the contrary, a market consistent valuation does attach value to the (tax) advantage in negative scenarios.

- Accounting standards (partially) recognize DTA's if a firm can prove future profitability using subjective assumptions. Market consistent valuation uses market data to encompass all future scenarios and thereby solely relies on objective parameters observed in the market.

The main novelty of this paper is to attach a market consistent valuation to deferred taxes arising from loss carryforward, loss carryback and temporary differences in a continuous time framework. Hereby, we find that market consistent valuation techniques yield significantly different estimates in comparison with conventional accounting valuation methods. This is because the market consistent approach takes all future profit and loss scenarios into account.

In a final step, our methodology is applied to investigate the loss absorbing capacity of European insurers. The model in this paper supports how insurance companies can value their deferred taxes according to the market consistent valuation principles of Solvency II. On top of that, the model provides insights in the loss absorbing capacity of deferred taxes, an element that lowers the Solvency II capital requirements. The market consistent valuation model indicates the following regarding the loss absorbing capacity of deferred taxes:

- DTL's indeed have loss absorbing capacity; when an insurer, or any other firm suffers a loss, part of this loss is being compensated by a lower market valuation of the DTL after the loss.

- DTA's may have loss absorbing capacity if the company has sufficient potential, resources and/or own funds to generate future profits; in that case the value of the total DTA increases after a loss. However, if insufficient own funds are available, such a firm would experience a decrease in its DTA. In these situations, a reduction of the Solvency II capital requirements for European insurance companies does not reflect the actual loss due to a decrease in their DTA.

We find that the loss absorbing capacity is, on average, less than extant estimates and under some circumstances can even be negative, since so much potential is lost after a severe (negative) shock. Not only is the market consistent valuation of deferred taxes important for insurers, but it is also relevant to mergers and acquisitions, when buyers have to value a firm. The common accounting valuation of deferred taxes does not necessarily reflect the market price of the tax advantage or disadvantage.

2 Literature Review

Current valuation methods of deferred taxes are generally based on GAAP (Generally Accepted Accounting Principles) or IAS12 (International Accounting Standard, Income Taxes). Sansing (1998) remarks that these approaches tend to overestimate the true value of deferred taxes appearing on financial statements, as they are future benefits but not discounted. Moreover, the appropriate discount factor is an open question, since the materialization of deferred taxes is not risk free. Sansing (1998) derives a discount factor for deferred tax liabilities, however assuming an average tax liability, thereby ignoring the dynamics over longer periods of time. Waegenaere et al. (2003) obtain closed form formulas for tax carry forward, including additional parameters like the duration period. This framework leads to the surprising conclusion that the market-to-book ratio of carry forward can exceed one, depending on the skewness of the underlying income distribution. This suggests that discounting deferred tax assets may not always be appropriate. But, Waegenaere et al. (2003) use the stringent assumption that income is generated in perpetuity, which is rather unrealistic. Empirical studies of deferred taxes are conducted by Amir et al. (1997); Ayers (1998); Givoly and Hayn (1992). Givoly and Hayn (1992) use a linear regression approach, where abnormal returns are regressed on the reduction in deferred tax liabilities during a period of tax reforms. Hereby, Givoly and Hayn (1992) find that investors discount the liability based on likelihood and timing of the settlement. Amir et al. (1997) use a regression approach as well, but splitting the deferred taxes into seven categories, which renders a more precise estimate of the influence of deferred taxes on equity. All regression coefficients of the deferred tax assets are found to be greater than one, which contradicts the hypothesis that DTA's ought to be discounted, as this would imply a regression coefficient between zero and one. A similar approach and conclusion is reached by Ayers (1998). However, as Waegenaere et al. (2003) point out, this only holds if the disparity between book and market value is solely due to discounting.

3 Four different types of deferred taxes

We now turn attention to the four types of deferred tax that we analyze. In this section we explain the key distinguishing characteristics of deferred tax coming from: *loss carryforward*, *loss carryback* and *temporary differences*. Throughout this paper we make the following important assumption about corporate tax payments, which is needed to model the contingent characteristics of DTA's/DTL's.

Assumption 3.1. Taxable profit is measured by the difference in asset value over two consecutive periods. Taxable income consists of net profit if this is a positive quantity and is zero otherwise.

3.1 DTA from carry forward

Carry forward is the allowance to carry forward losses to offset future taxable income. A firm recognizes that losses can be seen as an asset, since part of the loss will result in lower tax payments compared to a similar firm without any tax history. The extent to which a company is able to settle the carry forward in the future determines the eventual value of this type of DTA.

Consider a company with no fiscal history, so that it does not have any deferred taxes on the balance sheet. Let A_t denote the asset value of a company at time t before taxes are levied. At $t = 1$, a firm pays taxes only if $A_1 > A_0$ (by Assumption 3.1), in which case the total asset value is reduced. Let τ denote the tax rate, then the asset value at $t = 1$ after tax can be expressed as

$$\widetilde{A}_1 = A_1 - \tau(A_1 - A_0)^+. \tag{3.2}$$

Here $\widetilde{A}_t$ denotes the value of an asset after taxes in period t and $(x)^+ \triangleq \max(x, 0)$. The second term in (3.2) has the same structure as the payoff of an at-the-money European call option with strike A_0.

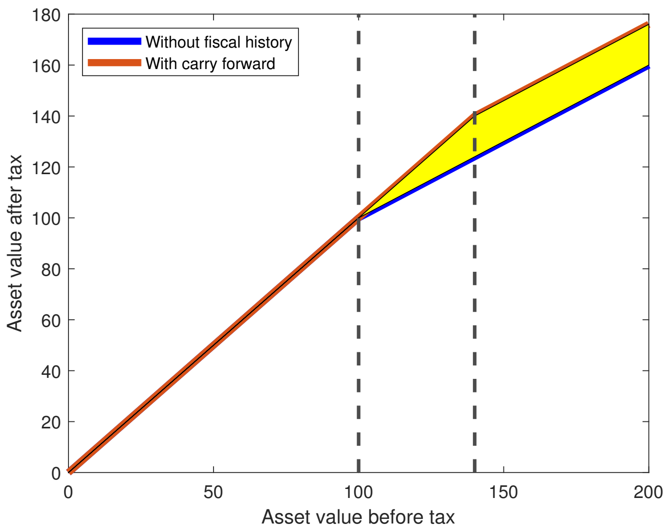
Consider a firm with an identical balance sheet, but with the additional benefit of carry forward as a deferred tax asset. Carry forward can be used to offset taxable income in case net profit is positive. Let CF_t be the loss carry forward available in year t. Then in period $t = 1$ we have the following post-tax asset value

$$\widetilde{A}_1 = A_1 - \tau(A_1 - A_0 - CF_1)^+. \tag{3.3}$$

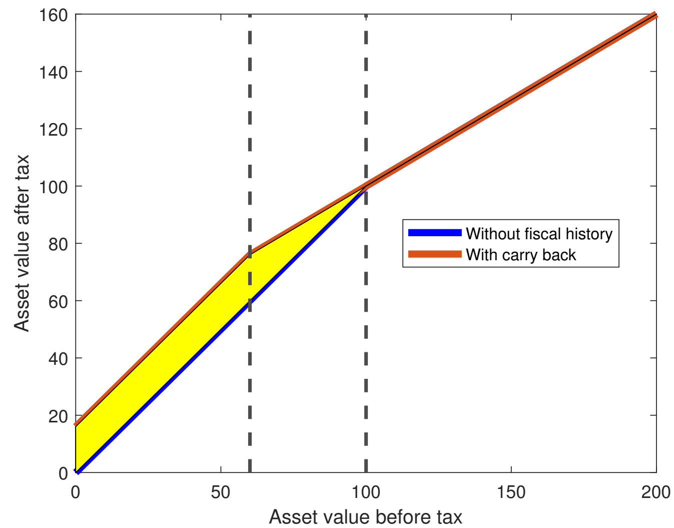
A firm with carry forward pays taxes from the moment $A_1 > A_0 + CF_1$, which differs from a firm without having this tax asset, as they pay taxes as soon as $A_1 > A_0$. The asset value after tax (3.3) versus (3.2) is shown in Figure 1a. The difference between the two asset values becomes relevant as soon as $A_1 > A_0$, corresponding to the moment that a firm without fiscal history has to pay taxes. The point from which the two firms start paying taxes is depicted by the vertical dashed lines. The yellow area in Figure 1a is what fundamentally determines the DTA (or DTL) value, as this concerns all profit scenarios where the DTA can be realized.

3.2 DTA from carry back

Tax carry back is the possibility to receive a refund of corporate tax paid in the past, due to current losses. The maximum amount that can be claimed as refund equals the current loss times the tax rate, although it may be less when historical profits are insufficient to offset the current loss. Carry back renders the firm with an immediate cash flow that (partly) compensates current loss, but expires worthless when a company makes profit in period one.



(a)



(b)

Figure 1: Panel (a) shows the post-tax asset value of a counterfactual firm with $A_0 = 100$ and a firm with $CF_1 = 40$. The yellow area denotes the tax benefit of the firm with a DTA. Panel (b) shows the post-tax asset value of a counterfactual firm with $A_0 = 100$ and a firm with $CB_1 = 40$.

This gives rise to the following asset value of a firm in period one

$$\widetilde{A}_1 = A_1 + \tau CB_1 - \tau(A_1 - A_0 + CB_1)^+, \tag{3.4}$$

where CB_1 denotes the loss carry back in year one. In general we write CB_t to denote loss carry back in year t. The asset value after tax (3.4) and (3.2) are shown in Figure 1b. The value of the DTA manifests itself when the company incurs a loss, in which one observes a positive difference between the two graphs, as depicted by the yellow area. This difference vanishes as soon as A_1 is bigger than or equal to A_0, which corresponds to a factual profit. In that situation, there is no loss that can be carried back anymore.

3.3 DTA from temporary differences

Temporary differences arise as a consequence of the difference between accounting and fiscal valuation principles. Fiscal accounting is based on historical cost. Initially, assets are valued at their market price on the fiscal balance sheet. If increases in the market price are not reflected on the fiscal balance sheet, the firm does not need to pay taxes on this profit now, but only when it sells. In other cases, like for fixed income, the tax advantage materializes over the lifetime of the fixed income asset. As a result, a company can make an accounting profit/loss, which is not recognized yet under fiscal accounting principles. The following example illustrates this.

Example 3.5. Consider a company without any fiscal history, so that the market consistent and fiscal account are exactly the same as in the T-account below.

Market consistent Account		Fiscal Account	
Stock 100	Equity 100	Stock 100	Equity 100

Next, assume that the company's assets rise in value to 120 due to an increase of stock investment, so that a profit of 20 is realized. However, this profit is not recognized on the fiscal account, because stocks are valued on the basis of historical costs. The company now faces a deferred tax liability (DTL), because it does not yet pay taxes, but is obliged to do so in the future. Suppose that the tax rate $\tau = 25\%$, so that the amount of tax paid on profit would be 5. This amount needs to be reserved on the market account, because it needs to be paid in the future. The fiscal balance sheet remains unchanged. The new situation leads to the following T-account.

Market consistent Account		Fiscal Account	
Stock 120	Equity 115	Stock 100	Equity 100
	DTL 5		

Now, suppose a negative shock occurs, reducing total assets by 40%. As a result, total assets fall to 72, which means that the earlier profit of 20 and DTL of 5 disappear. When selling the assets, a loss of 28 would be realized. This loss can be carried forward to offset the next 28 (taxable) profit, creating a tax advantage of $28\tau = 7$. This amount is put on the market consistent account as a deferred tax asset (DTA). The new situation leads to the following T-accounts.

Market consistent Account

Stock 72	Equity 79
DTA 7	

Fiscal Account

Stock 100	Equity 100

Because of deferred taxes, the loss is *not* 48 but 36. This is the *loss absorbing capacity of deferred taxes* (LAC DT), since part of the loss can be transferred to the tax authority, thereby mitigating the overall loss. However, the DTA can only be settled if future profits are sufficient to claim the full amount from the tax authority.

Example 3.5 details that a deferred tax asset arising from temporary differences might occur due to shocks in the asset value. In this case, the loss incurred now can offset future taxable income. Hence, the post-tax asset value of a firm having a DTA from temporary differences is equal to

$$\widetilde{A}_1 = A_1 - \tau(A_1 - A_0 - ATD_1)^+, \tag{3.6}$$

where ATD_1 (asset from temporary differences) is the nominal DTA value. We observe that the post-tax value of such a firm has the exact same structure as a firm having some carry forward available (see Equation (3.3)). Hence, when considering a one year time horizon, the analysis of DTA's arising from temporary differences is equivalent to finding a market consistent value of carry forward.

3.4 DTL from temporary differences

Here we consider scenarios in which a firm makes profit under applicable valuation principles, which is not recognized under fiscal valuation principles. The firm knows it is obliged to pay extra taxes in the future and reserves an appropriate amount on the balance sheet, as in

Example 3.5. In the one period case, the asset value after tax of a company having a DTL arising from temporary differences can be expressed as

$$\widetilde{A}_1 = A_1 - \tau(A_1 - A_0 + LTD_1)^+. \tag{3.7}$$

The parameter LTD_1 (liability from temporary differences) is the amount of taxable profit recognized under market consistent accounting, but not under fiscal accounting standards in period one. In general, we write LTD_t for the (total) unrecognized fiscal profit in time period t. The asset value (3.7) is always less than or equal to (3.2), because of deferred tax liabilities.

4 Two-period model unlevered firms

In a multi-period framework, the payoff structure of post-tax asset values becomes significantly more complicated, due to the presence of additional parameters. For example, the post-tax asset value depends on the settlement term of carryforward, whether carry back is allowed or not, settlement term of carryback etc.[1] In general, one would expect that deferred tax assets become more valuable over longer time periods, since the probability that the entire DTA is settled increases. An important change is that taxes are settled every year, which creates path dependency. In this section we present some formulas for the post-tax asset value of firms in a two-period model, which serve to illustrate the dynamics of the asset process over longer periods of time.

[1] The settlement term denotes the number of years that losses can be carried forward/back.

4.1 Carry forward

At the end of year two, the following post-tax asset value holds for a firm without deferred taxes and excluding carry back possibilities

$$\widetilde{A}_2 = A_2 - \tau(A_2 - \widetilde{A}_1 - \underbrace{1_{A_1<A_0}(A_0 - A_1)}_{=CF_2})^+. \tag{4.1}$$

In this formula, $\widetilde{A}_1$ is given by (3.2) and the indicator function is included to account for carry forward possibilities.[2] In case a firm has carry forward ($= CF_1$) which has a settlement term of one year, the formula is similar, except that $\widetilde{A}_1$ is now given by (3.3) Allowing carry back results in a linear combination of options multiplied by indicator functions to keep track of carry forward and carry back situations.

$$\widetilde{A}_2 = A_2 - \underbrace{1_{A_1<A_0}\tau(A_2 - A_0)^+}_{I} - \underbrace{1_{A_0<A_1<A_0+CF_1}\tau(A_2 - A_1)^+}_{II} + \underbrace{1_{A_1>A_0+CF_1}(\tau CB_2 - \tau(A_2 - \widetilde{A}_1 + CB_2)^+)}_{III},$$

where $\widetilde{A}_1$ is given by (3.3). This formula follows from considering 3 separate cases.

(i) Term I: If $A_1 < A_0$, a loss is incurred, carry forward expires worthless and no taxes are paid. However, the loss incurred in period one $(A_0 - A_1)$ can be carried forward, so that the strike of the call option in year two equals $A_1 + (A_0 - A_1) = A_0 > A_1$.

(ii) Term II: This concerns a situation in which profit is made which is less than the total carry forward, so that no taxes are paid and no carry forward nor carry back is taken to period two.

(iii) Term III: If profit is greater than carry forward, the entire carry forward is used and

[2] Alternatively, we could rewrite the last term as $1_{A_1<A_0}(A_0 - A_1) = (A_0 - A_1)^+$ to highlight the *nested* option like nature of the payoff. This would be more consistent with previous notation, but we refrain from doing so for notational convenience.

corporate tax is paid over the amount $A_1 - A_0 - CF_1$. This amount can subsequently be taken to period two, where it can be used as carry back (CB_2). Hence, in the two-period model, the payoff structure of the assets after tax already becomes quite involved.

Generalizing the formulas above for time periods $t \geq 3$ is certainly possible, but will not be pursued here. Excluding carry back and assuming a two year settlement term of carry forward still results in (4.1) for companies without deferred taxes, but a firm with carry forward in period $t = 0$ gives rise to

$$\widetilde{A}_2 = A_2 - \tau(A_2 - \widetilde{A}_1 - 1_{A_1 < A_0 + CF_1}(A_0 + CF_1 - A_1))^+.$$

This formula holds, since, if at time $t = 1$ the asset value is less than $A_0 + CF_1$, some part of carry forward has not been used and can be carried over to period two. Also, $\widetilde{A}_1$ is given by (3.3). The payoff after tax still has the structure of a call option, but the path dependency complicates analytical tractability.

4.2 Carry back

When determining the value of carry back, we always assume that loss carry forward can be created in subsequent years, as there is no country in the world that allows carry back but no carry forward. The opposite situation is ubiquitous in many tax regimes (see also Table 2). We can express the asset value after tax of a firm with carry back in the two year model as

$$\widetilde{A}_2 = A_2 - 1_{A_1 < A_0 - CB_1}\left(\tau(A_2 - \widetilde{A}_1^{(1)} - \underbrace{(A_0 - CB_1 - A_1)}_{=CF_2})^+\right) \tag{4.2}$$
$$- 1_{A_0 - CB_1 \le A_1 \le A_0}\left(\tau(A_2 - \widetilde{A}_0)^+\right)$$
$$+ 1_{A_1 > A_0}\left(\tau \underbrace{(A_1 - A_0)}_{=CB_2} - \tau(A_2 - \widetilde{A}_1^{(2)} + \underbrace{(A_1 - A_0)}_{=CB_2})^+\right).$$

where $\widetilde{A}_1^{(1)}, \widetilde{A}_2^{(2)}$ are given by (3.4) and (3.2) respectively. The first line follows since $A_1 < A_0 - CB_1$, which means that the complete carry back can be settled and additional carry forward in the amount of $A_0 - CB_1 - A_1$ is taken to period two. The second line considers $A_0 - CB_1 \le A_1 \le A_0$, which means that only part of the carry back is settled. Since carry back is only one year valid, the remaining carry back expires worthless as it cannot be taken to period two. The last line treats the condition $A_1 > A_0$, which means the firm made profit and the entire carry back expires worthless. However, additional carry back in the amount of $A_1 - A_0$ can be taken to period two and can be used when incurring a loss in that period.

At last, we consider the situation in which carry back is two years valid. This yields the after-tax asset value in period two

$$\widetilde{A}_2 = A_2 - 1_{A_1 < A_0 - CB_1}\left(\tau(A_2 - \widetilde{A}_1^{(1)} - \underbrace{(A_0 - CB_1 - A_1)}_{=CF_2})^+\right) \tag{4.3}$$
$$+ 1_{A_1 - CB_1 \le A_1 \le A_0}\left(\tau \underbrace{(CB_1 - (A_0 - A_1))}_{=CB_2} - \tau(A_2 - A_0 + \underbrace{(CB_1 - (A_0 - A_1))}_{=CB_2})^+\right)$$
$$+ 1_{A_1 > A_0}\left(\tau \underbrace{(A_1 - A_0 + CB_1)}_{=CB_2} - \tau(A_2 - \widetilde{A}_1^{(2)} + \underbrace{(A_1 - A_0 + CB_1)}_{=CB_2})^+\right).$$

Again $\widetilde{A}_1^{(1)}, \widetilde{A}_2^{(2)}$ are given by (3.4) and (3.2). The difference between (4.2) and (4.3) comes from the last two lines. The first line in (4.3) considers a situation in which the entire carry forward is used in year one and additional carry forward can be taken to year two. This situation is identical to the one where carry back is one year valid. The second line results from the situation in which part of carry back is used, but unlike (4.2), this time the remaining carry back can be taken to year two. The total amount of carry back left for period two equals $CB_1 - (A_0 - A_1)$. Finally, if a firm makes a profit in period one, then CB_1 cannot be used, but because it can be settled in two years, the amount can be taken to period two. Tax is levied over the amount $A_1 - A_0$, and these tax payments can also be taken to period two and used as carry back. This means that $CB_2 = CB_1 + A_1 - A_0$ and explains the last line of (4.3).

4.3 DTL

As opposed to DTA's arising from carry forward/back, there are no regulations on settlement terms of DTL's. A DTL is put on the balance sheet to reflect future tax expenses, but it depends on the specific characteristic of the profit stream when those untaxed profits are materialized. As there are no regulations to guide us here, we consider two different scenarios. In a basic setup, we assume that no intermediate tax payments occur and the DTL is settled at maturity. This gives the post-tax asset value

$$\widetilde{A}_2 = A_2 - (A_2 - A_0 + LTD_1)^+$$

In a more realistic setup, we assume that the DTL is reduced in period one whenever the firm incurs a loss in that period. The reduction in (nominal) DTL value is equal to the corresponding loss. If the loss in period one exceeds the entire DTL value, the DTL disappears in its entirety and carry forward is created over the remaining loss. In case the company makes a profit in period one $= (A_1 - A_0)$, taxes are paid over that profit and the DTL remains the same. The DTL value left is taken to period two, in which settlement is due. Considering each of these three scenarios yields the post-tax asset

$$
\begin{aligned}
\widetilde{A}_2 = A_2 &- 1_{A_1 < A_0 - LTD_1}(A_2 - A_1 - \underbrace{(A_0 - LTD_1 - A_1)}_{=CF_2})^+ \\
&- 1_{A_0 - LTD_1 \leq A_1 \leq A_0}(A_2 - A_1 + \underbrace{(LTD_1 - A_0 + A_1)}_{=gain_2})^+ \\
&- 1_{A_1 > A_0}(A_2 - \widetilde{A}_1 + LTD_1)^+,
\end{aligned}
$$

where $\widetilde{A}_1$ is the post-tax asset value in period one, as given by (3.2).

5 Extension to include levered firms

So far, we ignored the capital structure of a firm. However, the way in which a firm is financed has repercussions for tax payments. Our approach to give a market consistent valuation of deferred taxes depends on comparing tax payments of a counterfactual firm without fiscal history and a firm having the same characteristics with deferred taxes. Introducing debt financing alters tax payments, since coupon payments can be deducted from taxable income, creating the so-called *tax shield*. In the following subsections we examine the effect of coupon payments on DTA's/DTL's.

5.1 Carry forward

We assume that coupon payments are deducted from taxable income before deferred taxes are used. A counterfactual firm (without deferred taxes), making yearly coupon payments due to leverage, has the following post-tax asset value

$$\widetilde{A}_1 = A_1 - \mathcal{C} - \tau(A_1 - A_0 - \mathcal{C})^+, \tag{5.1}$$

where $\mathcal{C}$ is the coupon payment on debt. The rationale behind (5.1) is the following; part of taxable income is reduced by coupon payments, this is the *tax shield* and appears in the $(\cdot)^+$ term. Equation (5.1) contains (3.2) as a special case when debt (D) is zero, since $\mathcal{C} = 0$ in that case.

Remark 5.2. The amount of coupon payment a firm can deduct from taxable income is quite country specific. For example, countries like Italy have a limit on the amount of coupon a firm can deduct, in order to eschew perverse incentives arising from debt financing. Because we aim for some generality in our analysis, we model the amount of interest payments the firm can deduct by an exogenous parameter $\gamma \in [0, 1]$. By doing so, (5.1) is replaced by

$$\widetilde{A}_1 = A_1 - \mathcal{C} - \tau(A_1 - A_0 - \gamma\mathcal{C})^+. \tag{5.3}$$

In analogy with Section 3 and by Remark 5.2 we get the following asset value after tax for levered firms having some carry forward

$$\widetilde{A}_1 = A_1 - \mathcal{C} - \tau(A_1 - A_0 - CF_1 - \gamma\mathcal{C})^+. \tag{5.4}$$

Deducting interest payments from net profits has repercussions for the carry forward value, as the following example shows.

Example 5.5. Suppose an unlevered firm has 20 carry forward available ($CF_1 = 20$) and makes 10 profit in period one, i.e. $A_1 - A_0 = 10$. The firm can use 10 of the carry forward to offset all taxable income. Now consider an identical firm, which is levered and pays 10 interest each year, i.e. $\mathcal{C} = 10$ (and $\gamma = 1$). This means that the factual profit in period one is zero, because $A_1 - A_0 - \mathcal{C} = 0$. As a result, none of the carry forward can be used and expires worthless. Hence, the DTA arising from carry forward is less valuable for levered firms.

Thus, with leverage, generating fiscal loss becomes more likely, which decreases the probability of settling carry forward.

5.2 Carry back

The post-tax asset value of a levered firm with carry back is equal to

$$\widetilde{A}_1 = A_1 - \mathcal{C} + \tau CB_1 - \tau(A_1 - A_0 + CB_1 - \gamma\mathcal{C})^+. \tag{5.6}$$

In contrast to carry forward, tax carry back increases in value as a result of leverage. This is because fiscal profit/loss, as calculated by the difference in asset value less coupon payments, is always less for a levered firm. Thus, it becomes more likely that carry back is materialized, as the following example shows.

Example 5.7. Suppose that an unlevered firm has $CB_1 = 20$ and incurs a loss of 15 in period one , i.e. $A_1 - A_0 = -15$. It can use 15 of the carry back to neutralize the loss in period one. Now consider an identical levered firm carrying interest cost of $\mathcal{C} = 10$ each year. For this firm, the net "profit" is $-15 - 10 = -25$. With 20 carry back available, it can materialize the complete DTA. Hence, in this case carry back is more valuable when a firm is levered.

5.3 DTL

Finally, for firms carrying some tax liabilities (DTL), the post-tax asset value is given by

$$\widetilde{A}_1 = A_1 - \mathcal{C} - \tau(A_1 - A_0 + LTD_1 - \gamma\mathcal{C})^+. \tag{5.8}$$

The following example shows that levered firms are less likely to repay the entire DTL compared to unlevered firms.

Example 5.9. Take a firm having a deferred tax liability of 20 (i.e. $LTD_1 = 20$). If a firm makes a profit of 50 ($A_1 - A_0 = 50$) in the next period, then it has to pay taxes over

70, instead of paying taxes over 50 if it did not have a tax obligation. Consider again an identical firm, which is levered and makes interest payments of $\mathcal{C} = 10$. Consequentially, taxable income is equal to $A_1 - A_0 - \mathcal{C} + LTD_1 = 60$. Hence, the amount of taxes paid is less for levered firms, so that the DTL value is greater for such firms.

6 Market consistent valuation of deferred taxes

In this section we derive explicit market consistent pricing formulas for the various deferred tax items. Since the payoffs of all tax deferrals is reminiscent to the payoff of a European call option, we use the "*ideal market assumptions*" from Merton (1974). In particular, this implies that the value of the assets follow the stochastic differential equation

$$dA_t = \mu A_t dt + \sigma A_t dW_t^P \qquad \mu \in \mathbb{R},\ \sigma > 0, \tag{6.1}$$

where μ is the growth rate, σ is the volatility of the assets and W_t^P is the value of the Brownian motion at time t under physical (probability) measure P.

These assumptions allow us to find a market consistent value of deferred taxes by discounting the post-tax asset value at maturity under risk-neutral measure Q. The assumptions above essentially imply that the valuation of deferred taxes is isomorphic to the valuation of European call options. Since we adopt all necessary assumptions from Merton (1974), we find that the market consistent value of tax deferrals is expressed as a linear combination of Black-Scholes call/put option prices. For future reference, we state the Black-Scholes pricing formula for European call options, expressed in the variables tailored to our model.

$$C^{\mathrm{BS}}(K,T,A_t,\sigma,r,t) = A_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2), \quad \text{where} \tag{6.2}$$
$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\log\left(\frac{A_t}{K}\right) + (r+\frac{\sigma^2}{2})(T-t)\right]$$
$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

In this formula A_t is the starting value of the company in year t, K is the strike price, T is the exercise date, σ is the volatility of the assets, r is the risk-free interest rate and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal. We interpret the strike K in (6.2) as the threshold from which a firm pays taxes. For example, for a firm without fiscal history (see (3.2)), $K = A_0$, since a firm pays taxes whenever $A_1 > A_0$. As such, the quantity $\log(A_t/K)$ for $0 < t \leq 1$ in (6.2) represents the return on the assets when substituting $K = A_0$. Similarly, if a company has carry forward, (3.3) reveals that the strike will be $K = A_0 + CF_1$. In this and subsequent sections we use the following assumption and notation

Assumption 6.3. The following parameters are *fixed*: σ, t, r, A_0. To emphasize the dependency on the varying parameter K, we write $C^{\mathrm{BS}}(K) \triangleq C^{\mathrm{BS}}(K, T = 1, A_0, \sigma, r, t = 0)$. Similarly, $V, V_{\mathrm{cb}}, V_{\mathrm{cf}}, V_A, V_{\mathrm{L}}$ denote the market consistent value of a firm without fiscal history, with carry back, with carry forward, with a DTA from temporary differences and with a DTL from temporary differences respectively.

The valuation of deferred taxes is based on comparing the value of two hypothetical firms having the same assets, with the only difference that one firm has deferred taxes on the balance sheet. We frequently refer to the firm without deferred taxes as the *counterfactual firm*. Our terminology is inspired by the potential outcome framework of Rubin (1974).

Definition 6.4 (Market consistent valuation)**.** The market consistent value of a DTA/DTL is defined as the difference in firm value between a company with deferred taxes and a counterfactual firm with the same assets, without having deferred taxes. The precise value of the DTA/DTL then follows from comparing the discounted payoff of the post-tax assets at final time $t = T$ under martingale measure. We will henceforth refer to this quantity by ξ_a, where the subscript refers to the type of DTA/DTL.

6.1 Carry forward

Let us now turn to the original quest of determining a market consistent valuation of deferred taxes. First, we take a firm without fiscal history, whose asset value after tax at time one is given by (3.2). This is a contingent T-claim, whose value at time zero is given by[3]

$$V = e^{-r}\mathbb{E}^Q\big(A_1 - \tau(A_1 - A_0)^+|\mathcal{F}_0\big) = A_0 - \tau C^{\mathrm{BS}}(K = A_0), \tag{6.5}$$

where C^{BS} is the Black-Scholes price of a European at-the-money call option. In (6.5) we have $K = A_0$, so that by virtue of (6.2) we get $d_1 = \frac{1}{\sigma}(r + \frac{\sigma^2}{2})$ and similarly $d_2 = \frac{1}{\sigma}(r - \frac{\sigma^2}{2})$. It is well known that the Black-Scholes call option value is greater than or equal to the payoff received at expiry. Hence, V is always less than the actual asset value minus tax at expiry in (3.2).

Similar analysis allows us to find the market consistent value of a company with carry forward. Again, by martingale pricing, the no-arbitrage value of a company with carry forward is found by discounting (3.3)

$$\begin{aligned} V_{\mathrm{cf}} &= e^{-r}\mathbb{E}^Q\big(A_1 - \tau(A_1 - A_0 - CF_1)^+|\mathcal{F}_0\big) \\ &= A_0 - \tau C^{\mathrm{BS}}(K = A_0 + CF_1). \end{aligned} \tag{6.6}$$

Notice that V_{cf} in (6.6) is always greater than or equal to V appearing in (6.5). This makes sense, because a company having future tax deduction possibilities should be more valuable

[3]Throughout the paper, we always assume $\mathcal{F}_t = \sigma(W_s|0 \leq s \leq t)$, i.e. the sigma algebra generated by the Brownian motion up to time t.

than a company that doesn't have these possibilities. By Definition 6.4, the market consistent DTA value of carry forward follows by comparing (6.6) with (6.5), which yields

$$\xi_{\mathrm{cf}} \triangleq V_{\mathrm{cf}} - V = \tau\Big(C^{\mathrm{BS}}(A_0) - C^{\mathrm{BS}}(A_0 + CF_1)\Big). \tag{6.7}$$

Equation (6.7) is monotonically increasing in CF_1 and always bigger than zero, however for large values of CF_1 the additional benefit of extra carry forward is rather limited. This can also be seen from Figure 2a, which shows the diminishing marginal returns of carry forward for various tax rates. The reason is that expected profits are insufficient to utilize additional carry forward.

It is instructive to analyze the sensitivity of the DTA (or DTL) value with respect to the variable from which the DTA arises (such as CF_1). The following proposition facilitates these computations, which is a standard result in option pricing theory, see e.g. Dupire et al. (1994).[4]

Proposition 6.8. *For a standard European call option with constant interest rate we have the following expression for the first and second derivative with respect to the call price*

$$\frac{\partial}{\partial K}C(K,T,A_t,\sigma,r,t) = -e^{-r(T-t)}(1-F(K))$$
$$\frac{\partial^2}{\partial K^2}C(K,T,A_t,\sigma,r,t) = e^{-r(T-t)}f(K),$$

where $F(K)$ and $f(K)$ are the risk-neutral CDF and PDF of the underlying asset respectively.

By Proposition 6.8, the derivative of (6.7) is given by

$$\frac{\partial}{\partial CF_1}\xi_{\mathrm{cf}} = \tau e^{-r}(1 - F(A_0 + CF_1)). \tag{6.9}$$

Equation (6.9) leads to an interesting interpretation. Because $F(x)$ is the risk-neutral prob-

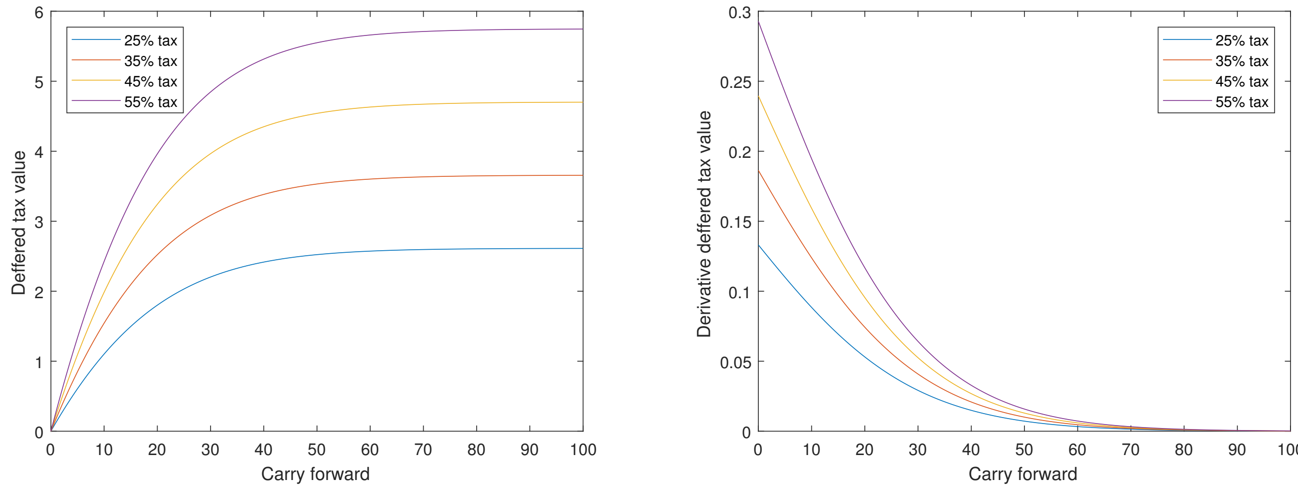
[4] These expressions are independent of the Black-Scholes model and can be derived irrespective of modeling issues.

ability that assets at time one are less than x, we can rewrite (6.9) to $\tau \exp(-r) Q(A_1 > A_0 + CF_1)$, where Q is the risk-neutral measure.[5] In other words, the sensitivity w.r.t. CF_1 is equal to the probability of $\{\omega \in \Omega : A_1(\omega) > A_0 + CF_1\}$ under the risk-neutral measure Q, weighted by a discount factor consisting of the tax rate τ and the risk-free rate. The event $\{\omega \in \Omega : A_1(\omega) > A_0 + CF_1\}$ corresponds to the risk-neutral probability that the entire carry forward will be used. This bears some resemblance to current valuation methods, which are discussed in Section 7.1. The variable τ works as a kind of amplification factor; higher tax rates increase the sensitivity of the DTA to CF_1 because a change in CF_1 has a more pronounced effect on firm value.

Since distribution functions are always bounded by one (and non decreasing), it follows that (6.9) is always positive. Taking the (formal) limit $CF_1 \to \infty$ renders that (6.9) goes to zero. Initially, if CF_1 is small, there is a high probability that the entire carry forward can be used for tax deduction, so a small change leads to a relatively big change in DTA value. On the contrary, if CF_1 is high, it is not likely that the entire carry forward will be used for tax deduction (since future profits are unlikely to settle the complete carry forward), so that a change in CF_1 does not have a considerable effect on the DTA value. Figure 2b illustrates the behavior of (6.9) for different tax rates.

Figure 3a shows the value of CF_1 according to (6.7) together with the value of CF_1 at maturity. Initially the market consistent value of carry forward is worth more than the final value at maturity. This is because there still exists a probability that some of the carry forward will be used. At some point, however, the upward potential is not enough to offset the guaranteed carry forward value at maturity, causing the graphs to intersect so that the market consistent value is worth less than the guaranteed payoff for large values of A_1. The structure of the payoff has the same form as that of a *bull call spread*, which corresponds to the option trading strategy used by investors to profit from the limited rise of an underlying security.

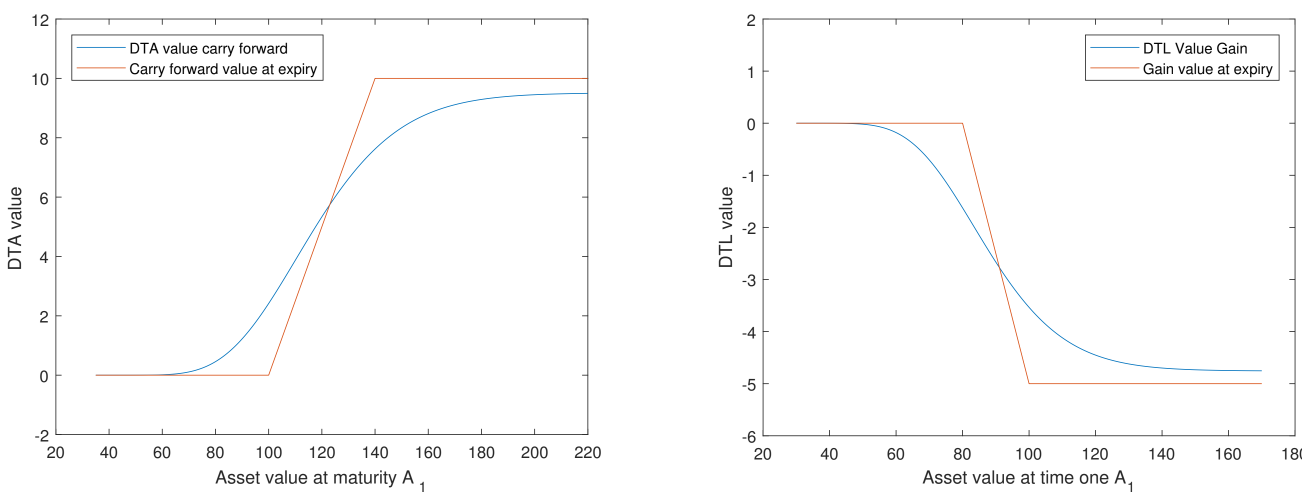
[5] In fact, we know that in the Black-Scholes model $F(x)$ is the CDF of the Log-normal distribution.



(a) DTA value given from carry forward (6.7) as function of carry forward (CF_1) for different tax rates. Parameters: $A_0 = 100, r = 0.05, \sigma = 0.2$.

(b) Derivative of DTA from carry forward (6.9) as a function of carry forward (CF_1) for different tax rates. Parameters: $A_0 = 100, r = 0.05, \sigma = 0.2$.

Figure 2



(a) Value of CF_1 under Black-Scholes vs. payoff at maturity. Parameters: $r = 0.05, \sigma = 0.2, A_0 = 100, CF_1 = 40, \tau = 0.25$.

(b) Value of DTL under Black-Scholes vs. payoff at maturity. Parameters: $r = 0.05, \sigma = 0.2, A_0 = 100, LTD_1 = 20$.

Figure 3

6.2 Carry back

The market consistent value of a company with carry back follows by discounting (3.4), which gives

$$
\begin{aligned}
V_{\text{cb}} &= e^{-r}\mathbb{E}^Q(A_1 + \tau CB_1 - \tau(A_1 - A_0 + CB_1)^+ | \mathcal{F}_0) \\
&= A_0 + e^{-r}\tau CB_1 - \tau C^{\text{BS}}(K = A_0 - CB_1). \qquad (6.10)
\end{aligned}
$$

In a similar vein, we obtain the market consistent DTA value of carry back by comparing the difference (6.10) and (6.5)

$$
\xi_{\text{cb}} \triangleq V_{\text{cb}} - V = e^{-r}\tau CB_1 - \tau\Big(C^{\text{BS}}(A_0 - CB_1) - C^{\text{BS}}(A_0)\Big). \qquad (6.11)
$$

Equation (6.11) expresses the DTA value of carry back as a linear combination of two factors. The first one corresponds to the value of carry back today if no settlement risk were involved. However, because it is not guaranteed that the (entire) carry back will be materialized at period one, the second term is subtracted to take this risk into account. By Proposition 6.8, the sensitivity of carry back to its DTA value is expressed by

$$
\begin{aligned}
\frac{\partial}{\partial CB_1}\xi_{\text{cb}} &= \tau e^{-r}\Big(1 - \big(1 - F(A_0 - CB_1)\big)\Big) \\
&= \tau e^{-r} F(A_0 - CB_1). \qquad (6.12)
\end{aligned}
$$

The factor after the tax rate is the probability that A_1 exceeds the asset value at time zero minus the carry back under the risk neutral measure Q, i.e. $Q(A_1 < A_0 - CB_1)$. So alternatively we may write[6]

$$\frac{\partial}{\partial CB_1}\xi_{\text{cb}} = \tau e^{-r} Q(A_1 < A_0 - CB_1).$$

The probability of the event $\{\omega \in \Omega : A_1 < A_0 - CB_1\} = \{\omega \in \Omega : A_0 - A_1 \geq CB_1\}$ is the probability (under risk-neutrality) that the loss in period one is sufficient to materialize the complete carry back. Also, from here it follows that (6.12) is always greater than zero and decreasing in CB_1 until $CB_1 \leq A_0$ after which it vanishes since $\{\omega \in \Omega : A_1(\omega) \leq 0\}$ has (risk-neutral) probability measure zero. The latter observation holds since we always have the constraint $CB_1 \leq A_0$. If this condition is not satisfied, the asset value prior to time $t = 0$ (say $t = -1$) would be less than zero, i.e. $A_{-1} < 0$. This cannot happen with probability one since the asset value is always bigger than zero by definition. Figure 4 shows the value of CB_1 in the market consistent model together with its value at expiry. In contrast to carry forward, the DTA value is less than the payoff at maturity when A_1 is small. The "payoff" structure of carry back is similar to the *bear spread strategy*, which is used by option traders to profit from the limited decrease of an underlying security.

6.3 DTA from temporary differences

The post-tax asset value of a firm with a DTA from temporary differences is given by (3.6). Discounting under risk-neutral measure gives the firm value

$$\begin{aligned} V_{\text{A}} &= e^{-r}\mathbb{E}^Q(A_1 - \tau(A_1 - A_0 - ATD_1)^+ | \mathcal{F}_0) \\ &= A_0 - \tau C^{\text{BS}}(K = A_0 + ATD_1) \end{aligned}$$

[6]The event $Q(A_1 < A_0 - CB_1)$ can be written down in explicit terms. Let $X \sim N(0,1)$, then $Q(A_1 < A_0 - CB_1) = Q(A_0 \exp(r - \sigma^2/2 + \sigma X) < A_0 - CB_1) = Q(X \leq \frac{1}{\sigma}(\log(\frac{A_0 - CB_1}{A_0}) + \sigma^2/2 - r)) = \Phi\big(\frac{1}{\sigma}(\log(\frac{A_0 - CB_1}{A_0}) + \sigma^2/2 - r)\big)$.

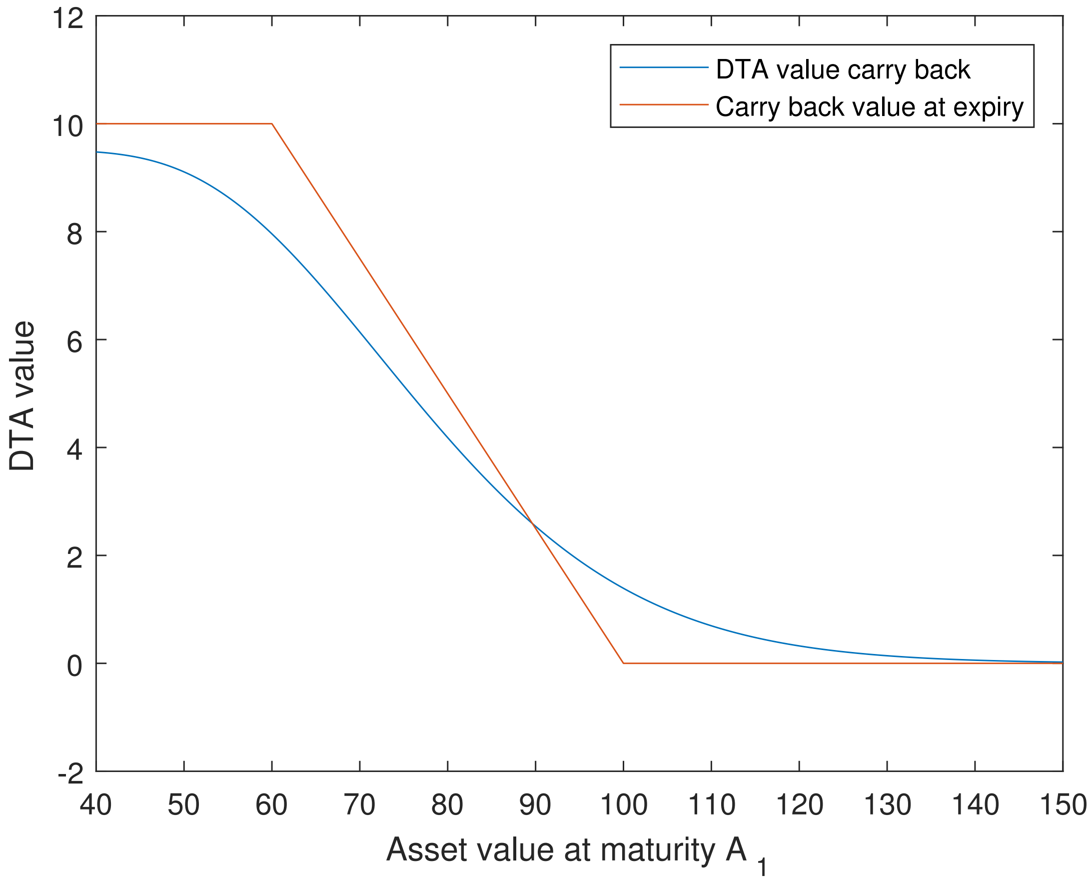


Figure 4: Value of CB_1 under Black-Scholes vs. payoff at maturity. Parameters: $r = 0.05, \sigma = 0.2, A_0 = 100, CB_1 = 40$.

Since the post-tax payoff is equivalent to that of carry forward, we find the same structure for the firm value of a company having a DTA from temporary differences. Therefore, the sensitivity analysis is completely equivalent and will be omitted.

6.4 DTL from temporary differences

The firm value of a company with a deferred tax liability follows by discounting (3.7) under martingale measure, which yields

$$\begin{aligned} V_{\mathrm{L}} &= e^{-r}\mathbb{E}^{Q}(A_1 - \tau(A_1 - A_0 + LTD_1)^{+}|\mathcal{F}_0) \\ &= A_0 - \tau C^{\mathrm{BS}}(A_0 - LTD_1). \end{aligned} \tag{6.13}$$

We stick to the convention of modeling the difference between normal tax conditions and a DTL by a negative quantity. Thus the market consistent DTL value is obtained by subtracting (6.13) from (6.5), which gives

$$\xi_L \triangleq V_{\mathrm{L}} - V^{\mathrm{BS}} = \tau\Big(C^{\mathrm{BS}}(A_0) - C^{\mathrm{BS}}(A_0 - LTD_1)\Big). \tag{6.14}$$

The shape of (6.14) as a function of A_1 is similar to that of carry back, which can be seen from Figure 3b. However, this time the values are negative, since the DTL is a tax obligation and hence a future liability. For large values of A_1, the untaxed profit (as measured by LTD_1) has to be paid in its entirety to the tax authority, by the amount of $\tau \cdot LTD_1$.

At last we turn to the sensitivity of the DTL to the untaxed profit LTD_1. Straightforward differentiation of (6.14) in conjunction with Proposition 6.8 gives

$$\frac{\partial}{\partial LTD_1}\xi_L = -\tau e^{-r}(1 - F(A_0 - LTD_1)) = -\tau e^{-r}Q(A_1 > A_0 - LTD_1). \tag{6.15}$$

Hence, higher values of LTD_1 increase the likelihood of $\{\omega \in \Omega : A_1(\omega) > A_0 - LTD_1\}$, which means that (6.15) is decreasing in LTD_1, but stabilizes when $LTD_1 \geq A_0$. The latter condition is excluded, since $LTD_1 < A_0$ by construction.[7] A company starts to lose value from the moment $A_1 > A_0 - LTD_1$, so higher values of LTD_1 lead to a more severe reduction in asset value compared to a firm with lower LTD_1. Hence, the slope of (6.14) ought to be decreasing.

6.5 Aggregate DTA value

We now outline how the valuation formulas for DTA's/DTL's can be combined into a single formula that incorporates all four market consistent DTA/DTL values.

Theorem 6.16 (Aggregate deferred tax value)**.** *The market consistent* net DTA *value is given by*

$$\begin{aligned}\xi = e^{-r}\tau(CF_1 + ATD_1 - LTD_1 + CB_1) \\ - \tau\Big(P^{BS}(A_0 + CF_1 + ATD_1 - LTD_1) - P^{BS}(A_0)\Big) \\ - \tau\Big(C^{BS}(A_0 - CB_1) - C^{BS}(A_0)\Big). \quad (6.17)\end{aligned}$$

Remark 6.18. There is no subscript for $\xi_{(\cdot)}$ in (6.17). Whenever the subscript is omitted, we indicate the aggregate DTA value.

Proof. First, combine the payoffs (3.3), (3.6) and (3.7) into the single equation

$$\widetilde{A}_1 = A_1 - \tau(A_1 - A_0 - CF_1 - ATD_1 + LTD_1)^+. \quad (6.19)$$

We interpret (6.19) as the net DTA position, apart from carry back. In this case the strike equals $K = CF_1 + ATD_1 - LTD_1$. Let us temporarily write $\Upsilon = CF_1 + ATD_1 - LTD_1$. Recall the *put-call parity*:

$$C(K) + Ke^{-r} = P(K) + A_0, \quad (6.20)$$

[7] Remember that LTD_1 is the untaxed profit made in period $t = 0$, so that $LTD_1 + A_{-1} = A_0 \implies LTD_1 < A_0$.

where $P(K)$ is the value of a European put option with strike K and we maintain the same notation as for the call option. The market consistent DTA value of carry forward (6.7) is the difference between two call options. Hence, put-call parity gives

$$\begin{aligned}
\xi &= \tau\Big(C^{\mathrm{BS}}(A_0) - C^{\mathrm{BS}}(A_0 + \Upsilon)\Big) \\
&\overset{\text{P-C parity}}{=} \tau\Big(P^{\mathrm{BS}}(A_0) + A_0 - A_0 e^{-r} - (P^{\mathrm{BS}}(A_0 + \Upsilon) + A_0 - (A_0 + \Upsilon)e^{-r})\Big) \\
&= e^{-r}\tau\Upsilon - \tau\Big(P^{\mathrm{BS}}(A_0 + \Upsilon) - P^{\mathrm{BS}}(A_0)\Big) \\
&= e^{-r}\tau(CF_1 + ATD_1 - LTD_1) - \tau\Big(P^{\mathrm{BS}}(A_0 + CF_1 + ATD_1 - LTD_1) - P^{\mathrm{BS}}(A_0)\Big).
\end{aligned} \tag{6.21}$$

As a result, the aggregate market consistent net DTA value is obtained by adding (6.21) and (6.11), which yields

$$\begin{aligned}
\xi &= e^{-r}\tau(CF_1 + ATD_1 - LTD_1 + CB_1) \\
&\quad - \tau\Big(P^{\mathrm{BS}}(A_0 + CF_1 + ATD_1 - LTD_1) - P^{\mathrm{BS}}(A_0)\Big) \\
&\quad - \tau\Big(C^{\mathrm{BS}}(A_0 - CB_1) - C^{\mathrm{BS}}(A_0)\Big).
\end{aligned}$$

□

Equation (6.17) is the sum of three components: the discounted value of deferred taxes when no settlement risk is involved, the difference between two put options and the difference between two call options both included to reflect settlement risk. By construction, carry back and carry forward are *mutually exclusive*, which means it is impossible to have carry forward and carry back at the same time. Our interpretation of (6.17) is therefore as follows:

- If the firm has loss carry forward, then $CB_1 = 0$ and (6.17) reduces to

$$\begin{aligned}\xi &= e^{-r}\tau(CF_1 + ATD_1 - LTD_1)\\ &\quad - \tau\Big(P^{\mathrm{BS}}(A_0 + CF_1 + ATD_1 - LTD_1) - P^{\mathrm{BS}}(A_0)\Big).\end{aligned}$$

- If the firm has loss carry back, then $CF_1 = 0$ and (6.17) reduces to

$$\begin{aligned}\xi &= e^{-r}\tau(ATD_1 - LTD_1 + CB_1)\\ &\quad - \tau\Big(P^{\mathrm{BS}}(A_0 + ATD_1 - LTD_1) - P^{\mathrm{BS}}(A_0)\Big)\\ &\quad - \tau\Big(C^{\mathrm{BS}}(A_0 - CB_1) - C^{\mathrm{BS}}(A_0)\Big).\end{aligned}$$

In both situations it is perfectly possible to have deferred taxes from temporary differences alongside carry back or carry forward.

7 Differences with current accounting and valuation practices

We now compare our market consistent approach to the extant accounting valuation techniques. These practices ignore value creation due to loss carry back, neither are deferred taxes coming from temporary differences covered. Hence, we only compare our valuation techniques based on loss carry forward. As a result, the aggregate net DTA value in (6.17) collapses to the market consistent carry forward value in (6.7).

7.1 GAAP and IAS12

In Section 3.3 we explained that extant valuation procedures acknowledge the underlying value of the different type of DTA/DTL as the nominal amount appearing on the balance sheet. We compare our method to two frequently employed accounting principles: GAAP and IAS12. Most other accounting guidelines use techniques for valuing deferred taxes by methods either based on GAAP or IAS12. However, both of these guidelines neglect carry back possibilities and the value creation as a result of this possibility. Hence, we only draw comparison to carry forward valuation.

Let us first analyze the way under which GAAP computes the value of carry forward. Under GAAP principles, carry forward is recognized whenever there is a more than 50% chance that future profit settles the complete carry forward (The Financial Accounting Standards Board (1992), Waegenaere et al. (2003)).

In other words, the DTA arising from carry forward is recognized completely if and only if the probability of materializing the entire carry forward has a more than 50% chance. If this is not the case, a valuation allowance (VA) is issued, which reduces the overall DTA value. In our model, this translates to the condition

$$P(A_1 - A_0 \leq A_1^*) = \frac{1}{2},$$

where A_1^* is the median profit and $P(\cdot)$ the physical probability measure. The geometric Brownian motion assumption of the asset process (6.1) renders the explicit expression $A_1^* = A_0 e^{\mu - \sigma^2/2} - A_0$. Hence, for carry forward, a valuation allowance is issued whenever

$$CF_1 > A_1^* \iff CF_1 > A_0(e^{\mu - \sigma^2/2} - 1). \tag{7.1}$$

This expression reveals that sufficiently large values of volatility always lead to the issuance of a valuation allowance, i.e. when $\sigma^2/2 \gg \mu$. Secondly, it is less likely that a valuation allowance is issued for large values of the starting value A_0 when $\mu - \sigma^2/2 > 0$. This is because the geometric Brownian motion assumption on the asset process is concerned with relative profits. So for higher values of A_0, a small percentage change leads to a more pronounced difference in absolute asset values, which makes it more likely that the complete carry forward will be settled. In analogy to Waegenaere et al. (2003), a valuation allowance (VA) is issued if (7.1) holds and the corresponding balance sheet value is given by

$$VA = \Big(\tau\big(CF_1 - A_1^*\big)\Big)^+.$$

The DTA value under the GAAP approach thus takes the form

$$\begin{aligned} \xi_{\mathrm{cf},GAAP} &\triangleq DTA - VA \\ &= \tau\Big(CF_1 - (CF_1 - A_1^*)^+\Big). \end{aligned} \tag{7.2}$$

It follows from (7.2) that the DTA value arising from carry forward stabilizes when it hits the threshold level for which it becomes more likely than not to settle the complete carry forward. Interestingly, in Figure 5a we see that the GAAP approach is higher for small values of CF_1 but is lower for high values of CF_1. However, this relation is ambiguous as A_1^* depends on the growth parameter μ. To see this, for large values of CF_1, the DTA value of carry forward $\xi_{\mathrm{cf}}^{\mathrm{BS}}$ in (6.7) goes to $\tau C^{\mathrm{BS}}(A_0)$. In contrast, $\xi_{\mathrm{cf},GAAP} \to \tau A_1^*$ for large values of CF_1. Therefore, the relation boils down to comparing $C^{\mathrm{BS}}(A_0)$ with A_1^*. However, the relation between these two quantities is inconclusive because A_1^* depends on μ. For the particular case shown in Figure 5a, μ is chosen small enough such that the market consistent value is eventually higher than the GAAP value. But we might equally well take μ so large that the relationship breaks down eventually.[8] On the other hand, it always holds true that the GAAP approach renders higher values for the DTA when $CF_1 < A_1^*$.

Valuation guidelines of deferred taxes under IAS12 are less flexible. According to these accounting principles, deferred taxes are recognized only if there is a more than 50% chance that the complete DTA will be materialized (Deloitte, 2017). Otherwise, the deferred tax asset is not recognized. Hence, we have the following value of the DTA arising from carry

[8]In fact, $\mu = 0.12$ is already sufficient in this example.

forward under IAS12

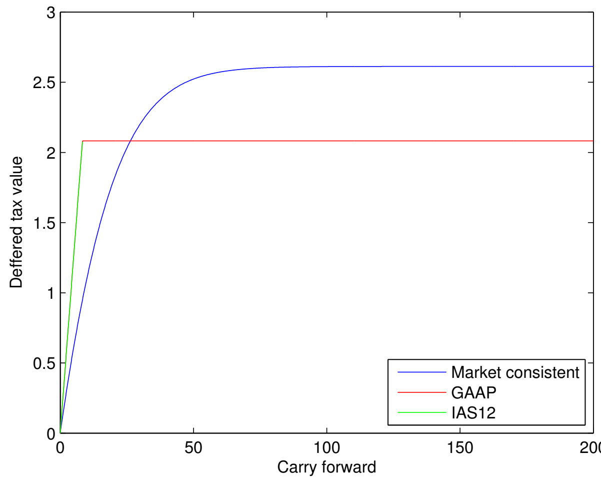
$$\xi_{\mathrm{cf},IAS12} = \tau 1_{\{CF_1 \le A_1^*\}} CF_1.$$

The carry forward value under IAS12 in Figure 5a (green line) concurs with the GAAP value, but vanishes as soon as carry forward exceeds the median profit. In some sense, our model contains the GAAP and IAS12 approach as a special case, namely if we take $\lim_{\sigma\to 0^+} \xi_{\mathrm{cf}}^{\mathrm{BS}} = \tau e^{-r} CF_1$, provided that $(e^r - 1)A_0 > CF_1$. This is almost equal to the GAAP and IAS12 value, apart from the discounting term. Our model not only encompasses the GAAP and IAS12 approach as special cases, but is preferred in certain other aspects:

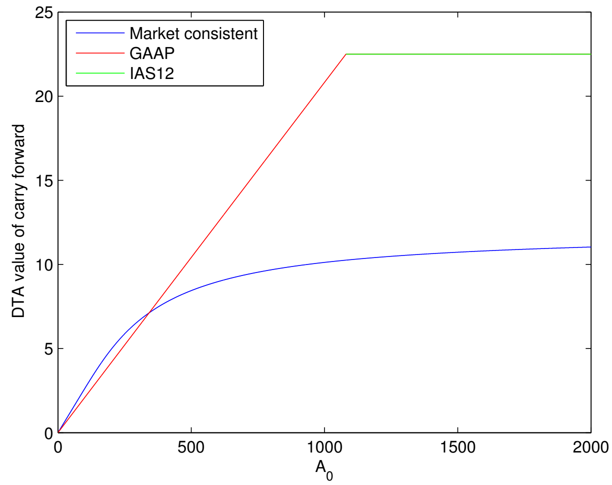
(i) The market consistent approach does not depend on the subjective substantiation of future profit. In our model, the uncertainty of future profit is implicitly measured by the volatility of the assets σ, which determines the likelihood of materializing the entire carry forward under all future scenarios.

(ii) If the probability of realizing the entire carry forward is considerable, the GAAP and IAS12 approach are not in line with conventional economic theory, which suggests that the nominal carry forward value should at least be discounted to reflect time preferences. However, the precise discounting value is somewhat diffuse and depends on parameters difficult to measure, such as likelihood and timing of the settlement (Givoly and Hayn, 1992). Other researchers even find evidence against discounting of deferred taxes (e.g. Amir et al. (1997)), which can be explained by assuming a skewed income distribution (Waegenaere et al., 2003).

As in the previous section(s), we analyze the sensitivity of the conventional accounting valuation principles to carry forward. The GAAP and IAS12 valuation methods are not classically differentiable. At least the GAAP approach gives rise to a function that is weakly



(a) DTA value of carry forward as a function of CF_1 under different valuation approaches. Parameters: $A_0 = 100, \tau = 0.25, r = 0.05, \sigma = 0.2, \mu = 0.1$.



(b) DTA value of carry forward as a function of A_0 under different valuation approaches. Parameters: $\tau = 0.25, CF_1 = 90, r = 0.05, \sigma = 0.2, \mu = 0.1$.

Figure 5

differentiable, where the weak derivative is given by[9]

$$\frac{\partial}{\partial CF_1}\xi_{\text{cf},GAAP} = \begin{cases} \tau, & \text{for } CF_1 \le A_1^* \\ 0, & \text{else.} \end{cases} \tag{7.3}$$

The sensitivity is larger compared to the market consistent sensitivity in (6.9) at each point on the support of (7.3).

Another illuminating quantity is the dependence on the initial start value A_0, which is shown in Figure 5b. Initially, the asset value is so small that materializing the complete carry forward is unlikely. Hence, under GAAP principles, a valuation allowance is issued, which reduces the nominal value of CF_1. However, at some point, the initial asset value is large enough so that the probability of materializing the complete carry forward is larger than 50%. At this point, $\xi_{\text{cf},GAAP}$ stabilizes and becomes constant. This is also the point where $\xi_{\text{cf},IAS12}$ gets positive and concurs with IAS12 (green line in Figure 5b). Both $\xi_{\text{cf},GAAP}$

[9]Recall that a function f has weak derivative g if $\int_{\mathbb{R}} f(x)\varphi(x)'dx = -\int_{\mathbb{R}} g(x)\varphi(x)dx$ for all $\varphi \in C_c^\infty(\mathbb{R})$, which is the space of smooth functions with compact support.

and $\xi_{\text{cf},IAS12}$ converge to τCF_1 for large values of A_0.

Remark 7.4. The size of A_0 relative to CF_1 plays an important role in the loss absorbing capacity of deferred taxes. After a negative shock in the asset value, a loss is incurred, which can be used as carry forward. Hence, CF_1 increases, but the asset value decreases. The asset value after shock can be so low that the market consistent carry forward is actually worth less than it was before, even though the nominal value increased.

8 Valuation of deferred taxes including levered firms

8.1 Valuation of debt

We first discuss the valuation of debt before we introduce market consistent prices for tax deferrals of levered claims. Since debt is *not* assumed to be risk-free we cannot assume that the coupon payment in year one equals $\mathcal{C} = (e^r - 1)D$. Instead, we opt for a market consistent valuation of debt, which incorporates the *limited liability* of bondholders, just as in Merton (1974). However, we cannot directly apply Merton's result, since tax payments may influence bankruptcy conditions.

In the following, we always assume that the debt value at time $t = 0$ (denoted by D_0) is less than the asset value at time zero, i.e. $D_0 \leq A_0$. Moreover, we also assume that claims by the tax authority rank above those of general creditors and equity holders. In the U.S., this is called the *absolute priority rule* (Brouwer, 2006). This is important, since it may happen that the firm is solvent before paying taxes, but insolvent after tax is levied. Over a one year time horizon, this can only happen if $\gamma < 1$.

Example 8.1. Suppose $A_0 = 100, D = 95, \mathcal{C} = 20, \gamma = 0.5$ and $\tau = 0.5$. In addition, assume that $A_1 = 118$ (before tax). Notice that before tax, the company is solvent, since $A_1 > D + \mathcal{C} = 115$. However, the post-tax asset value is $\widetilde{A}_1 = A_1 - \tau(A_1 - A_0 - \gamma\mathcal{C})^+ = 114 < D + \mathcal{C}$ and the company is bankrupt.

Remark 8.2. One cannot concoct similar examples if $\gamma = 1$.

Hence, we assume that bankruptcy is triggered whenever $\widetilde{A}_1 < D + \mathcal{C}$. Therefore, in period one, debtholders receive

$$\mathcal{C} + \min(\widetilde{A}_1, D).$$

Using the risk-neutral pricing paradigm, we obtain the market consistent value of debt in period zero

$$D_0 = e^{-r}\mathcal{C} + e^{-r}\mathbb{E}^Q\big(\min(D, \widetilde{A}_1)\big), \tag{8.3}$$

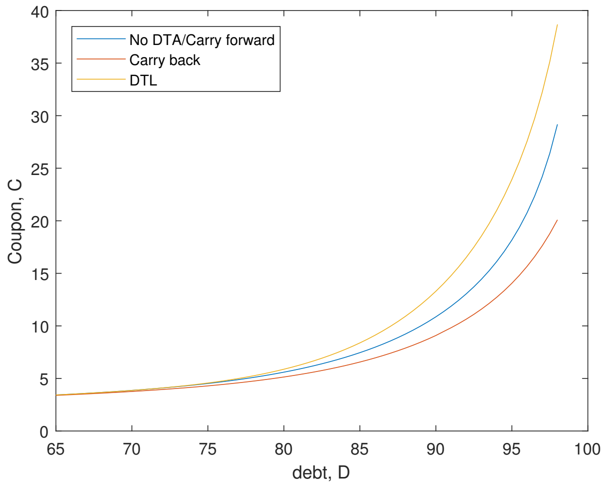
which is recognized as Merton's debt value (Merton, 1974) with coupon payments. The precise form of $\widetilde{A}_1$ depends on the availability of deferred taxes.

8.2 Carry forward

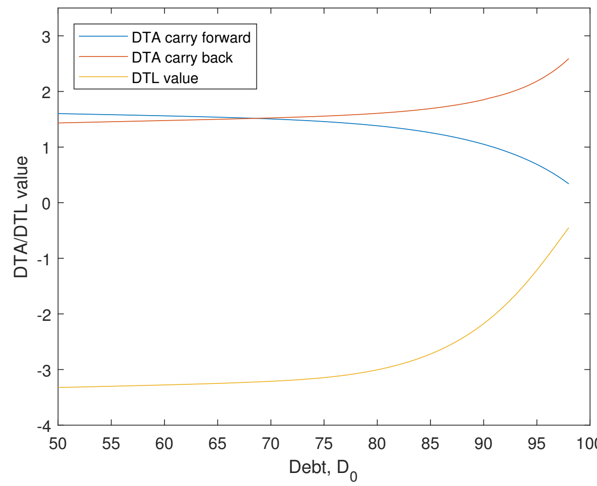
Initially, we suppose that $\gamma = 1$, so that a firm can subtract all interest payments from taxable income. In this case, carry forward has no influence on the bankruptcy condition, since bankruptcy only occurs if the firm incurs a loss. This is because we assume that $D_0 \leq A_0$. Hence, by writing $\min(D, \widetilde{A}_1) = D - (D - \widetilde{A}_1)^+$, (8.3) equals

$$\begin{aligned} D_0 &= e^{-r}(\mathcal{C} + D) - e^{-r}\mathbb{E}^Q\big((D - \widetilde{A}_1)^+\big) \\ &= e^{-r}(\mathcal{C} + D) - e^{-r}\mathbb{E}^Q\big((D + \mathcal{C} - A_1)^+\big) \\ &= e^{-r}(D + \mathcal{C}) - P^{BS}(D + \mathcal{C}). \end{aligned} \tag{8.4}$$

The second line follows from $\widetilde{A}_1 < D \iff A_1 - \mathcal{C} < D$, since no taxes have to be paid in case of a loss. The notation $P^{BS}(K)$ denotes the Black-Scholes price of a European call option with strike K. We choose $\mathcal{C}$ such that $D_0 = D$. The solution can be obtained by numerical methods, e.g. Newton-Raphson iteration. The resulting coupon payment is shown in Figure 6a as a function of D. For sufficiently high levels of debt, the coupon $\mathcal{C}$ is seen to rise exponentially as a consequence of the imminence of bankruptcy.



(a) Coupon payment $\mathcal{C}$ on risky debt as a function of debt D for companies with different deferred taxes. Parameters: $A_0 = 100, r = 0.05, \sigma = 0.2, \tau = 0.25, CB_1 = 10, LTD_1 = 30, \gamma = 1$.



(b) DTA/DTL value of levered firms seen as a function of debt. The coupon payment corresponding to debt level D_0 is determined via numerical methods and depends on the deferred tax. Parameters: $A_0 = 100, r = 0.05, \sigma = 0.2, \tau = 0.25, CF_1 = CB_1 = LTD_1 = 20$.

Figure 6

The more general case corresponding to $\gamma \in [0, 1]$ yields a more complicated form than (8.4). In particular, D_0 for fixed $\mathcal{C}$ corresponding to a firm without deferred taxes follows from the next theorem.

Theorem 8.5. *The debt value at time zero for a firm without deferred taxes is given by*

$$D_0 = e^{-r}(D + \mathcal{C}) - e^{-r}(D + \mathcal{C})\Phi(\theta_3) + A_0\Phi(\theta_3 - \sigma),$$

if $D + \mathcal{C} \leq A_0 + \gamma\mathcal{C}$. Otherwise

$$\begin{aligned} D_0 &= e^{-r}(D + \mathcal{C}) - e^{-r}(D + \mathcal{C})\Phi(\theta_2) + A_0\Phi(\theta_2 - \sigma) \\ &\quad - \tau A_0[\Phi(\theta_2 - \sigma) - \Phi(\theta_1 - \sigma)] + e^{-r}\tau(A_0 + \gamma\mathcal{C})[\Phi(\theta_2) - \Phi(\theta_1)]. \end{aligned}$$

In these expressions

$$\theta_1 = \frac{1}{\sigma}\left[\log\left(\frac{A_0+\gamma\mathcal{C}}{A_0}\right) - r + \sigma^2/2\right]$$
$$\theta_2 = \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}-\tau(A_0+\gamma\mathcal{C})}{(1-\tau)A_0}\right) - r + \sigma^2/2\right]$$
$$\theta_3 = \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}}{A_0}\right) - r + \sigma^2/2\right].$$

Proof. See Appendix A.1. □

Likewise, debt at time zero for companies with carry forward follows from the next theorem.

Theorem 8.6. *The debt value at time zero for a firm having CF_1 is given by*[10]

$$D_0 = e^{-r}(D+\mathcal{C}_{\mathtt{cf}}) - e^{-r}(D+\mathcal{C}_{\mathtt{cf}})\Phi(\theta_3) + A_0\Phi(\theta_3-\sigma),$$

if $D+\mathcal{C}_{\mathtt{cf}} \leq A_0 + CF_1 + \gamma\mathcal{C}_{\mathtt{cf}}$. Otherwise

$$D_0 = e^{-r}(D+\mathcal{C}_{\mathtt{cf}}) - e^{-r}(D+\mathcal{C}_{\mathtt{cf}})\Phi(\theta_2) + A_0\Phi(\theta_2-\sigma)$$
$$-\tau A_0[\Phi(\theta_2-\sigma)-\Phi(\theta_1-\sigma)] + e^{-r}\tau(A_0+CF_1+\gamma\mathcal{C}_{\mathtt{cf}})[\Phi(\theta_2)-\Phi(\theta_1)].$$

In these expressions

$$\theta_1 = \frac{1}{\sigma}\left[\log\left(\frac{A_0+CF_1+\gamma\mathcal{C}_{\mathtt{cf}}}{A_0}\right) - r + \sigma^2/2\right]$$
$$\theta_2 = \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}_{\mathtt{cf}}-\tau(A_0+CF_1+\gamma\mathcal{C}_{\mathtt{cf}})}{(1-\tau)A_0}\right) - r + \sigma^2/2\right]$$
$$\theta_3 = \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}_{\mathtt{cf}}}{A_0}\right) - r + \sigma^2/2\right].$$

Proof. See Appendix A.2. □

[10] The subscript `a` in $\mathcal{C}_{\mathtt{a}}$ refers to the specific type of deferred tax. No subscript indicates a firm without deferred taxes.

From Theorem 8.5 and Theorem 8.6, the Newton-Raphson method can be employed to find $\mathcal{C}$ (or $\mathcal{C}_{\mathtt{cf}}$) such that $D_0 = D$. In this case, there is a difference between the coupon paid by firms without deferred taxes and one that has carry forward. The disparity arises if $0 \leq \gamma < 1$. To see this, note that if $D + \mathcal{C} > A_0$, a firm can go bankrupt even if it makes a profit in period one. When interest payments are fully deductible ($\gamma = 1$), the firm never has to pay taxes over that profit. However, when $\gamma < 1$, some profit will be taxed, and this tax deduction might be enough to trigger the bankruptcy condition.[11]

Having established the coupon $\mathcal{C}$, we now turn to the valuation of levered firms and their deferred taxes. The value of a levered firm without deferred taxes is given by discounting the assets at time one (5.1), plus the coupon payment to creditors

$$\mathcal{V} = e^{-r}\mathbb{E}(\widetilde{A}_1 + \mathcal{C}|\mathcal{F}_0) = A_0 - \tau C^{\mathrm{BS}}(K = A_0 + \gamma\mathcal{C}). \tag{8.7}$$

We henceforth denote the value of levered firms by $\mathcal{V}_a$, to distinguish it from unlevered firms. Consistent with previous notation, subscript a depicts the deferred tax available at the starting period. The strike value in (8.7) is higher compared to unlevered firms, since interest deductions lead to a tax advantage. The interest deduction also contains an option component, as it is not certain that the entire interest payment can be deducted from taxable income, e.g. when a firm incurs a loss, so there is no taxable income to offset the interest payment.

The market consistent pricing approach gives the value of a levered firm having some carry forward by discounting (5.4) and adding the coupon payment

$$\mathcal{V}_{\mathrm{cf}} = e^{-r}\mathbb{E}^{Q}(\widetilde{A}_1 + \mathcal{C}_{\mathtt{cf}}|\mathcal{F}_0) = A_0 - \tau C^{\mathrm{BS}}(K = A_0 + CF_1 + \gamma\mathcal{C}_{\mathtt{cf}}). \tag{8.8}$$

Remark 8.9. Since our approach to valuing DTA's/DTL's has been to compare firm values with an otherwise identical firm, which does not have deferred taxes (Definition 6.4), we

[11] The absolute priority rule ensures that taxes are levied first, before bondholders can file for bankruptcy.

must assume that coupon payments for the reference firm are the same to avoid circular reasoning. To facilitate subsequent sensitivity calculations, we therefore chose to set $\mathcal{C}_{\mathtt{cf}} = \mathcal{C}$ when valuing carry forward.

As a result, the DTA value arising from carry forward for levered firms is given by the difference (8.8) and (8.7)

$$\xi_{\text{cf}} = \tau\Big(C^{\text{BS}}(A_0 + \gamma\mathcal{C}) - C^{\text{BS}}(A_0 + CF_1 + \gamma\mathcal{C})\Big). \tag{8.10}$$

Equation (8.10) contains the unlevered DTA value of carry forward (6.7) as a special case when $\gamma = 0$ or $D = 0$. The value of carry forward for levered firms in (8.10) is smaller in comparison to the value of carry forward for unlevered firms in (6.7). Mathematically, this is evident from Proposition 6.8, as the derivative of a European call option to the strike is decreasing in absolute value. The slope of the call option seen as a function of the strike is steeper, so the difference between two call options is greater compared to the difference between two call options further in the tail. There is also some economic rationale behind this result. The value of a DTA coming from carry forward is positively dependent on the amount of tax payments. In case a firm is levered, less tax is paid due to the interest tax shield. Hence, the overall DTA is reduced in value compared to unlevered firms. The relation between debt and the DTA value of carry forward is shown in Figure 6b. The negative relationship between the DTA value arising from carry forward and debt is clearly visible. The DTA value even tends to zero when a firm is extremely leveraged, because coupon payments are excessive in those cases.

The sensitivity of the DTA value for levered firms is given by

$$\frac{\partial}{\partial CF_1}\xi_{\text{cf}} = \tau e^{-r}(1 - F(A_0 + CF_1 + \gamma\mathcal{C})) = \tau e^{-r} Q(A_1 > A_0 + CF_1 + \gamma\mathcal{C}).$$

This is lower than the sensitivity for unlevered firms, since the probability that A_1 exceeds the term on the right is lower when interest payments are included. Ceteris paribus, a levered firm is less likely to profit from the full carry forward than an unlevered firm, so a small change in carry forward has less impact on the overall value for levered firms.

8.3 Carry back

In case a firm has CB_1 at year one, bankruptcy can be avoided in the event of a severe loss, by reclaiming previous tax expenses. This suggests that the coupon payment for companies with carry back should be lower compared to firms without deferred taxes (or firms with carry forward when $\gamma = 1$). In this case, the debt value at $t = 0$ follows from

Theorem 8.11. *The debt value at time zero for a company having CB_1 is given by*

$$D_0 = e^{-r}(D + \mathcal{C}_{\mathtt{cb}}) - e^{-r}(D + \mathcal{C}_{\mathtt{cb}} - \tau CB_1)\Phi(\theta_3) + A_0\Phi(\theta_3 - \sigma),$$

if $D + \mathcal{C}_{\mathtt{cb}} \leq A_0 - CB_1 + \gamma\mathcal{C}_{\mathtt{cb}}$. Otherwise

$$\begin{aligned}
D_0 &= e^{-r}(D + \mathcal{C}_{\mathtt{cb}}) - e^{-r}(D + \mathcal{C}_{\mathtt{cb}} - \tau CB_1)\Phi(\theta_2) + A_0\Phi(\theta_2 - \sigma) \\
&- e^{-r}\tau(CB_1 - A_0 - \gamma\mathcal{C}_{\mathtt{cb}})[\Phi(\theta_2) - \Phi(\theta_1)] - \tau A_0[\Phi(\theta_2 - \sigma) - \Phi(\theta_1 - \sigma)].
\end{aligned}$$

In these expressions

$$\begin{aligned}
\theta_1 &= \frac{1}{\sigma}\left[\log\left(\frac{A_0 - CB_1 + \gamma\mathcal{C}_{\mathtt{cb}}}{A_0}\right) - r + \sigma^2/2\right] \\
\theta_2 &= \frac{1}{\sigma}\left[\log\left(\frac{D + \mathcal{C}_{\mathtt{cb}} - \tau(A_0 + \gamma\mathcal{C}_{\mathtt{cb}})}{(1-\tau)A_0}\right) - r + \sigma^2/2\right] \\
\theta_3 &= \frac{1}{\sigma}\left[\log\left(\frac{D + \mathcal{C}_{\mathtt{cb}} - \tau CB_1}{A_0}\right) - r + \sigma^2/2\right].
\end{aligned}$$

Proof. See Appendix A.3. □

Again, Newton-Raphson can be used to find $\mathcal{C}_{\mathtt{cb}}$ such that $D_0 = D$. Even if $\gamma = 1$, the coupon payments for firms with carry back are lower compared to those of the reference firm (see Figure 6a). As a result, the value of a firm having some carry back is calculated by discounting (5.6) and adding the coupon

$$\mathcal{V}_{\text{cb}} = e^{-r}\mathbb{E}(\widetilde{A}_1 + \mathcal{C}_{\mathtt{cb}}|\mathcal{F}_0) = A_0 + e^{-r}\tau CB_1 - \tau C^{\text{BS}}(K = A_0 + \gamma\mathcal{C}_{\mathtt{cb}} - CB_1). \tag{8.12}$$

To avoid circular reasoning, we assume once more that $\mathcal{C}_{\mathtt{cb}} = \mathcal{C}$. The DTA value for carry back is then given by the difference between (8.12) and (8.7)

$$\xi_{\text{cb}} = \tau e^{-r}CB_1 - \tau\Big(C^{\text{BS}}(A_0 + \gamma\mathcal{C} - CB_1) - C^{\text{BS}}(A_0 + \gamma\mathcal{C})\Big). \tag{8.13}$$

In contrast to carry forward, the DTA value arising from carry back is actually more valuable when a firm is increasingly leveraged. The last two terms in (8.13) are smaller in difference compared to the last two terms appearing in (6.11) for unlevered firms. This is due to the higher strike value of the call option, which is also visible in Figure 6b. The economic reason behind this phenomenon comes from the coupon payments, which decreases fiscal loss even further. Hence, in case of a loss, it is more likely that a higher part of the carry back will be materialized, which increases the value of the DTA. The derivative of the DTA value to the carry back is given by

$$\frac{\partial}{\partial CB_1}\xi_{\text{cb}} = \tau e^{-r}F(A_0 + \gamma\mathcal{C} - CB_1) = \tau e^{-r}Q(A_1 < A_0 + \gamma\mathcal{C} - CB_1).$$

This is higher compared to unlevered firms. The same carry back value has higher probability of being realized, so that a small change in the carry back has more influence on the DTA value when a firm is levered.

8.4 DTL

A firm having a deferred tax liability is more likely to go bankrupt, since even in case of a loss the firm might be obliged to pay taxes. Thus, it can potentially happen that $A_1 > D + \mathcal{C}_{\mathtt{L}}$, but after taxes $\widetilde{A}_1 < D+\mathcal{C}_{\mathrm{L}}$. This should be taken into account when calculating the coupon $\mathcal{C}_{\mathrm{L}}$ and implies that $\mathcal{C}_{\mathrm{L}}$ is generally higher for firms having a DTL in comparison to firms without deferred tax obligations.

Theorem 8.14. *The debt value at time zero for a firm having LTD_1 is given by*

$$D_0 = e^{-r}(D+\mathcal{C}_{\mathrm{L}}) - e^{-r}(D+\mathcal{C}_{\mathrm{L}})\Phi(\theta_3) + A_0\Phi(\theta_3-\sigma),$$

if $D+\mathcal{C}_{\mathrm{L}} \le A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}$. Otherwise

$$\begin{aligned} D_0 &= e^{-r}(D+\mathcal{C}_{\mathrm{L}}) - e^{-r}(D+\mathcal{C}_{\mathrm{L}})\Phi(\theta_2) + A_0\Phi(\theta_2-\sigma) \\ &\quad - e^{-r}\tau(LTD_1 - A_0 - \gamma\mathcal{C}_{\mathrm{L}})[\Phi(\theta_2)-\Phi(\theta_1)] - \tau A_0[\Phi(\theta_2-\sigma)-\Phi(\theta_1-\sigma)]. \end{aligned}$$

In these expressions

$$\begin{aligned} \theta_1 &= \frac{1}{\sigma}\left[\log\left(\frac{A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}}{A_0}\right) - r + \sigma^2/2\right] \\ \theta_2 &= \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}_{\mathrm{L}} - \tau(A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}})}{(1-\tau)A_0}\right) - r + \sigma^2/2\right] \\ \theta_3 &= \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}_{\mathrm{L}}}{A_0}\right) - r + \sigma^2/2\right]. \end{aligned}$$

Proof. See Appendix A.4. ☐

The resulting coupon payments are higher compared to those of the reference firm, which can be seen from Figure 6a. The market consistent firm value follows from discounting (5.8) and adding the coupon payment

$$\mathcal{V}_{\mathrm{L}} = e^{-r}\mathbb{E}^Q(\widetilde{A}_1 + \mathcal{C}_{\mathrm{L}}|\mathcal{F}_0) = A_0 - \tau C^{\mathrm{BS}}(K = A_0 + \gamma\mathcal{C}_{\mathrm{L}} - LTD_1). \tag{8.15}$$

Once again we impose that $\mathcal{C}_{\mathrm{L}} = \mathcal{C}$ to avoid circular reasoning. The DTL value is given by comparing (8.15) and (8.7)

$$\xi_{\mathrm{L}} = \mathcal{V}_{\mathrm{L}} - \mathcal{V} = \tau\bigg(C^{\mathrm{BS}}(A_0 + \gamma\mathcal{C}) - C^{\mathrm{BS}}(A_0 + \gamma\mathcal{C} - LTD_1)\bigg). \tag{8.16}$$

The DTL value arising from temporary differences for levered firms is greater compared to unlevered firms due to higher strike values. Economically, this holds since coupon payments reduce fiscal profits, which makes it less likely that untaxed profit remains in period one.

The sensitivity of the DTL value (8.16) to a change in untaxed profit equals

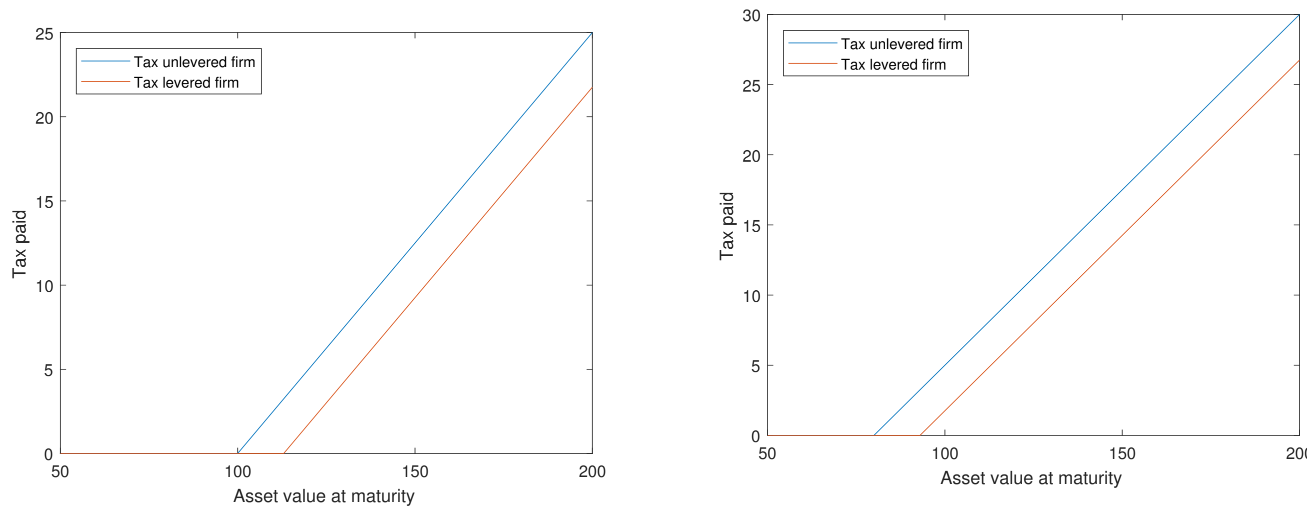
$$\frac{\partial}{\partial LTD_1}\xi_L = \tau e^{-r}(1 - F(A_0 + \gamma\mathcal{C}_{\mathrm{L}} - LTD_1)) = \tau e^{-r}Q(A_1 > A_0 + \gamma\mathcal{C}_{\mathrm{L}} - LTD_1).$$

Hence, the DTL value arising from temporary differences is less sensitive to a change in the untaxed profit when a firm is levered. A change in LTD_1 has less influence on tax payments as coupon payments make it less likely that such tax payments are materialized.

8.5 Valuation interest tax shield

We can obtain a market consistent value of the tax shield under Assumption **??** by adapting the valuation formulas for levered firms. The interest tax shield derives its value from the uncertainty related to the fact that not all of the interest tax shield will be materialized. However, the extent to which a firm is able to profit from the tax shield depends on the fiscal history. Figure 7 shows the difference in tax payments for levered and unlevered firms for two type of companies; a reference firm without fiscal history and a firm having a DTL. The difference between each of the graphs is what determines the tax shield value.

There is no general consensus in the literature about tax shield valuation. This topic started off with the classical article of Modigliani and Miller (1963), which suffers from some serious drawbacks such as risk-free debt and the tacit assumption that the tax shield will be completely materialized each year. Numerous investigations have tried to improve upon this work, such as Kemsley and Nissim (2002) who assess the impact of debt financing



(a) Reference firm without fiscal history, $\mathcal{C} = 12$.

(b) Firm with a DTL ($LTD_1 = 20$). The coupon $\mathcal{C}_{\mathrm{L}}$ equals 13.

Figure 7: Tax payments for levered and unlevered firms with different fiscal history. Parameters: $A_0 = 100, \tau = 0.25$.

by cross sectional regression, concluding that firm value is a strongly positive function of debt. Arzac and Glosten (2005) take a more theoretical approach, in which future cash flows arising from tax payments are discounted by a pricing kernel. The value of the tax shield is subsequently obtained as the difference in tax payments for levered and unlevered firms. Arzac and Glosten (2005) show that their framework contains the Modigliani-Miller theorem as a special case, by making specific assumptions about the dynamics of the free cash flow process. In line with Arzac and Glosten (2005), we calculate the value of the tax shield by subtracting the enterprise value of an unlevered firm from a levered firm. In particular, the following theorem presents the valuation of the two type of interest tax shields that might arise in our model.

Theorem 8.17. *The interest tax shield corresponding to firms with different fiscal history is given by*

(i) For firms with no fiscal history, carry forward or carry back

$$R \triangleq \mathcal{V} - V = \tau\Big(C^{BS}(A_0) - C^{BS}(A_0 + \gamma\mathcal{C})\Big). \tag{8.18}$$

(ii) For firms with a DTL arising from temporary differences

$$R_L \triangleq \mathcal{V}_L - V_L = \tau\Big(C^{BS}(A_0 - LTD_1) - C^{BS}(A_0 - LTD_1 + \gamma\mathcal{C}\Big). \tag{8.19}$$

Proof. For (i), simply subtract (6.5) from (8.7). In Section 5.1 we assumed a specific order for the coupon payments. After accounting for coupon payments, carry forward/back can be used for the remaining profit/loss. Hence, under this assumption, deferred tax assets like carry forward/back are immaterial for tax shield valuations. They only have an influence on the coupon payments, but to avoid circular reasoning coupons are taken to be the same as those of the reference firm. Finally, (ii) takes into consideration that, *ex-ante*, it is known that additional taxes are levied over LTD_1 in period one. Therefore, a DTL makes it more likely that part of the tax shield is materialized and the value is obtained by subtracting (6.13) from (8.15). □

Remark 8.20. This theorem is markedly different than the conventional Modigliani-Miller theorem, which (in our notation) states that

$$R = e^{-r}\tau\mathcal{C}. \tag{8.21}$$

Modigliani and Miller (1963) tacitly assume that the full interest tax shield can be deducted each year, which is not generally valid since taxable income might not be sufficient to compensate the entire tax advantage. Our model does account for this risk, which is reflected by the option price formulas in the previous theorem. The model for firms without fiscal history (8.18) contains the Modigliani-Miller theorem as a special case, which can be seen by taking $\sigma \to 0^+$ in (8.18).[12] Using the Black-Scholes formula (6.2) gives that (8.18) converges to

$$\lim_{\sigma\to 0^+} R^{\mathrm{BS}} = \tau e^{-r}\gamma\mathcal{C},$$

[12]There is no explicit σ in (8.18) since we assumed it was constant throughout our analysis. However, the DTA/DTL values do depend on this quantity.

provided $(e^r - 1)A_0 > \gamma\mathcal{C}$. Mathematically, this condition derives from the fact that d_2 in (6.2) goes to infinity if and only if $\log(A_0/(A_0 + \gamma\mathcal{C})) + r > 0$ when $\sigma \to 0^+$.[13] In our model, the condition expresses that taxable income (as measured by $(e^r - 1)A_0$) should be greater than $\gamma\mathcal{C}$ in order to benefit completely from the tax shield. If all interest payments can be deducted from taxable income (i.e. $\gamma = 1$), this is precisely the (continuously) discounted interest tax shield.

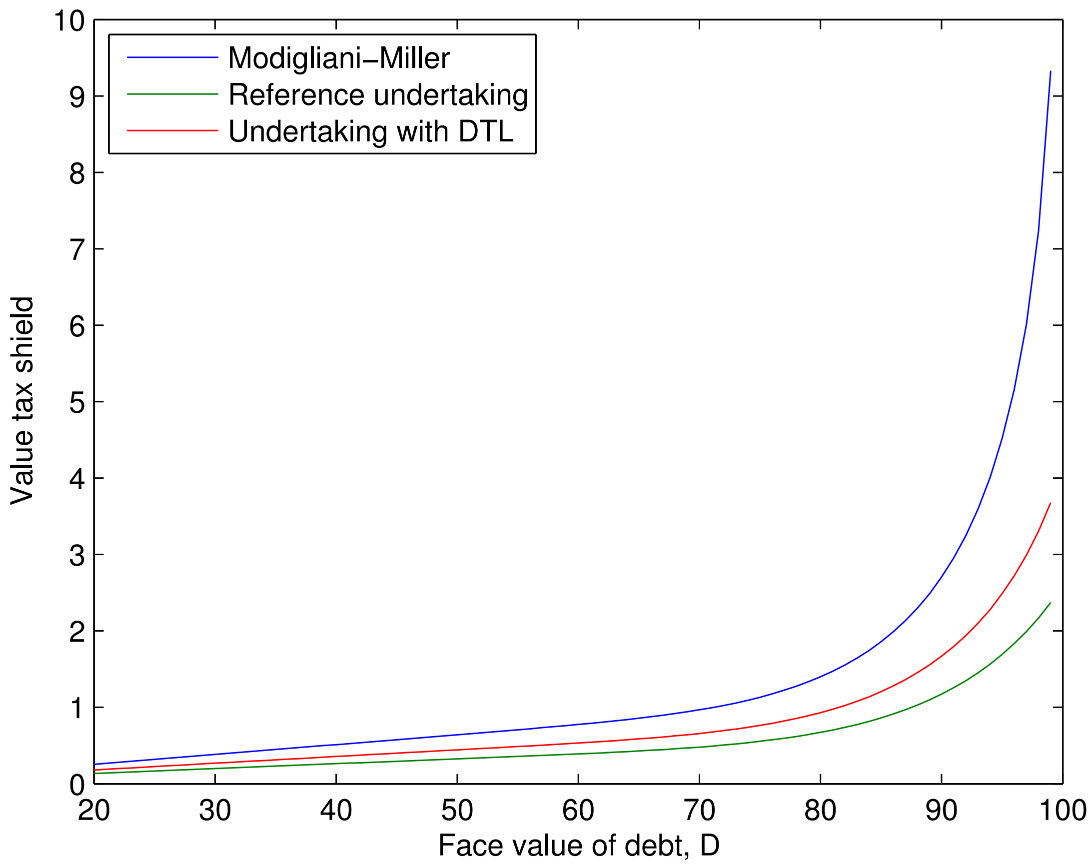


Figure 8: Interest tax shield seen as a function of debt. The Modigliani-Miller value is given by (8.21), whereas the value of the reference firm and the firm with DTL are given by R and R_L respectively. Parameters: $r = 0.05, \sigma = 0.2, \tau = 0.25, A_0 = 100, LTD_1 = 10, \gamma = 1$. Bankruptcy cost are zero ($\alpha = 0$).

Figure 8 shows the value of the tax shield as a function of debt. The option interpretation of the tax shield renders values lower than the Modigliani-Miller approach, since this model

[13] The strike value of the option is $K = A_0 + \gamma\mathcal{C}$.

takes into account that part of the tax shield may not be settled in period one. The valuation difference is most pronounced when a firm is highly leveraged, due to the exponential rise in coupon payments. Figure 8 also shows that the tax shield is more valuable for firms with a DTL, although the difference with a reference firm is relatively moderate.

9 Loss absorbing capacity of deferred taxes (LAC DT) for European insurers

As a practical implementation of the theory developed in previous chapters, we analyze the impact of our new valuation approach to the *loss absorbing capacity* of deferred taxes on European insurance undertakings. The recently established *Solvency* II regulations (analogue of Basel III for insurers) dictate that European insurers should maintain a *Solvency ratio* ratio greater than one, where

$$\text{Solvency ratio} = \frac{\text{Eligible own funds}}{\text{Solvency capital requirements}}.$$

The *Solvency capital requirements* (SCR) are calculated such that an insurer can withstand a shock that occurs once every 200 years (this is essentially the 99.5% VaR). The *standard formula* used to calculate the SCR makes use of a modular approach (EIOPA, 2014). This means that the overall risk is subdivided into sub risks and sub-sub risks. For each sub risk (or sub-sub risk) one calculates the capital requirements (corresponding to a 99.5% VaR over a one year period). All these capital requirements are aggregated using correlation matrices, which results in the Solvency capital requirements (EIOPA, 2014). Part of a shock is absorbed by deferred taxes, as such anomalies mitigate DTL's or create additional carry forward when the net DTA position is positive. Basically, undertakings transfer part of the loss to the tax authority, as it reduces future taxable income. The Solvency II guidelines take this *loss absorbing capacity of deferred taxes* (LAC DT) into account by subtracting this from the Solvency capital requirements. Hence, when taking LAC DT into account, the Solvency ratio follows from

$$\text{Solvency ratio} = \frac{\text{Eligible own funds}}{\text{Solvency capital requirements} - \text{LAC DT}}. \tag{9.1}$$

Suppose that LAC DT is 25% of the Solvency capital requirements, then incorporating LAC DT can increase the Solvency ratio from 100% to 133%. By definition, LAC DT is the difference in net DTA position *post* and *ex-ante* shock, i.e. LAC DT = post shock net DTA − ex-ante shock net DTA. The maximum LAC DT is equal to the tax rate multiplied by the magnitude of the shock, but in reality these values are often lower since insurers cannot substantiate enough future profits to prove that the deferred tax asset will be settled completely.

Example 9.2. Suppose $A_0 = 100, CF_1 = 10$ and $SCR = 40$, i.e. the undertaking loses maximally 40 following a shock that is bound to occur once every 200 years. The loss of 40 immediately raises carry forward to $CF_1 = 50$. If the undertaking can substantiate enough future profits to prove that the additional carry forward will be settled completely, then LAC DT $= \tau(50 - 10) = \tau \cdot 40$, which is the entire shock loss times the corporate tax rate. If the undertaking expects to settle only 20 of the shock loss of 40 (because future profits are not sufficient), then LAC DT $= \tau(30 - 10) = \tau \cdot 20$.

9.1 Market consistent approach

We use a data set provided by EIOPA, which contains a list of the largest European insurers, together with a detailed number of variables needed to calculate LAC DT and the Solvency ratio.[14] We briefly outline all observables included in the data set, which are needed for our computations:

- **Eligible own funds (EOF)**: This is the sum of Tier I capital, the eligible Tier II and

[14] These data are secret, and we refrain from discussing them in detail.

III capital, which are subject to quantitative restrictions as outlined in the delegated acts of *Article* 98 in the Solvency II directive. The eligible own funds appear in the numerator of the Solvency ratio in (9.1).

- **Assets**: This is the sum of all balance sheet items and will be used as A_0, for the input of our valuation approach.

- **Liabilities**: These are the *technical provisions*, which by definition include all insurance obligations, policyholder commitments and other beneficiaries. The technical provisions enter our model as D (face value of debt), which is needed for the determination of DTA's/DTL's for levered firms.

- **Duration liabilities**: This is the average duration of all outstanding debt. We round the average duration to the nearest whole number T and use this to simulate the T-period model.

- **Forward rate**: This is the future yield on a bond, calculated by interpolating the curve constructed from swap rates. Forward rates beyond 20 years are no longer calculated market consistent by EIOPA, since the swap rate market is considered illiquid beyond this point. This rate enters the model as input for the variable $r_{\text{forward},T}$, where T stands for the T-period model.

- **net DTA**: This is the aggregated deferred tax position on the balance sheet. Negative quantities denote DTL's ($= LTD_1$) , whereas positive quantities denote DTA's ($= CF_1$). For the simulation, we assume that a positive net DTA comes from carry forward, as there are no accounting values attached to carry back.

- **Solvency capital requirements (SCR)**: This variable appears in the denominator of the Solvency ratio in (9.1) and is calibrated using a 99.5% VaR of the normal distribution with mean $\mu = 0$ and variance σ^2.

- **LAC DT**: This is the Solvency II LAC DT. The variable is used to compare our computations with those of EIOPA.
- **Tax rate**: This is the applicable (corporate) tax rate τ, which is country specific. The tax rate for each member state of the European Union is shown in the first column of Table 2.
- **Dummy variable carry back**: This binary variable indicates whether carry back is allowed in each respective country, which is shown in the second column of Table 2.
- **Duration carry forward**: This concerns the number of years that losses can be carried forward in each member state of the European Union (see third column of Table 2).

To make the data amenable to our computations, we set all negative *duration liabilities* to one and put a cap of 30 years on it. The model is fully operational once we have the volatility of the assets σ and the coupon payment $\mathcal{C}$. Since the SCR is the 99.5% VaR of the normal distribution with mean zero and variance σ^2, we have $SCR = \hat{\sigma} \cdot x_\alpha$, where x_α is the quantile of a standard normal distribution with tail level α. In our case $\alpha = 0.995$, so that $x_\alpha = \Phi^{-1}(0.995) = 2.58$. The SCR are provided in the data sheet, hence we solve for the volatility to obtain $\hat{\sigma} = SCR/x_\alpha$. This, however, renders the absolute volatility depending on the unit of measurement. The simulation of the geometric Brownian motion requires a relative volatility. Therefore, we rescale by A_0 to obtain the relative volatility of the assets, which is used for subsequent valuation inferences

$$\hat{\sigma} = \frac{SCR}{x_\alpha A_0}.$$

The total number of insurance undertakings taken into consideration is 2851. The coupon is determined *exogenously*. Since no data are available on the risk premium of debt for each insurer, we assume that liabilities are risk free. In particular, this means that $\mathcal{C} = \mathcal{C}_{\mathtt{cf}} = \mathcal{C}_{\mathtt{L}} =$

$(\exp(r_{\text{forward},T})-1)\cdot D$. This assumption finds partial justification by the fact that European countries are inclined to bailout systemically important insurers when bankruptcy is imminent. Most insurance undertakings considered in our sample belong to this class. However, historical evidence suggests that countries do not always intervene when insurers are close to default, which is best illustrated by the bankruptcy of the Equitable Life Assurance in the UK (O'Brien, 2006). The risk-free debt assumption forces $K = 0$, so that bankruptcy prior to maturity is excluded. Finally, since no data are available for bankruptcy cost, we enforce them to be zero, i.e. $\alpha = 0$.

We now outline how to implement these data in our simulation approach. First, a **simulation before shock** is carried out to (re-)calculate the net DTA before shock in a market consistent manner. We simulate the asset paths of an insurer with a DTL according to the **more realistic case** (ii).

1. In case the insurer has a DTA (coming from carry forward)

   (a) Simulate asset paths of (hypothetical) insurer without carry forward, using the parameter tuple $(A_0, T, r_{\text{forward},T}, \hat{\sigma}, D, \mathcal{C}, \tau)$. This gives an estimate of the insurers value $\widehat{\mathcal{V}}^{\text{BS}}$.

   (b) Simulate asset paths of insurer having carry forward with parameter tuple $(A_0, T, r_{\text{forward},T}, \hat{\sigma}, D,$ This gives an estimate of the insurers value $\widehat{\mathcal{V}}^{\text{BS}}_{\text{cf}}$. The DTA value is then computed by $\hat{\xi}^{\text{BS}}_{\text{cf}} = \widehat{\mathcal{V}}^{\text{BS}}_{\text{cf}} - \widehat{\mathcal{V}}^{\text{BS}}$.

2. In case the insurer has a DTL (coming from temporary differences)

   (a) Simulate asset paths of reference undertaking without a DTL, using the parameter tuple $(A_0, T, r_{\text{forward},T}, \hat{\sigma}, D, \mathcal{C}, \tau)$. Again, this yields the insurers value $\widehat{\mathcal{V}}^{\text{BS}}$.

   (b) Simulate asset paths of insurer having a DTL (using the **more realistic case** (ii)) with parameter tuple $(A_0, T, r_{\text{forward},T}, \hat{\sigma}, D, \mathcal{C}, \tau, LTD_1)$. This gives $\widehat{\mathcal{V}}^{\text{BS}}_{L}$. The DTL value follows from $\hat{\xi}^{\text{BS}}_{L} = \widehat{\mathcal{V}}^{\text{BS}}_{L} - \widehat{\mathcal{V}}^{\text{BS}}$.

In this way, the **net DTA** values coming from EIOPA are reassessed in a market consistent way. Panel **(a)** of Figure 9 shows the result of these computations in a scatter plot. You can see that the market consistent approach yields less negative DTL values and less positive DTA values. In a second step, LAC DT data are re-evaluated using the market consistent approach. This time, DTA/DTL values are computed after a shock that is bound to occur every 200 years. The magnitude of this shock equals the SCR and is calculated by internal models of EIOPA. The insurer instantly creates loss carry forward equal to the SCR after such a shock occurs. This means that the asset value goes down by SCR, but the net DTA goes up by SCR. The post-shock net DTA value is calculated using a **simulation after shock**

1. In case the insurer has a DTA

   (a) Simulate asset paths of reference undertaking without carry forward, this time using the post-shock asset value $A_0 \to A_0 - SCR$. This gives the new estimate $\widehat{\mathcal{V}}^{\mathrm{BS}}$.

   (b) Simulate asset paths of insurer, which now has carry forward equal to $CF_1 \to CF_1 + SCR$ and starting value of the assets $A_0 \to A_0 - SCR$. This renders the firm value $\widehat{V}_{\mathrm{cf}}^{\mathrm{BS}}$. The post-shock DTA value is calculated by $\hat{\xi}_{\mathrm{cf,post\text{-}shock}} = \widehat{\mathcal{V}}_{\mathrm{cf}}^{\mathrm{BS}} - \widehat{\mathcal{V}}^{\mathrm{BS}}$.

2. In case the insurer has a DTL and the shock SCR is less than LTD_1 (this condition means that the insurer still has a DTL post-shock)

   (a) Simulate asset paths of reference undertaking without a DTL, using the new asset value $A_0 \to A_0 - SCR$. This gives the new estimate $\widehat{\mathcal{V}}^{\mathrm{BS}}$.

   (b) Simulate asset paths of the insurer having a post-shock DTL equaling $LTD_1 \to LTD_1 - SCR$ and asset value $A_0 \to A_0 - SCR$. The corresponding firm value is equal to $\widehat{\mathcal{V}}_{L}^{\mathrm{BS}}$. As a result, the new DTL value equals $\hat{\xi}_{\mathrm{L,post\text{-}shock}} = \widehat{\mathcal{V}}_{\mathrm{L}}^{\mathrm{BS}} - \widehat{\mathcal{V}}^{\mathrm{BS}}$.

3. In case the insurer has a DTL and the shock is greater than LTD_1. Following a shock,

this means that the complete DTL disappeared and additional carry forward is created in the amount $SCR - LTD_1$.

(a) Simulate asset paths of reference undertaking without a DTL, using the starting value of the assets $A_0 \to A_0 - SCR$. This gives the estimate $\widehat{\mathcal{V}}^{\mathrm{BS}}$.

(b) Simulate asset paths of insurer, where the DTL has disappeared and carry forward is created equal to the amount $LTD_1 \to CF_1 = SCR - LTD_1$ and starting value of the assets $A_0 \to A_0 - SCR$. In this case, the DTA value is given by $\hat{\xi}_{\mathrm{L,post\text{-}shock}} = \widehat{\mathcal{V}}^{\mathrm{BS}}_{\mathrm{cf}} - \widehat{\mathcal{V}}^{\mathrm{BS}}$.

In this way, we compute the net DTA values *ex-ante* and *post* shock in a market consistent manner. By definition, the market consistent LAC DT (LAC DT*) follows from

$$\text{LAC DT}^* \triangleq \begin{cases} \hat{\xi}^{BS}_{\mathrm{cf,post\text{-}shock}} - \hat{\xi}^{BS}_{\mathrm{cf}}, & \text{if } \textbf{net DTA } > 0 \\ \hat{\xi}^{BS}_{\mathrm{L,post\text{-}shock}} - \hat{\xi}^{BS}_{\mathrm{L}}, & \text{if } \textbf{net DTA } < 0. \end{cases} \tag{9.3}$$

The second case depends on the condition $LTD_1 \lesseqgtr SCR$. The LAC DT values obtained in this way are shown in panel **(b)** of Figure 9. The market consistent approach renders smaller LAC DT values on average in comparison to those of EIOPA. We observe that the majority of scatter points are below the 45° line, which indicates that the market consistent approach renders lower LAC DT values. However, there are also points above the 45° line. This is possible, since in case of a DTA, the pre-and post shock values move down when calculated market consistently and this can result in a higher difference compared to EIOPA's calculations. The opposite effects hold for DTL's. It is perfectly possible to obtain negative LAC DT values in the market consistent approach, since the loss of potential arising from a shock can be so severe that the additional carry forward doesn't weigh up against the loss of potential. This is not possible in EIOPA's approach.

In a second step, the Solvency II ratios are reassessed using the market consistent interpretation of deferred taxes. The Solvency ratios computed by EIOPA follow from (9.1). The market consistent Solvency ratios are calculated according to

$$\text{Solvency ratio}^* = \frac{\text{Eligible own funds}^*}{\text{Solvency capital requirements} - \text{LAC DT}^*}. \tag{9.4}$$

In this formula, LAC DT* is given by (9.3). The new Eligible own funds (Eligible own funds*) are given by

$$\begin{aligned}
&\text{Eligible own funds}^* = \text{Eligible own funds} \\
&- \min\big(\max(\textbf{net DTA}, 0), 0.15 \cdot (\text{SCR} - \text{LAC DT})\big) \\
&- \min(\textbf{net DTA}, 0) \\
&+ \min\big(\max(\textbf{net DTA}^*, 0), 0.15 \cdot (\text{SCR} - \text{LAC DT}^*)\big) \\
&+ \min(\textbf{net DTA}^*, 0).
\end{aligned}$$

The Eligible own funds must be adjusted since the **net DTA** variable (before shock) is included in the Eligible own funds calculated by EIOPA. For prudential reasons, the maximum DTA included in the EOF is 15% of the $SCR -$ LAC DT. There is no cap on the DTL values, since they mitigate the EOF. Whence, the market consistent Eligible own funds (Eligible own funds*) are obtained by removing the **net DTA** from EIOPA and adding the market consistent **net DTA*** before shock, which are obtained by the **simulation before shock**. The results of our computations are summarized in the scatter plots of Figure 10. Panel **(a)** reflects the impact when the insurer has a DTL, whereas panel **(b)** shows the result for insurers with a DTA. In both cases, the impact is moderate. In fact, a simple regression of the form

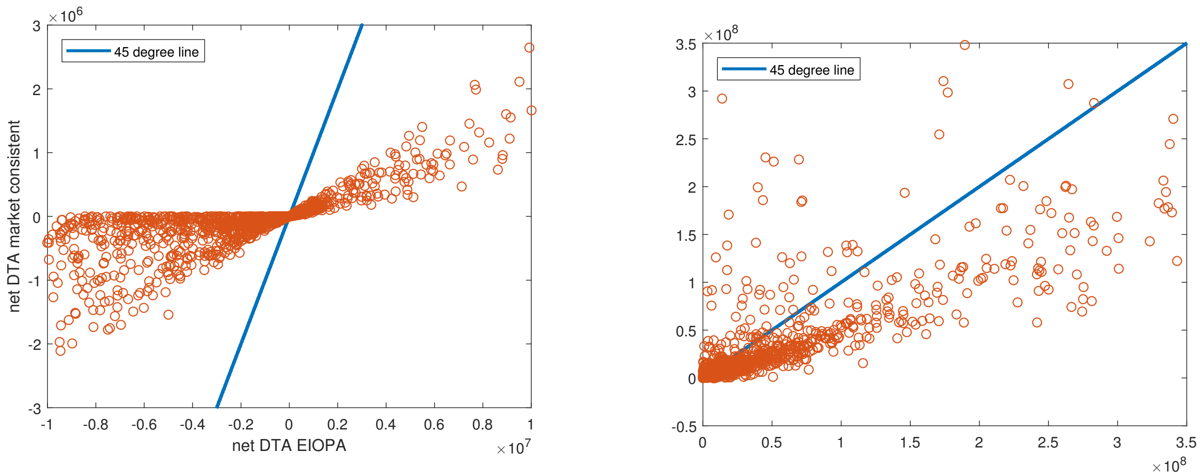
$$\text{Solvency ratio}^* = \beta \cdot \text{Solvency ratio EIOPA} + \varepsilon,$$

renders an OLS estimate $\hat{\beta}$ not significantly different from one based on a t-test. Another t-

Table 1: Forward rates T-periods from now, expressed in percentages (%).

T	1	2	3	4	5	6	7	8	9	10
$r_{\text{forward},T}$	-0.30	-0.22	-0.10	0.13	0.37	0.67	0.96	1.22	1.43	1.57
T	11	12	13	14	15	16	17	18	19	20
$r_{\text{forward},T}$	1.68	1.74	1.82	1.78	1.66	1.52	1.44	1.51	1.64	1.88

test on the mean of the Solvency II ratios yields that both sample means are not significantly different.



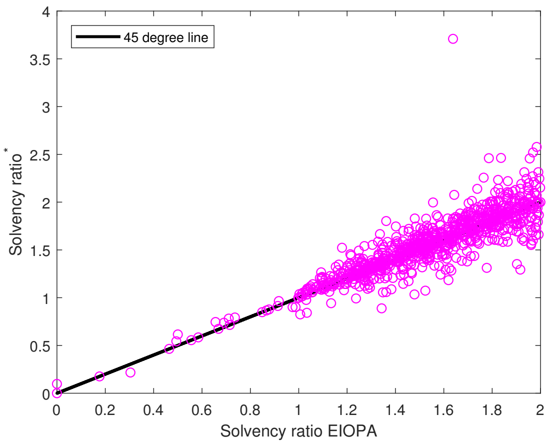
(a) Scatter diagram of market consistent calculations of **net DTA** vs. the **net DTA** values calculated by EIOPA.

(b) Estimated LAC DT EIOPA (x-axis) vs. LAC DT market consistent (y-axis).

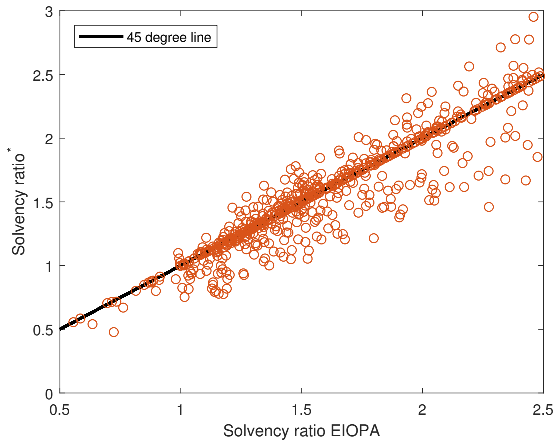
Figure 9: Estimates of net DTA and LAC DT in a market consistent framework vs. those obtained by EIOPA.

9.2 Implication for policymakers

Even though the overall impact of the market consistent approach is relatively moderate, it can have major repercussions for individual insurers. Indeed, we find that 29 more insurance undertakings are not well capitalized to withstand an anomalous shock. For these undertakings, one could impose *recapitalization* or *de-risk* measures or both. Recapitalization and de-risking have different objectives. Recapitalization is aimed to increase the EOF, whereas de-risking is employed with the intent to reduce $\hat{\sigma}$ by selling risky portfolios. Hence, de-



(a) Solvency ratio when net DTA is negative.



(b) Solvency ratio when net DTA is positive.

Figure 10: Scatter plot of market consistent estimates of Solvency ratio vs. Solvency ratio calculated by EIOPA when net DTA is negative and positive.

risking leads to a decrease in the SCR, which is positively dependent on $\hat{\sigma}$. Both measures, or a combination of the two, will lead to an increase in the Solvency II ratio.

The influence of recapitalization in the market consistent framework will roughly be similar to EIOPA's projections. However, the influence of de-risking in the market consistent approach is more subtle. Namely, a change in $\hat{\sigma}$ influences the DTA/DTL value (EIOPA does not take this effect into account). The DTA/DTL value can either increase or decrease, depending on the relation between A_0, CF_1 and $r_{\text{forward},T}$. In the 1-period model, this can be proved by a careful inspection of the call option *vega* ($= \partial_\sigma C^{\text{BS}}$). Simulation in the T-period model for $T \geq 2$ renders a similar conclusion, namely that the DTA/DTL value can move up or down following a decrease in $\hat{\sigma}$. The transition in $\hat{\sigma}$ with the resulting change in DTA/DTL value affects the market consistent EOF* and LAC DT*. Hence, it can potentially happen that the Solvency II ratio goes down after de-risking. However, in most reasonable scenarios this in unlikely to occur since the DTA/DTL represent only a small fraction of the SCR. Nevertheless, it is important to be aware of these dynamics, as the effect of de-risking may not lead to the projected outcome.

10 Conclusion

We propose a market consistent valuation of deferred taxes based on the option interpretation of tax payments. This leads us to express DTA/DTL values as the difference between Black-Scholes call option formulas over a one year time horizon. These formulas offer clear insight in the contingent nature of deferred taxes and avoid the necessity of subjective profit forecasts that are needed for extant accounting valuation techniques. Moreover, the market consistent model acknowledges value creation due to loss carryback, which is not recognized by applicable accounting standards.

Over multiple time periods, valuation results for deferred taxes are obtained by simulation, since path dependency eschews the tractability of analytical formulæ. The option interpretation is no longer exact, but the shape of the pricing formulas for different parameter values bears strong resemblance to the one-period model. In this case, the valuation also depends on the settlement term of carry forward/back and whether carry back is allowed or not. Extending the T-period model by an extra year negatively influences DTA value, since an undertaking with the initial tax advantage is expected to pay more taxes after the DTA has been settled, compared to a reference undertaking that does not have the tax benefit.

Moreover, we make a clear distinction between levered and unlevered firms. Coupon payments resulting from debt financing mitigates taxable income, thereby negatively influencing carry forward values, but igniting a positive effect on carry back value. Leverage also reduces tax liabilities (in absolute terms) arising from temporary differences. The option interpretation of deferred taxes can also be applied to tax shield valuations, which leads us to an alternative version of the Modigliani-Miller theorem (Modigliani and Miller, 1963), essentially containing the latter as a special case.

Lastly, the model is flexible enough to cover a wide range of practical applications. The *loss absorbing capacity* of deferred taxes of European insurance undertakings is reassessed using the market consistent approach. Hereby, we find that the loss absorbing capacity is less than anticipated, but the overall effect is relatively moderate. In conclusion, we offer an omnibus framework that offers clear insight in the contingent nature of deferred taxes relevant to virtually all firms with deferred taxes on the balance sheet.

A Proofs

A.1 No deferred taxes

Theorem A.1. *The debt value at time zero for a company without deferred taxes is given by*

$$D_0 = e^{-r}(D+\mathcal{C}) - e^{-r}(D+\mathcal{C})\Phi(\theta_3) + A_0\Phi(\theta_3-\sigma),$$

if $D+\mathcal{C} \leq A_0+\gamma\mathcal{C}$. Otherwise

$$\begin{aligned} D_0 &= e^{-r}(D+\mathcal{C}) - e^{-r}(D+\mathcal{C})\Phi(\theta_2) + A_0\Phi(\theta_2-\sigma) \\ &\quad - \tau A_0[\Phi(\theta_2-\sigma)-\Phi(\theta_1-\sigma)] + e^{-r}\tau(A_0+\gamma\mathcal{C})[\Phi(\theta_2)-\Phi(\theta_1)]. \end{aligned}$$

In these expressions

$$\begin{aligned} \theta_1 &= \frac{1}{\sigma}\left[\log\left(\frac{A_0+\gamma\mathcal{C}}{A_0}\right) - r + \sigma^2/2\right] \\ \theta_2 &= \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}-\tau(A_0+\gamma\mathcal{C})}{(1-\tau)A_0}\right) - r + \sigma^2/2\right] \\ \theta_3 &= \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}}{A_0}\right) - r + \sigma^2/2\right]. \end{aligned}$$

Proof. A firm without deferred taxes gives rise to the following debt value at time zero

$$D_0 = e^{-r}(D+\mathcal{C}) - e^{-r}\mathbb{E}^Q((D+\mathcal{C}-[A_1-\tau(A_1-A_0-\gamma\mathcal{C})^+])^+).$$

To evaluate this expression, we assume that $\mathcal{C}$ is fixed. Introduce θ_1 as the solution to $A_1 = A_0+\gamma\mathcal{C}$ (point from which the firm pays taxes). Write $A_1 = A_0\exp(r-\sigma^2/2+\sigma X)$, where $X \sim N(0,1)$. The analytical solution for θ_1 follows from

$$\begin{aligned} A_0 \exp(r - \sigma^2/2 + \sigma\theta_1) = A_0 + \gamma\mathcal{C} \implies \\ \theta_1 = \frac{1}{\sigma}[\log\left(\frac{A_0 + \gamma\mathcal{C}}{A_0}\right) - r + \sigma^2/2]. \end{aligned}$$

Let θ_2 be the solution to $D + \mathcal{C} = A_1 - \tau(A_1 - A_0 - \gamma\mathcal{C})$. Rearranging leads to

$$\begin{aligned} (1-\tau)A_1 = D + \mathcal{C} - \tau(A_0 + \gamma\mathcal{C}) \implies \\ (1-\tau)A_0 \exp(r - \sigma^2/2 + \sigma\theta_2) = D + \mathcal{C} - \tau(A_0 + \gamma\mathcal{C}) \implies \\ \theta_2 = \frac{1}{\sigma}[\log\left(\frac{D + \mathcal{C} - \tau(A_0 + \gamma\mathcal{C})}{(1-\tau)A_0}\right) - r + \sigma^2/2]. \end{aligned}$$

Finally, θ_3 is defined to be the solution to $D + \mathcal{C} = A_1$. This gives

$$\begin{aligned} A_0 \exp(r - \sigma^2/2 + \sigma\theta_3) = D + \mathcal{C} \implies \\ \theta_3 = \frac{1}{\sigma}[\log\left(\frac{D + \mathcal{C}}{A_0}\right) - r + \sigma^2/2]. \end{aligned}$$

We now distinguish the following two cases

(i) $D + \mathcal{C} \leq A_0 + \gamma\mathcal{C}$.

(ii) $D + \mathcal{C} > A_0 + \gamma\mathcal{C}$.

In case (i), the expectation equals

$$\begin{aligned} & e^{-r}\mathbb{E}^Q((D + \mathcal{C} - [A_1 - \tau(A_1 - A_0 - \gamma\mathcal{C})^+])^+) \\ & = e^{-r}\int_{-\infty}^{\theta_3} D + \mathcal{C} - A_1 dQ \\ & = e^{-r}\int_{-\infty}^{\theta_3} (D + \mathcal{C} - A_0 e^{r - \sigma^2/2 + \sigma x})\varphi(x)dx \\ & = e^{-r}(D + \mathcal{C})\Phi(\theta_3) - A_0\Phi(\theta_3 - \sigma). \end{aligned}$$

Case (ii) yields

$$
\begin{aligned}
& e^{-r}\mathbb{E}^Q((D+\mathcal{C}-[A_1-\tau(A_1-A_0-\gamma\mathcal{C})^+])^+) \\
& = e^{-r}\int_{-\infty}^{\theta_1}(D+\mathcal{C}-A_1)dQ \\
& + e^{-r}\int_{\theta_1}^{\theta_2}(D+\mathcal{C}-A_1+\tau(A_1-A_0-\gamma\mathcal{C}))dQ \\
& = e^{-r}(D+\mathcal{C})\Phi(\theta_2)-A_0\Phi(\theta_2-\sigma) \\
& + \tau A_0[\Phi(\theta_2-\sigma)-\Phi(\theta_1-\sigma)] - e^{-r}\tau(A_0+\gamma\mathcal{C})[\Phi(\theta_2)-\Phi(\theta_1)].
\end{aligned}
$$

□

A.2 Carry forward

Theorem A.2. *The debt value at time zero for a firm having CF_1 is given by*

$$D_0 = e^{-r}(D+\mathcal{C}_{\texttt{cf}}) - e^{-r}(D+\mathcal{C}_{\texttt{cf}})\Phi(\theta_3) + A_0\Phi(\theta_3-\sigma),$$

if $D+\mathcal{C}_{\texttt{cf}} \le A_0 + CF_1 + \gamma\mathcal{C}_{\texttt{cf}}$. Otherwise

$$
\begin{aligned}
& D_0 = e^{-r}(D+\mathcal{C}_{\texttt{cf}}) - e^{-r}(D+\mathcal{C}_{\texttt{cf}})\Phi(\theta_2) + A_0\Phi(\theta_2-\sigma) \\
& - \tau A_0[\Phi(\theta_2-\sigma)-\Phi(\theta_1-\sigma)] + e^{-r}\tau(A_0+CF_1+\gamma\mathcal{C}_{\texttt{cf}})[\Phi(\theta_2)-\Phi(\theta_1)].
\end{aligned}
$$

In these expressions

$$
\begin{aligned}
\theta_1 &= \frac{1}{\sigma}\left[\log\left(\frac{A_0+CF_1+\gamma\mathcal{C}_{\texttt{cf}}}{A_0}\right) - r + \sigma^2/2\right] \\
\theta_2 &= \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}_{\texttt{cf}}-\tau(A_0+CF_1+\gamma\mathcal{C}_{\texttt{cf}})}{(1-\tau)A_0}\right) - r + \sigma^2/2\right] \\
\theta_3 &= \frac{1}{\sigma}\left[\log\left(\frac{D+\mathcal{C}_{\texttt{cf}}}{A_0}\right) - r + \sigma^2/2\right].
\end{aligned}
$$

Proof. For a company with carry forward, debt at time zero equals

$$D_0 = (D + \mathcal{C}_{\texttt{cf}}) - e^{-r}\mathbb{E}^Q((D + \mathcal{C}_{\texttt{cf}} - [A_1 - \tau(A_1 - A_0 - CF_1 - \gamma\mathcal{C}_{\texttt{cf}})^+])^+).$$

To compute the expectation explicitly, we proceed as in the previous section. Let θ_1 be the solution to $A_1 = A_0 + CF_1 + \gamma\mathcal{C}_{\texttt{cf}}$ (moment from which firm pays taxes). In particular

$$\begin{aligned} A_0 \exp(r - \sigma^2/2 + \sigma\theta_1) = A_0 + CF_1 + \gamma\mathcal{C}_{\texttt{cf}} \implies \\ \theta_1 = \frac{1}{\sigma}[\log\left(\frac{A_0 + CF_1 + \gamma\mathcal{C}_{\texttt{cf}}}{A_0}\right) - r + \sigma^2/2]. \end{aligned}$$

In addition θ_2 is the solution to $D + \mathcal{C}_{\texttt{cf}} = A_1 - \tau(A_1 - A_0 - CF_1 - \gamma\mathcal{C}_{\texttt{cf}})$. Rearranging terms yields

$$\begin{aligned} (1-\tau)A_0 = D + \mathcal{C}_{\texttt{cf}} - \tau(A_0 + CF_1 + \gamma\mathcal{C}_{\texttt{cf}}) \implies \\ \theta_2 = \frac{1}{\sigma}[\log\left(\frac{D + \mathcal{C}_{\texttt{cf}} - \tau(A_0 + CF_1 + \gamma\mathcal{C}_{\texttt{cf}})}{(1-\tau)A_0}\right) - r + \sigma^2/2]. \end{aligned}$$

Finally, θ_3 is the solution to $D + \mathcal{C}_{\texttt{cf}} = A_1$.

$$\begin{aligned} A_0 \exp(r - \sigma^2/2 + \sigma\theta_3) = D + \mathcal{C}_{\texttt{cf}} \implies \\ \theta_3 = \frac{1}{\sigma}[\log\left(\frac{D + \mathcal{C}_{\texttt{cf}}}{A_0}\right) - r + \sigma^2/2]. \end{aligned}$$

This time we distinguish between

(i) $D + \mathcal{C}_{\texttt{cf}} \leq A_0 + CF_1 + \gamma\mathcal{C}_{\texttt{cf}}$.

(ii) $D + \mathcal{C}_{\texttt{cf}} > A_0 + CF_1 + \gamma\mathcal{C}_{\texttt{cf}}$.

Case (i) renders the following solution to the expectation

$$
\begin{aligned}
&e^{-r}\mathbb{E}^{Q}((D+\mathcal{C}_{\texttt{cf}}-[A_1-\tau(A_1-A_0-CF_1-\gamma\mathcal{C}_{\texttt{cf}})^+])^+)\\
&=e^{-r}\int_{-\infty}^{\theta_3} D+\mathcal{C}_{\texttt{cf}}-A_1 dQ\\
&=e^{-r}(D+\mathcal{C}_{\texttt{cf}})\Phi(\theta_3)-A_0\Phi(\theta_3-\sigma).
\end{aligned}
$$

Case (ii) yields

$$
\begin{aligned}
&e^{-r}\mathbb{E}^{Q}((D+\mathcal{C}_{\texttt{cf}}-[A_1-\tau(A_1-A_0-CF_1-\gamma\mathcal{C}_{\texttt{cf}})^+])^+)\\
&=e^{-r}\int_{-\infty}^{\theta_1}(D+\mathcal{C}_{\texttt{cf}}-A_1)dQ\\
&+e^{-r}\int_{\theta_1}^{\theta_2}(D+\mathcal{C}_{\texttt{cf}}-A_1+\tau(A_1-A_0-CF_1-\gamma\mathcal{C}_{\texttt{cf}}))dQ\\
&=e^{-r}(D+\mathcal{C}_{\texttt{cf}})\Phi(\theta_2)-A_0\Phi(\theta_2-\sigma)\\
&+\tau A_0[\Phi(\theta_2-\sigma)-\Phi(\theta_1-\sigma)]-e^{-r}\tau(A_0+CF_1+\gamma\mathcal{C}_{\texttt{cf}})[\Phi(\theta_2)-\Phi(\theta_1)].
\end{aligned}
$$

□

A.3 Carry back

Theorem A.3. *The debt value at time zero for a firm having CB_1 is given by*

$$D_0=e^{-r}(D+\mathcal{C}_{\texttt{cb}})-e^{-r}(D+\mathcal{C}_{\texttt{cb}}-\tau CB_1)\Phi(\theta_3)+A_0\Phi(\theta_3-\sigma),$$

if $D+\mathcal{C}_{\texttt{cb}}\le A_0-CB_1+\gamma\mathcal{C}_{\texttt{cb}}$. Otherwise

$$
\begin{aligned}
D_0&=e^{-r}(D+\mathcal{C}_{\texttt{cb}})-e^{-r}(D+\mathcal{C}_{\texttt{cb}}-\tau CB_1)\Phi(\theta_2)+A_0\Phi(\theta_2-\sigma)\\
&-e^{-r}\tau(CB_1-A_0-\gamma\mathcal{C}_{\texttt{cb}})[\Phi(\theta_2)-\Phi(\theta_1)]-\tau A_0[\Phi(\theta_2-\sigma)-\Phi(\theta_1-\sigma)].
\end{aligned}
$$

In these expressions

$$
\begin{aligned}
\theta_1 &= \frac{1}{\sigma}\left[\log\left(\frac{A_0 - CB_1 + \gamma\mathcal{C}_{\texttt{cb}}}{A_0}\right) - r + \sigma^2/2\right]\\
\theta_2 &= \frac{1}{\sigma}\left[\log\left(\frac{D + \mathcal{C}_{\texttt{cb}} - \tau(A_0 + \gamma\mathcal{C}_{\texttt{cb}})}{(1-\tau)A_0}\right) - r + \sigma^2/2\right]\\
\theta_3 &= \frac{1}{\sigma}\left[\log\left(\frac{D + \mathcal{C}_{\texttt{cb}} - \tau CB_1}{A_0}\right) - r + \sigma^2/2\right].
\end{aligned}
$$

Proof. We know that debt at time zero is given by

$$D_0 = e^{-r}(D + \mathcal{C}_{\texttt{cb}}) - e^{-r}\mathbb{E}^Q((D + \mathcal{C}_{\texttt{cb}} - [A_1 + \tau CB_1 - \tau(A_1 - A_0 + CB_1 - \gamma\mathcal{C}_{\texttt{cb}})^+])^+). \quad \text{(A.4)}$$

Let us denote the solution to $A_1 = A_0 - CB_1 + \gamma\mathcal{C}_{\texttt{cb}}$ by θ_1.

$$
\begin{aligned}
A_0 \exp(r - \sigma^2/2 + \sigma\theta_1) = A_0 - CB_1 + \gamma\mathcal{C}_{\texttt{cb}} &\implies\\
\theta_1 = \frac{1}{\sigma}[\log\left(\frac{A_0 - CB_1 + \gamma\mathcal{C}_{\texttt{cb}}}{A_0}\right) - r + \sigma^2/2].
\end{aligned}
$$

Also, set θ_2 to be the solution to $D+\mathcal{C}_{\texttt{cb}} = A_1+\tau CB_1-\tau(A_1-A_0+CB_1-\gamma\mathcal{C}_{\texttt{cb}})$. Rearranging leads to

$$
\begin{aligned}
(1-\tau)A_1 &= D + \mathcal{C}_{\texttt{cb}} - \tau(A_0 + \gamma\mathcal{C}_{\texttt{cb}}) \implies\\
(1-\tau)A_0 \exp(r - \sigma^2/2 + \sigma\theta_2) &= D + \mathcal{C}_{\texttt{cb}} - \tau(A_0 + \gamma\mathcal{C}_{\texttt{cb}}) \implies\\
\theta_2 &= \frac{1}{\sigma}[\log\left(\frac{D + \mathcal{C}_{\texttt{cb}} - \tau(A_0 + \gamma\mathcal{C}_{\texttt{cb}})}{(1-\tau)A_0}\right) - r + \sigma^2/2].
\end{aligned}
$$

Finally, define θ_3 to be the solution to $D + \mathcal{C}_{\texttt{cb}} = A_1 + \tau CB_1$, which gives

$$
\begin{aligned}
A_0 \exp(r - \sigma^2/2 + \sigma\theta_3) &= D + \mathcal{C}_{\texttt{cb}} - \tau CB_1 \implies\\
\theta_3 &= \frac{1}{\sigma}[\log\left(\frac{D + \mathcal{C}_{\texttt{cb}} - \tau CB_1}{A_0}\right) - r + \sigma^2/2].
\end{aligned}
$$

Distinguish the following cases

(i) $D + \mathcal{C}_{\mathtt{cb}} \leq A_0 - CB_1 + \gamma\mathcal{C}_{\mathtt{cb}}$.

(ii) $D + \mathcal{C}_{\mathtt{cb}} > A_0 - CB_1 + \gamma\mathcal{C}_{\mathtt{cb}}$.

In case (i), the expectation in (A.4) is easily evaluated, as the firm can only go bankrupt if $A_1 \leq A_0 + CB_1 - \gamma\mathcal{C}_{\mathtt{cb}}$, which means debt is so low compared to A_0 that if bankruptcy is triggered, a firm can reclaim the complete carry back. The expectation then follows from

$$
\begin{aligned}
& e^{-r}\mathbb{E}^Q((D + \mathcal{C}_{\mathtt{cb}} - [A_1 + \tau CB_1 - \tau(A_1 - A_0 + CB_1 - \gamma\mathcal{C}_{\mathtt{cb}})^+])^+) \\
&= e^{-r}\int_{-\infty}^{\theta_3} D + \mathcal{C}_{\mathtt{cb}} - A_1 - \tau CB_1 dQ \\
&= e^{-r}\int_{-\infty}^{\theta_3} (D + \mathcal{C}_{\mathtt{cb}} - A_0 e^{r-\sigma^2/2+\sigma x} - \tau CB_1)\varphi(x)dx \\
&= e^{-r}(D + \mathcal{C}_{\mathtt{cb}} - \tau CB_1)\Phi(\theta_3) - A_0\Phi(\theta_3 - \sigma).
\end{aligned}
$$

In case (ii), bankruptcy is triggered already when a firm can only reclaim part of the carry back from the tax authority. In this case the expectation in (A.4) is found by splitting the integral

$$
\begin{aligned}
& e^{-r}\mathbb{E}^Q((D + \mathcal{C}_{\mathtt{cb}} - [A_1 + \tau CB_1 - \tau(A_1 - A_0 + CB_1 - \gamma\mathcal{C}_{\mathtt{cb}})^+])^+) \\
&= e^{-r}\int_{-\infty}^{\theta_1} D + \mathcal{C}_{\mathtt{cb}} - A_1 - \tau CB_1 dQ \\
&+ e^{-r}\int_{\theta_1}^{\theta_2} D + \mathcal{C}_{\mathtt{cb}} - A_1 - \tau CB_1 + \tau(A_1 - A_0 + CB_1 - \gamma\mathcal{C}_{\mathtt{cb}})dQ \\
&= e^{-r}(D + \mathcal{C}_{\mathtt{cb}} - \tau CB_1)\Phi(\theta_2) - A_0\Phi(\theta_2 - \sigma) + e^{-r}\tau\int_{\theta_1}^{\theta_2}(A_1 - A_0 + CB_1 - \gamma\mathcal{C}_{\mathtt{cb}})dQ \\
&= e^{-r}(D + \mathcal{C}_{\mathtt{cb}} - \tau CB_1)\Phi(\theta_2) - A_0\Phi(\theta_2 - \sigma) \\
&+ e^{-r}\tau(CB_1 - A_0 - \gamma\mathcal{C}_{\mathtt{cb}})[\Phi(\theta_2) - \Phi(\theta_1)] + \tau A_0[\Phi(\theta_2 - \sigma) - \Phi(\theta_1 - \sigma)].
\end{aligned}
$$

□

A.4 DTL

Theorem A.5. *The debt value at time zero for a firm having LTD_1 is given by*

$$D_0 = e^{-r}(D + \mathcal{C}_{\mathrm{L}}) - e^{-r}(D + \mathcal{C}_{\mathrm{L}})\Phi(\theta_3) + A_0\Phi(\theta_3 - \sigma),$$

if $D + \mathcal{C}_{\mathrm{L}} \leq A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}$. Otherwise

$$\begin{aligned} D_0 &= e^{-r}(D + \mathcal{C}_{\mathrm{L}}) - e^{-r}(D + \mathcal{C}_{\mathrm{L}})\Phi(\theta_2) + A_0\Phi(\theta_2 - \sigma) \\ &\quad - e^{-r}\tau(LTD_1 - A_0 - \gamma\mathcal{C}_{\mathrm{L}})[\Phi(\theta_2) - \Phi(\theta_1)] - \tau A_0[\Phi(\theta_2 - \sigma) - \Phi(\theta_1 - \sigma)]. \end{aligned}$$

In these expressions

$$\begin{aligned} \theta_1 &= \frac{1}{\sigma}\left[\log\left(\frac{A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}}{A_0}\right) - r + \sigma^2/2\right] \\ \theta_2 &= \frac{1}{\sigma}\left[\log\left(\frac{D + \mathcal{C}_{\mathrm{L}} - \tau(A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}})}{(1-\tau)A_0}\right) - r + \sigma^2/2\right] \\ \theta_3 &= \frac{1}{\sigma}\left[\log\left(\frac{D + \mathcal{C}_{\mathrm{L}}}{A_0}\right) - r + \sigma^2/2\right]. \end{aligned}$$

Proof. The DTL increases the probability of bankruptcy and thus influences coupon payments. To see this, we write

$$D_0 = e^{-r}(D + \mathcal{C}_{\mathrm{L}}) - e^{-r}\mathbb{E}^Q((D + \mathcal{C}_{\mathrm{L}} - [A_1 - \tau(A_1 - A_0 + LTD_1 - \gamma\mathcal{C}_{\mathrm{L}})^+])^+). \tag{A.6}$$

Now we define θ_1 to be the solution to $A_1 = A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}$ (moment from which levered firm with DTL pays taxes). Solving gives

$$\begin{aligned} A_0 \exp(r - \sigma^2/2 + \sigma\theta_1) = A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}} &\implies \\ \theta_1 &= \frac{1}{\sigma}[\log\left(\frac{A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}}{A_0}\right) - r + \sigma^2/2]. \end{aligned}$$

Similarly, θ_2 is the solution to $D + \mathcal{C}_{\mathrm{L}} = A_1 - \tau(A_1 - A_0 + LTD_1 - \gamma\mathcal{C}_{\mathrm{L}})$. Rearranging leads to

$$
\begin{aligned}
(1-\tau)A_1 &= D + \mathcal{C}_{\mathrm{L}} - \tau(A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}) \implies \\
(1-\tau)A_0\exp(r - \sigma^2/2 + \sigma\theta_2) &= D + \mathcal{C}_{\mathrm{L}} - \tau(A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}) \implies \\
\theta_2 &= \frac{1}{\sigma}[\log\left(\frac{D + \mathcal{C}_{\mathrm{L}} - \tau(A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}})}{(1-\tau)A_0}\right) - r + \sigma^2/2].
\end{aligned}
$$

Finally, θ_3 is the solution to $D + \mathcal{C}_{\mathrm{L}} = A_1$. Solving renders

$$
\begin{aligned}
A_0\exp(r - \sigma^2/2 + \sigma\theta_3) &= D + \mathcal{C}_{\mathrm{L}} \implies \\
\theta_3 &= \frac{1}{\sigma}[\log\left(\frac{D + \mathcal{C}_{\mathrm{L}}}{A_0}\right) - r + \sigma^2/2].
\end{aligned}
$$

The expectation in (A.6) follows by distinguishing two cases

(i) $D + \mathcal{C}_{\mathrm{L}} \leq A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}$.

(ii) $D + \mathcal{C}_{\mathrm{L}} > A_0 - LTD_1 + \gamma\mathcal{C}_{\mathrm{L}}$.

Case (i) renders the solution

$$
\begin{aligned}
&e^{-r}\mathbb{E}^Q((D + \mathcal{C}_{\mathrm{L}} - [A_1 - \tau(A_1 - A_0 + LTD_1 - \gamma\mathcal{C}_{\mathrm{L}})^+])^+) \\
&= e^{-r}\int_{-\infty}^{\theta_3}(D + \mathcal{C}_{\mathrm{L}} - A_1)dQ \\
&= e^{-r}(D + \mathcal{C}_{\mathrm{L}})\Phi(\theta_3) - A_0\Phi(\theta_3 - \sigma).
\end{aligned}
$$

In case (ii), we have

$$
\begin{aligned}
& e^{-r}\mathbb{E}^Q((D + \mathcal{C}_{\mathtt{L}} - [A_1 - \tau(A_1 - A_0 + LTD_1 - \gamma\mathcal{C}_{\mathtt{L}})^+])^+) \\
& = e^{-r}\int_{-\infty}^{\theta_1} D + \mathcal{C}_{\mathtt{L}} - A_1 dQ \\
& + e^{-r}\int_{\theta_1}^{\theta_2} D + \mathcal{C}_{\mathtt{L}} - A_1 + \tau(A_1 - A_0 + LTD_1 - \gamma\mathcal{C}_{\mathtt{L}})dQ \\
& = e^{-r}(D + \mathcal{C}_{\mathtt{L}})\Phi(\theta_2) - A_0\Phi(\theta_2 - \sigma) \\
& + e^{-r}\tau(LTD_1 - A_0 - \gamma\mathcal{C}_{\mathtt{L}})[\Phi(\theta_2) - \Phi(\theta_1)] + \tau A_0[\Phi(\theta_2 - \sigma) - \Phi(\theta_1 - \sigma)].
\end{aligned}
$$

$\square$

B Tax regimes

Table 2: Tax regimes for member states of the European Union.

	Tax rate	Carry back	Carry forward	
			Duration	Deductible
Austria	0.25	no	∞	0.75
Belgium	0.34	no	∞	1.00
Bulgaria	0.10	no	5	1.00
Croatia	0.20	no	5	1.00
Cyprus	0.13	no	5	1.00
Czech Republic	0.19	no	5	1.00
Denmark	0.22	no	∞	0.60
Estonia	0.25	no	NA	NA
Finland	0.20	no	10	1.00
France	0.34	no	∞	0.50
Germany	0.30	no	∞	0.60
Greece	0.29	no	5	1.00
Hungary	0.19	no	5	0.50
Ireland	0.13	yes	∞	1.00
Italy	0.24	no	∞	0.80
Latvia	0.15	no	∞	1.00
Liechtenstein	0.13	no	∞	1.00
Lithuania	0.15	no	∞	0.70
Luxembourg	0.27	no	∞	1.00
Malta	0.35	no	∞	1.00
Netherlands	0.25	yes	9	1.00
Norway	0.25	no	∞	1.00
Poland	0.19	no	5	0.50
Portugal	0.30	no	5	0.70
Romania	0.16	no	7	1.00
Slovakia	0.22	no	4	1.00
Slovenia	0.19	no	∞	0.50
Spain	0.30	no	∞	1.00
Sweden	0.22	no	∞	1.00
United Kingdom	0.20	yes	∞	1.00

Note: Tax rate *denotes the corporate tax rate in each respective country;* Carry back *denotes whether carry back is allowed or not. The settlement term of carry back is only one year when allowed;* Carry forward duration *concerns the number of years that losses can be carried forward ("∞" means there is no time limit);* Carry forward deductible *gives the fraction of carry forward that can be used to mitigate tax expenses next year.*

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