A Modal Approach to the Space Time Dynamics of Cognitive Biomarkers

T. Griffith

Defense

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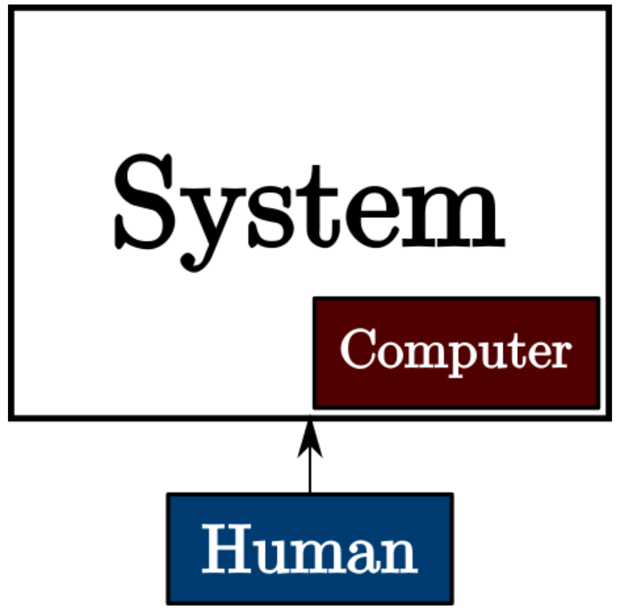
Outline

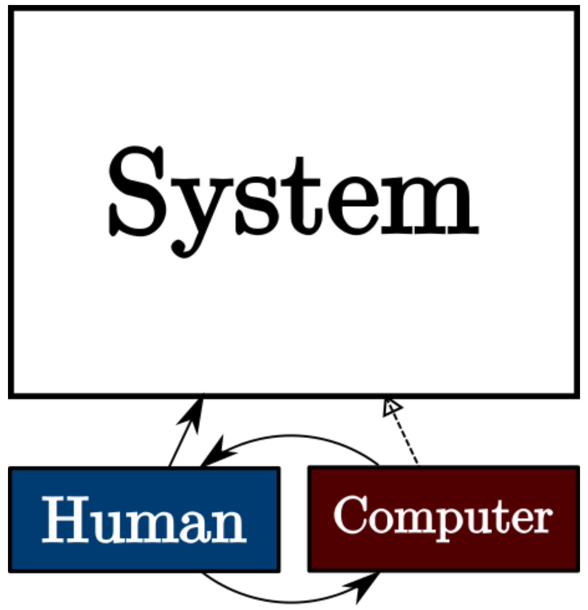
1. Introduction & Motivation
2. A Dynamic Systems View of Brain Waves
3. System Identification of Brain Wave Modes Using EEG
4. Modal Analysis of Brain Wave Dynamics
5. Adaptive Unknown Input Estimators
6. Reconstructing the Unknown Input Using EEG
7. Significant Contributions

Modern systems demands bi-directional flow of information

- Modern systems feature humans as supervisors, not sole actors
  - Teaming becomes important for safety and performance
- Comparatively less research analyzes the information flow from human to computer.
- **This work investigates the use of canonical engineering principles for estimation of human state/cognition.**



The computer as part of the system



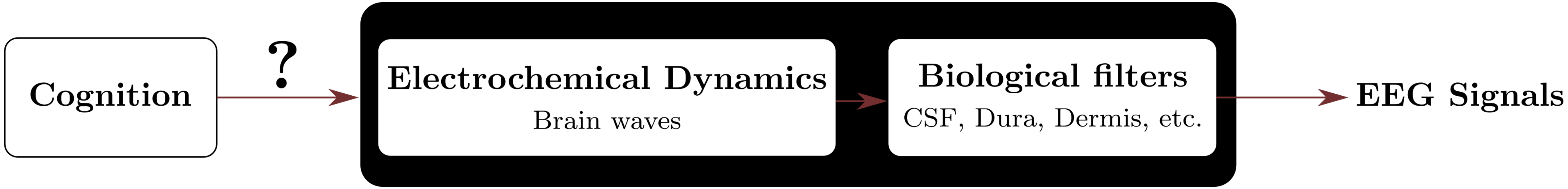
The computer as a teaming member

Hardware and experimental design can be used to interpret information about human states from noisy physiological data.

- Physiological signals are historically not portable
  - Prevented ecologically valid experiments
- Human state (e.g. SA) reduced to discrete self reports
- **There is new potential for modeling techniques to interpret human state from noisy physiological signals.**

Time Domain fNIRS from Kernel Flow

Cognition as a black box



- Cognition gives rise to EEG signals
  - but it is **noisy** and only **loosely** correlated with cognition
- Cellular activity can only be measured invasively
- **Can we say something about cognition from dynamic EEG signals**?

Nonstationary, nonlinear signals make the **dynamics** complicated

- Historically,
  - stationary analysis is used with a sliding window
  - or case by case highly derived models are developed
- This work seeks a method to address in engineering dynamics terms
  - with eye towards cognitive outcomes
  - because there are many existing analytical tools in engineering dynamics

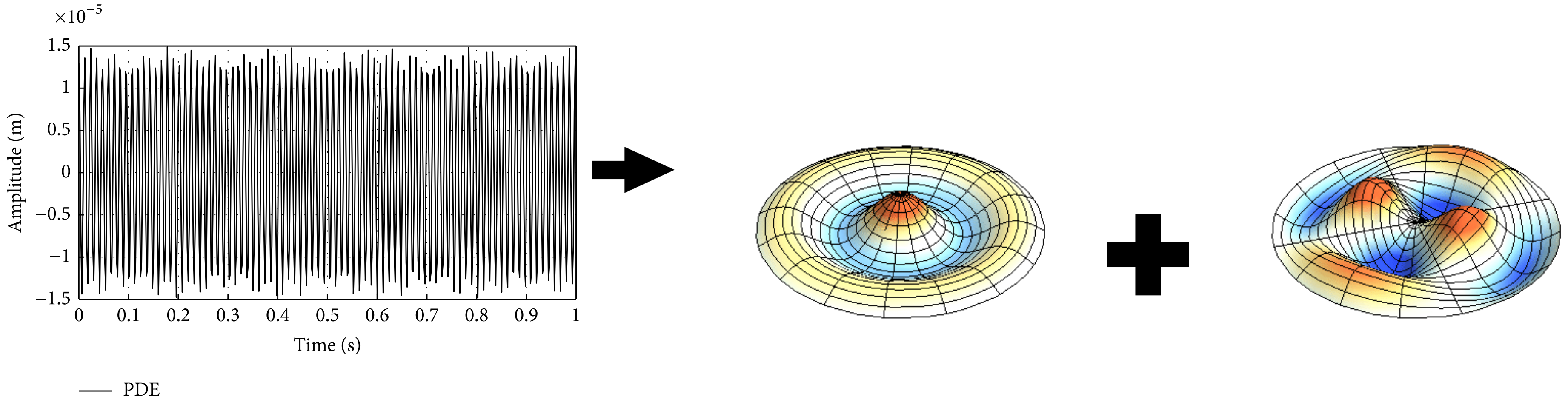
a dynamic analysis in which no assumptions about stationarity are made, is required.

Hindriks, Rikkert, et al. “Latency analysis of resting-state BOLD-fMRI reveals traveling waves in visual cortex linking task-positive and task-negative networks.” Neuroimage 200 (2019): 259-274.

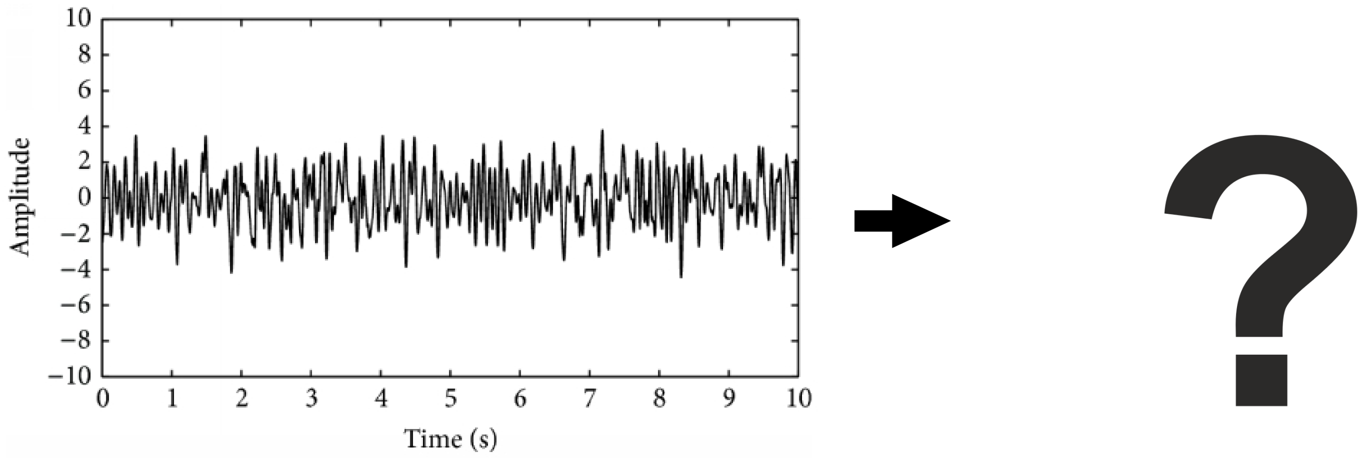
Guardrails: This is not a model of the brain!

but you can measure EEG signals and say something about the system

- Engineering mechanics does not require atomic level analysis to evaluate stress and strain.
  - Can we extend this analogy to spatio-temporal dynamics of human cognition?
  - Because brain wave dynamics are also spectral and **admit** spatio-temporal modes
  - Without the need to model the connectome.



Surface recordings of membranes yield useful engineering information.

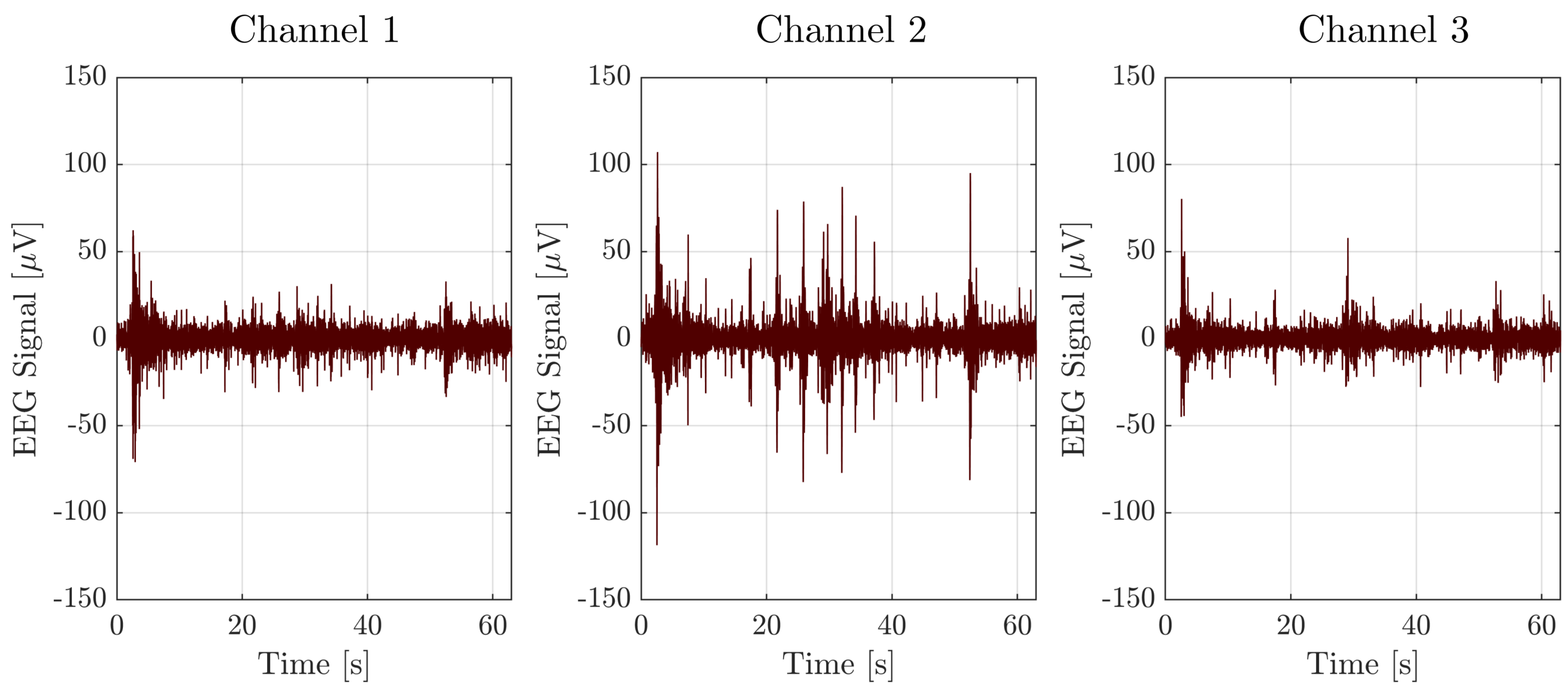


Is this notion relevant to brain waves?

2D membrane modes

2. A Dynamic Systems View of Brain Waves

A canonical approach:



Linearized brain wave “plant”: $\begin{cases} \dot{x} = Ax + Bu + v_x \\ y = Cx \end{cases}$

linearization is **around an operating point** (i.e. a cognitive state)

but $A,\ B,\ C,\ v_x,\ x$, and u are all unknown.

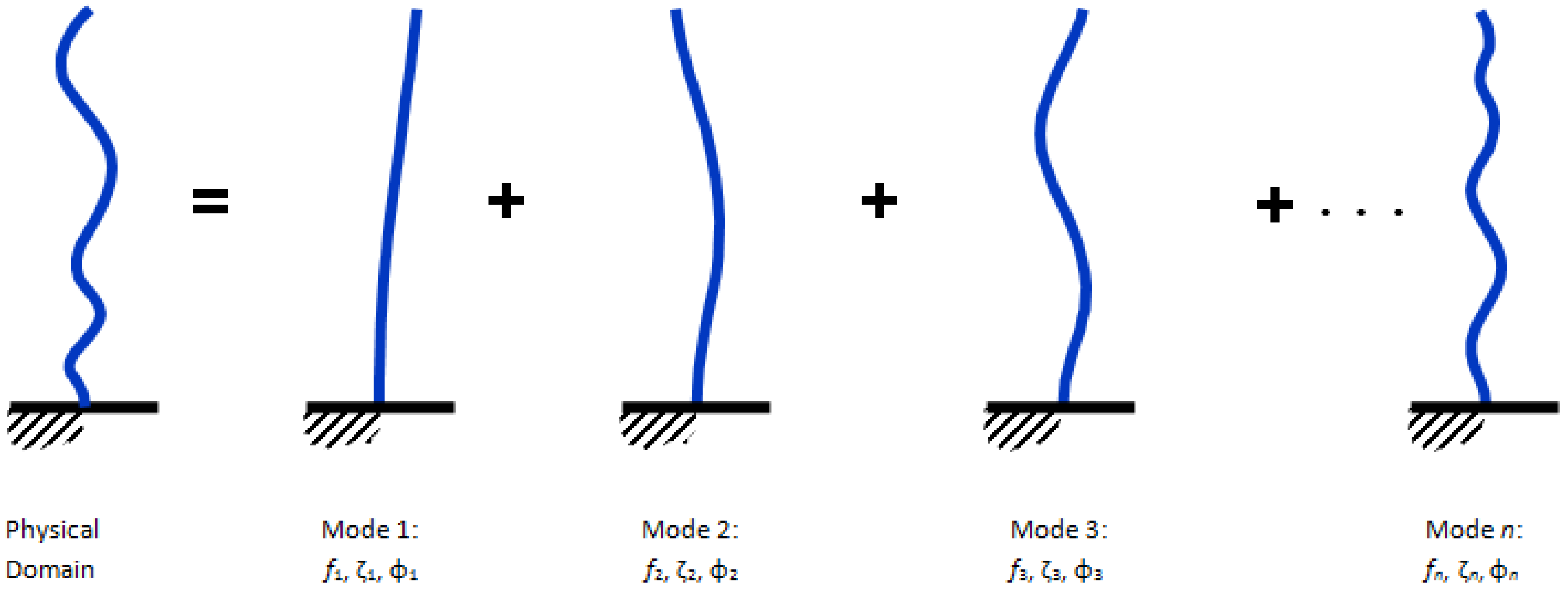
$A,\ B,\ C,\ v_x,\ x$, and u are **all unknown**.

This level of uncertainty is an unsovled problem

Identify the plant: $\begin{cases} \dot{x}_m = A_m x + v_x \\ y_m = C x_m \end{cases}$,

accepting the uncertainty in A_m.

A modal transformation yields a discrete set of spatio-temporal modes which are useful for brain wave analysis and mapping

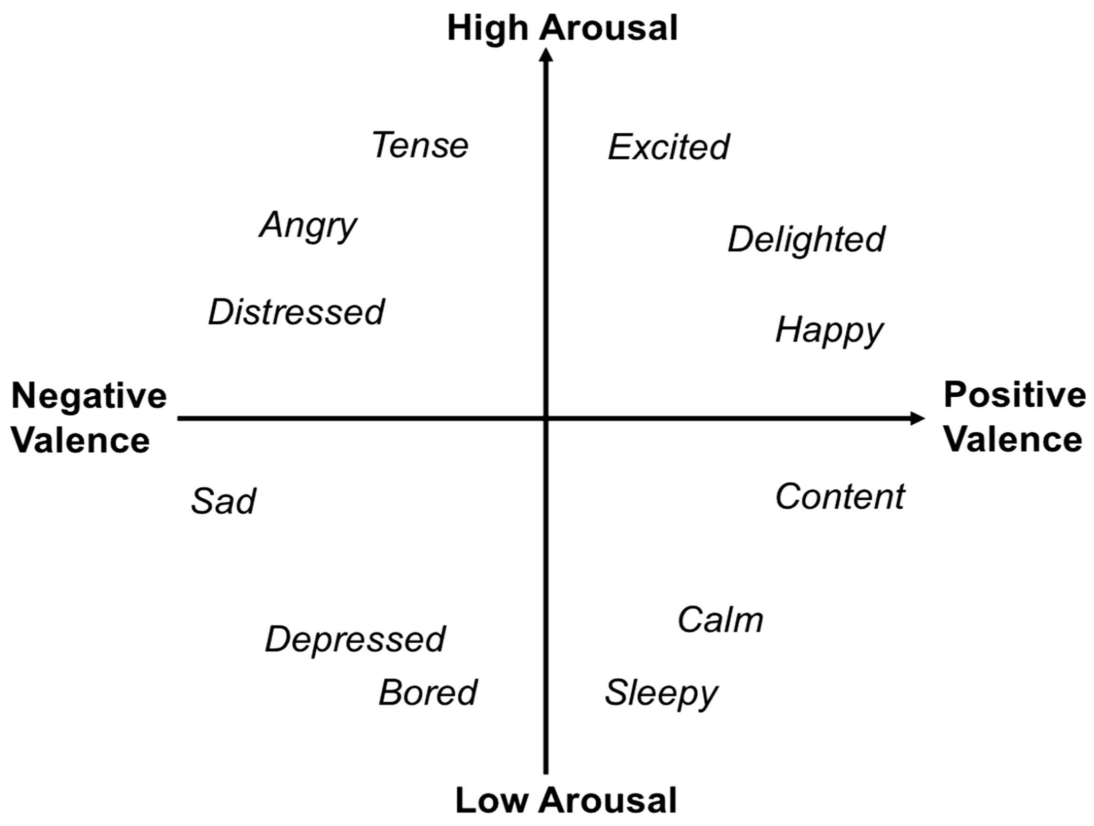


- A giant (A, C) may not be useful!
- Modes have:
  - Frequency (f)
  - Damping (ζ)
  - Mode shape (ϕ)
  - Complexity ($\%$)
- Modal dynamics are equivalent to original model

3. System Identification of Brain Wave Modes Using EEG

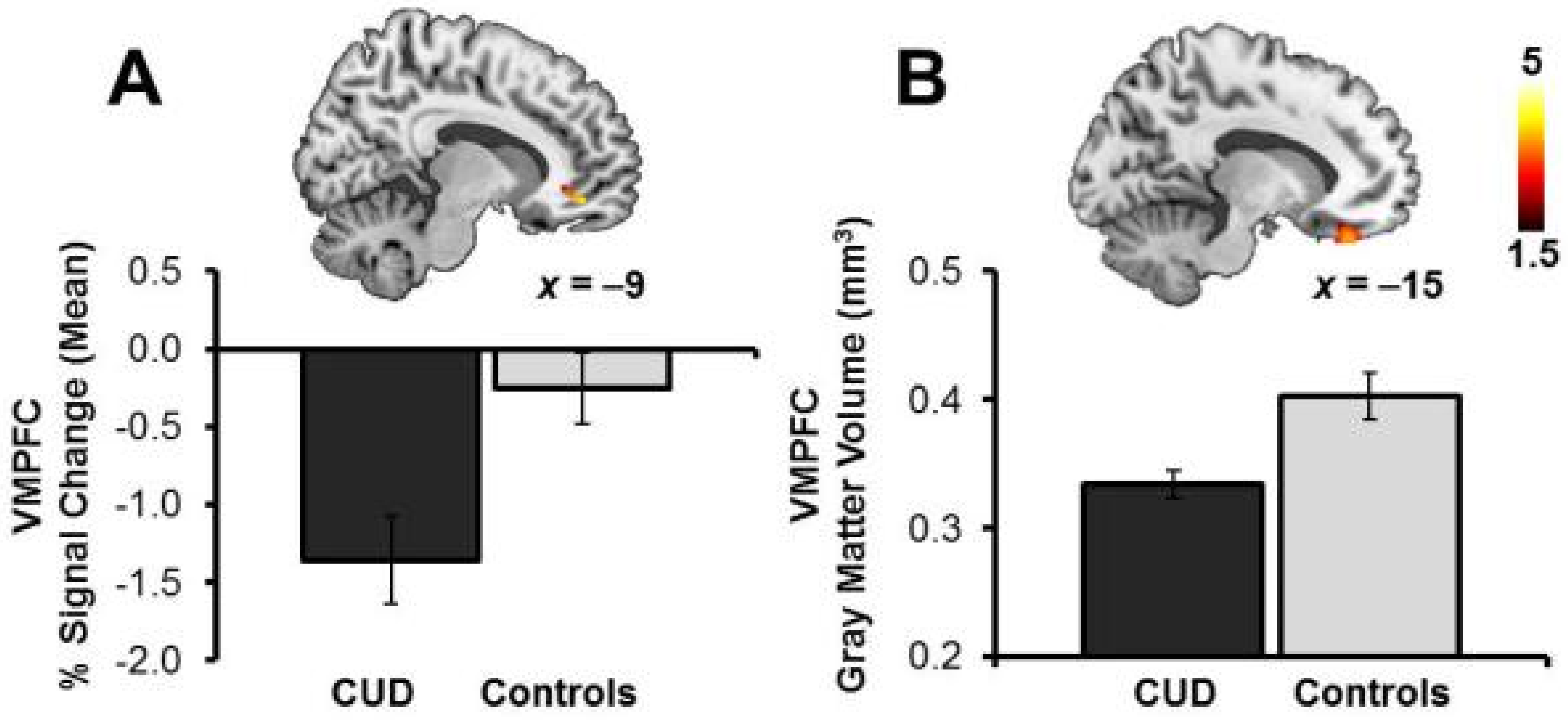
Datasets considered

DEAP: A Database for Emotion Analysis using Physiological Signals



- 32 sensors
- 32 subjects watch 40 videos
- Subjects self report Valence and Arousal

NARC: Neuropsychoimaging of Addiction and Related Conditions Dataset



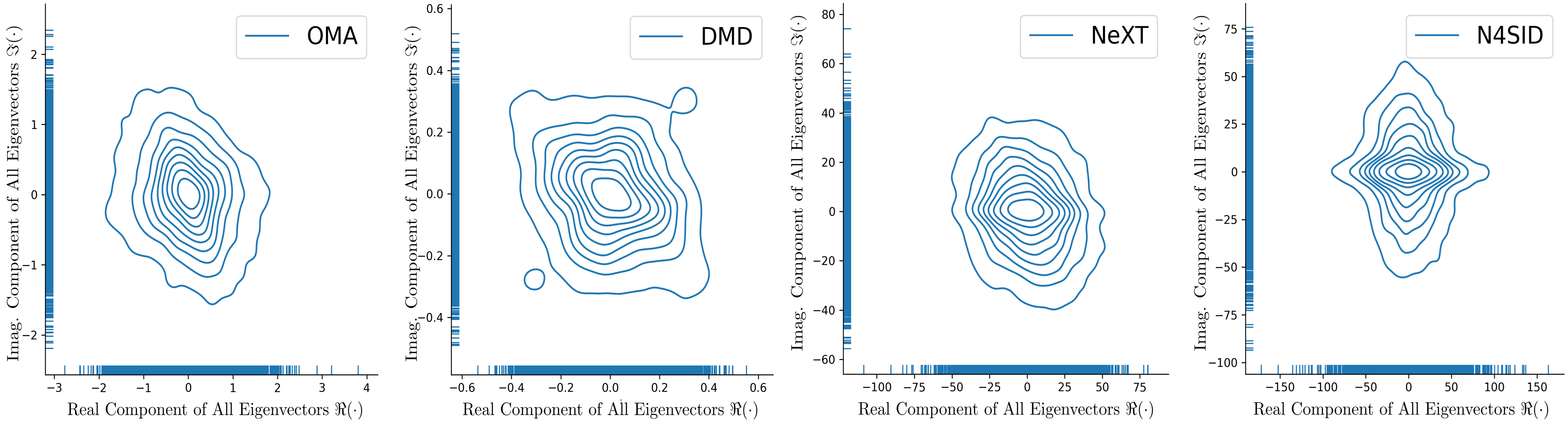
- 60 sensors
- 100 subjects at rest state
- Subjects self report craving scale

Koelstra, Sander, et al. "Deap: A database for emotion analysis; using physiological signals." IEEE transactions on affective computing 3.1 (2011): 18-31.

Konova, Anna B., et al. "Structural and behavioral correlates of abnormal encoding of money value in the sensorimotor striatum in cocaine addiction." European Journal of Neuroscience 36.7 (2012): 2979-2988.

Output only modal analysis is well suited to EEG waves

- OMA (stochastic, zero mean)
- DMD (deterministic, full state)
- NeXT (deterministic, modal)
- N4SID (stochastic, Kalman states)



Bivariate distribution of identified modes in DEAP dataset

Between 40 and 50 modes are needed for brain wave modeling

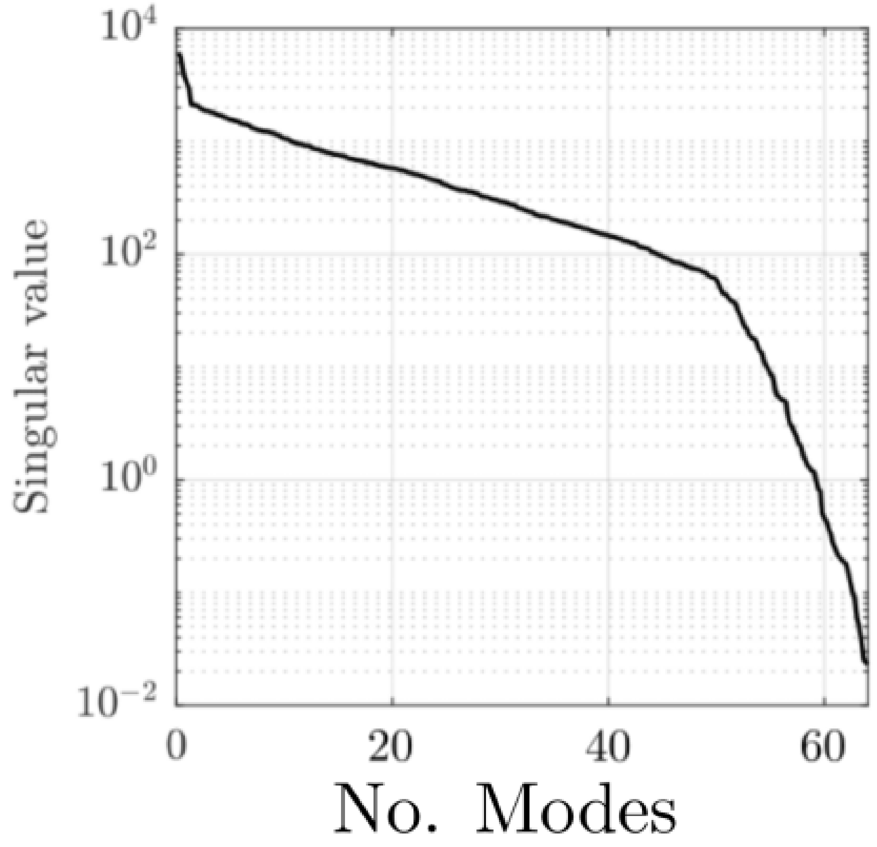
Identify the plant: $\begin{cases} \dot{x}_m &= A_m x + v_x \\ y_m &= C x_m \end{cases}$

$$O = \begin{bmatrix} C \\ CA_m \\ CA_m^2 \\ \vdots \\ CA_m^{s-1} \end{bmatrix} X_0$$

$$= \Gamma X_0$$

$$\hat{\Gamma} = US^{1/2} \; \hat{X}_0 = S^{1/2} V^*$$

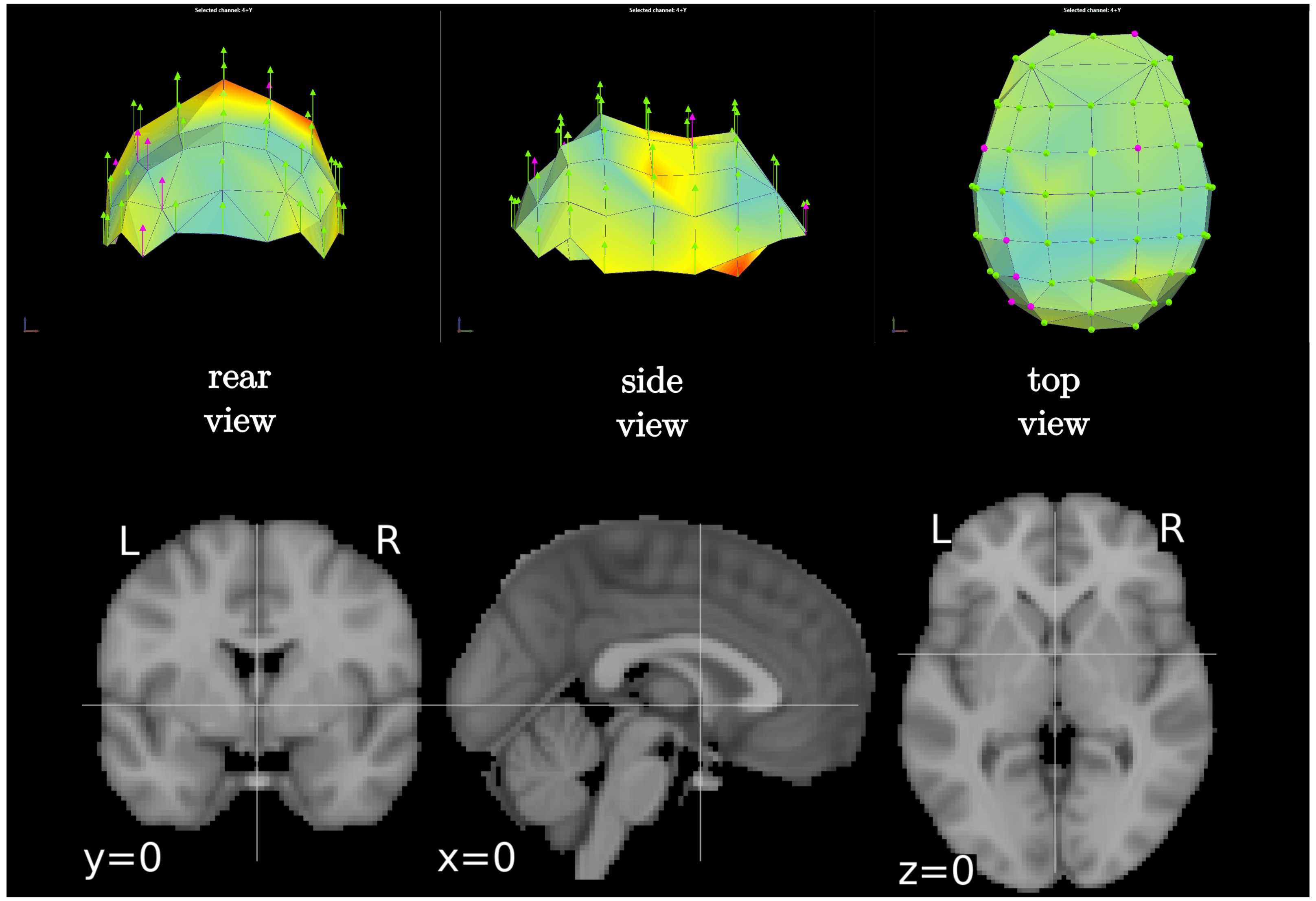
Observability is important!



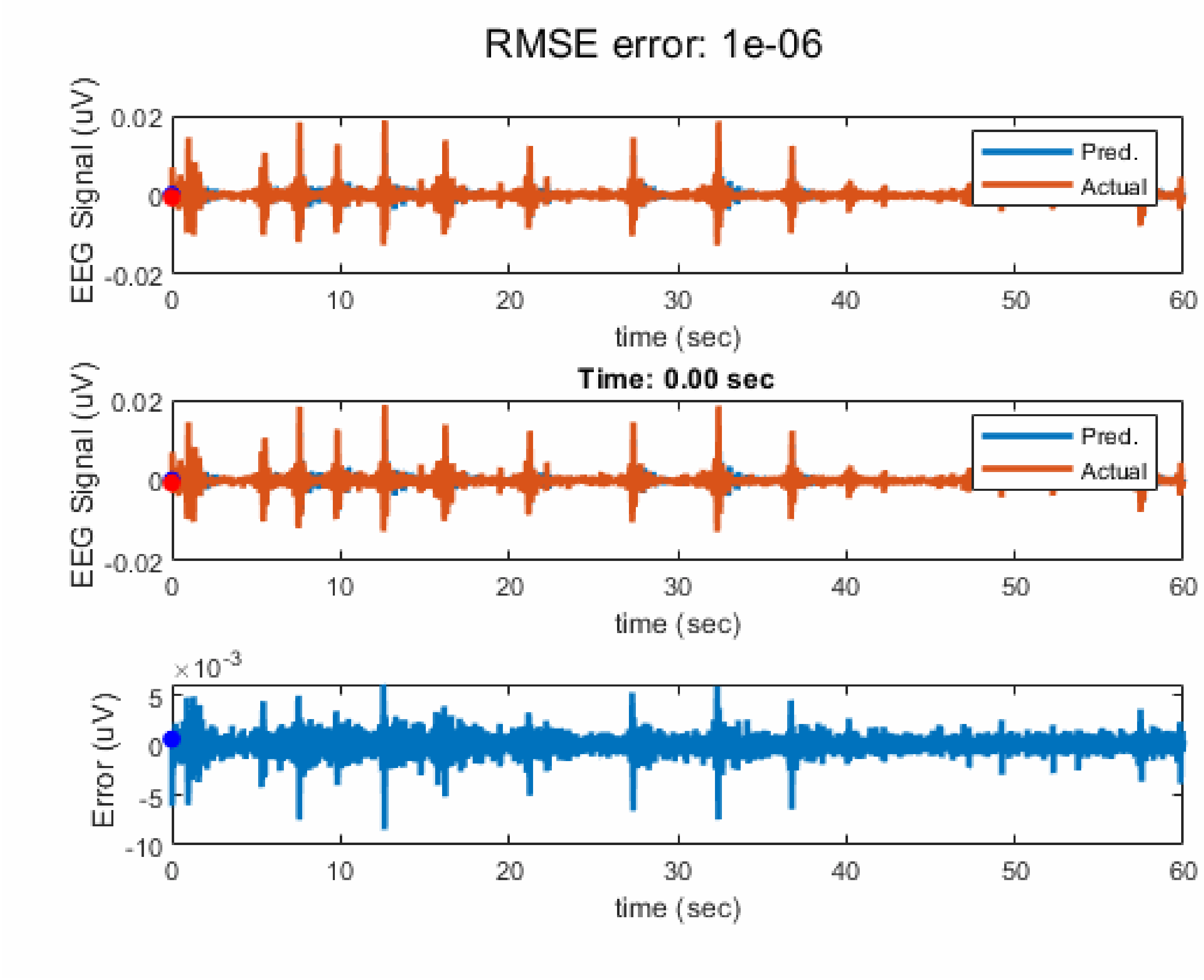
The singular values (i.e. importance) of each mode rolls off after 50 modes.

Modal superposition recreates the measured data

Example from Mt. Sinai CUD database



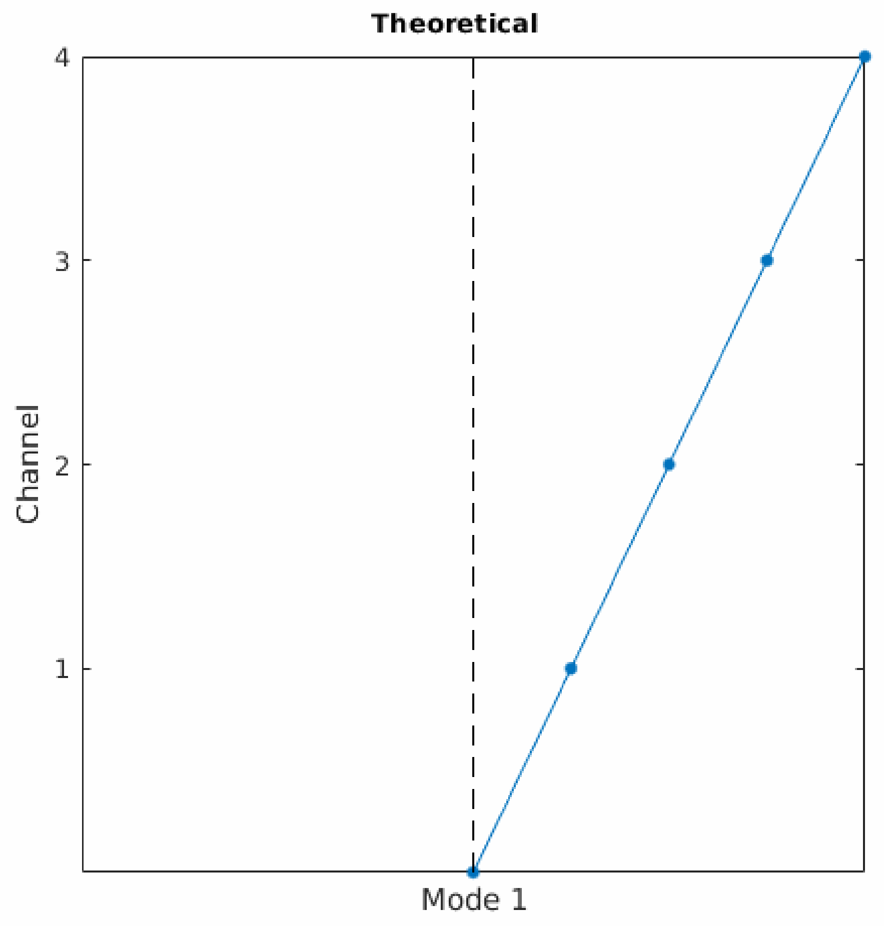
Example mode from Mt. Sinai data: ($f = 23$ hz, $\zeta = 0.12$, $C_r = 22\%$)



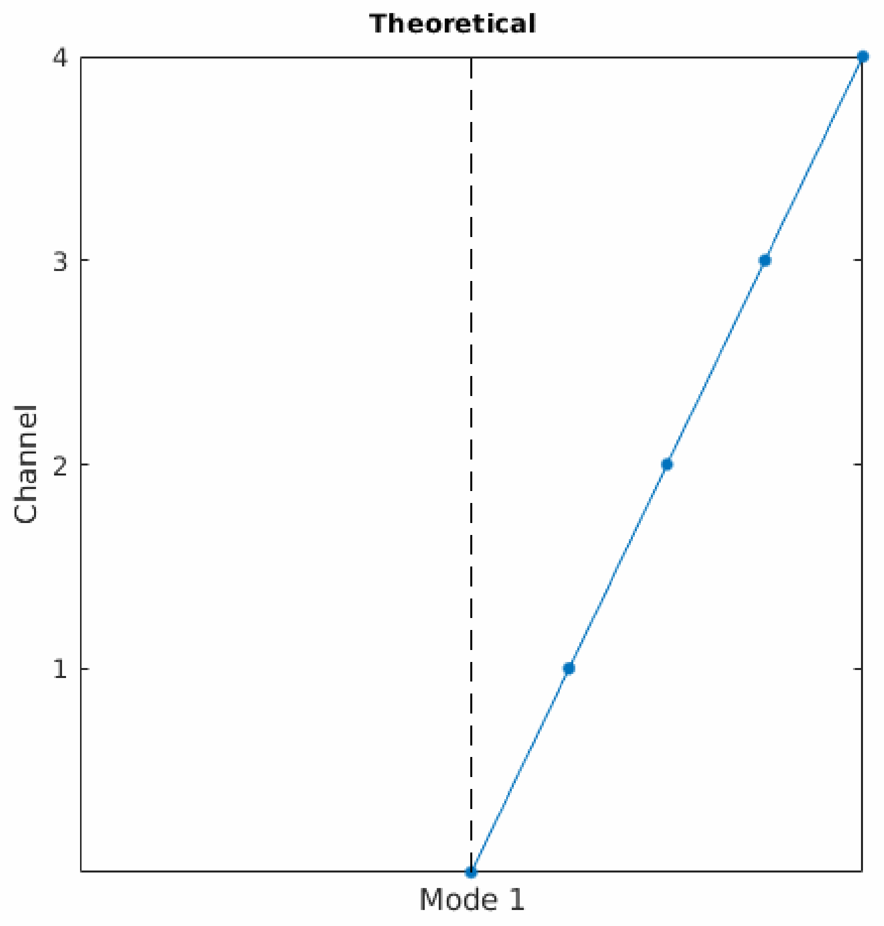
A single channel example of how modes superpose to recreate the observed EEG data.

4. Modal Analysis of Brain Wave Dynamics

Modal complexity and traveling waves

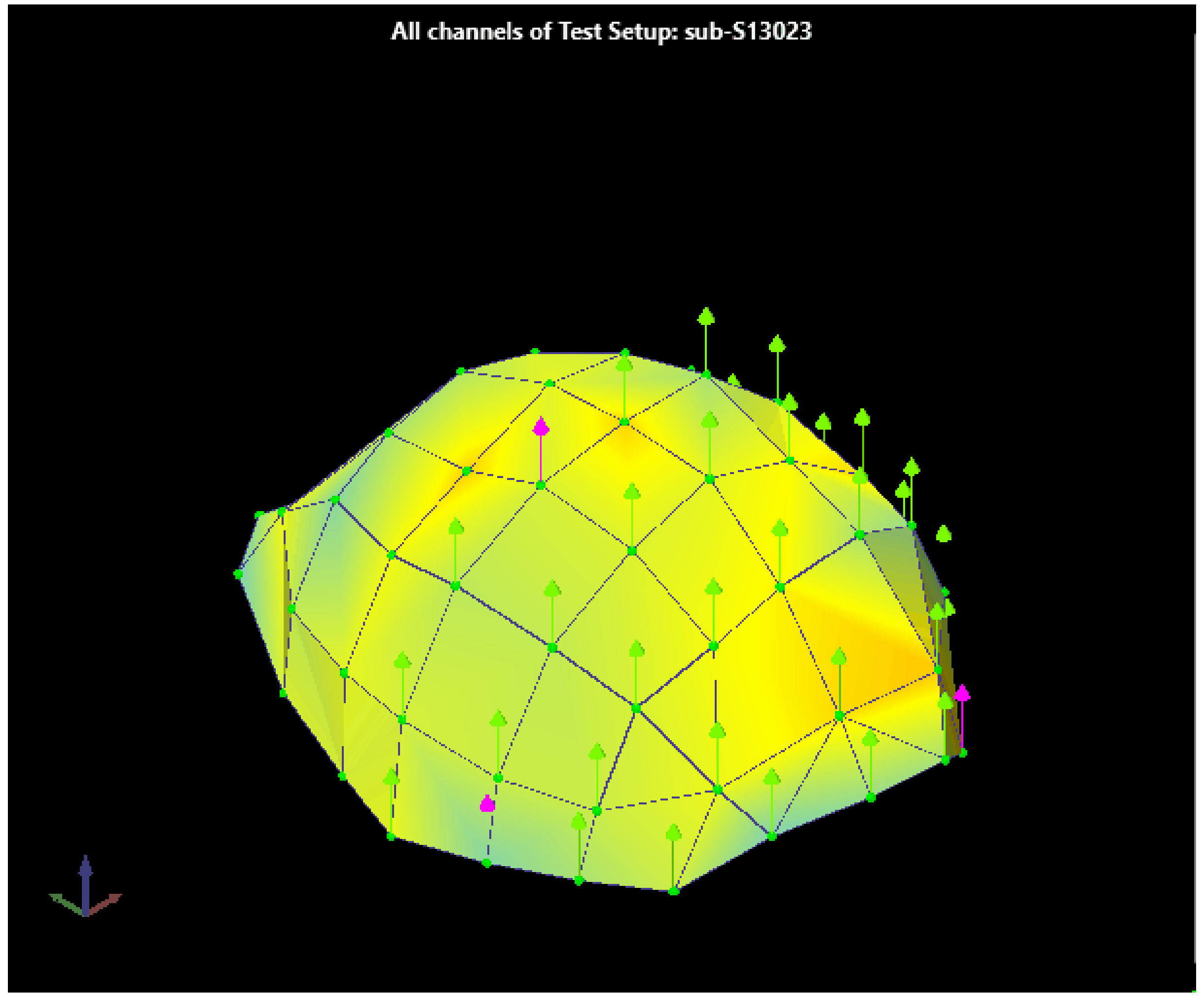


Theoretical standing wave mode shape $C_r = 0\%$

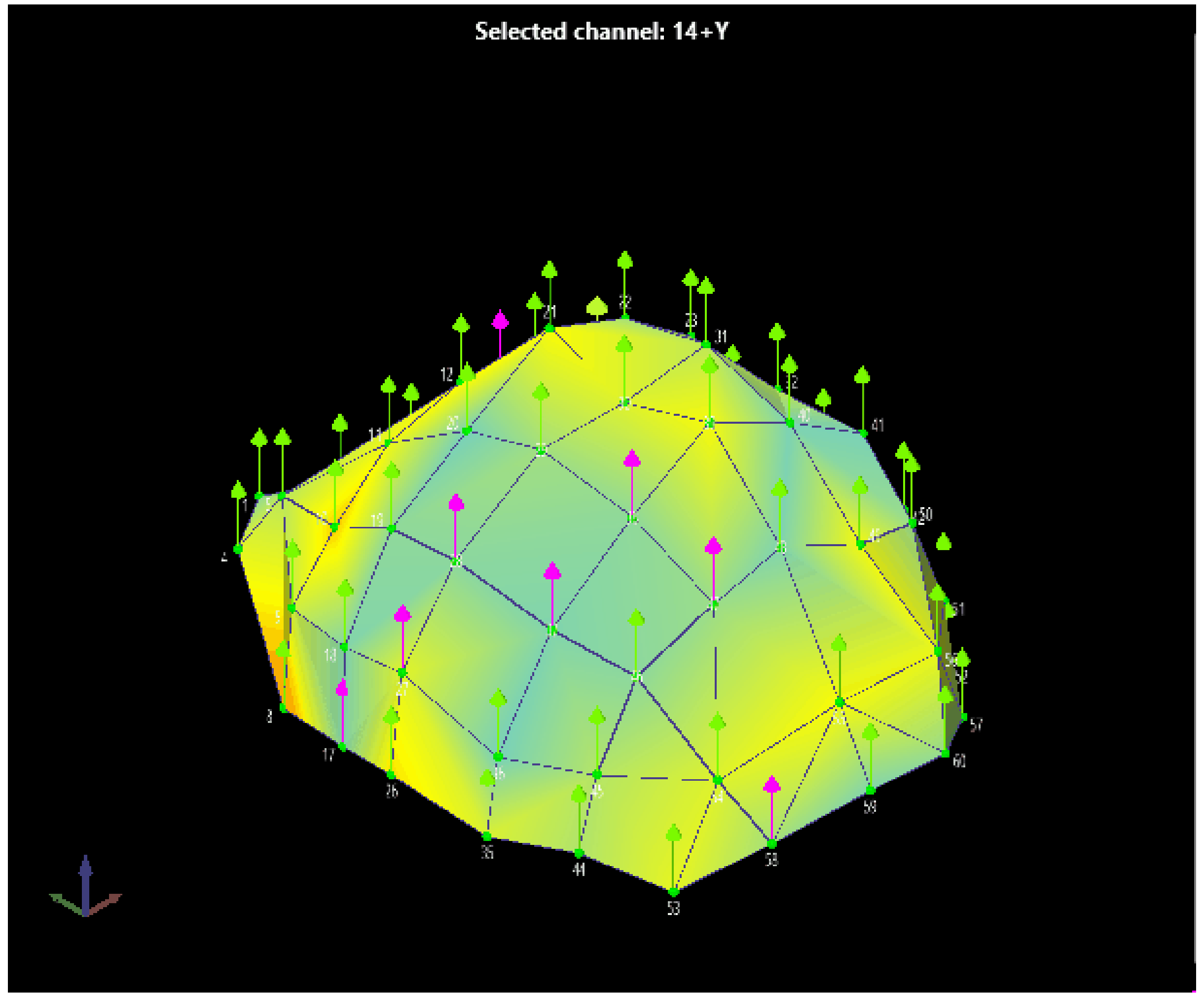


Theoretical standing wave mode shape $C_r = 15\%$

Brain wave modes can be standing or traveling

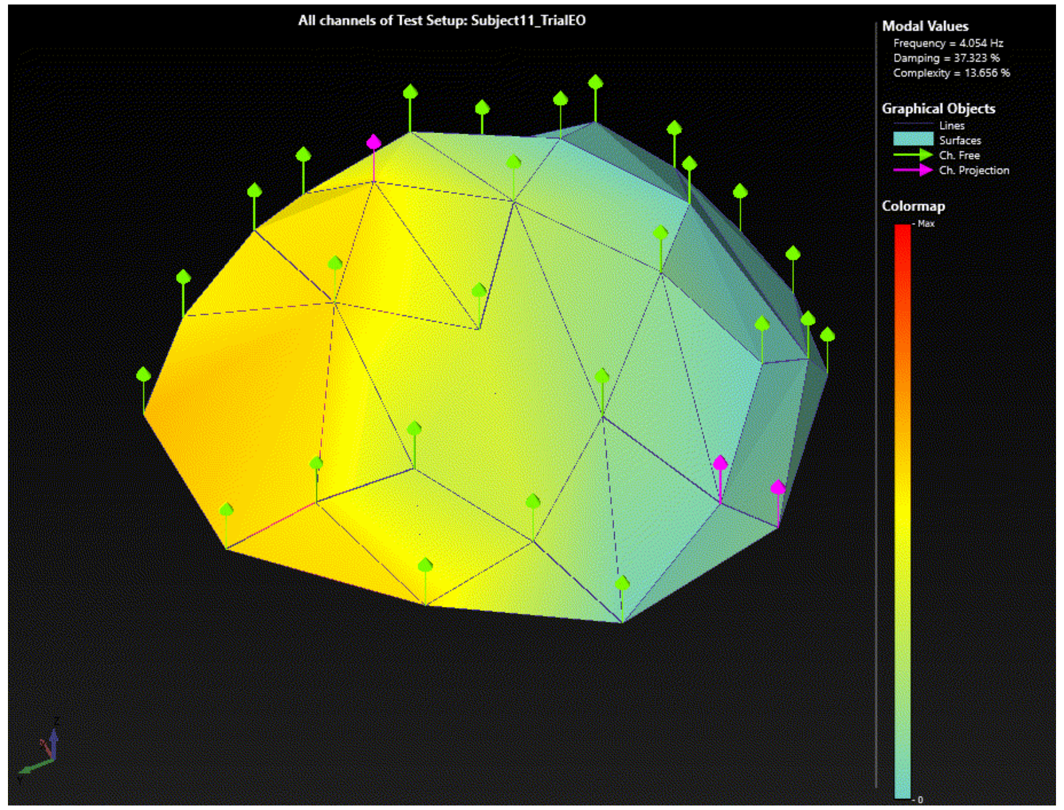


An example standing wave ($C_r = 5\%$) from the Mt. Sinai database.
Standing waves are most prevalent in rest conditions.

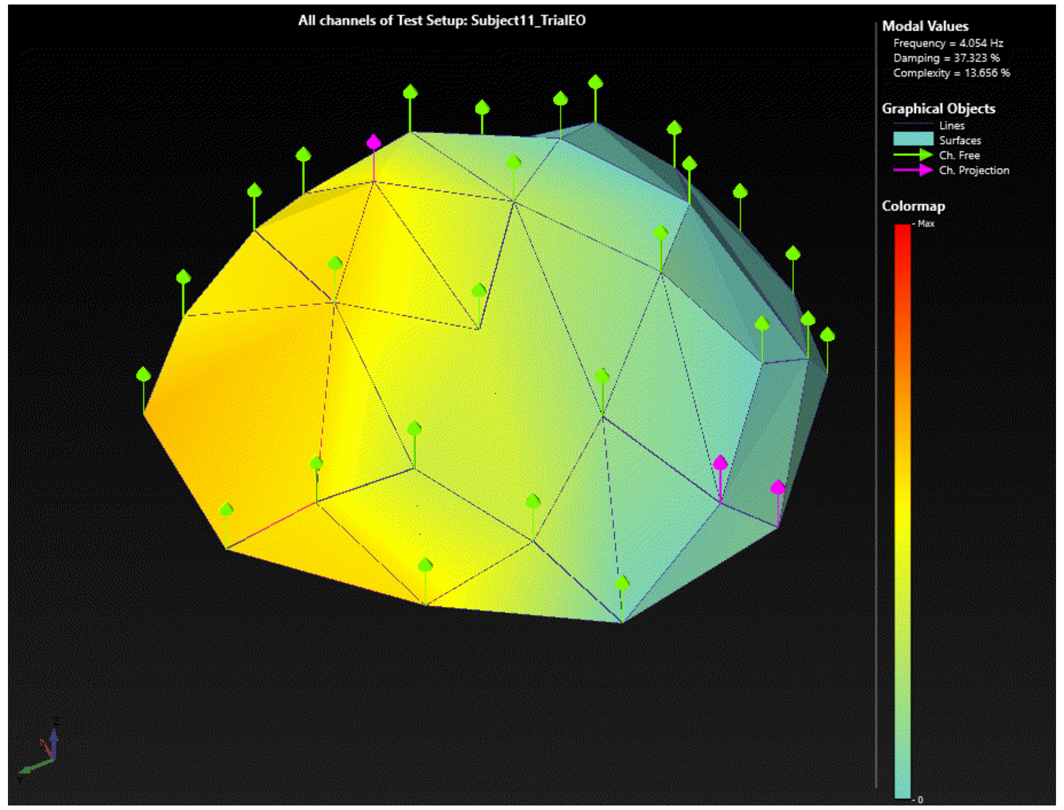


An example traveling wave ($C_r = 83\%$) from the Mt. Sinai database.
Traveling waves are most prevalent in active conditions.

Humans share certain modes



Alpha Mode 1 from Subject 19, **Trial 6** in the DEAP database.

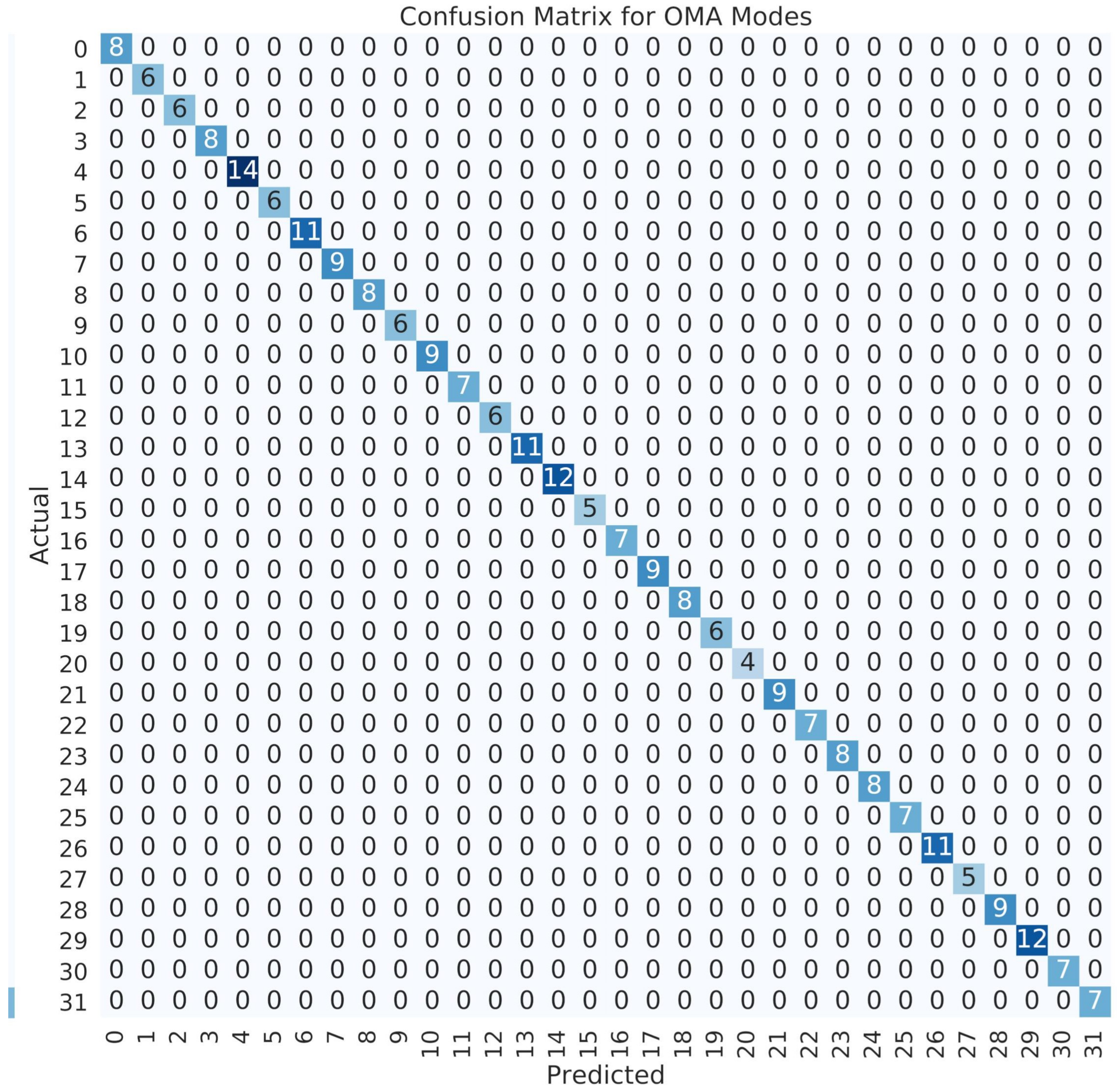


Alpha Mode 1 from Subject 19, **Trial 20** in the DEAP database.

Common mode frequencies are aligned with the Rest State Network

- *Alpha Mode 1:* 4.34 ± 0.03 *hz*
- *Beta Mode 2:* 21.83 ± 0.22 *hz*
- *Gamma Mode 3:* 40.39 ± 0.26 *hz*
- *Gamma Mode 4:* 44.19 ± 0.24 *hz*

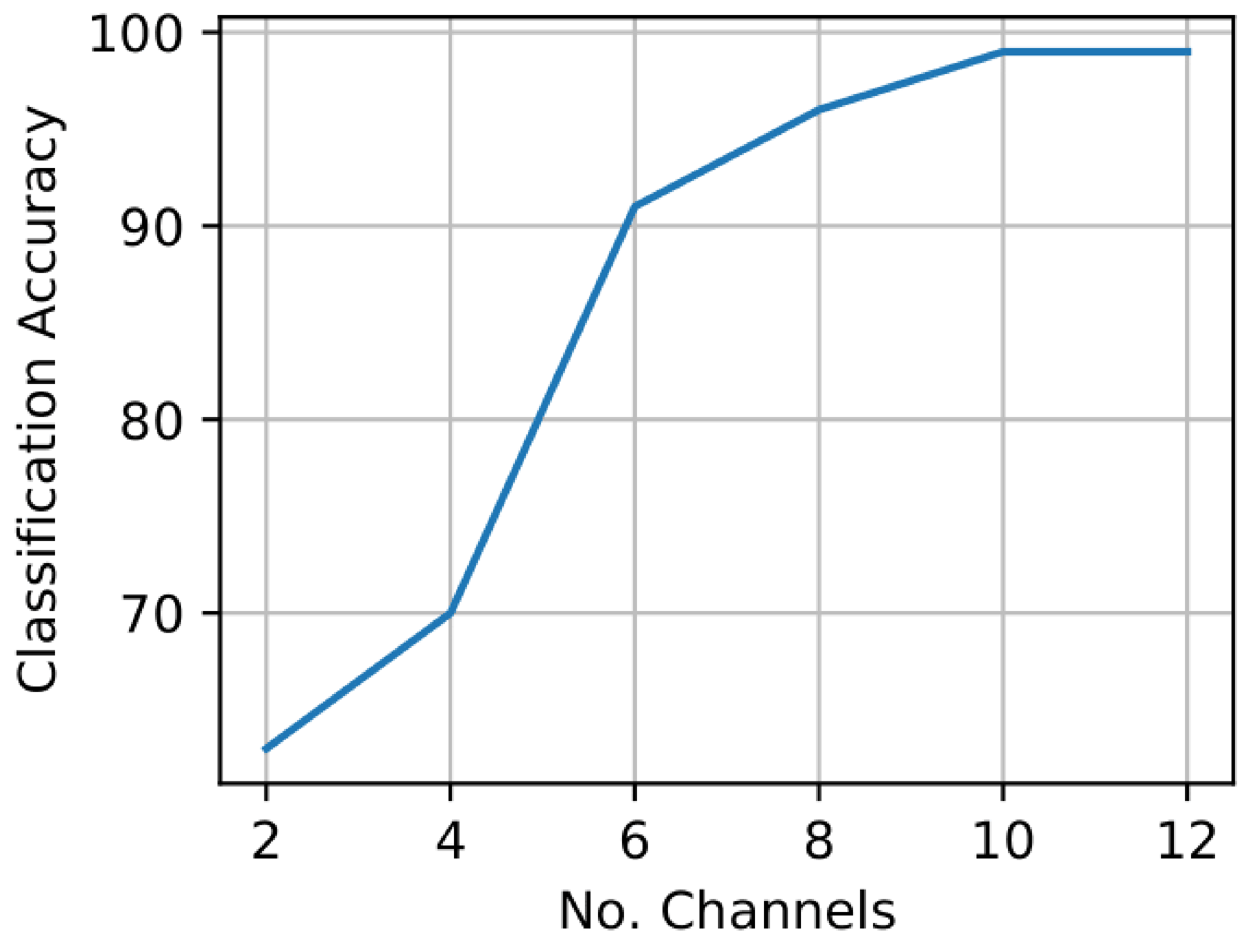
These brain wave modes can be used to fingerprint or identify individual subjects



The subject identification confusion matrix for brain wave modes in the DEAP database. The algorithm can view a set of modes and identify the subject they came from.

Reference	Accuracy [%]
This work	99.85
Wilaiprasitporn et al.	99.90
DelPozo-Banos et al.	97.97

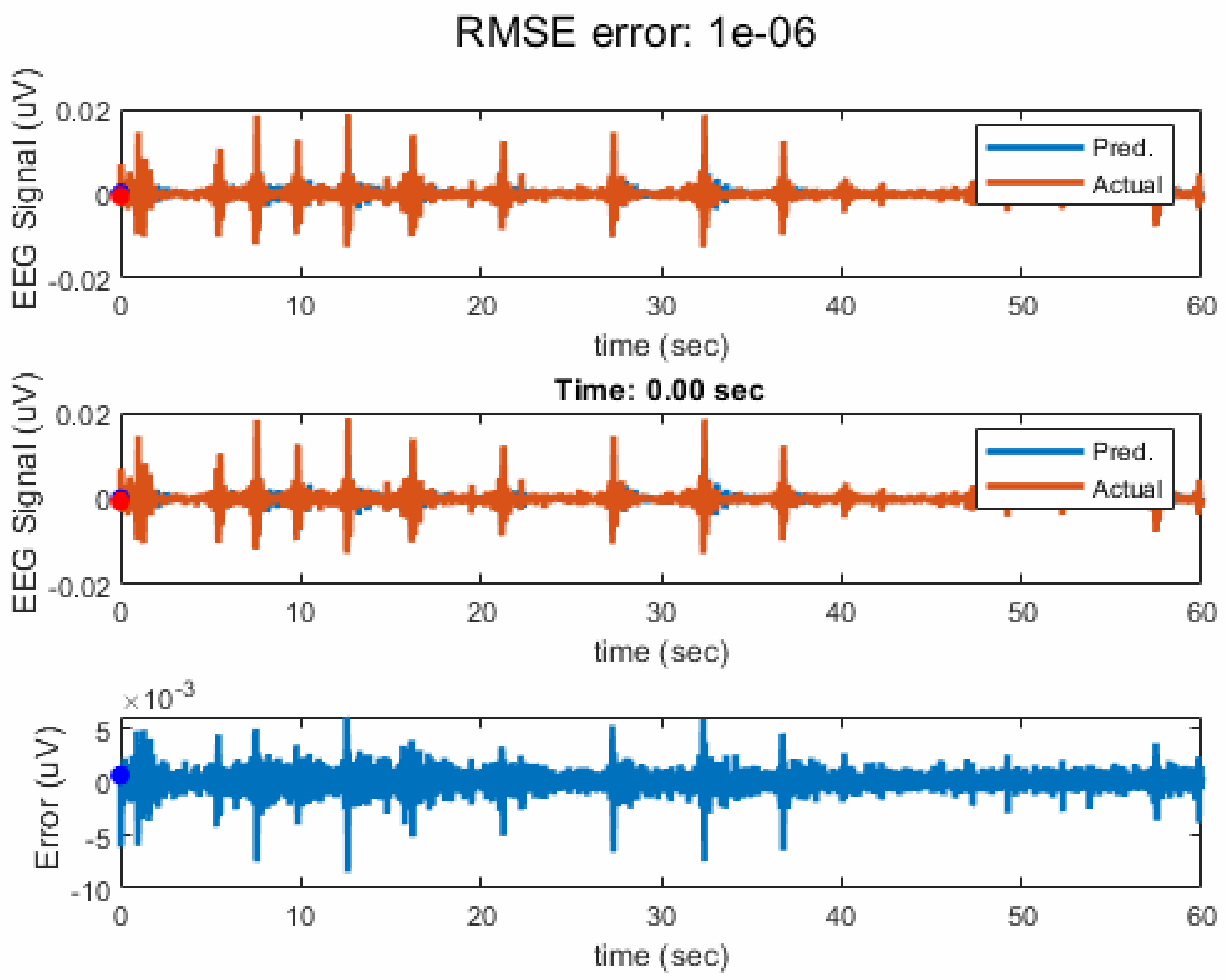
Not all EEG channels are needed for subject identification



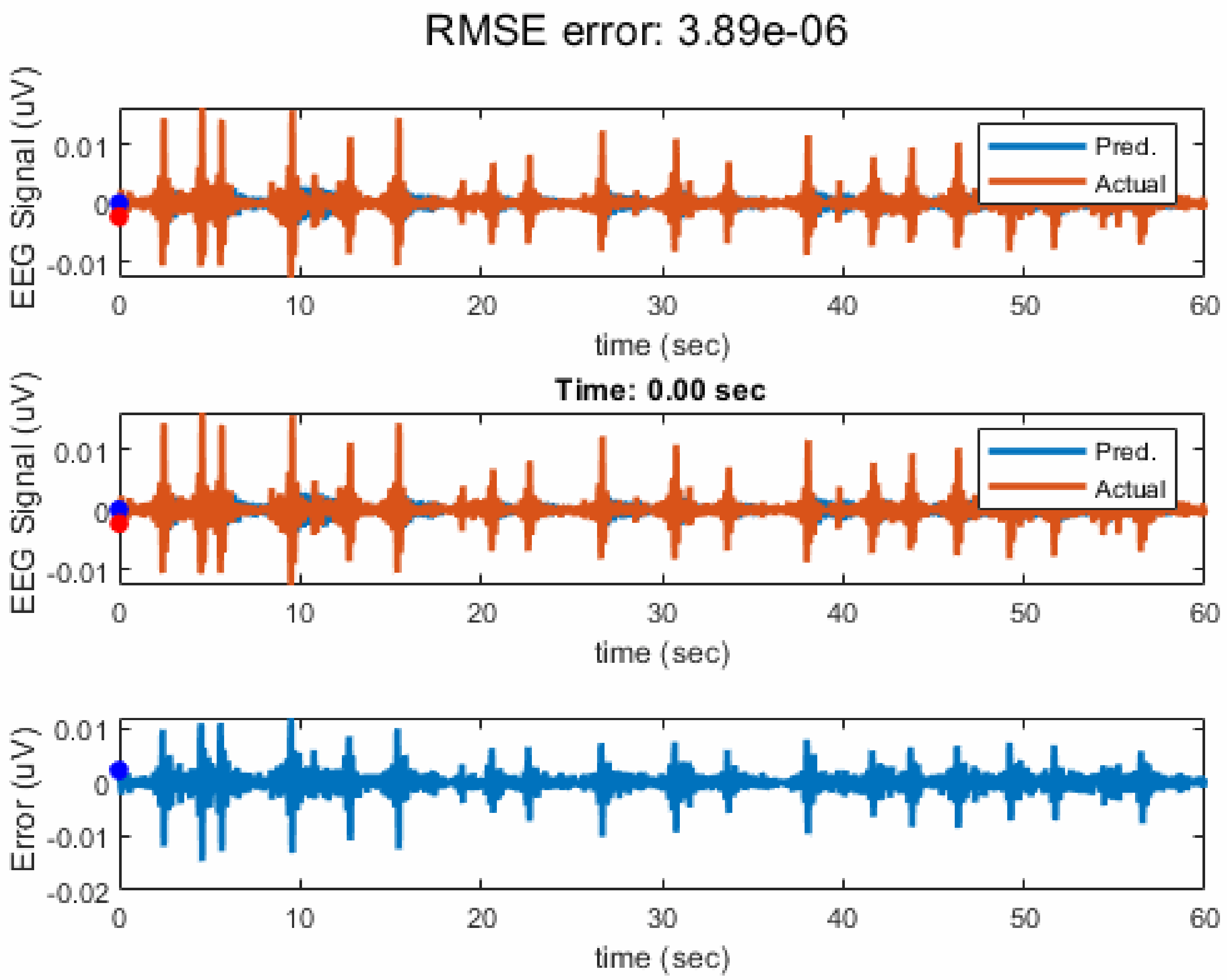
Subject identification accuracy vs. the number of channels in the EEG recording.

Reference	No. Channels	Accuracy [%]
This work	8	96.45
Wilaiprasitporn et al.	5	99.1

Brain wave modes poorly match nonlinear dynamics



Superposed modes recreate the data they came from.



Superposed modes do not match unseen data well. An adaptive update is needed.

5. Adaptive Unknown Input Estimators

Adaptive Unknown Input Estimators

Estimator overview

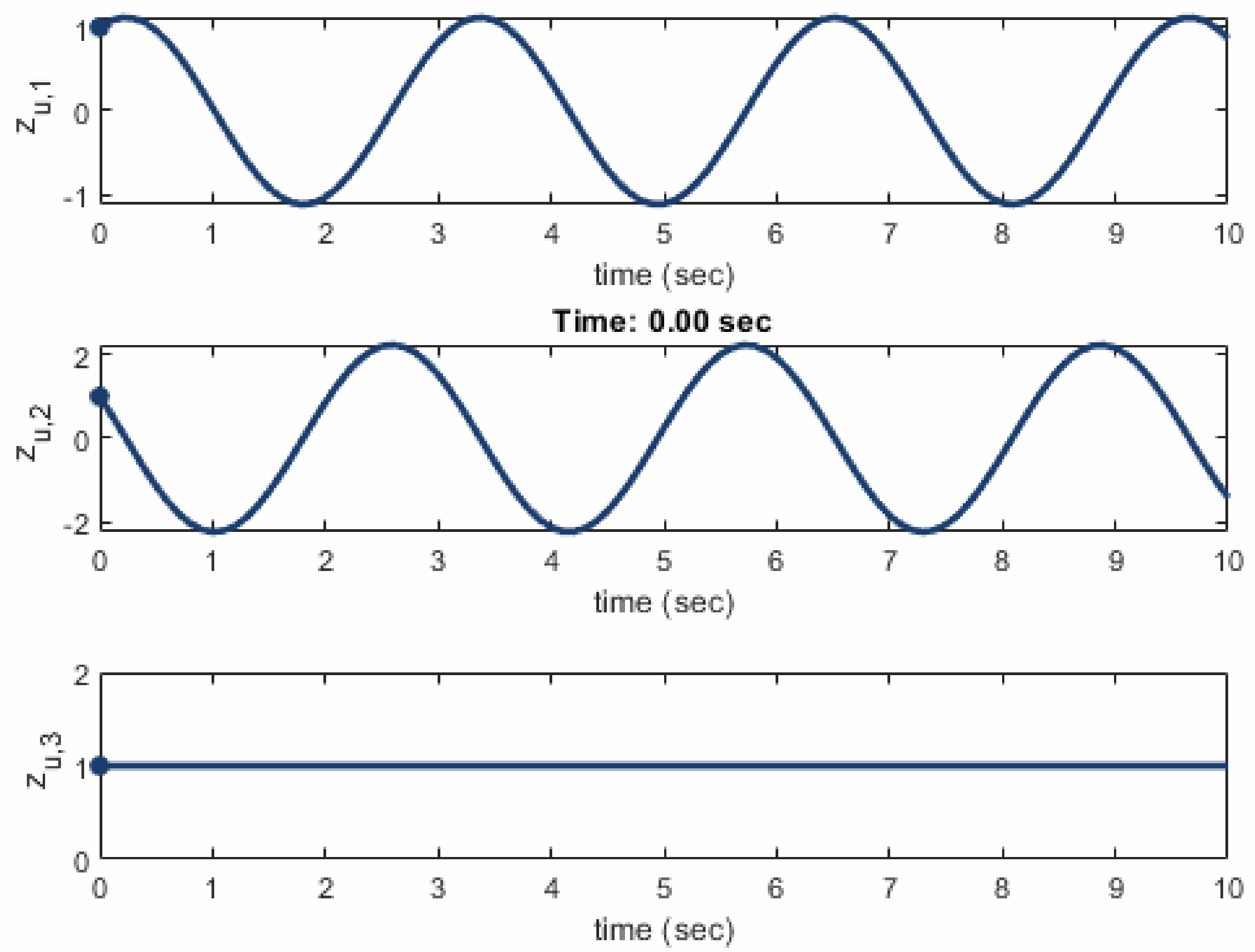
- Three significant uncertainties
  - Input u is unknown, external, deterministic
  - State matrix A may have uncertainty
  - General process uncertainty v_x
- **Can we synthesize u and correct A?**

$$\dot{x} = Ax + Bu + v_x$$
$$y = Cx$$

Adaptive Unknown Input Estimators

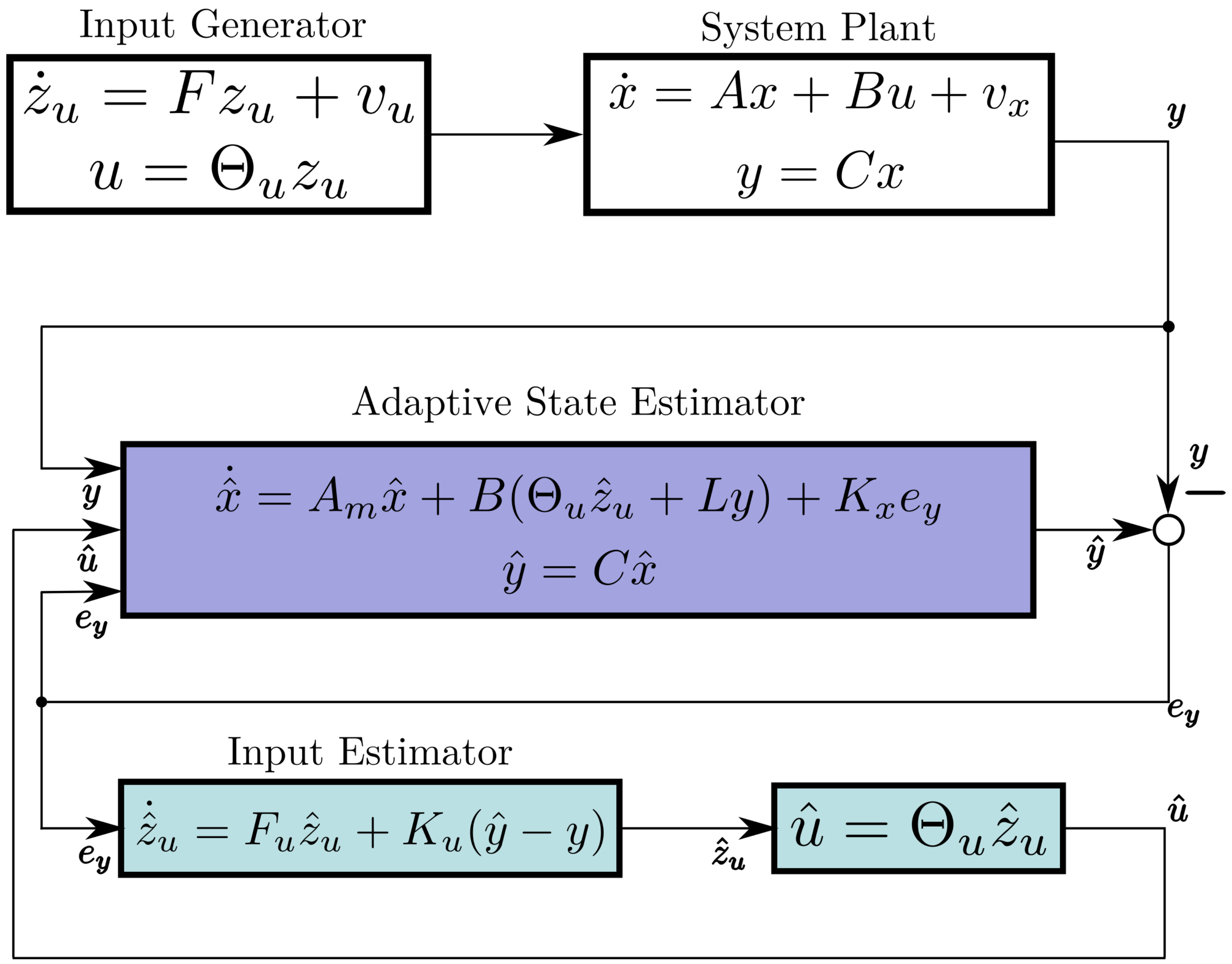
Modeling unknown inputs

- Approximate input space $\mathbb{U}$
  - $\hat{u} = \sum_{i=1}^{N} c_i f_i(t)$
- Persistent Inputs
  - $\dot{z}_u = F_u z_u$
  - $\hat{u} = \Theta_u z_u$
  - $F_u = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$



Adaptive Unknown Input Estimators

Architecture and estimator error



Recover A with adaptive scheme

$$A \equiv A_m + BL_* C$$

$$\dot{L} = -e_y y^* \gamma_e - \alpha L; \ \alpha > 0, \ \gamma_e > 0$$

Error dynamics

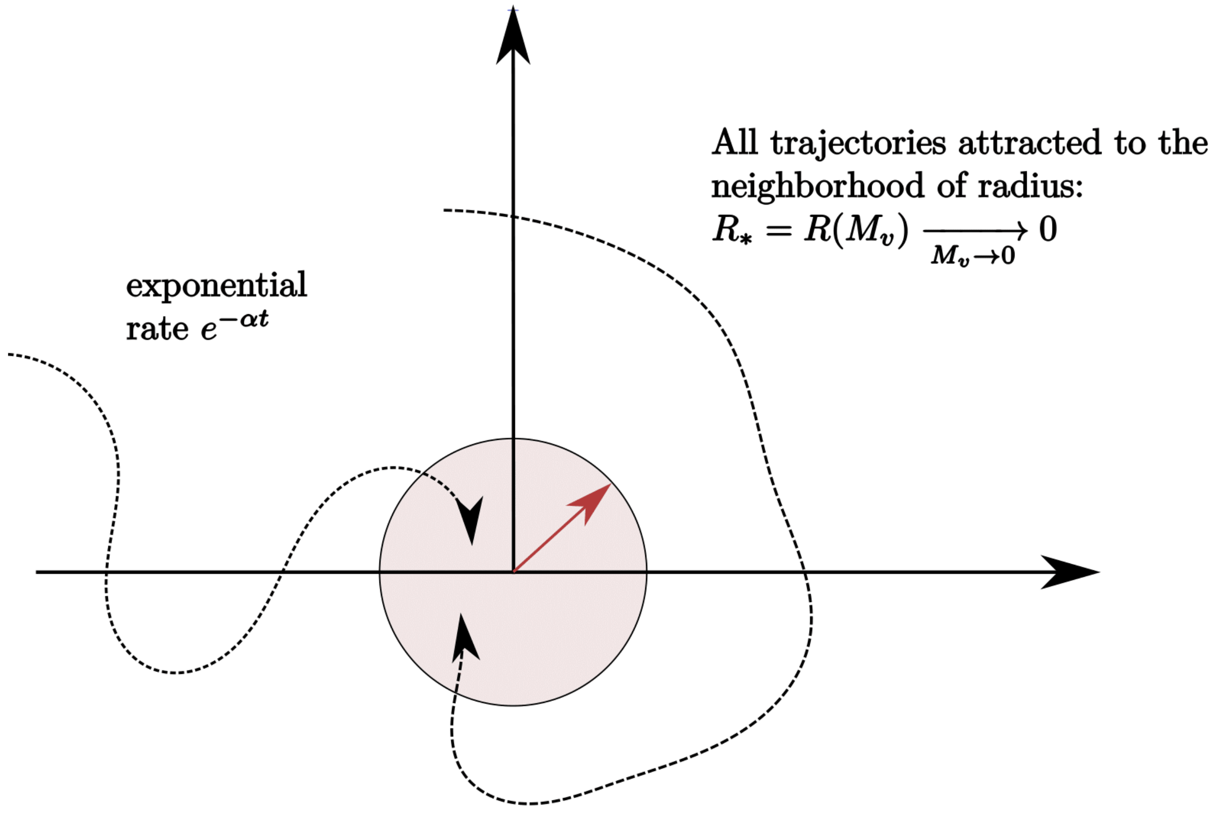
$$\dot{e} = (\bar{A} + \bar{K}\bar{C})e + \bar{B}\underbrace{\Delta L y}_{w} + v$$

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_z \end{bmatrix} = \underbrace{\begin{bmatrix} A_m + K_x C & B\Theta_u \\ K_u C & F_u \end{bmatrix}}_{\bar{A}_c} \begin{bmatrix} e_x \\ e_z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w + \begin{bmatrix} v_x \\ v_u \end{bmatrix}$$

Adaptive Unknown Input Estimators

Architecture and estimator error

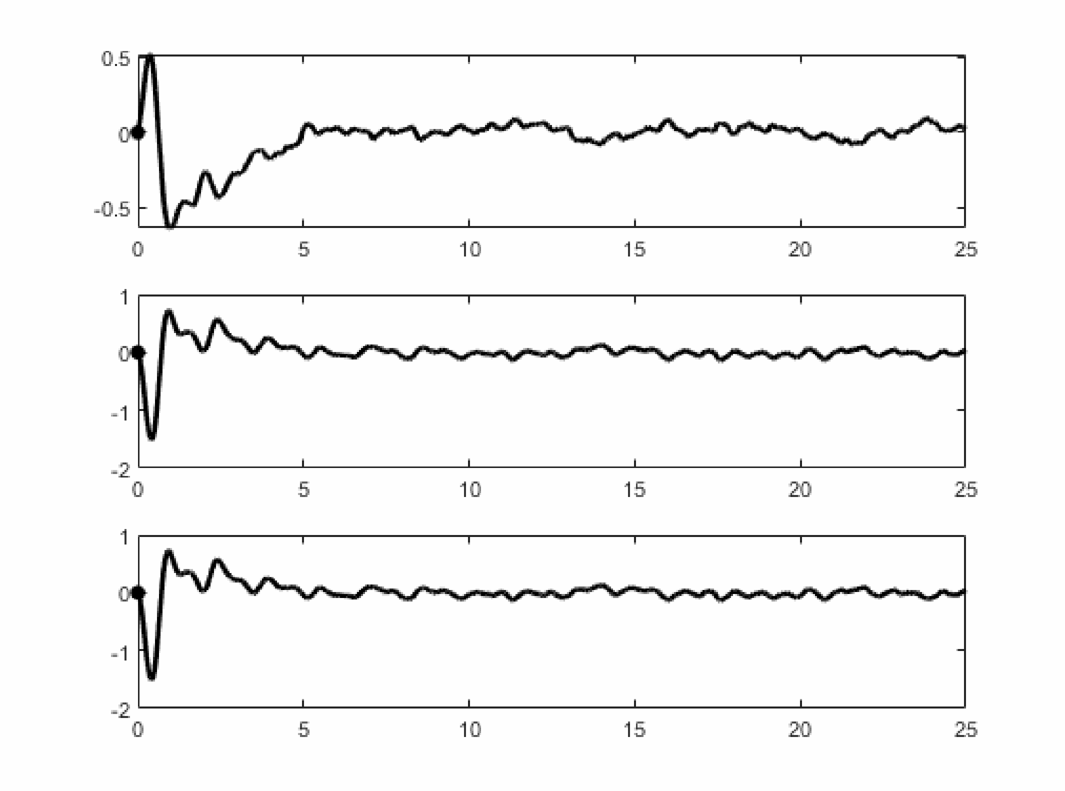
- ASD plant dynamics
  - $A_c^* P + P A_c = -Q$
  - $PB = C^*$
- Bounded L_*, v, and γ_e
- Error in state and input converges to an neighborood centered at zero
  - $V(e, \Delta L) = \frac{1}{2} e^* \bar{P} e + \frac{1}{2} \mathrm{tr}(\Delta L \gamma_e^{-1} \Delta L^*)$
  - $\boldsymbol{\lim_{t\to\infty} \sup ||e(t)|| \leq \frac{1+\sqrt{\lambda_{\max}\bar{P}}}{\alpha\sqrt{\lambda_{\min}\bar{P}}} M_v \equiv R^*}$



Illustrative example

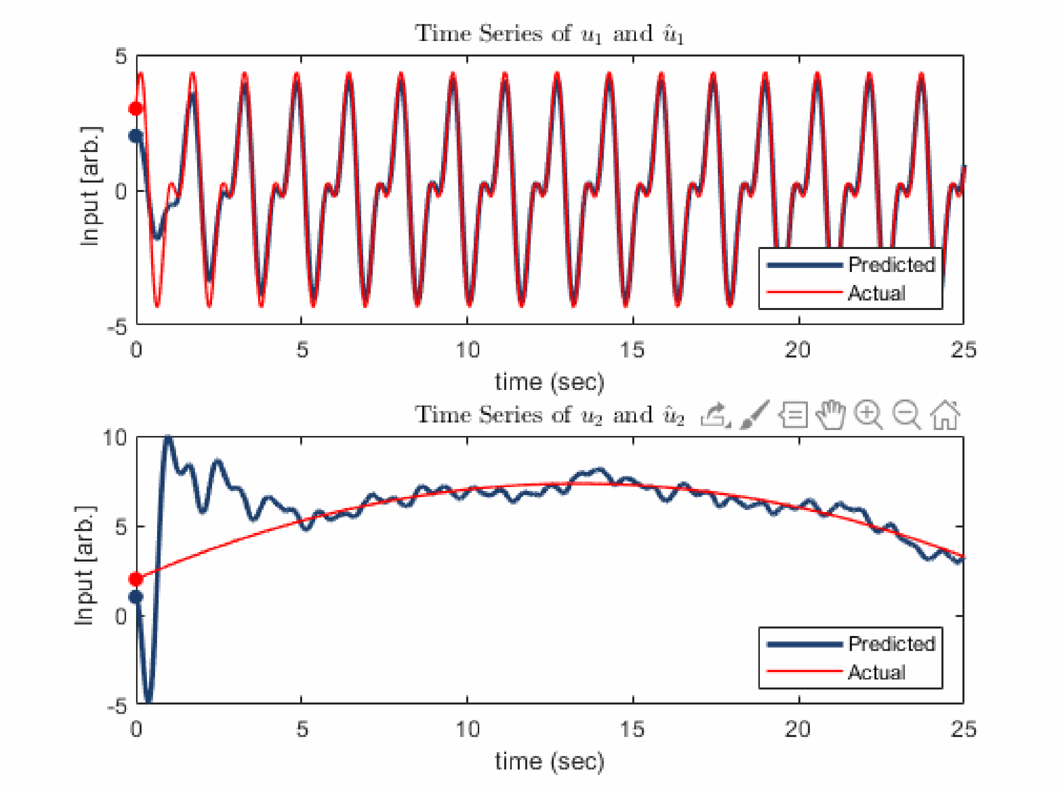
$$
\begin{aligned}
\dot{x} &= A_m x + Bu + v_x \\
&= \begin{bmatrix} -4 & 1 & 2 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} x + Bu + v_x \\
y &= Cx
\end{aligned}
$$

Internal state error time series



$$
\begin{aligned}
\dot{x} &= Ax + Bu + v_x \\
&= \begin{bmatrix} -2.86 & 1 & 4.7 \\ 1.8 & -1 & 6.7 \\ -9 & 1 & -17.2 \end{bmatrix} x + Bu + v_x \\
y &= Cx
\end{aligned}
$$

Estimating the unknown input

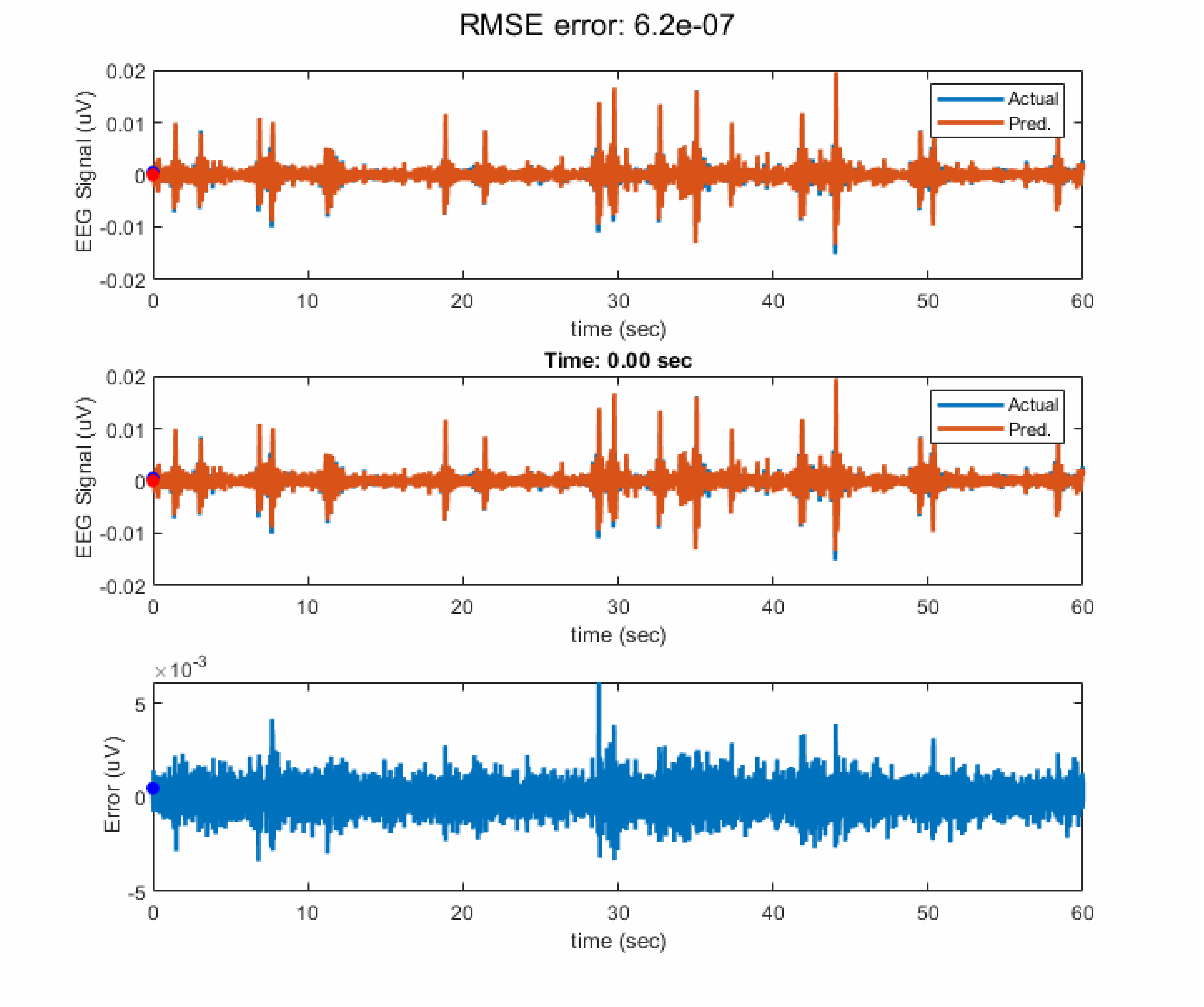


6. Reconstructing the Unknown Input Using EEG

Recall: Solving the nonstationary problem

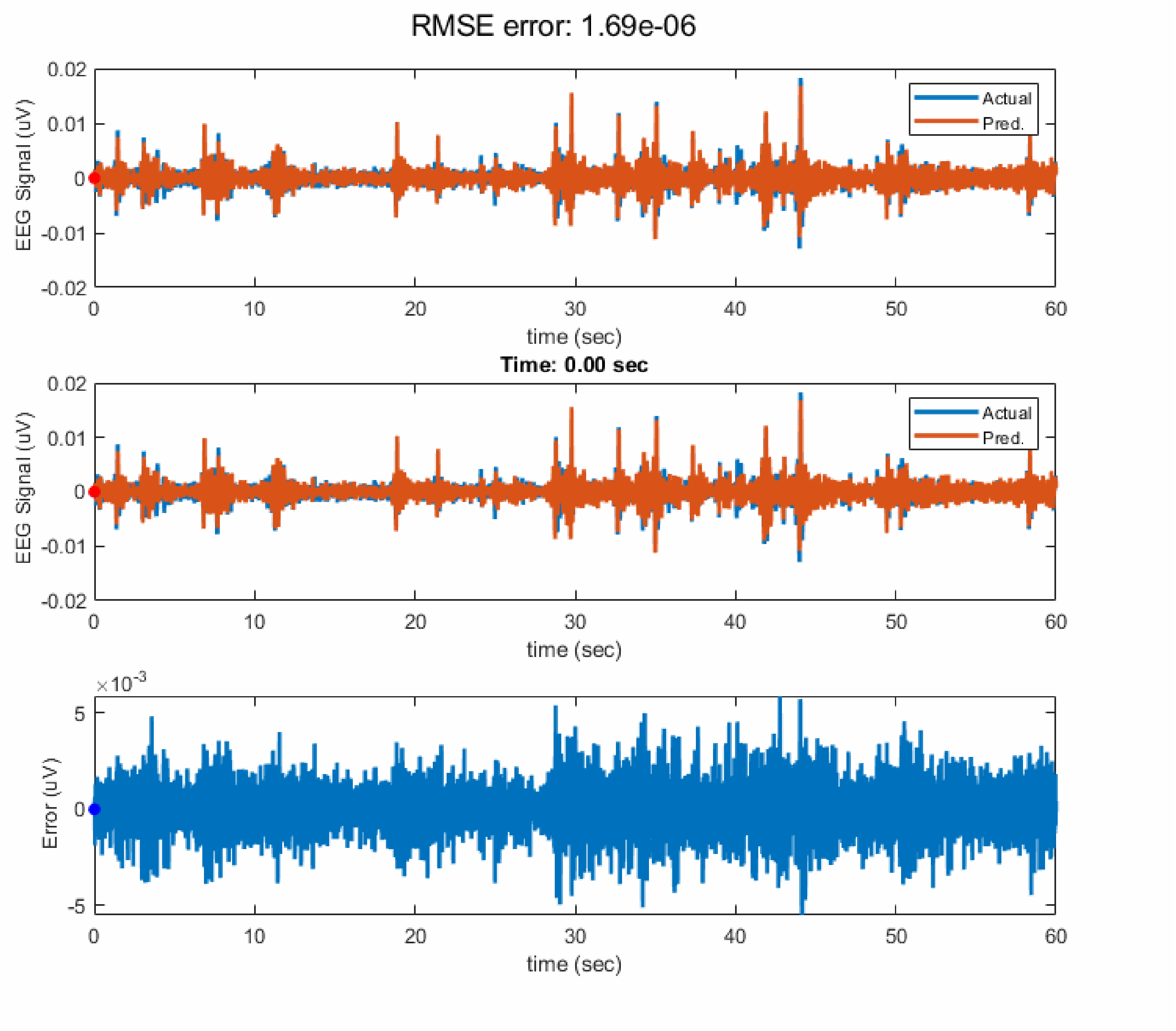
aUIO outperforms static modes

aUIO on unseen data



Adaptive input estimator performance for the unseen data.

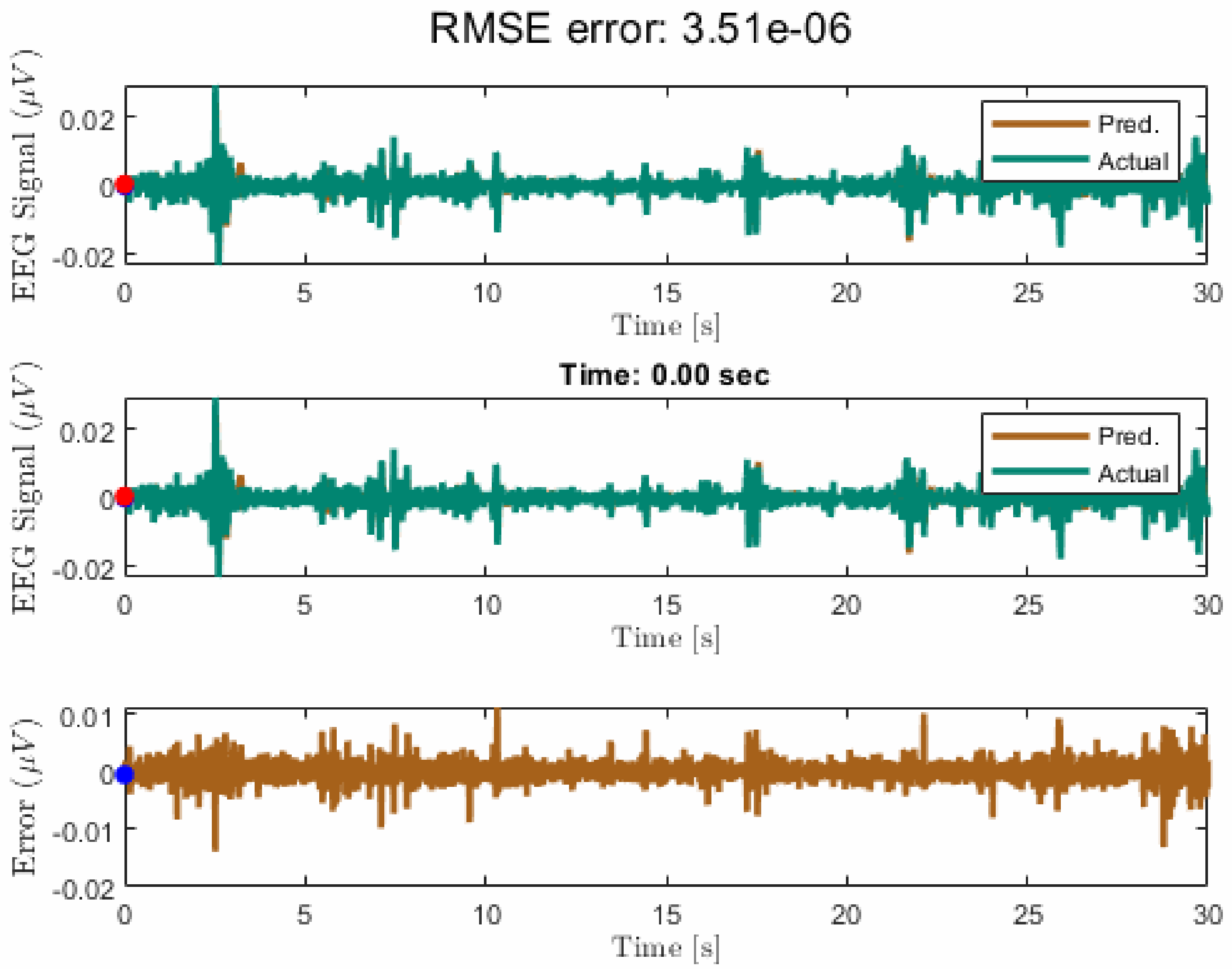
Weighted modes on seen data



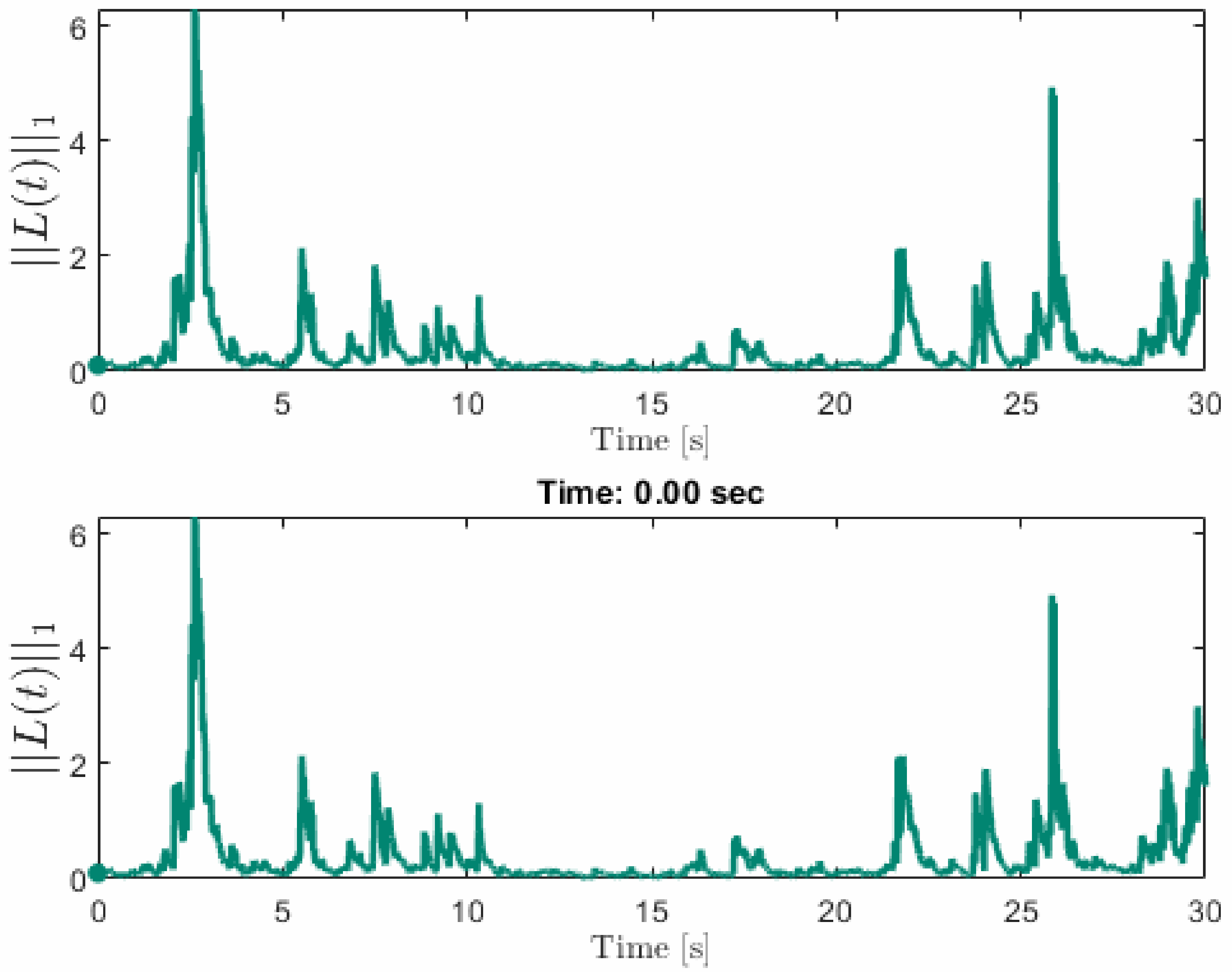
Superposition of modes decomposed from this data.

aUIO critically updates model as needed

aUIO on unseen data



Adaptive gain matrix 1-norm



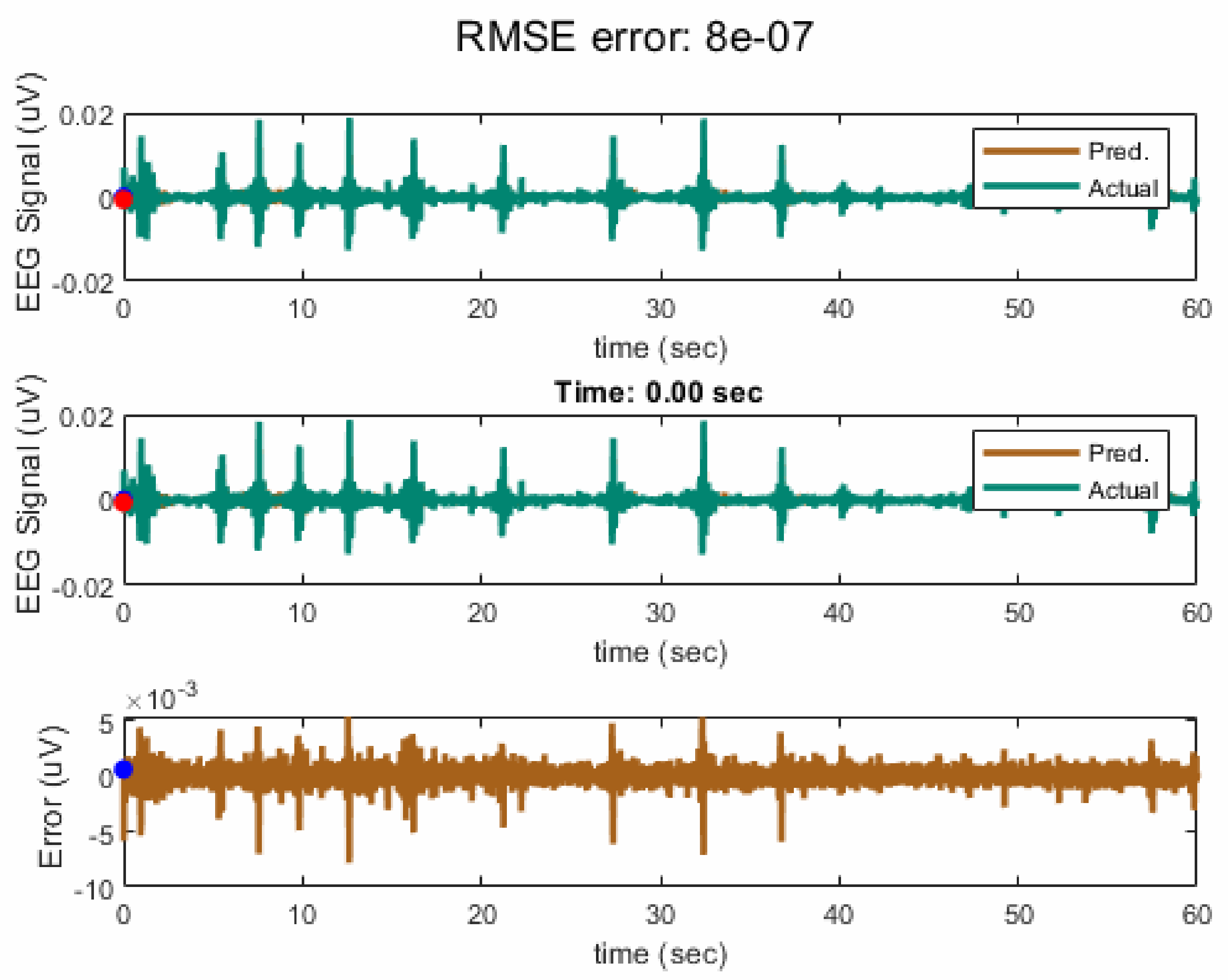
The norm measures “how much” adapting the estimator is doing

Unknown Input Modeling Assumptions

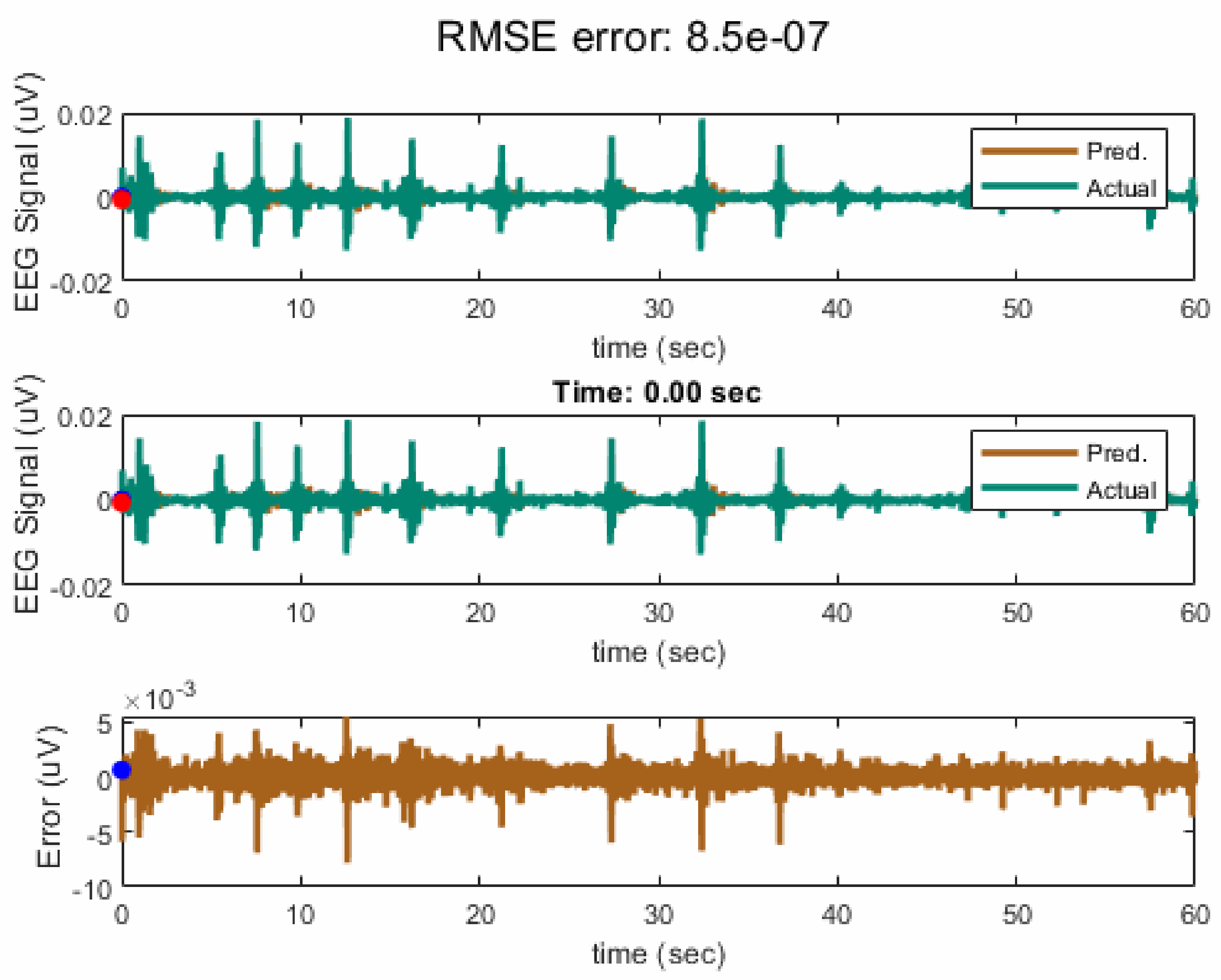
- Unknown input acts evenly over spatial domain
- F_u generates sine-cosine basis
- Static gains per LQR
- **Unknown input is “external information”**

aUIO is tolerant to some parametric uncertainty in the modes

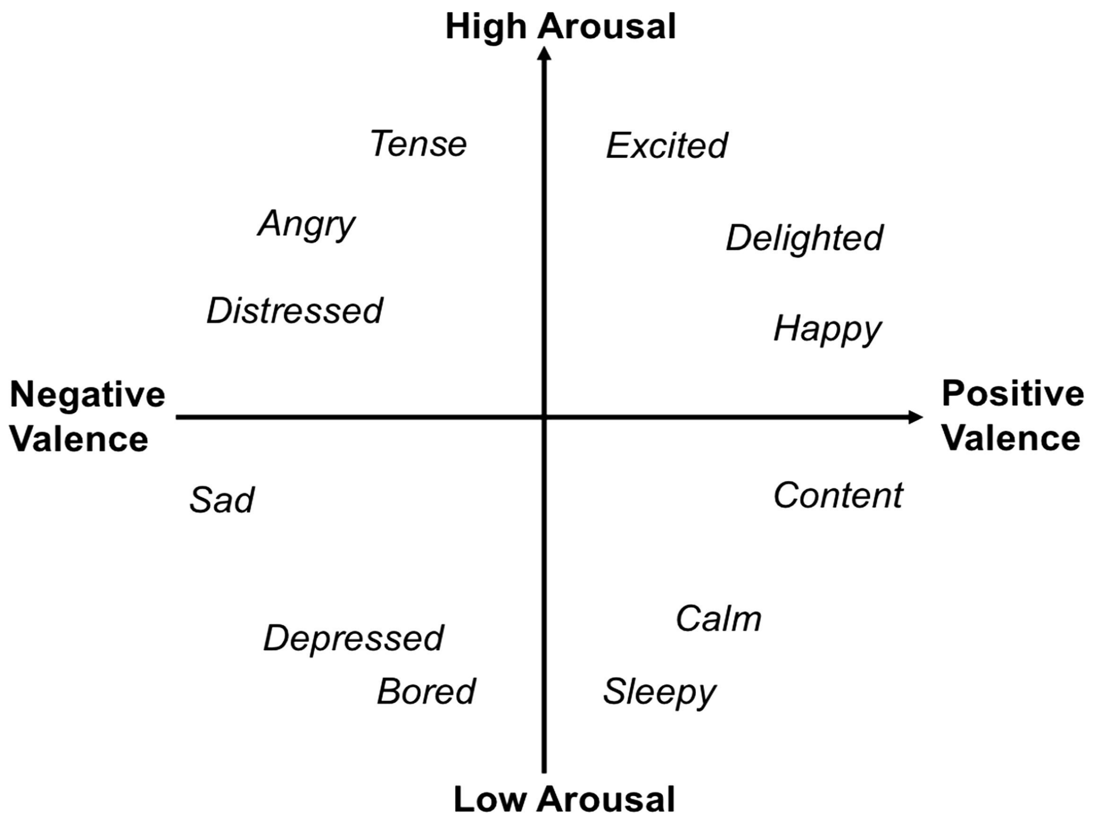
aUIO on unseen data



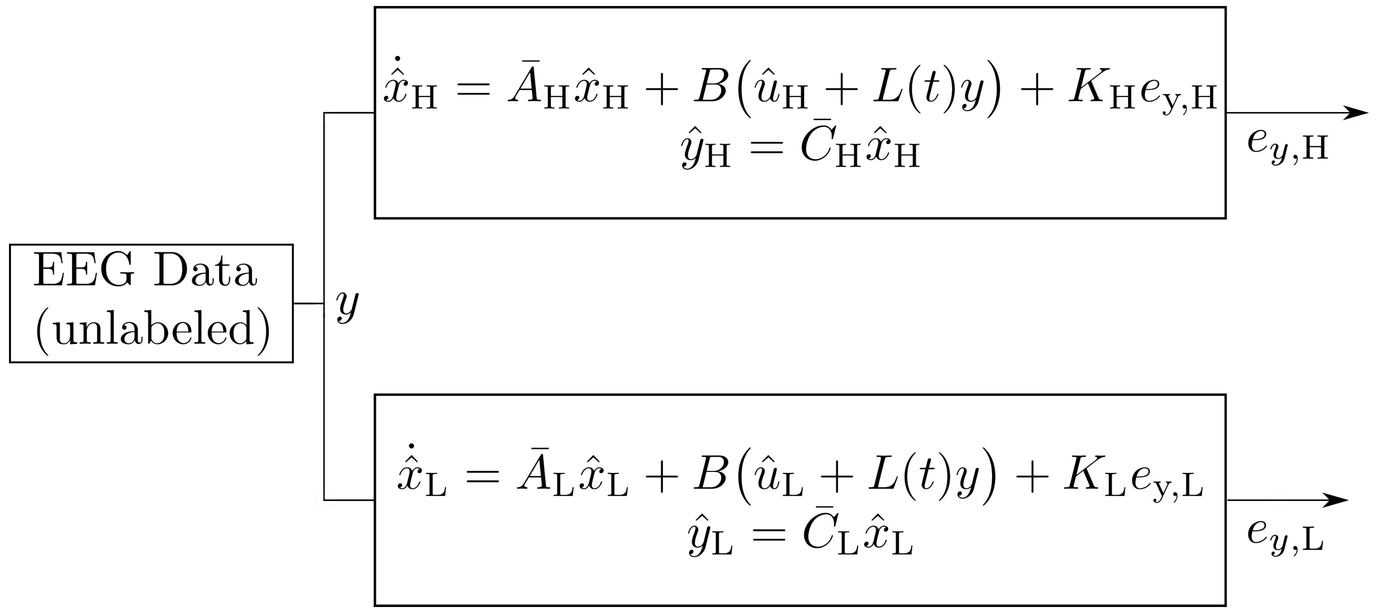
aUIO with modes from another subject



Classification via estimation



- DEAP: Two self reported variables
- F_u generates sine-cosine basis
- Static gains per LQR

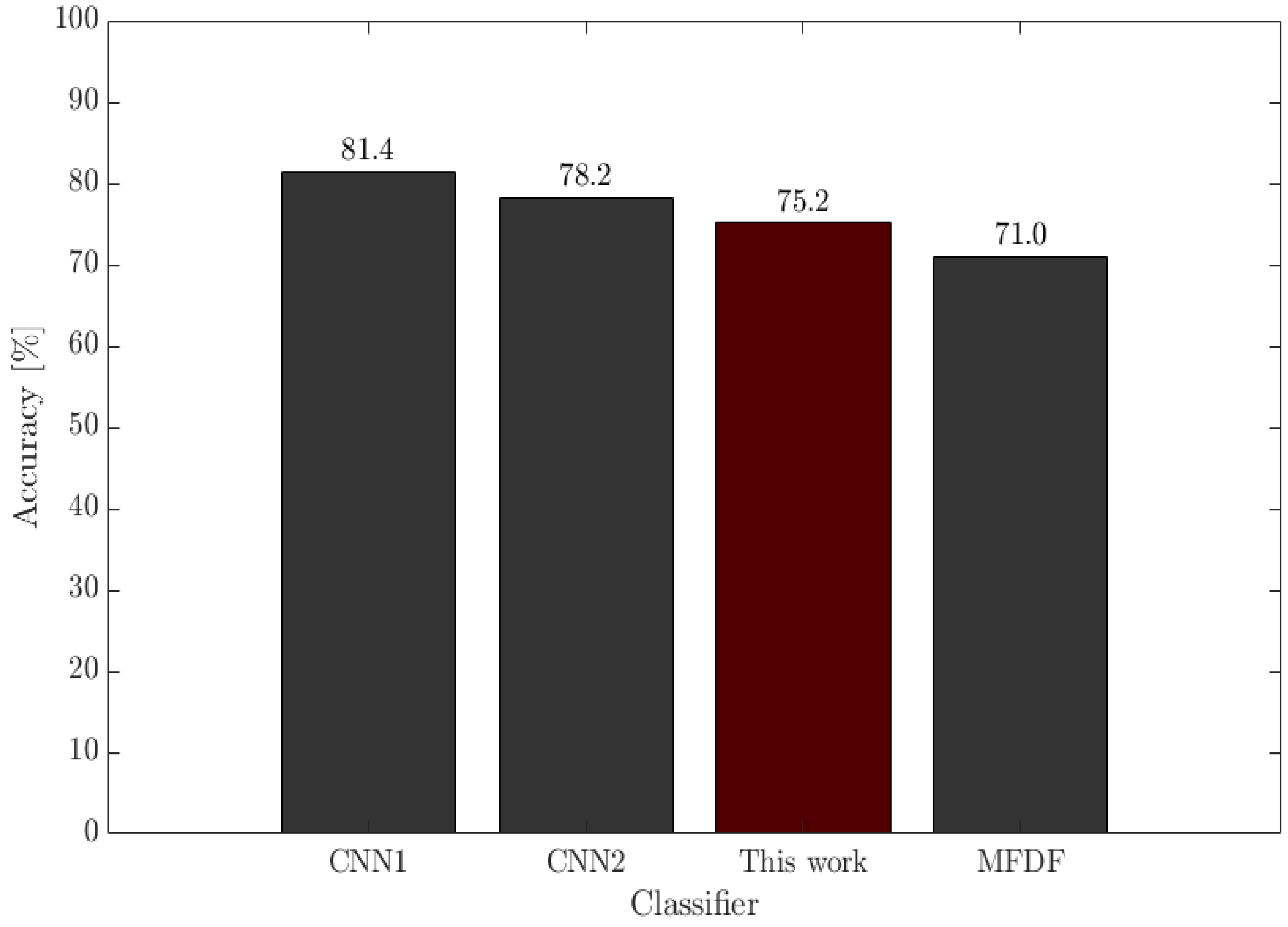


- Hypothesis:
  - modes are correlated with human state/cognition, so
  - same state should have similar modes, so
  - you can take the average modes in a state,
  - and the estimator will perform better than the other averaged model
- **This is a inter-individual approach**

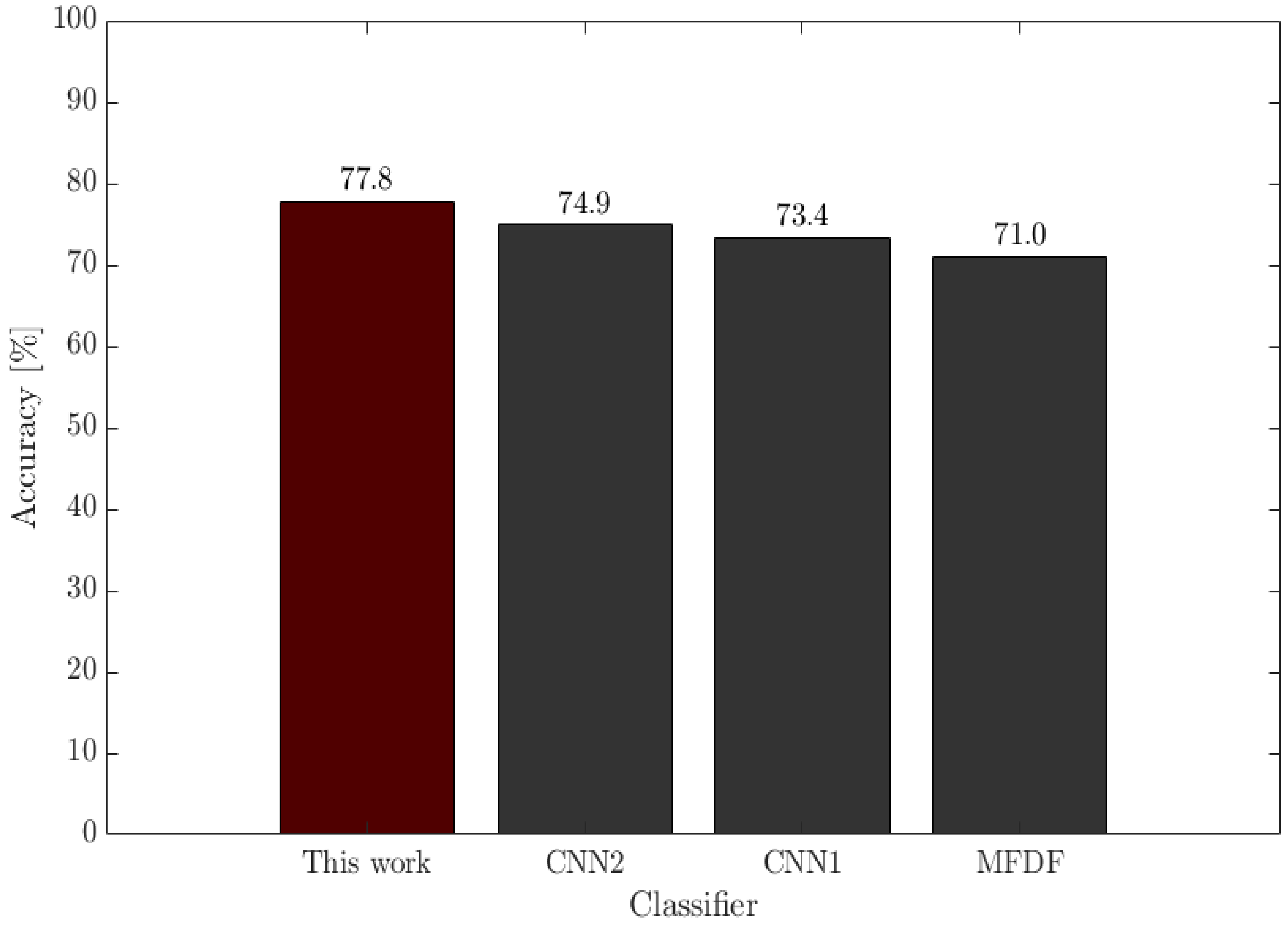
This method is comparable to state of the art deep learning approaches

- Computational input and time is lower
- **Analytical information is greater**
- Accuracy is comparable

Valence Classification



Arousal Classification



CNN1, CNN2, MFDF

7. Significant Contributions

Contributions of this dissertation

- Output only system identification techniques are suitable for linear models of brain wave dynamics via EEG around an operating state
- Real time spatio-temporal brain wave imaging via modal analysis
- A novel brain wave fingerprinting algorithm on par with state of the art deep learning approaches
- A complete body of adaptive, highly nonlinear unknown input estimator work
- Real time brain wave imaging that accounts for nonstationary, nonlinear dynamics by updating the modes in real time
- A novel recreation of the unknown brain wave plant's input
- Valence-arousal emotion classification from the DEAP database on par with cutting edge deep learning approaches

Publications & Presentations

- **T. Griffith**, J.E. Hubbard. System identification methods for dynamic models of brain activity. Biomedical Signal Processing and Control
- **T. Griffith**, M. J. Balas. An Adaptive Control Framework for Unknown Input Estimation. ASME IMECE 2021 Proceedings
- **T. Griffith**, V.P. Gehlot, M. J. Balas. Robust Adaptive Unknown Input Estimation with Uncertain System Realization. AIAA SciTech 2022 Forum
- **T. Griffith**, V.P. Gehlot, M. J. Balas. Adaptive Estimation of Unknown Inputs with Weakly Nonlinear Dynamics. ACC 2022 [Accepted]
- **T. Griffith**, V.P. Gehlot, M. J. Balas. On the Observability of Quantum Dynamical Systems. ASME IMECE 2022 Proceedings [Accepted]
- **T. Griffith**, V.P. Gehlot, M. J. Balas, J.E. Hubbard. An Adaptive Approach to Real Time EEG Estimation. Biomedical Signal Processing and Control [In-Review]
- **T. Griffith**, J.E. Hubbard. System Identification of Brain Wave Modes Using EEG. Journal of Neural Engineering [In-Revision]

Future work

- Multiple data types
- Improved analysis and classification
- Probablistic considerations

A Modal Approach to the Space Time Dynamics of Cognitive Biomarkers

The willow submits to the wind and prospers until one day it is many willows – a wall against the wind.

- Dune