

# Adaptive Control is Not Complicated

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Seminar Interview

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# Outline

1. Why Study Adaptive Control?
2. Augmentation Example
3. Adaptive Control is Not Complicated
4. Adaptive Unknown Input Estimators
5. Some Applications of Note
6. Open Problems
7. Conclusions

# Why Study Adaptive Control?

# Why Study Adaptive Control?

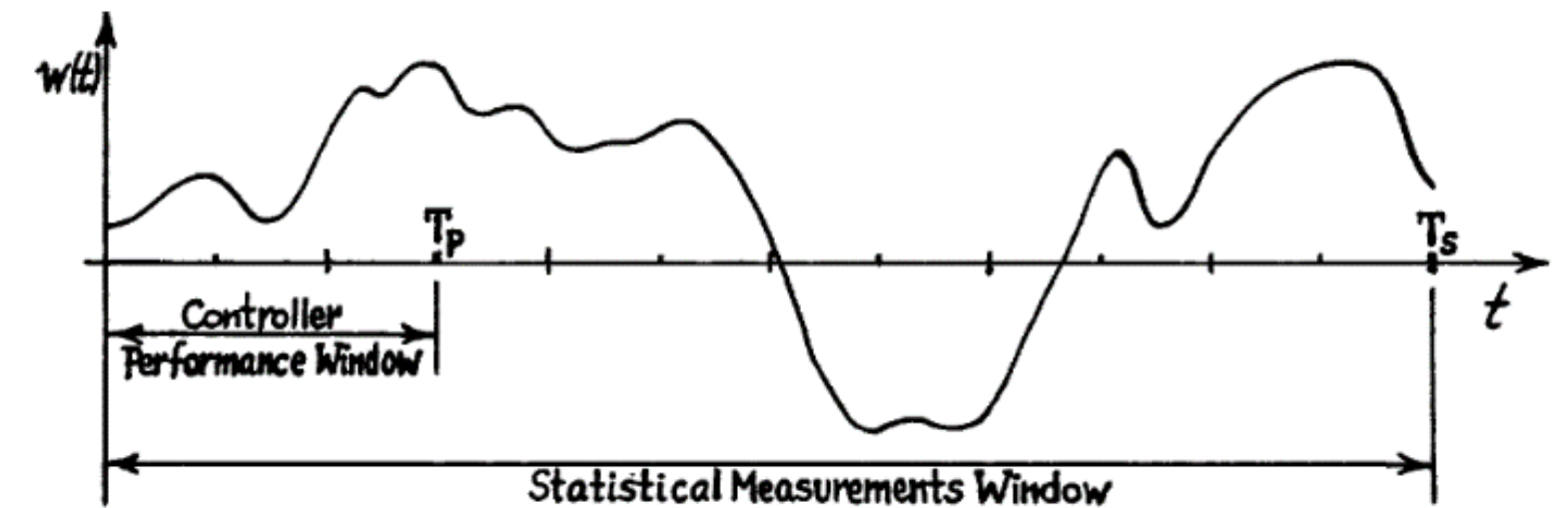
What this talk is and what it is not

- This talk is not:
  - An argument for adaptive control in every system always
  - A chance for me to look smart by being confusing
  - An overview of everything adaptive control
- This talk is:
  - Something I like talking about
  - Technically relevant
  - Presents compelling theoretical challenges

# Why Study Adaptive Control?

## Some perspectives

- Classical vs. stochastic vs. adaptive control
- Flight and Space Structure Needs:
  - Operating in a poorly known environment
  - Are experiments equivalent to actual operation?
  - Many degrees of freedom
  - Finite element models are only as good as the physics
  - Changing situations: takeoff, deployment, landing
  - Control schemes based on reduced order models
- **Greatly emphasizes local vs. global, linear vs. nonlinear thinking**



Gambling in function space from [1].

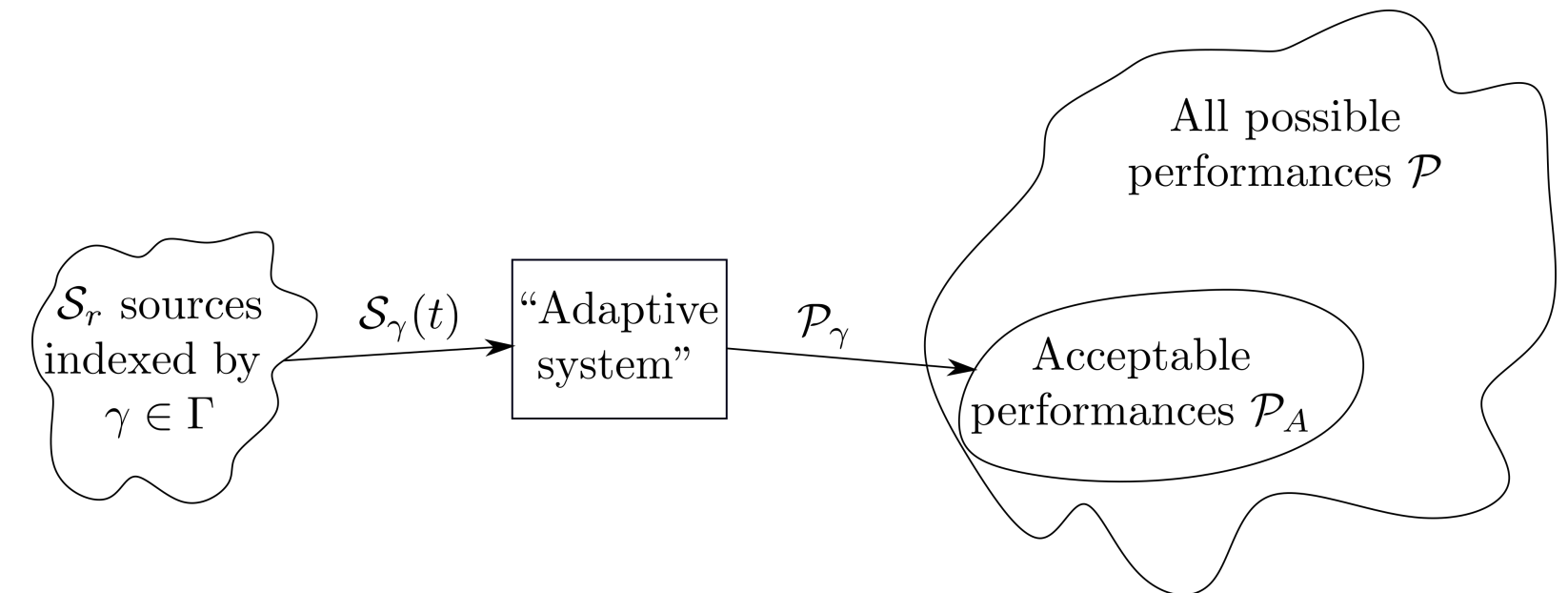
# Why Study Adaptive Control?

## Defining an adaptive system

- Conceptually:
  - A system with knowledge of its performance and the potency to improve it.
- OR, more mathematically

*A map  $\mathcal{J}$  from  $\mathcal{S}_r$  to  $\mathcal{P}$  ( $\mathcal{J} : \mathcal{S}_r \rightarrow \mathcal{P}$ ) with  
range  $\mathcal{J}(\mathcal{S}) \subseteq \mathcal{P}_A$*

- Remark: All systems are adaptive in this definition with respect to some  $\mathcal{S}_r$  and  $\mathcal{P}_A$



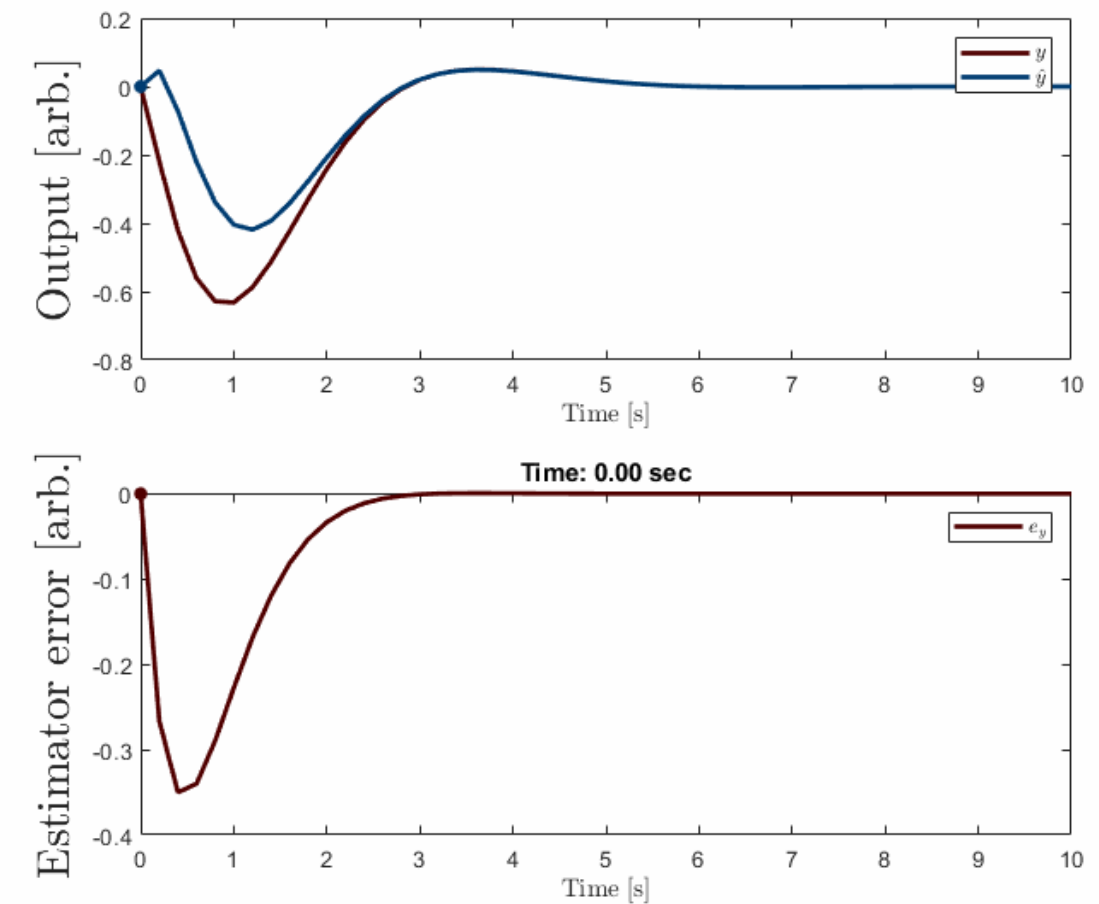
L. Zadeh, "Optimality and non-scalar-valued performance criteria [2].

# Augmentation example

# Augmentation example

## Recovering $\mathcal{P}_A$

- Double integrator:
  - $\dot{x} = Ax + Bu, y = Cx$
  - $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 1]$
  - Min. phase with  $Z(A, B, C) = -1$
- Separation principle controller:
  - $u = G\hat{x}$
  - $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
  - $\hat{y} = C\hat{x}$
- With set gains:
  - $\sigma(A + BG) = -1 \pm j \Rightarrow G = \begin{bmatrix} -2 & -2 \end{bmatrix}$
  - $\sigma(A - KC) = -2 \pm j \Rightarrow K = \begin{bmatrix} -1 & 5 \end{bmatrix}^T$



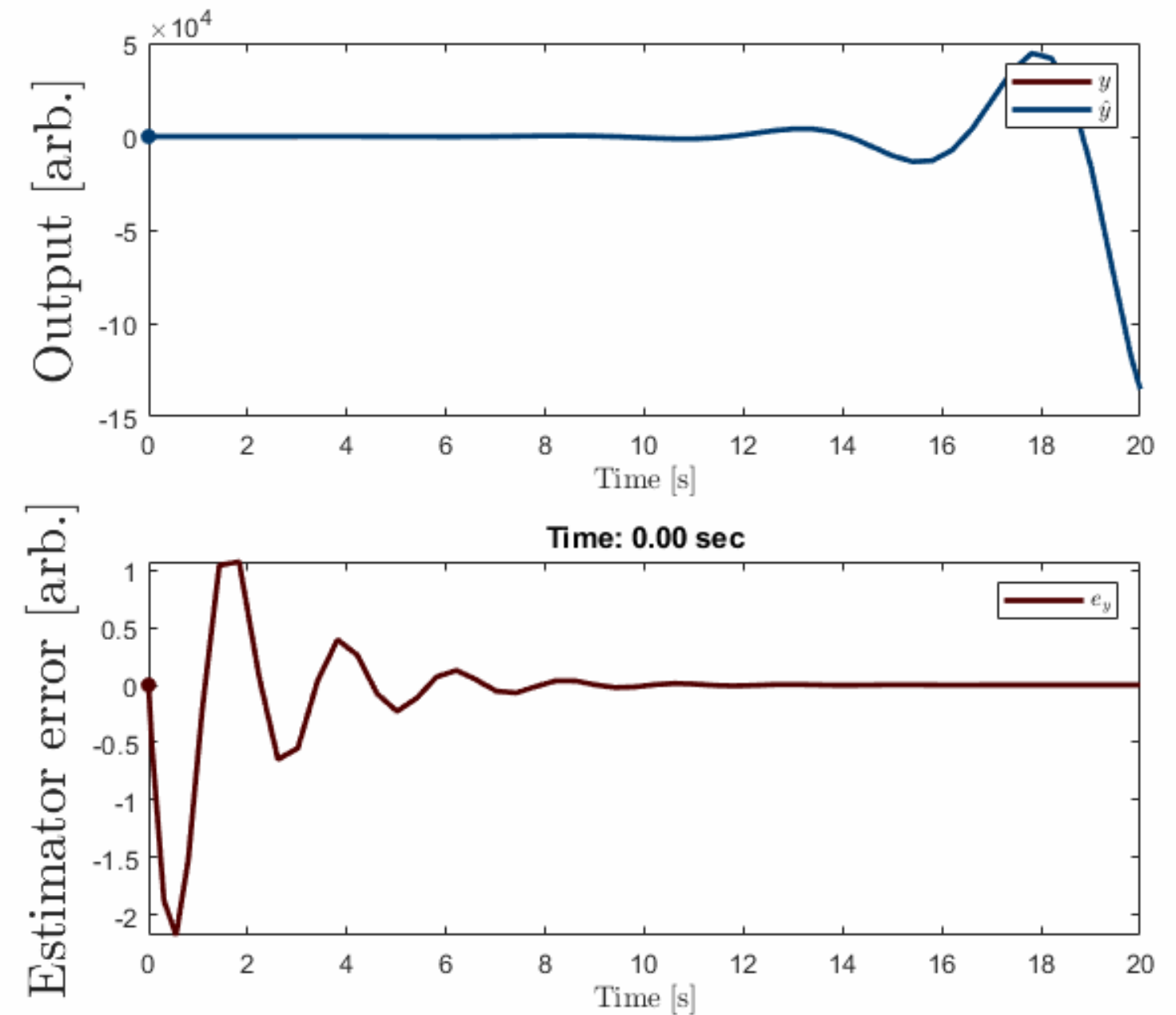
Separation principle controller is stable.



# Augmentation example

## Recovering $\mathcal{P}_A$

- But suppose  $A$  became  $\tilde{A}$ :
  - $\dot{x} = \tilde{A}x + Bu, y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & \mathbf{3} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
  - Min. phase with  $Z(A, B, C) = -1$
- Separation principle controller:
  - $u = G\hat{x}$
  - $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
  - $\hat{y} = C\hat{x}$
- With set gains:
  - $\sigma(\tilde{A} + BG) = 0.5 \pm 1.3j \Rightarrow G = \begin{bmatrix} -2 & -2 \end{bmatrix}$
  - $\sigma(\tilde{A} - KC) = -0.5 \pm 2.7j \Rightarrow K = \begin{bmatrix} -1 & 5 \end{bmatrix}^T$



Perturbed separation principle controller is not stable.

# Augmentation example

## Recovering $\mathcal{P}_A$

- But suppose  $A$  became  $\tilde{A}$  and I have augmented the system with an adaptive outer loop:

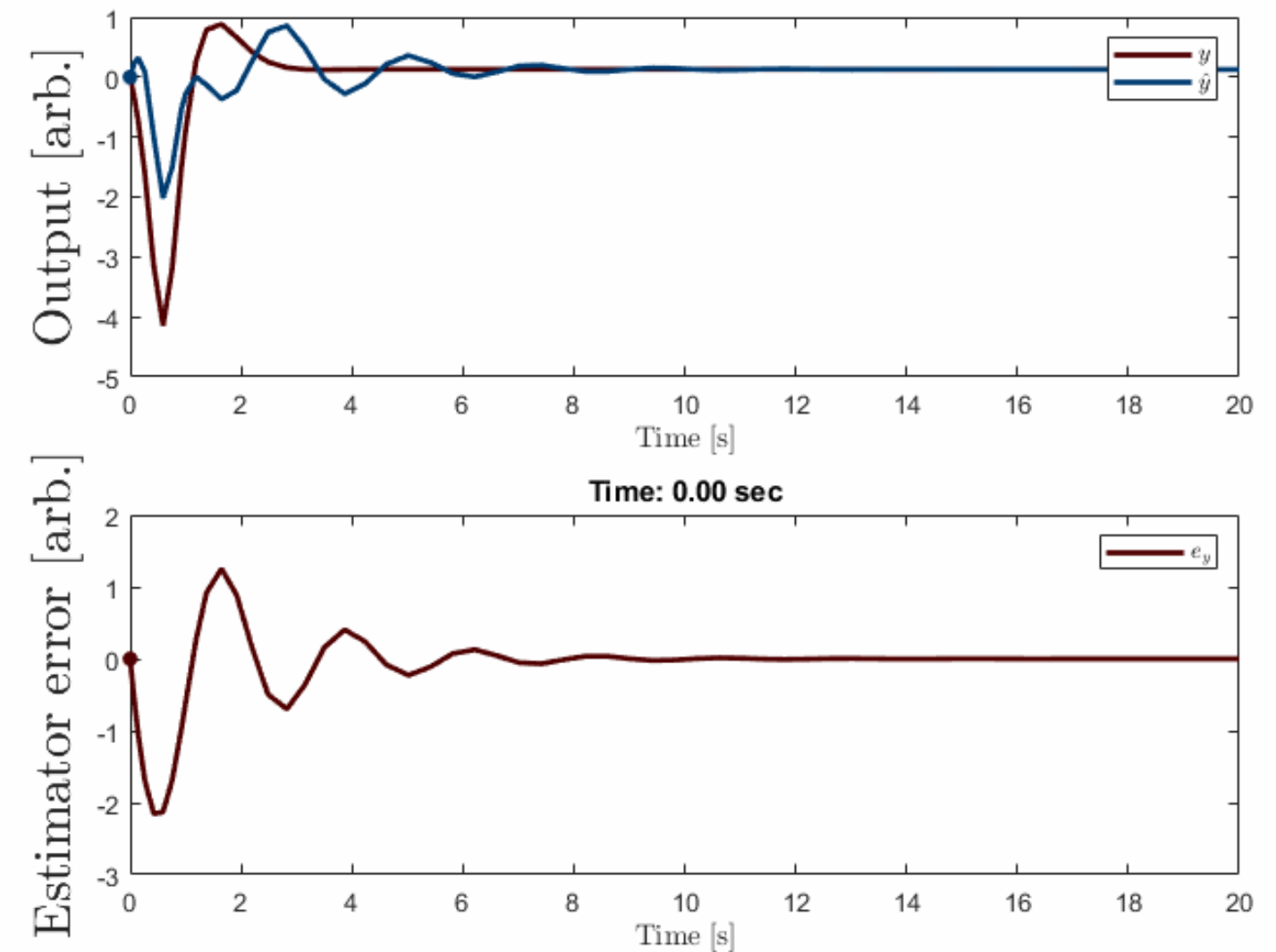
- $\dot{x} = \tilde{A}x + Bu, y = Cx$
- $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & \mathbf{3} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
- Min. phase with  $Z(A, B, C) = -1$

- Adaptive separation principle controller:

- $u = G\hat{x} + Ly$
- $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
- $\hat{y} = C\hat{x}$

- With same set gains and adaptive law:

- $\dot{L} = -yy^T\sigma, \sigma > 0$

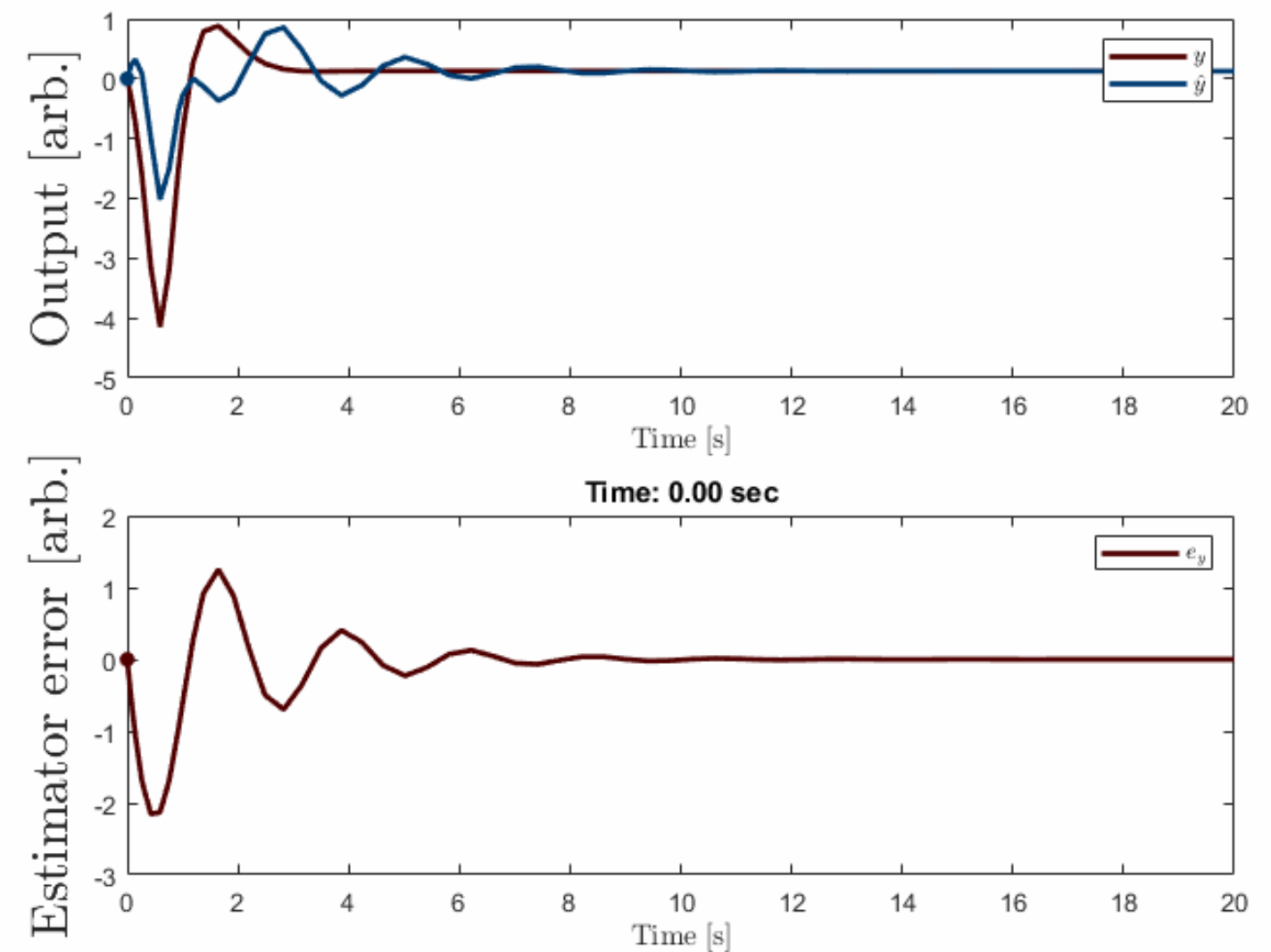


Adaptive separation principle controller is stable.

# Augmentation example

Recovering  $\mathcal{P}_A$

- Does this happen with gain scheduled controllers?
- We treated a significant constant perturbation adaptively
- Remark: Adaptive controllers are especially good at handling significant, slower disturbances
  - Robust controllers are especially good at small, fast disturbances
  - **$\therefore$  we should generally consider the adaptive augmentation of robust controllers.**



Adaptive separation principle controller is stable.

Adaptive Control is not Complicated

# Adaptive Control is not Complicated

- Given:

- $\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases}$

- $(A, B, C)$  ctrb/obsv (i.e. **minimal** description of  $P(s) = C(sI - A)^{-1}B$ )

- Recall Kimura-Davison sufficient conditions:

- $M \equiv \text{rank } B = \text{rank } C = M$  (square)

- $(A, B, C)$  ctrb/obsv

- $M \geq \frac{N+1}{2}$ ;  $N = \dim x$

- $\exists G_* \ni \sigma(A + BG_*C)$  that assigns pole locations arbitrarily

# Adaptive Control is not Complicated

- Sufficient conditions for arb. pole placement but we must **know**  $(A, B, C)$  in detail to find  $G_*$ !
- This can be onerous, but if  $G_*$  exists, the system is called output feedback stabilizable
- Ex:

- $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad \varepsilon]$

- With  $G_* = -g$ ,  $A + BG_*C = \begin{bmatrix} 0 & 1 \\ -g & -g\varepsilon \end{bmatrix}$

- $\det(\lambda I - A_c) = \lambda^2 + g\varepsilon\lambda + g$

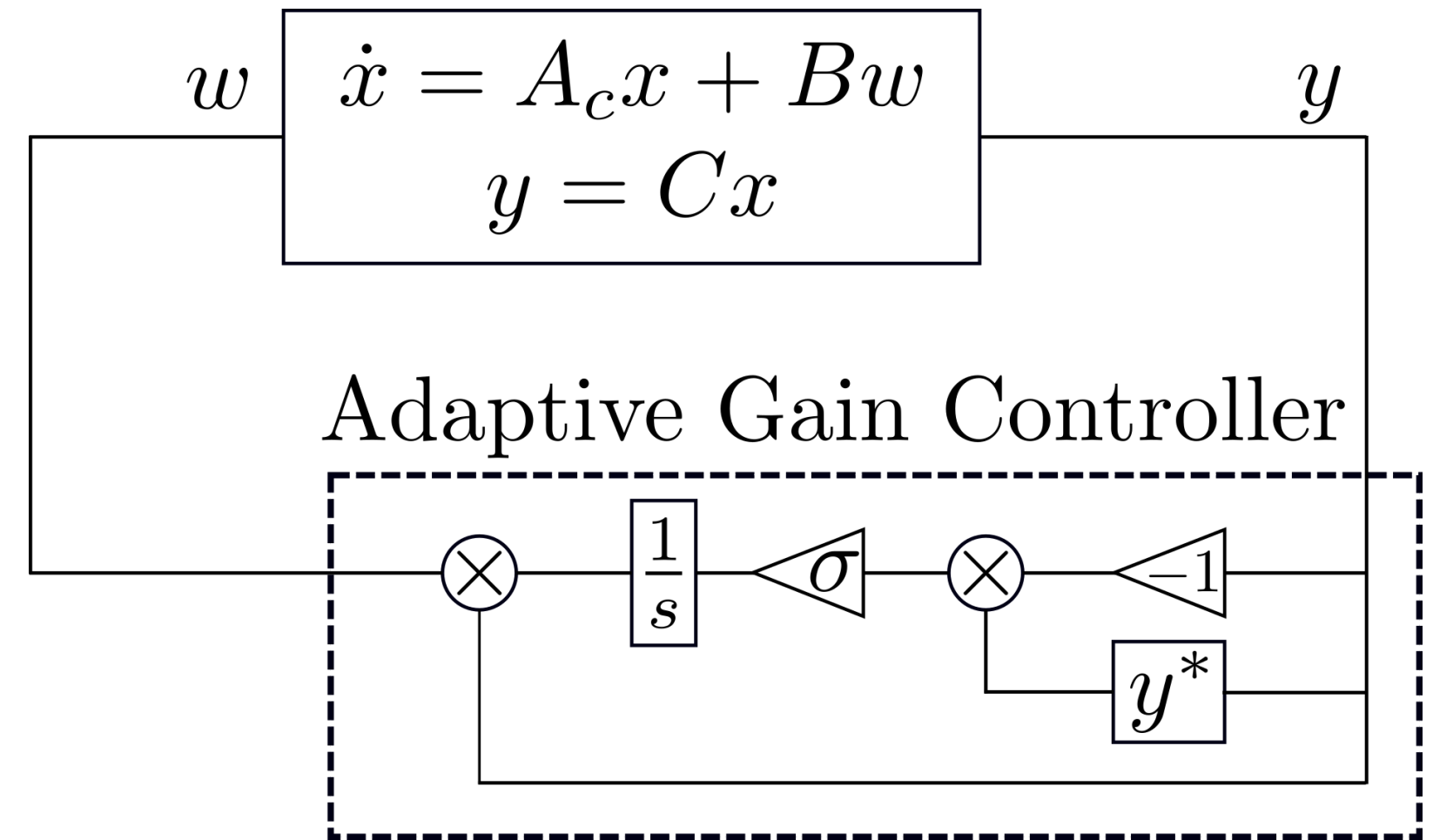
- $\therefore$  output feedback stabilizable when  $\varepsilon > 0$  only!

- Note:  $\exists P > 0 \ni A_c^T P + P A_c = -Q, Q > 0$

# Adaptive Control is not Complicated

## Adaptive Regulator using Output Feedback Only

- Plant:  $\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases}$  (square)
- Regulator:  $\begin{cases} u &= Gy \\ \dot{G} &= -yy^T \sigma \end{cases}$
- Let  $G \equiv G_* + \Delta G$ . Closed loop system is:
  - $$\begin{cases} \dot{x} &= \underbrace{(A + BG_*C)}_{A_c} x + \underbrace{B\Delta G}_{w} y \\ y &= Cx \\ \Delta \dot{G} &= \dot{G} = -yy^T \sigma, \sigma > 0 \end{cases}$$



Adaptive regulator architecture.

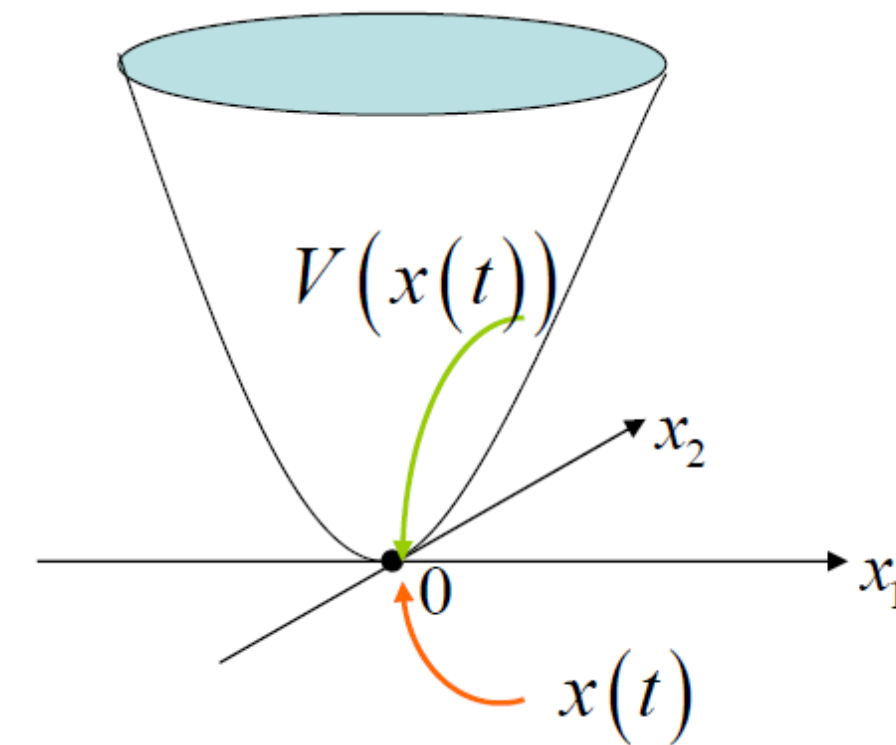




# Adaptive Control is not Complicated

## Lyapunov Stability Argument

- If a scalar function  $V(x, t)$  satisfies
  - function is lower bounded
  - Time derivative  $\dot{V}(x, t)$  is negative semidefinite:  $\sigma(\dot{V}(x, t)) \leq 0$
  - Time derivative  $\dot{V}(x, t)$  is uniformly continuous in  $t$ : derivative is bounded
- Then  $\lim_{t \rightarrow \infty} \dot{V}(x, t) = 0$
- and we have a theoretical stability guarantee.



Example Lyapunov candidate function

# Adaptive Control is not Complicated

## Lyapunov Stability Argument

- Here,  $P$  from  $A_c^T P + P A_c = -Q$  yields a quadratic, lower bounded function

- $\frac{\lambda_{\min}(P)}{2} \|x\|^2 \leq V_1(x) \equiv \frac{1}{2} x^* P x \leq \frac{\lambda_{\max}(P)}{2} \|x\|^2$

- which meets our first requirement.

- Notice

$$\begin{aligned} \dot{V}_1(x) &\equiv \triangle V_1 \dot{x} = x^* P [A_c x + B w] \\ &= x^* P A_c x + x^* P B w \\ &\leq -\frac{1}{2} x^* Q x + x^* C^* w \\ &\leq -1/2 \lambda_{\min}(Q) \|x\|^2 + (y, w) \end{aligned}$$

- which may or may not be negative semidefinite, but is bound.



James Joseph Sylvester

# Adaptive Control is not Complicated

## Lyapunov Stability Argument

- $\dot{V}_1(x) \leq -1/2\lambda_{\min}(Q)||x||^2 + (y, w)$ 
  - which may or may not be negative semidefinite, but is bound.
- However, we have not checked the stability of the adaptive gain  $G$

- Consider  $V_2(\Delta G) \equiv \frac{1}{2}\text{tr}(\Delta G\sigma^{-1}\Delta G^*)$

$$\dot{V}_2 = \text{tr}(\Delta \dot{G}\sigma^{-1}\Delta G^*)$$

$$= \text{tr}(-yy^*\sigma\sigma^{-1}\Delta G^*)$$

- $= -\text{tr}(yy^*\underbrace{\Delta G^*}_{w^*}) = -\text{tr}(w^*y)$  scalar!

$$= -(y, w)$$

- Which “conveniently” yields:

$$\dot{V}(x, \Delta G, t) = \dot{V}_1(x, t) + \dot{V}_2(\Delta G, t)$$

- $\leq -1/2\lambda_{\min}(Q)||x||^2 + (y, w) - (y, w)$   
 $\leq -1/2\lambda_{\min}(Q)||x||^2$

- Since  $x, G$  are now bound, composite system is bound.  $V$  is negative semidefinite. Therefore, by Lyapunov,  $x \Rightarrow 0$ .

# Augmentation example

## Double integrator example

- Returning to our double integrator example:

- $\dot{x} = \tilde{A}x + Bu, y = Cx$

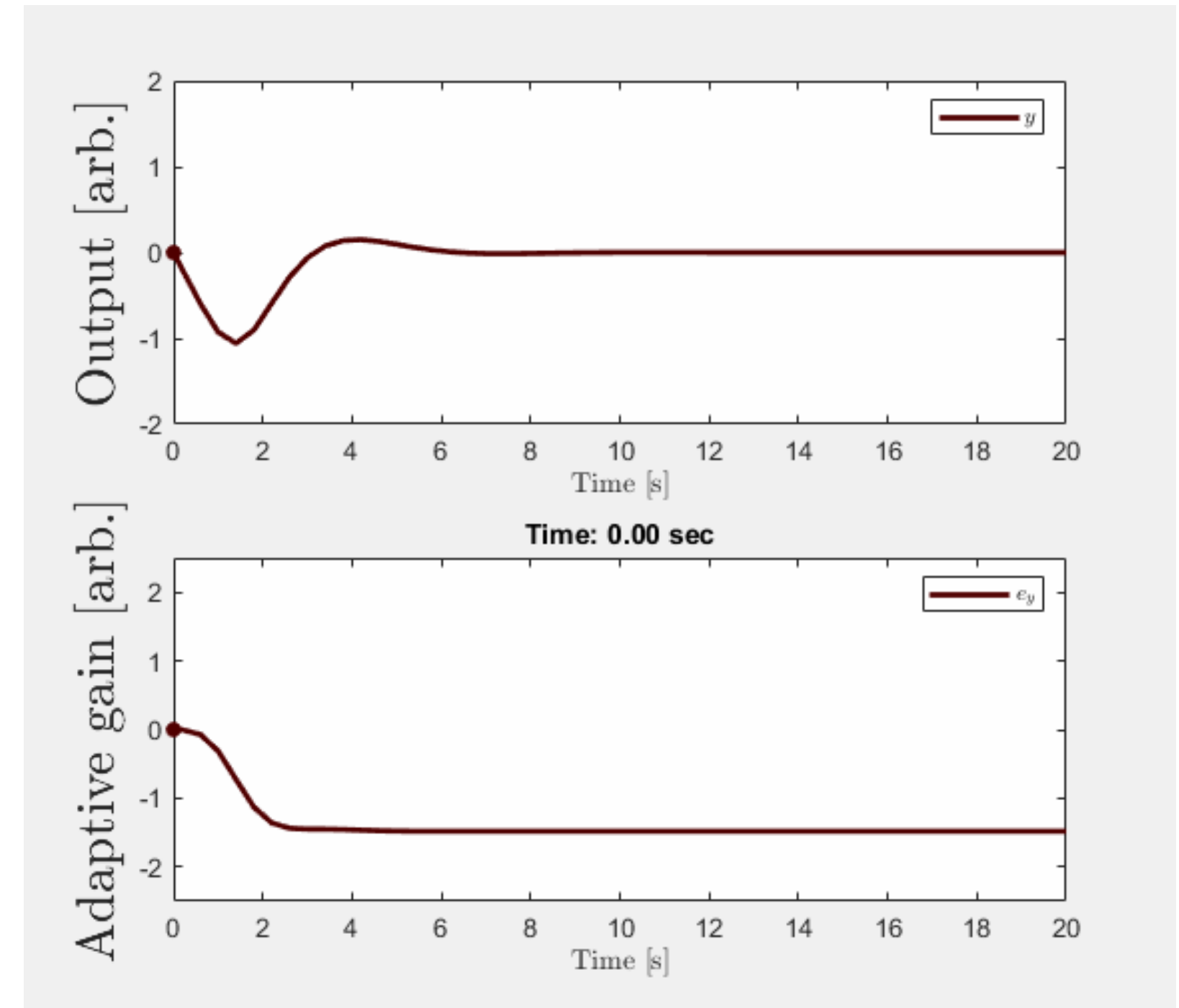
- $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 1]$

- Adaptive regulator:

- $u = Gy$

- With adaptive law:

- $\dot{G} = -yy^T \sigma, \sigma > 0$



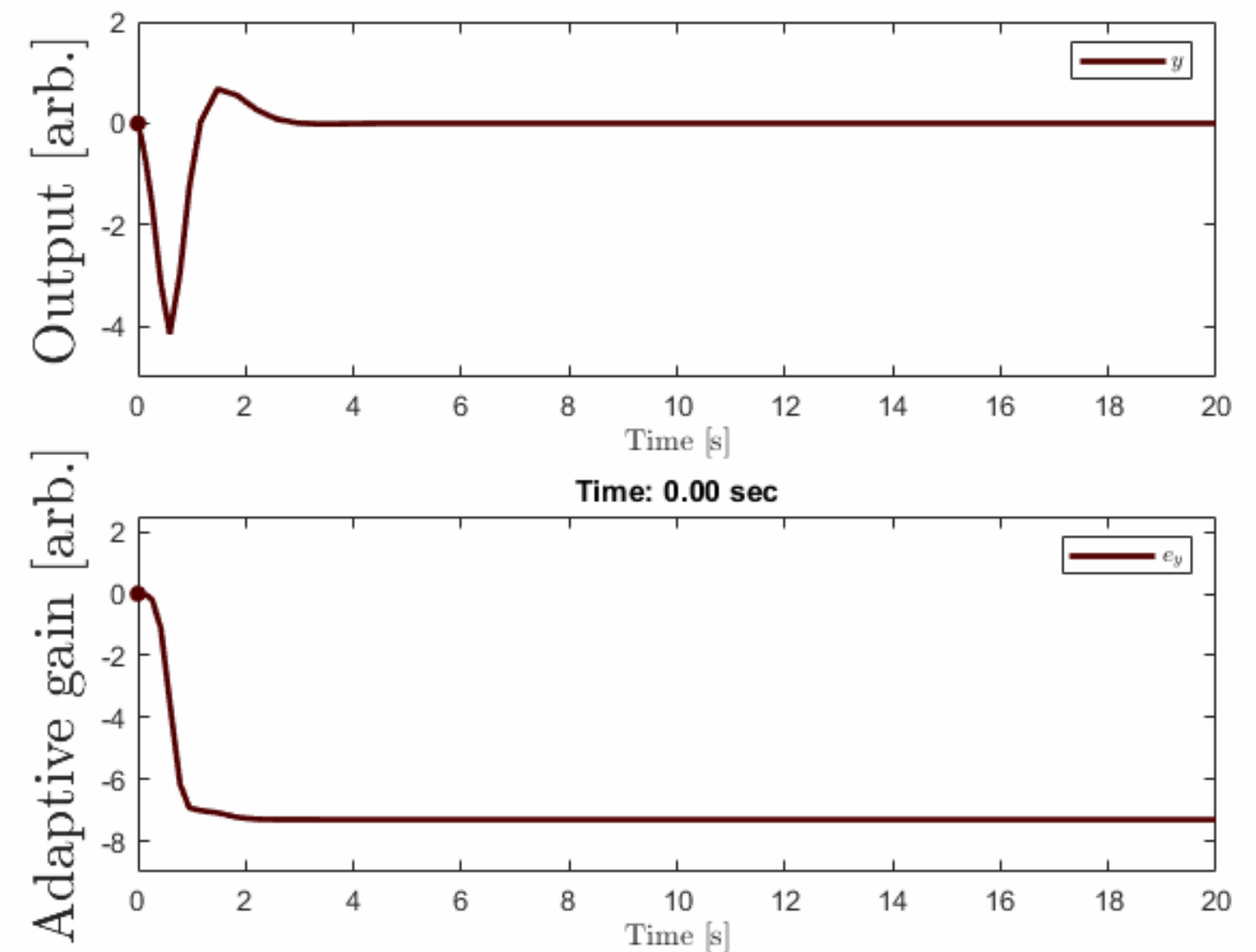
Adaptive controller is stable.

# Augmentation example

## Double integrator example

- Returning to our double integrator example:
  - $\dot{x} = \tilde{A}x + Bu, y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 1]$
- Adaptive regulator:
  - $u = Gy$
- With adaptive law:
  - $\dot{G} = -yy^T\sigma, \sigma > 0$

*Achieve exponential stability with exactly the same controller!*



Same controller is stable for a different plant.

# Adaptive Unknown Input Estimators

# Adaptive Unknown Input Estimators

## Estimator overview

- Three significant uncertainties
  - Input  $u$  is unknown, external, deterministic
  - State matrix  $A$  may have uncertainty
  - Known, Lipschitz nonlinear internal dynamics  $g(x)$
- Can we synthesize  $u$  and correct  $A$ ?

$$\begin{aligned}\dot{x} &= Ax + g(x) + Bu \\ y &= Cx\end{aligned}$$

# Adaptive Unknown Input Estimators

## Modeling unknown inputs

- Approximate input space  $\mathbb{U}$

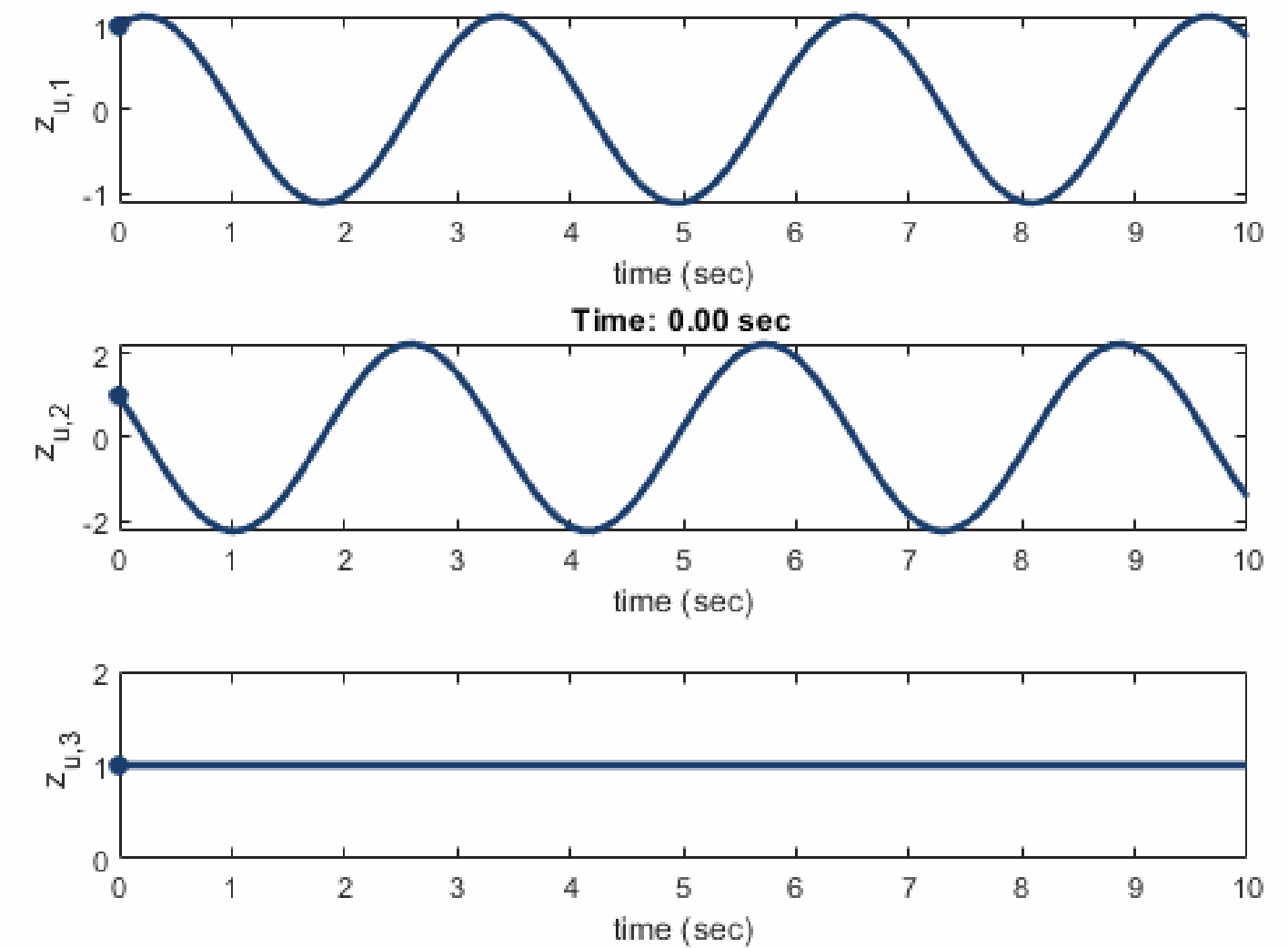
- $\hat{u} = \sum_{i=1}^N c_i f_i(t)$

- Persistent Inputs

- $\dot{z}_u = F_u z_u$

- $\hat{u} = \Theta_u z_u$

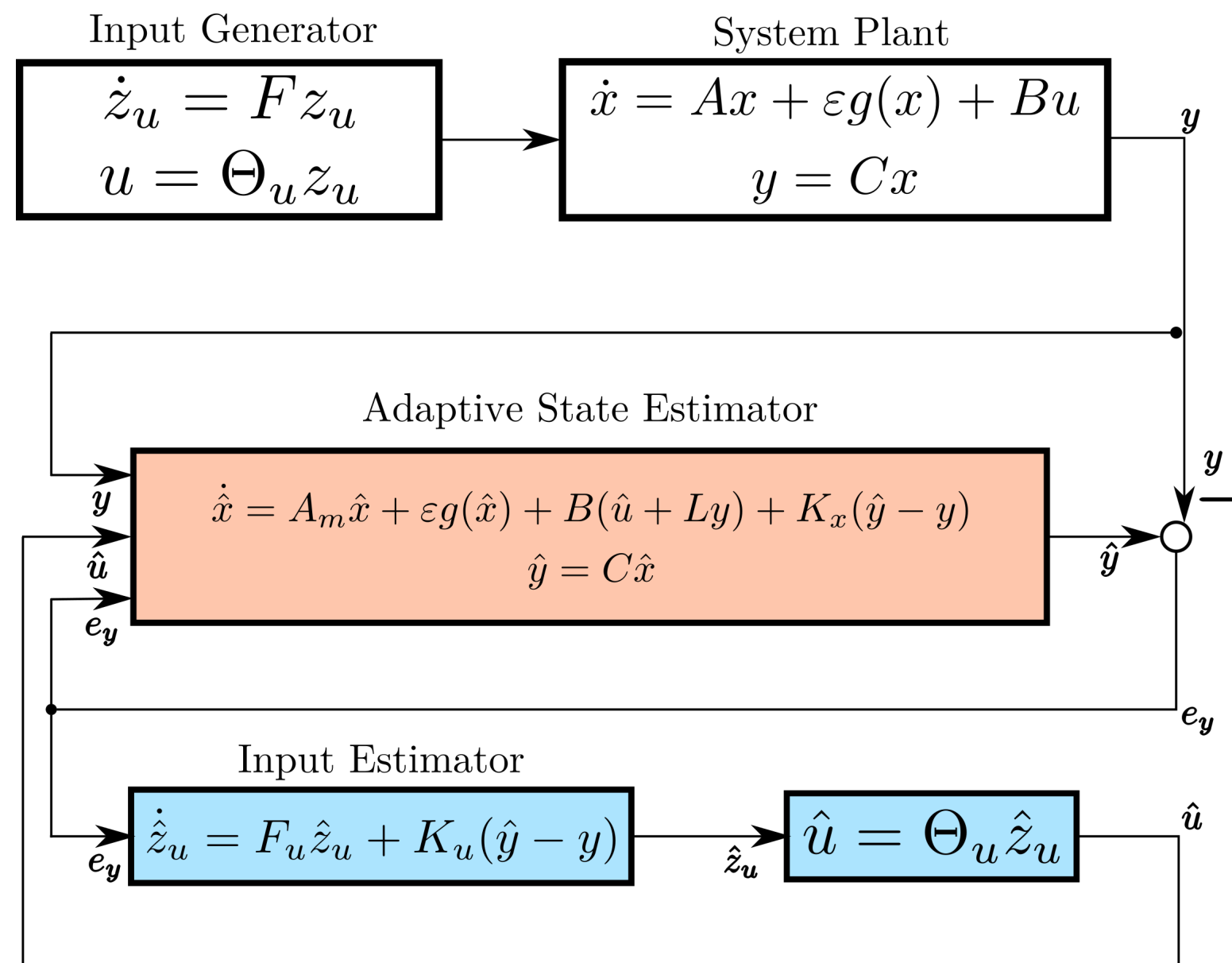
- $F_u = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$





# Adaptive Unknown Input Estimators

## Architecture and estimator error



Recover  $A$  with adaptive scheme

$$A \equiv A_m + BL_*C$$

$$\dot{L} = -e_y y^* \gamma_e; \gamma_e > 0$$

Error dynamics

$$\begin{aligned} \dot{e} &= (\bar{A} + \bar{K}\bar{C})e + \bar{B}\Delta L y + \varepsilon \Delta g \\ \begin{bmatrix} \dot{e}_x \\ \dot{e}_z \end{bmatrix} &= \underbrace{\begin{bmatrix} A_m + K_x C & B\Theta_u \\ K_u C & F \end{bmatrix}}_{\bar{A}_c} \begin{bmatrix} e_x \\ e_z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w + \varepsilon \begin{bmatrix} g(\hat{x}) - g(x) \\ 0 \end{bmatrix} \end{aligned}$$

# Adaptive Unknown Input Estimators

## Architecture and estimator error

- ASD plant dynamics
  - $\bar{A}_c^* \bar{P} + \bar{P} \bar{A}_c = -\bar{Q}$
  - $\bar{P} \bar{B} = \bar{C}^*$
- $A$  Hurwitz
- Bounded  $L_*$
- Error in state and input converges to zero
  - $V(e, \Delta L) = \frac{1}{2} e^* \bar{P} e + \frac{1}{2} \text{tr}(\Delta L \gamma_e^{-1} \Delta L^*)$
  - $\dot{V}(e, \Delta L) \leq - \underbrace{\left( \frac{1}{2} \lambda_{\min}(\bar{Q}) - \varepsilon \mu \lambda_{\max}(\bar{P}) \right)}_{\bar{\alpha} > 0} \|e\|^2$

$$0 < \varepsilon < \frac{\lambda_{\min}(\bar{Q})}{2\mu\lambda_{\max}(\bar{P})} \iff \bar{\alpha} > 0.$$

## Illustrative example

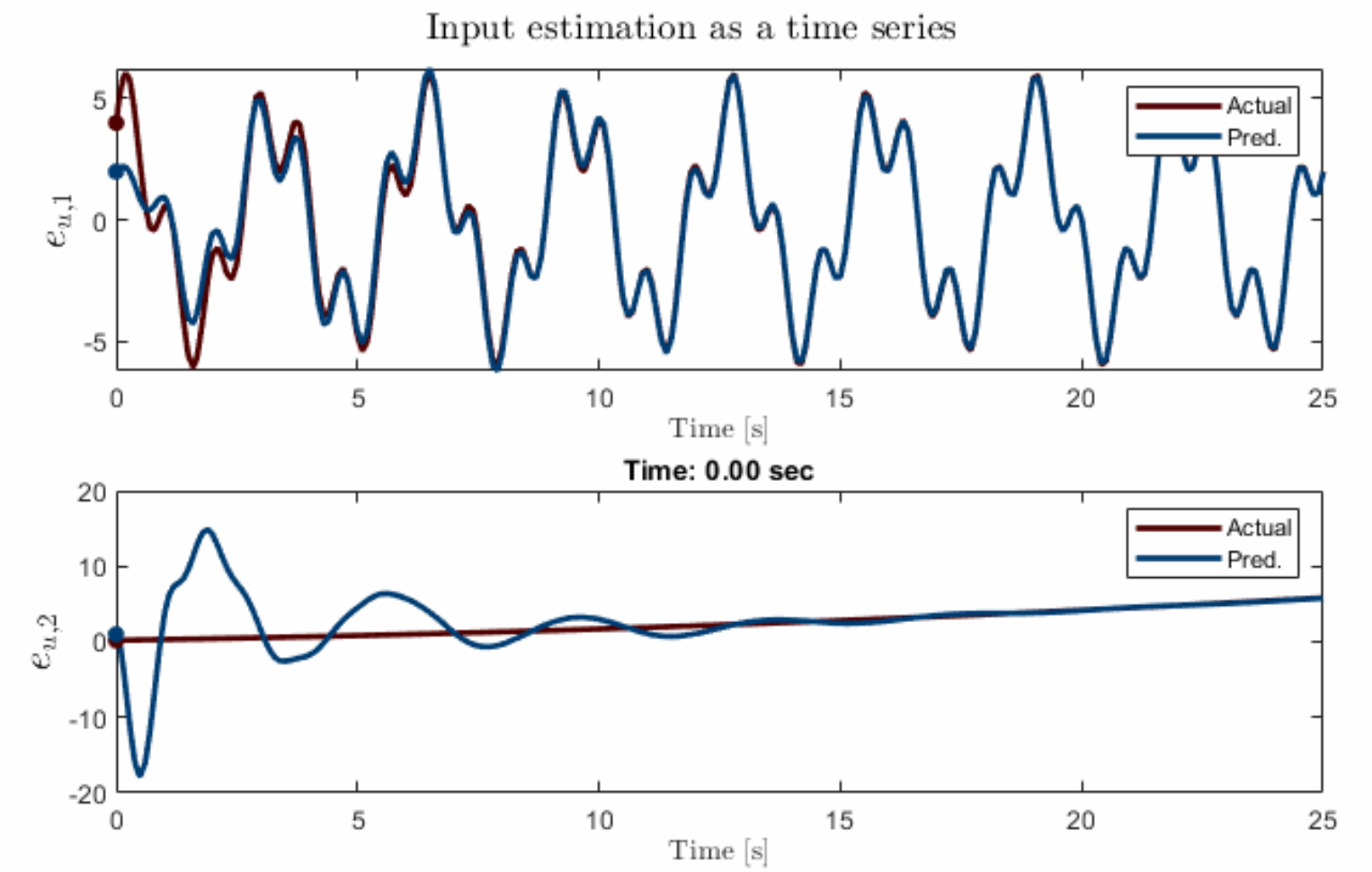
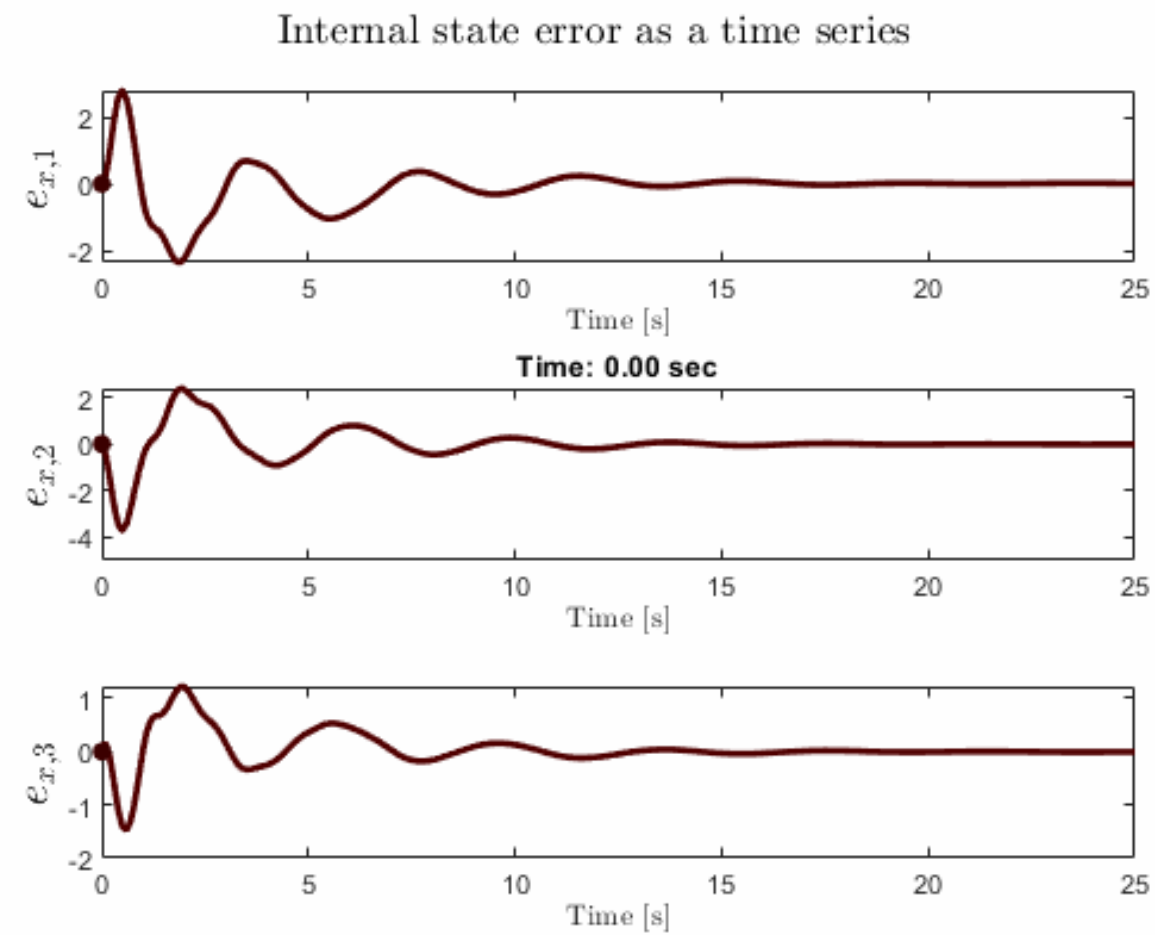
$$\begin{aligned}\dot{x} &= A_m x + \varepsilon g(x) + Bu \\ &= \begin{bmatrix} -4 & 1 & 2 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} x + \sin(x) + Bu \\ y &= Cx\end{aligned}$$

$$\begin{aligned}\dot{x} &= Ax + \varepsilon g(x) + Bu \\ &= \begin{bmatrix} -2.86 & 1 & 4.7 \\ 1.8 & -1 & 6.7 \\ -9 & 1 & -17.2 \end{bmatrix} x + \sin(x) + Bu \\ y &= Cx\end{aligned}$$

$$\begin{aligned}L^* &= \begin{bmatrix} -8 & 1 \\ 2 & -7 \end{bmatrix} \\ u_1(t) &= c_{11} \sin(2t) + c_{12} \cos(2t) + c_{13} \sin(7t) + c_{14} \cos(7t) \\ u_2(t) &= c_{11} + c_{22}t + c_{23}t^2 + c_{24}t^3\end{aligned}$$

# Illustrative example

Both the state error and the input error converge simultaneously

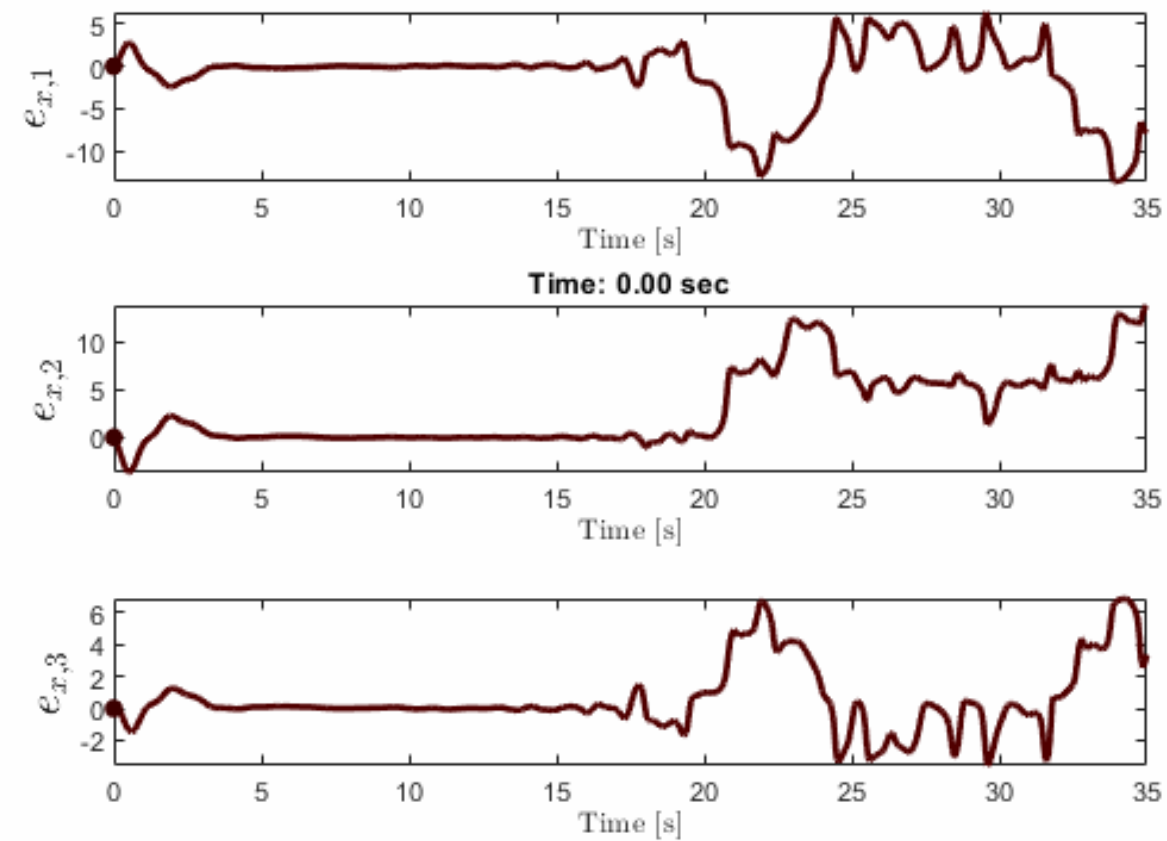


# Illustrative example

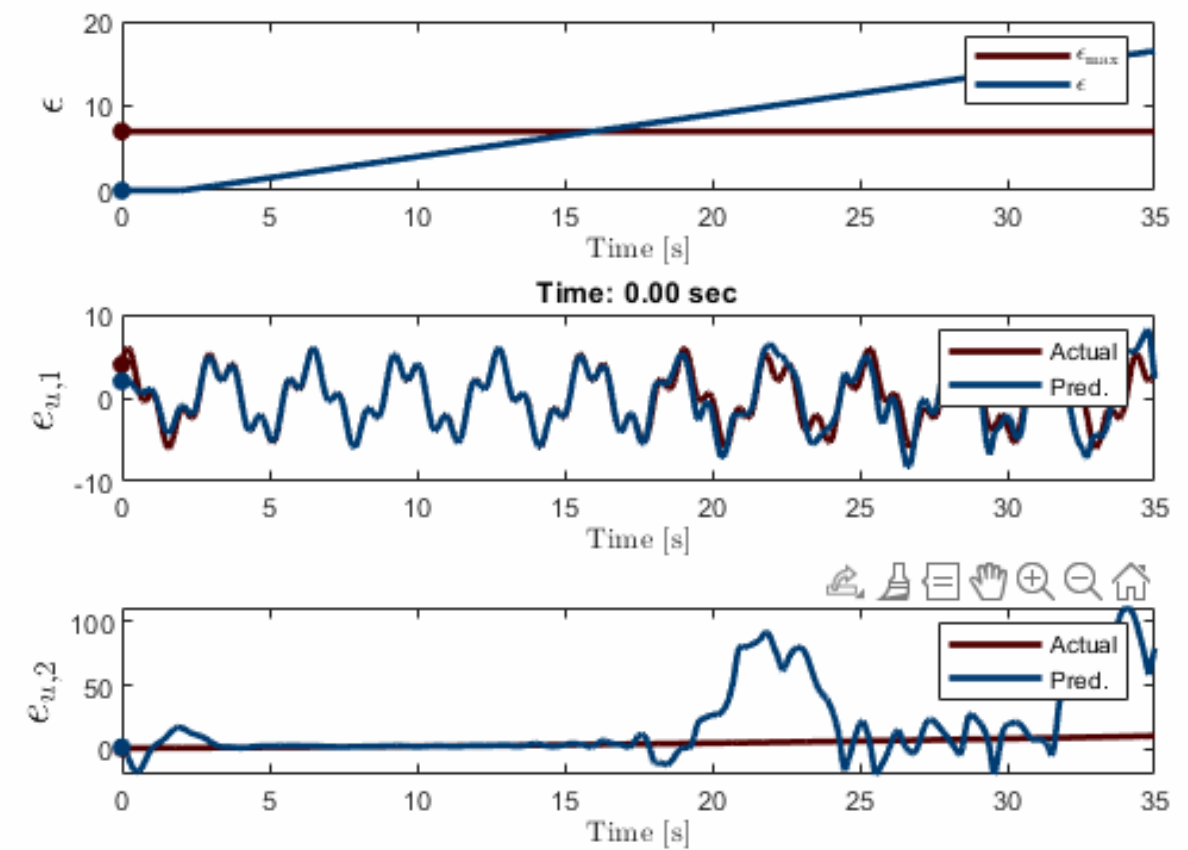
provided  $\epsilon$  is not too great

$$0 < \epsilon < \frac{\lambda_{\min}(\bar{Q})}{2\mu\lambda_{\max}(\bar{P})}$$

Internal state error as a time series

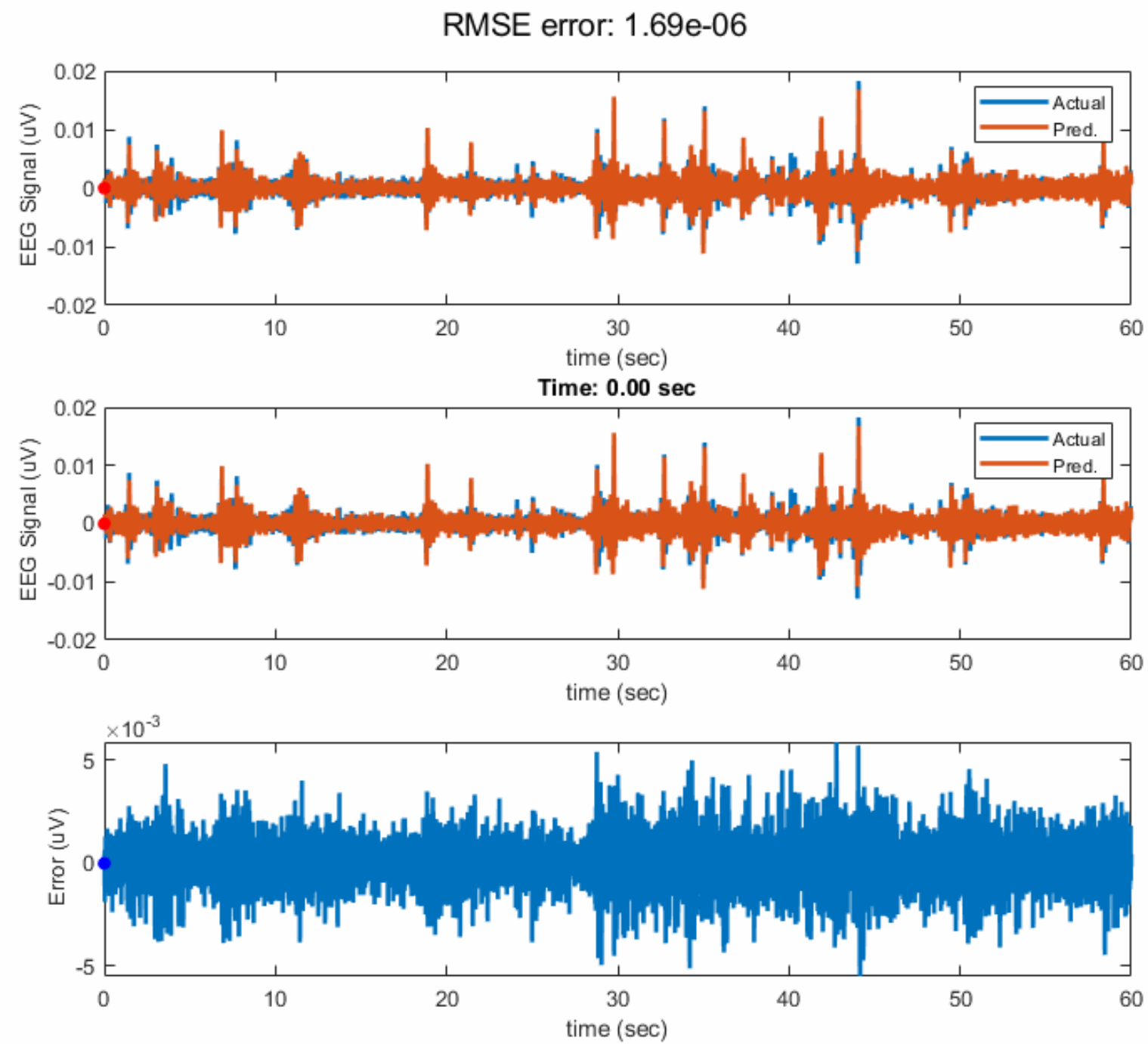


Input estimation as a time series

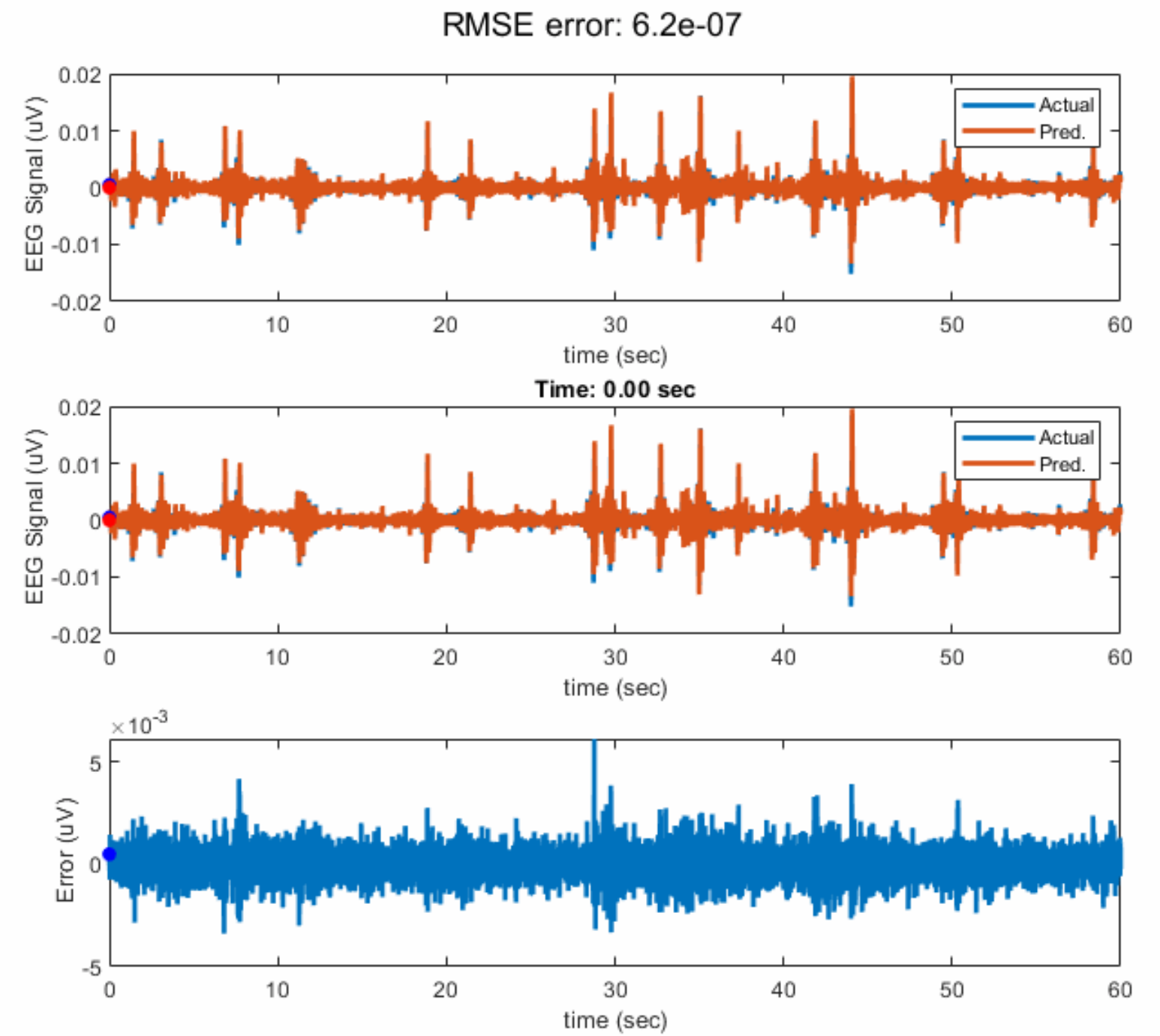


# Application: Biomarker dynamics

Kalman filtering

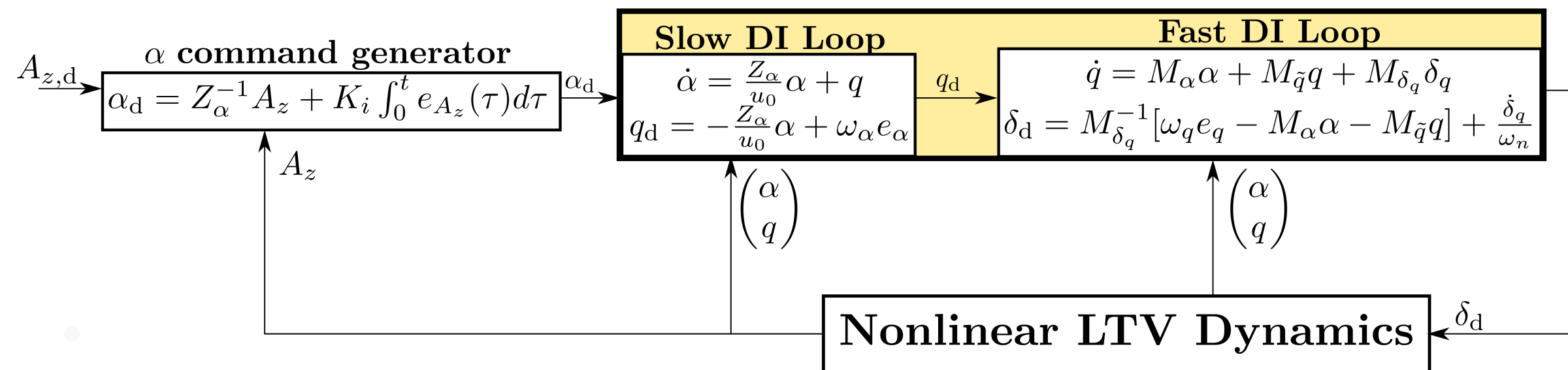


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# Application: Dynamic inversion for High Speed Projectile

Adaptive DI scheme



Most sensitive to error in outer loop coefficients:  $\dot{Z}_\alpha^{-1} = e_{A_z} A_z \sigma$

## Open problems:

- Methods to certify flight critical systems not readily available
  - Existing validation methods are analogous but not immediate.
  - Stability margins? Validation of closed loop performance?



We lived in a sloppy world,  
but we were precise, very precise.

- Carrying the Fire