Adaptive Control is Not Complicated

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Seminar Interview

July 30, 2022

Outline

Why Study Adaptive Control?

Why Study Adaptive Control?

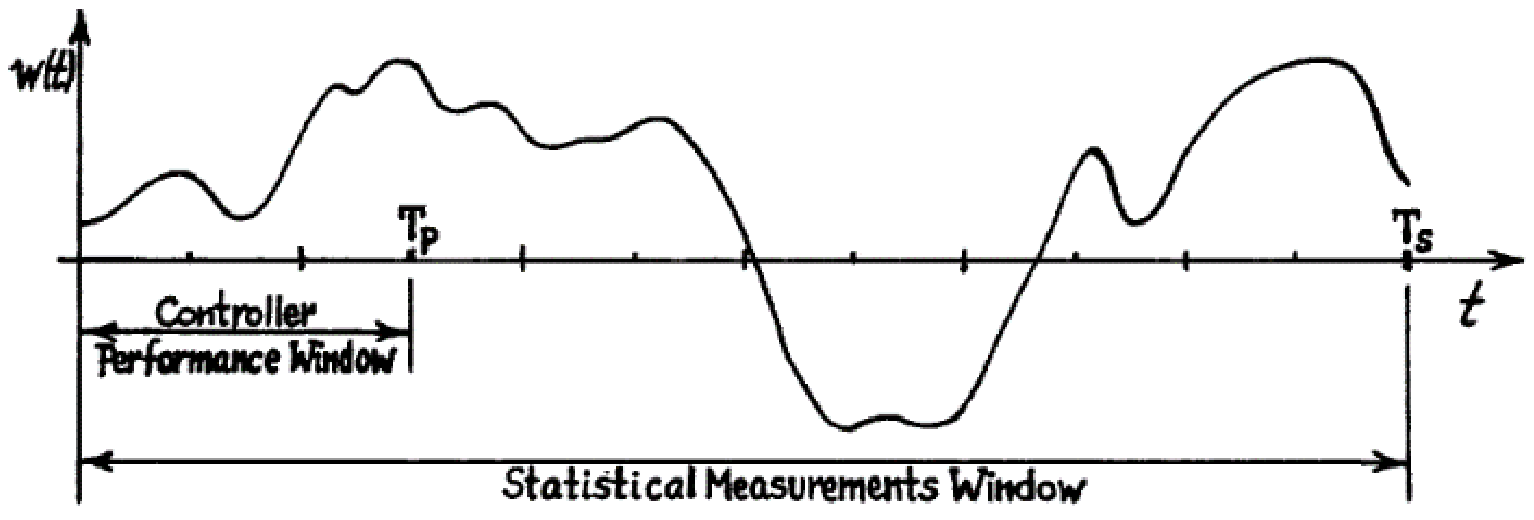
What this talk is and what it is not

- This talk is not:
  - An argument for adaptive control in every system always
  - A chance for me to look smart by being confusing
  - An overview of everything adaptive control
- This talk is:
  - Something I like talking about
  - Technically relevant
  - Presents compelling theoretical challenges

Why Study Adaptive Control?

Some perspectives

- Classical vs. stochastic vs. adaptive control
- Flight and Space Structure Needs:
  - Operating in a poorly known environment
  - Are experiments equivalent to actual operation?
  - Many degrees of freedom
  - Finite element models are only as good as the physics
  - Changing situations: takeoff, deployment, landing
  - Control schemes based on reduced order models
- Greatly emphasizes local vs. global, linear vs. nonlinear thinking



Gambling in function space from [1].

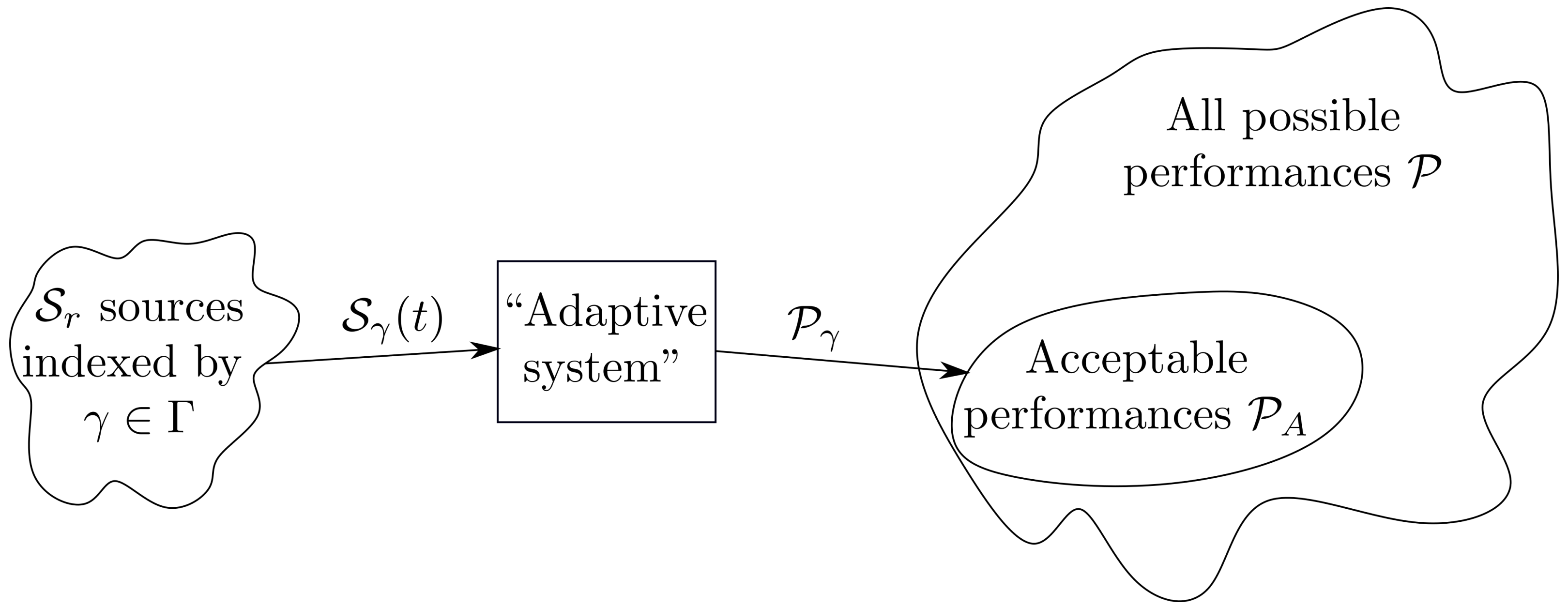
Why Study Adaptive Control?

Defining an adaptive system

- Conceptually:
  - A system with knowledge of its performance and the potency to improve it.
- OR, more mathematically

> *A map $\mathcal{J}$ from $\mathcal{S}_r$ to $\mathcal{P}$ ($\mathcal{J} : \mathcal{S}_r \to \mathcal{P}$) with range $\mathcal{J}(\mathcal{S}) \subseteq \mathcal{P}_A$*

  - Remark: All systems are adaptive in this definition with respect to some $\mathcal{S}_r$ and $\mathcal{P}_A$



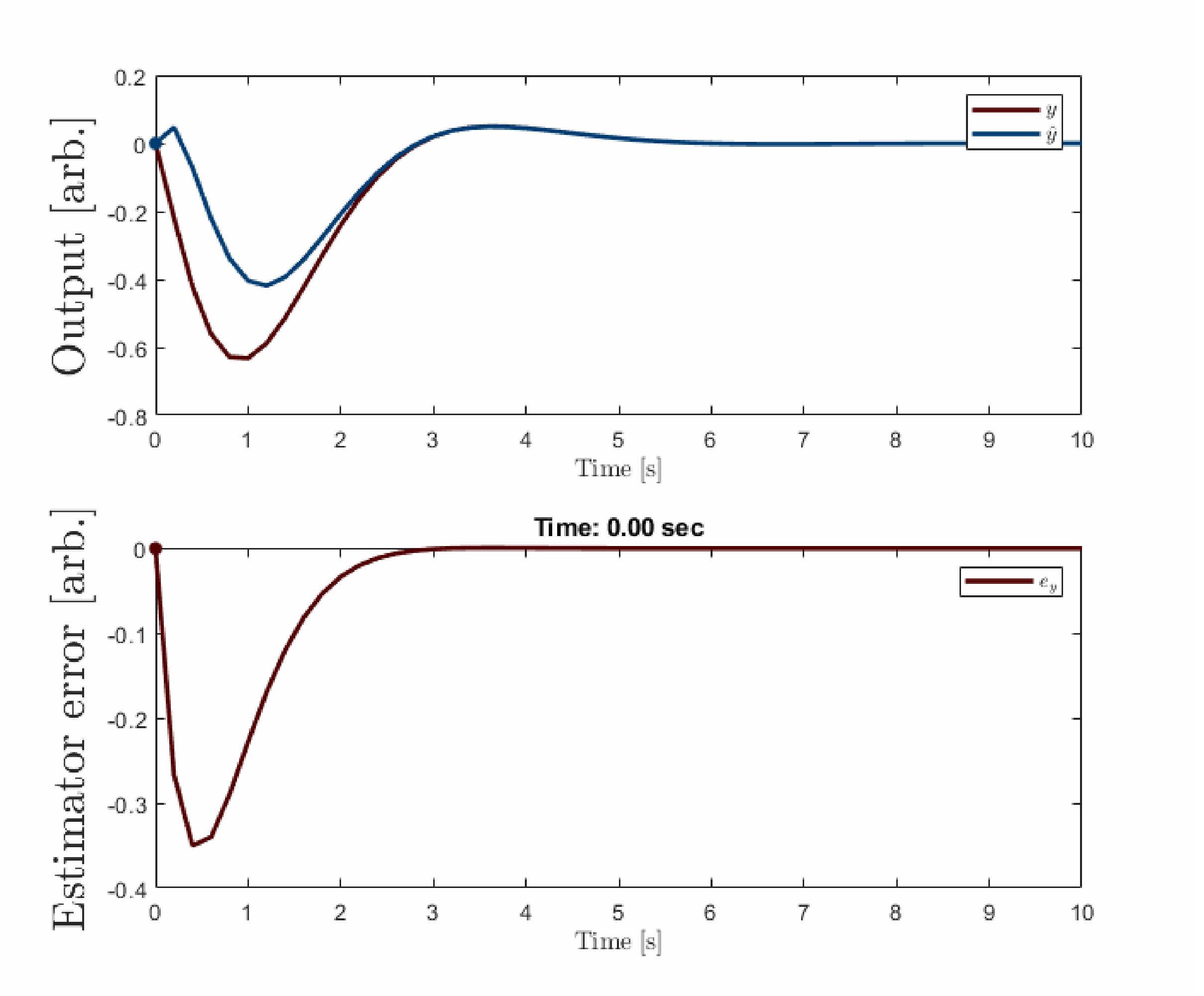
L. Zadeh, "Optimality and non-scalar-valued performance criteria [2].

Augmentation example

Augmentation example

Recovering $\mathcal{P}_A$

- Double integrator:
  - $\dot{x} = Ax + Bu,\ y = Cx$
  - $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
  - Min. phase with $Z(A, B, C) = -1$
- Separation principle controller:
  - $u = G\hat{x}$
  - $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
  - $\hat{y} = C\hat{x}$
- With set gains:
  - $\sigma(A + BG) = -1 \pm j \Rightarrow G = \begin{bmatrix} -2 & -2 \end{bmatrix}$
  - $\sigma(A - KC) = -2 \pm j \Rightarrow K = \begin{bmatrix} -1 & 5 \end{bmatrix}^T$

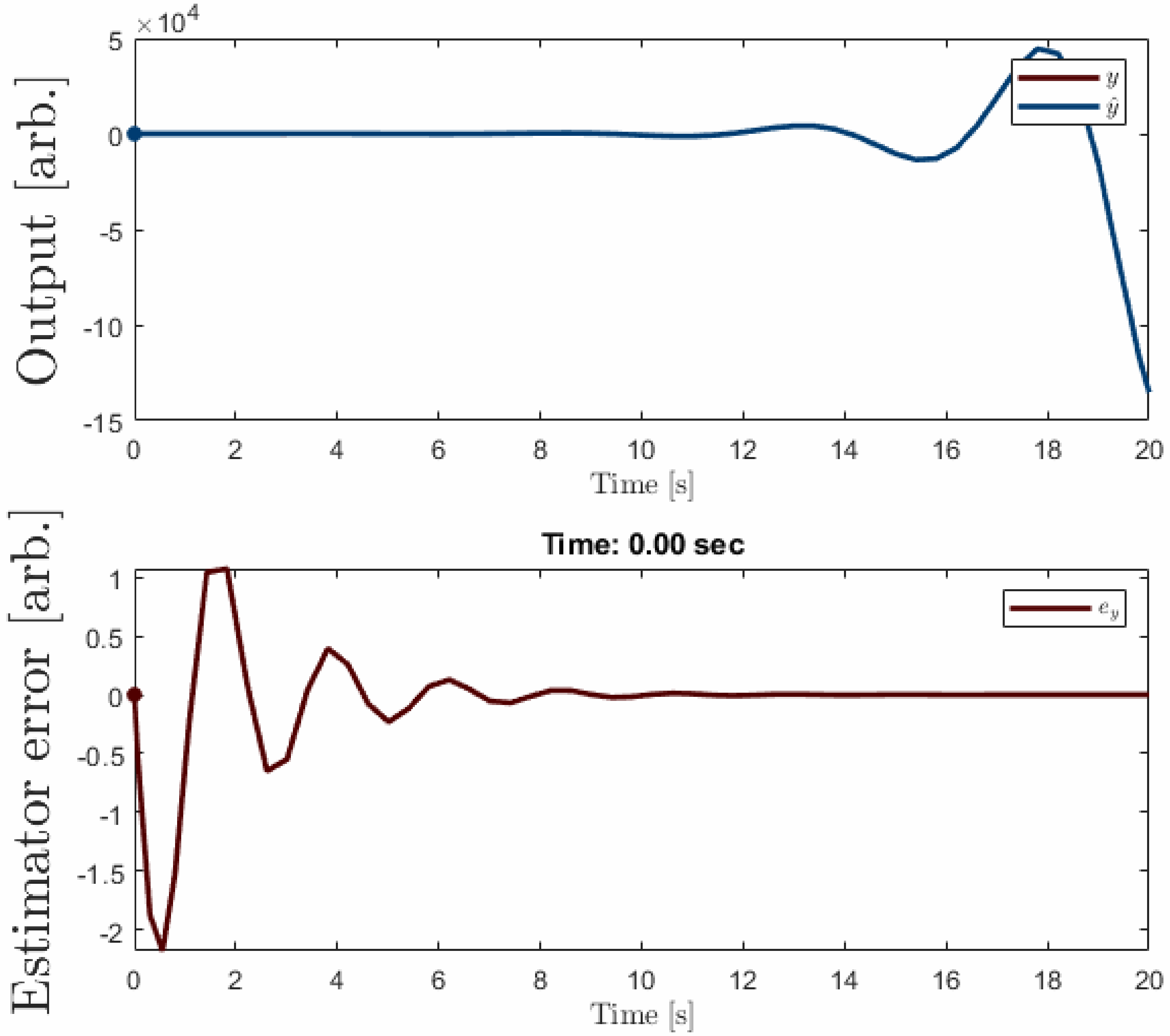


Separation principle controller is stable.

Augmentation example

Recovering $\mathcal{P}_A$

- But suppose A became $\tilde{A}$:
  - $\dot{x} = \tilde{A}x + Bu, \; y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
  - Min. phase with $Z(A, B, C) = -1$
- Separation principle controller:
  - $u = G\hat{x}$
  - $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
  - $\hat{y} = C\hat{x}$
- With set gains:
  - $\sigma(\tilde{A} + BG) = 0.5 \pm 1.3j \Rightarrow G = \begin{bmatrix} -2 & -2 \end{bmatrix}$
  - $\sigma(\tilde{A} - KC) = -0.5 \pm 2.7j \Rightarrow K = \begin{bmatrix} 4 & 5 \end{bmatrix}^T$

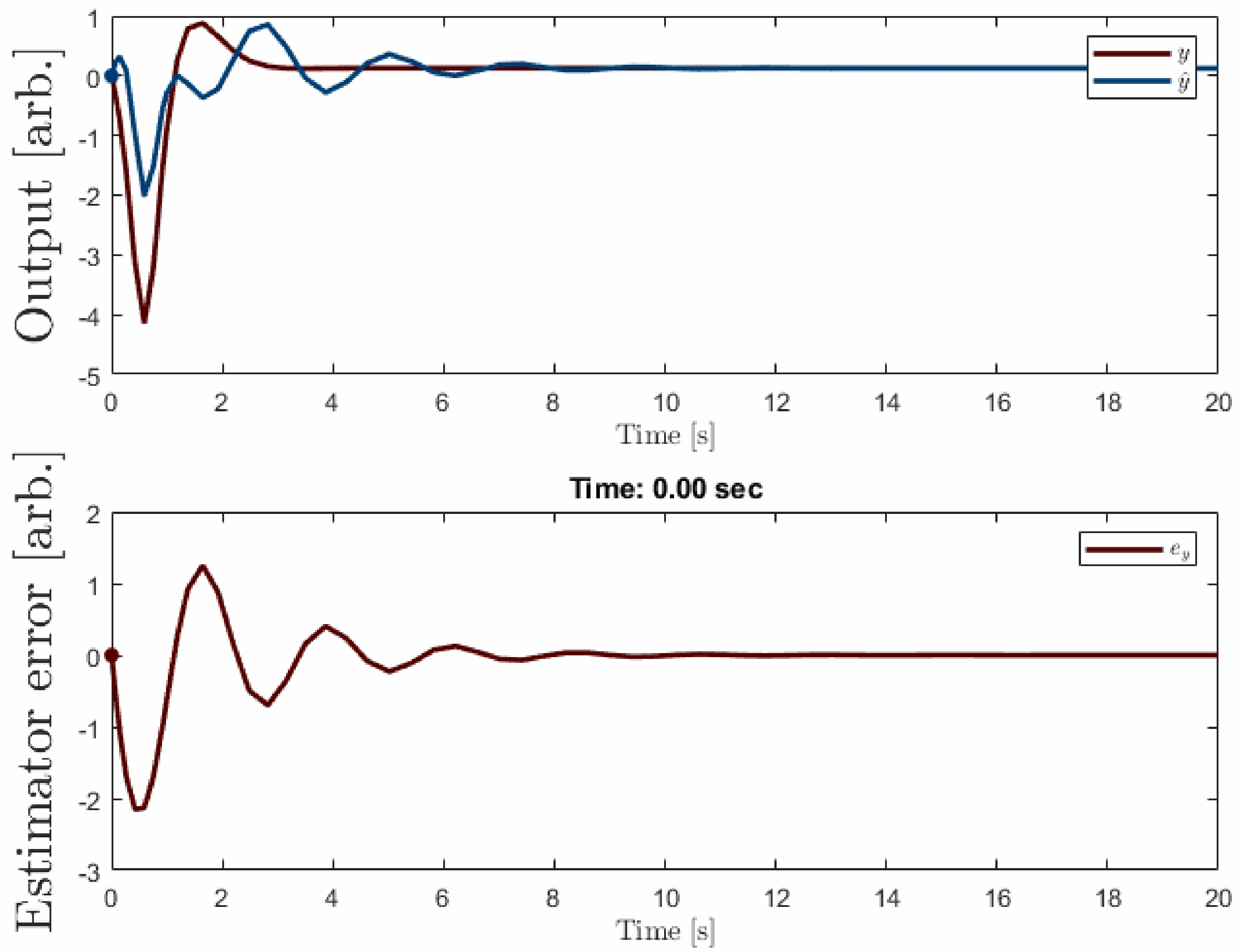


Perturbed separation principle controller is not stable.

Augmentation example

Recovering $\mathcal{P}_A$

- But suppose A became $\tilde{A}$ and I have augmented the system with an adaptive outer loop:
  - $\dot{x} = \tilde{A}x + Bu,\ y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
  - Min. phase with $Z(A, B, C) = -1$
- Adaptive separation principle controller:
  - $u = G\hat{x} + G_A y$
  - $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
  - $\hat{y} = C\hat{x}$
- With same set gains and adaptive law:
  - $\dot{G}_A = -yy^T\sigma,\ \sigma > 0$

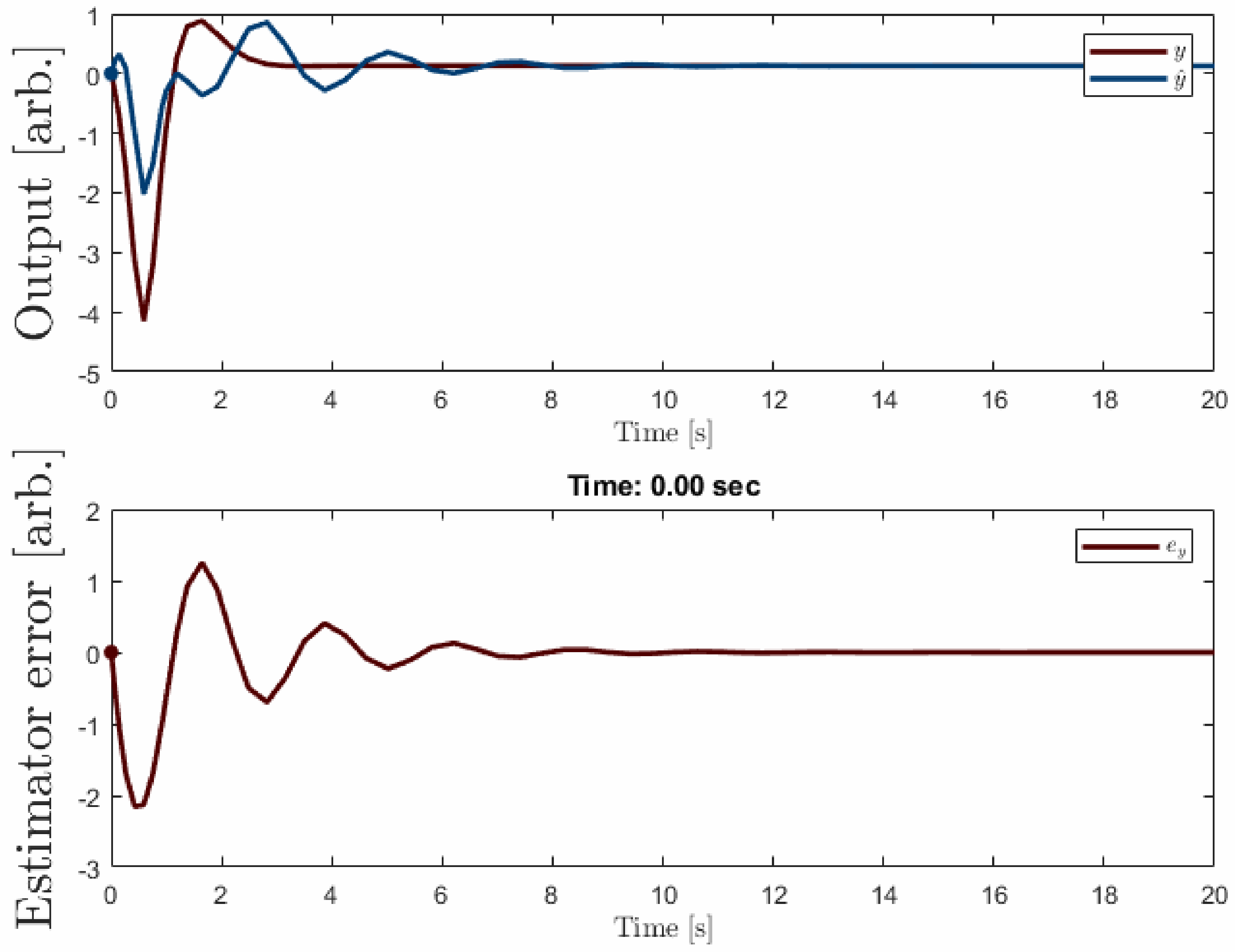


Adaptive separation principle controller is stable.

Augmentation example

Recovering $\mathcal{P}_A$

- Does this happen with gain scheduled controllers?
- We treated a significant constant perturbation adaptively
- Remark: Adaptive controllers are especially good at handling significant, slower disturbances
  - Robust controllers are especially good at small, fast disturbances
  - $\therefore$ **we should generally consider the adaptive augmentation of robust controllers.**



Adaptive separation principle controller is stable.

Adaptive Control is not Complicated

Adaptive Control is not Complicated

- Given:
  - $\begin{cases} \dot{x} & = Ax + Bu \\ y & = Cx \end{cases}$
  - (A, B, C) ctrb/obsv (i.e. minimal description of $P(s) = C(sI - A)^{-1}B$)
- Recall Kimura-Davison sufficient conditions:
  - $M \equiv \text{rank } B = \text{rank } C = M$ (square)
  - (A, B, C) ctrb/obsv
  - $M + P > N = \dim x$
    - $\exists G_* \ni \sigma(A + BG_*C)$ that assigns pole locations arbitrarily

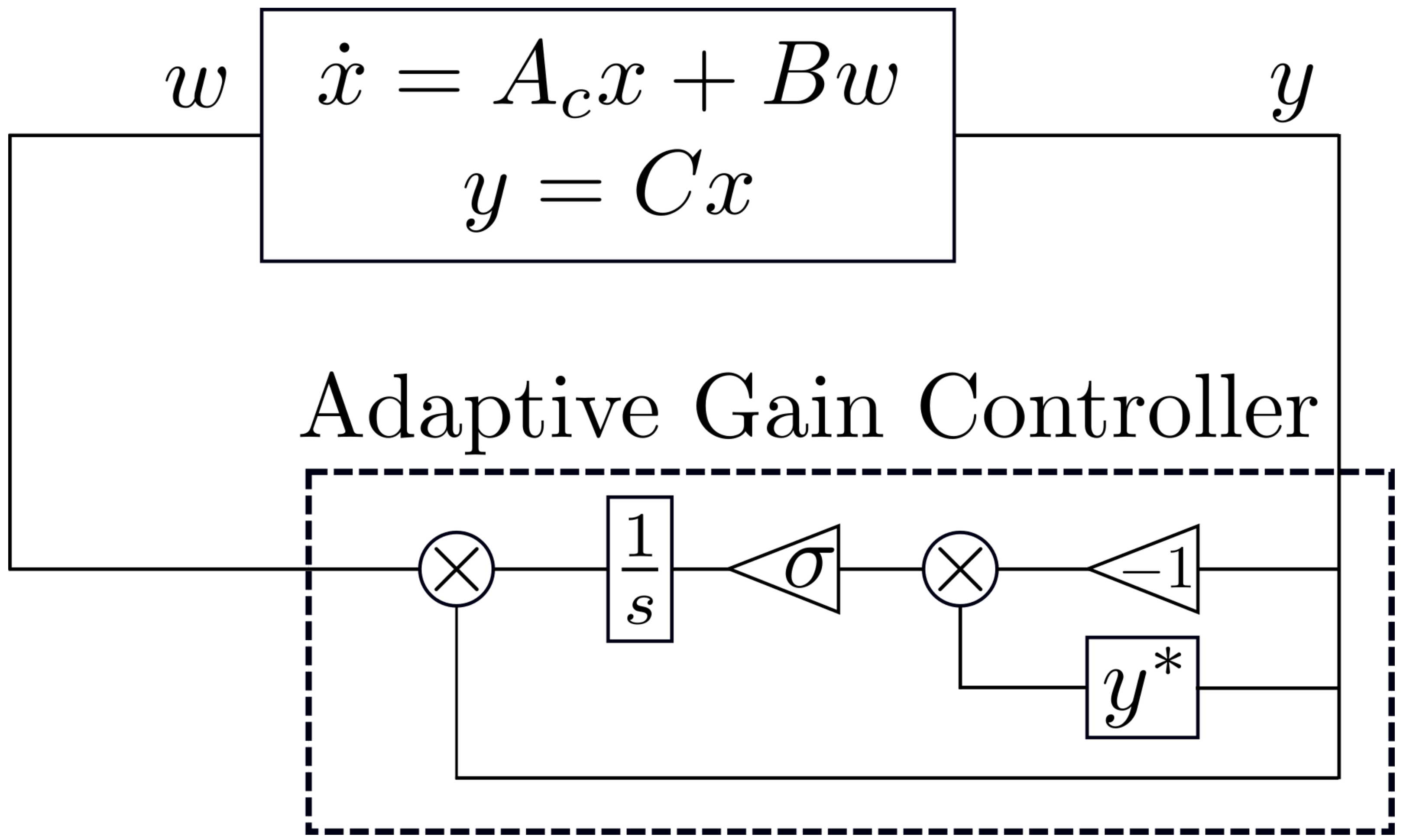
Adaptive Control is not Complicated

- Sufficient conditions for arb. pole placement but we must **know** (A, B, C) in detail to find G_*!
- This can be onerous, but if G_* exists, the system is called output feedback stabilizable
- Ex:
  - $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & \varepsilon \end{bmatrix}$
  - With $G_* = -g$, $A + BGC = \begin{bmatrix} 0 & 1 \\ -g & -g\varepsilon \end{bmatrix}$
  - $\det(\lambda I - A_c) = \lambda^2 + g\varepsilon\lambda + g$
  - $\therefore$ output feedback stabilizable when $\varepsilon > 0$ only!
    - Note: $\exists P > 0 \ni A_c^T P + PA_c = -Q > 0$

Adaptive Control is not Complicated

Adaptive Regulator using Output Feedback Only

- Plant: $\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases}$ (square)
- Regulator: $\begin{cases} u &= Gy \\ \dot{G} &= -yy^T\sigma \end{cases}$
- Let $G \equiv G_* + \Delta G$. Closed loop system is:
  - $\begin{cases} \dot{x} &= \underbrace{(A + BG_*C)}_{A_c}x + B\underbrace{\Delta G y}_{w} \\ y &= Cx \\ \Delta\dot{G} &= \dot{G} = -yy^T\sigma,\ \sigma > 0 \end{cases}$

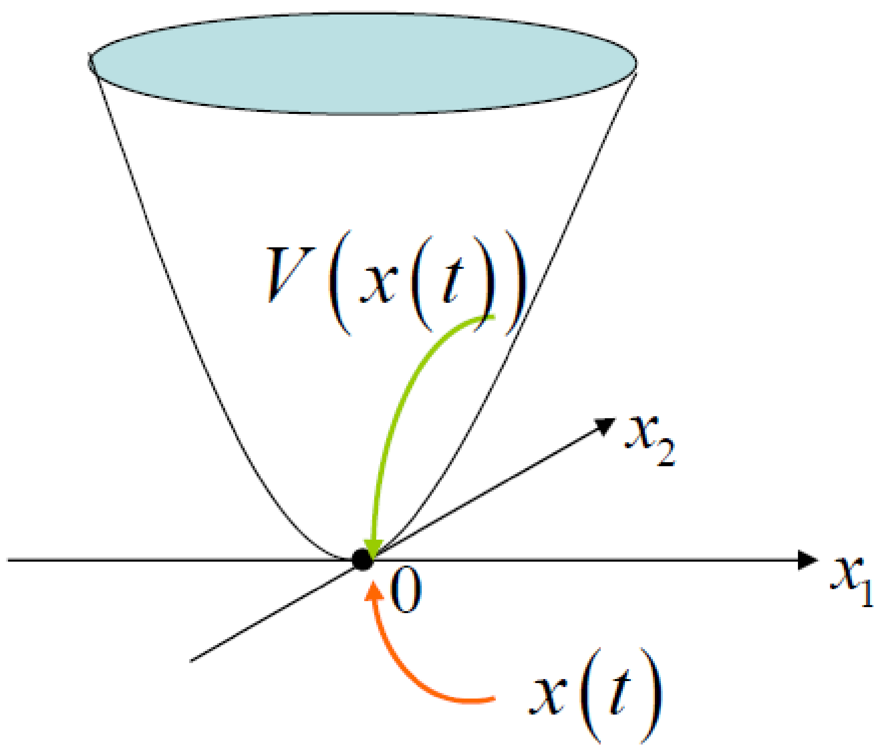


Adaptive regulator architecture.

Adaptive Control is not Complicated

Lyapunov Stability Argument

- If a scalar function $V(x,t)$ satisfies
  - function is lower bounded
  - Time derivative $\dot{V}(x,t)$ is negative semidefinite: $\sigma\left(\dot{V}(x,t)\right) \leq 0$
  - Time derivative $\dot{V}(x,t)$ is uniformly continuous in t: derivative is bounded
- Then $\lim_{t\to\infty} \dot{V}(x,t) = 0$
- and we have a theoretical stability guarantee.

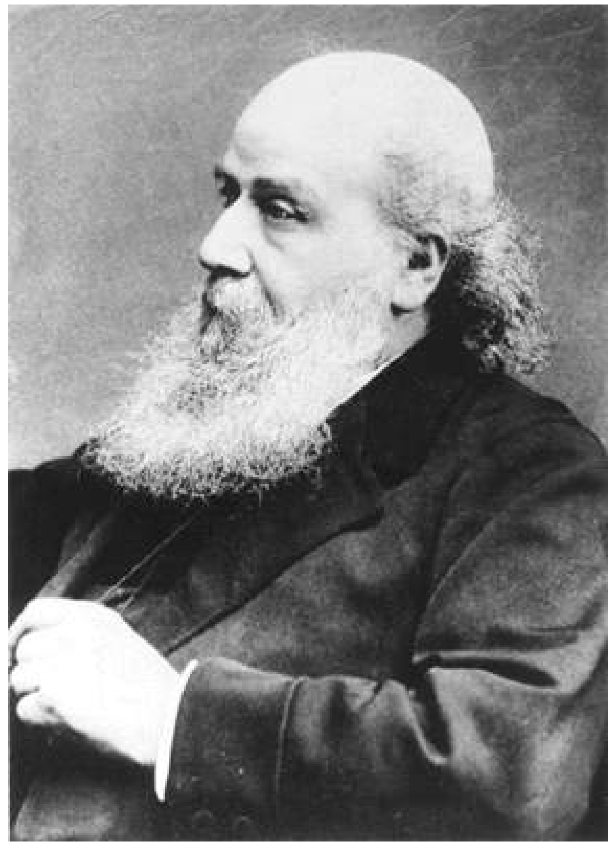


Example Lyapunov candidate function

Adaptive Control is not Complicated

Lyapunov Stability Argument

- Here, P from $A_c^T P + PA_c = -Q$ yields a quadratic, lower bounded function
  - $\frac{\lambda_{\min}(P)}{2}||x||^2 \leq V_1(x) \equiv \frac{1}{2}x^* P x \leq \frac{\lambda_{\max}(P)}{2}||x||^2$
  - which meets our first requirement.
- Notice
  - $$\begin{aligned} \dot{V}_1(x) \equiv \triangle V_1 \dot{x} &= x^* P[A_c x + Bw] \\ &= x^* P A_c x + x^* P B w \\ &\leq -\frac{1}{2} x^* Q x + x^* C^* w \\ &\leq -1/2 \lambda_{\min}(Q) ||x||^2 + (y, w) \end{aligned}$$
  - which may or may not be negative semidefinite, but is bound.



James Joseph Sylvester

Adaptive Control is not Complicated

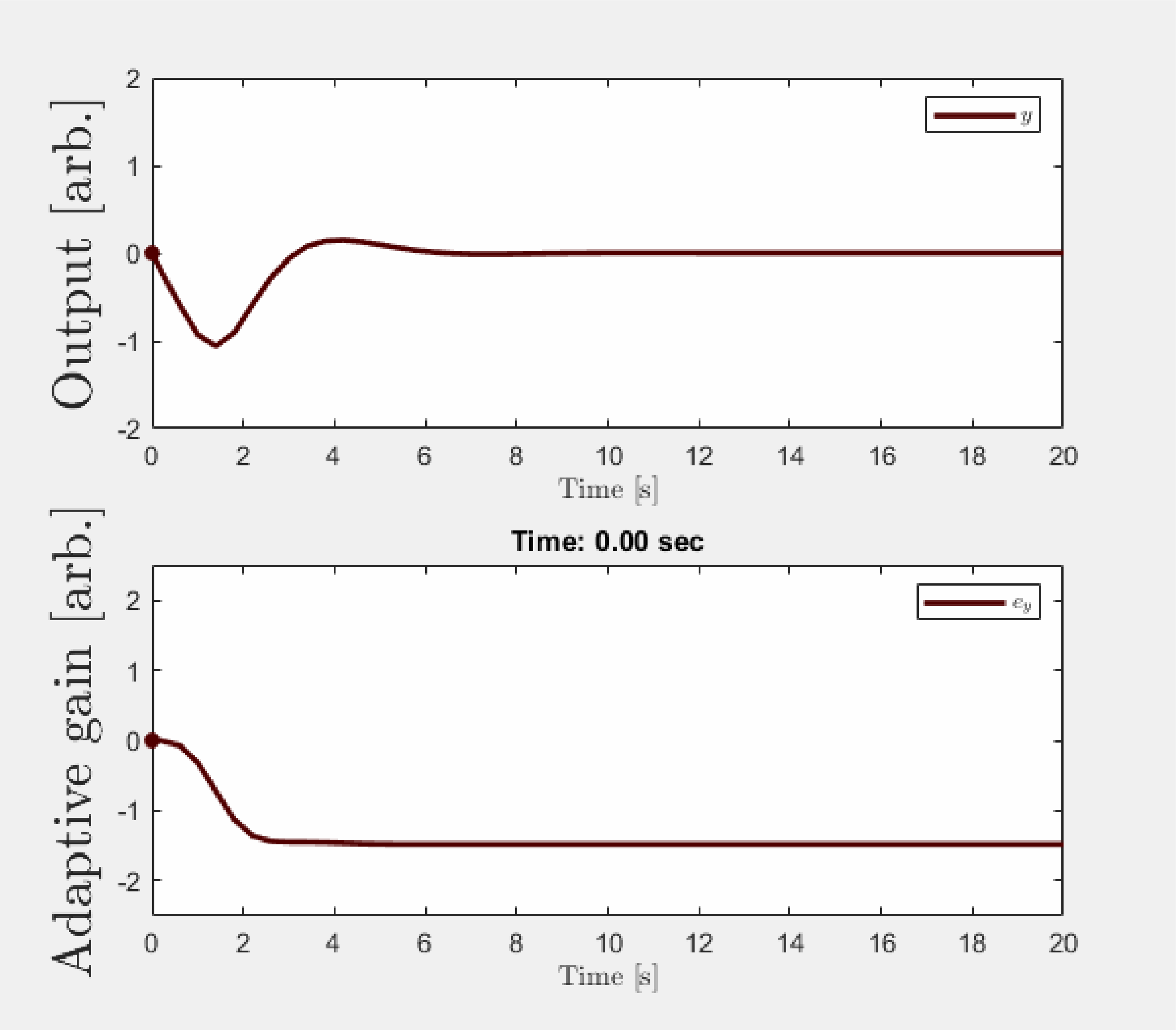
Lyapunov Stability Argument

- $\dot{V}_1(x) \leq -1/2\lambda_{\min}(Q)||x||^2 + (y, w)$
  - which may or may not be negative semidefinite, but is bound.
- However, we have not checked the stability of the adaptive gain G
  - Consider $V_2(\Delta G) \equiv \frac{1}{2}\mathrm{tr}(\Delta G \sigma^{-1} \Delta G^*)$
  - $$\begin{aligned}\dot{V}_2 &= \mathrm{tr}(\Delta\dot{G}\sigma^{-1}\Delta G^*) \\ &= \mathrm{tr}(-yy^*\sigma\sigma^{-1}\Delta G^*) \\ &= -\mathrm{tr}(y\underbrace{y^*\Delta G^*}_{w^*}) = -\mathrm{tr}(w^* y) \text{ scalar!} \\ &= -(y, w)\end{aligned}$$
- Which "conveniently" yields:
  - $$\begin{aligned}\dot{V}(x, \Delta G, t) &= \dot{V}_1(x, t) + \dot{V}_2(\Delta G, t) \\ &\leq -1/2\lambda_{\min}(Q)||x||^2 + (y, w) - (y, w) \\ &\leq -1/2\lambda_{\min}(Q)||x||^2\end{aligned}$$
- Since x, G are now bound, composite system is bound. V is negative semidefinite. Therefore, by Lyapunov, $x \Rightarrow 0$.

Augmentation example

Double integrator example

- Returning to our double integrator example:
  - $\dot{x} = \tilde{A}x + Bu,\ y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
- Adaptive regulator:
  - $u = Gy$
- With adaptive law:
  - $\dot{G} = -yy^T\sigma,\ \sigma > 0$



Adaptive controller is stable.

Augmentation example

Double integrator example

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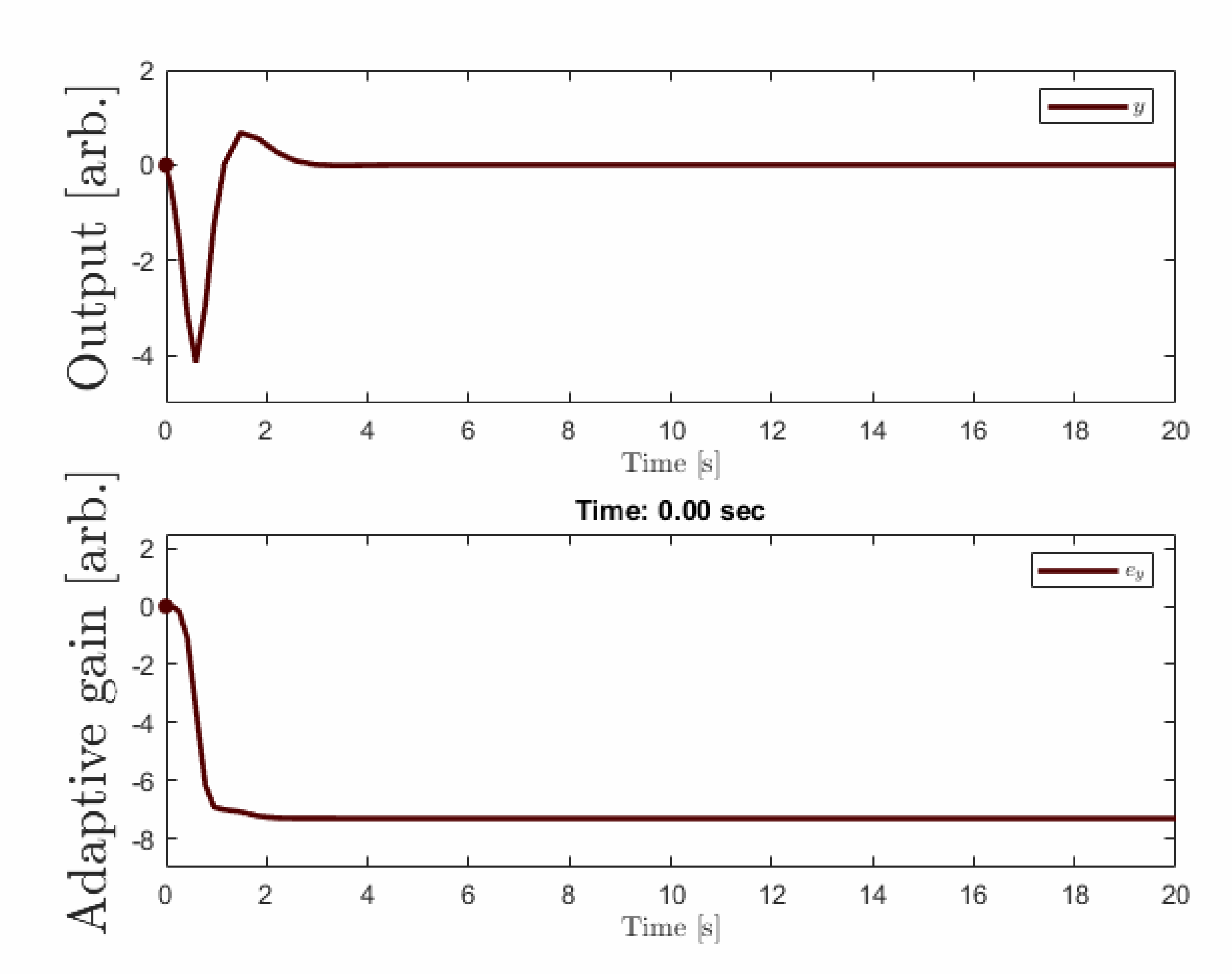
  - $\dot{x} = \tilde{A}x + Bu,\ y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
- Adaptive regulator:
  - $u = Gy$
- With adaptive law:
  - $\dot{G} = -yy^T\sigma,\ \sigma > 0$

Achieve exponential stability with exactly the same controller!



Same controller is stable for a different plant.

Adaptive Unknown Input Estimators

Adaptive Unknown Input Estimators

Estimator overview

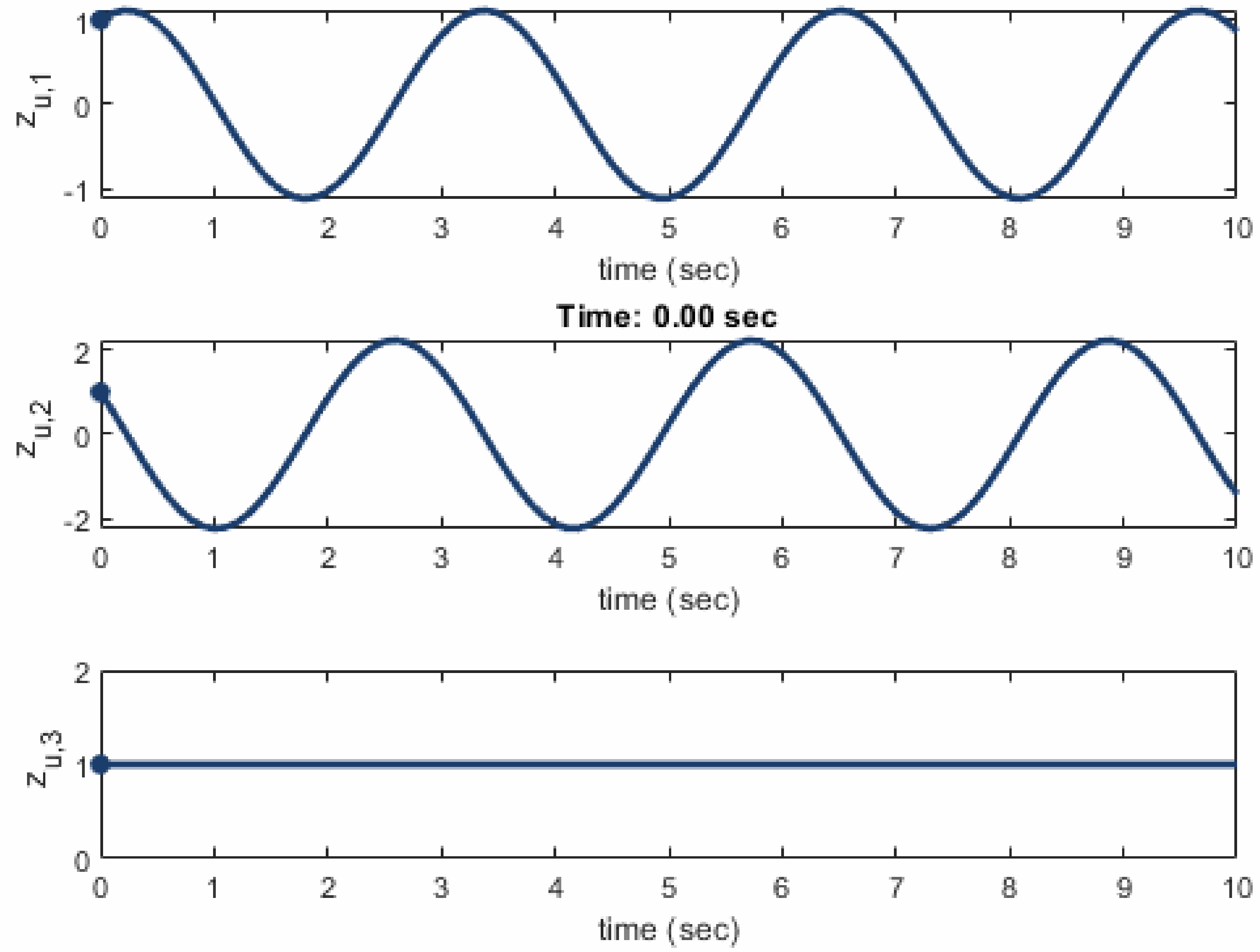
- Three significant uncertainties
  - Input u is unknown, external, deterministic
  - State matrix A may have uncertainty
  - Known, Lipschitz nonlinear internal dynamics $g(x)$
- **Can we synthesize $\boldsymbol{u}$ and correct $\boldsymbol{A}$?**

$$\dot{x} = Ax + g(x) + Bu$$
$$y = Cx$$

Adaptive Unknown Input Estimators

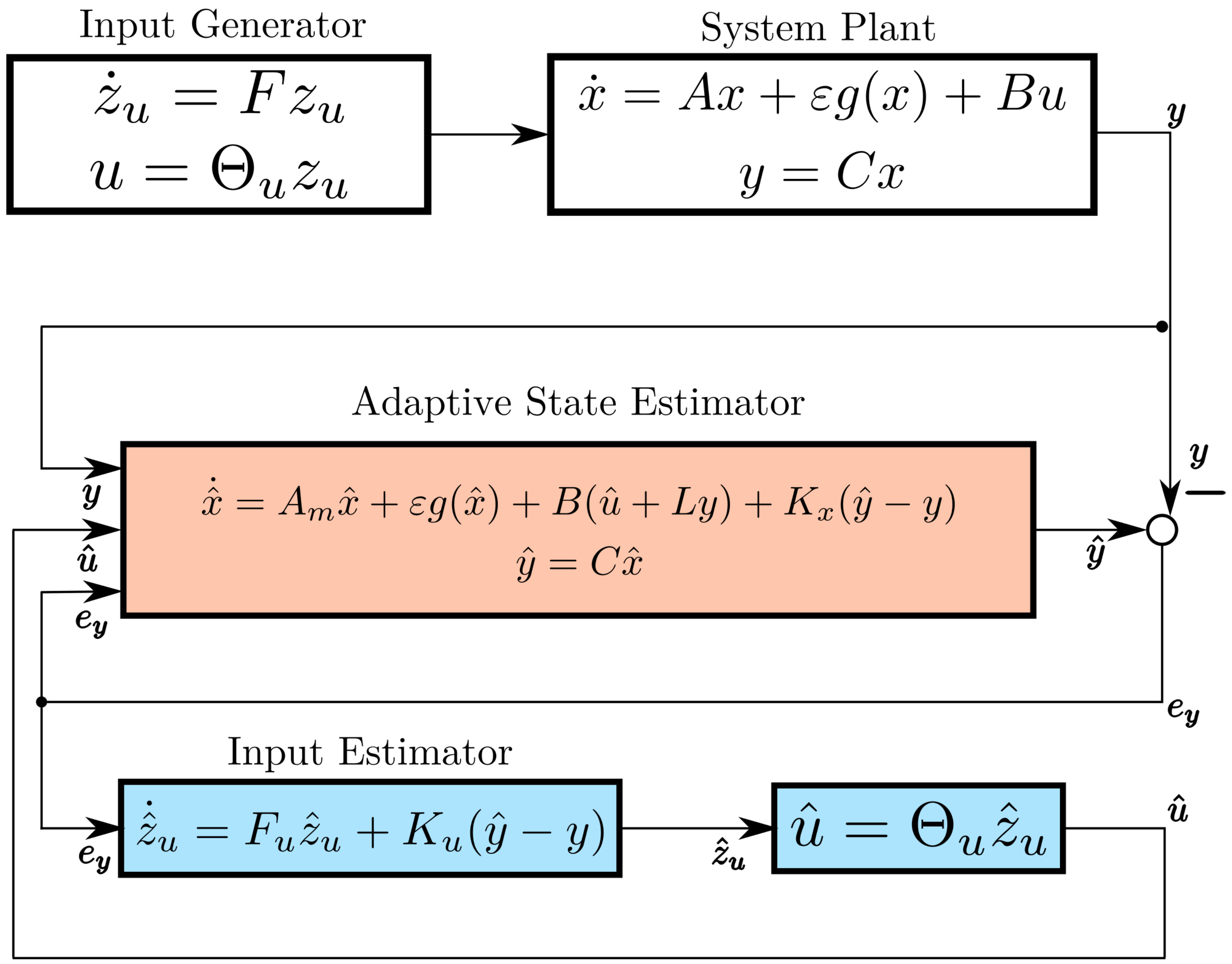
Modeling unknown inputs

- Approximate input space $\mathbb{U}$
  - $\hat{u} = \sum_{i=1}^{N} c_i f_i(t)$
- Persistent Inputs
  - $\dot{z}_u = F_u z_u$
  - $\hat{u} = \Theta_u z_u$
  - $F_u = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$



Adaptive Unknown Input Estimators

Architecture and estimator error



Recover A with adaptive scheme

$$A \equiv A_m + BL_* C$$

$$\dot{L} = -e_y y^* \gamma_e;\ \gamma_e > 0$$

Error dynamics

$$\dot{e} = (\bar{A} + \bar{K}\bar{C})e + \bar{B}\Delta L y + \varepsilon \Delta g$$

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_z \end{bmatrix} = \underbrace{\begin{bmatrix} A_m + K_x C & B\Theta_u \\ K_u C & F \end{bmatrix}}_{\bar{A}_c} \begin{bmatrix} e_x \\ e_z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w + \varepsilon \begin{bmatrix} g(\hat{x}) - g(x) \\ 0 \end{bmatrix}$$

Adaptive Unknown Input Estimators

Architecture and estimator error

- ASD plant dynamics
  - $\bar{A}_c^* \bar{P} + \bar{P}\bar{A}_c = -\bar{Q}$
  - $\bar{P}\bar{B} = \bar{C}^*$
- A Hurwitz
- Bounded L_*

- Error in state and input converges to zero
  - $V(e, \Delta L) = \frac{1}{2} e^* \bar{P} e + \frac{1}{2} \mathrm{tr}(\Delta L \gamma_e^{-1} \Delta L^*)$
  - $\dot{V}(e, \Delta L) \leq -\underbrace{\left(\frac{1}{2}\lambda_{\min}(\bar{Q}) - \varepsilon\mu\lambda_{\max}(\bar{P})\right)}_{\bar{\alpha}>0} ||e||^2$

$$0 < \varepsilon < \frac{\lambda_{\min}(\bar{Q})}{2\mu\lambda_{\max}(\bar{P})} \iff \bar{\alpha} > 0.$$

Illustrative example

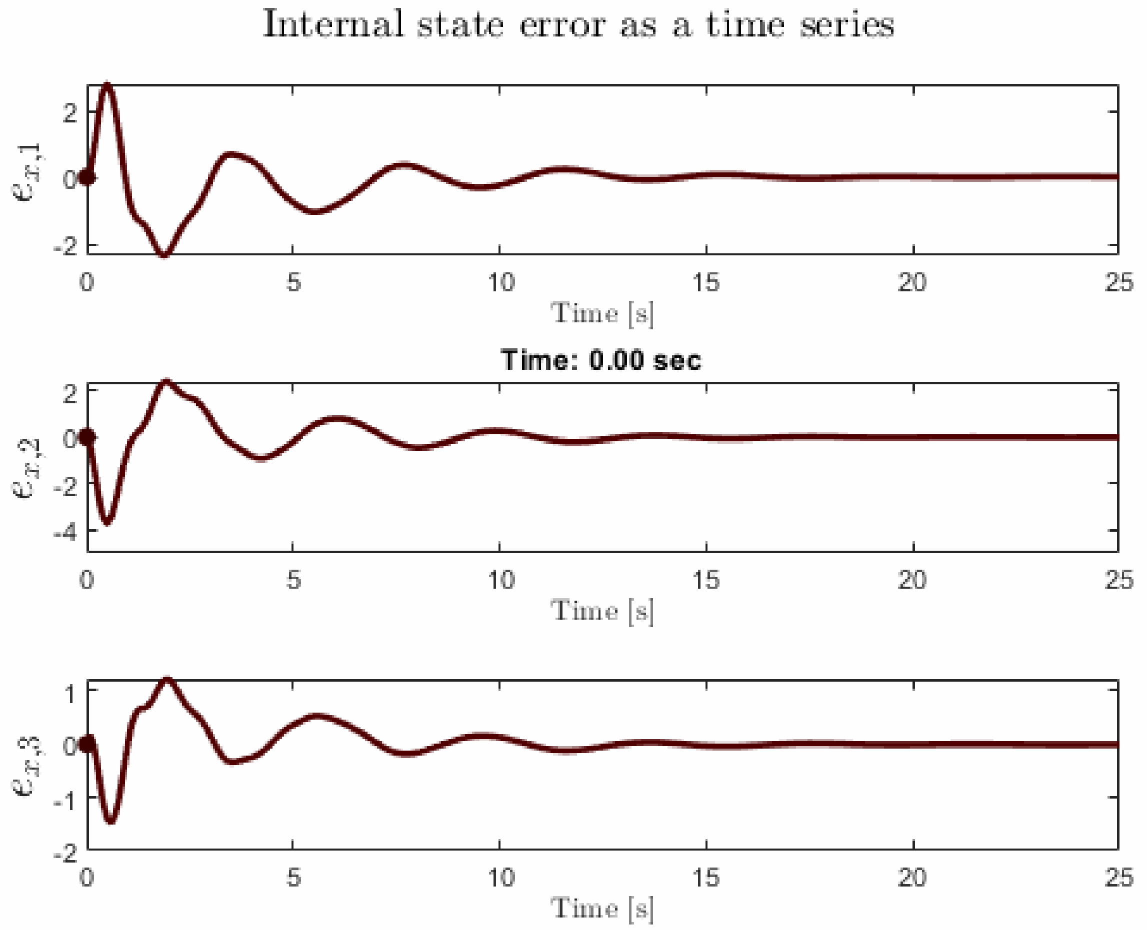
$$\dot{x} = A_m x + \varepsilon g(x) + Bu$$
$$= \begin{bmatrix} -4 & 1 & 2 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} x + \sin(x) + Bu$$
$$y = Cx$$

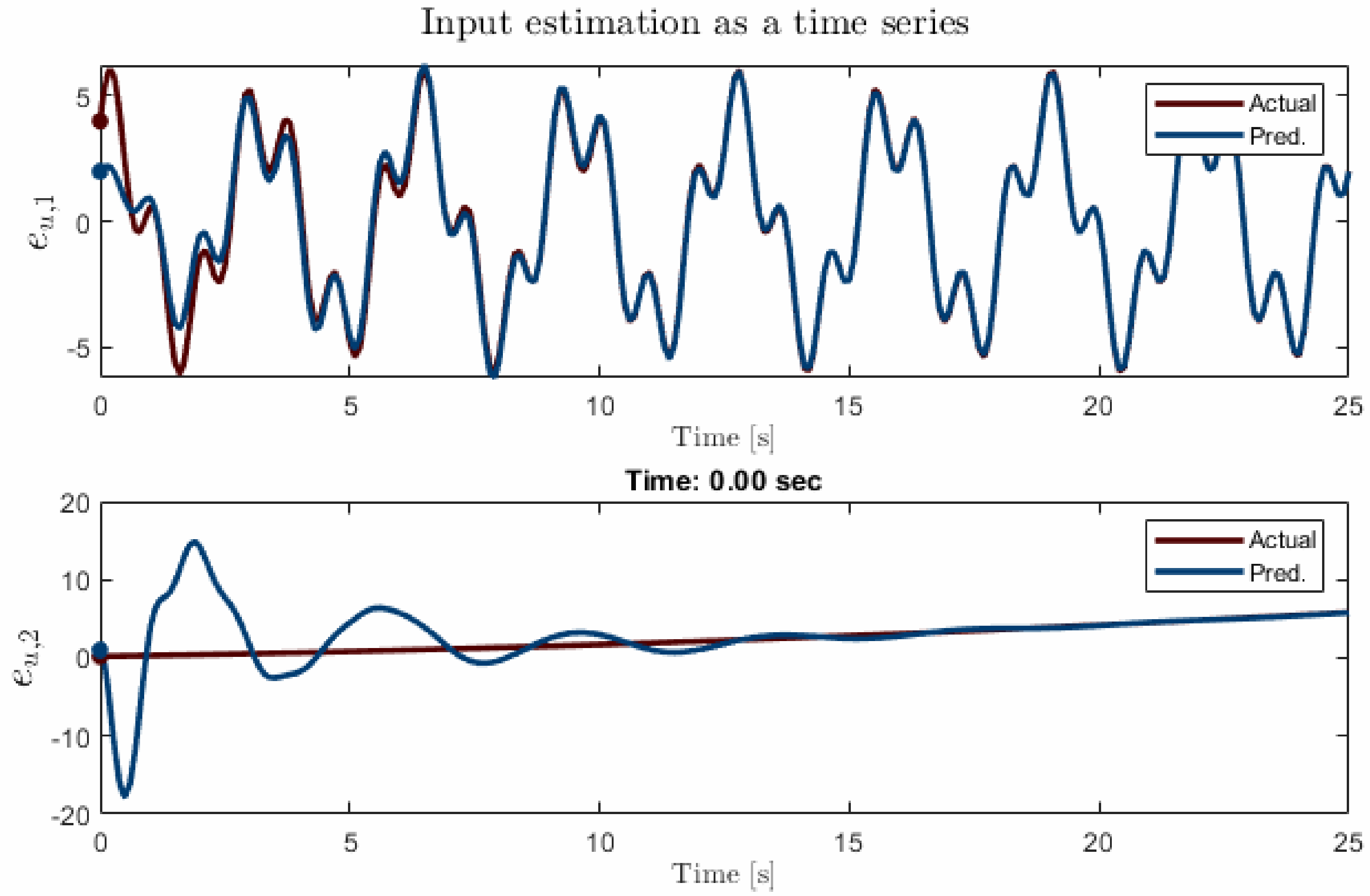
$$\dot{x} = Ax + \varepsilon g(x) + Bu$$
$$= \begin{bmatrix} -2.86 & 1 & 4.7 \\ 1.8 & -1 & 6.7 \\ -9 & 1 & -17.2 \end{bmatrix} x + \sin(x) + Bu$$
$$y = Cx$$

$$L* = \begin{bmatrix} -8 & 1 \\ 2 & -7 \end{bmatrix}$$
$$u_1(t) = c_{11}\sin(2t) + c_{12}\cos(2t) + c_{13}\sin(7t) + c_{14}\cos(7t)$$
$$u_2(t) = c_{11} + c_{22}t + c_{23}t^2 + c_{24}t^3$$

Illustrative example

Both the state error and the input error converge simultaneously

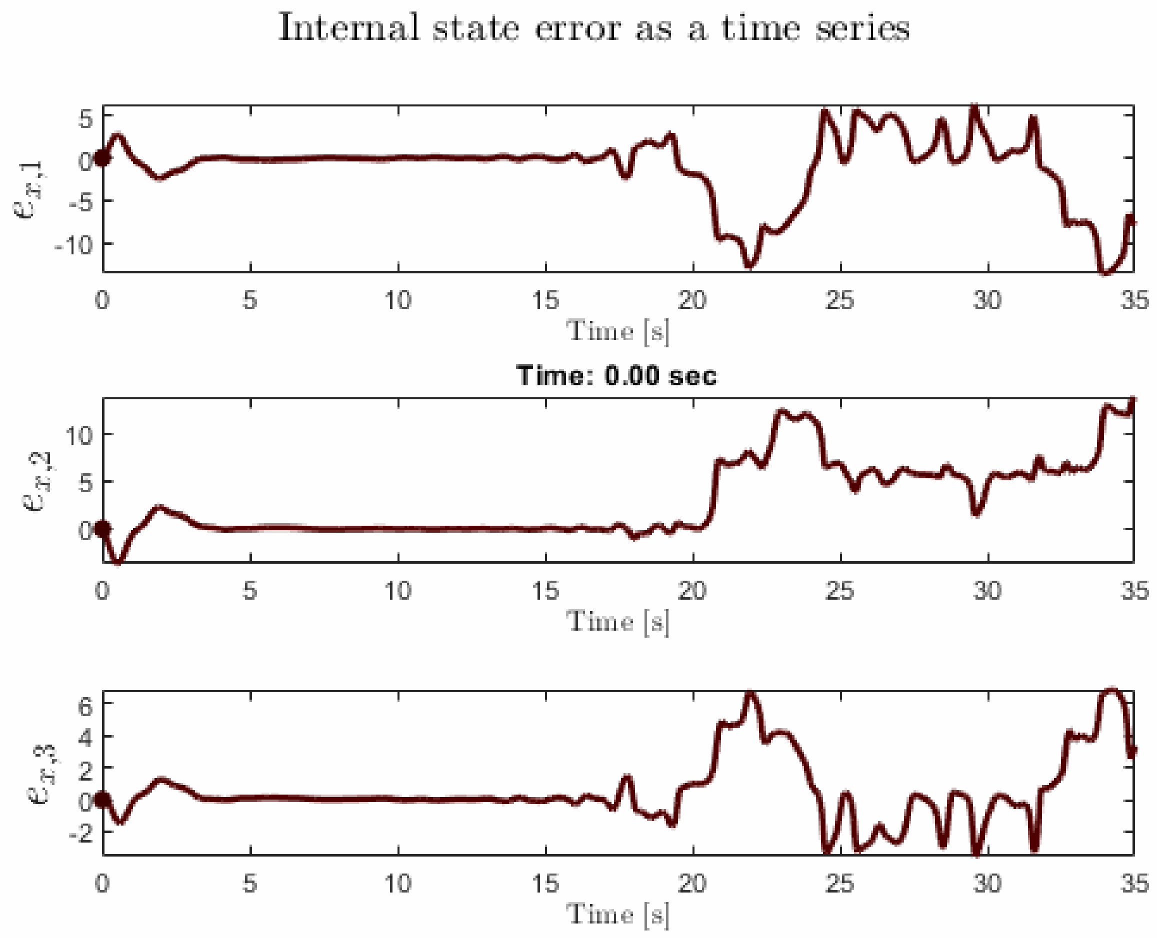


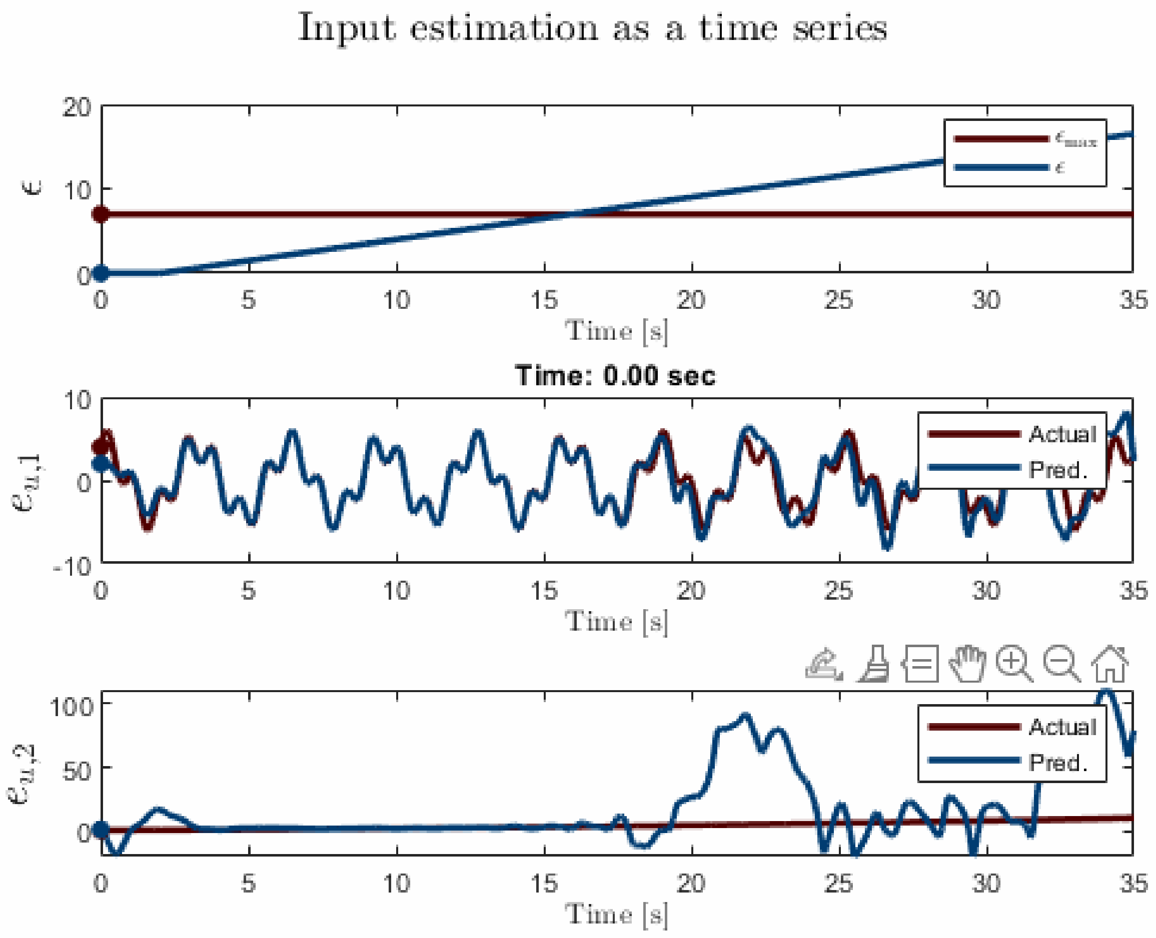


Illustrative example

provided ϵ is not too great

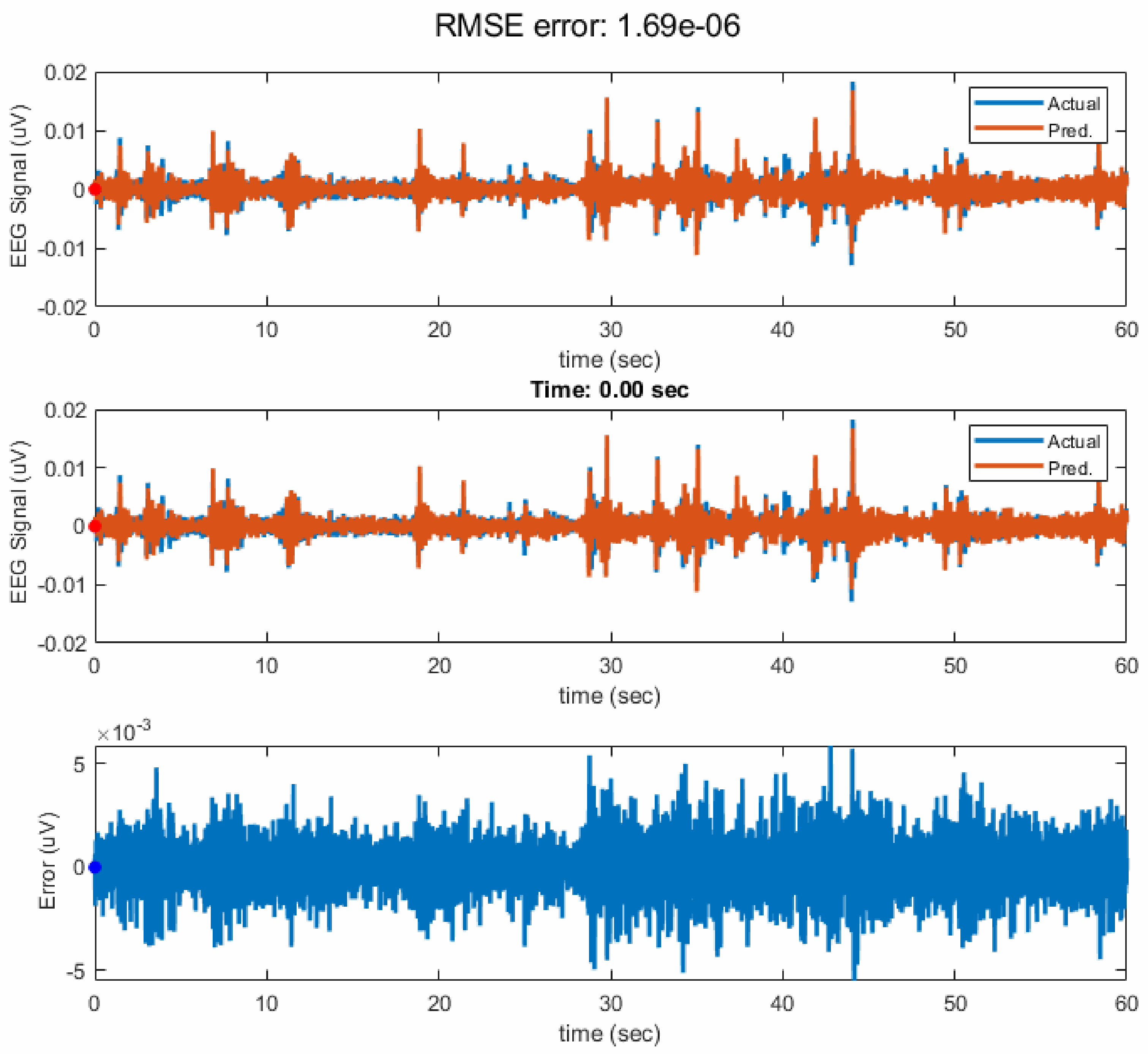
$$0 < \epsilon < \frac{\lambda_{\min}(\bar{Q})}{2\mu\lambda_{\max}(\bar{P})}$$





Application: Biomarker dynamics

Kalman filtering



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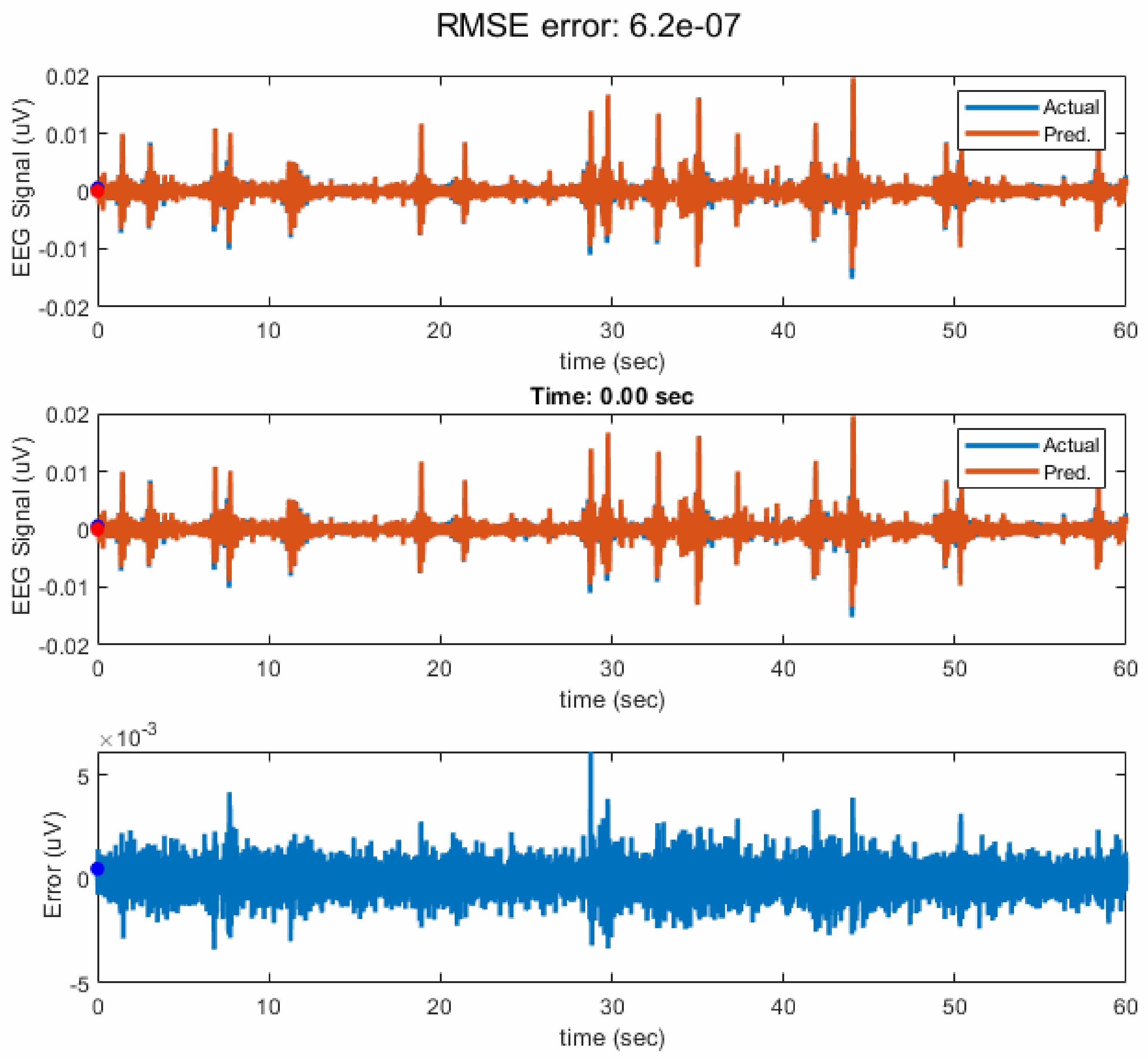
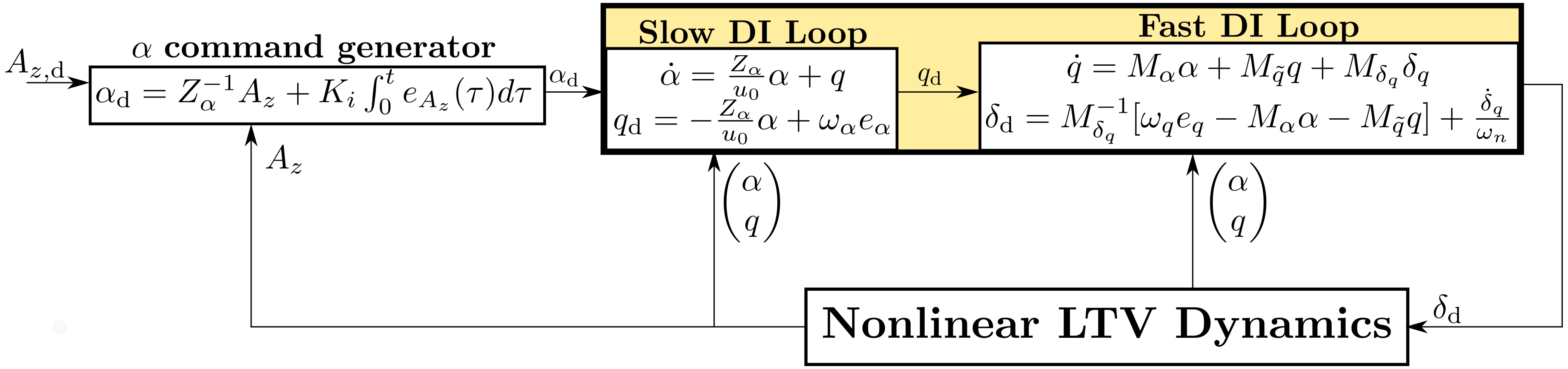


Application: Dynamic inversion for High Speed Projectile

Adaptive DI scheme



Most sensitive to error in outer loop coefficients: $\dot{Z}_\alpha^{-1} = e_{A_z} A_z \sigma$

Open problems:

- Methods to certify flight critical systems not readily available
  - Existing validation methods are analogous but not immediate.
  - Stability margins? Validation of closed loop performance?

We lived in a sloppy world,

but we were precise, very precise.

- Carrying the Fire