Adaptive Estimation of Unknown Inputs with Weakly Nonlinear Dynamics

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Adaptive Unknown Input Estimators

Estimator overview

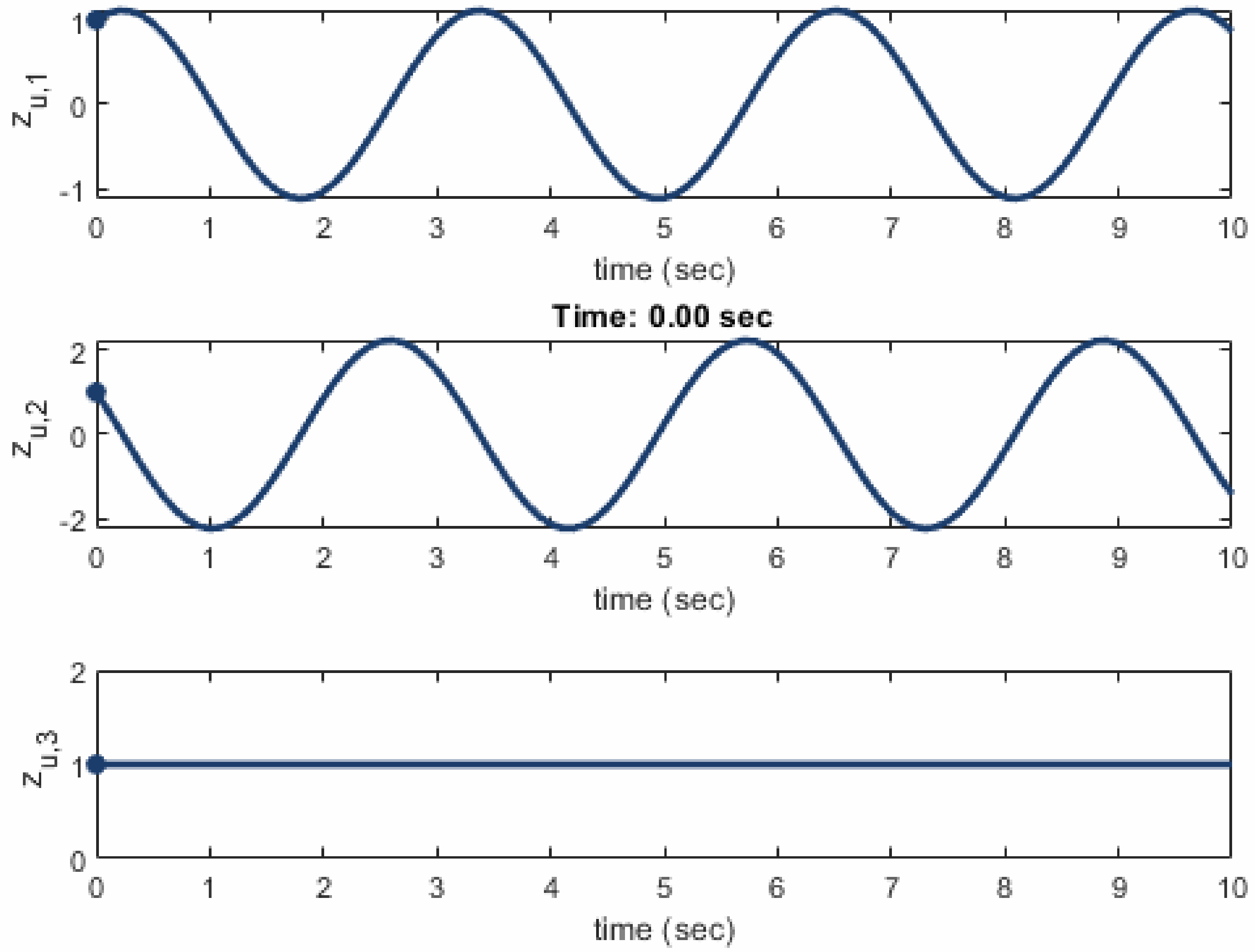
- Three significant uncertainties
  - Input u is unknown, external, deterministic
  - State matrix A may have uncertainty
  - Known, Lipschitz nonlinear internal dynamics $g(x)$
- **Can we synthesize u and correct A?**

$$\dot{x} = Ax + g(x) + Bu$$
$$y = Cx$$

Adaptive Unknown Input Estimators

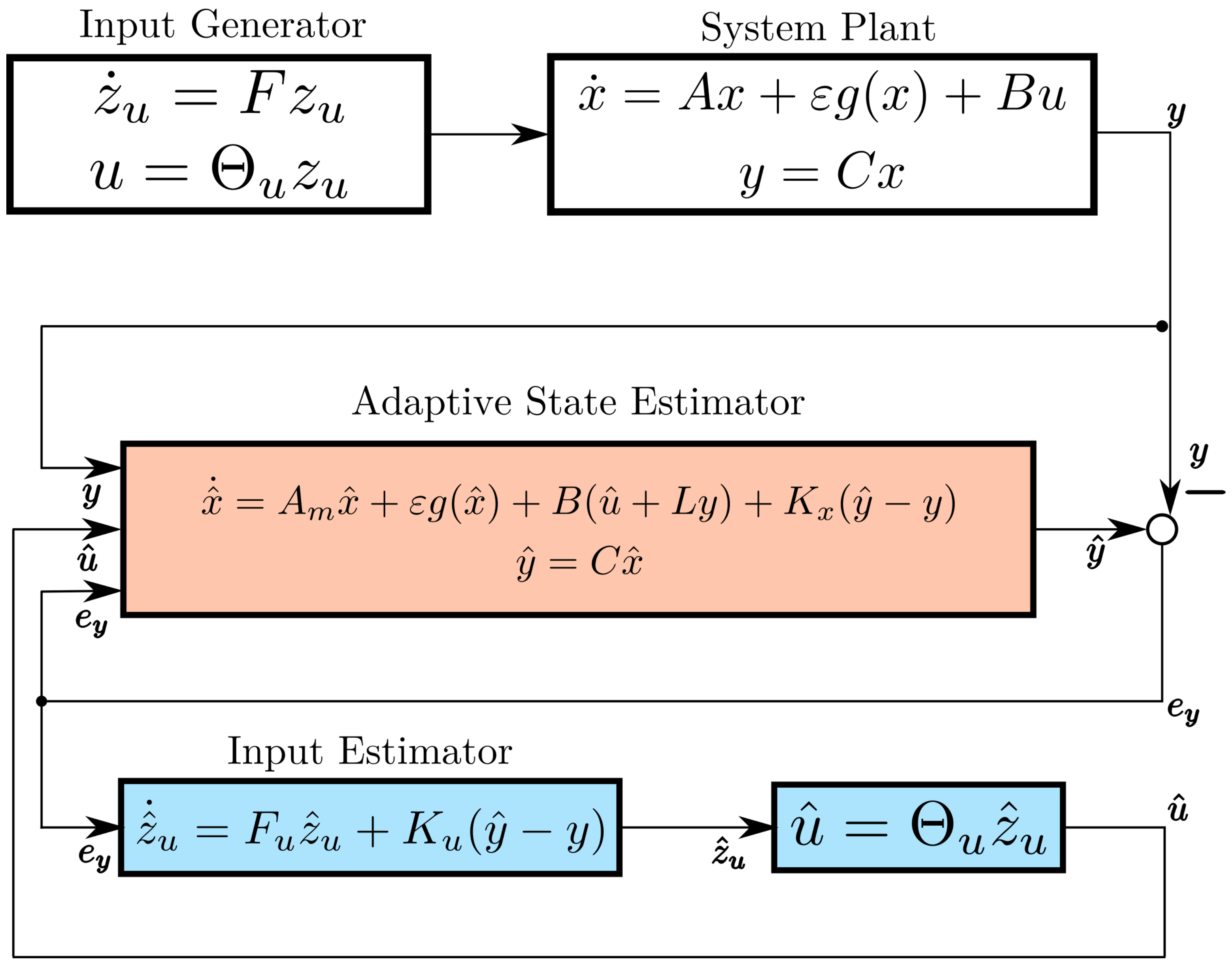
Modeling unknown inputs

- Approximate input space $\mathbb{U}$
  - $\hat{u} = \sum_{i=1}^{N} c_i f_i(t)$
- Persistent Inputs
  - $\dot{z}_u = F_u z_u$
  - $\hat{u} = \Theta_u z_u$
  - $F_u = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$



Adaptive Unknown Input Estimators

Architecture and estimator error



Recover A with adaptive scheme

$$A \equiv A_m + BL_* C$$
$$\dot{L} = -e_y y^* \gamma_e; \; \gamma_e > 0$$

Error dynamics

$$\dot{e} = (\bar{A} + \bar{K}\bar{C})e + \bar{B}\Delta L y + \varepsilon \Delta g$$
$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_z \end{bmatrix} = \underbrace{\begin{bmatrix} A_m + K_x C & B\Theta_u \\ K_u C & F \end{bmatrix}}_{\bar{A}_c} \begin{bmatrix} e_x \\ e_z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w + \varepsilon \begin{bmatrix} g(\hat{x}) - g(x) \\ 0 \end{bmatrix}$$

Adaptive Unknown Input Estimators

Architecture and estimator error

- ASD plant dynamics
- A Hurwitz
- Bounded L_*
- Error in state and input converges to zero
  - $V(e, \Delta L) = \frac{1}{2} e^* \bar{P} e + \frac{1}{2} \mathrm{tr}(\Delta L \gamma_e^{-1} \Delta L^*)$
  - $\dot{V}(e, \Delta L) \leq -\Big(\underbrace{\frac{1}{2} \lambda_{\min}(\bar{Q}) - \varepsilon \mu \lambda_{\max}(\bar{P})}_{\bar{\alpha}>0}\Big) ||e||^2$

$$0 < \varepsilon < \frac{\lambda_{\min}(\bar{Q})}{2\mu\lambda_{\max}(\bar{P})} \iff \bar{\alpha} > 0.$$

Illustrative example

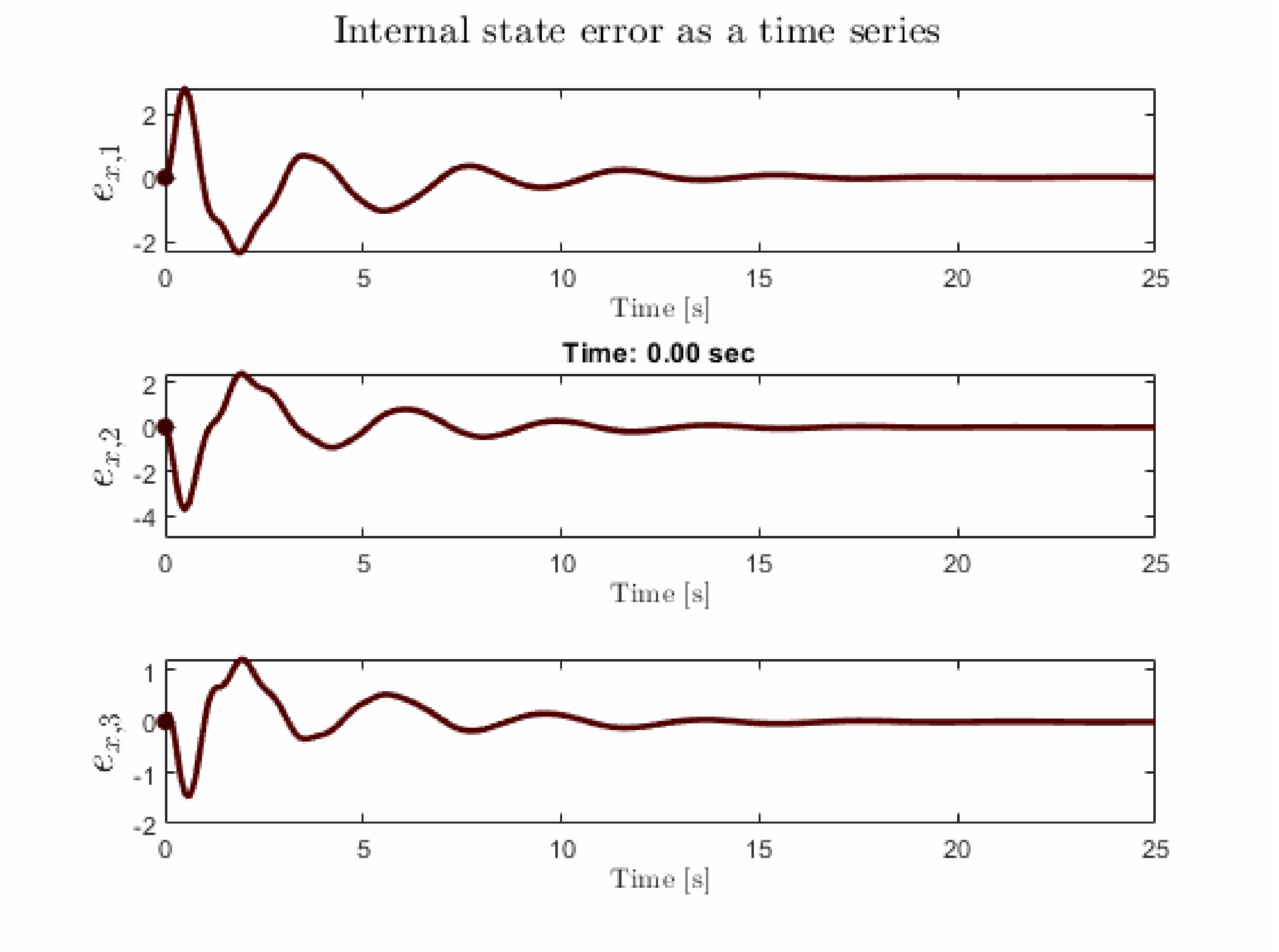
$$\dot{x} = A_m x + \varepsilon g(x) + Bu$$
$$= \begin{bmatrix} -4 & 1 & 2 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} x + \sin(x) + Bu$$
$$y = Cx$$

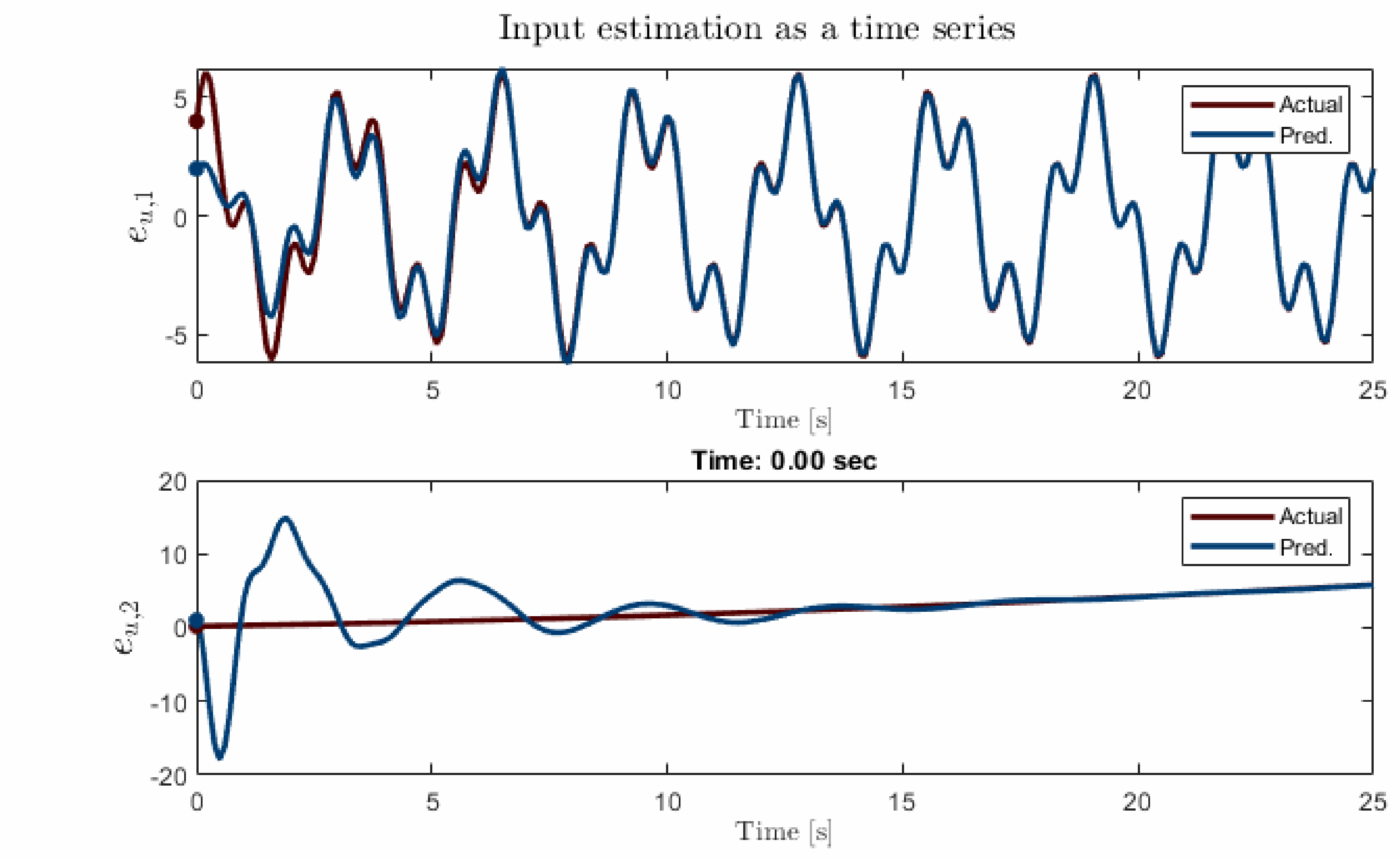
$$\dot{x} = Ax + \varepsilon g(x) + Bu$$
$$= \begin{bmatrix} -2.86 & 1 & 4.7 \\ 1.8 & -1 & 6.7 \\ -9 & 1 & -17.2 \end{bmatrix} x + \sin(x) + Bu$$
$$y = Cx$$

$$L* = \begin{bmatrix} -8 & 1 \\ 2 & -7 \end{bmatrix}$$
$$u_1(t) = c_{11} \sin(2t) + c_{12} \cos(2t) + c_{13} \sin(7t) + c_{14} \cos(7t)$$
$$u_2(t) = c_{11} + c_{22} t + c_{23} t^2 + c_{24} t^3$$

Illustrative example

Both the state error and the input error converge simultaneously

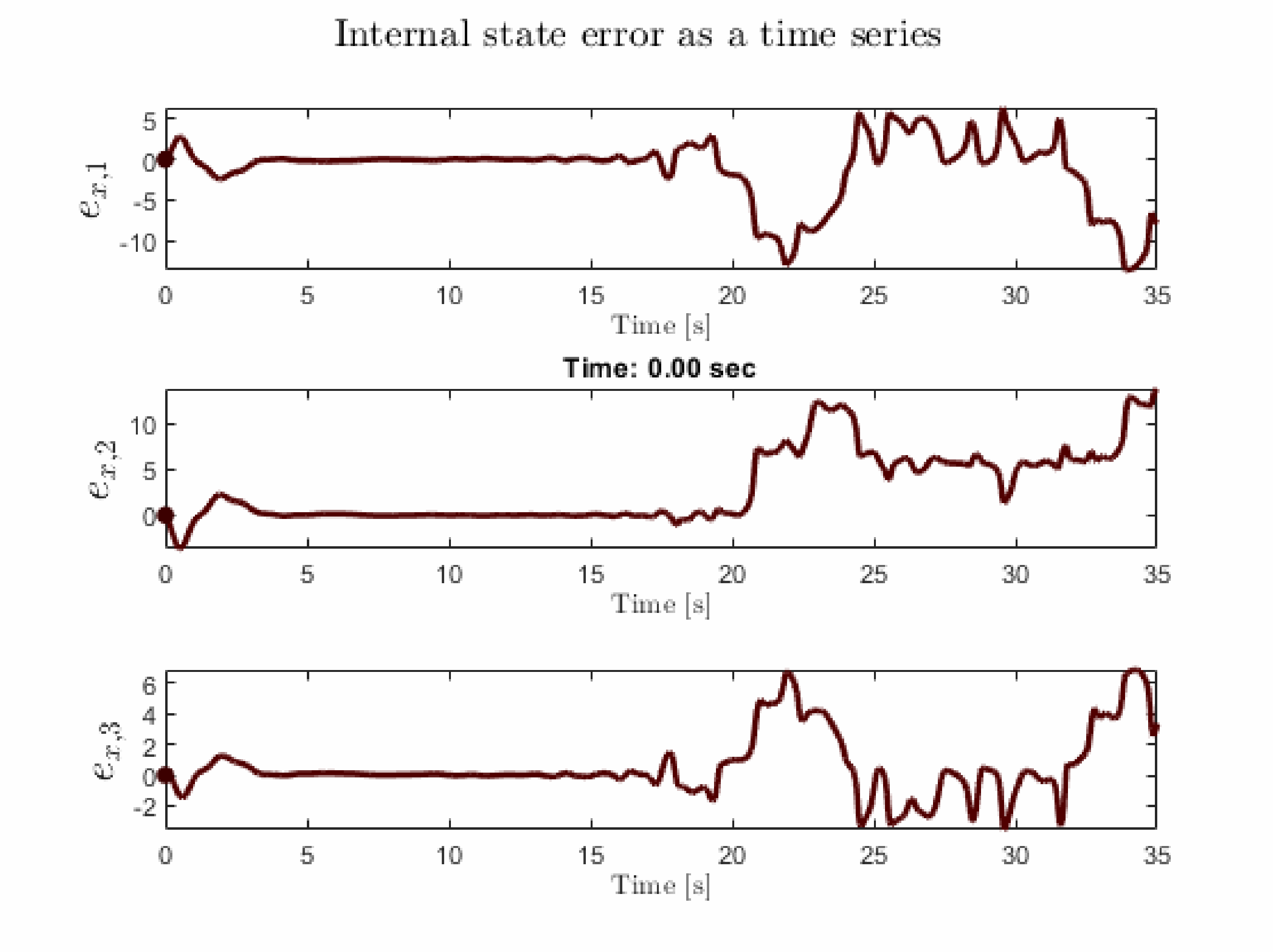


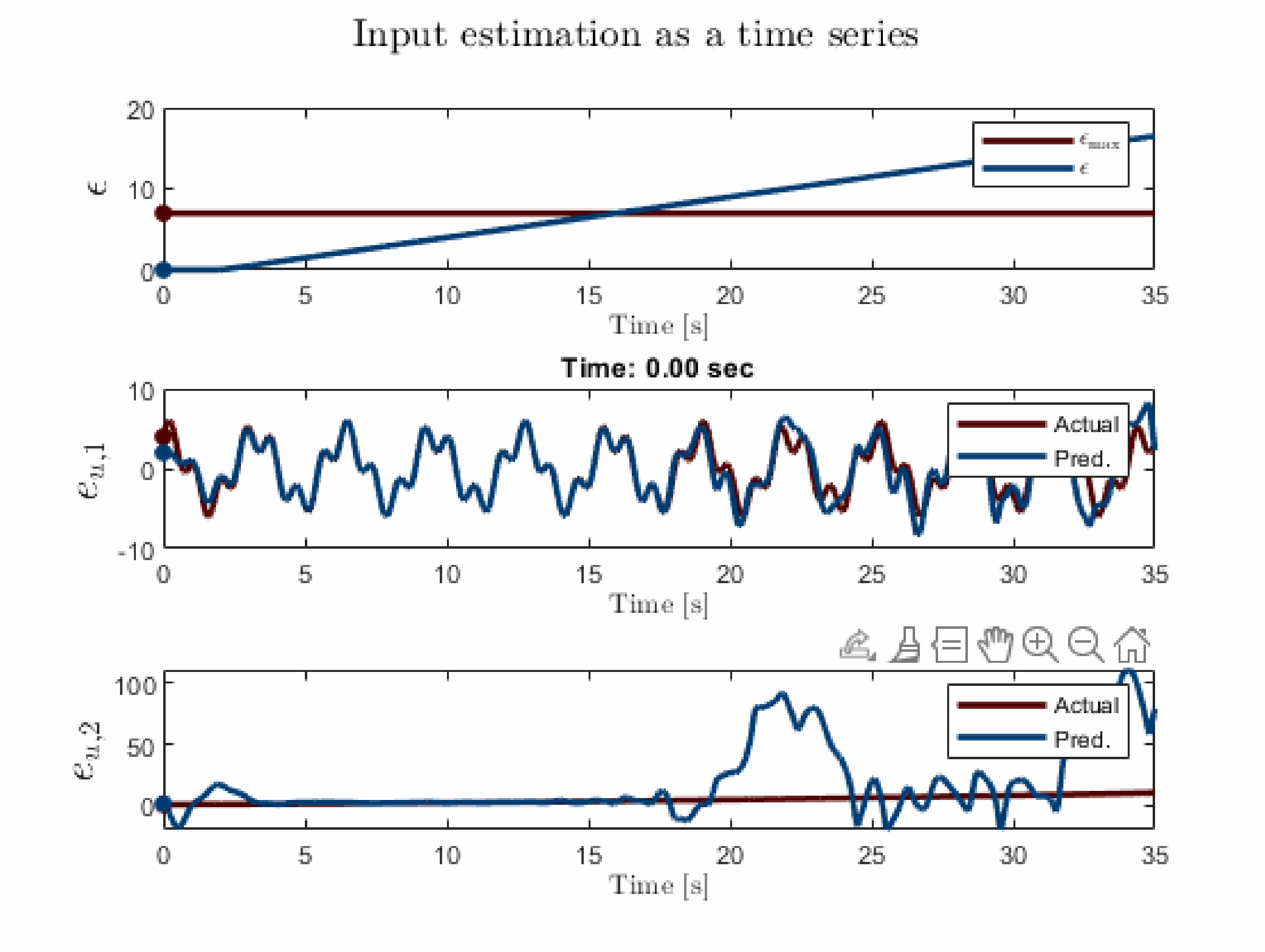


Illustrative example

provided ϵ is not too great

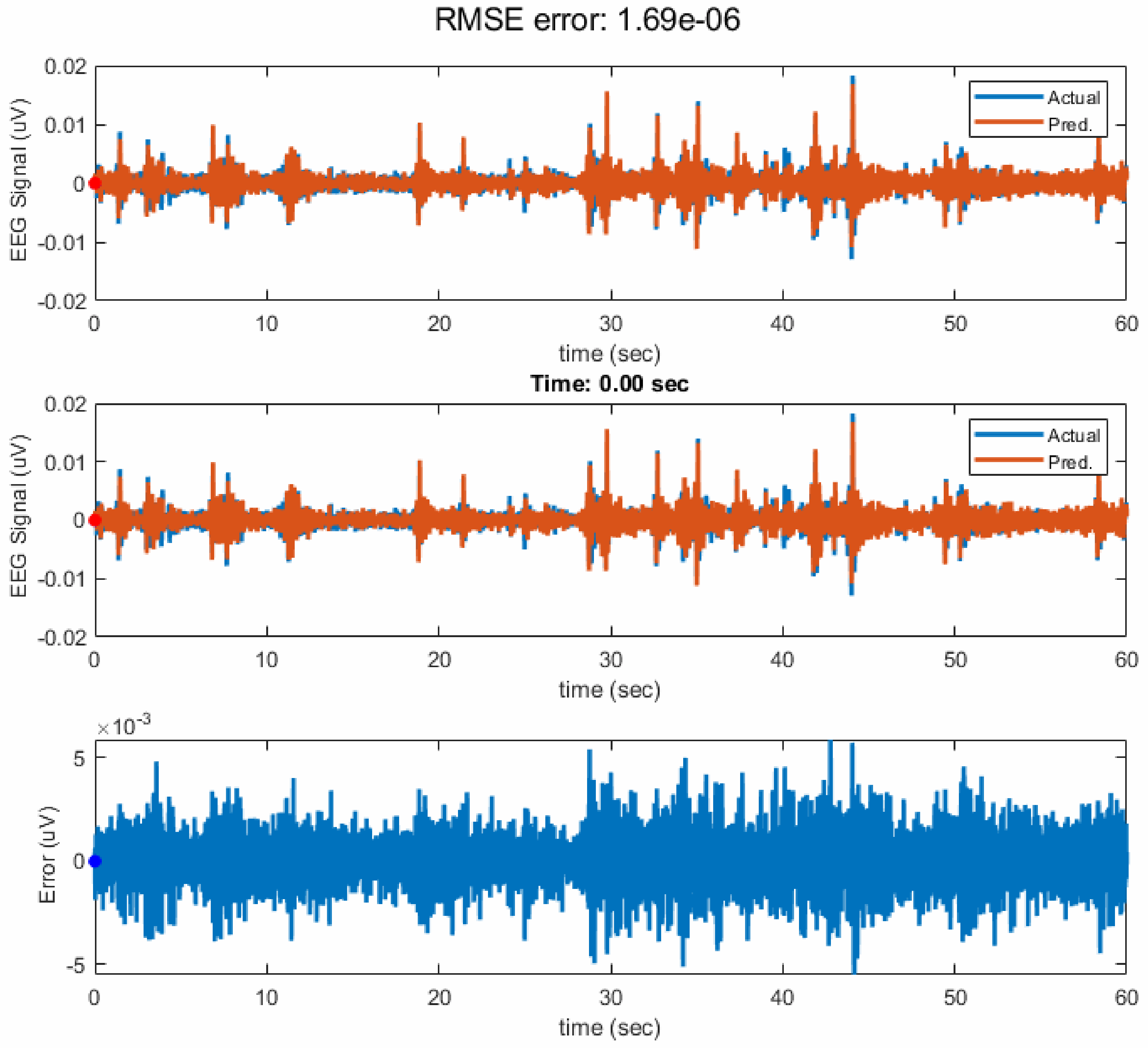
$$0 < \epsilon < \frac{\lambda_{\min}(\bar{Q})}{2\mu\lambda_{\max}(\bar{P})}$$





Application: Biomarker dynamics

Kalman filtering



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