A Modal Approach to the Space Time Dynamics of Cognitive Biomarkers

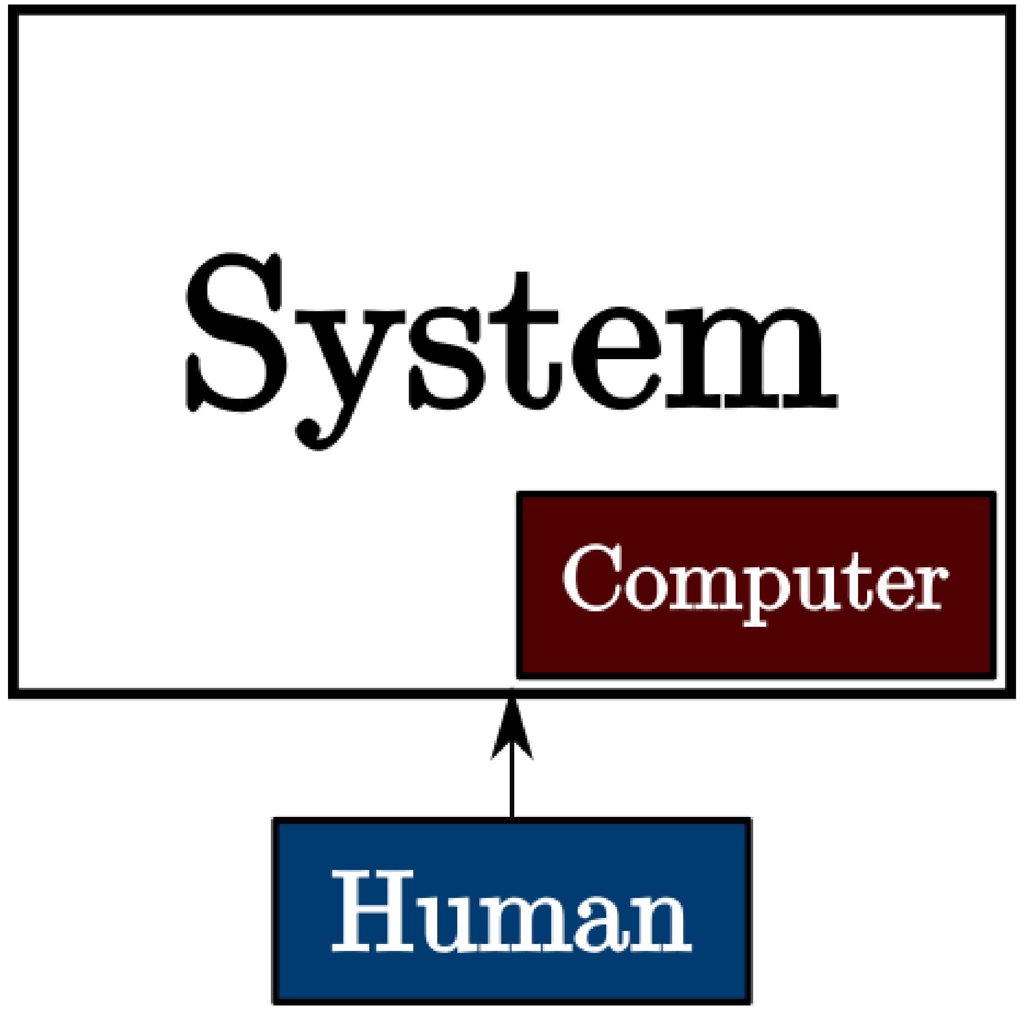
T. Griffith

Defense

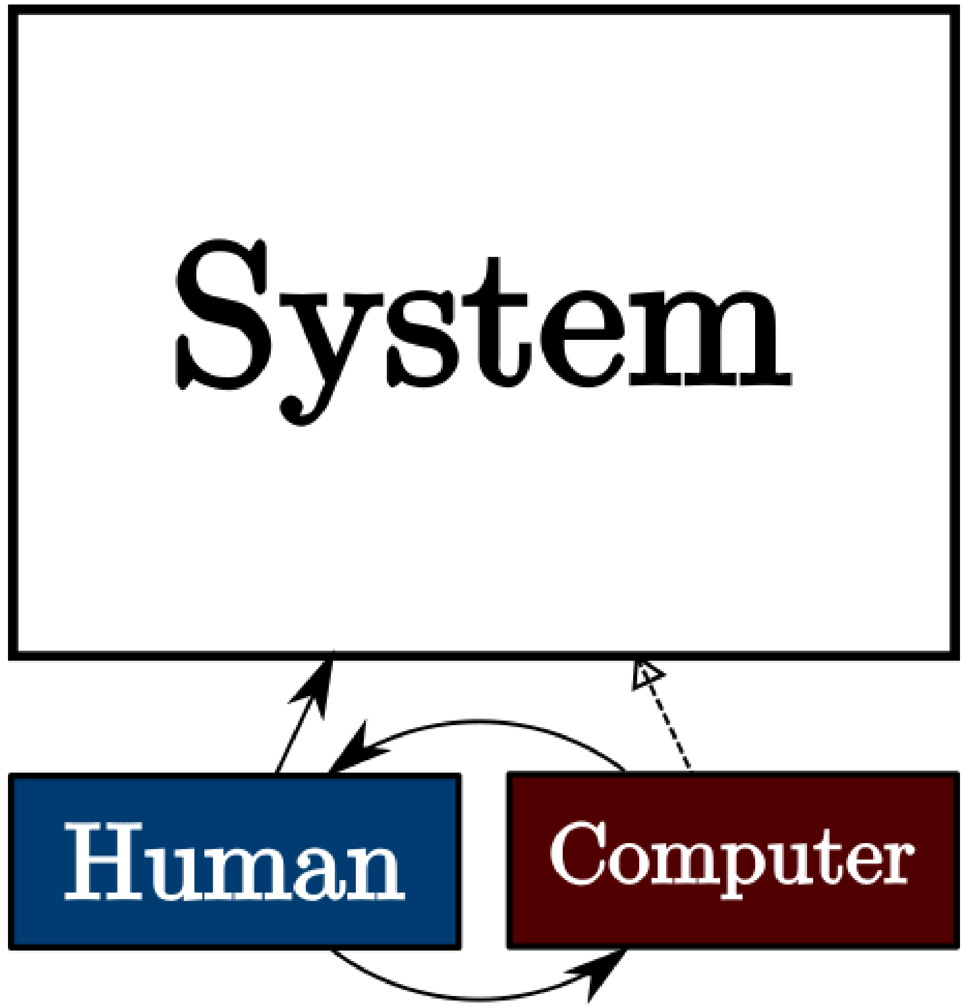
April 29, 2022

Outline

1. Introduction & Motivation



The computer as part of the system

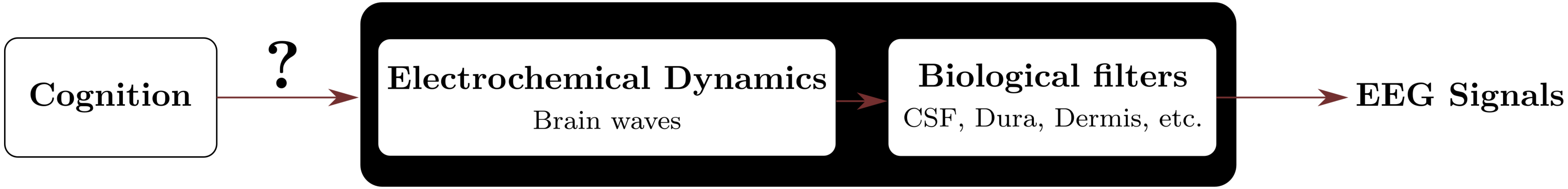


The computer as a teaming member

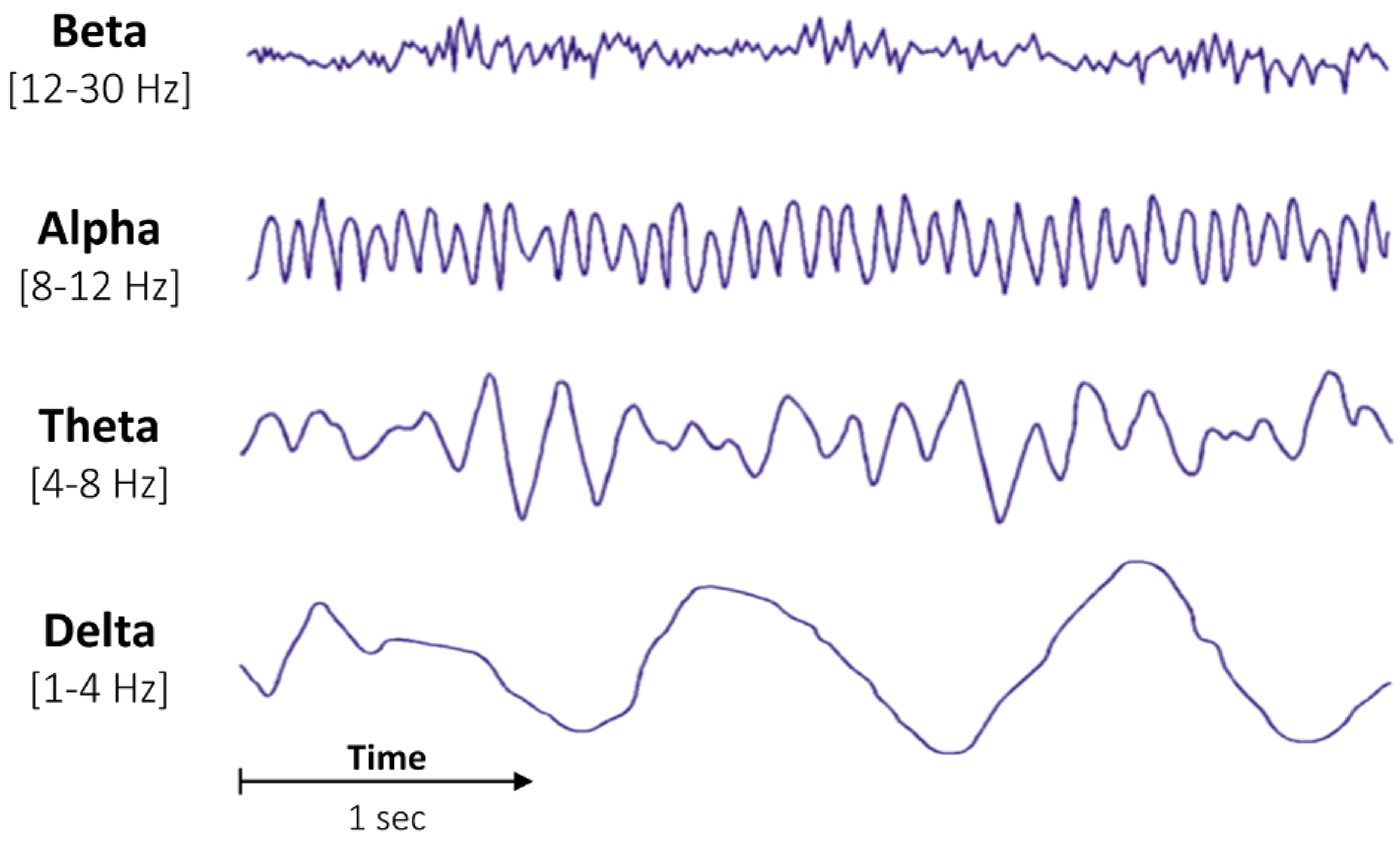
Novel potential

Bracken, B., Tobyne, S., Winder, A., Shamsi, N., & Endsley, M. R. (2021, July). Can Situation Awareness Be Measured Physiologically?. In International Conference on Applied Human Factors and Ergonomics (pp. 31-38). Springer, Cham.

Cognition as a black box



State of the art: surveys and orthogonal bases

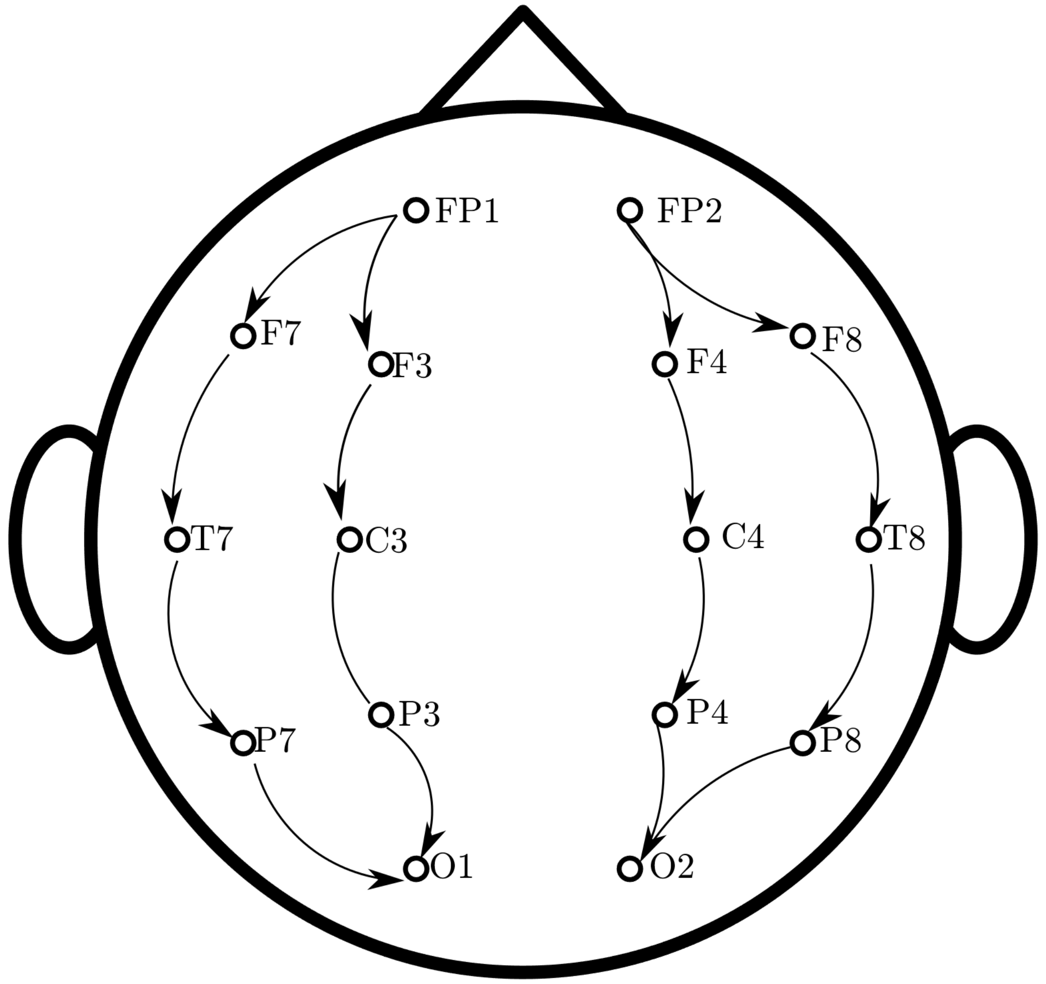


Hindriks, Rikkert, et al. "Latency analysis of resting-state BOLD-fMRI reveals traveling waves in visual cortex linking task-positive and task-negative networks." Neuroimage 200 (2019): 259-274.

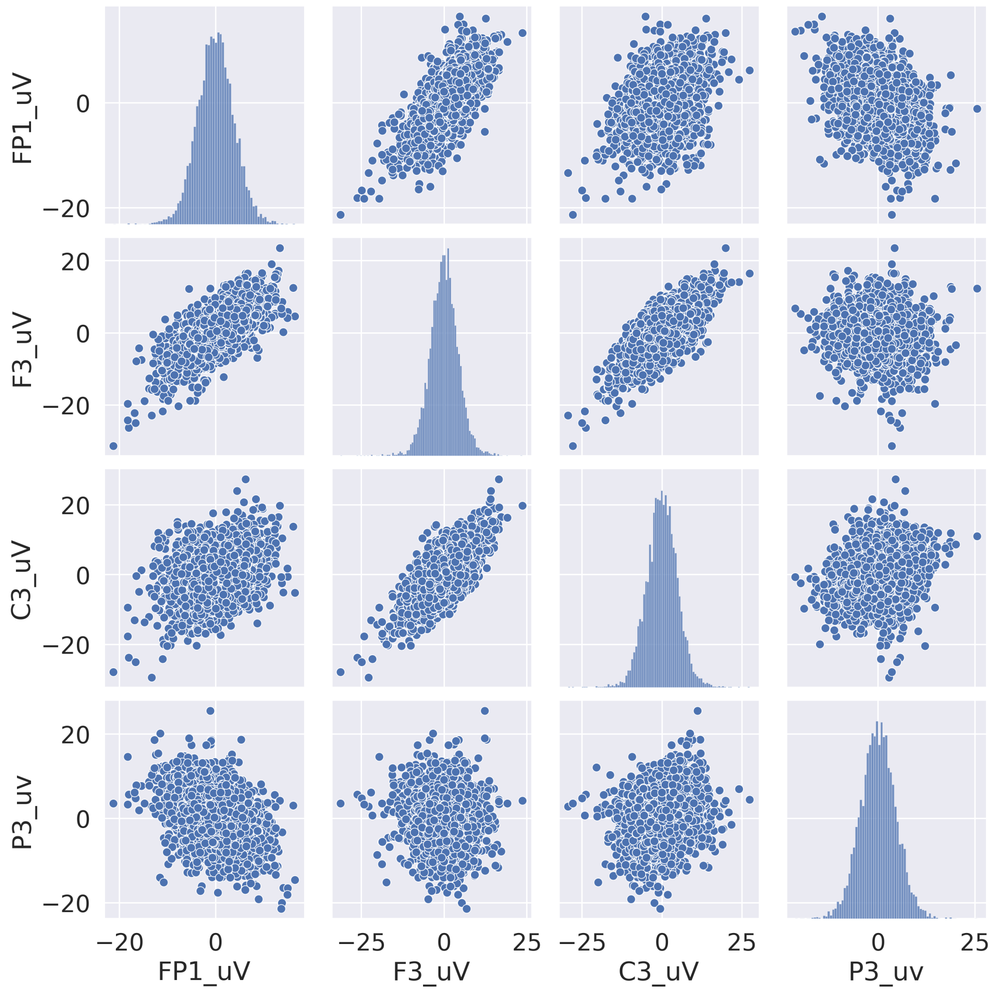
Hogarth de la Plante, Unsplash

2. A Dynamic Systems View of Brain Waves

Characteristics of EEG

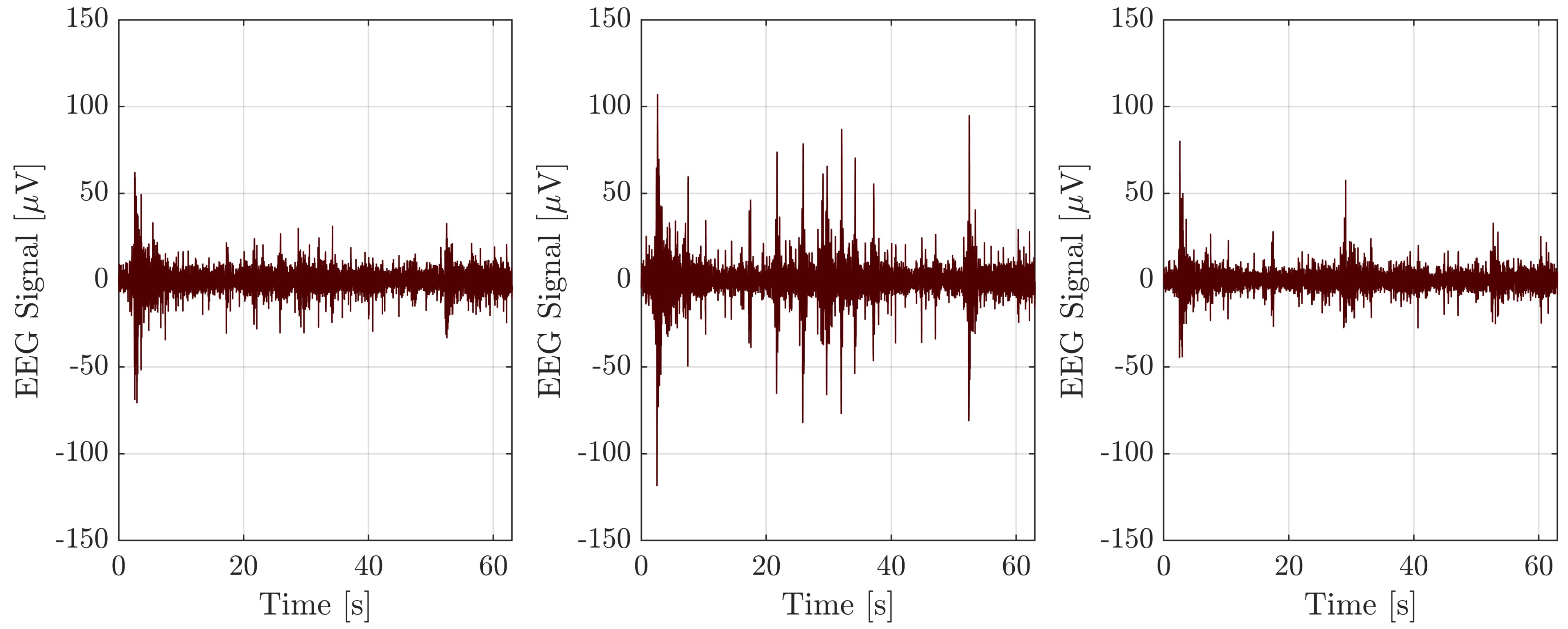


Longitudinal referencing



EEG channel pair plots

A canonical approach:



True brain wave plant: $\begin{cases} \dot{x} = Ax + Bu + v_x \\ y = Cx \end{cases}$

where $A,\ B,\ C,\ v_x,\ x$, and u are **all unknown**.

$A,\ B,\ C,\ v_x,\ x,$ and u are **all unknown**.

This level of uncertainty is an unsovled problem

Identify the plant: $\begin{cases} \dot{x}_m = A_m x + v_x \\ y_m = C x_m \end{cases}$,

accepting the uncertainty in A_m.

Treating nonlinear effects

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Indications of nonlinear structures in brain electrical activity

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(Received 6 August 2002; revised manuscript received 23 January 2003; published 10 April 2003)

Capturing time-varying brain dynamics

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[2] Helmholtz-Institute for Radiation and Nuclear Physics, University of Bonn, Nussallee 14–16, 53115 Bonn, Germany
[3] Interdisciplinary Center for Complex Systems, University of Bonn, Brühler Straße 7, 53175 Bonn, Germany

Indications of nonlinear deterministic and finite-dimensional structures in time series of brain electrical activity: Dependence on recording region and brain state

Ralph G. Andrzejak,[1,2,*] Klaus Lehnertz,[1,†] Florian Mormann,[1,2] Christoph Rieke,[1,2] Peter David,[2] and Christian E. Elger[1]

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(Received 14 May 2001; published 20 November 2001)

Adaptive Unknown Input Brain Wave Estimator:

$$\begin{cases} \dot{\hat{x}} = \big(A_m + BL(t)C\big)\hat{x} + B\hat{u} + K_x e_y; \\ \hat{y} = C\hat{x}. \end{cases}$$

Modes elegantly capture the spatio-temporal dynamics

True brain wave plant

$$\begin{cases} \dot{x} = Ax + Bu + v_x \\ y = Cx \end{cases}$$

Modal brain wave plant

$$\begin{cases} \dot{\eta} = \Lambda\eta + V^{-1}Bu + V^{-1}v_x \\ y = CV\eta \end{cases}$$

Some important analytical properties:

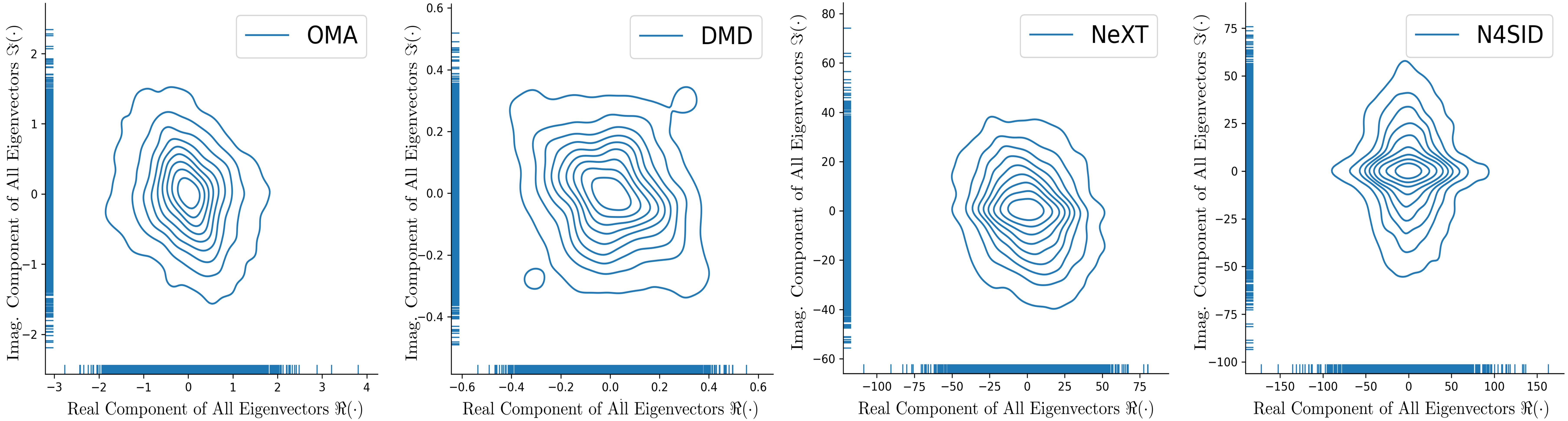
- Frequency
- Damping
- Mode shape
- Complexity

3. System Identification of Brain Wave Modes Using EEG

System Identification of Brain Wave Modes Using EEG

Identifying linear patterns

Identify the plant: $\begin{cases} \dot{x}_m &= A_m x + v_x \\ y_m &= C x_m \end{cases}$



System Identification of Brain Wave Modes Using EEG

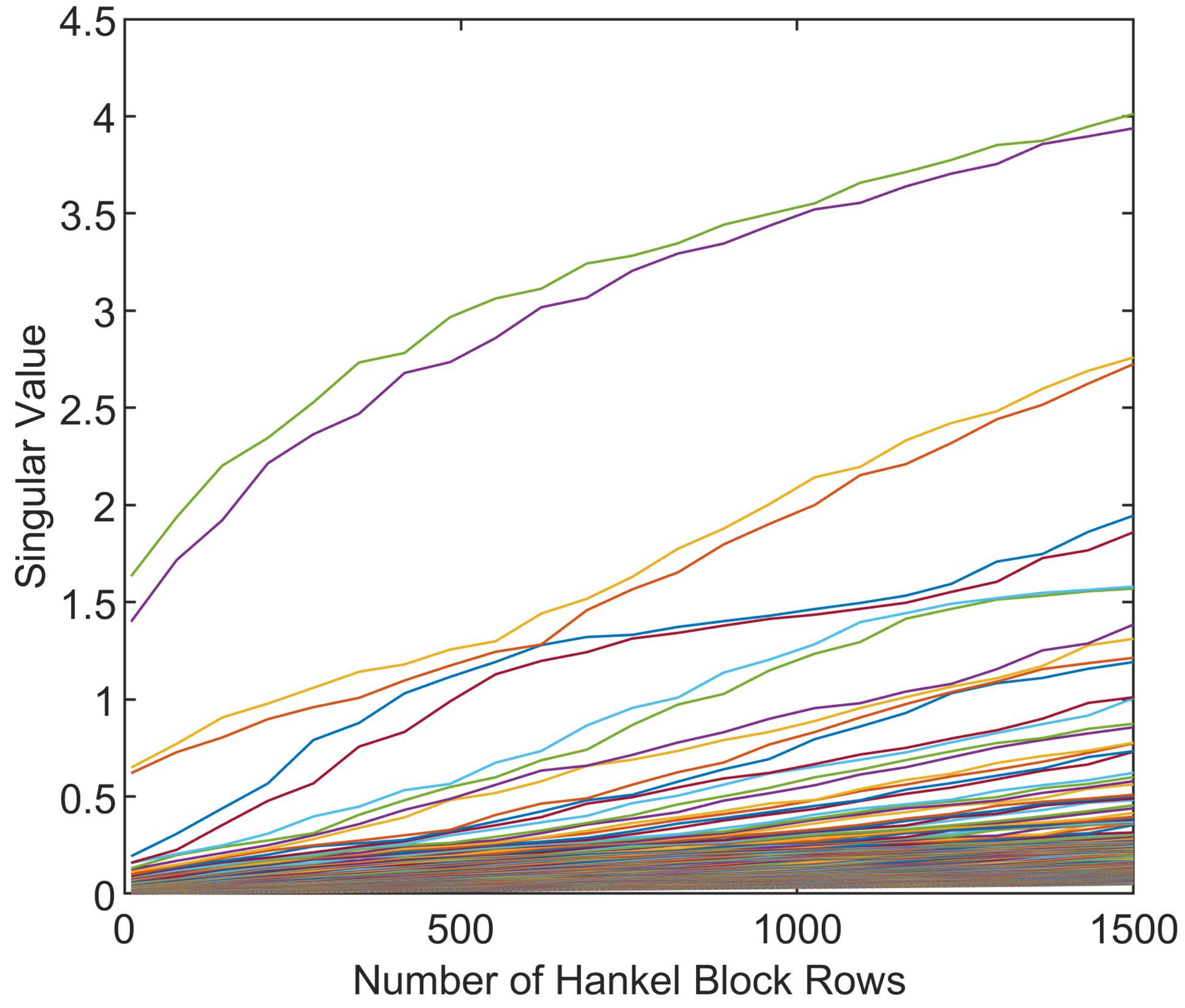
Identifying linear patterns

Identify the plant: $\begin{cases} \dot{x}_m = A_m x + v_x \\ y_m = C x_m \end{cases}$

$$O = \begin{bmatrix} C \\ CA_m \\ CA_m^2 \\ \vdots \\ CA_m^{s-1} \end{bmatrix} X_0$$

$$= \Gamma X_0$$

$$\hat{\Gamma} = US^{1/2} \quad \hat{X}_0 = S^{1/2} V^*$$



System Identification of Brain Wave Modes Using EEG

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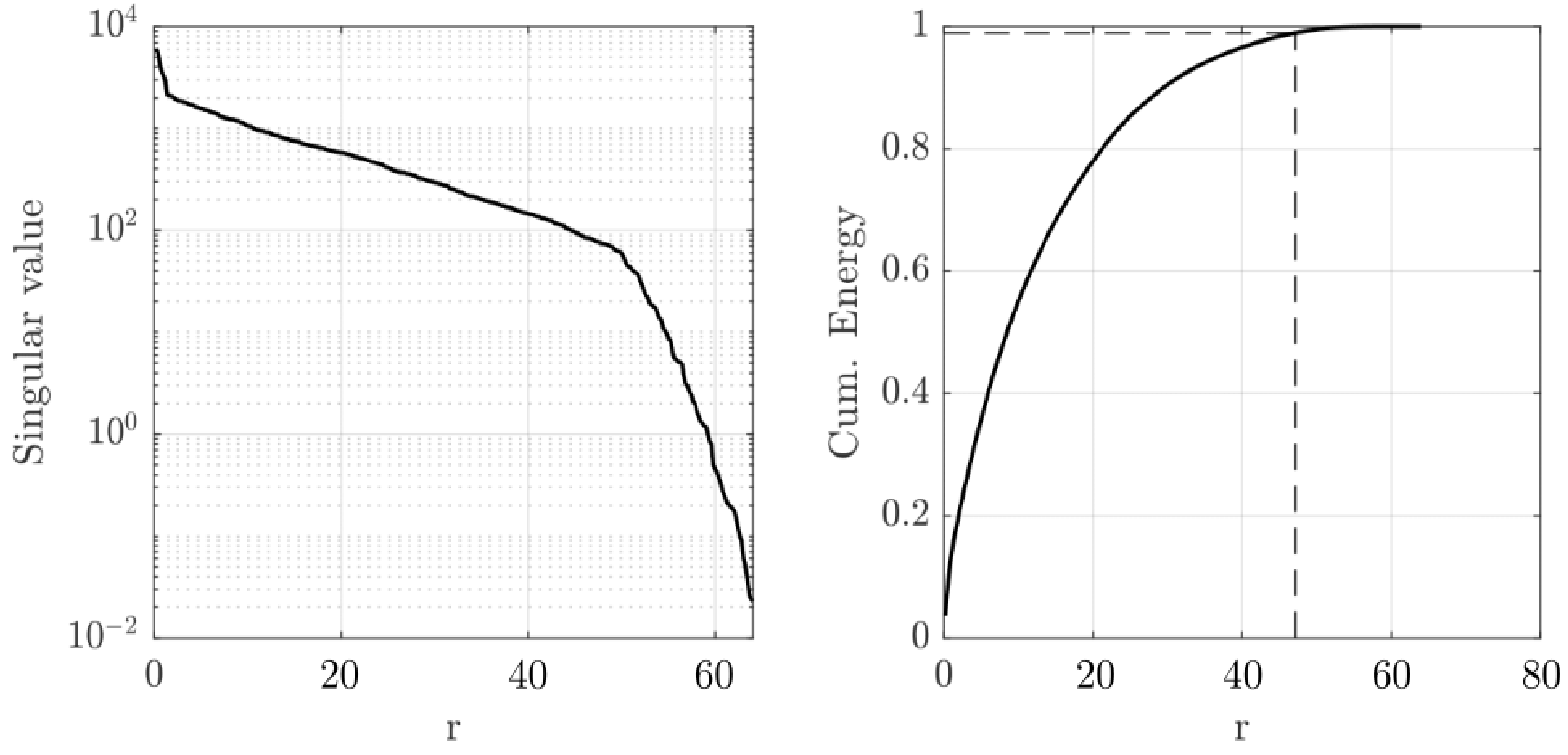
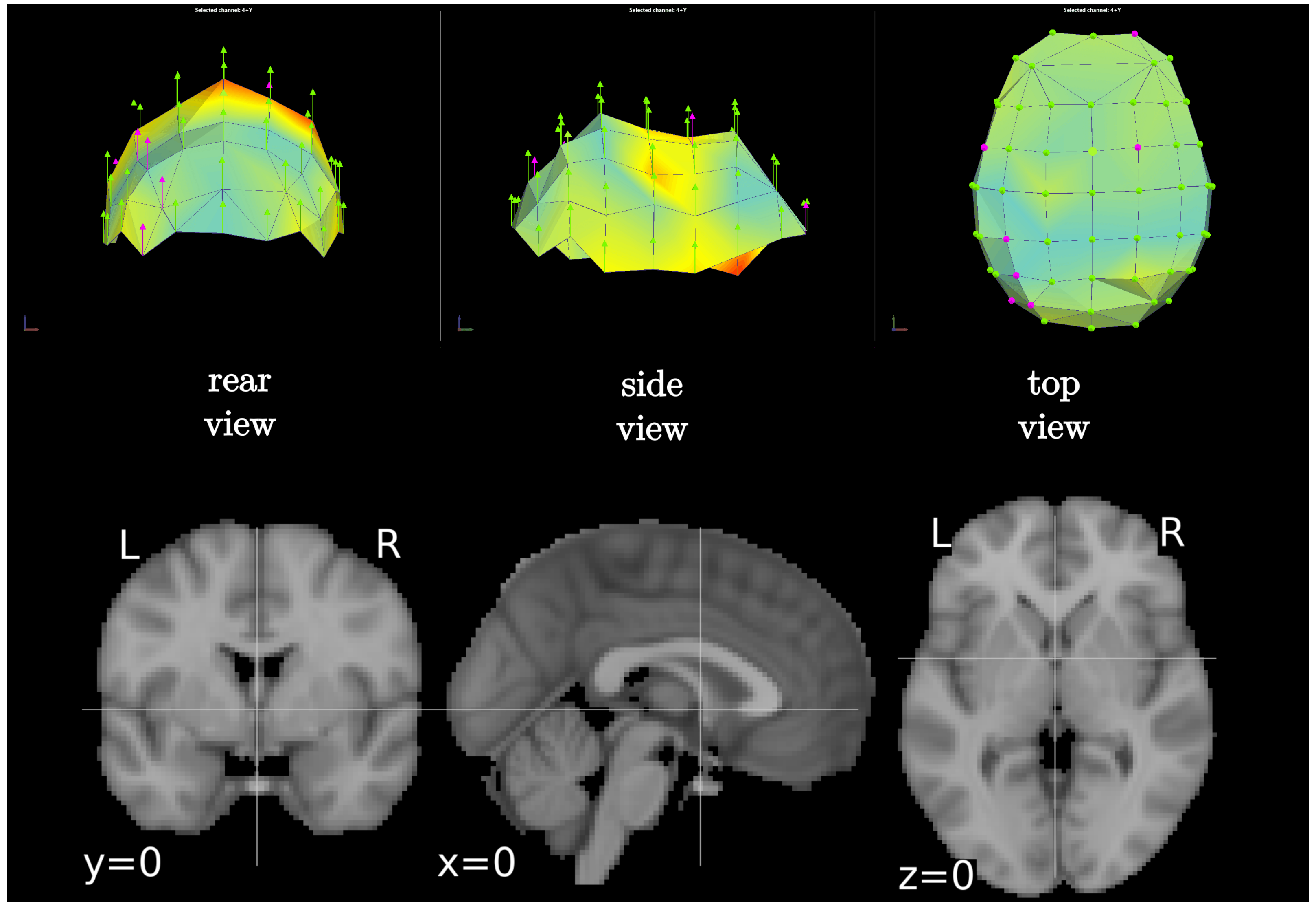
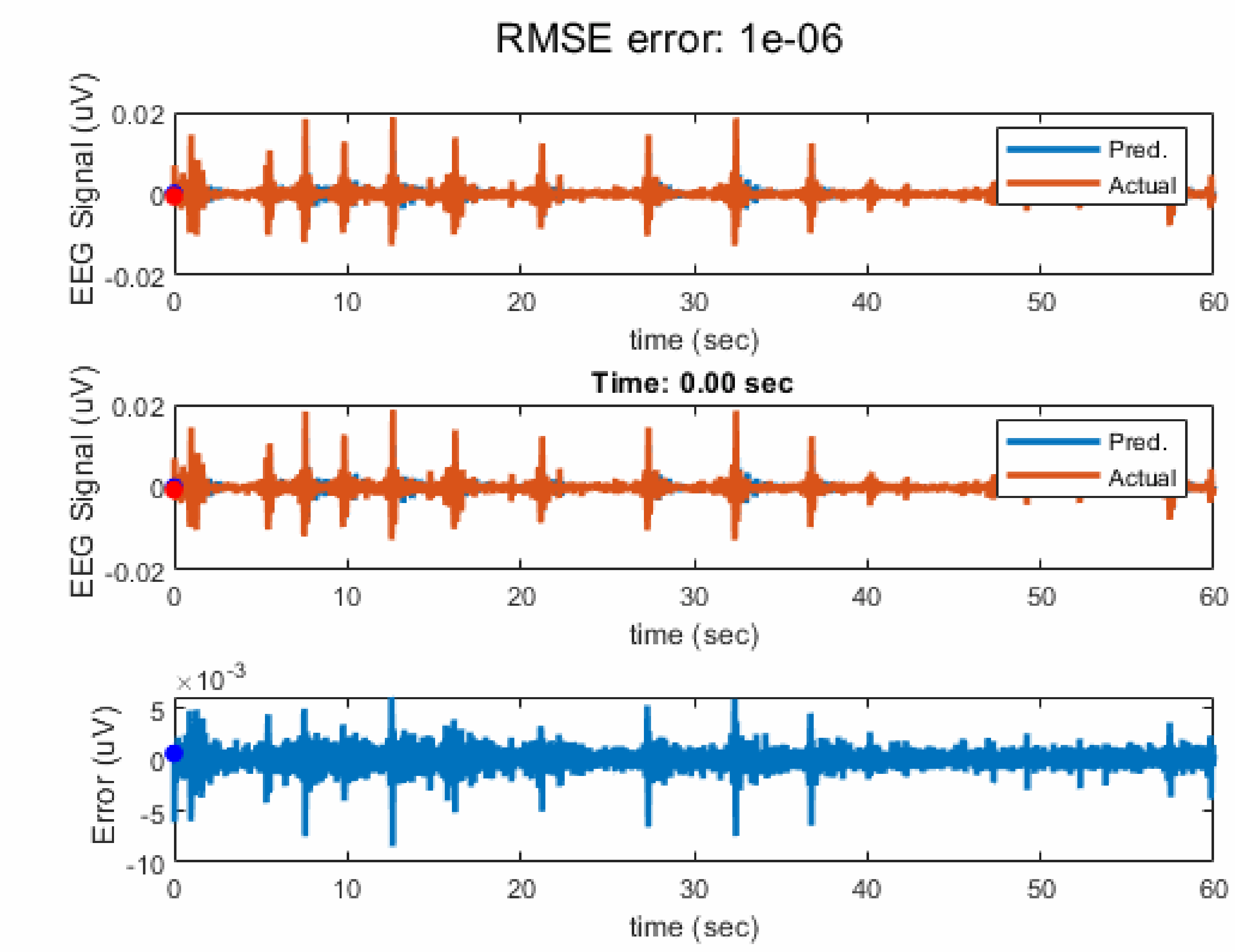


System Identification of Brain Wave Modes Using EEG

Identifying linear patterns

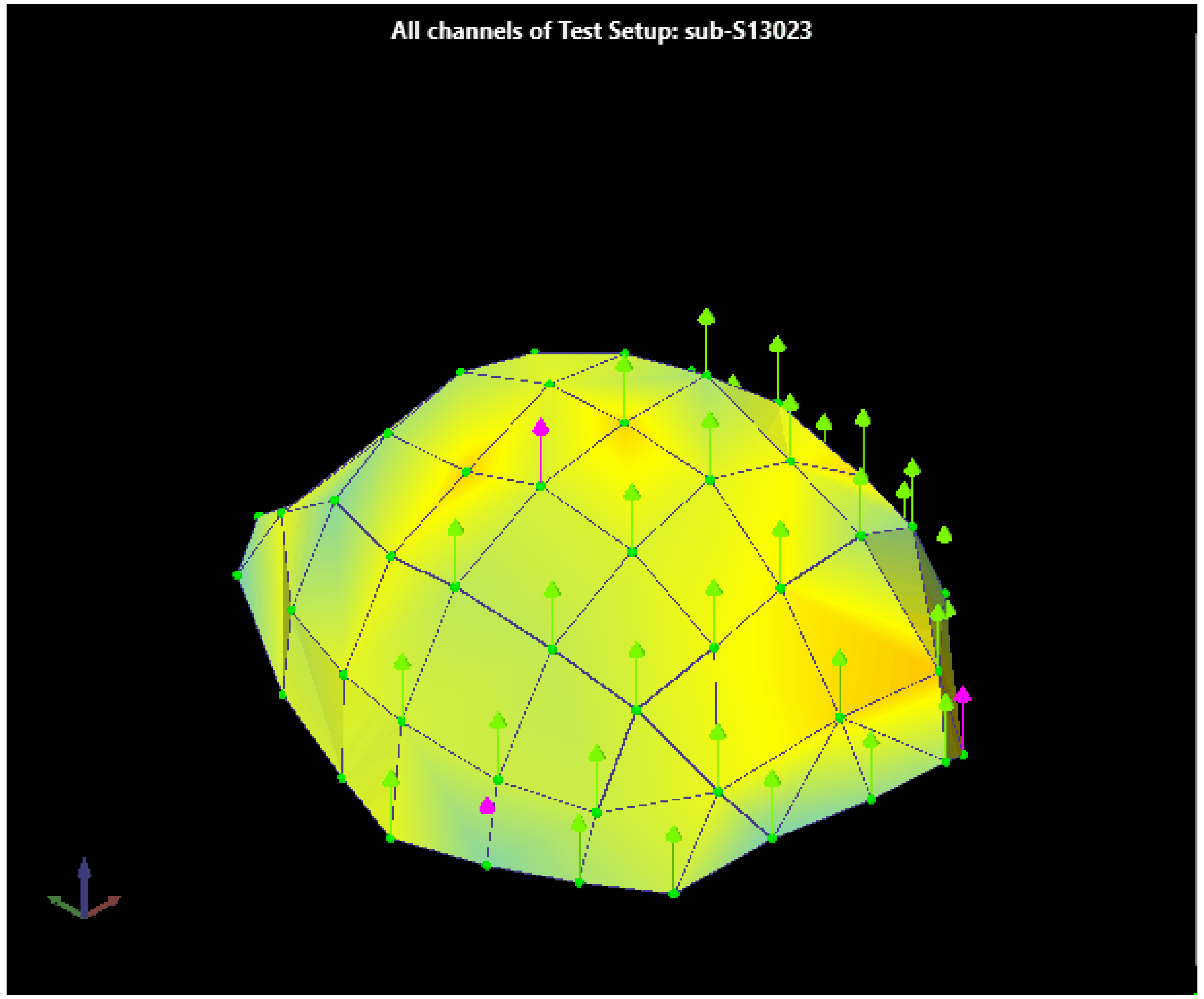


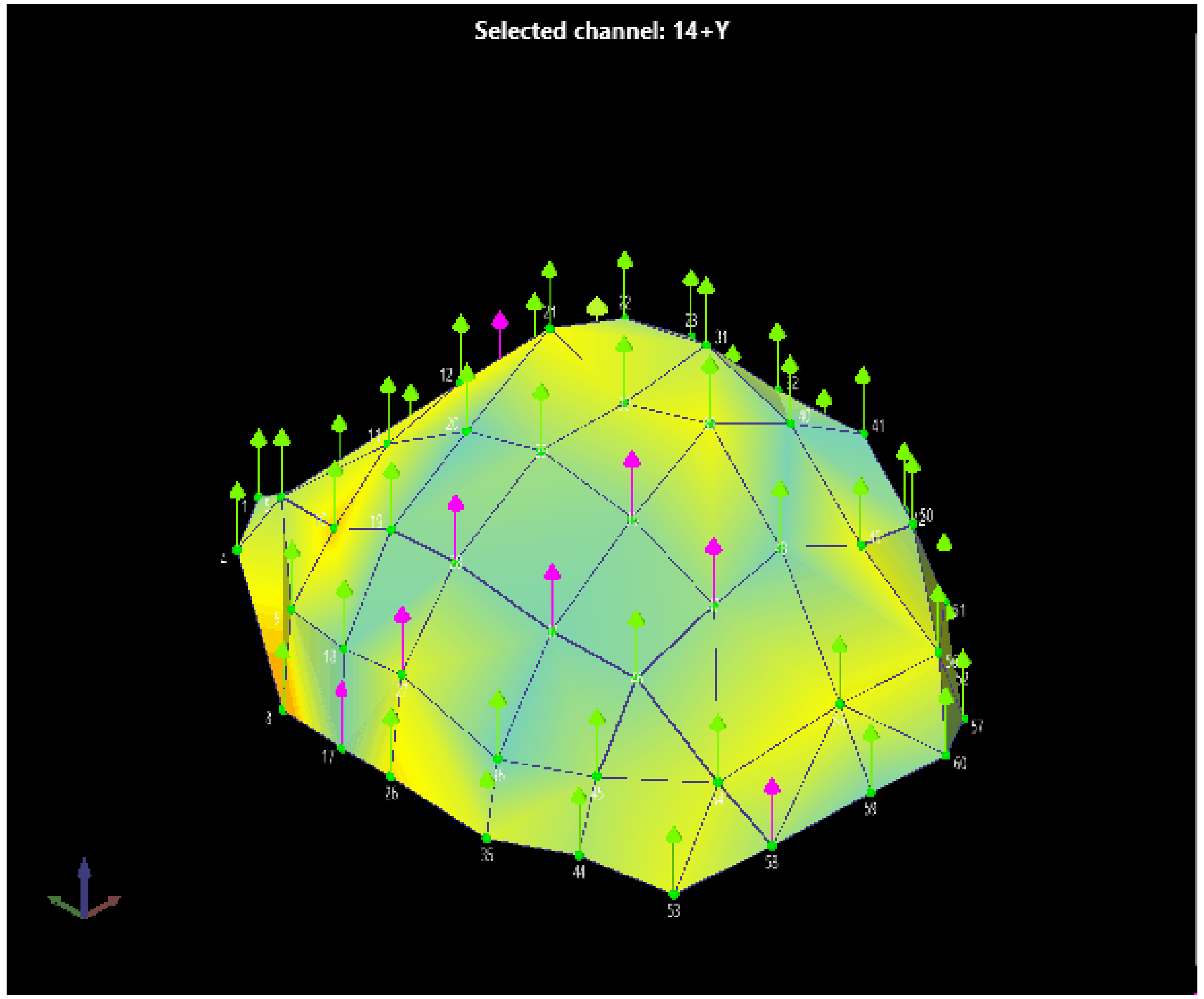


4. Modal Analysis of Brain Wave Dynamics

Modal Analysis of Brain Wave Dynamics

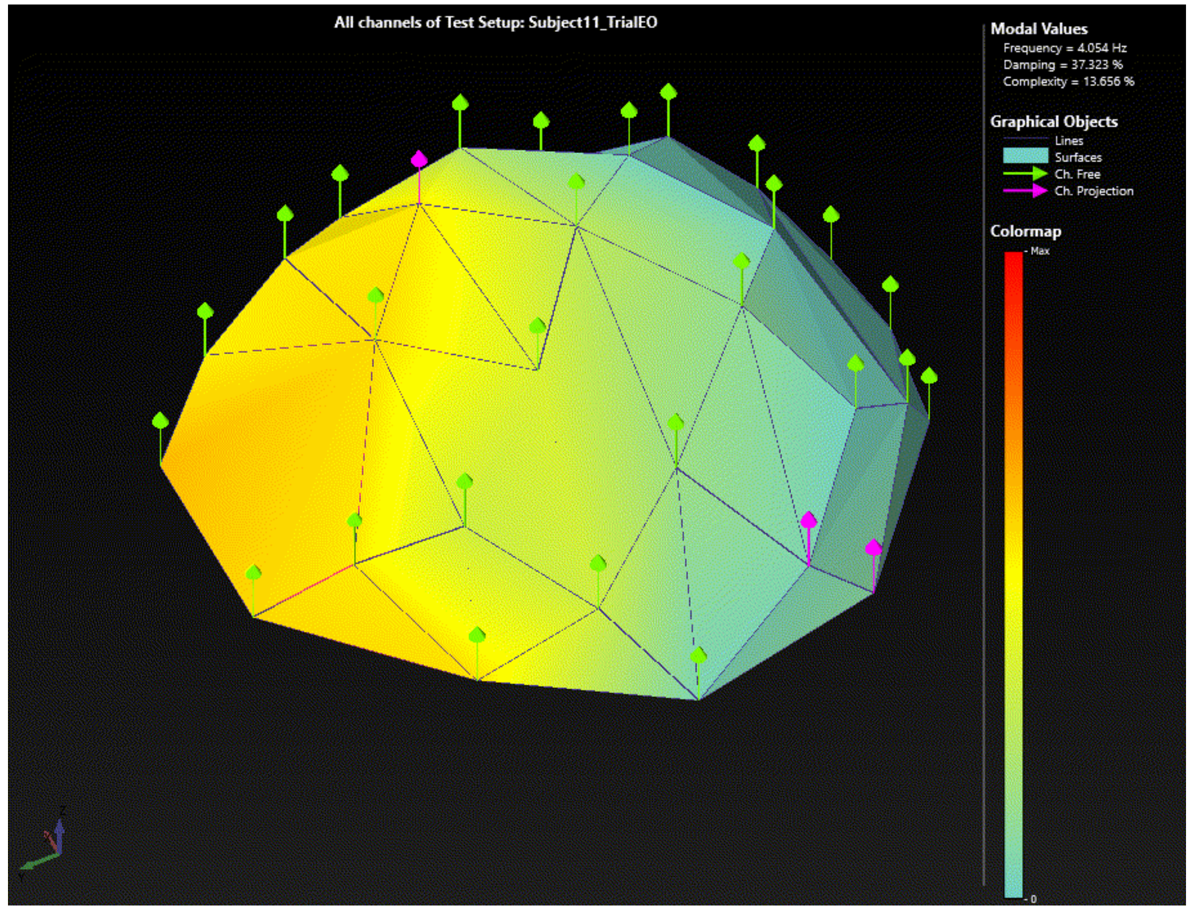
Brain wave modes are standing and traveling

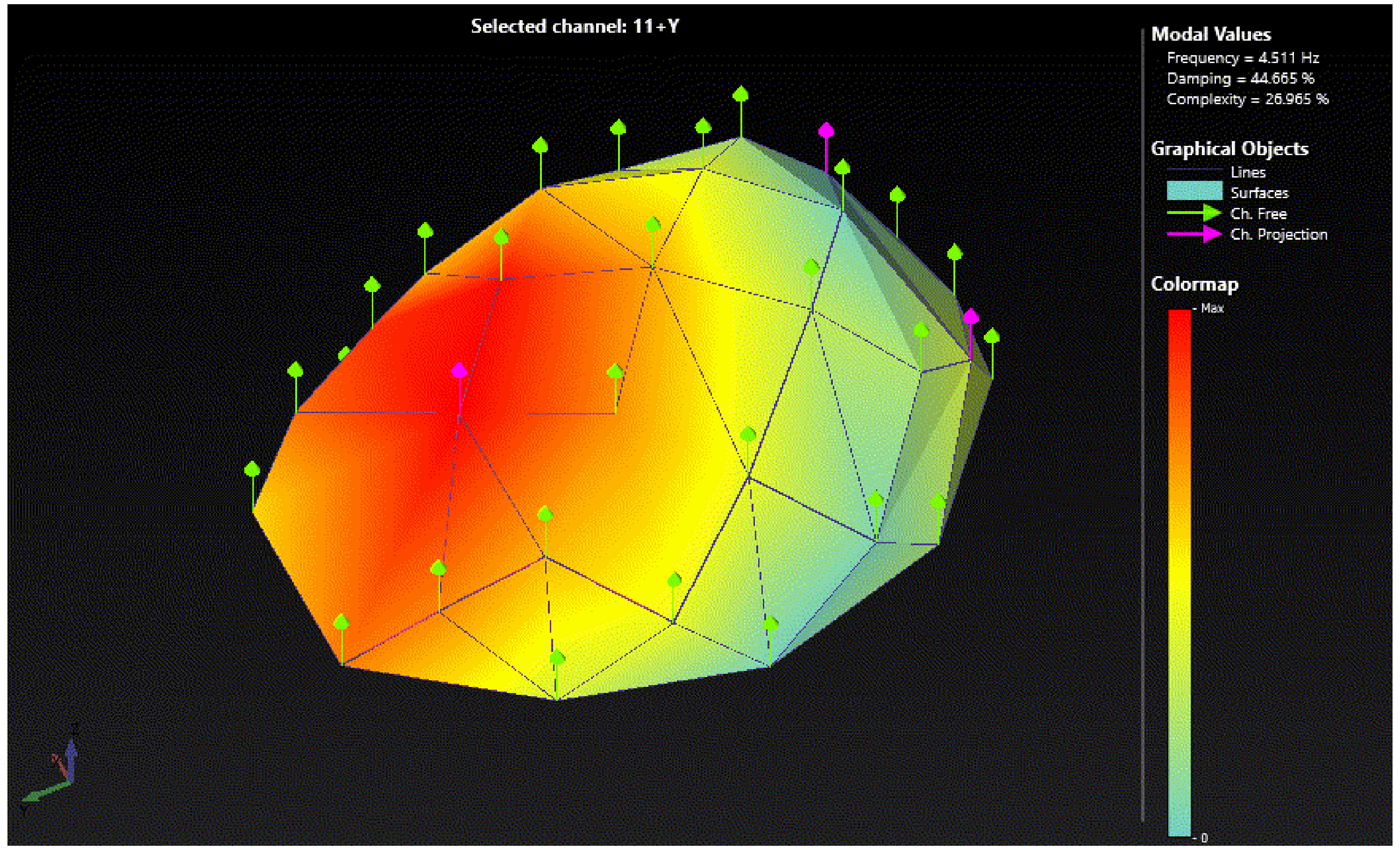




Modal Analysis of Brain Wave Dynamics

Some brain wave modes are task independent

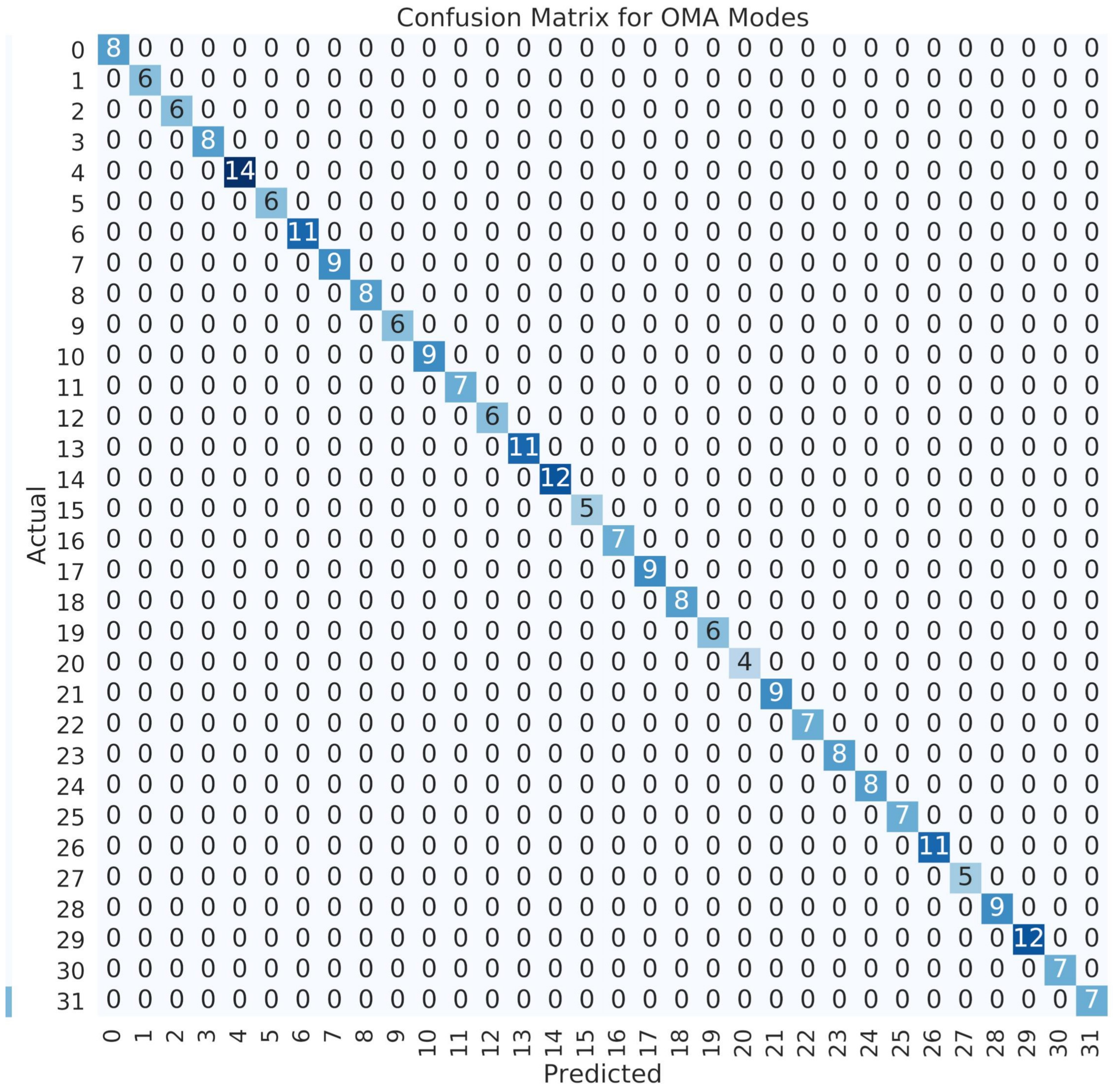




	Frequency	**Damping [%]**	**Complexity [%]**	**Shape Correl.**
Alpha Mode 1	4.34 ± 0.03	8.20 ± 1.20	11.47 ± 17.59	0.97 ± 0.016
Beta Mode 2	21.83 ± 0.22	1.98 ± 2.63	32.29 ± 35.67	0.96 ± 0.018
Gamma Mode 3	40.39 ± 0.26	11.87 ± 7.49	12.42 ± 16.88	0.99 ± 0.010
Gamma Mode 4	44.19 ± 0.24	2.52 ± 1.39	2.93 ± 5.69	0.99 ± 0.012

Modal Analysis of Brain Wave Dynamics

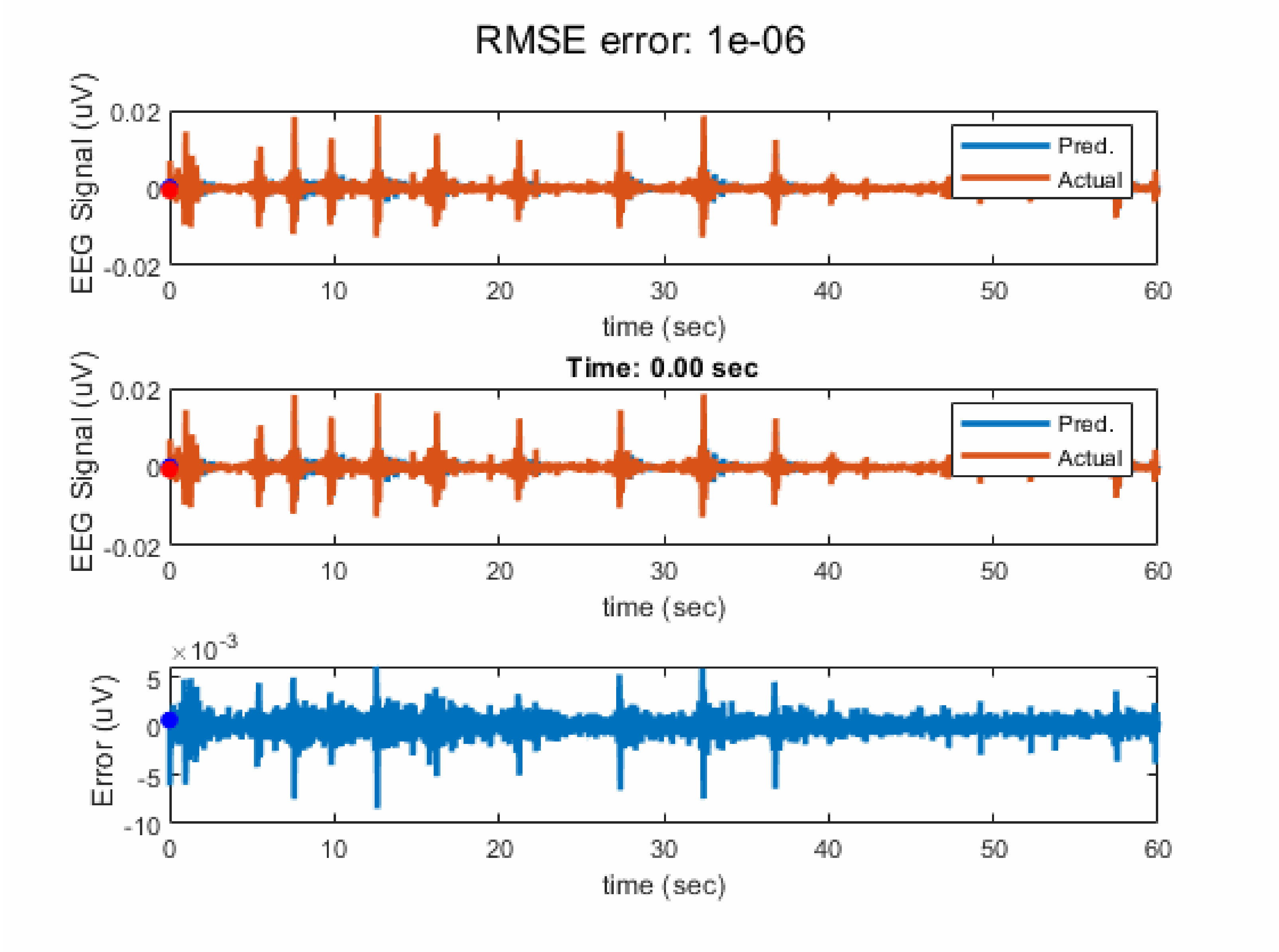
Brain wave modes are interindividual

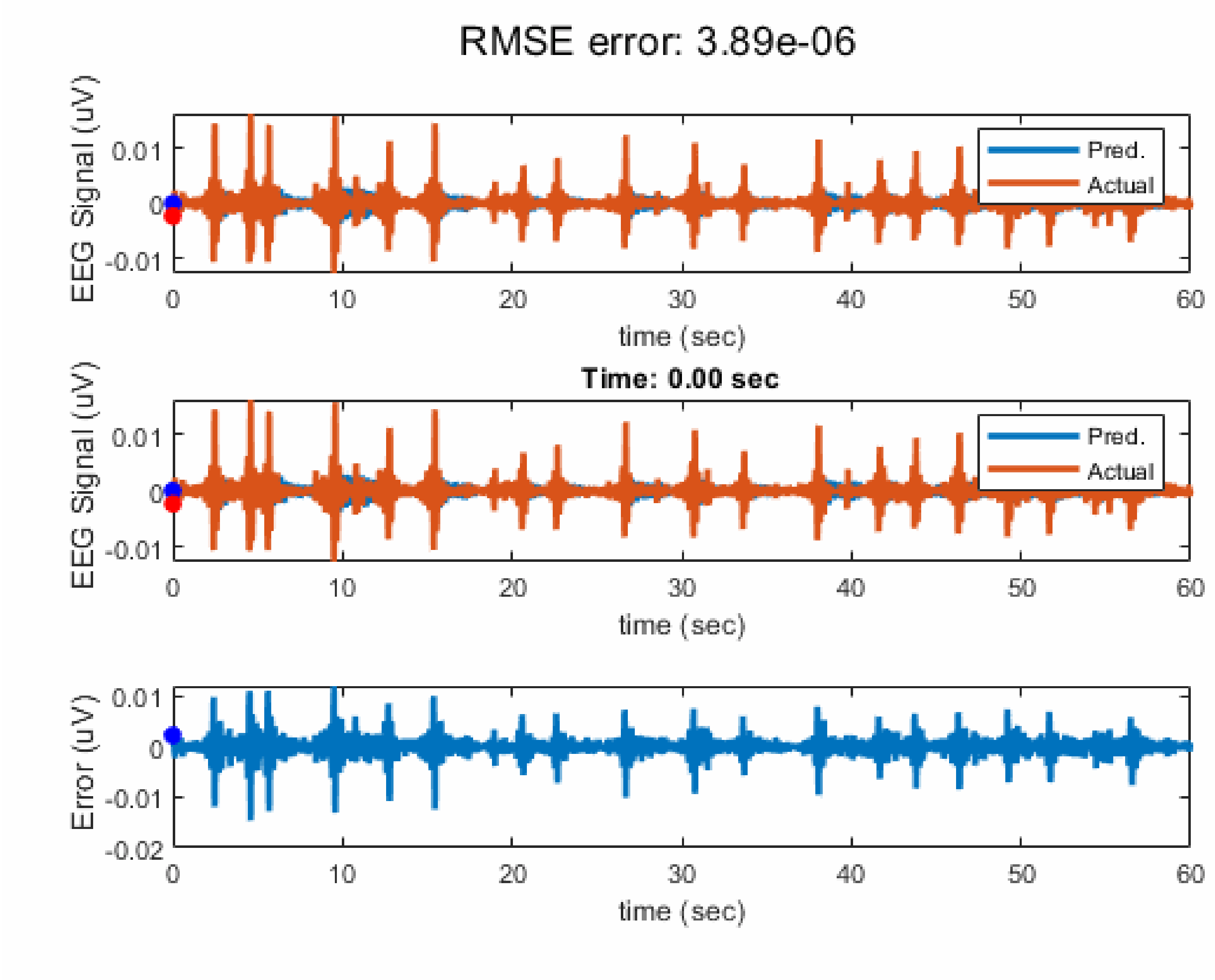


Reference	No. of Electrodes	Accuracy [%]
This work	32	99.85
This work	8	96.45
Wilaiprasitporn et al.	32	99.90
Wilaiprasitporn et al.	5	99.1
DelPozo-Banos et al.	32	97.97

Modal Analysis of Brain Wave Dynamics

Brain wave modes poorly match nonlinear dynamics





5. Adaptive Unknown Input Estimators

Adaptive Unknown Input Estimators

Estimator overview

Three significant uncertainties

- Input u is unknown, external
- State matrix A may have uncertainty
- General process uncertainty v_x

Can we synthesize u and correct A?

$$\dot{x} = Ax + Bu + v_x$$
$$y = Cx$$

Adaptive Unknown Input Estimators

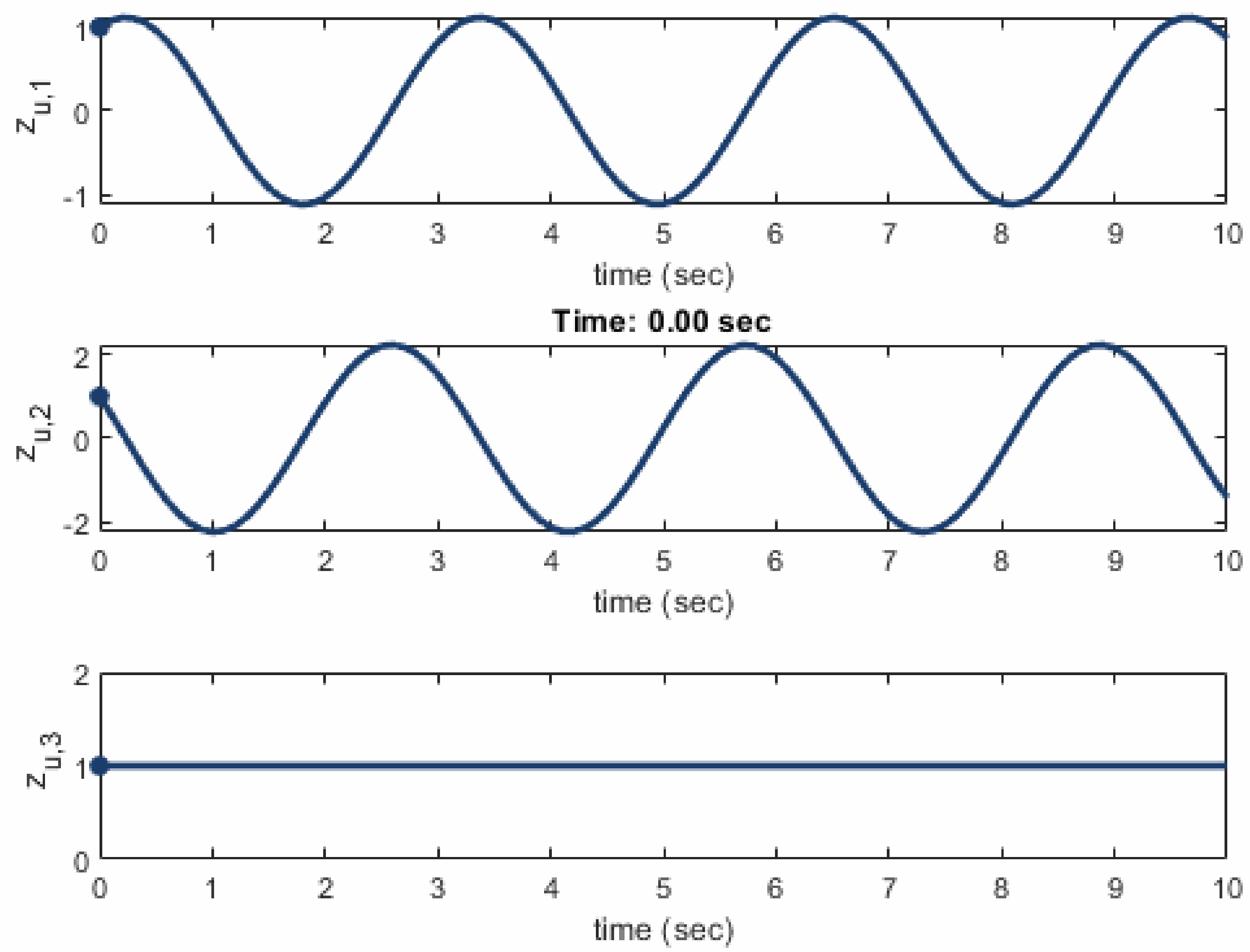
Modeling unknown inputs

Approximate input space $\mathbb{U}$

- $\hat{u} = \sum_{i=1}^{N} c_i f_i(t)$

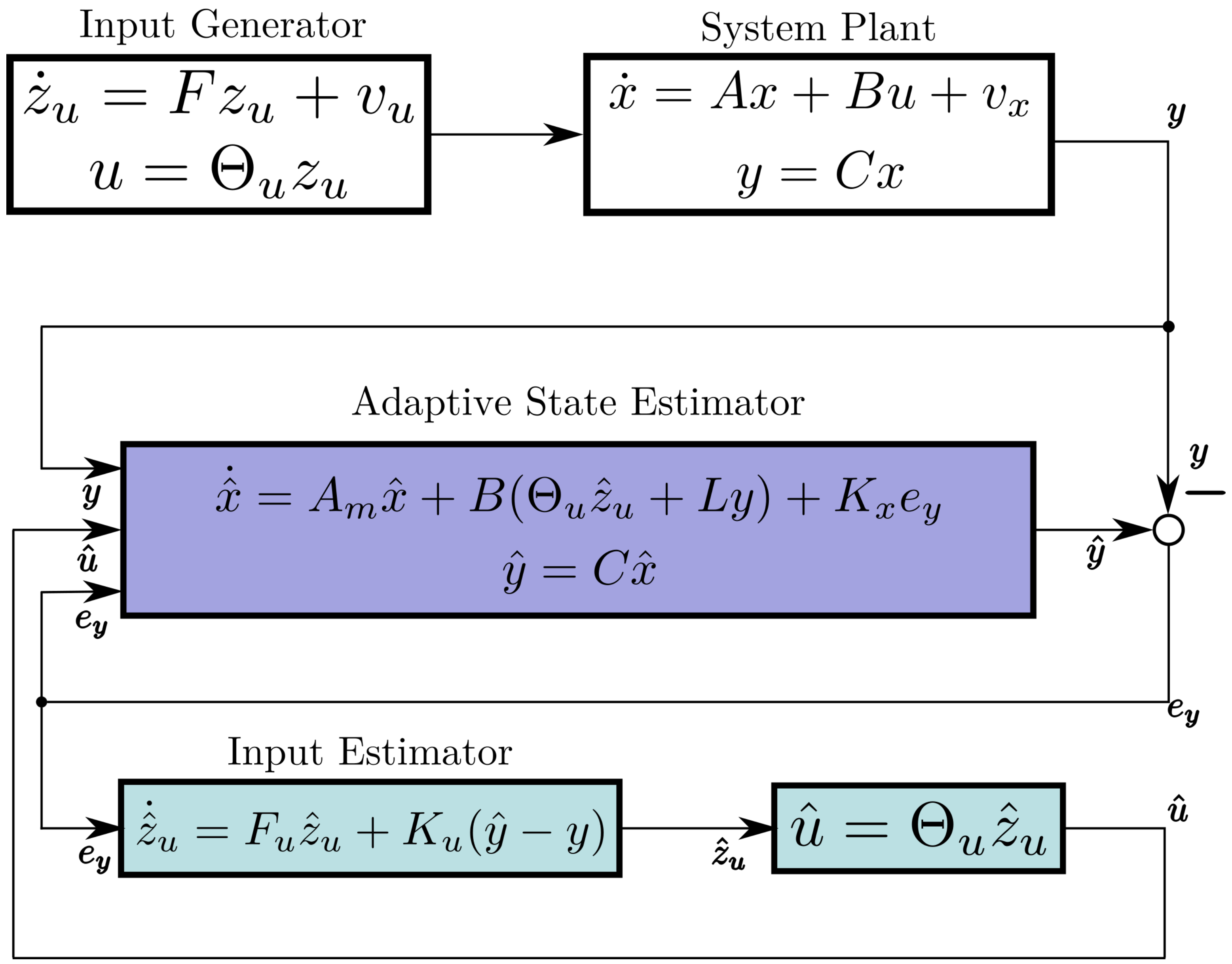
Persisten Inputs

- $\dot{z}_u = F_u z_u$
- $\hat{u} = \Theta_u z_u$
- $F_u = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$



Adaptive Unknown Input Estimators

Architecture and estimator error



Recover A with adaptive scheme

$$A \equiv A_m + BL_* C$$

$$\dot L = -e_y y^* \gamma_e - \alpha L;\ \alpha > 0,\ \gamma_e > 0$$

Error dynamics

$$\dot e = (\bar A + \bar K \bar C)e + \bar B \underbrace{\Delta L y}_{w} + v$$

$$\begin{bmatrix} \dot e_x \\ \dot e_z \end{bmatrix} = \underbrace{\begin{bmatrix} A_m + K_x C & B\Theta_u \\ K_u C & F_u \end{bmatrix}}_{\bar A_c} \begin{bmatrix} e_x \\ e_z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w + \begin{bmatrix} v_x \\ v_u \end{bmatrix}$$

Adaptive Unknown Input Estimators

Architecture and estimator error

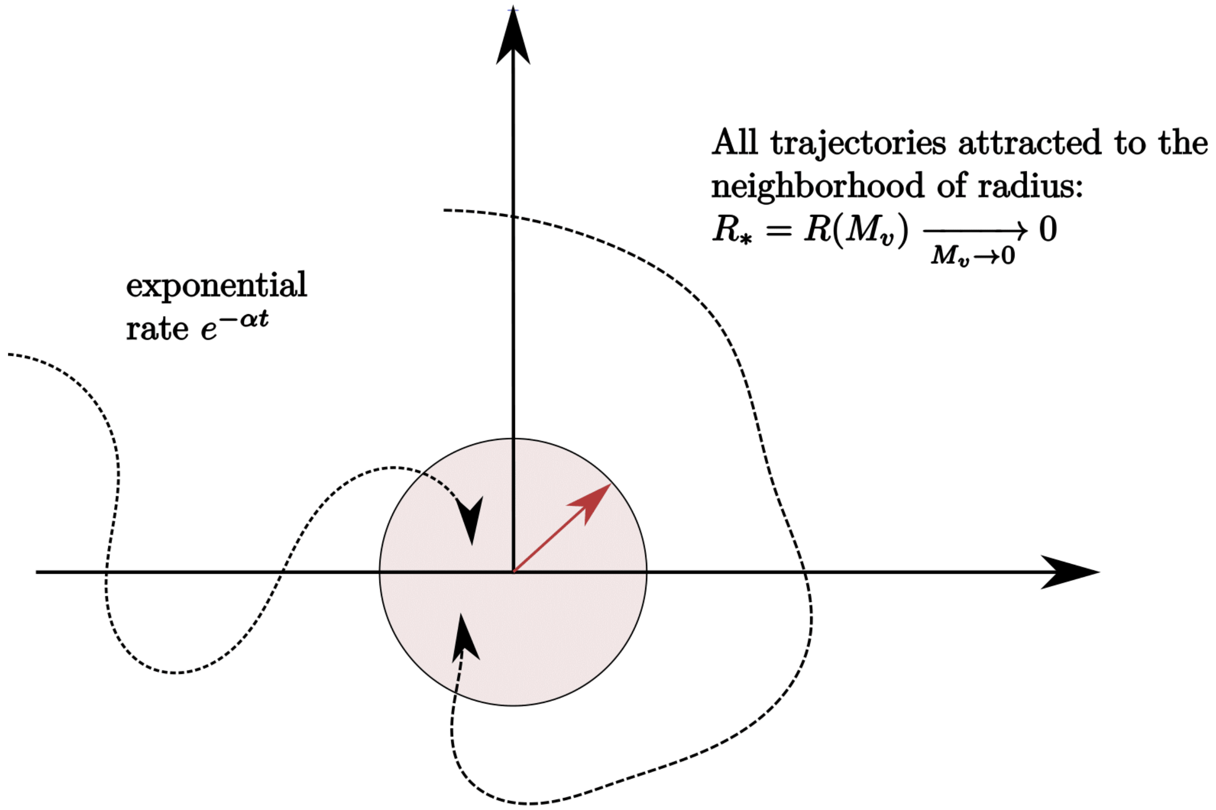
ASD plant dynamics

$$A_c^* P + P A_c = -Q$$
$$PB = C^*$$

Bounded L_*, v, and γ_e

Error in state and input converges to an neighborood centered at zero

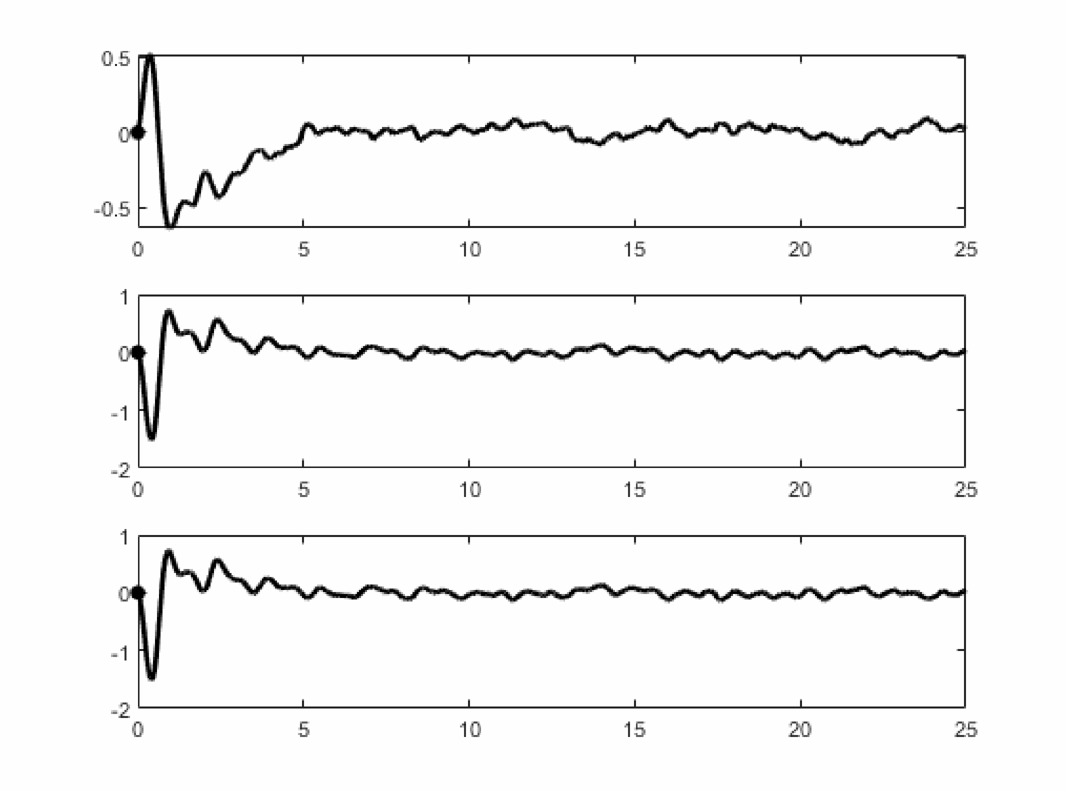
$$V(e, \Delta L) = \frac{1}{2} e^* \bar{P} e + \frac{1}{2}\mathrm{tr}(\Delta L \gamma_e^{-1} \Delta L^*)$$
$$\lim_{t\to\infty} \sup ||e(t)|| \leq \frac{1+\sqrt{\lambda_{\max}\bar{P}}}{\alpha\sqrt{\lambda_{\min}\bar{P}}} M_v \equiv R^*$$



Illustrative example

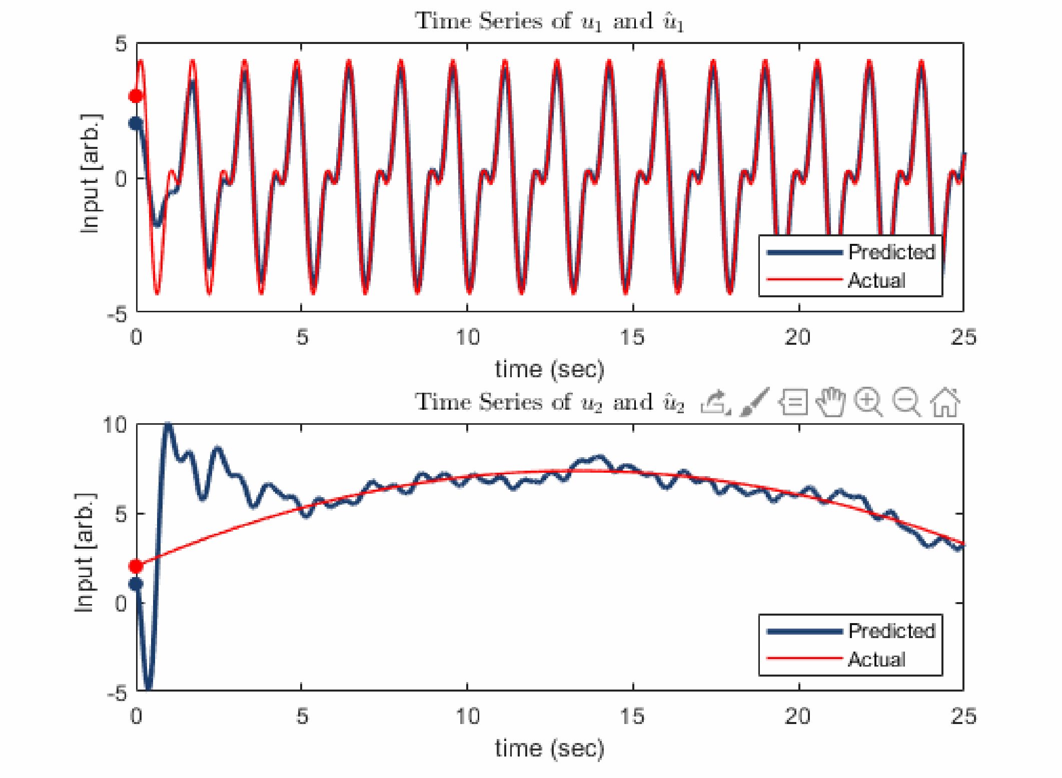
$$\dot{x} = A_m x + Bu + v_x = \begin{bmatrix} -4 & 1 & 2 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} x + Bu + v_x$$
$$y = Cx$$

Internal state error time series



$$\dot{x} = Ax + Bu + v_x = \begin{bmatrix} -2.86 & 1 & 4.7 \\ 1.8 & -1 & 6.7 \\ -9 & 1 & -17.2 \end{bmatrix} x + Bu + v_x$$
$$y = Cx$$

Estimating the unknown input



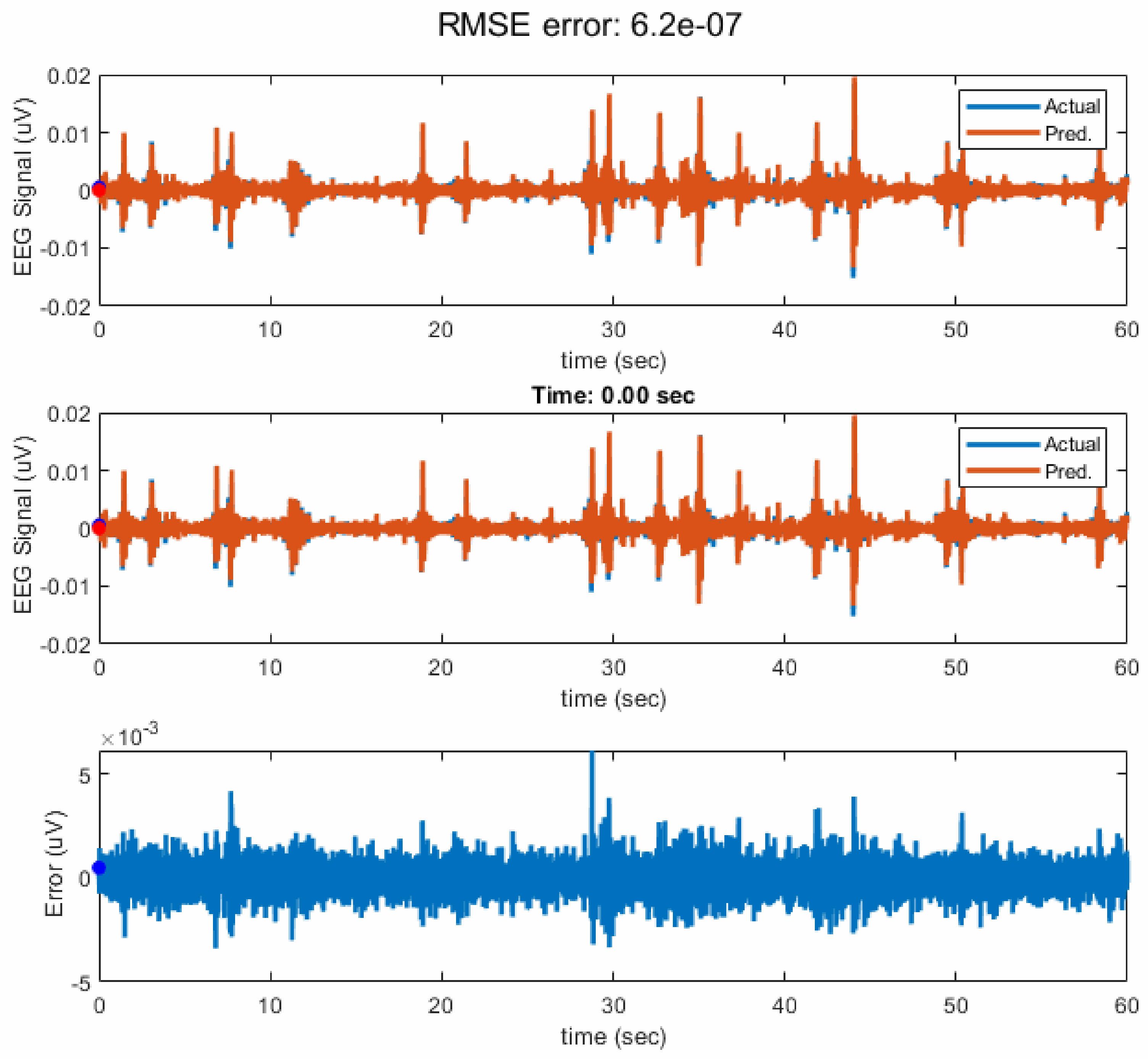
6. Reconstructing the Brain’s Unknown Input

Recall: Solving the nonstationary problem

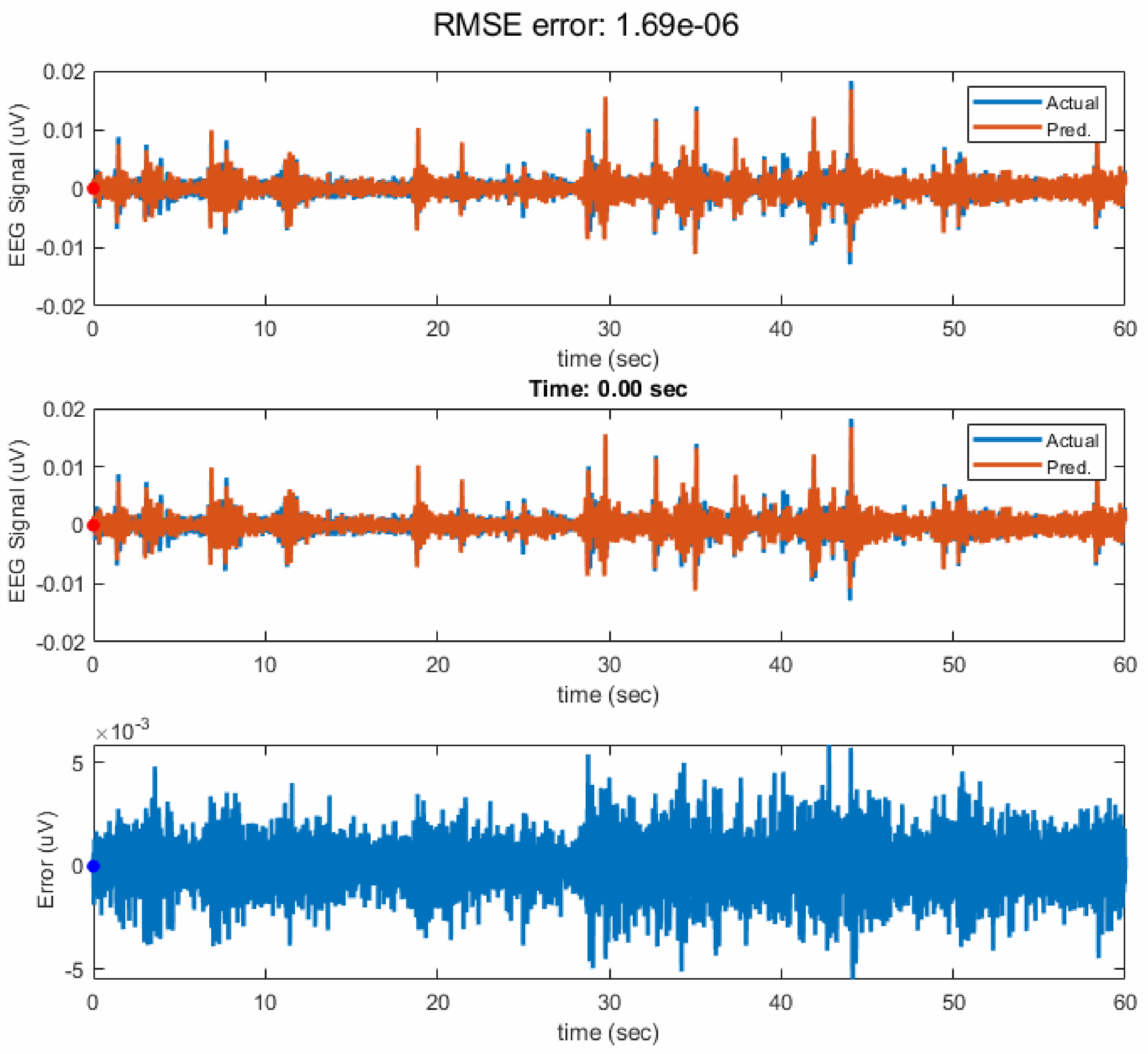
Reconstructing the Brain's Unknown Input

aUIO outperforms static modes

aUIO on unseen data



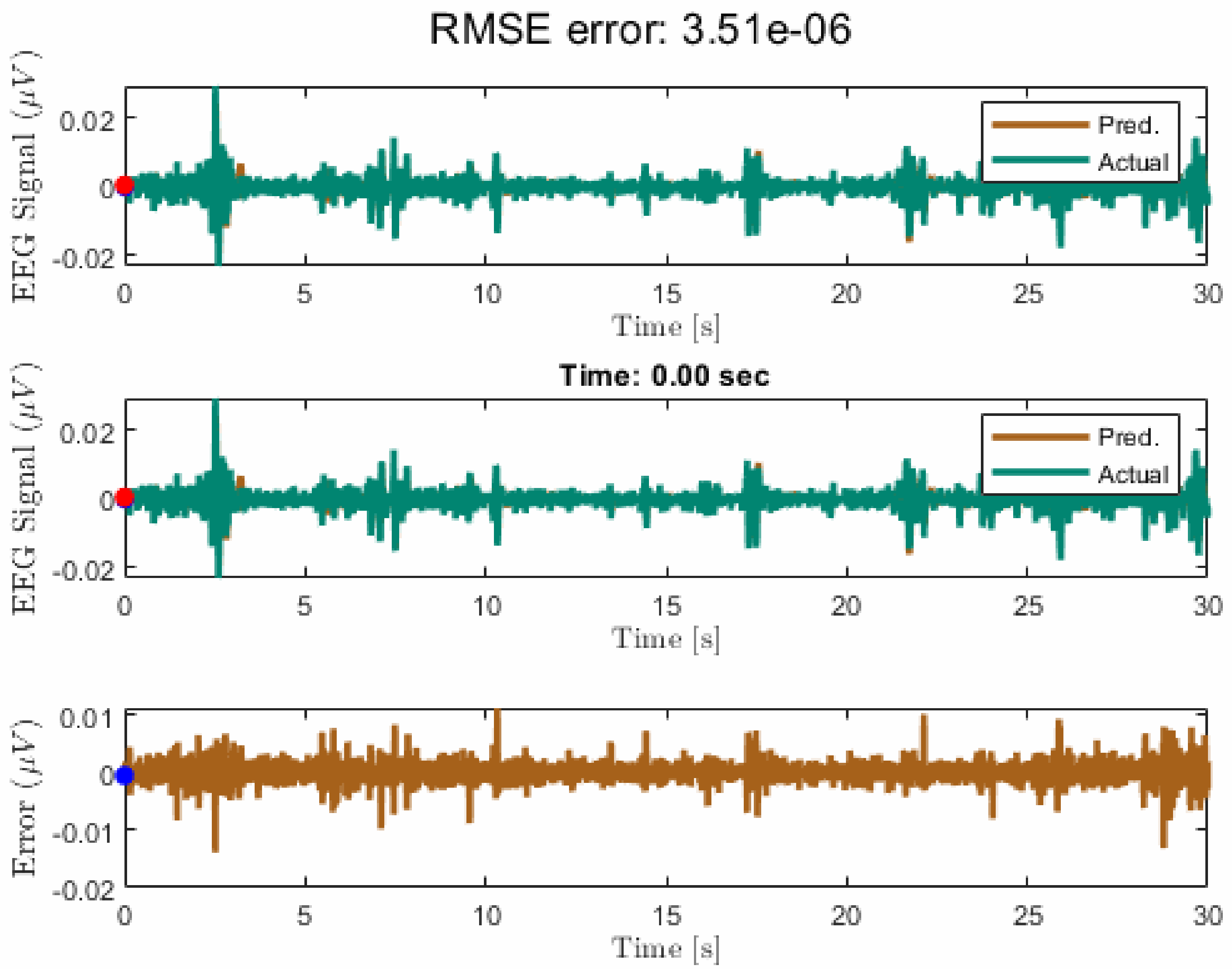
Weighted modes on seen data



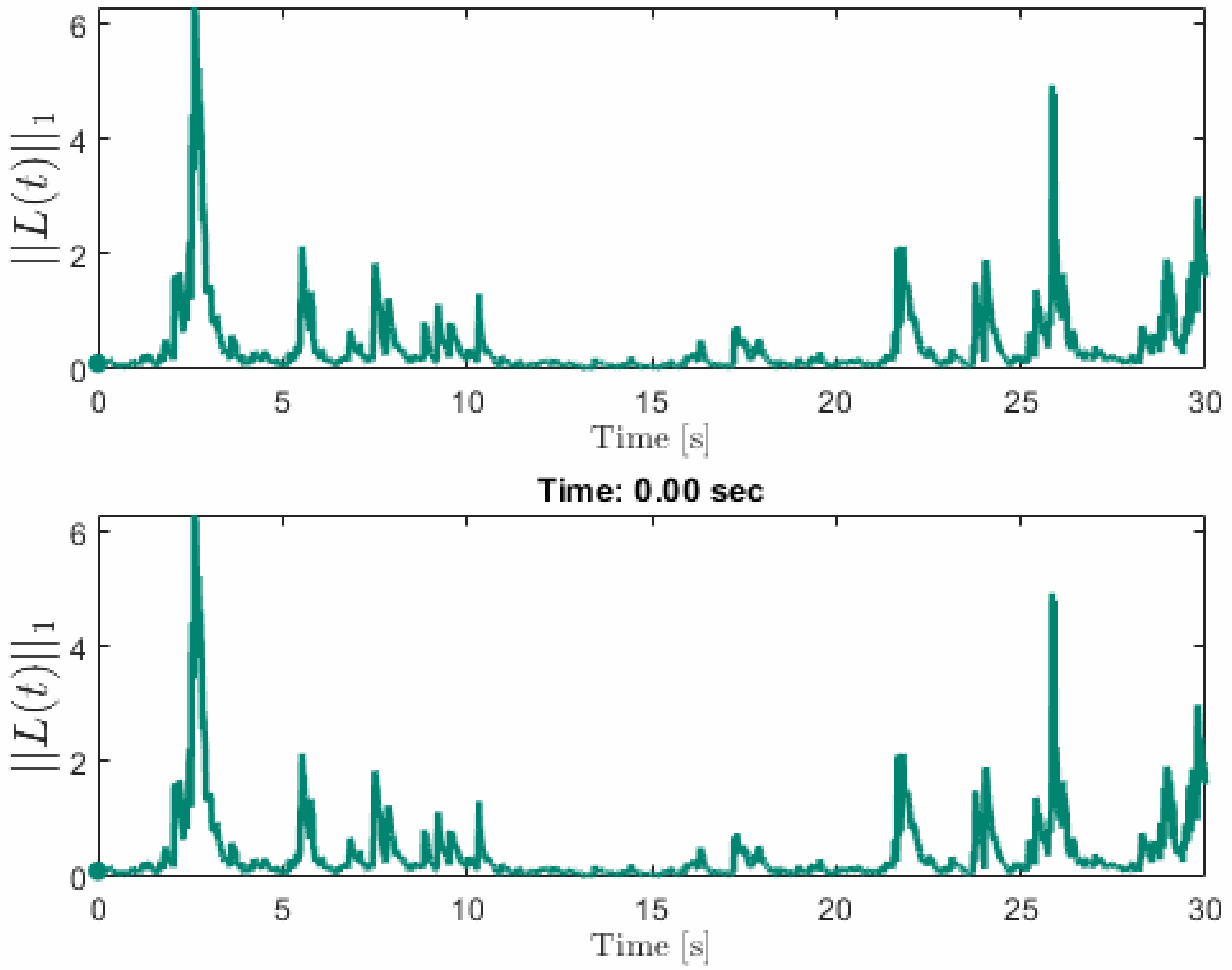
Reconstructing the Brain's Unknown Input

aUIO critically updates model as needed

aUIO on unseen data



Adaptive gain matrix 2-norm



Reconstructing the Brain’s Unknown Input

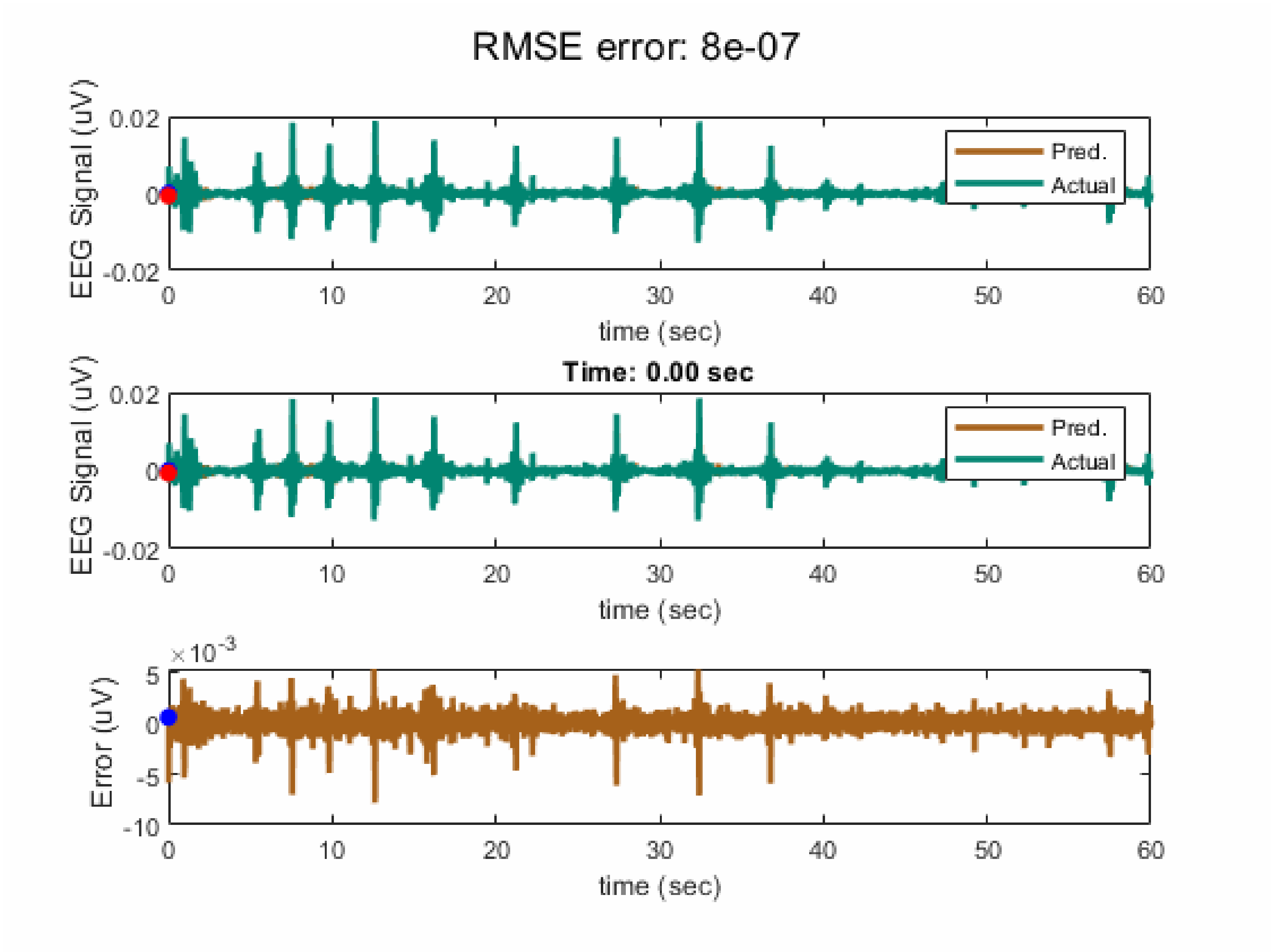
Modeling details

- Unknown input acts evenly over spatial domain
- F_u generates sine-cosine basis
- Static gains per LQR

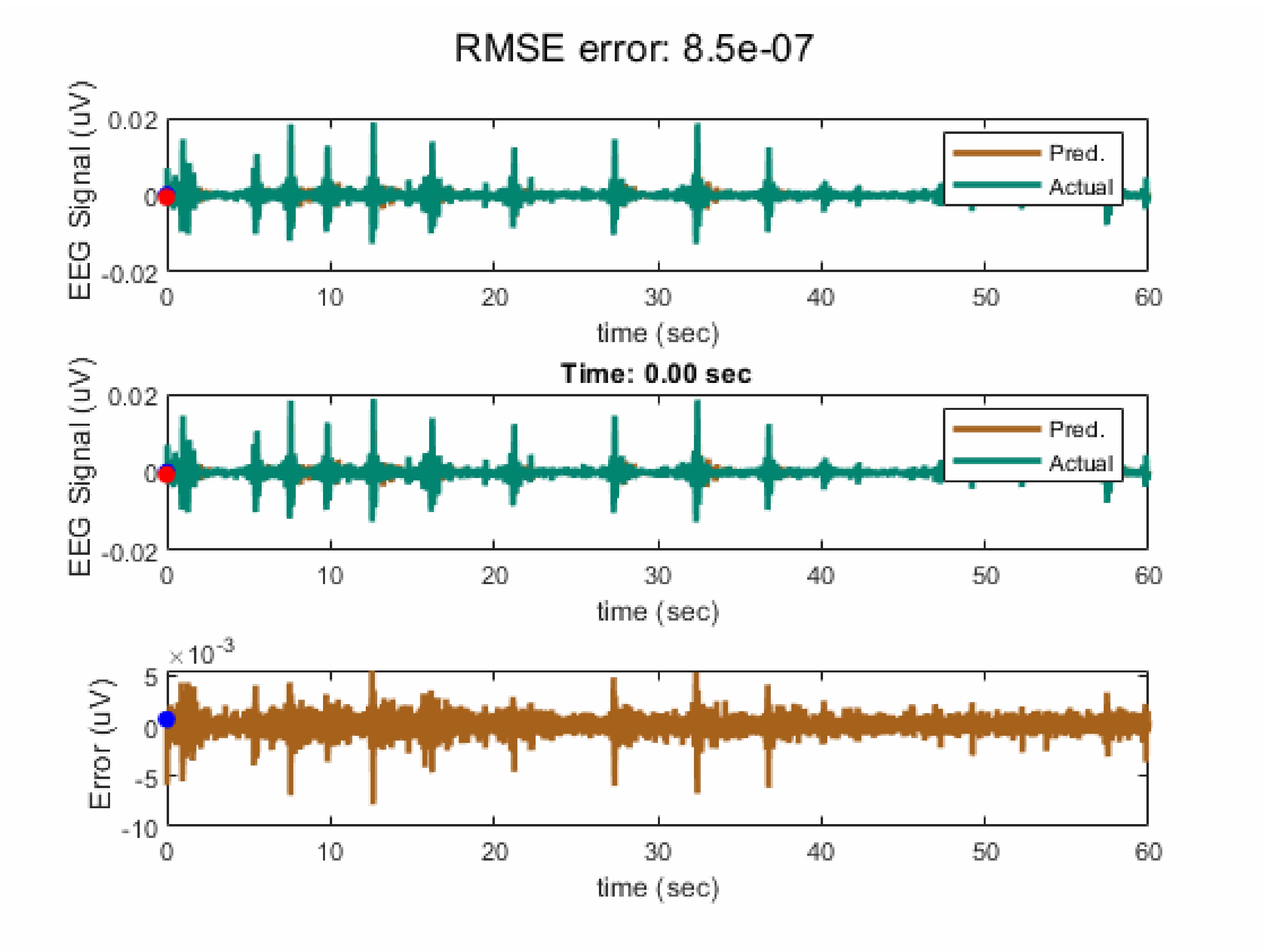
Reconstructing the Brain's Unknown Input

aUIO is tolerant to parametric uncertainty in modes

aUIO on unseen data

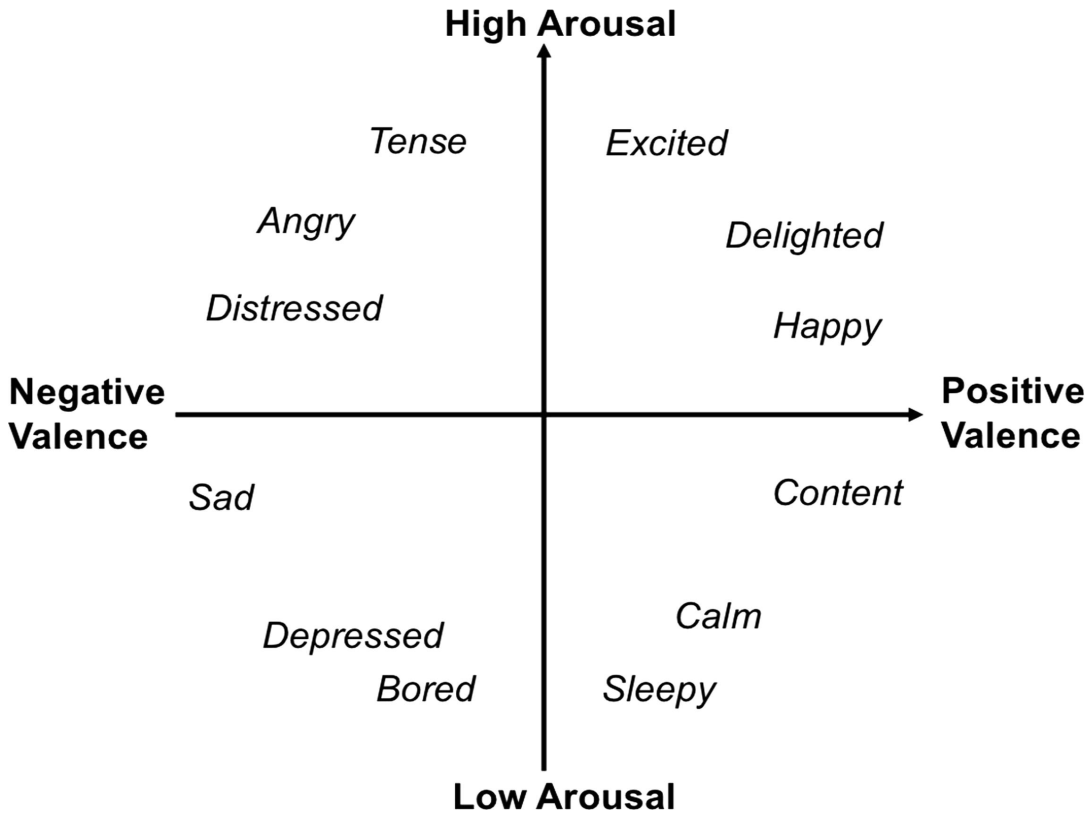


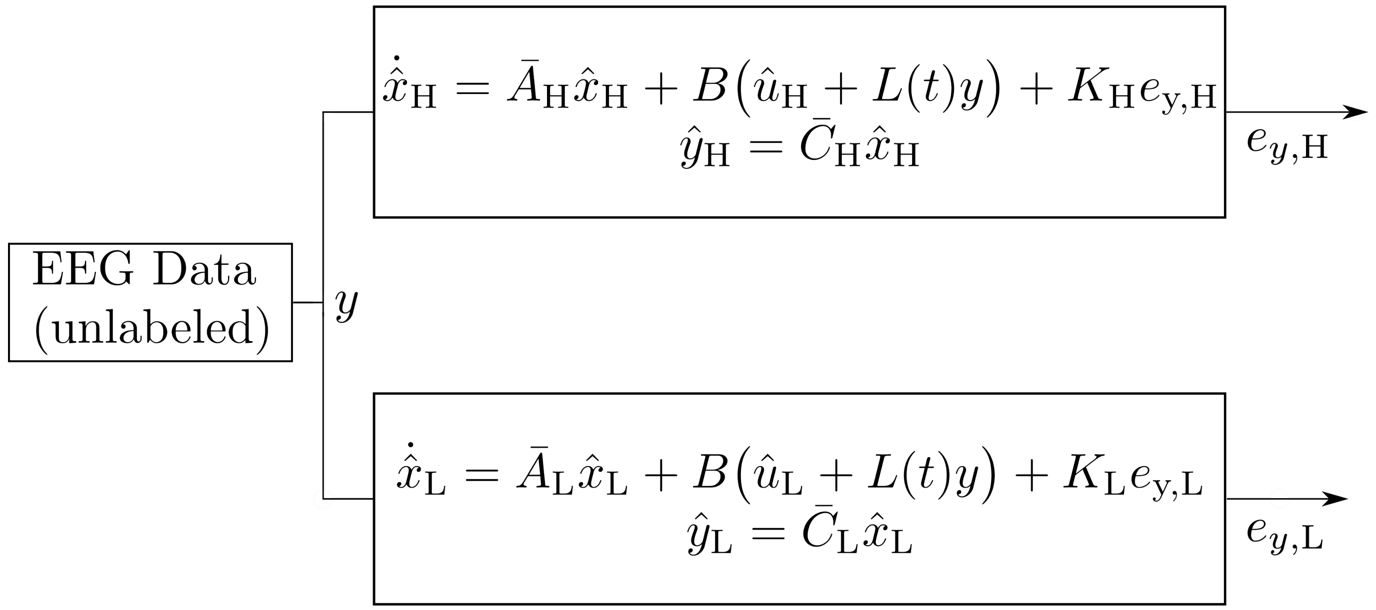
aUIO with wrong A_m



Reconstructing the Brain's Unknown Input

Classification via estimation

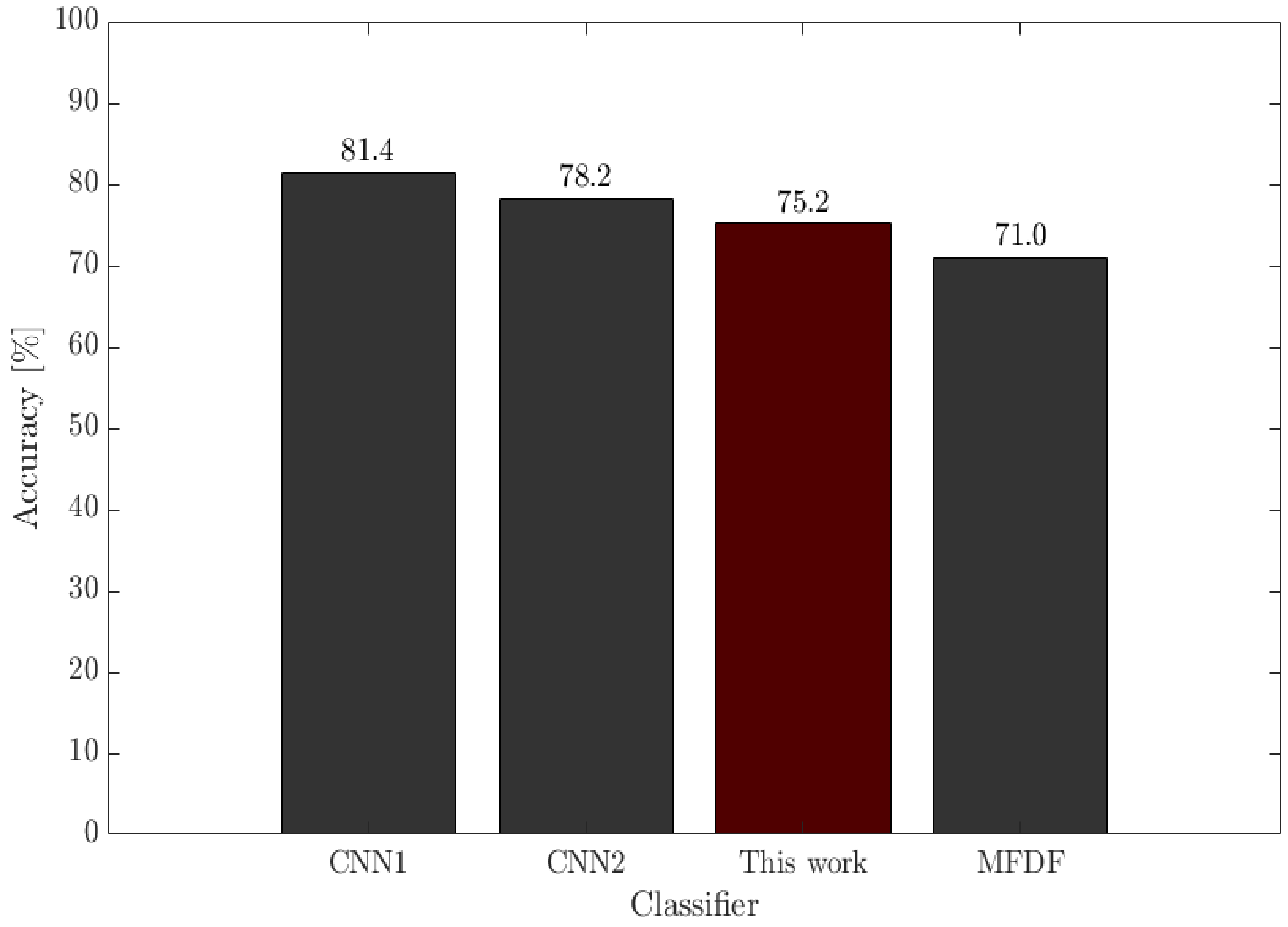




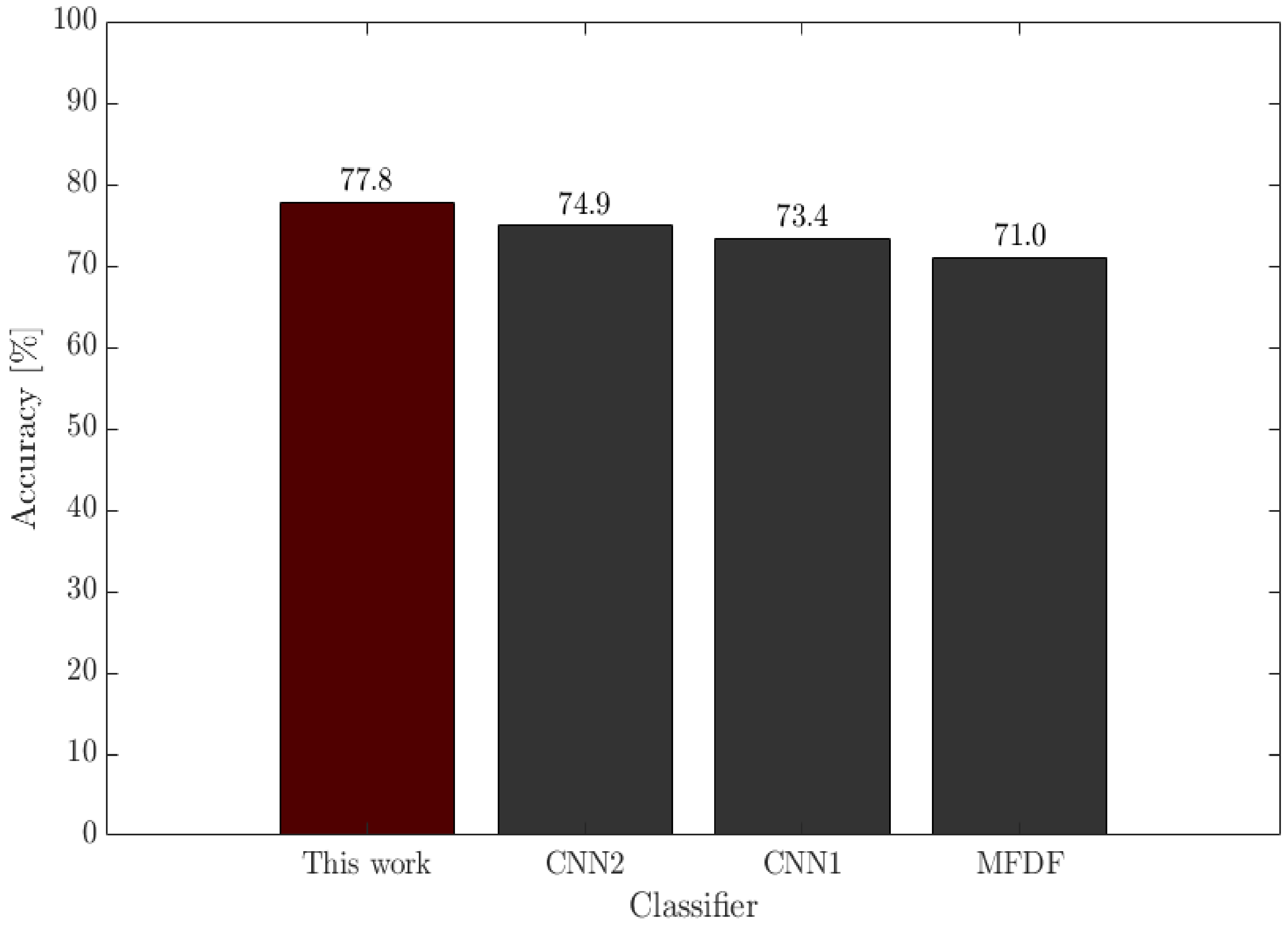
Reconstructing the Brain's Unknown Input

Classification via estimation

Valence Classification



Arousal Classification



CNN1, CNN2, MFDF

7. Conclusions

Summary

- Modern, sys-id techniques work on biomarker data
- Modal representation aids interpreation and analysis
- Complete body of UIO work
- Online estimation of nonlinear brain wave dynamics

Future work

- Multiple data types
- Improved analysis and classification
- Probablistic considerations

Acknowledgements

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> *The willow submits to the wind and prospers until one day it is many willows - a wall against the wind.*