

A Modal Approach to the Space Time Dynamics of Cognitive Biomarkers

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Defense

April 29, 2022

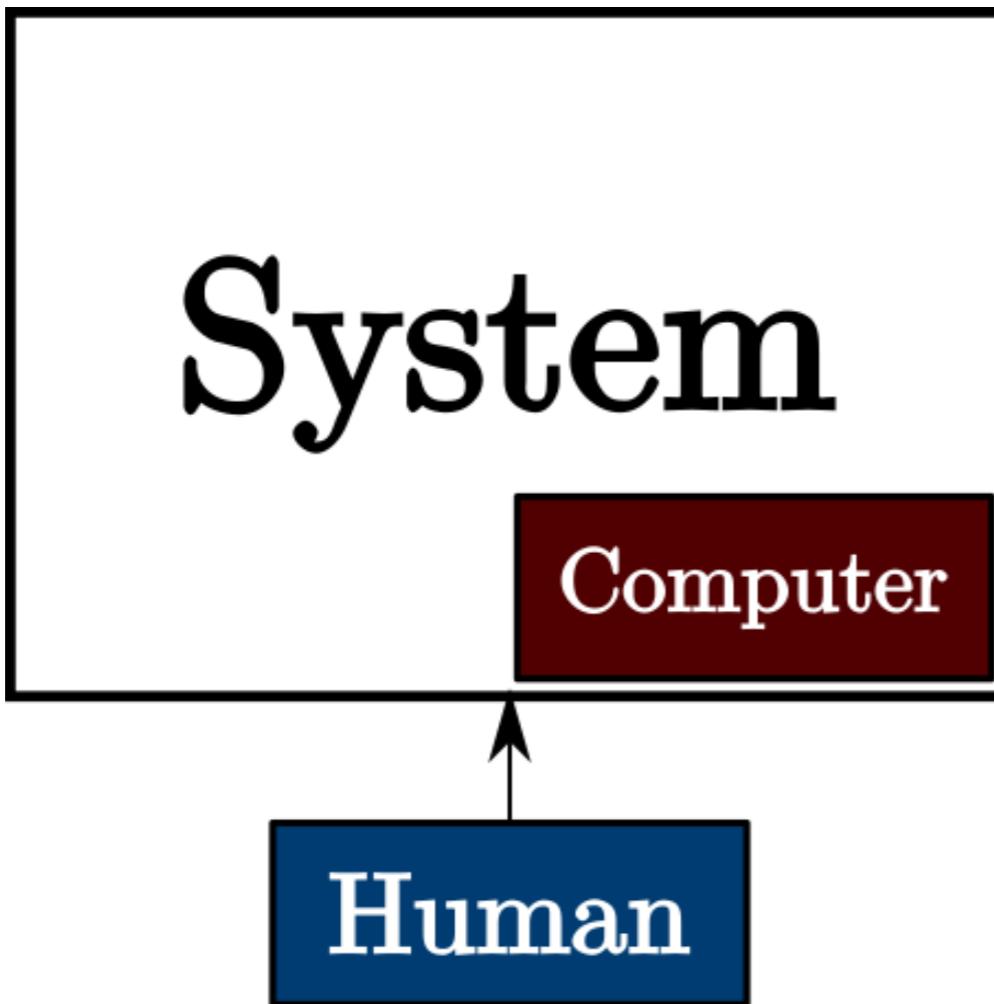


Outline

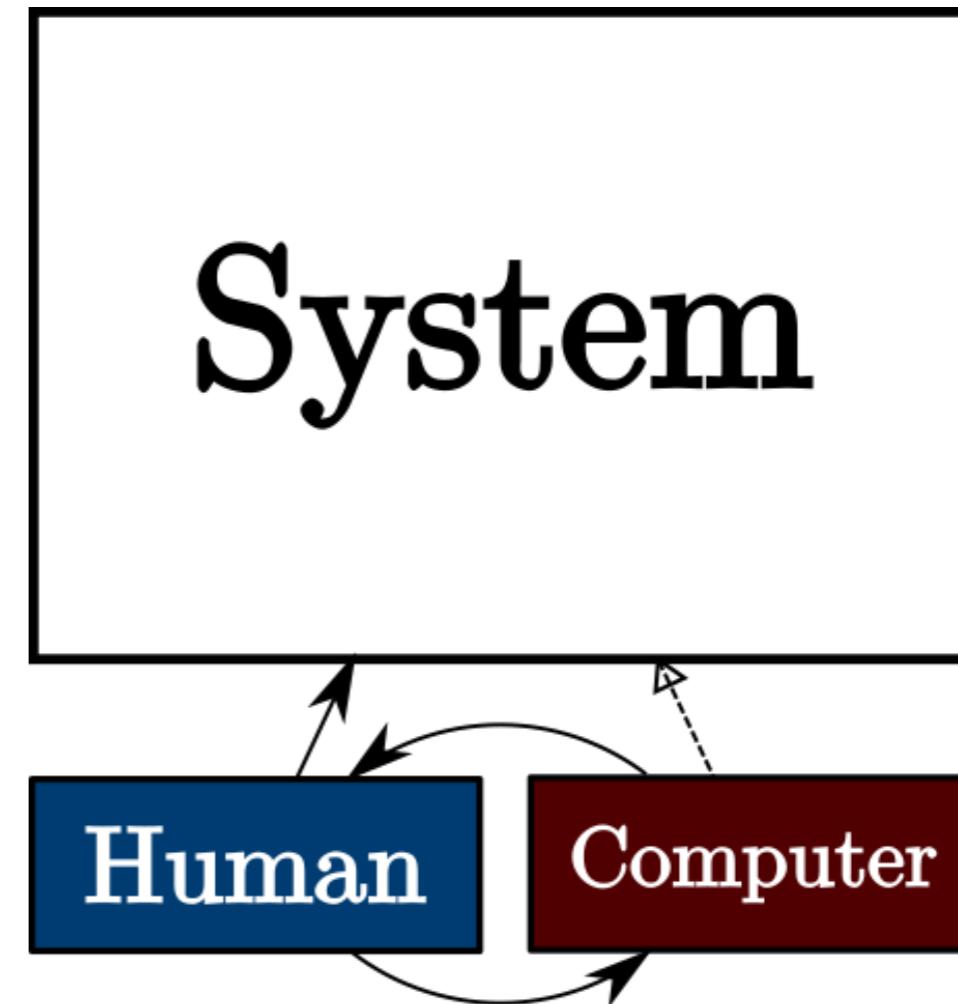
1. Introduction & Motivation
2. A Dynamic Systems View of Brain Waves
3. System Identification of Brain Wave Modes Using EEG
4. Modal Analysis of Brain Wave Dynamics
5. Adaptive Unknown Input Estimators
6. Reconstructing the Brain's Unknown Input
7. Conclusions



1. Introduction & Motivation



The computer as part of the system



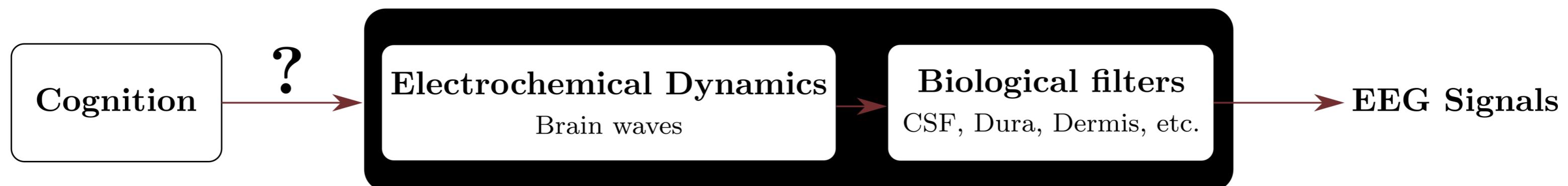
The computer as a teaming member

Novel potential

There has been a recent proliferation of more rugged and durable sensor devices (e.g., fNIRS sensors) that can be used while people take part in ecologically valid activities to assess changes in neurophysiology, physiology, and behavior that correlate with cognitive state. In addition, recent advances in machine learning and modeling techniques can be used to interpret information about human states (e.g., SA) from noisy data acquired in such environments that previously was unusable.

Bracken, B., Tobyne, S., Winder, A., Shamsi, N., & Endsley, M. R. (2021, July). Can Situation Awareness Be Measured Physiologically?. In International Conference on Applied Human Factors and Ergonomics (pp. 31-38). Springer, Cham.

Cognition as a black box



State of the art: surveys and orthogonal bases

Recent modeling work, however, using large-scale dynamical models on the human connectome, suggests that cortical flow patterns are multistable and exhibit phase-transitions. To study such phenomena, a dynamic analysis in which no assumptions about stationarity are made, is required.

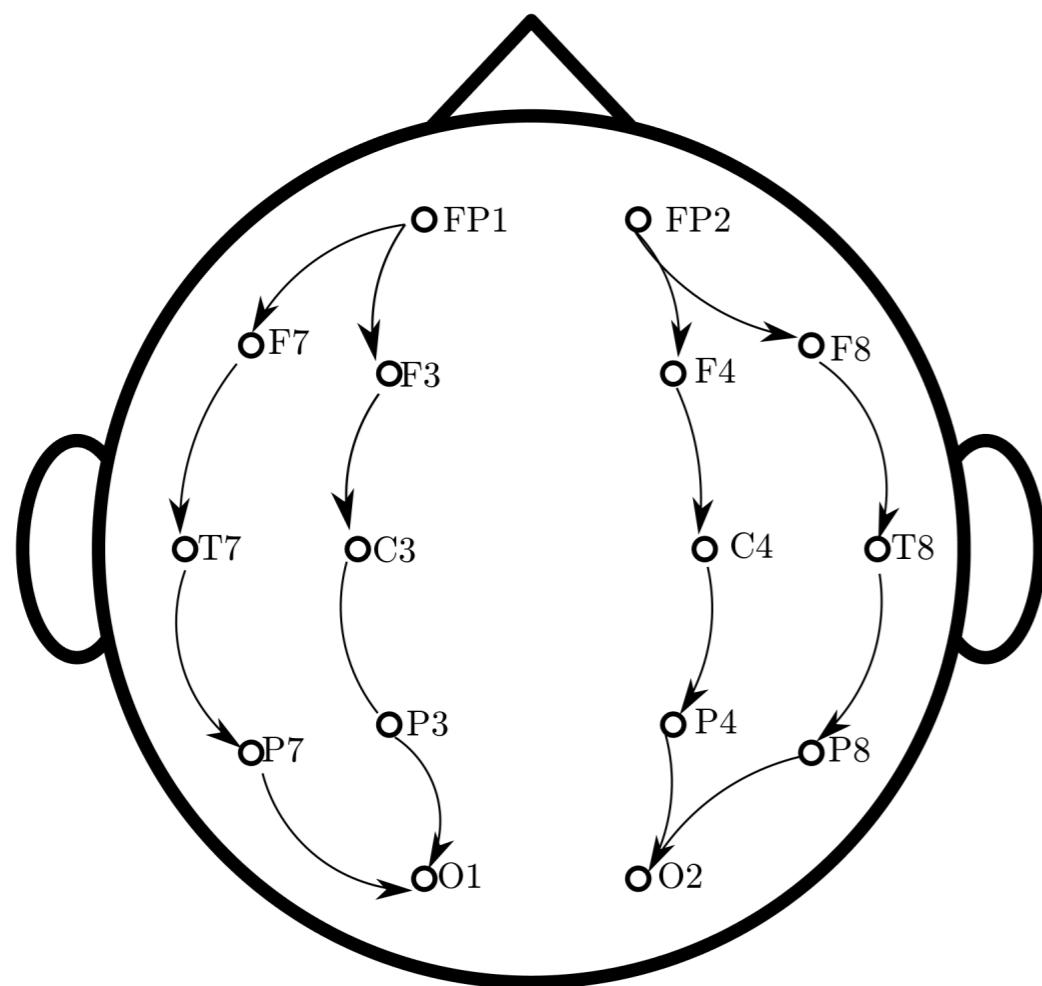
Hindriks, Rikkert, et al. "Latency analysis of resting-state BOLD-fMRI reveals traveling waves in visual cortex linking task-positive and task-negative networks." Neuroimage 200 (2019): 259-274.



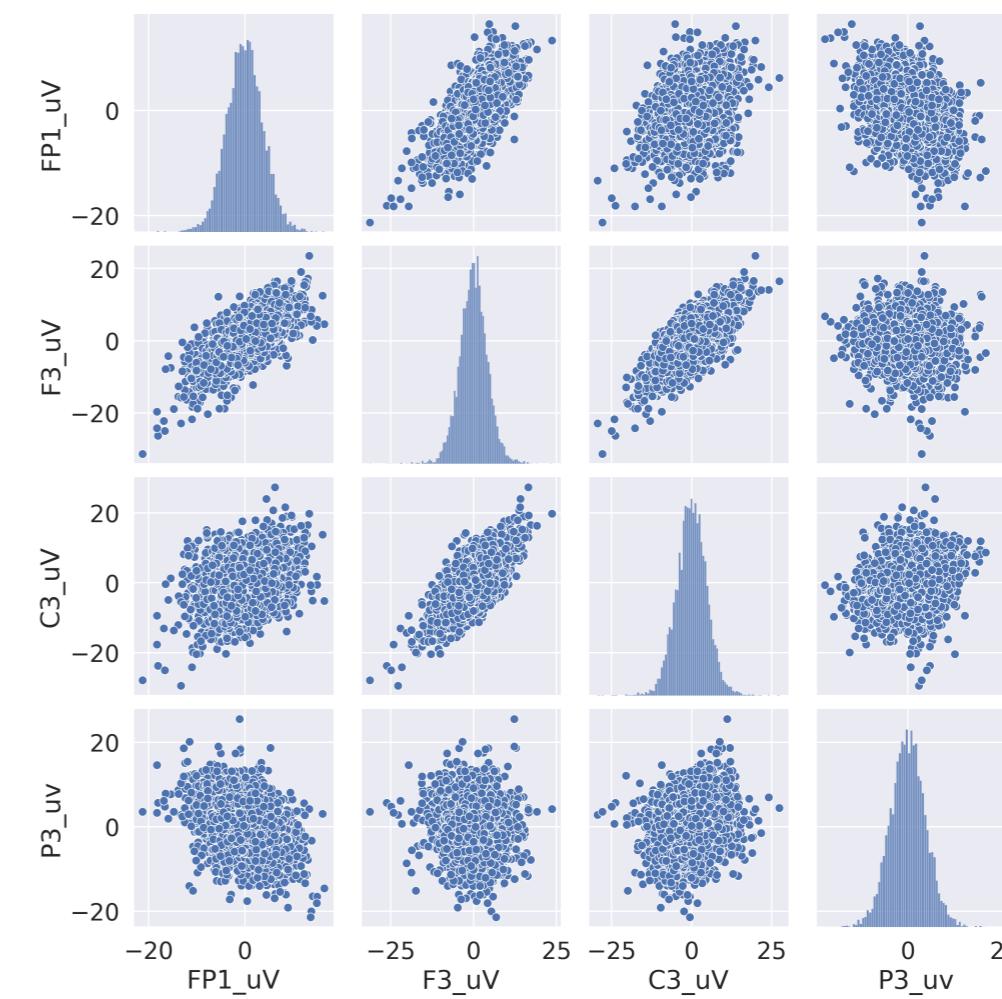
Hogarth de la Plante, Unsplash

2. A Dynamic Systems View of Brain Waves

Characteristics of EEG

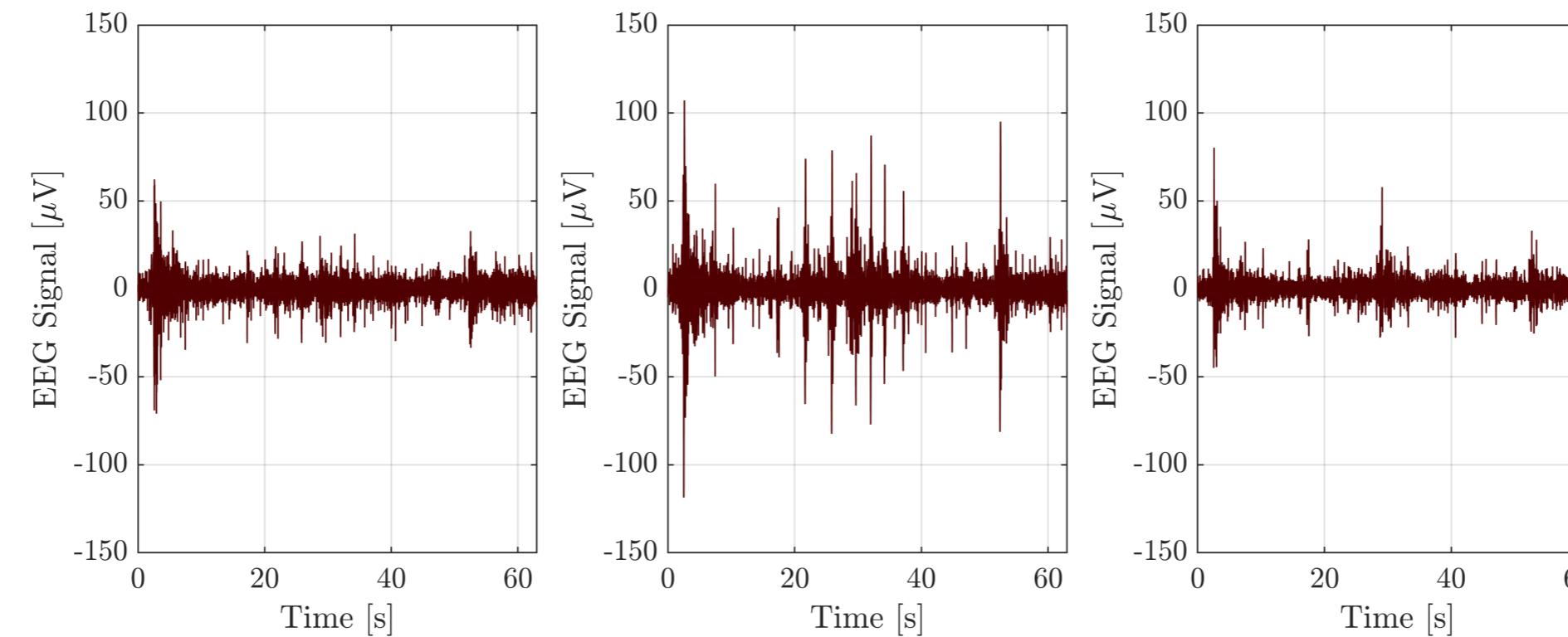


Longitudinal referencing



EEG channel pair plots

A canonical approach:



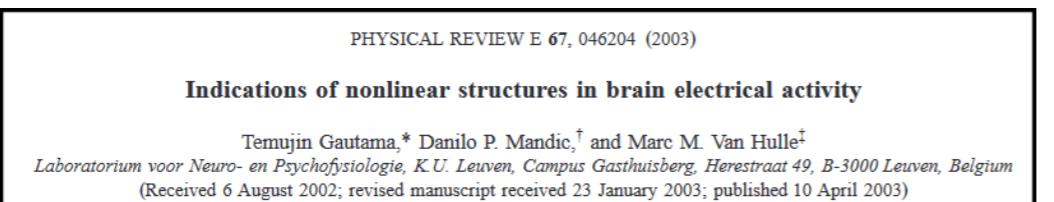
True brain wave plant: $\begin{cases} \dot{x} = Ax + Bu + v_x \\ y = Cx \end{cases}$
where A , B , C , v_x , x , and u are **all unknown**.

A , B , C , v_x , x , and u are **all unknown**.

This level of uncertainty is an unsolved problem

Identify the plant: $\begin{cases} \dot{x}_m = A_m x + v_x \\ y_m = C x_m \end{cases}$,
accepting the uncertainty in A_m .

Treating nonlinear effects



Capturing time-varying brain dynamics

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Indications of nonlinear deterministic and finite-dimensional structures in time series of brain electrical activity: Dependence on recording region and brain state

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(Received 14 May 2001; published 20 November 2001)

Adaptive Unknown Input Brain Wave Estimator:

$$\begin{cases} \dot{\hat{x}} = (A_m + BL(t)C)\hat{x} + B\hat{u} + K_x e_y; \\ \hat{y} = C\hat{x}. \end{cases}$$

Modes elegantly capture the spatio-temporal dynamics

True brain wave plant

$$\begin{cases} \dot{x} = Ax + Bu + v_x \\ y = Cx \end{cases}$$



Modal brain wave plant

$$\begin{cases} \dot{\eta} = \Lambda\eta + V^{-1}Bu + V^{-1}v_x \\ y = CV\eta \end{cases}$$

Some important analytical properties:

- Frequency
- Damping
- Mode shape
- Complexity

3. System Identification of Brain Wave Modes Using EEG



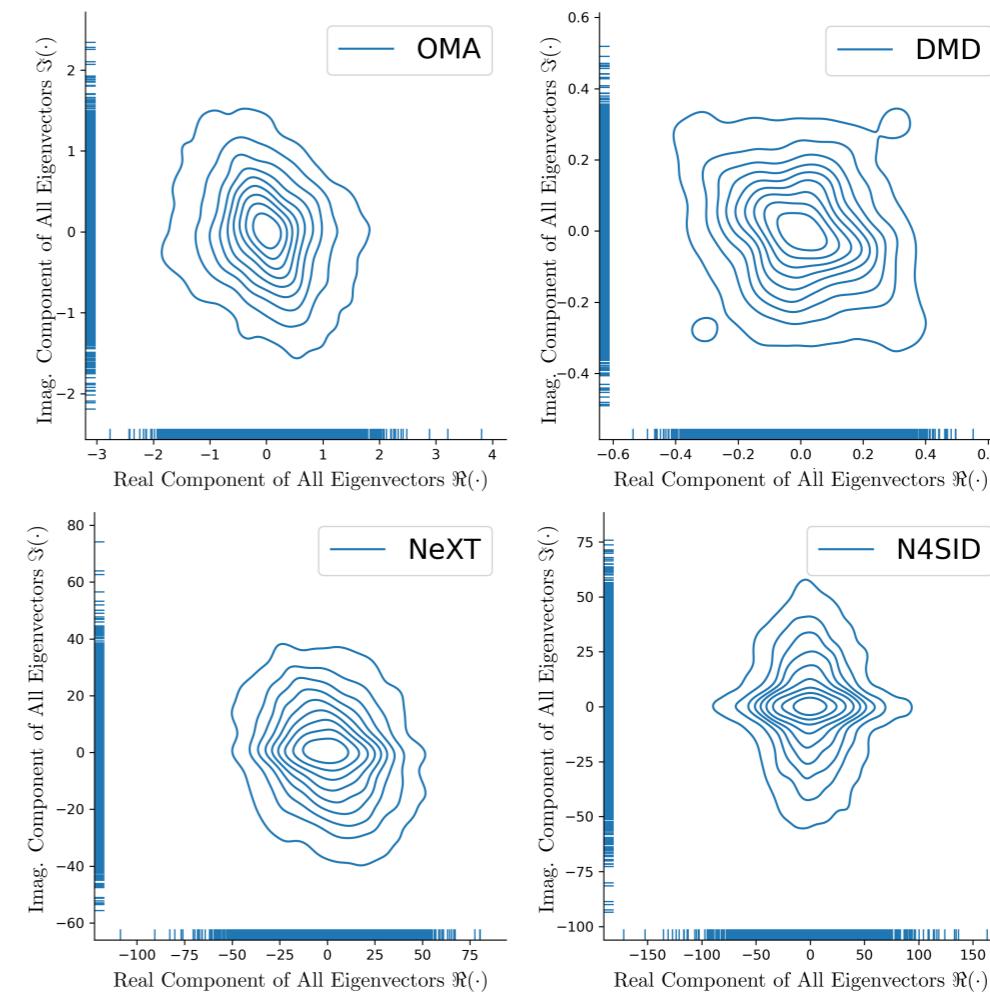
System Identification of Brain Wave Modes Using EEG

Identifying linear patterns

Identify the plant: $\begin{cases} \dot{x}_m = A_m x + v_x \\ y_m = C x_m \end{cases}$

Considered algorithms

- OMA
- NeXT-ERA
- n4sid
- DMD



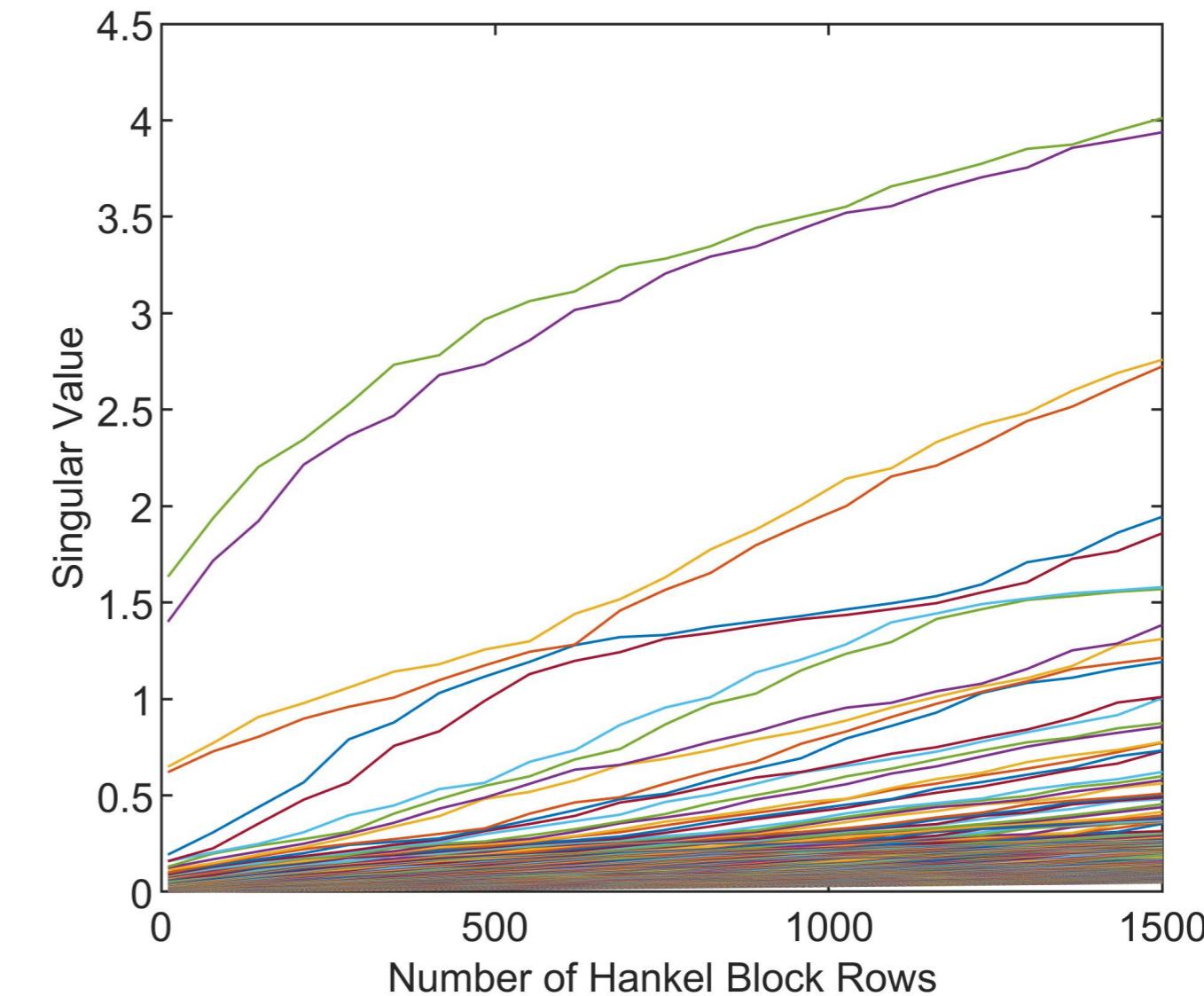
System Identification of Brain Wave Modes Using EEG

Identifying linear patterns

Identify the plant: $\begin{cases} \dot{x}_m = A_m x + v_x \\ y_m = C x_m \end{cases}$

$$O = \begin{bmatrix} C \\ CA_m \\ CA_m^2 \\ \vdots \\ CA_m^{s-1} \end{bmatrix} X_0 \\ = \Gamma X_0$$

$$\hat{\Gamma} = U S^{1/2} \hat{X}_0 = S^{1/2} V^*$$



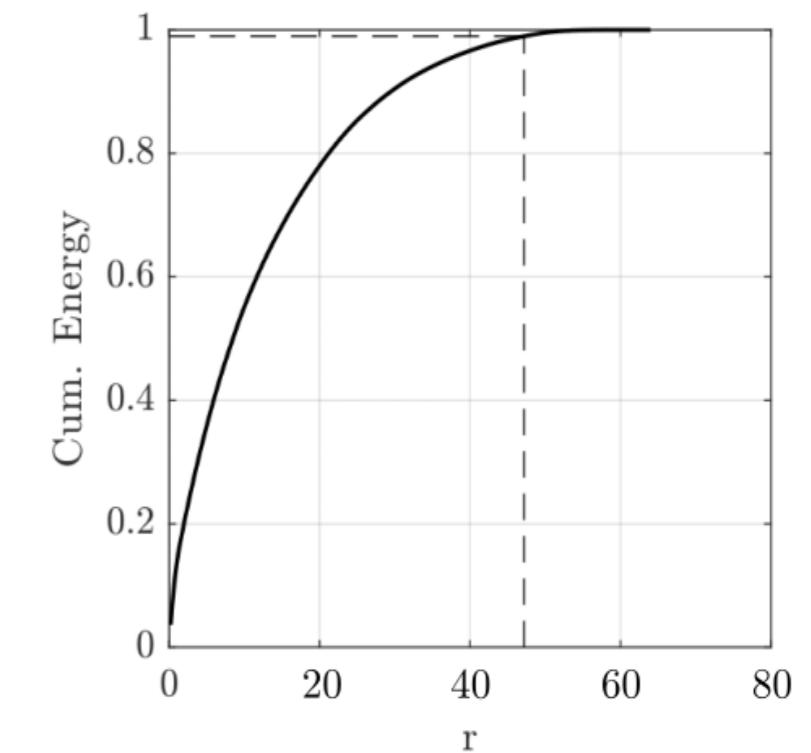
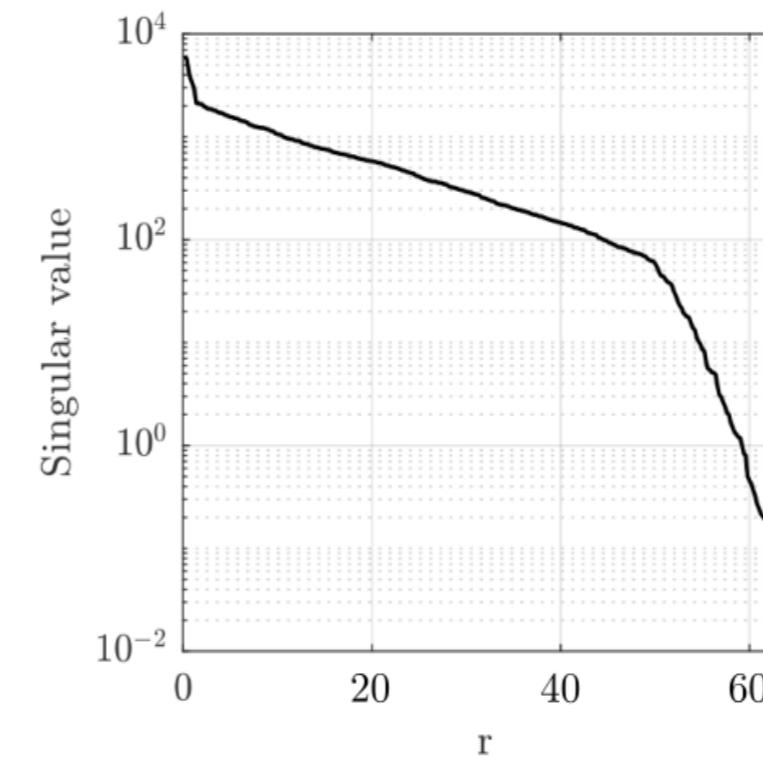
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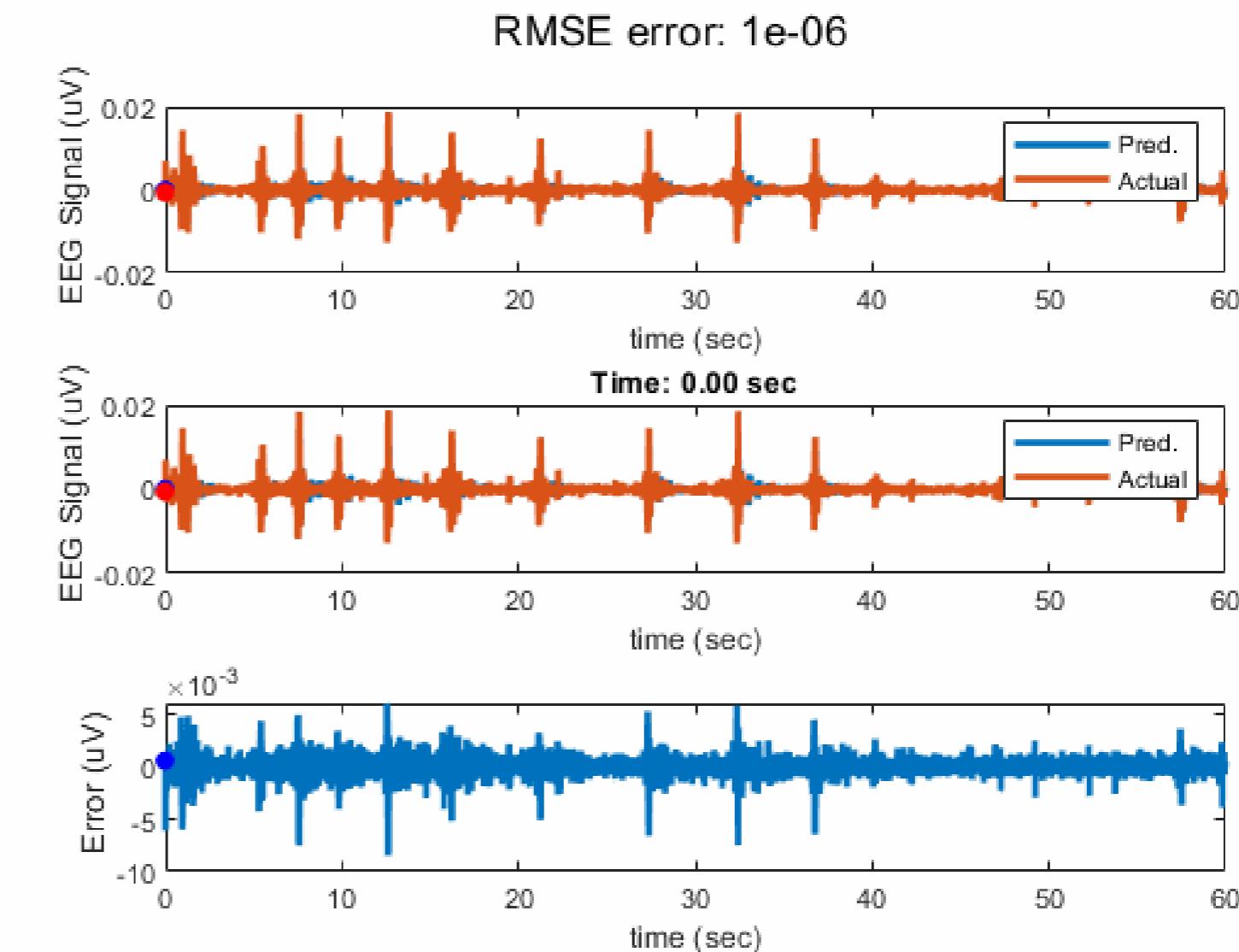
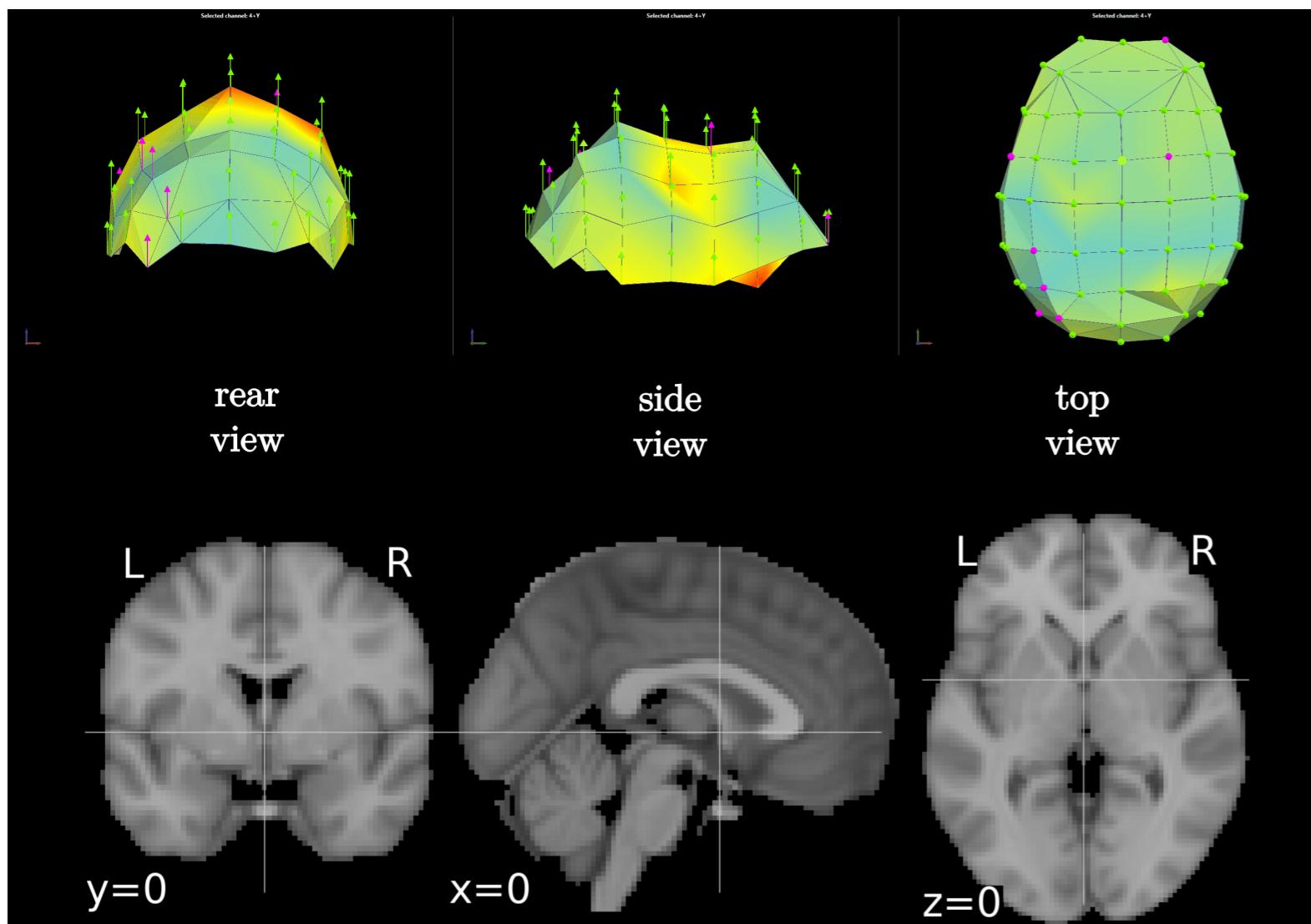
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System Identification of Brain Wave Modes Using EEG

Identifying linear patterns

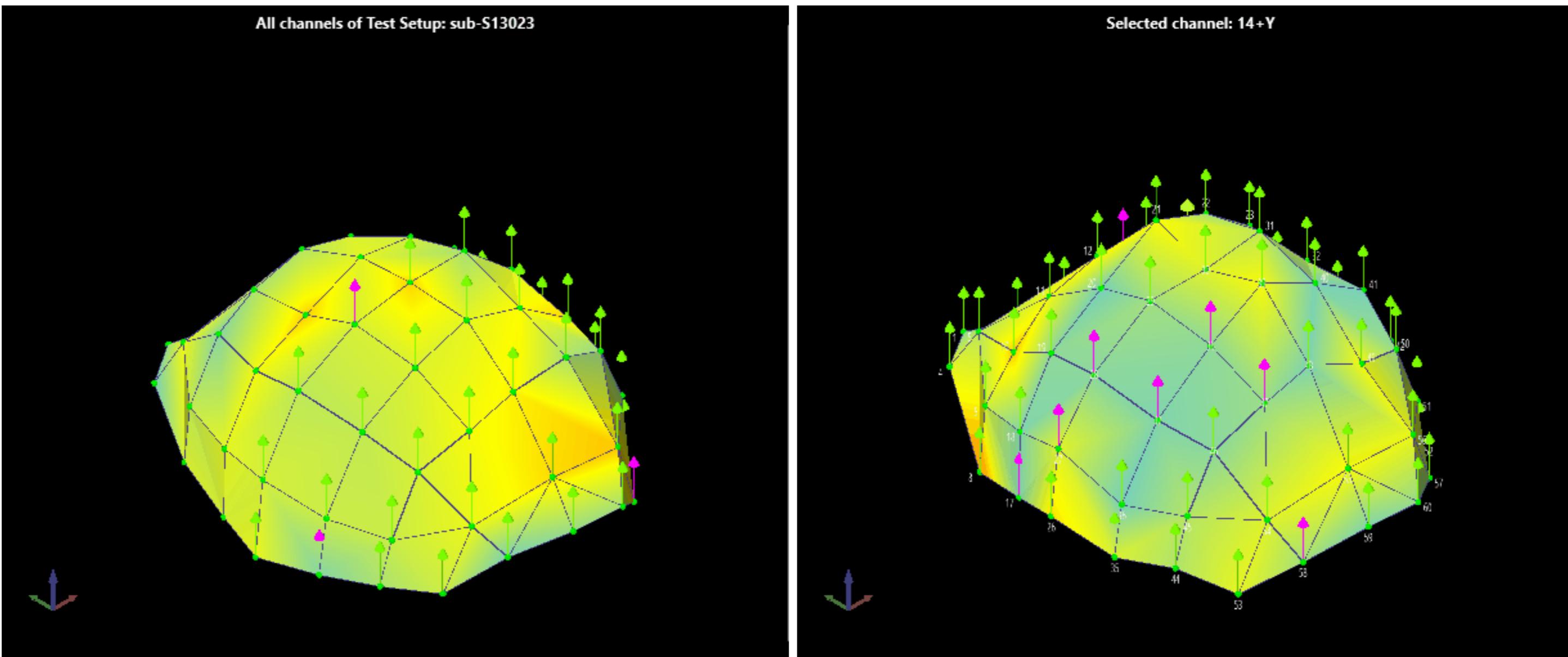


4. Modal Analysis of Brain Wave Dynamics



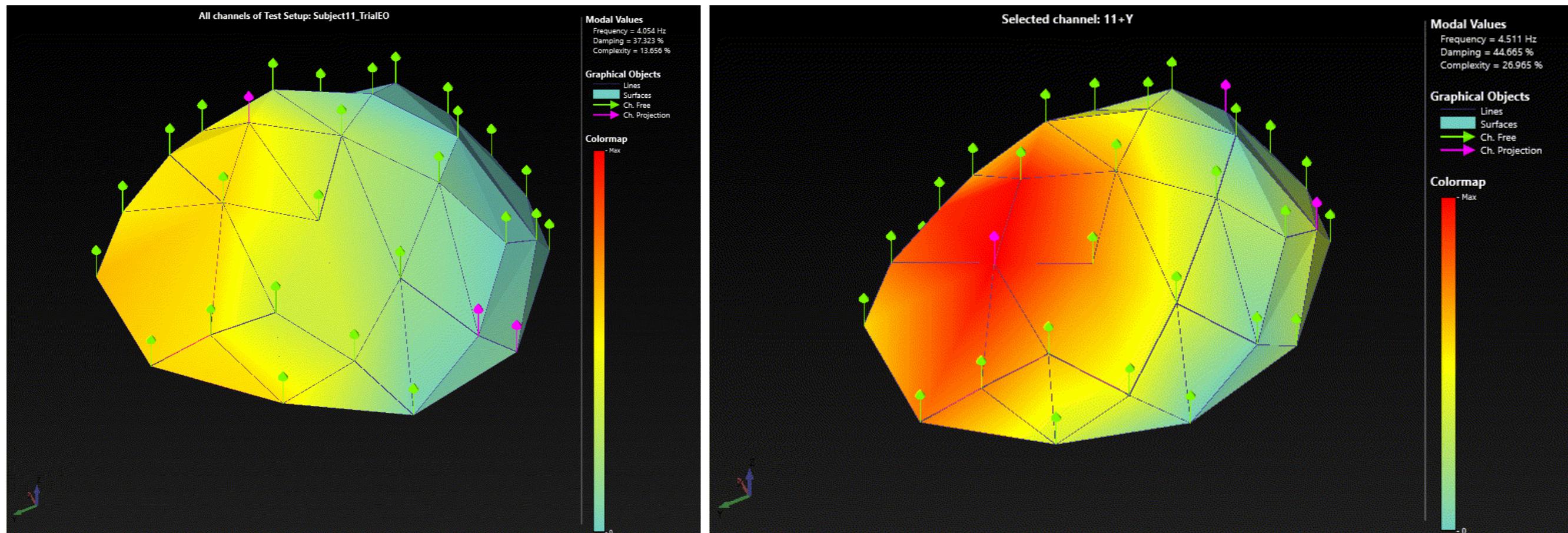
Modal Analysis of Brain Wave Dynamics

Brain wave modes are traveling and standing



Modal Analysis of Brain Wave Dynamics

Some brain wave modes are task independent



	Frequency	Damping [%]	Complexity [%]	Shape Correl.
Alpha Mode 1	4.34 ± 0.03	8.20 ± 1.20	11.47 ± 17.59	0.97 ± 0.016
Beta Mode 2	21.83 ± 0.22	1.98 ± 2.63	32.29 ± 35.67	0.96 ± 0.018
Gamma Mode 3	40.39 ± 0.26	11.87 ± 7.49	12.42 ± 16.88	0.99 ± 0.010
Gamma Mode 4	44.19 ± 0.24	2.52 ± 1.39	2.93 ± 5.69	0.99 ± 0.012

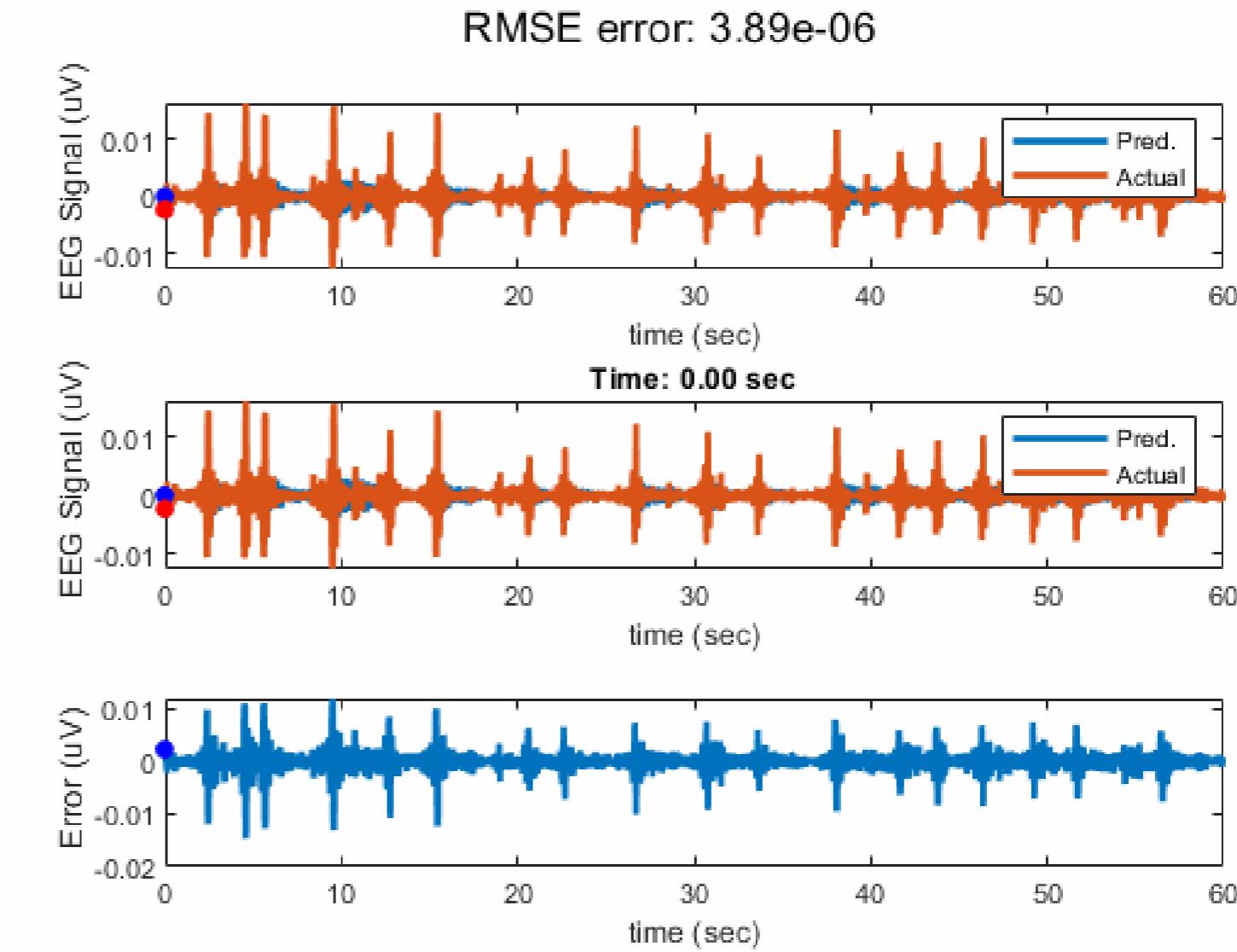
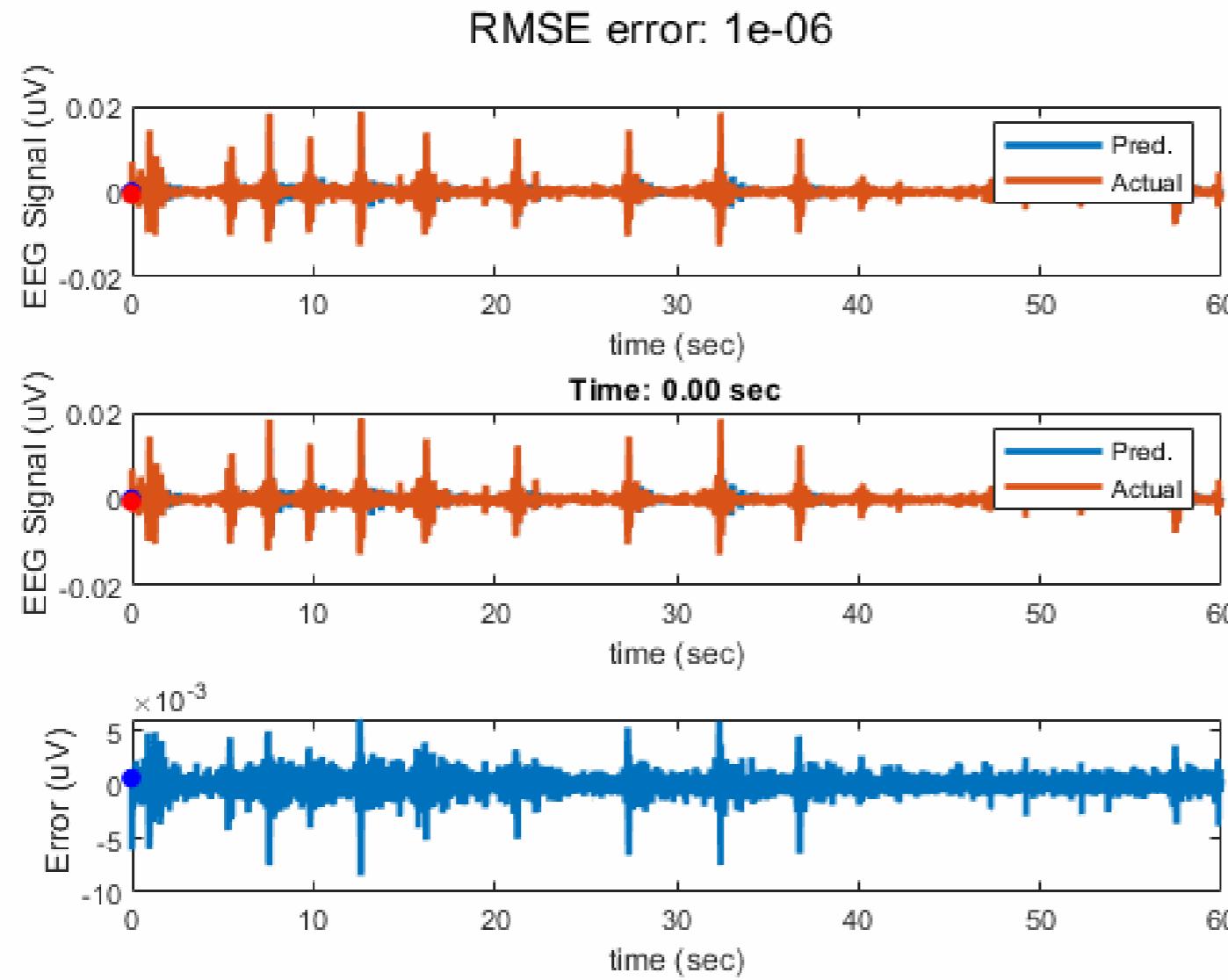
Modal Analysis of Brain Wave Dynamics

Brain wave modes are interindividual

Reference	No. of Electrodes	Accuracy [%]
This work	32	99.85
This work	8	96.45
Wilaiprasitporn et al.	32	99.90
Wilaiprasitporn et al.	5	99.1
DelPozo-Banos et al.	32	97.97

Modal Analysis of Brain Wave Dynamics

Brain wave modes poorly match nonlinear dynamics



5. Adaptive Unknown Input Estimators

Adaptive Unknown Input Estimators

Estimator overview

Three significant uncertainties

- Input u is unknown, external
- State matrix A may have uncertainty
- General process uncertainty v_x

Can we synthesize u and correct A ?

$$\begin{aligned}\dot{x} &= Ax + Bu + v_x \\ y &= Cx\end{aligned}$$

Adaptive Unknown Input Estimators

Modeling unknown inputs

Approximate input space \mathbb{U}

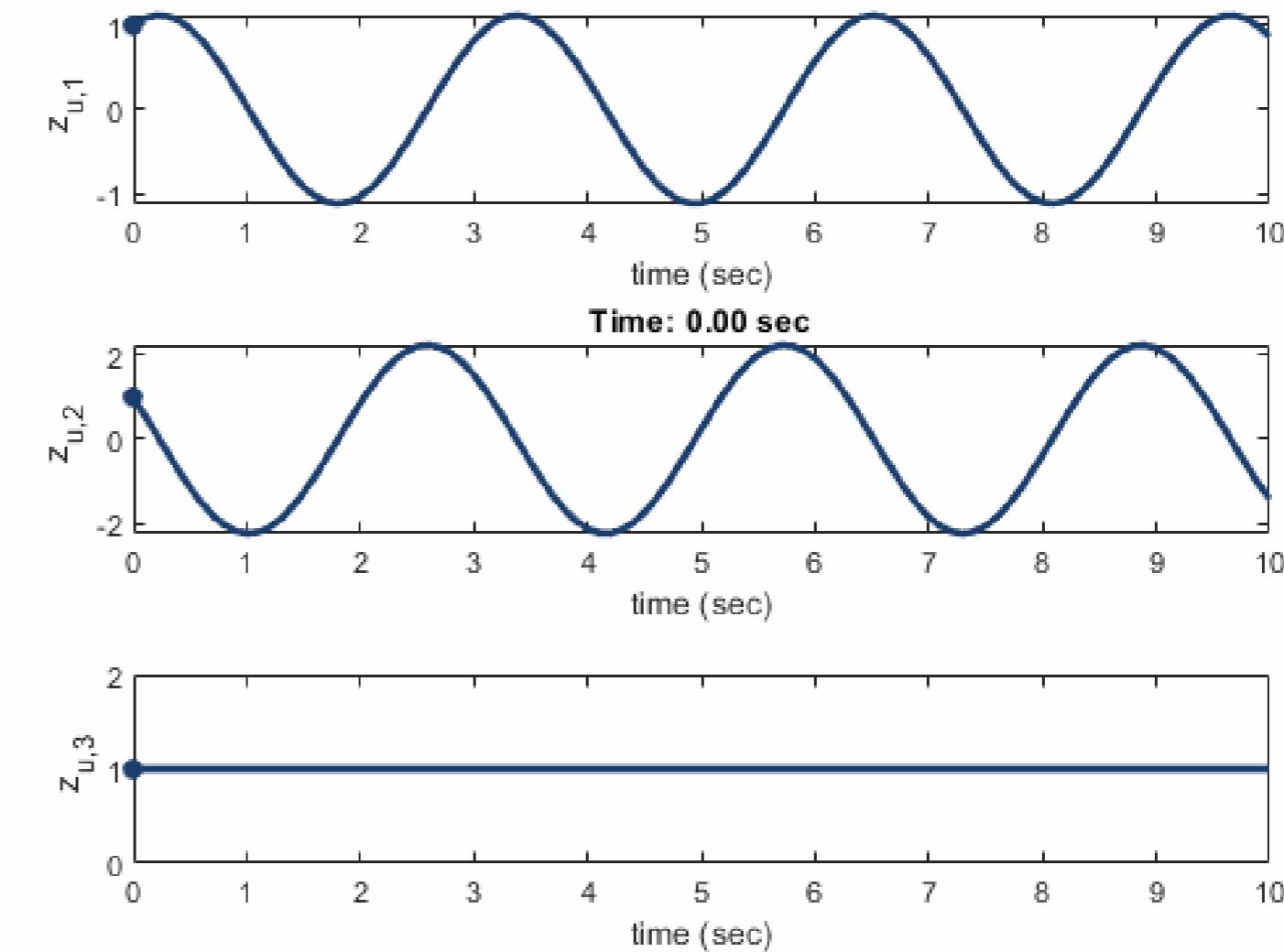
$$- \hat{u} = \sum_{i=1}^N c_i f_i(t)$$

Persistent Inputs

$$- \dot{z}_u = F_u z_u$$

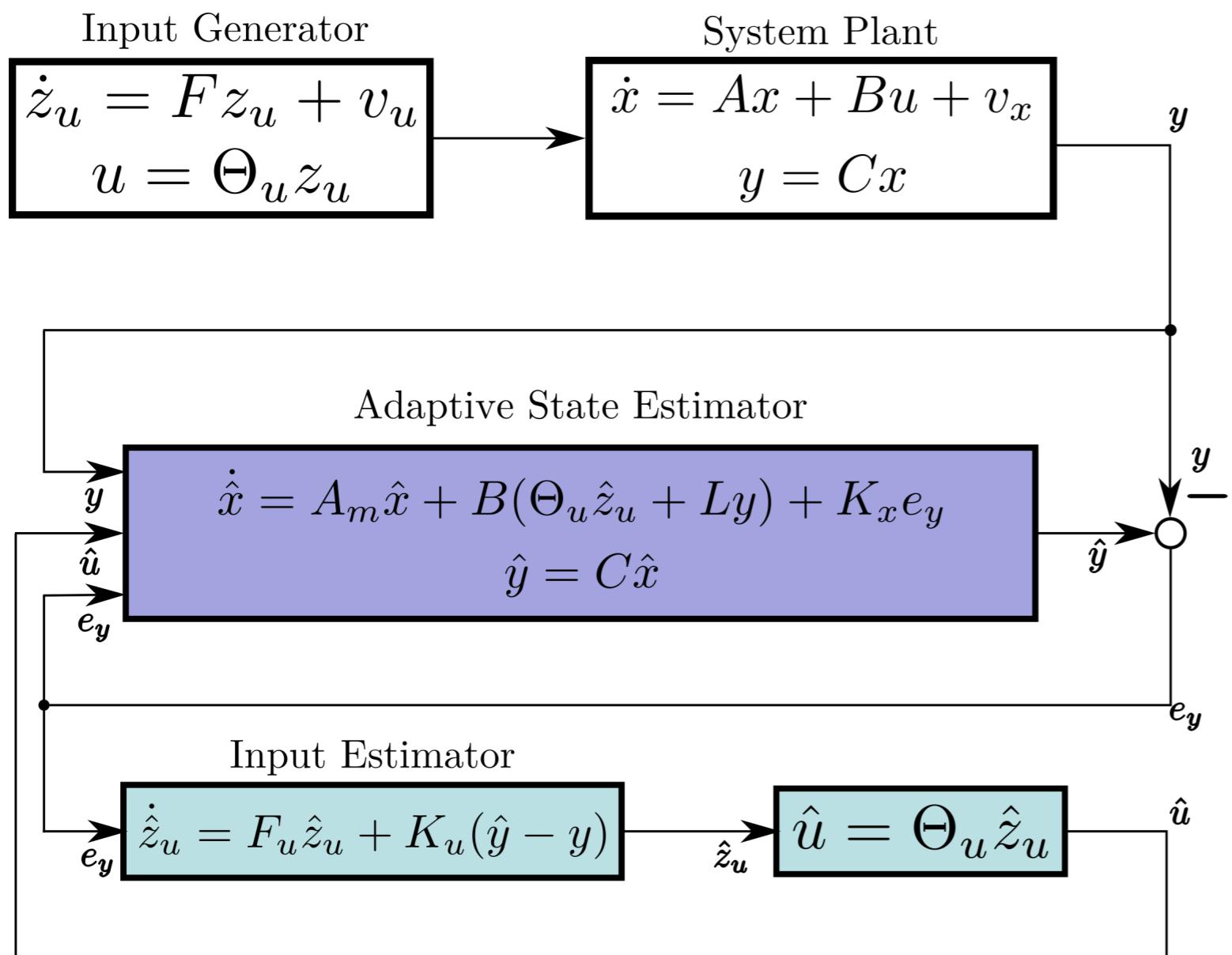
$$- \hat{u} = \Theta_u z_u$$

$$- F_u = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Adaptive Unknown Input Estimators

Architecture and estimator error



Recover A with adaptive scheme

$$A \equiv A_m + BL_*C$$

$$\dot{L} = -e_y y^* \gamma_e - \alpha L; \quad \alpha > 0, \quad \gamma_e > 0$$

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_z \end{bmatrix} = \underbrace{\begin{bmatrix} A_m + K_x C & B \Theta_u \\ K_u C & F_u \end{bmatrix}}_{\bar{A}_c} \begin{bmatrix} e_x \\ e_z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w + \begin{bmatrix} v_x \\ v_u \end{bmatrix}$$

Adaptive Unknown Input Estimators

Architecture and estimator error

ASD plant dynamics

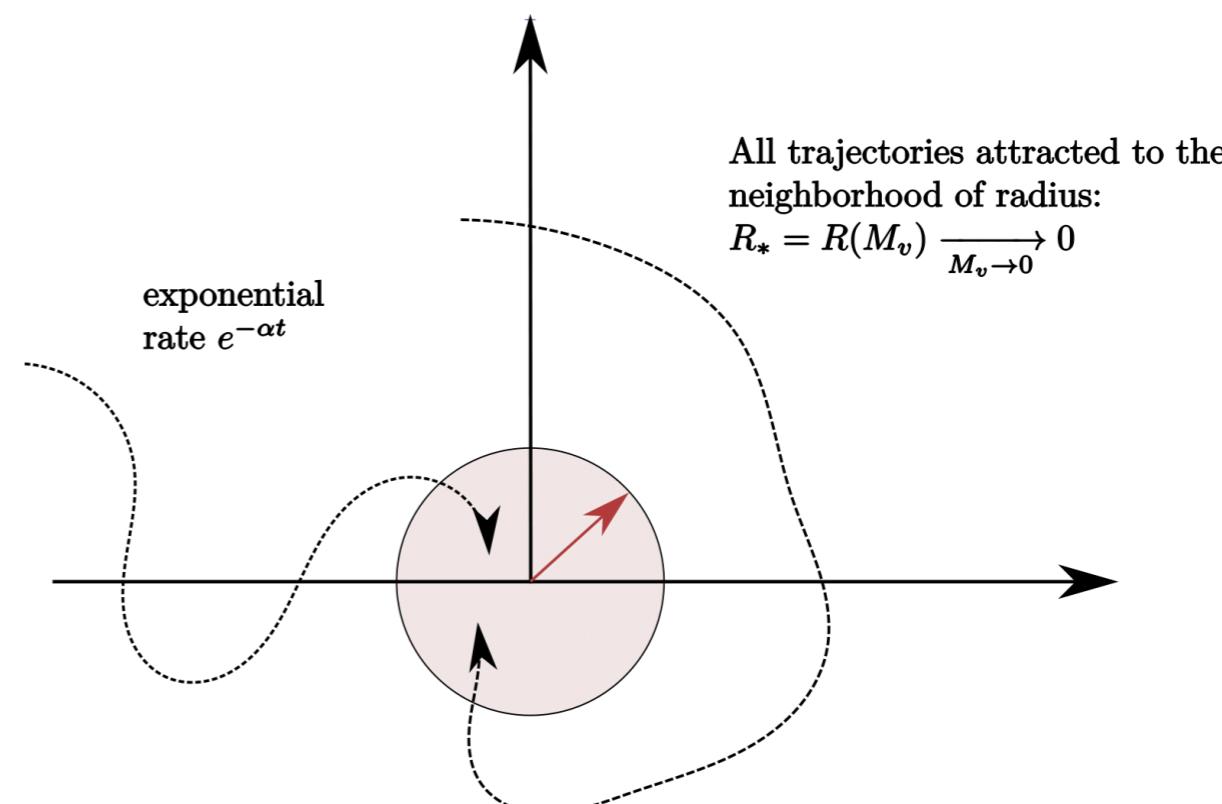
$$\begin{aligned} A_c^* P + P A_c &= -Q \\ P B &= C^* \end{aligned}$$

Bounded L_* , v , and γ_e

Error in state and input converges to an neighborhood centered at zero

$$V(e, \Delta L) = \frac{1}{2} e^* \bar{P} e + \frac{1}{2} \text{tr}(\Delta L \gamma_e^{-1} \Delta L^*)$$

$$\lim_{t \rightarrow \infty} \sup ||e(t)|| \leq \frac{1 + \sqrt{\lambda_{\max} \bar{P}}}{\alpha \sqrt{\lambda_{\min} \bar{P}}} M_v \equiv R^*$$



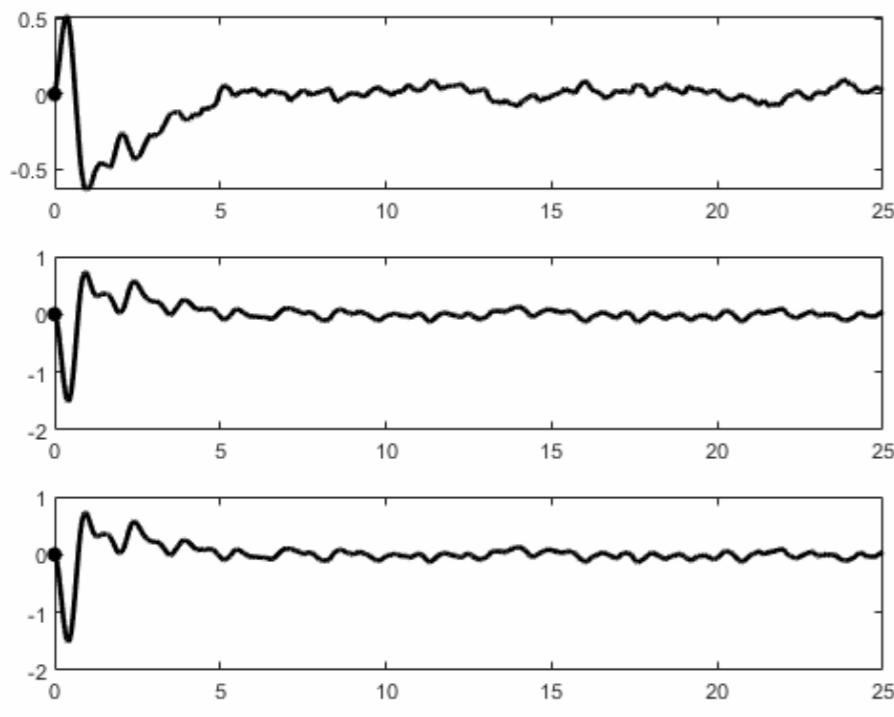
Illustrative example

$$\dot{x} = A_m x + Bu + v_x$$

$$= \begin{bmatrix} -4 & 1 & 2 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} x + Bu + v_x$$

$$y = Cx$$

Internal state error time series

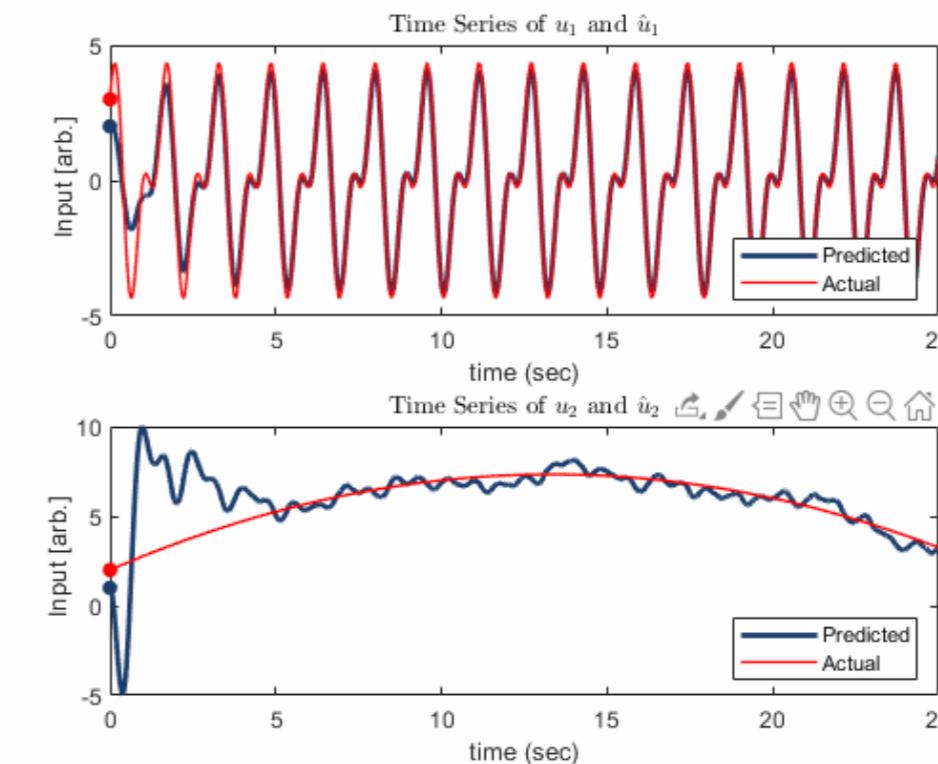


$$\dot{x} = Ax + Bu + v_x$$

$$= \begin{bmatrix} -2.86 & 1 & 4.7 \\ 1.8 & -1 & 6.7 \\ -9 & 1 & -17.2 \end{bmatrix} x + Bu + v_x$$

$$y = Cx$$

Estimating the unknown input



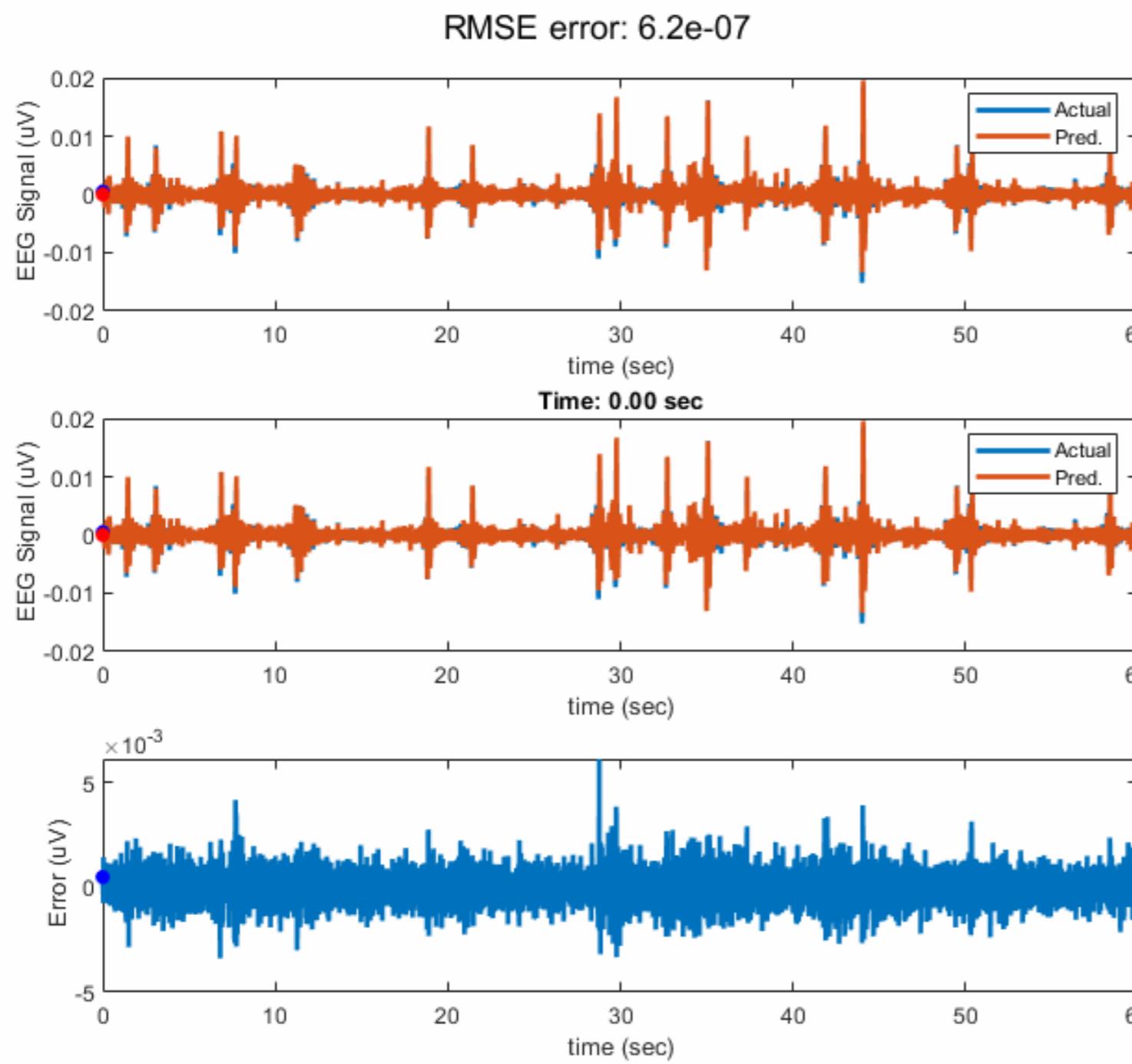
6. Reconstructing the Brain's Unknown Input

Recall: Solving the nonstationary problem

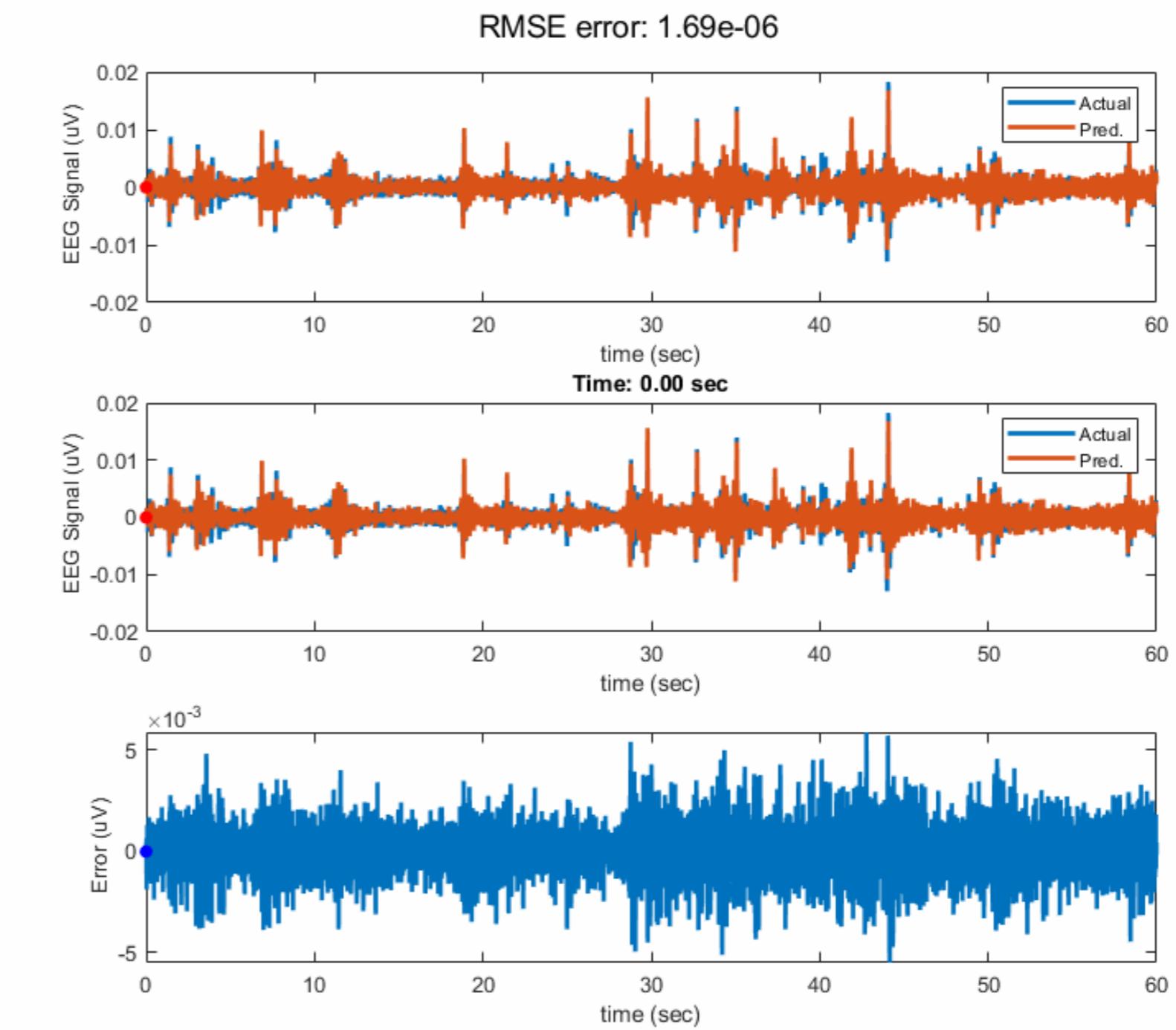
Reconstructing the Brain's Unknown Input

aUIO outperforms static modes

aUIO on unseen data



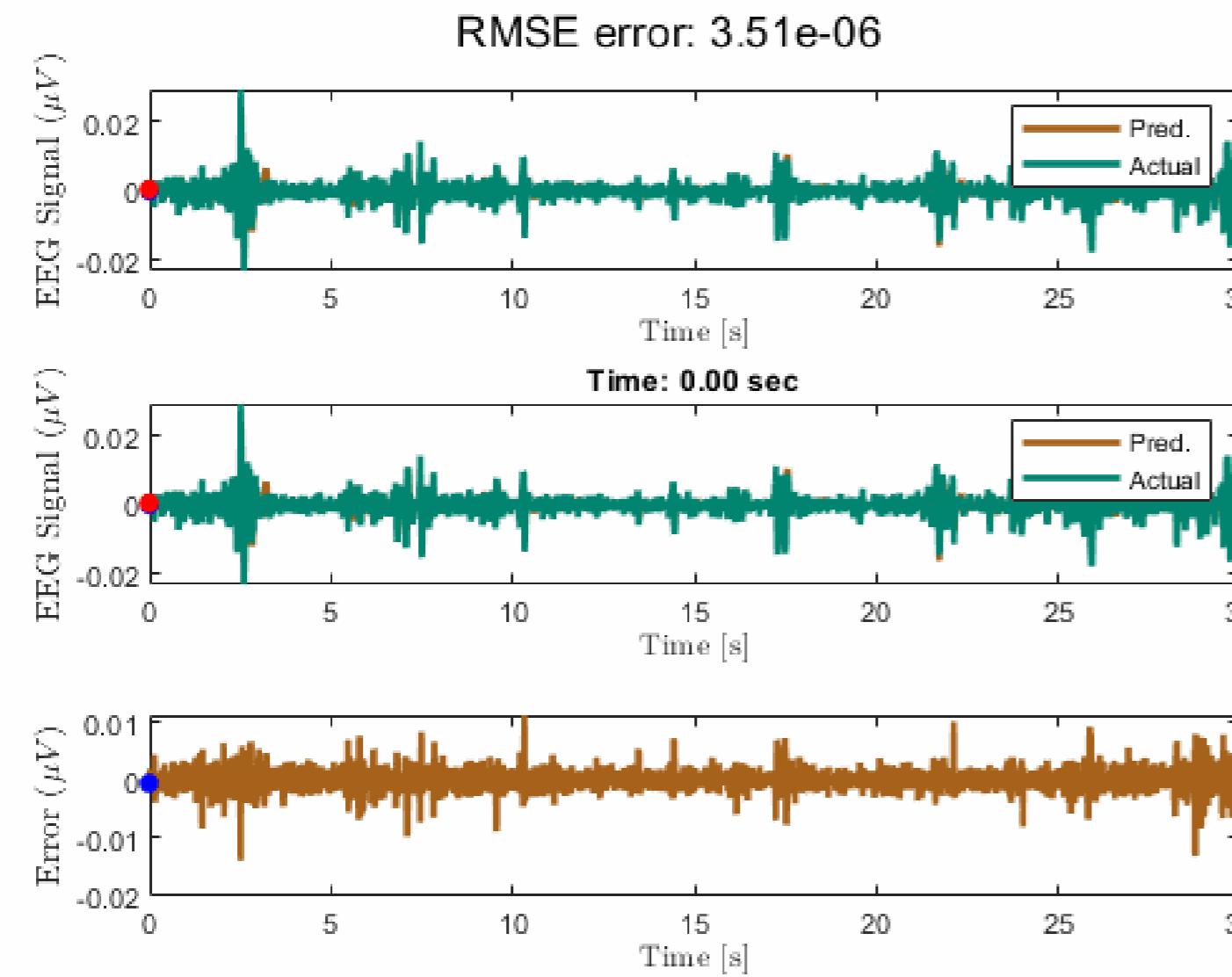
Weighted modes on seen data



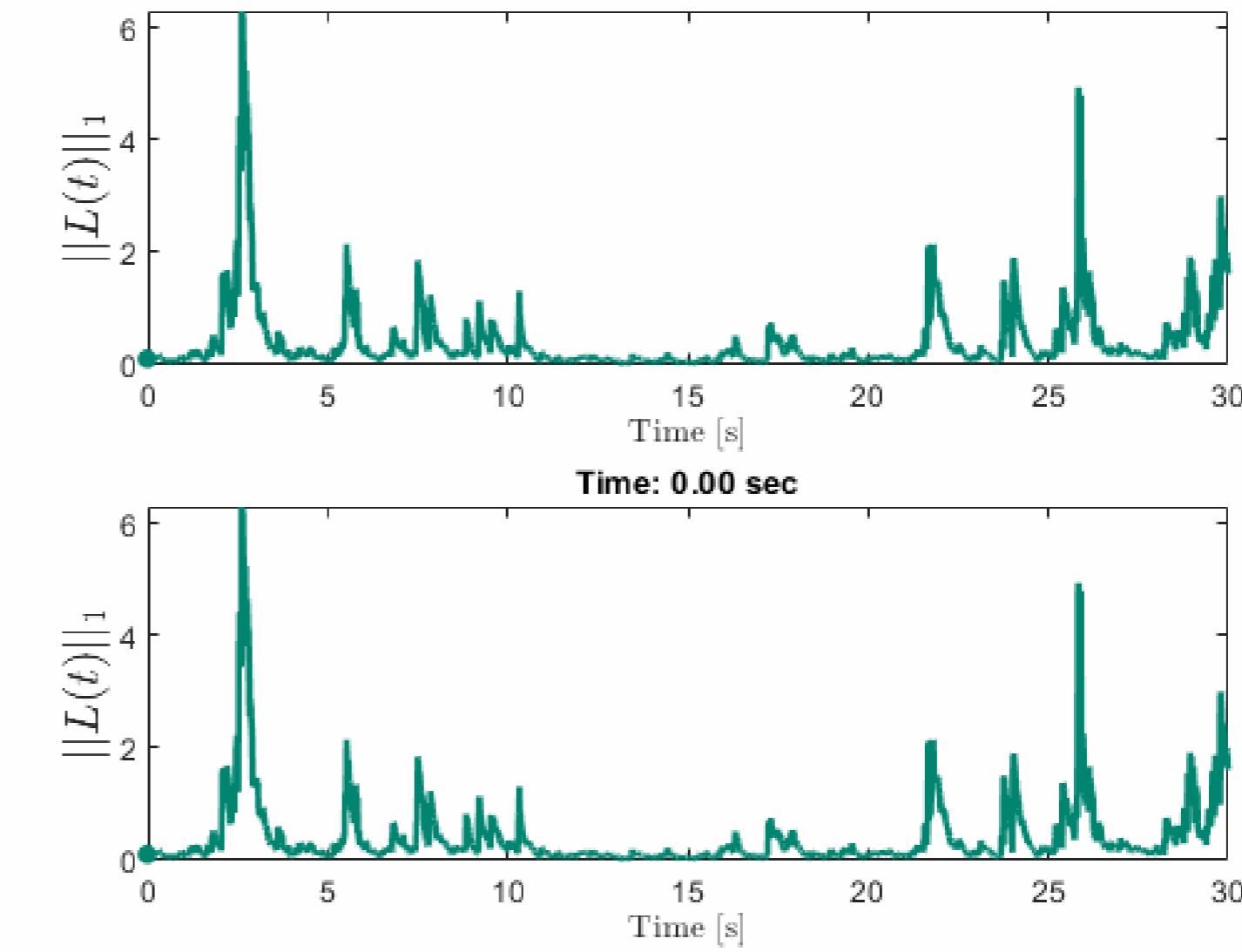
Reconstructing the Brain's Unknown Input

aUIO critically updates model as needed

aUIO on unseen data



Adaptive gain matrix 2-norm



Reconstructing the Brain's Unknown Input

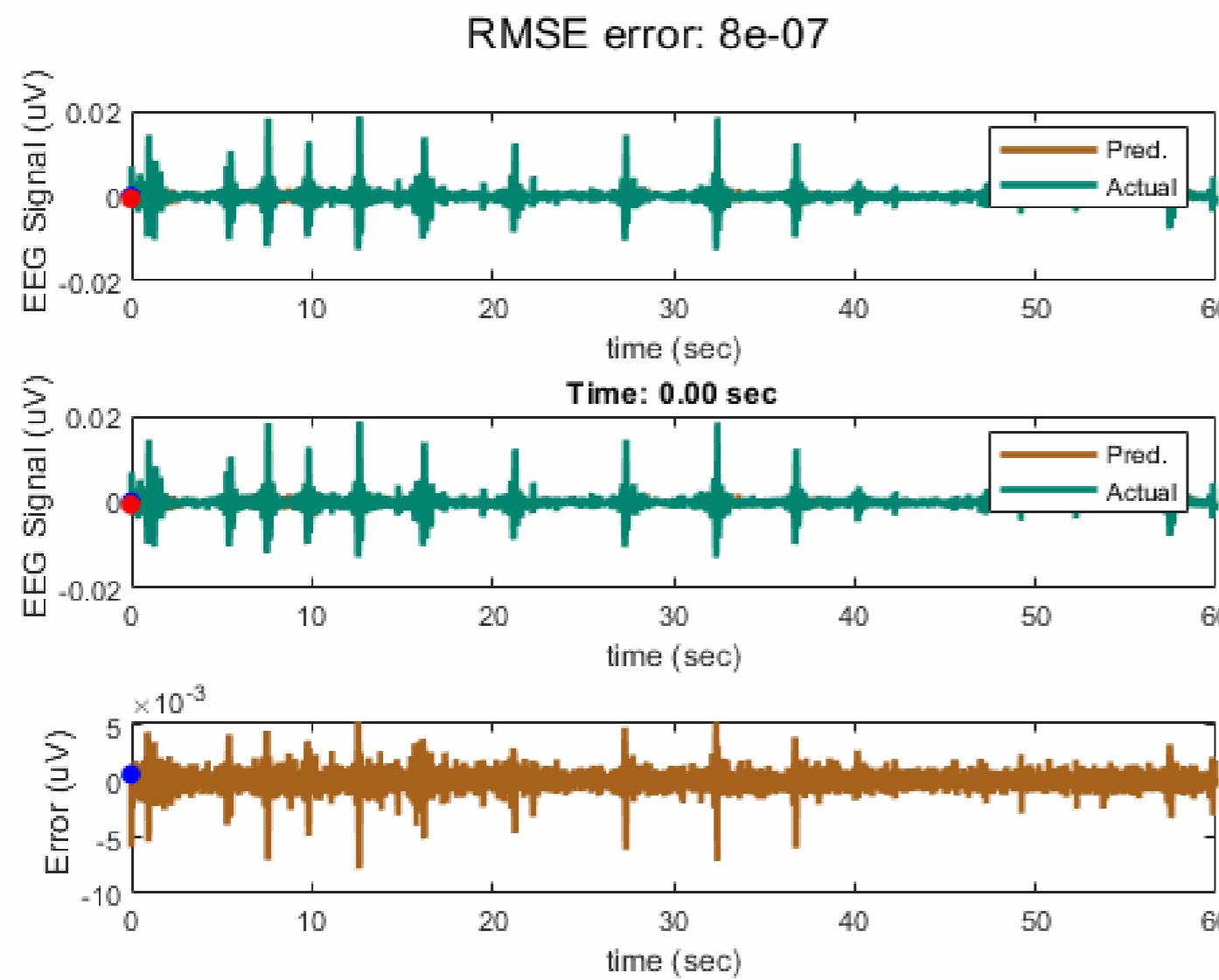
Modeling details

- Unknown input acts evenly over spatial domain
- F_u generates sine-cosine basis
- Static gains per LQR

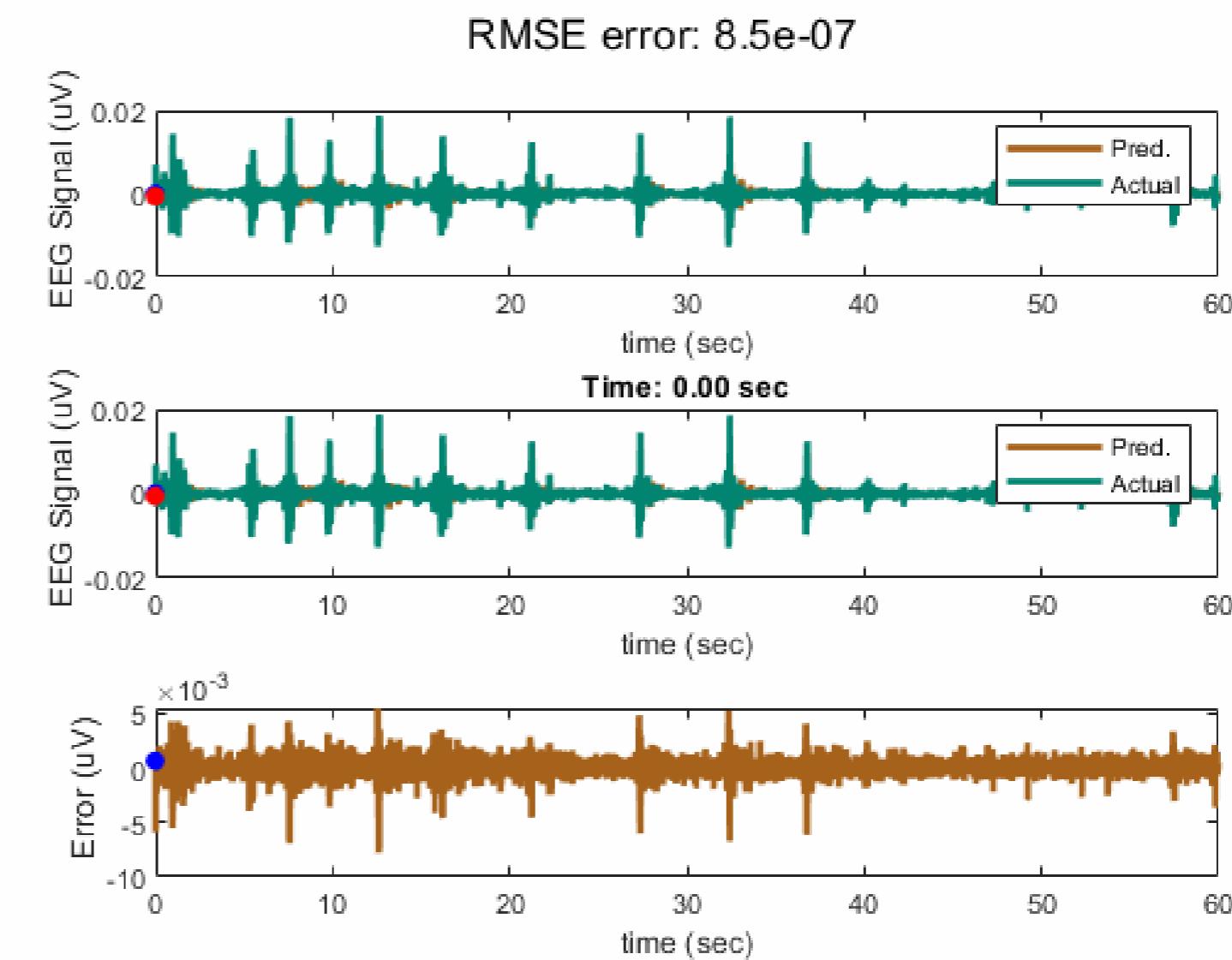
Reconstructing the Brain's Unknown Input

aUIO is tolerant to parametric uncertainty in modes

aUIO on unseen data

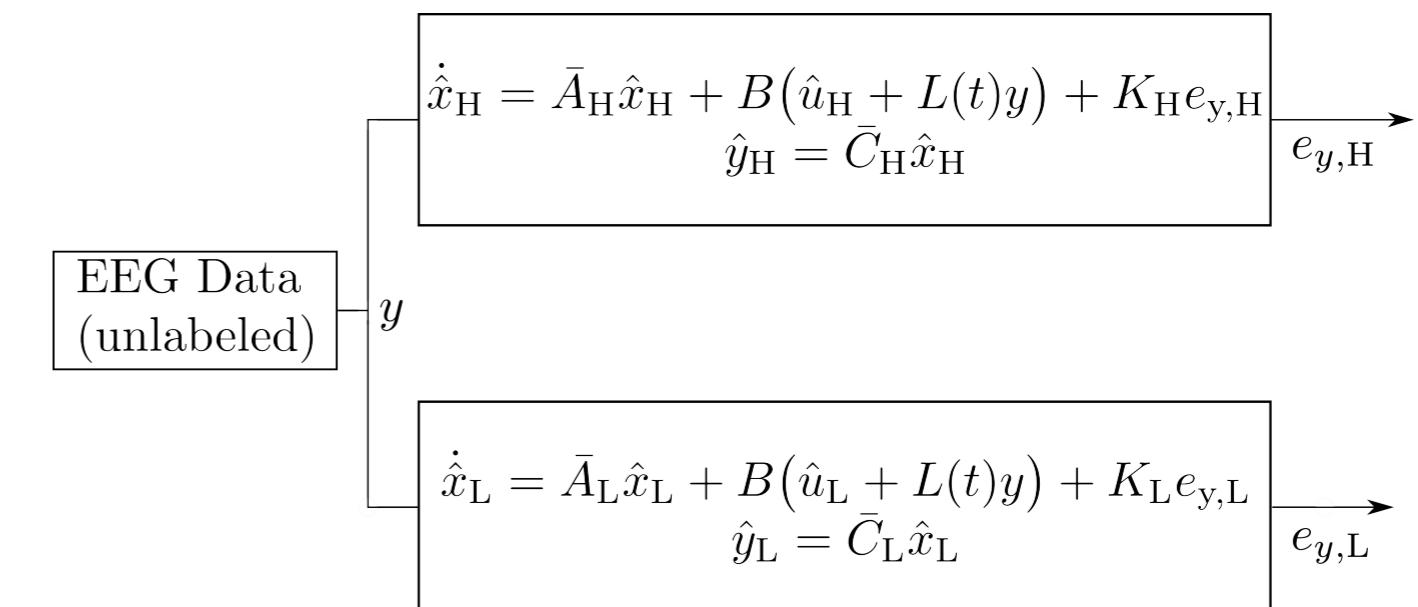
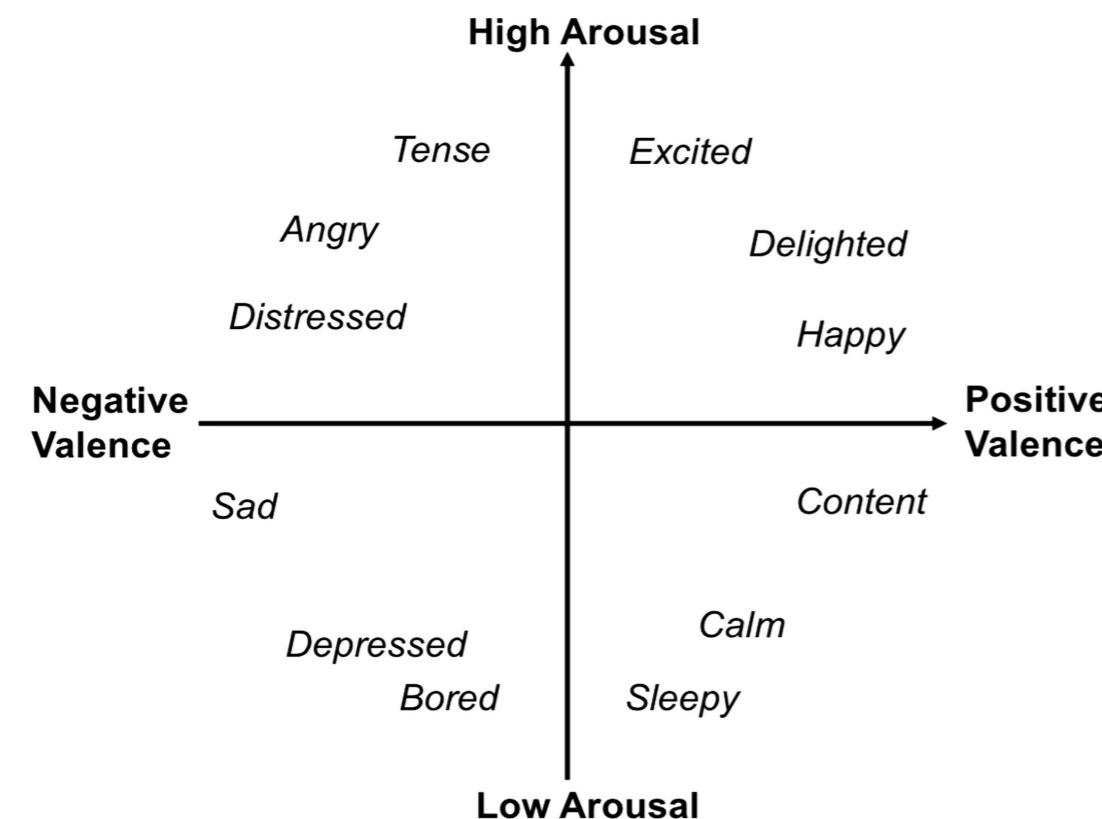


aUIO with wrong A_m



Reconstructing the Brain's Unknown Input

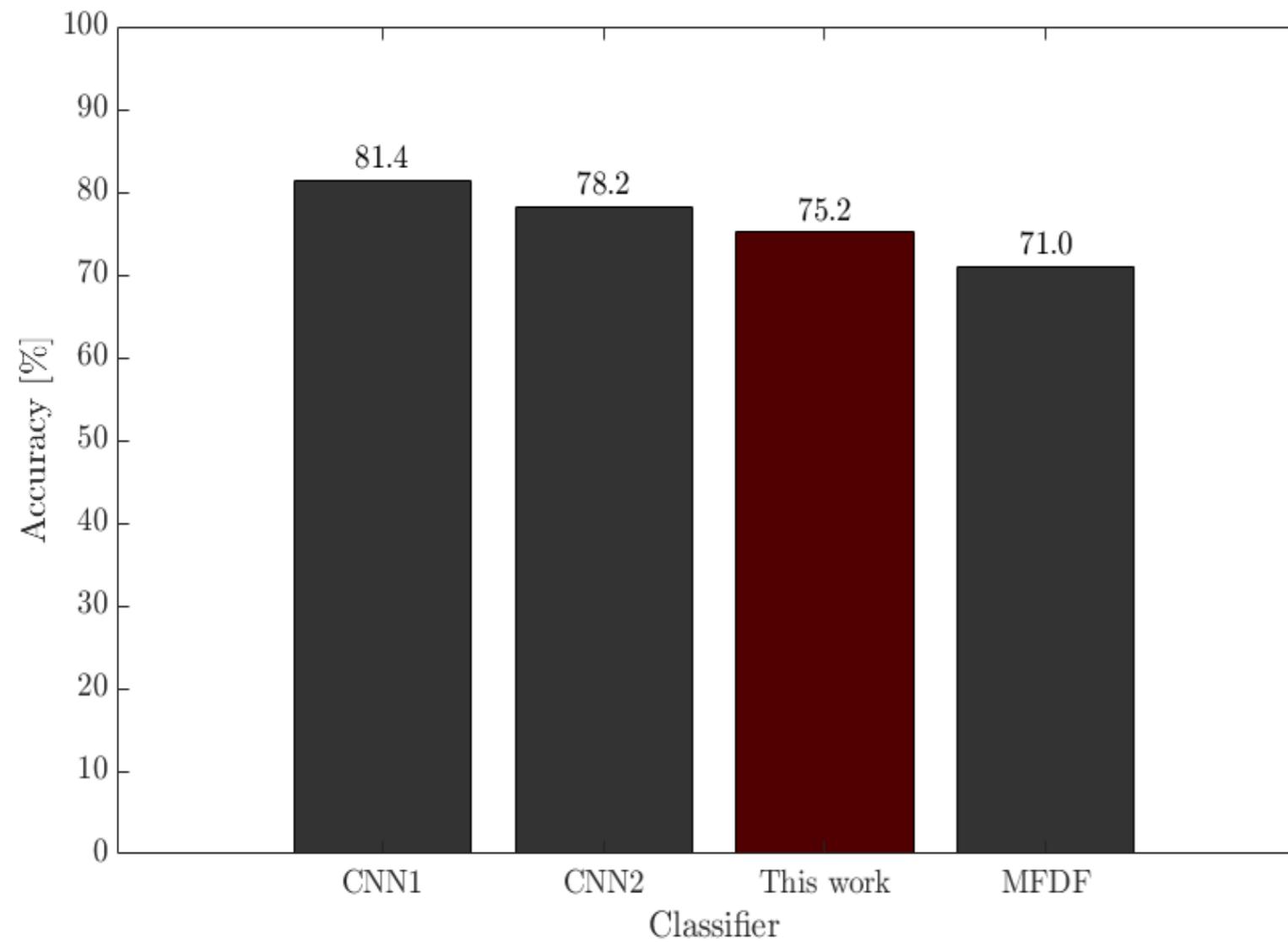
Classification via estimation



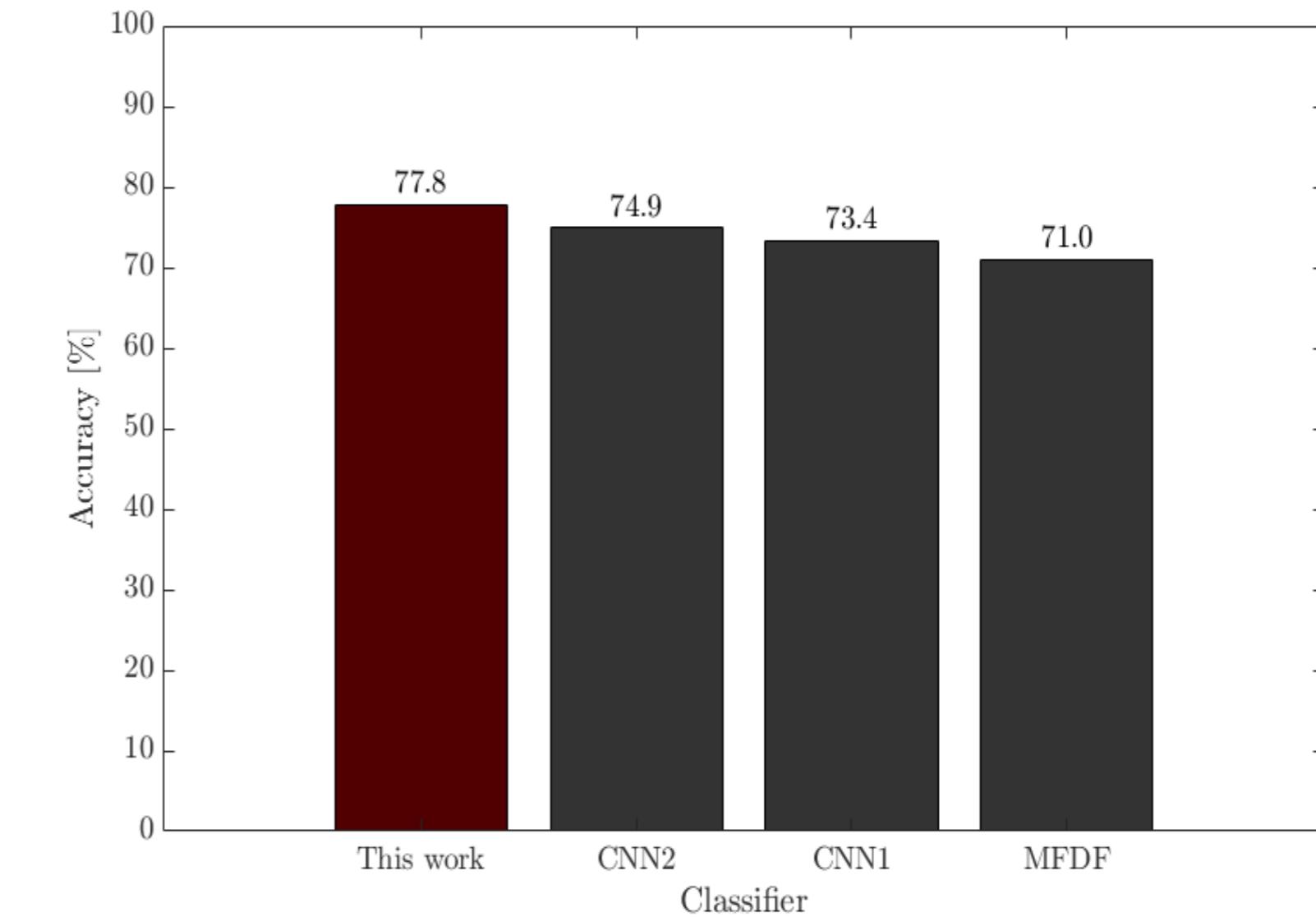
Reconstructing the Brain's Unknown Input

Classification via estimation

Valence Classification



Arousal Classification



Energy and Policy Considerations for Deep Learning in NLP

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[CNN1](#), [CNN2](#), [MFDF](#)

7. Conclusions

Summary

- Modern, sys-id techniques work on biomarker data
- Modal representation aids interpretation and analysis
- Complete body of UIO work
- Online estimation of nonlinear brain wave dynamics

Future work

- Multiple data types
- Improved analysis and classification
- Probabilistic considerations



Acknowledgements

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The willow submits to the wind and prospers until one day it is many willows - a wall against the wind.