A Modal Approach to the Space Time Dynamics of Cognitive Biomarkers

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Outline

Introduction & Motivation

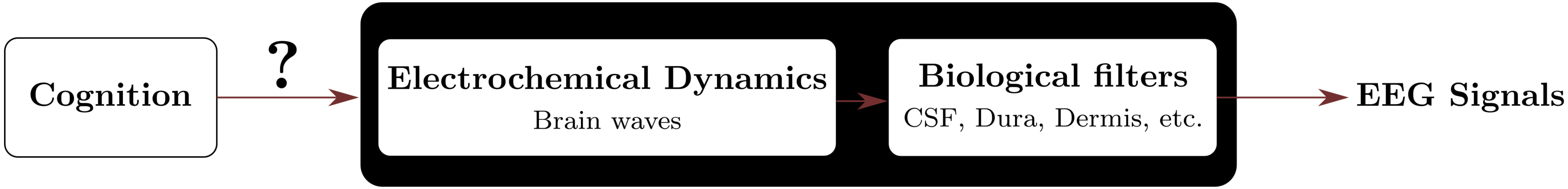


Fig 1. A Wholistic View of Brain Waves

> *Recent modeling work, however, using large-scale dynamical models on the human connectome, suggests that cortical flow patterns are multistable and exhibit phase-transitions. To study such phenomena, a dynamic analysis in which no assumptions about stationarity are made, is required.*

Hindriks, Rikkert, et al. "Latency analysis of resting-state BOLD-fMRI reveals traveling waves in visual cortex linking task-positive and task-negative networks." Neuroimage 200 (2019): 259-274.

A cannonical approach:

True brain wave plant

$$\begin{cases} \dot{x} = Ax + Bu + v_x \\ y = Cx \end{cases}$$

Modal brain wave plant

$$\begin{cases} \dot{\eta} = \Lambda\eta + V^{-1}Bu + V^{-1}v_x \\ y = CV\eta \end{cases}$$

where $A,\ B,\ C,\ v_x,\ x,$ and u are all unknown.

This level of uncertainty is an unsovled problem

Identify the plant: $\begin{cases} \dot{x}_m = A_m x \\ y_m = Cx_m \end{cases},$

Adaptive Unknown Input Brain Wave Estimator:

$$\begin{cases} \dot{\hat{x}} = \big(A_m + BL(t)C\big)\hat{x} + B\hat{u} + K_x e_y; \\ \hat{y} = C\hat{x}. \end{cases}$$

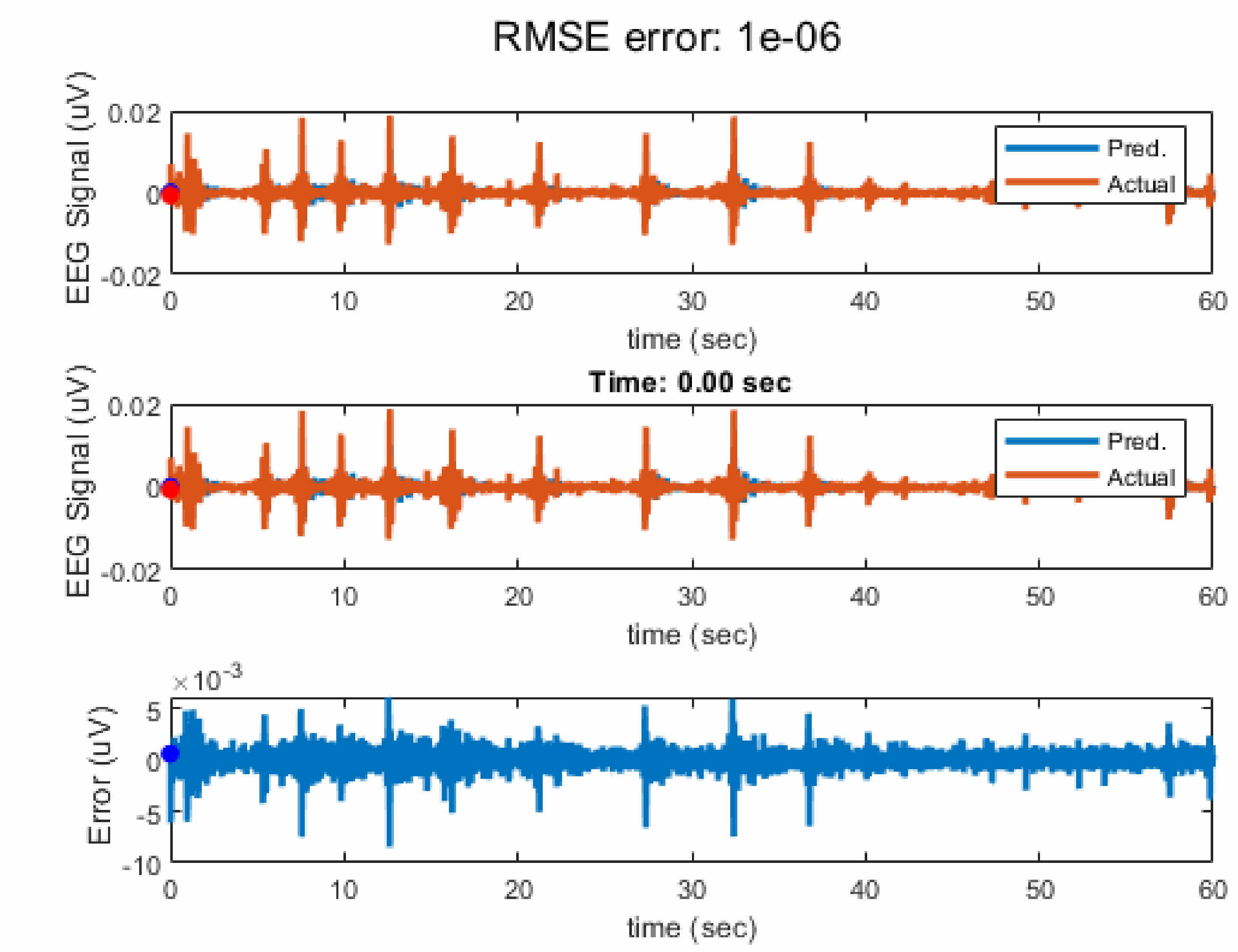
System Identification of Brain Wave Modes Using EEG

Identifying linear patterns

Identify the plant: $\begin{cases} \dot{x}_m = A_m x \\ y_m = C x_m \end{cases}$

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{s-1} \end{bmatrix} X_0$$

$$\hat{\Gamma} = U S^{1/2} \; \hat{X}_0 = S^{1/2} V^*$$



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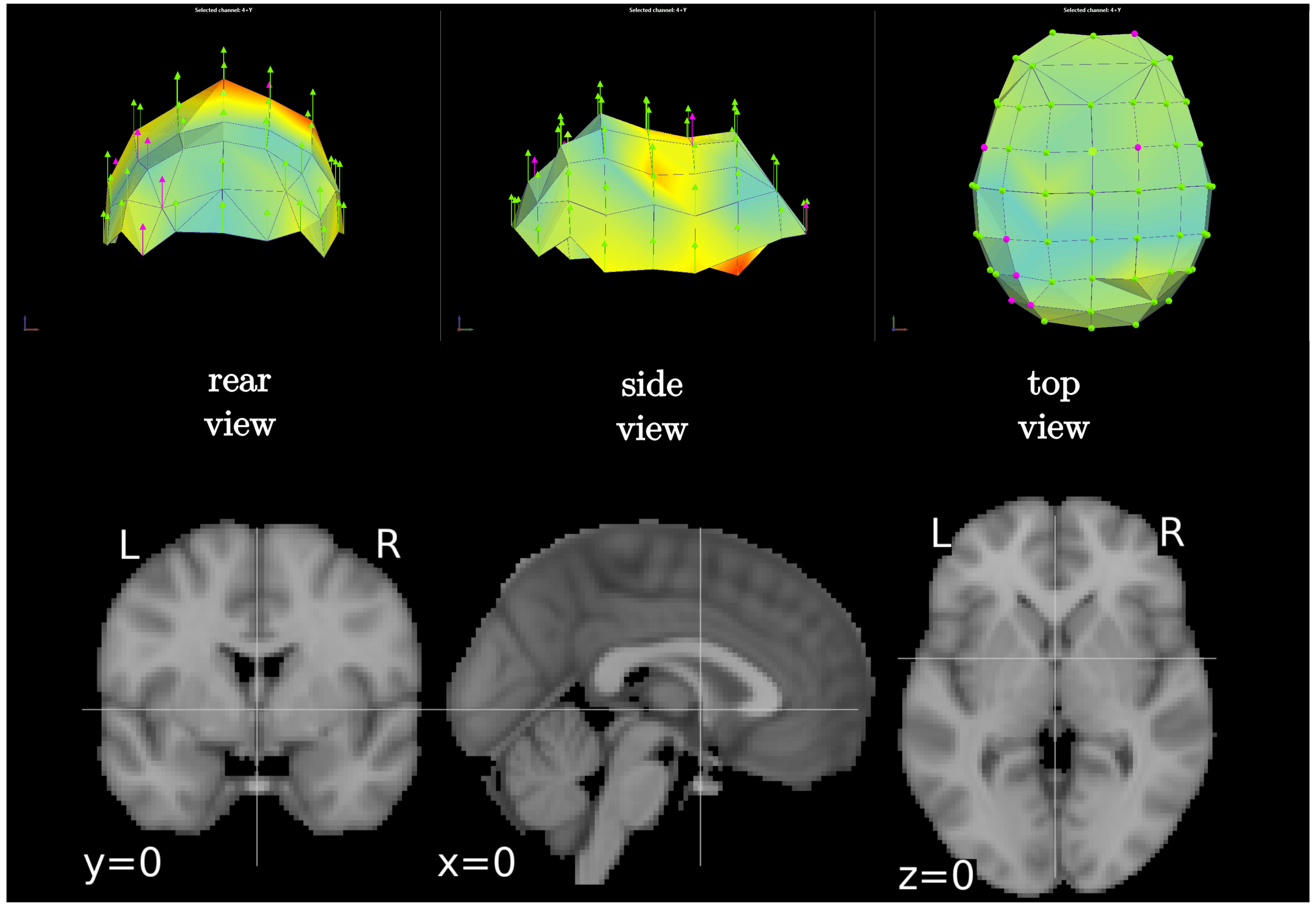
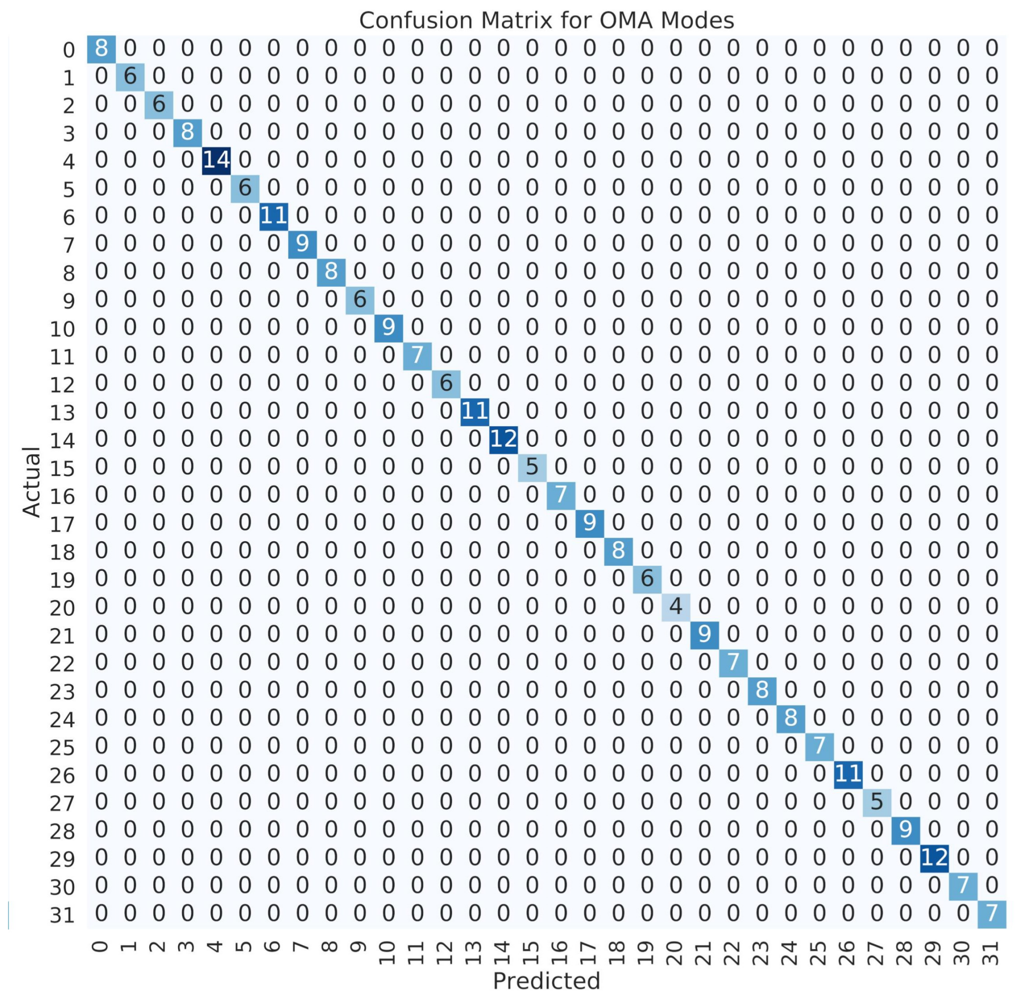


Analysis of Linear Brain Wave Modes



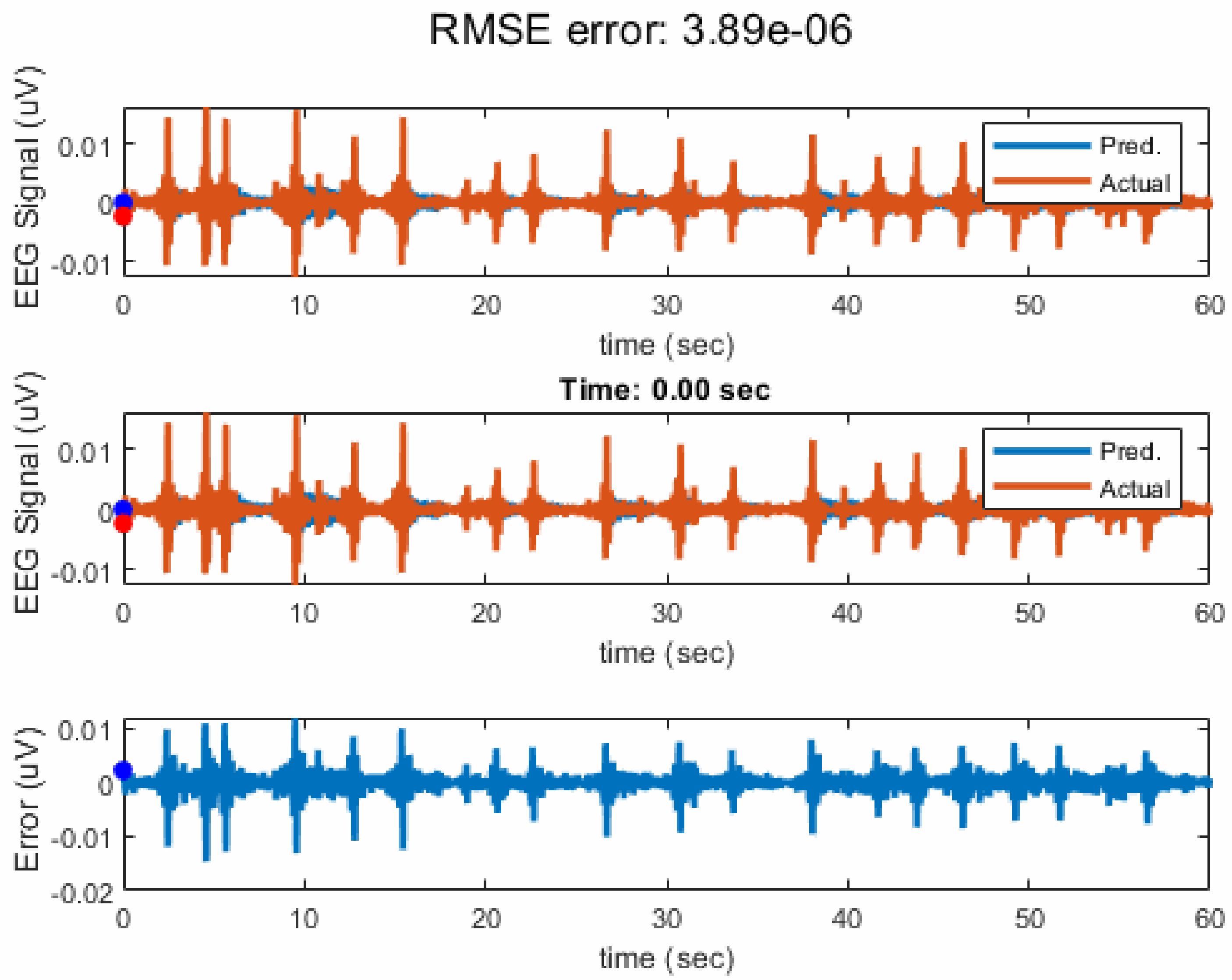
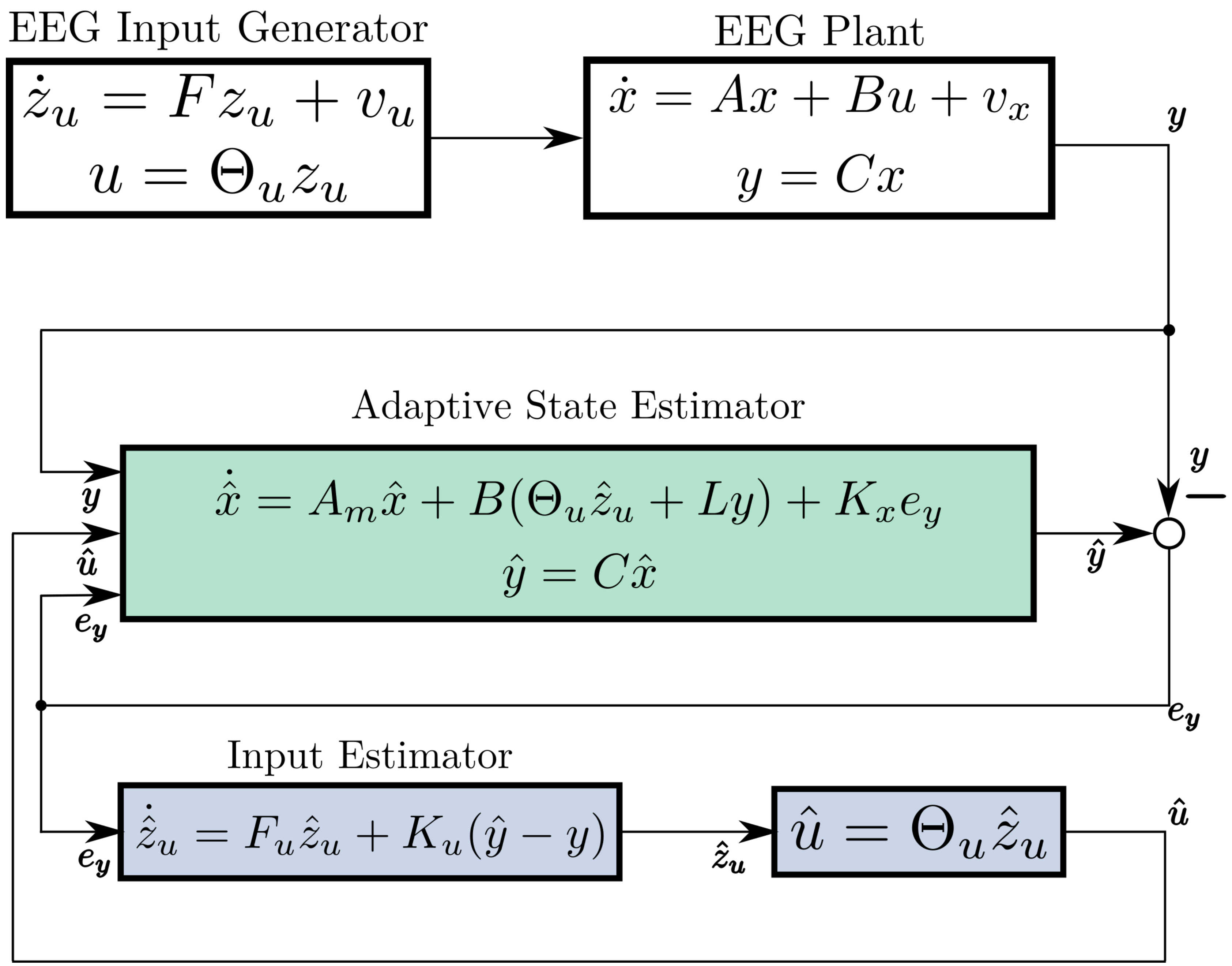
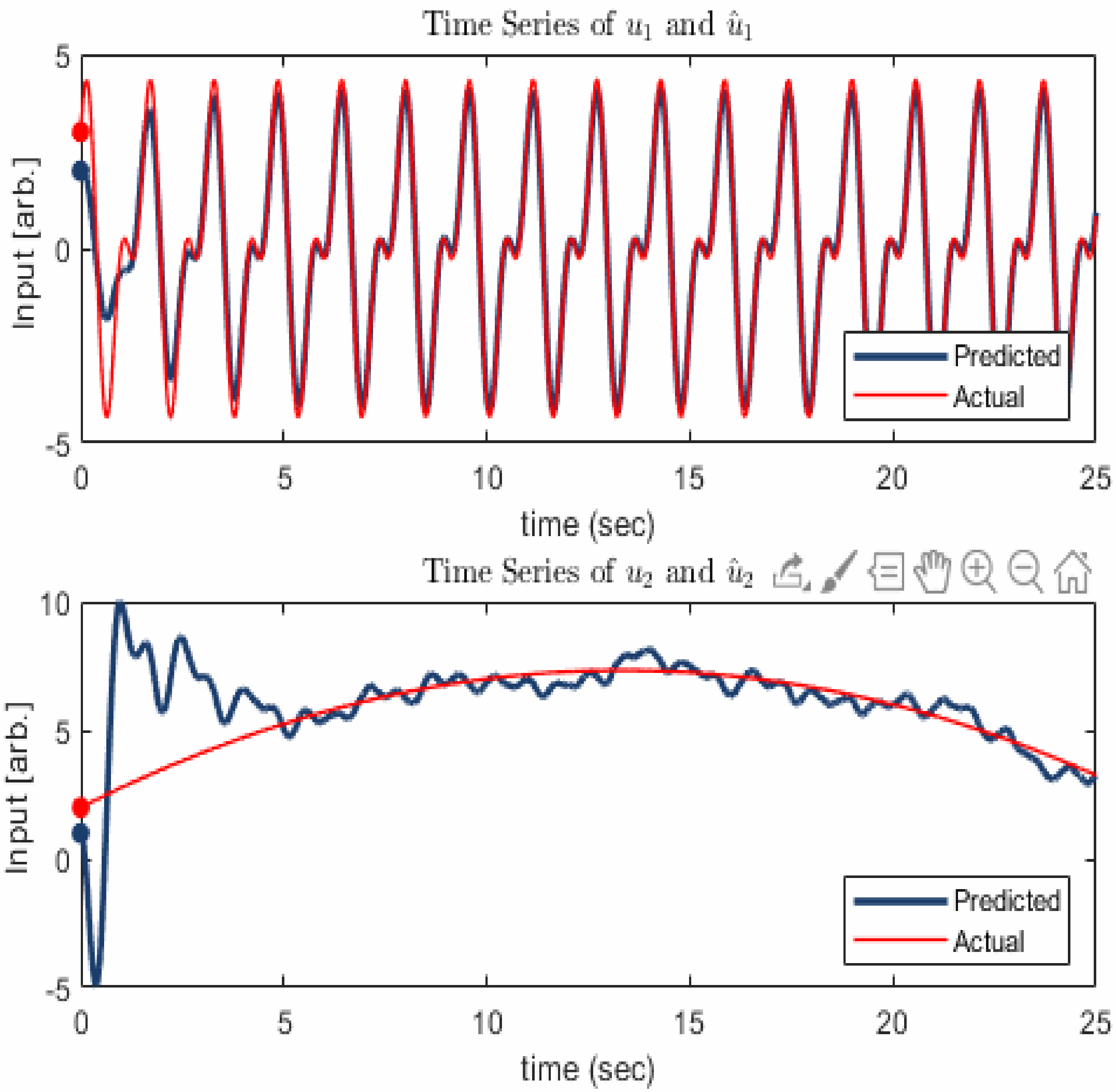


Table 1: Common OMA Brain Modes

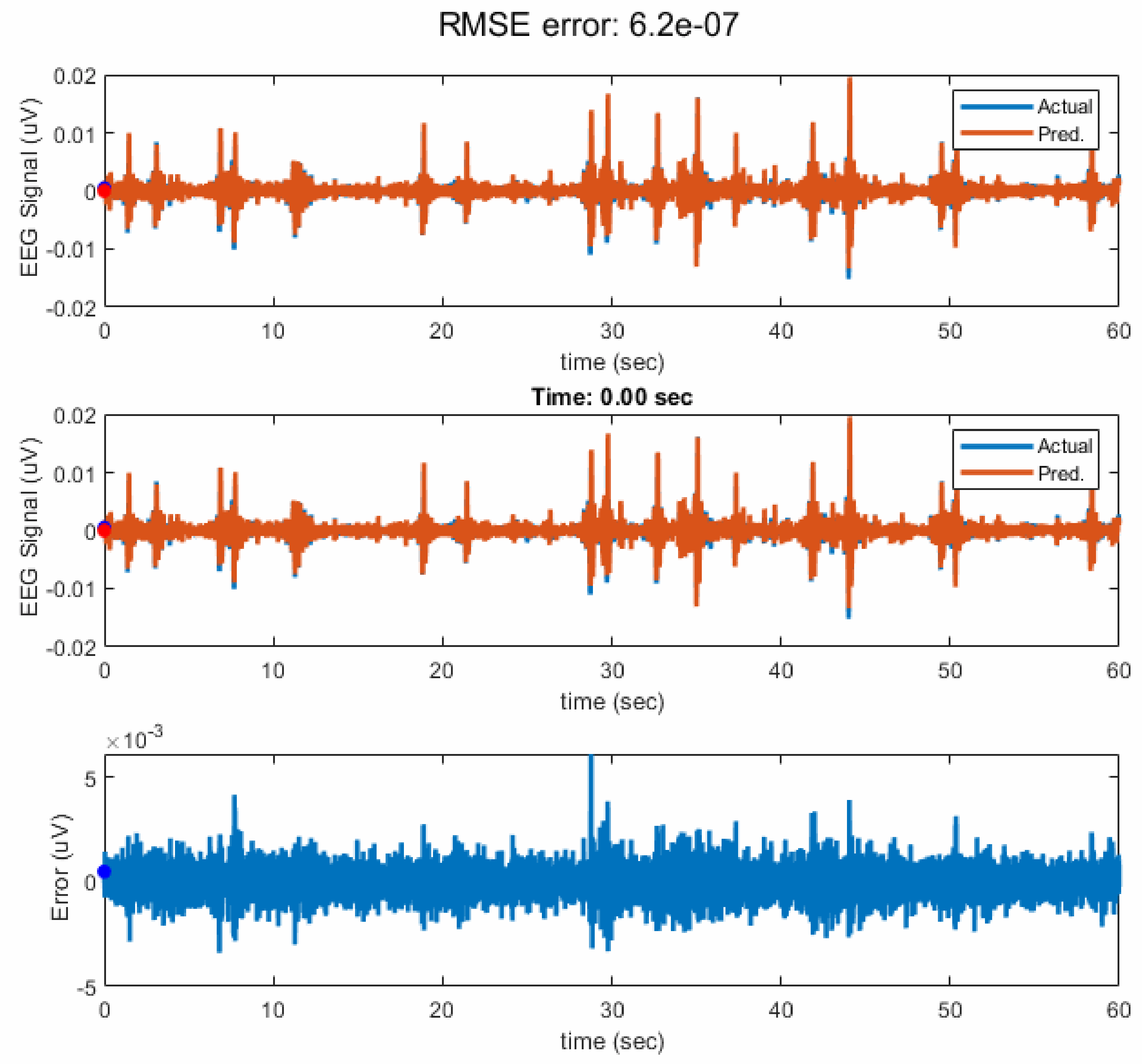
	Frequency	Damping [%]	Complexity [%]	Shape Correl.
Alpha Mode 1	4.34±0.03	8.20±1.20	11.47±17.59	0.97±0.016
Beta Mode 2	21.83±0.22	1.98±2.63	32.29±35.67	0.96±0.018
Gamma Mode 3	40.39±0.26	11.87±7.49	12.42±16.88	0.99±0.010
Gamma Mode 4	44.19±0.24	2.52±1.39	2.93±5.69	0.99±0.012

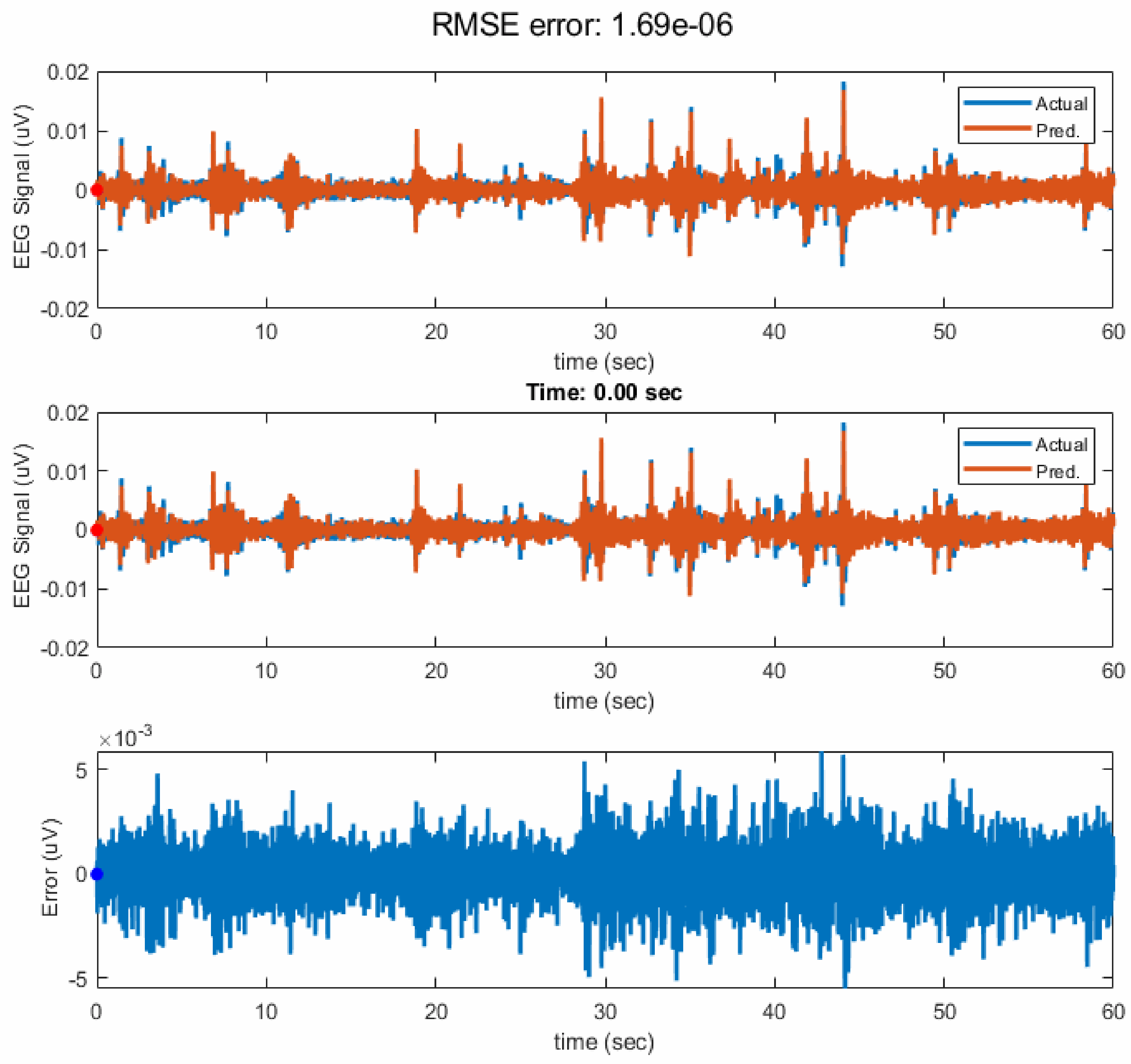
Unknown Input Estimators





aUIO Performance





Classification

EEG Data (unlabeled)

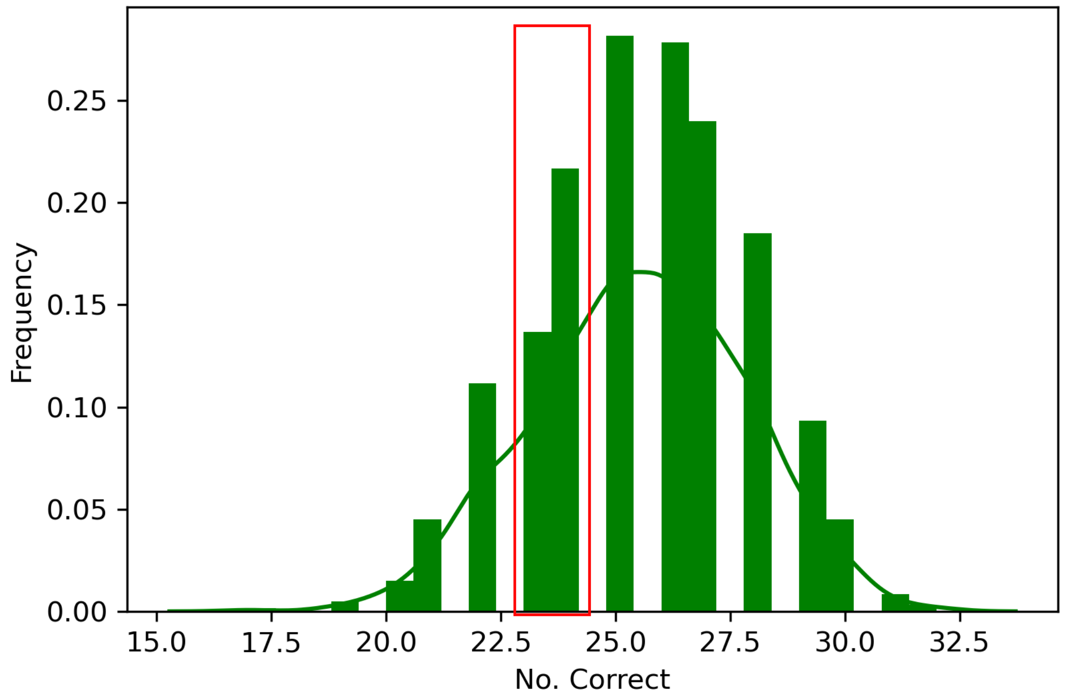
y

$$\dot{\hat{x}}_{\mathrm{H}} = \bar{A}_{\mathrm{H}}\hat{x}_{\mathrm{H}} + B\big(\hat{u}_{\mathrm{H}} + L(t)y\big) + K_{\mathrm{H}}e_{\mathrm{y,H}}$$
$$\hat{y}_{\mathrm{H}} = \bar{C}_{\mathrm{H}}\hat{x}_{\mathrm{H}}$$

$e_{y,\mathrm{H}}$

$$\dot{\hat{x}}_{\mathrm{L}} = \bar{A}_{\mathrm{L}}\hat{x}_{\mathrm{L}} + B\big(\hat{u}_{\mathrm{L}} + L(t)y\big) + K_{\mathrm{L}}e_{\mathrm{y,L}}$$
$$\hat{y}_{\mathrm{L}} = \bar{C}_{\mathrm{L}}\hat{x}_{\mathrm{L}}$$

$e_{y,\mathrm{L}}$



Quantum Observability

$$\begin{aligned}\dot{\rho}(t) &= \sum_{k=1}^{N} p_k \dot{P}_k(t) \\ &= -i[H\rho(t) - \rho(t)H] \\ &= (-i)[H,\rho] \equiv (-i)L\rho \\ y_i(t) &= \mathrm{tr}(C_i\rho)\end{aligned}$$

$$y^{(k)}(0) = (C, A^k\rho_0)_{\mathrm{tr}} = (\underbrace{(A^*)^k C}_{\Theta_k}, \rho_0)_{\mathrm{tr}};\ k = 0, 1, \ldots, N^2 - 1$$