An Unknown B Matrix with Unknown Inputs

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Quantum Seminar

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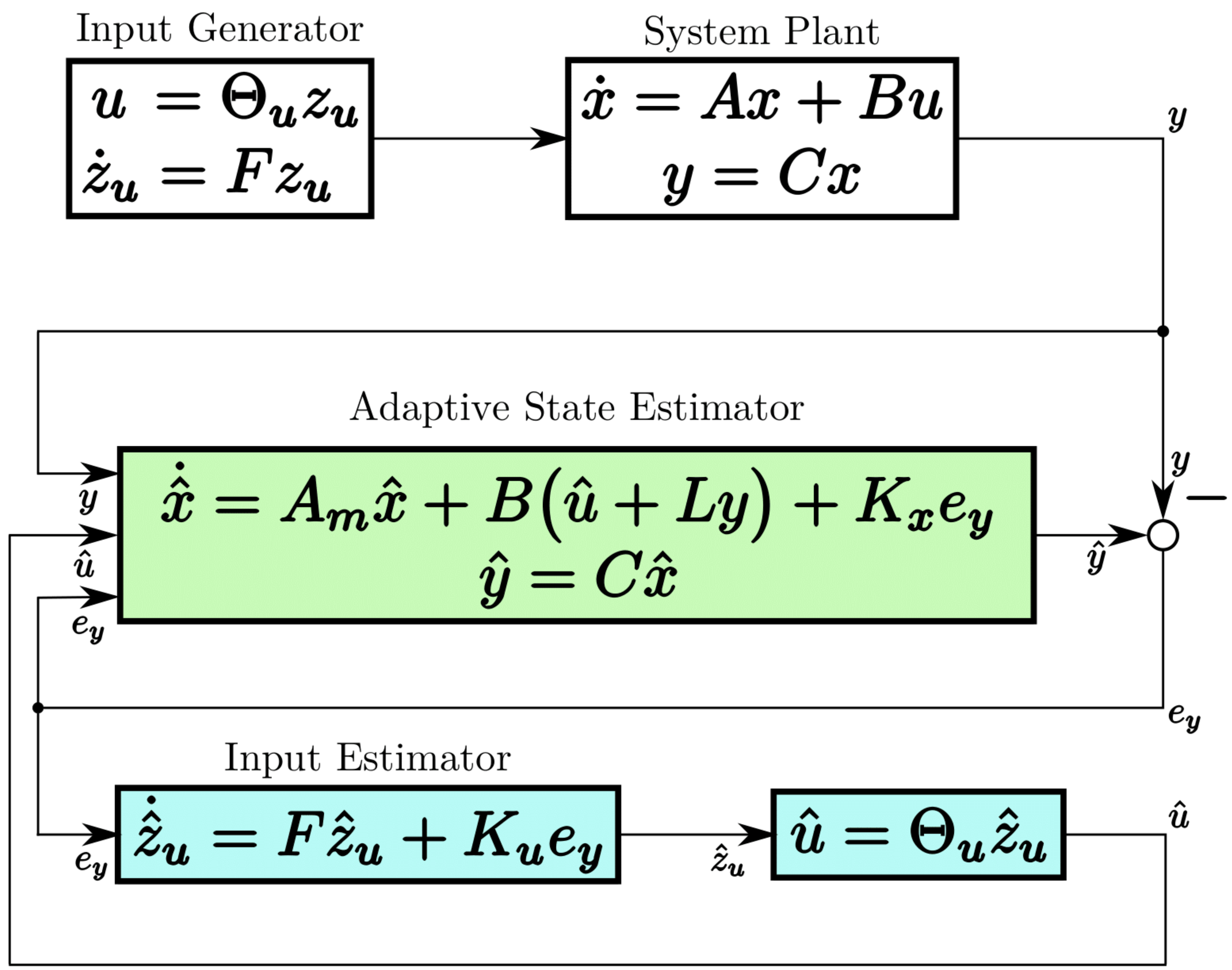
1. UIO Recap
2. Optimized Parameterizations
3. Identifying the “Best Fit” B Matrix for the Brain
4. Application to Emotion Data
5. Application to Movement Data
6. Perturbations

1. UIO Recap

UIO Review: Estimator Architecture

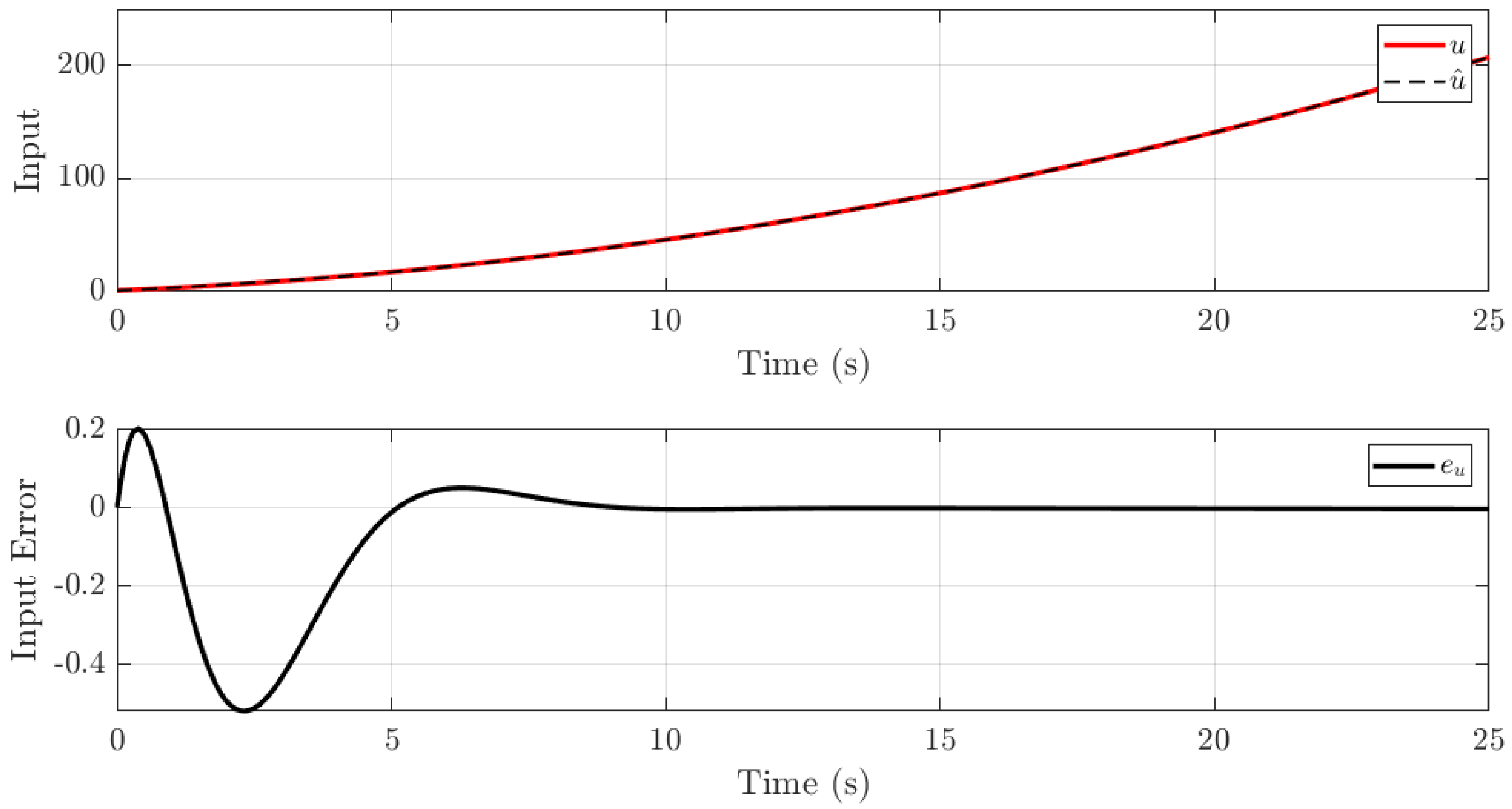


A Polynomial Basis

$$\{1, x, x^2, x^3, \ldots, x^n\}$$

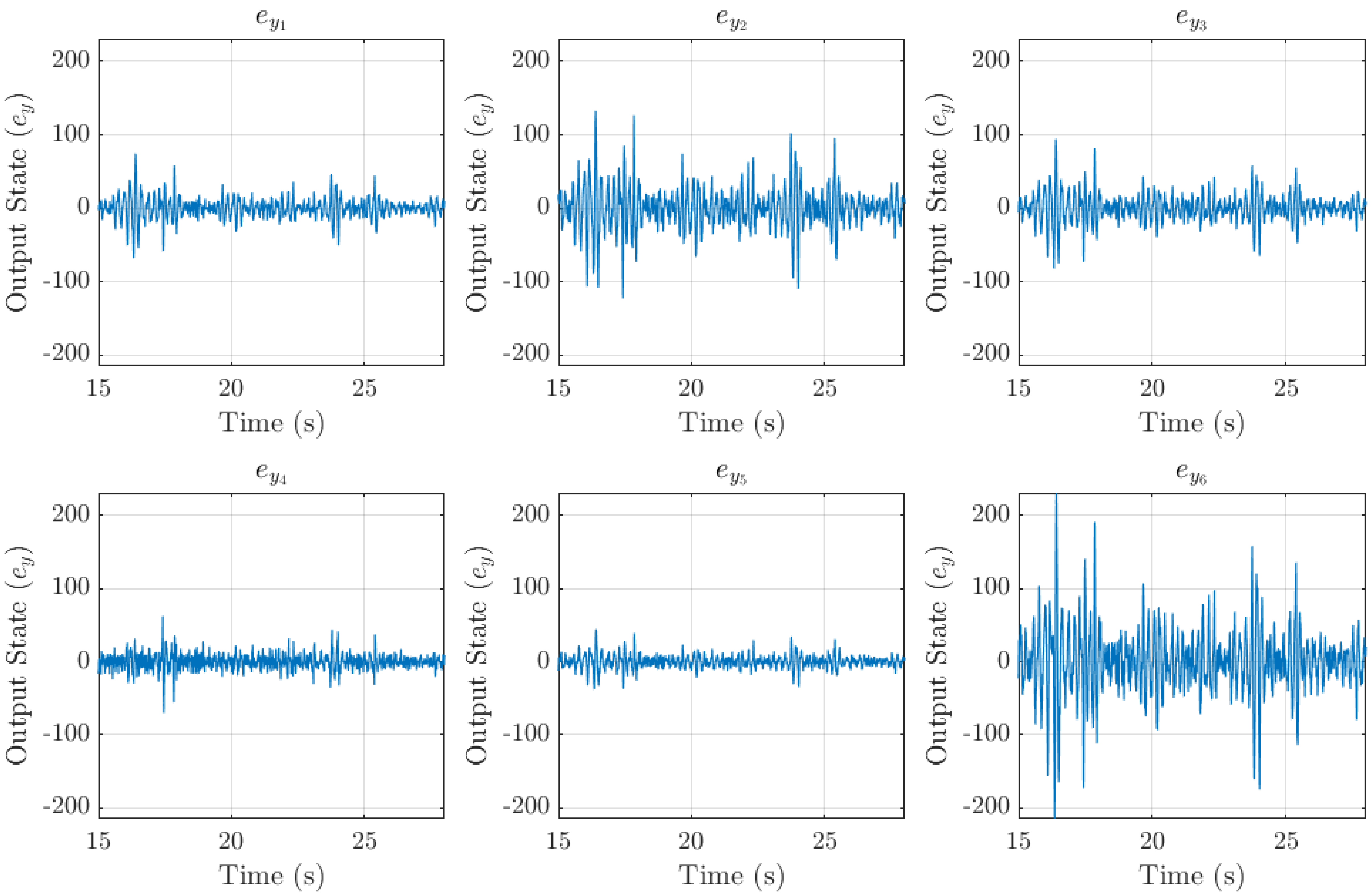
$$u(t) = 1 + 2t + 0.5t^2$$

In some cases the natural basis functions for $u(t)$ are not clearly defined by the available data. For such cases it is usually effective to use a polynomial spline waveform-model

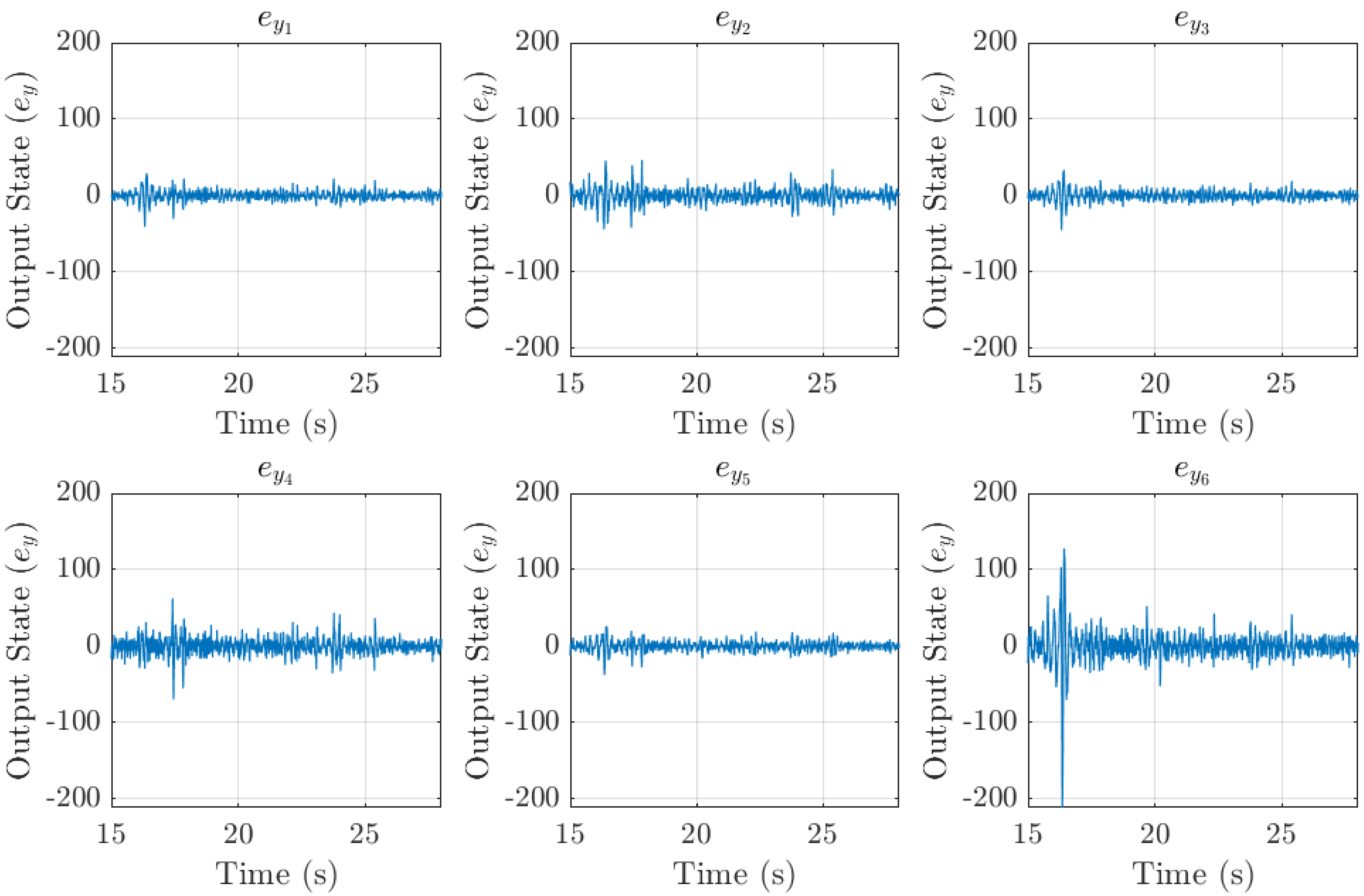


Johnson, C. D. (1989, January). Effective techniques for the identification and accommodation of disturbances. In Pros. 3rd Annual NASA/DOD Controls-Structures Interaction (CSI) Technical Conf. (p. 163).

Polynomial Basis Comparison: Fourier 1-300 hz



Polynomial Basis Comparison: 1-6th



Some Obscured Assumptions

- $B = I$
  - $\dot{x} = Ax + Bu$
- SIMO

A Better B Matrix?

- Alter B to improve modeling error e_y
  - Convex Optimization

2. Optimized Parameterizations

Convex Optimization

an optimization problem in which the objective function is a **convex function** and the feasible set is a **convex set**

the objective function is a **convex function**

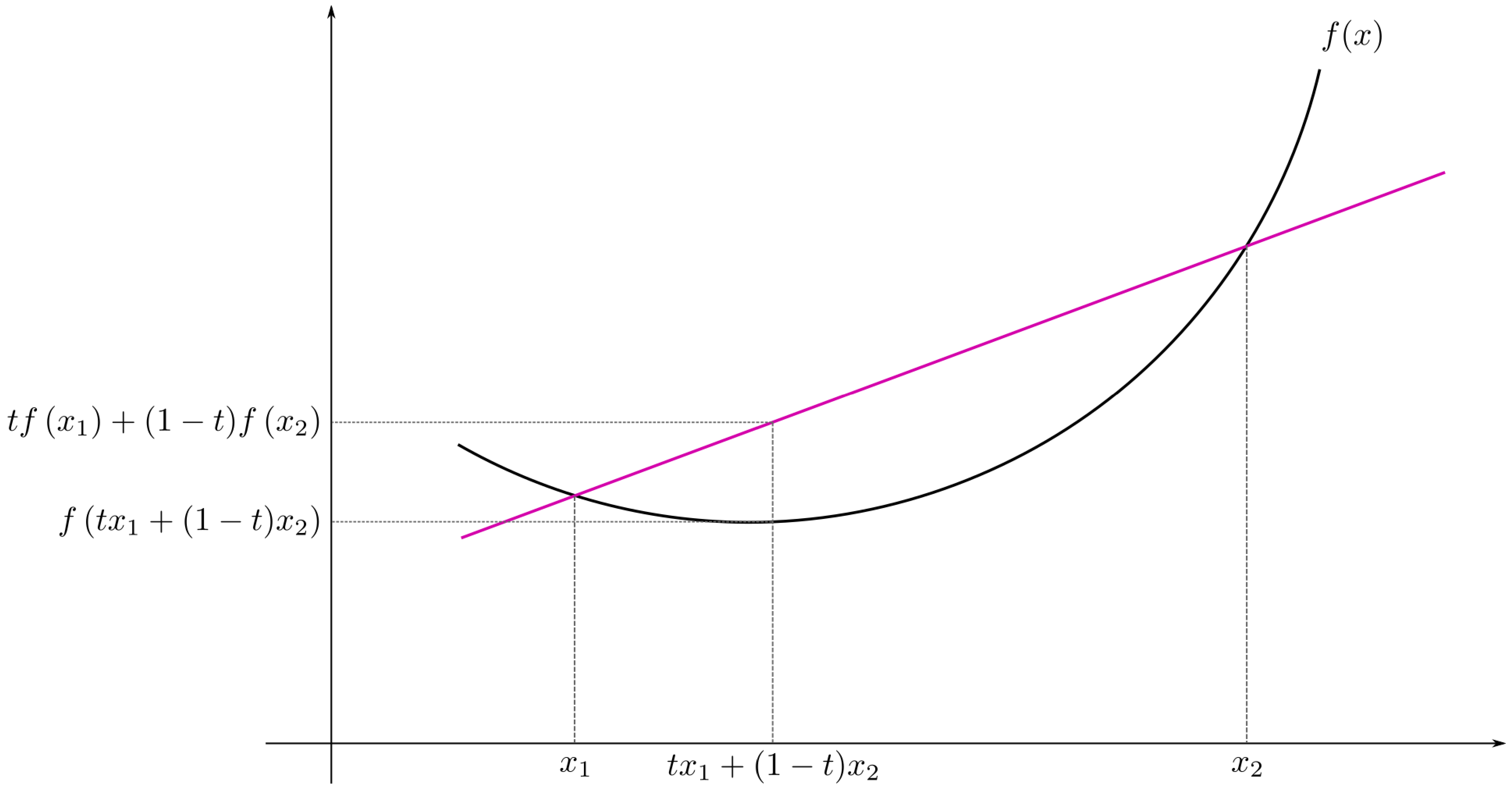
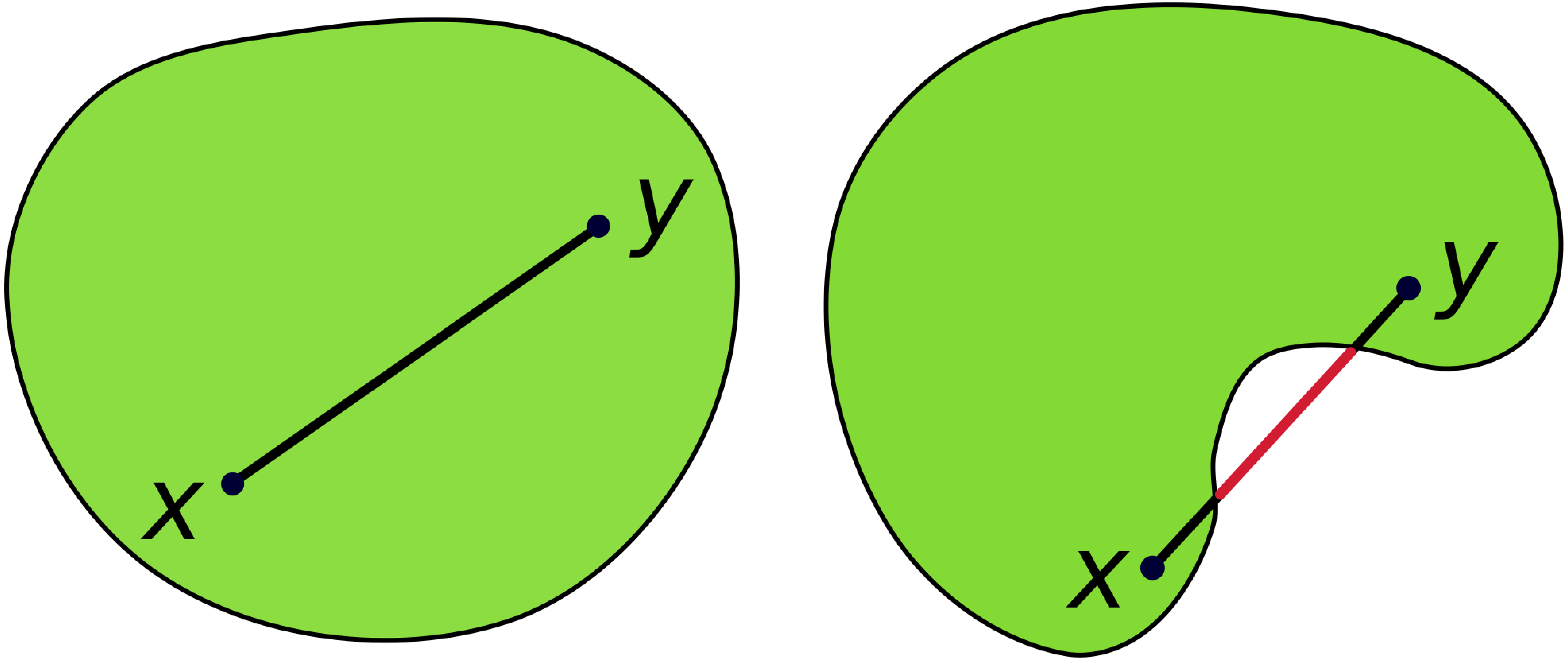


Image Source

the feasible set is a **convex set**



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Important Properties of Convex Problems

- every local minimum is a global minimum
- gradient descent converges in polynomial time

3. Best Fit B Matricies

A Convex Function for B matrix optimization

- $\min ||y - \hat{y} - C\Delta B\hat{u}||_2$
- **not** the only possible minimization

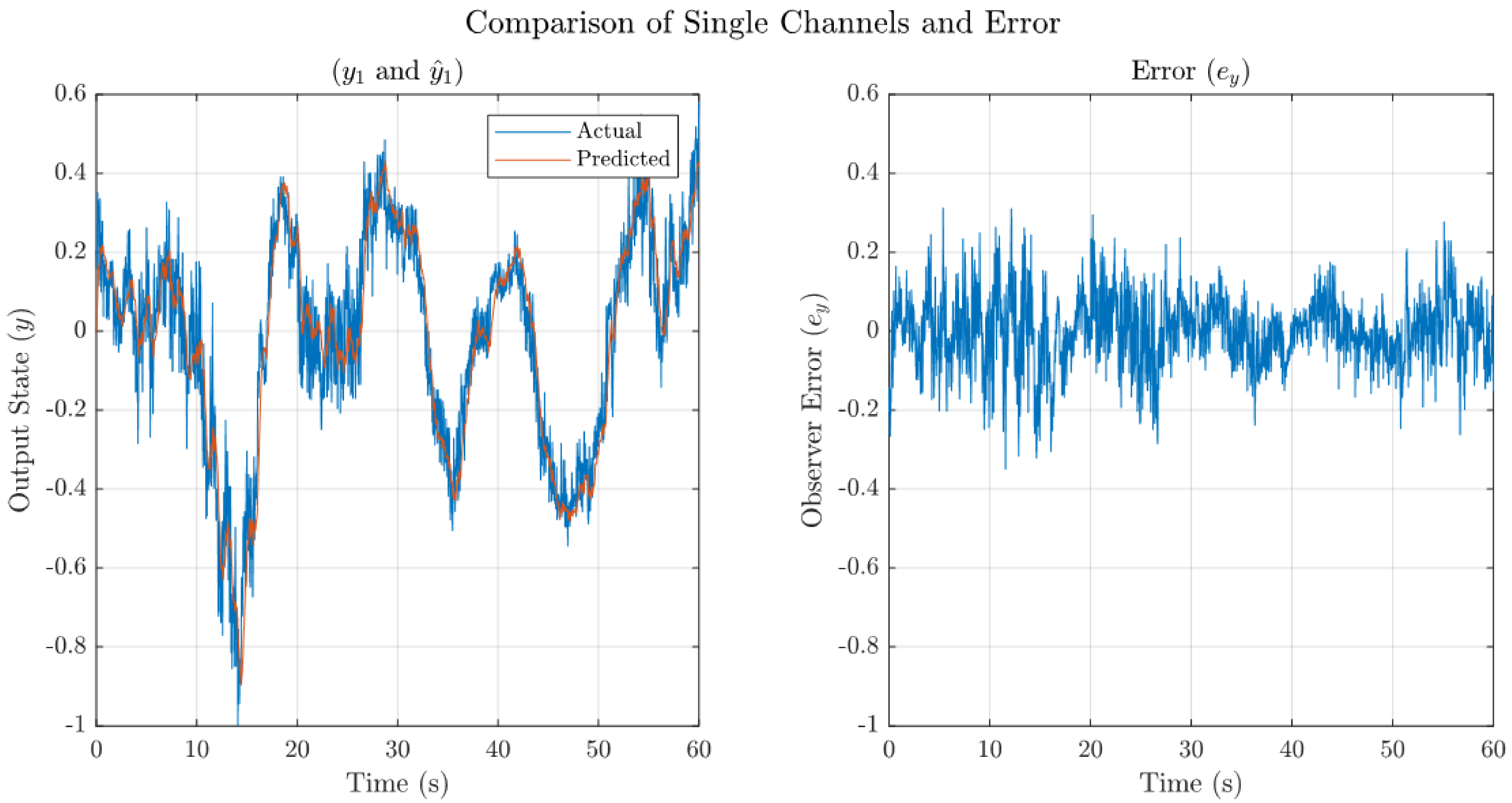
B Matrix Optimization Example

- 3x3 example
  - $\dot{\hat{x}} = A_m x + B\hat{u}$
  - $A_m \neq A$
- $\mathbf{min}\, ||\boldsymbol{y} - \hat{\boldsymbol{y}} - \boldsymbol{C\Delta B\hat{u}}||_2$
- $B = \begin{bmatrix} 1.2 \\ 1 \\ 1.6 \end{bmatrix},$
- $B_m = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

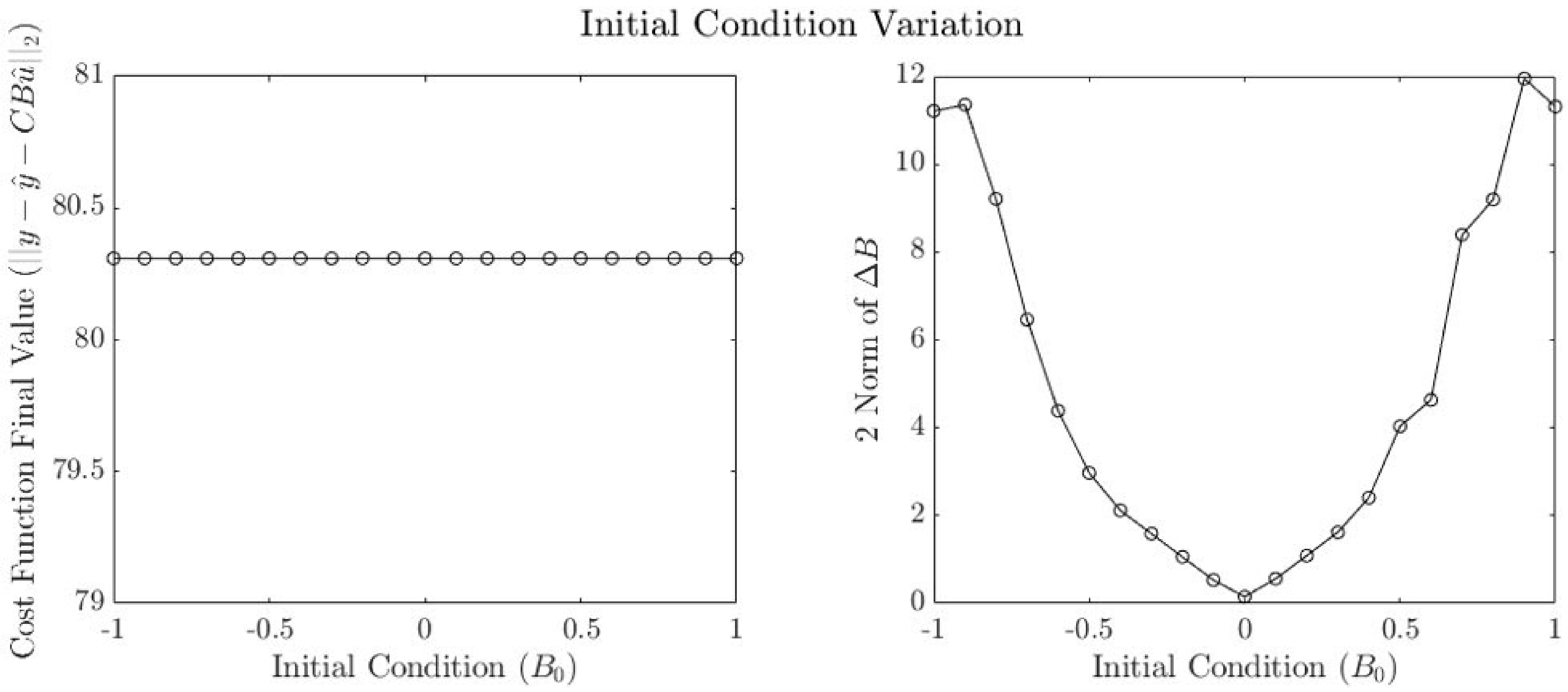
B Matrix Optimization Example

- $\min ||y - \hat{y} - C\Delta B\hat{u}||_2$
- $\Delta B = \begin{bmatrix} 0.18 \\ 0 \\ 0.37 \end{bmatrix},$
- $B_f = \begin{bmatrix} 1.18 \\ 1 \\ 1.37 \end{bmatrix}$

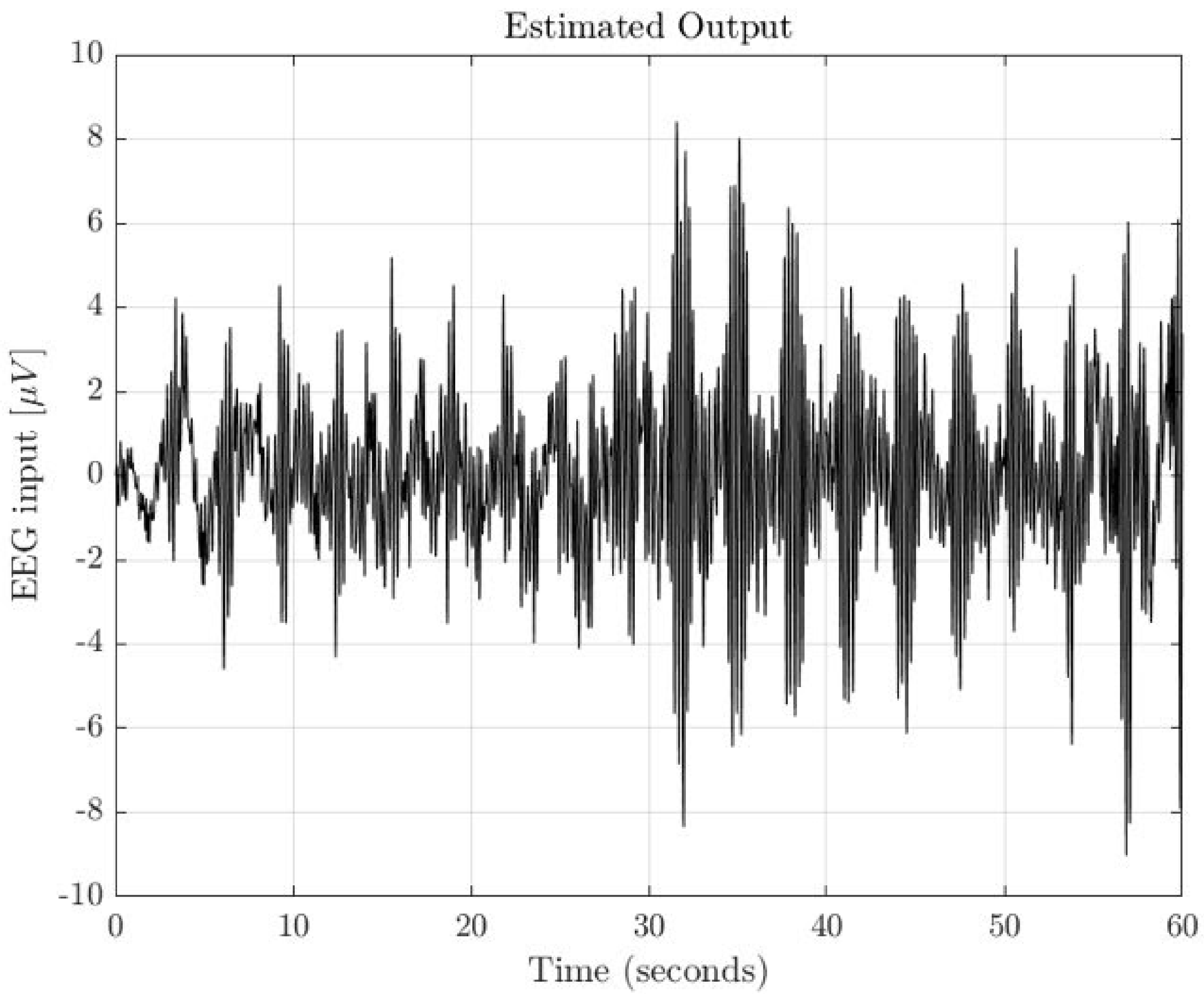
B Matrix on EEG Data

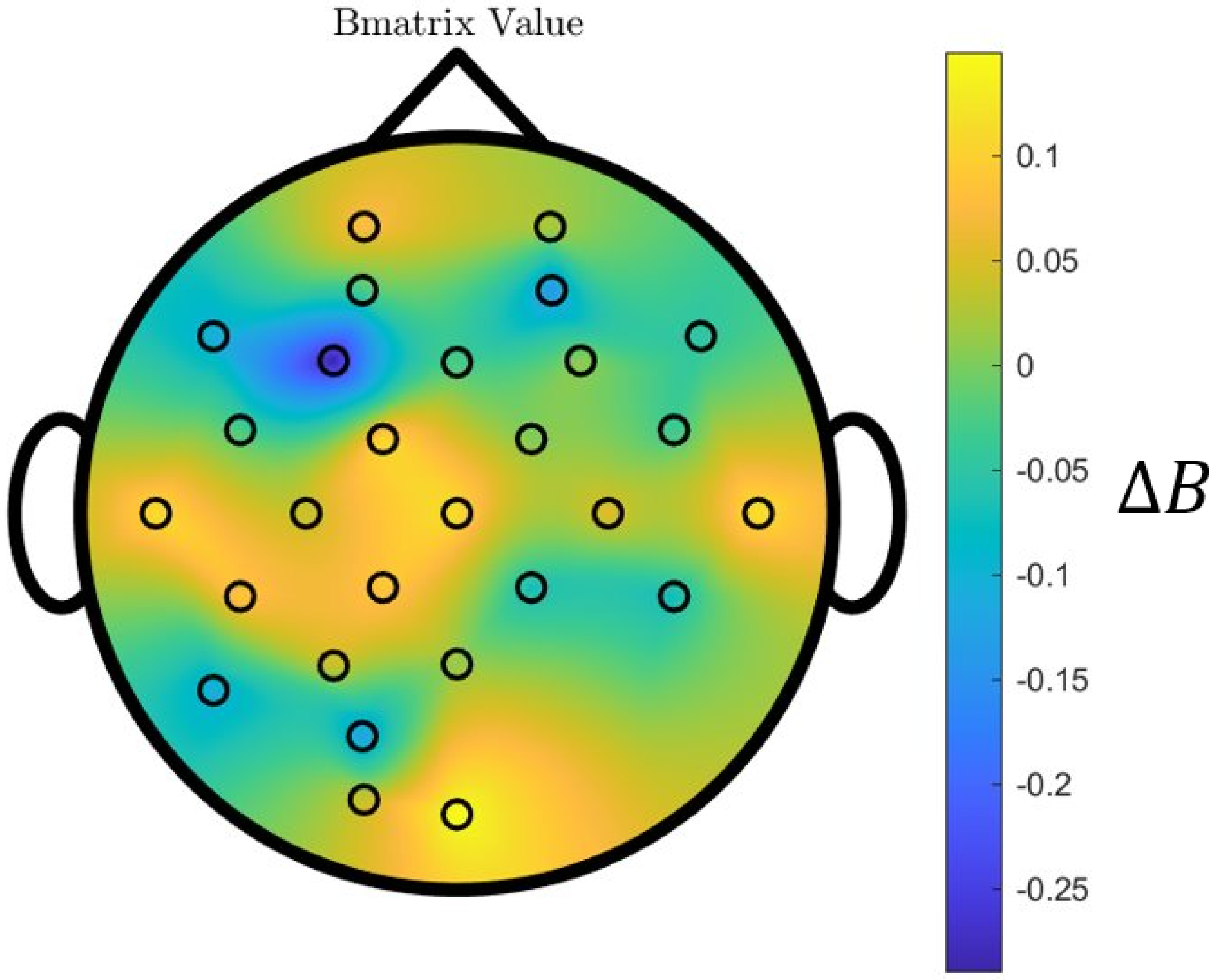


B Matrix on EEG Data



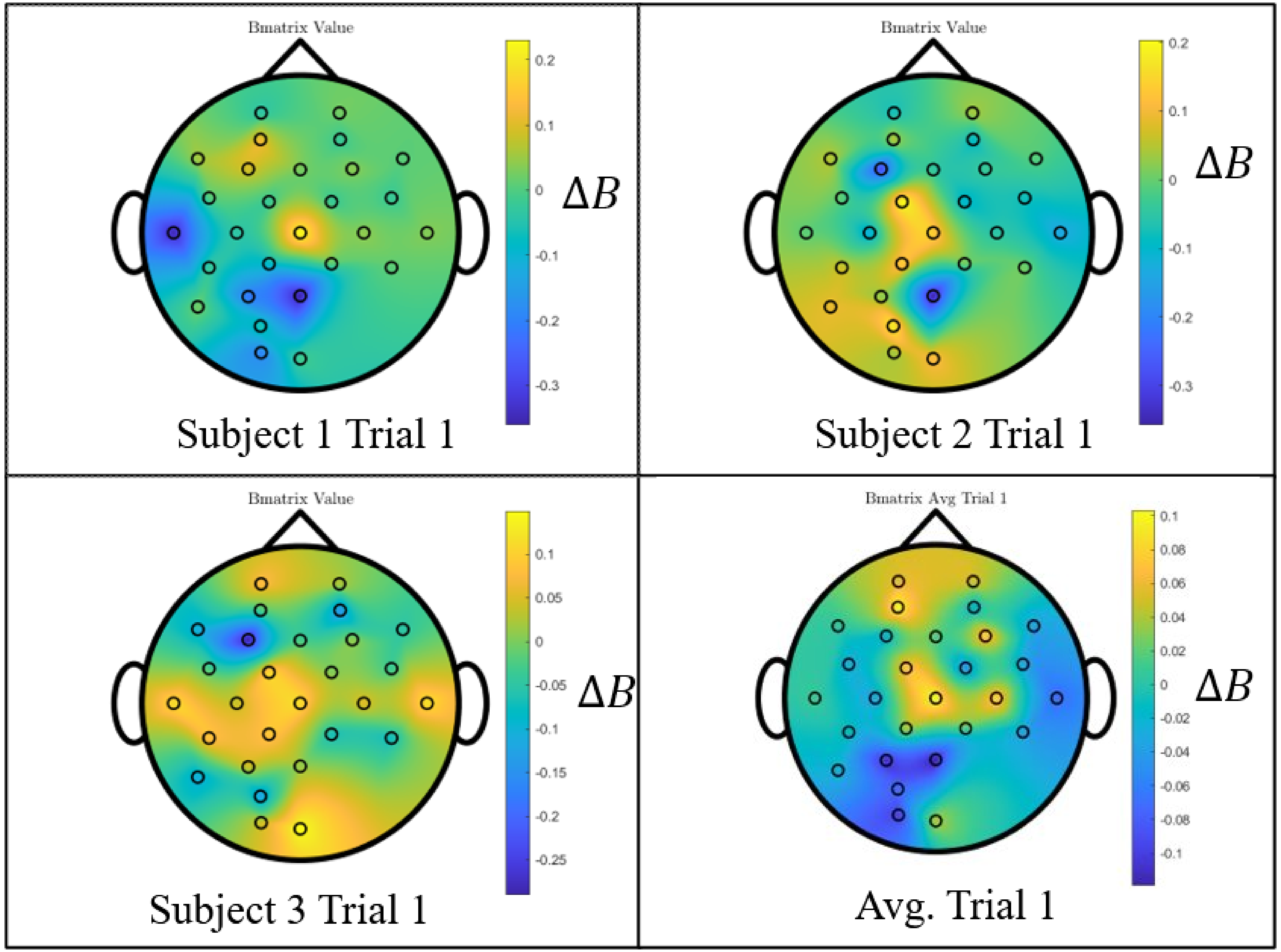
Current models



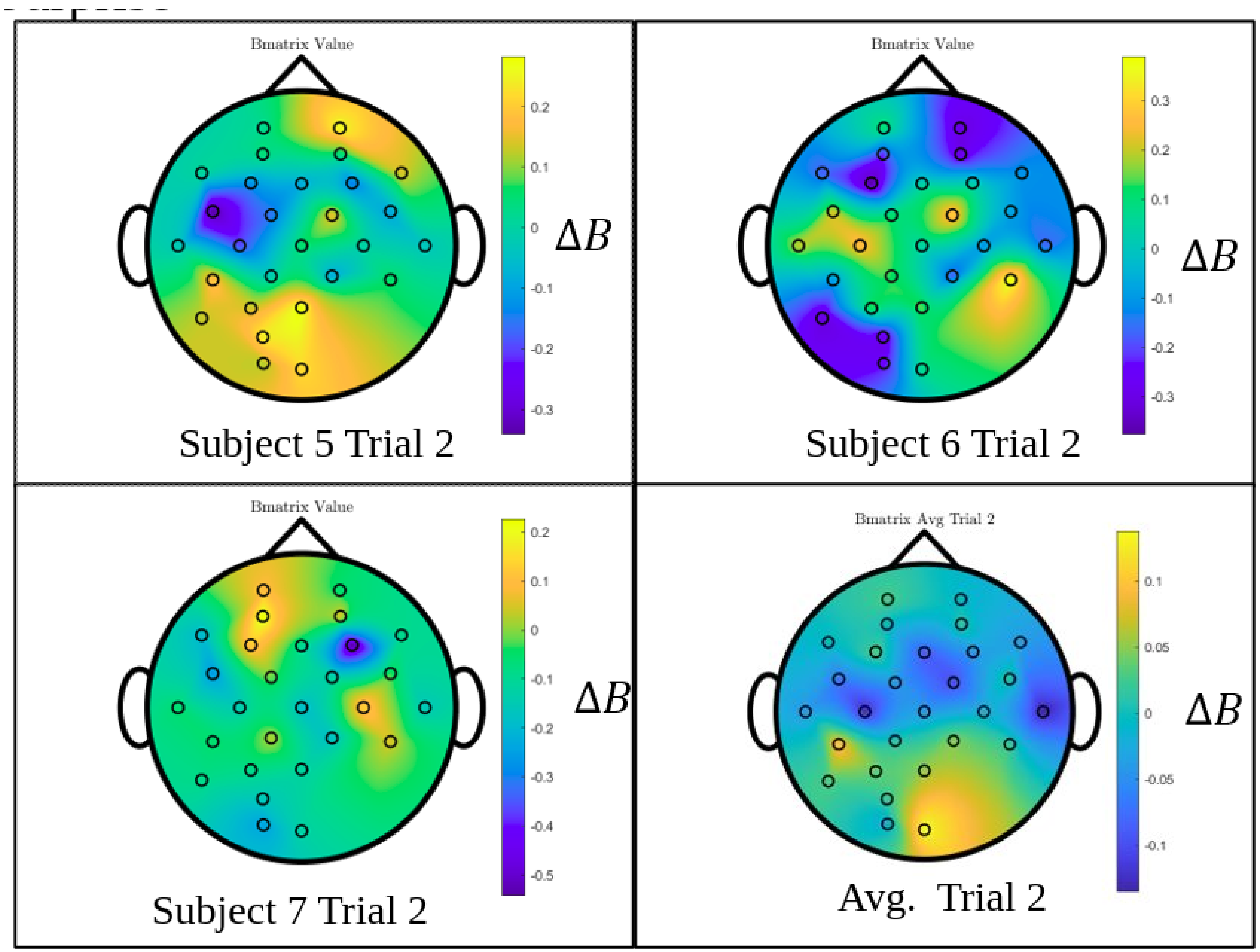


4. Application to Emotion Data

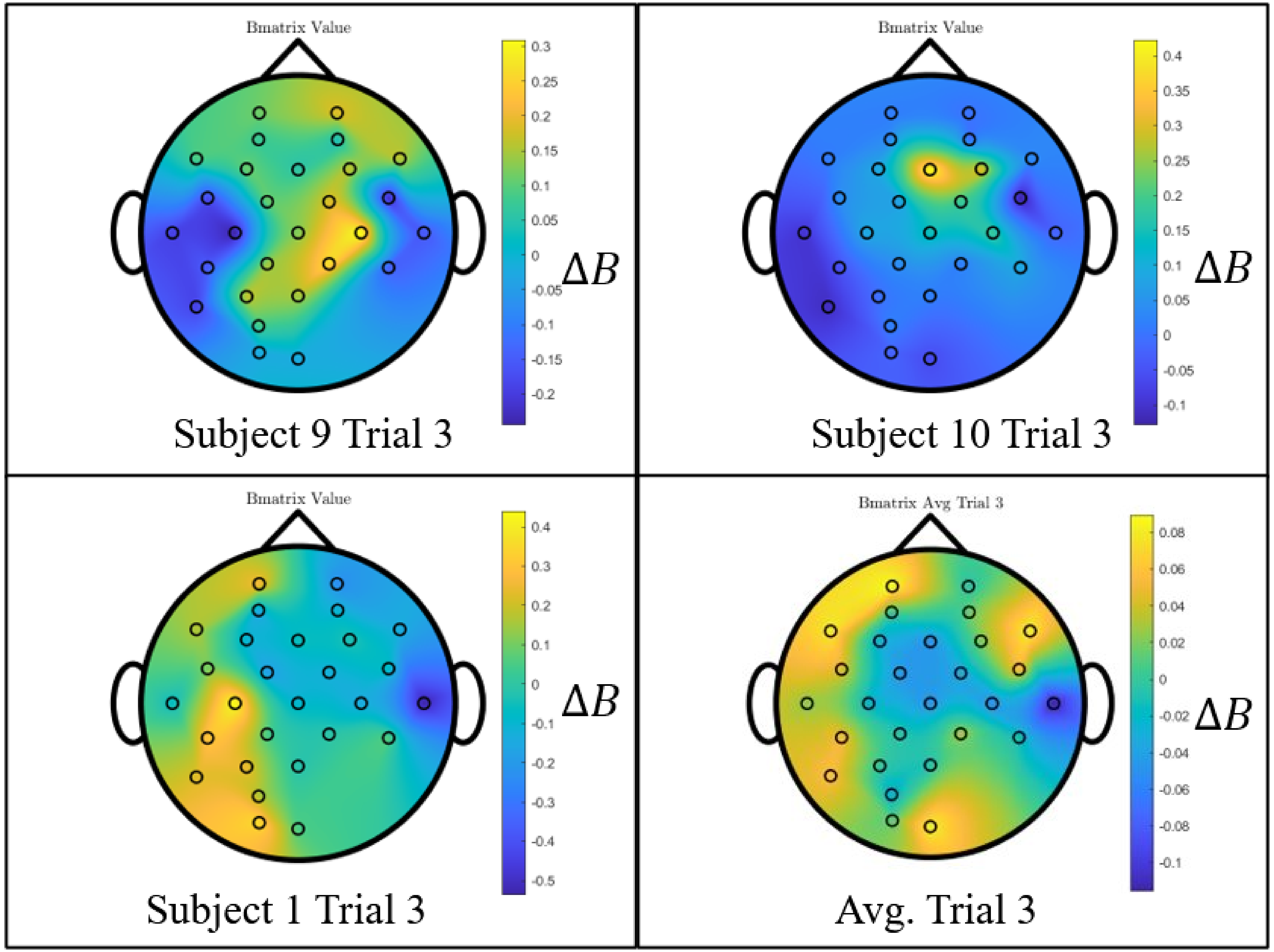
B Matrix on EEG Data: Satisfaction (T1)



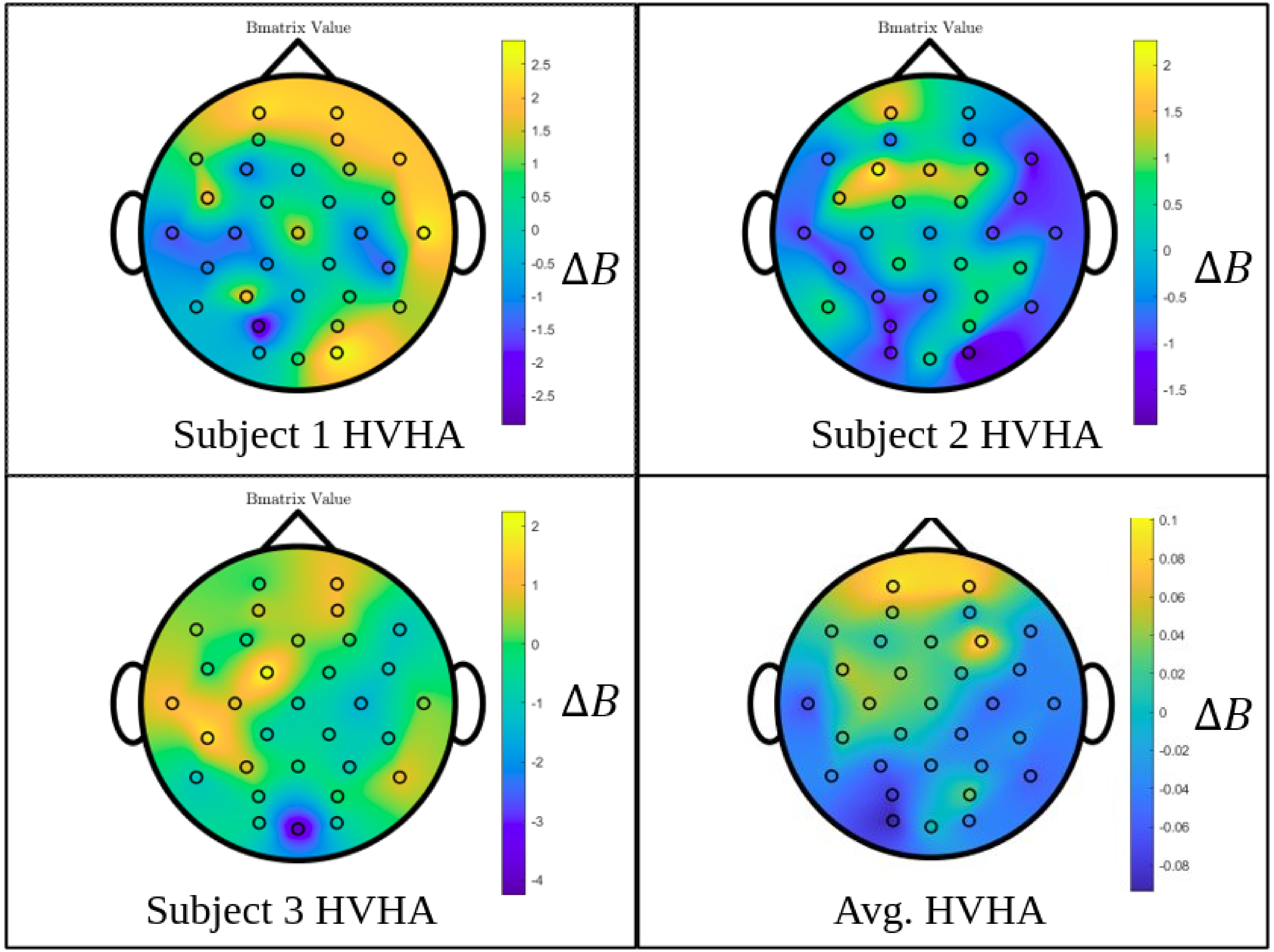
B Matrix on EEG Data: Surprise (T2)



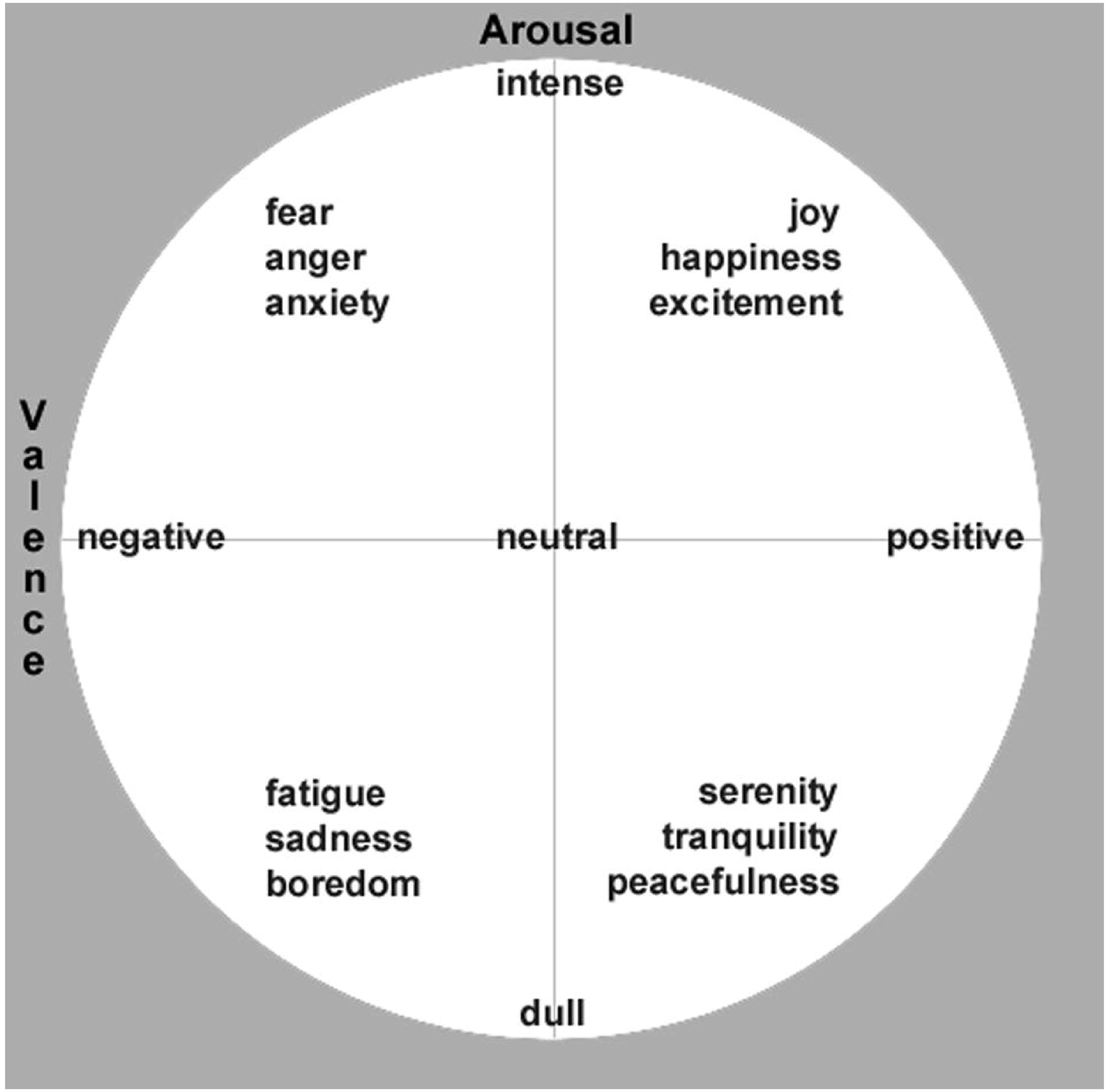
B Matrix on EEG Data: **Fear (T8)**



B Matrix on EEG Data: **HVHA (T13)**

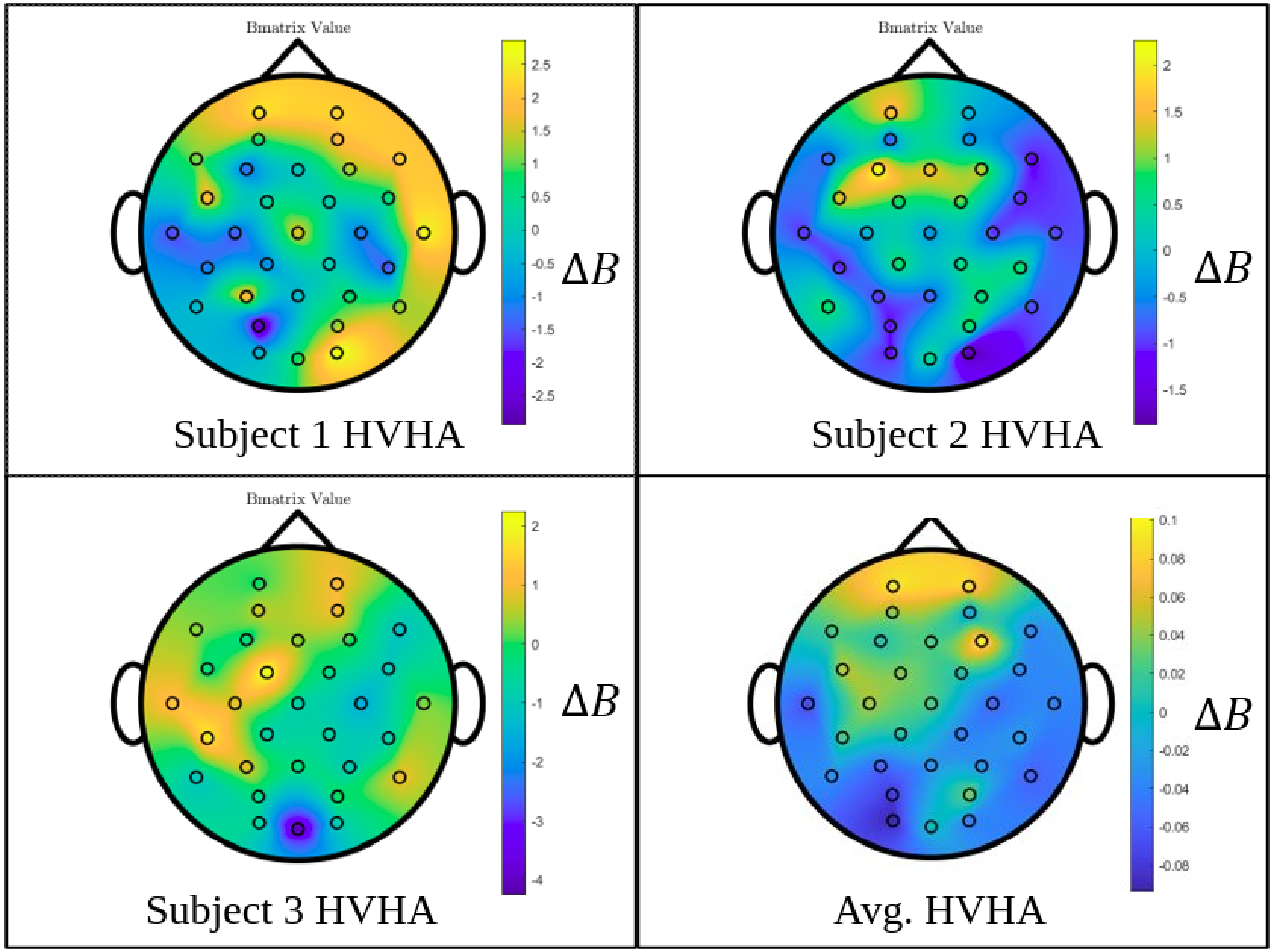


Comparing the same "emotion"

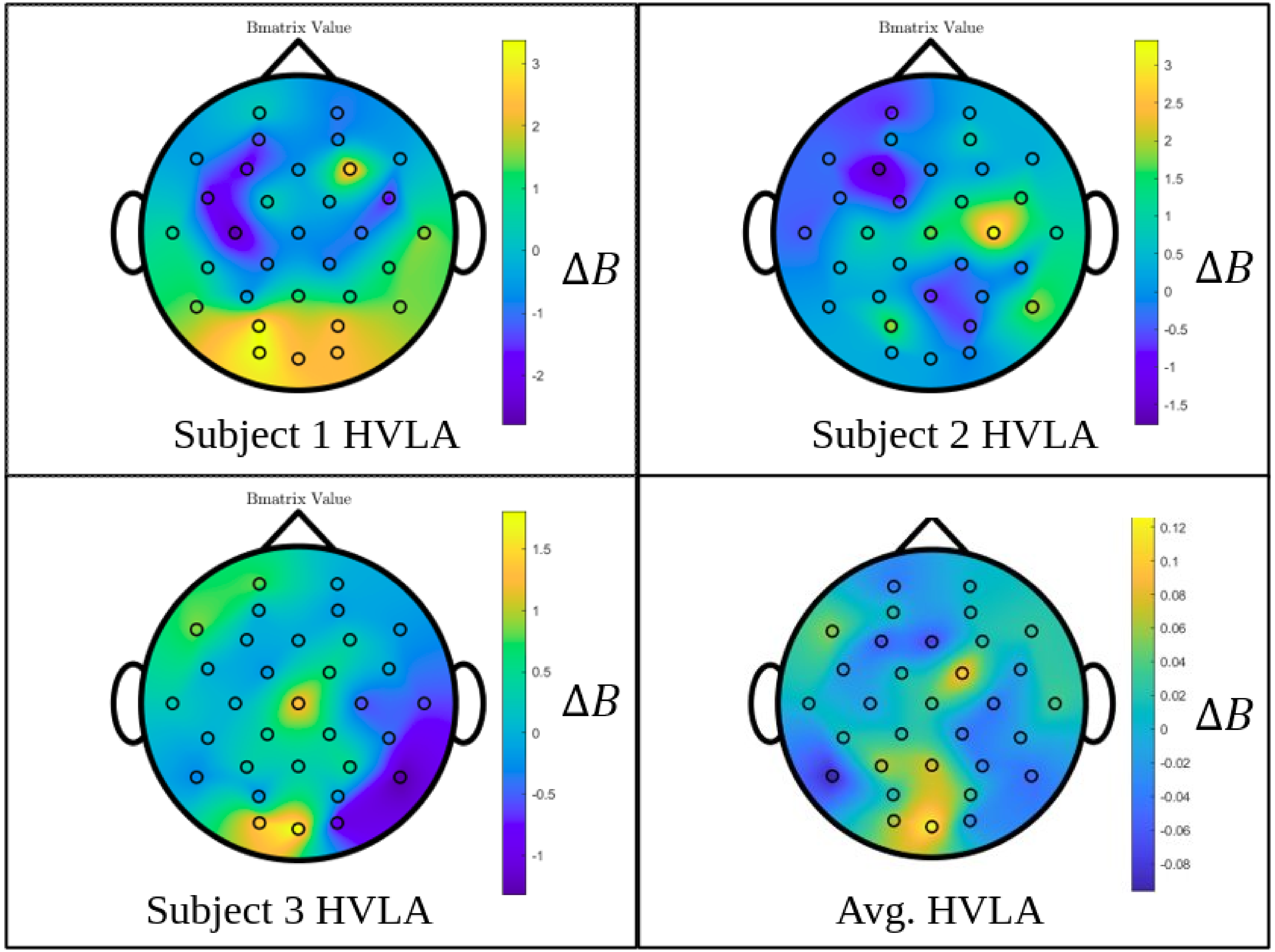


Mneimne, M., Powers, A. S., Walton, K. E., Kosson, D. S., Fonda, S., & Simonetti, J. (2010). Emotional valence and arousal effects on memory and hemispheric asymmetries. Brain and Cognition, 74(1), 10-17.

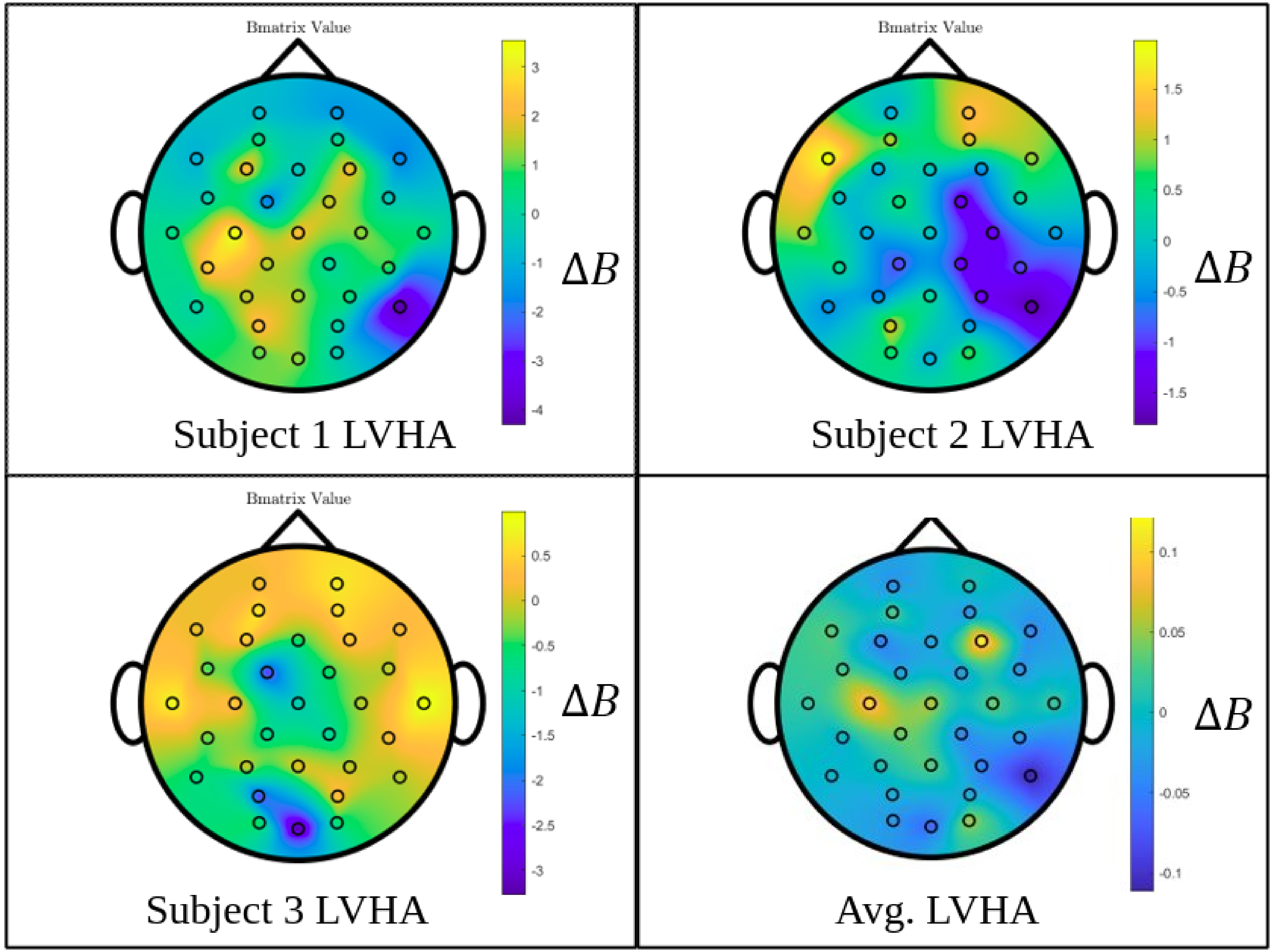
B Matrix on EEG Data: HVHA



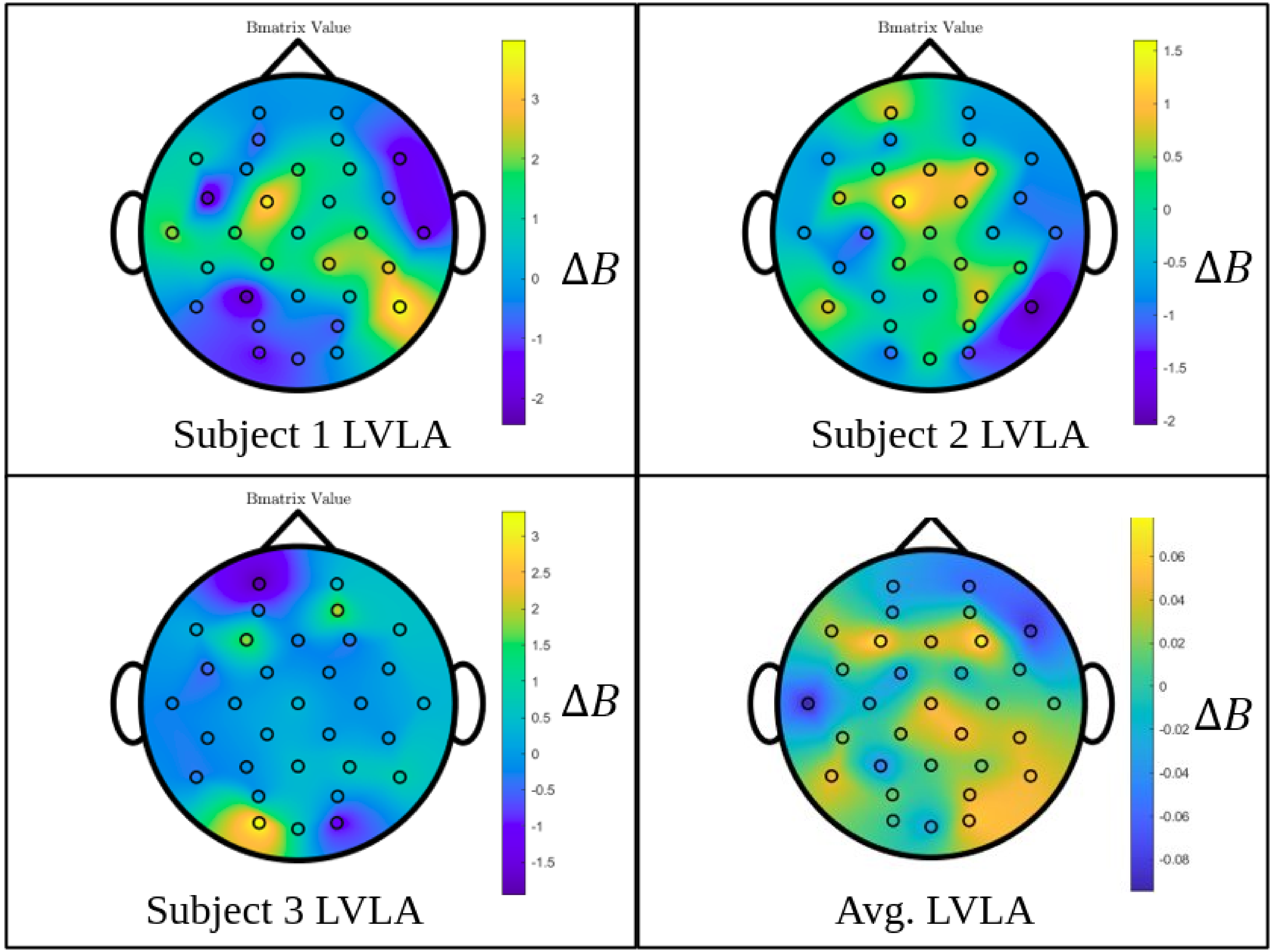
B Matrix on EEG Data: HVLA



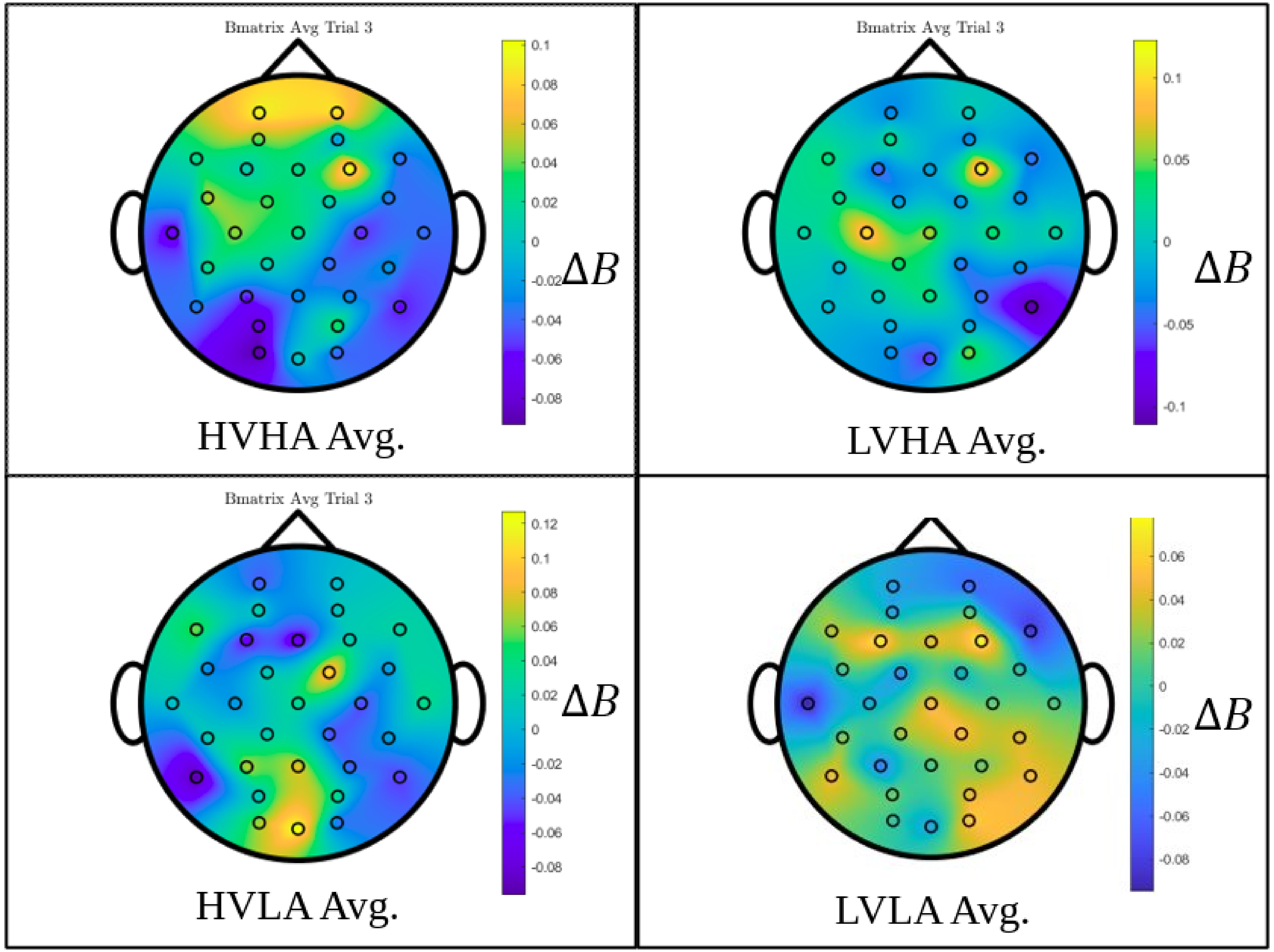
B Matrix on EEG Data: **LVHA**



B Matrix on EEG Data: LVLA

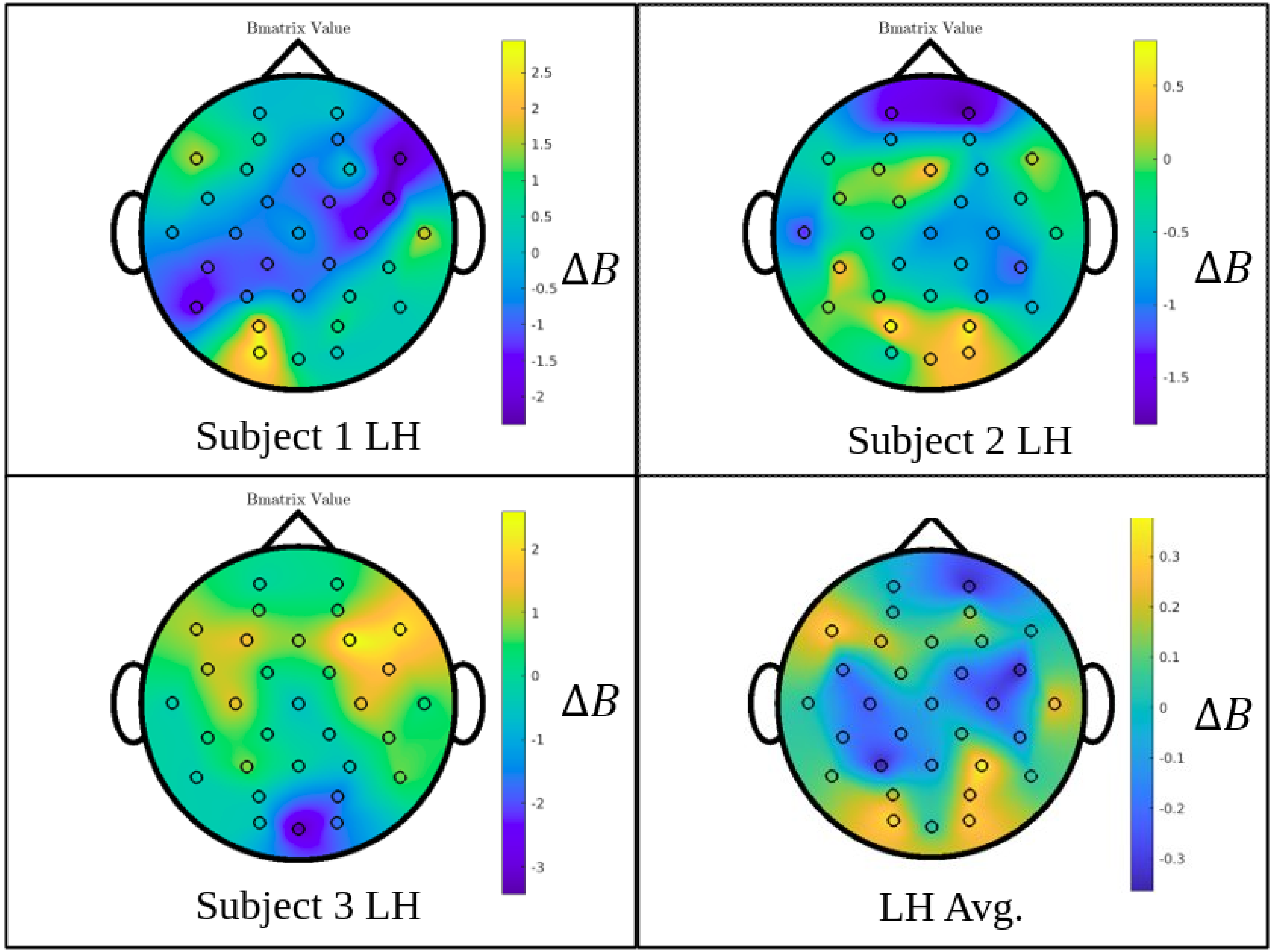


B Matrix on EEG Data: **Avg. Quadrants**

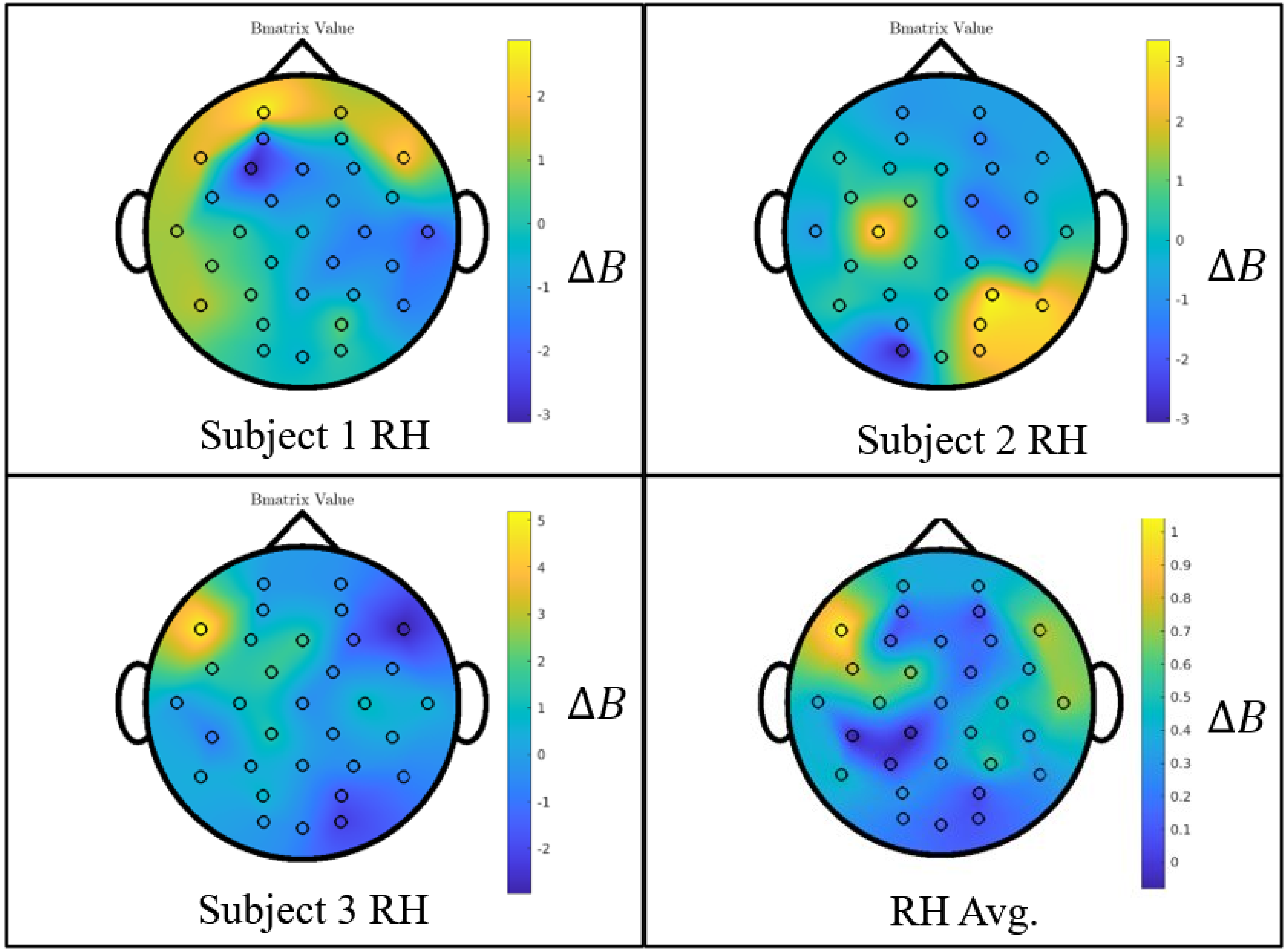


5. Application to Movement Data

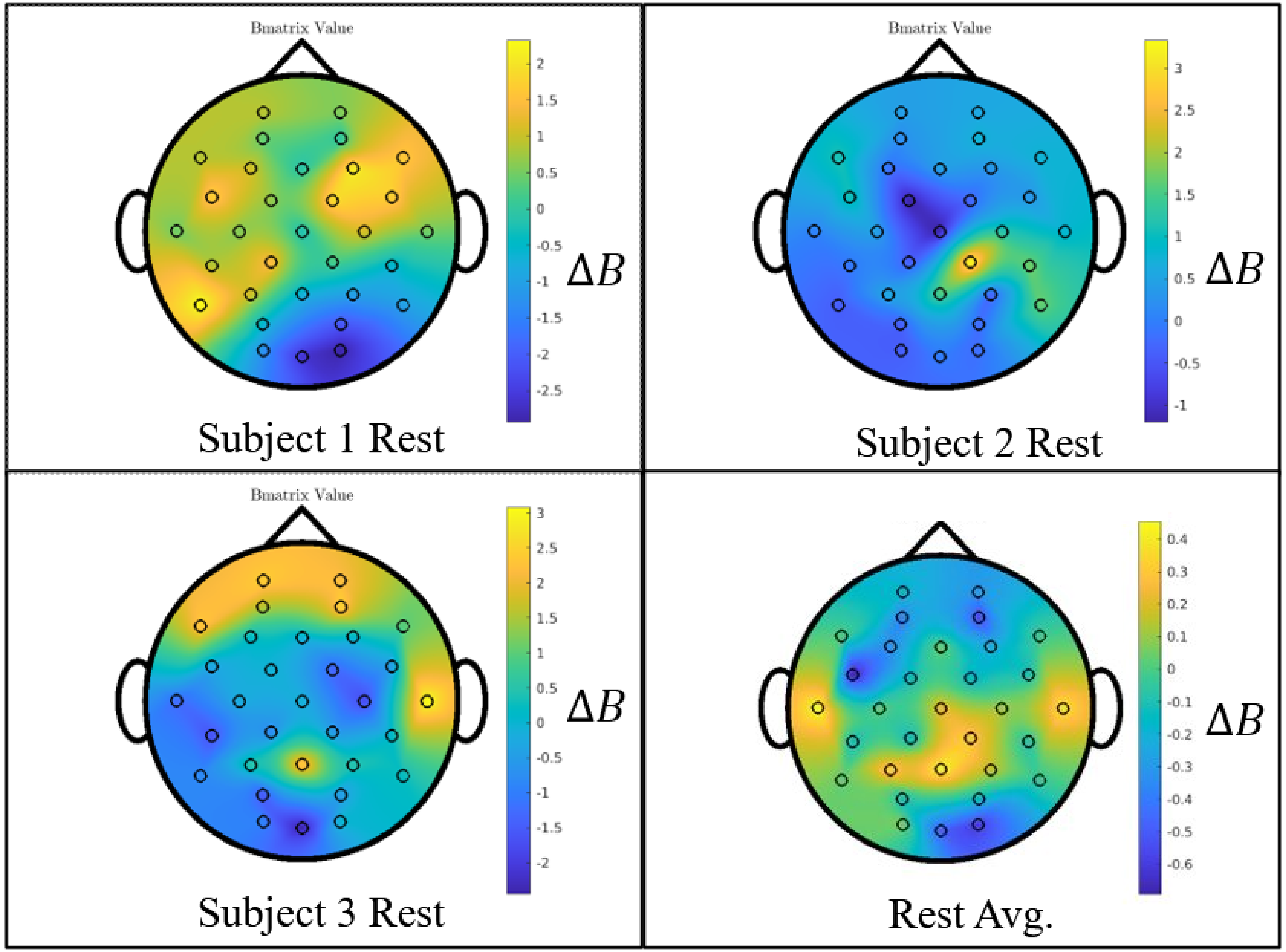
B Matrix on EEG Data: **Left Hand**



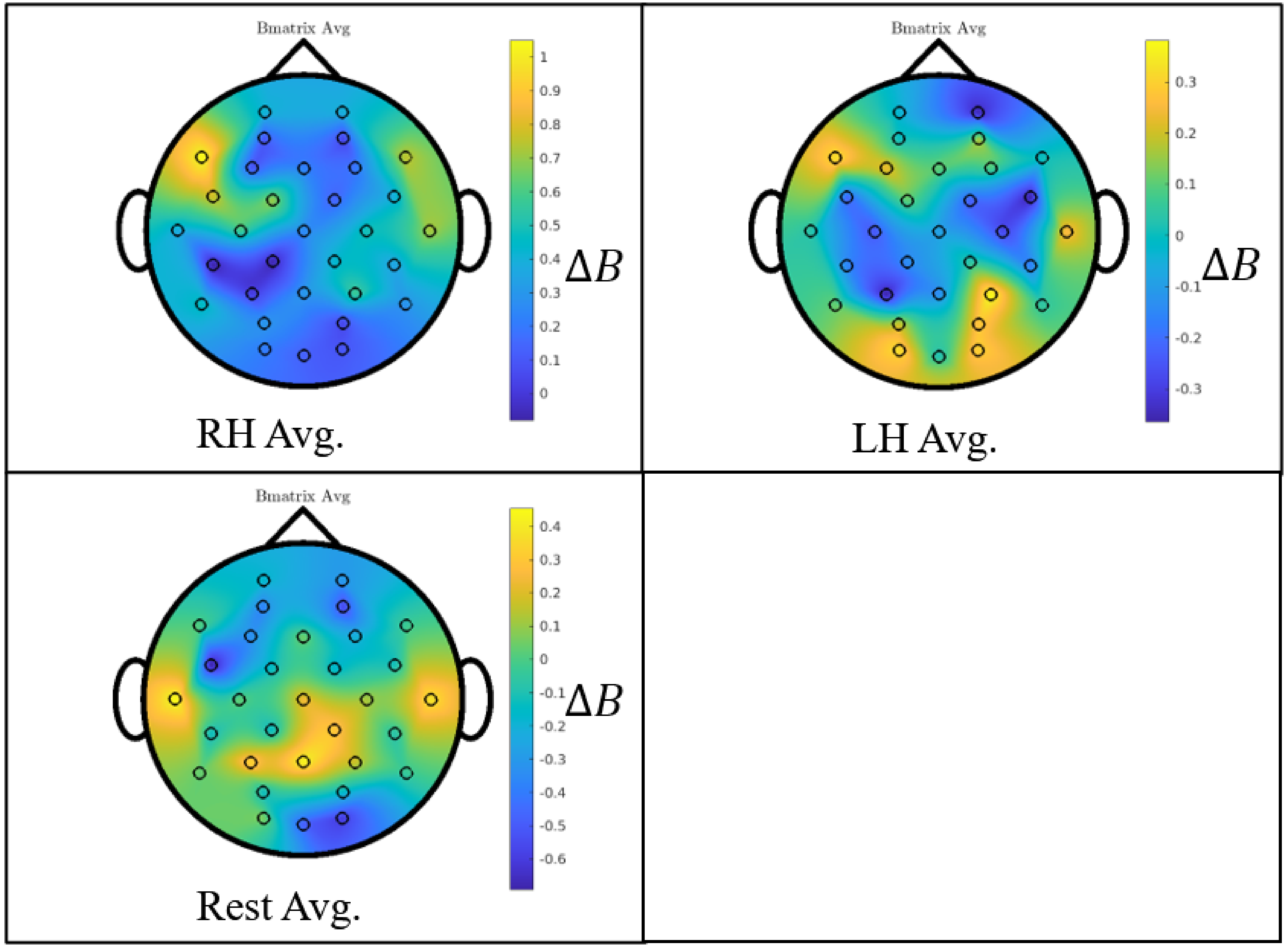
B Matrix on EEG Data: Right Hand



B Matrix on EEG Data: Resting



B Matrix on EEG Data: All Averages



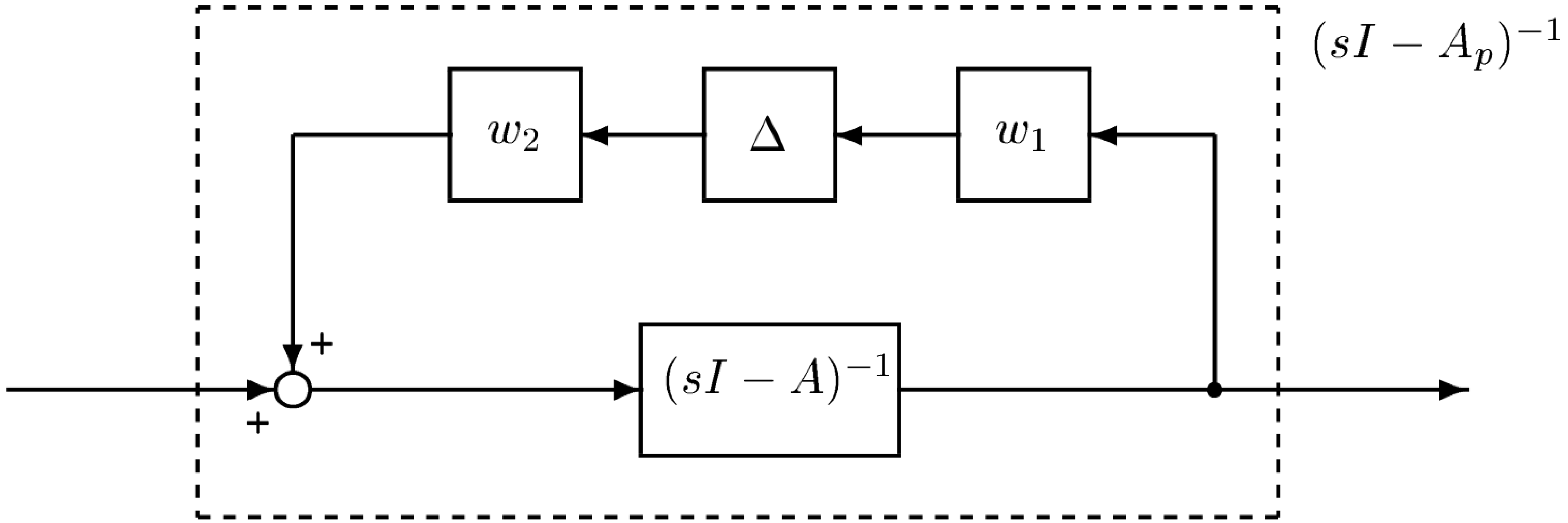
6. Perturbations (V.)

Systems with unmodeled dynamics

- $\dot{x} = A_m x + B_m u$
- $y = C_m x$
- b.c. of uncertainty in real parameters:
  - $A_m = A + \sum \delta_i A_i$
  - $B_m = B + \sum \delta_i B_i$
  - $C_m = C + \sum \delta_i C_i$

Collect the uncertainty

- $A_m = A + \sum \delta_i A_i = A + W_2 \Delta W_1$



Simple Example

- $A = \begin{bmatrix} -3 & -2 \\ 7 & -1 \end{bmatrix}$
- Parametric uncertainty in 2 degrees:
  - $A_1 = \begin{bmatrix} -w_1 & w_1 \\ w_1 & -w_1 \end{bmatrix}$
  - $A_2 = \begin{bmatrix} 0 & -w_2 \\ 2w_2 & 0 \end{bmatrix}$
- $A_m = A + \delta_1 \begin{bmatrix} -w_1 & w_1 \\ w_1 & -w_1 \end{bmatrix} + \delta_2 \begin{bmatrix} 0 & -w_2 \\ 2w_2 & 0 \end{bmatrix}$
- $A_m = A + \begin{bmatrix} -w_1 & 0 & -w_2 \\ w_1 & 2w_2 & 0 \end{bmatrix} \begin{bmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & \delta_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

EEG Example