Adaptive Control is Not Complicated

T. Griffith, Ph.D

Seminar Interview

July 30, 2022

Outline

Why Study Adaptive Control?

Why Study Adaptive Control?

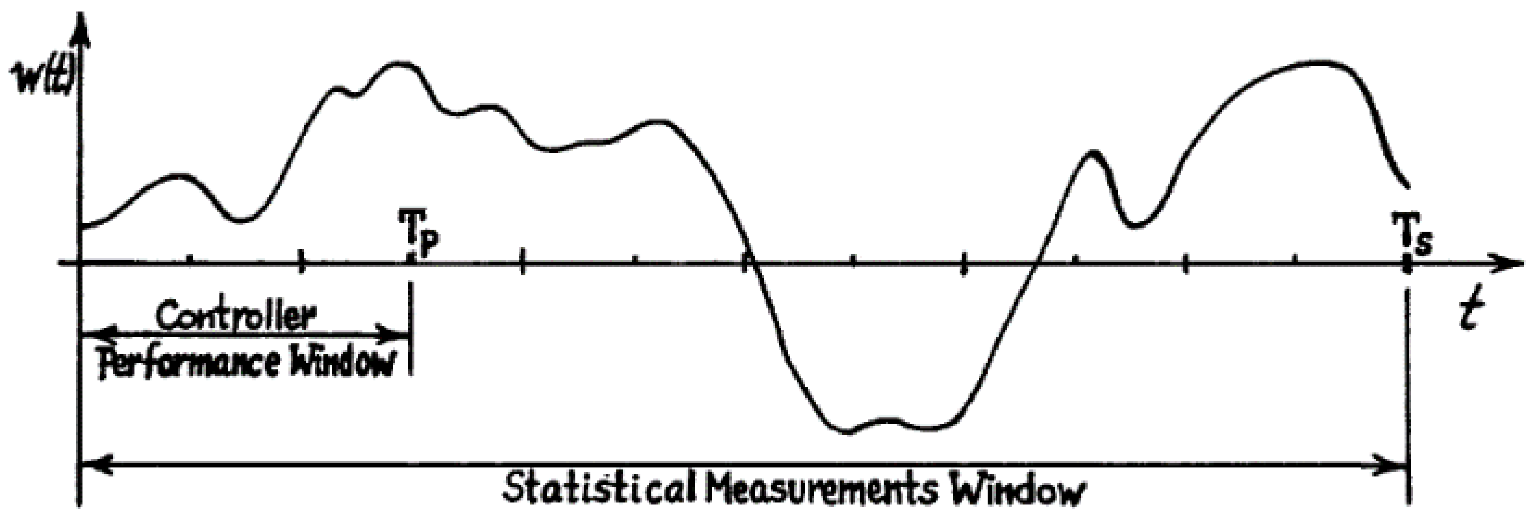
What this talk is and what it is not

- This talk is not:
  - An argument for adaptive control in every system always
  - A chance for me to look smart by being confusing
  - An overview of everything adaptive control
- This talk is:
  - Something I like talking about
  - Technically relevant
  - Presents compelling theoretical challenges

Why Study Adaptive Control?

Some perspectives

- Classical vs. stochastic vs. adaptive control
- Flight and Space Structure Needs:
  - Operating in a poorly known environment
  - Are experiments equivalent to actual operation?
  - Many degrees of freedom
  - Finite element models are only as good as the physics
  - Changing situations: takeoff, deployment, landing
  - Control schemes based on reduced order models
- Greatly emphasizes local vs. global, linear vs. nonlinear thinking



Gambling in function space from [1].

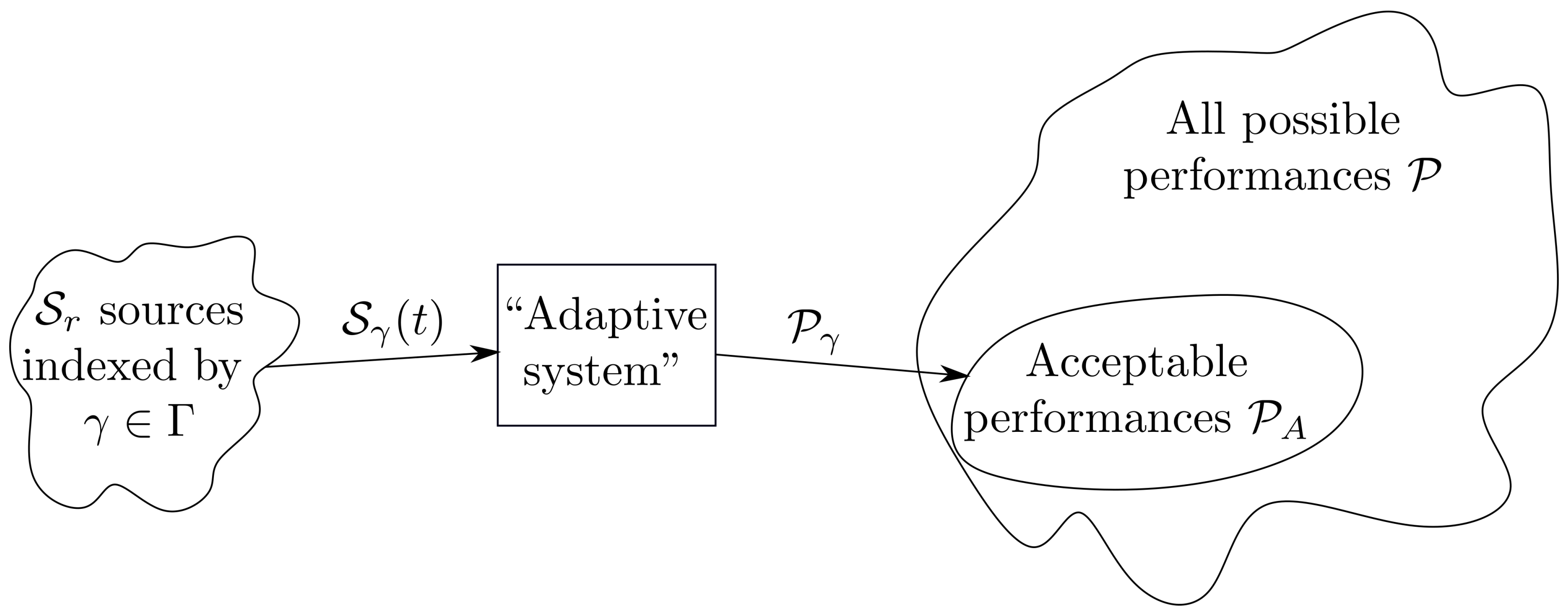
Why Study Adaptive Control?

Defining an adaptive system

- Conceptually:
  - A system with knowledge of its performance and the potency to improve it.
- OR, more mathematically

> *A map $\mathcal{J}$ from $\mathcal{S}_r$ to $\mathcal{P}$ ($\mathcal{J} : \mathcal{S}_r \to \mathcal{P}$) with range $\mathcal{J}(\mathcal{S}) \subseteq \mathcal{P}_A$*

  - Remark: All systems are adaptive in this definition with respect to some $\mathcal{S}_r$ and $\mathcal{P}_A$



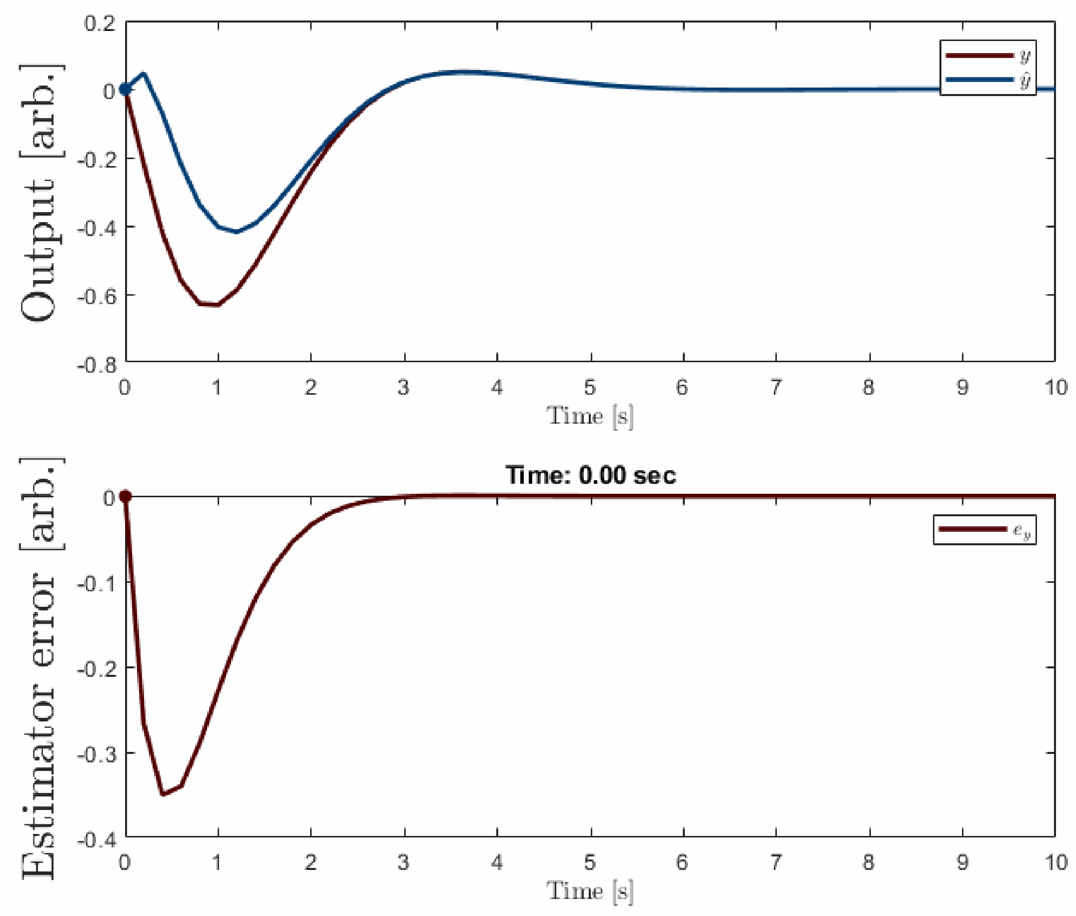
L. Zadeh, "Optimality and non-scalar-valued performance criteria [2].

Augmentation example

Augmentation example

Recovering $\mathcal{P}_A$

- Double integrator:
  - $\dot{x} = Ax + Bu,\ y = Cx$
  - $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
  - Min. phase with $Z(A, B, C) = -1$
- Separation principle controller:
  - $u = G\hat{x}$
  - $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
  - $\hat{y} = C\hat{x}$
- With set gains:
  - $\sigma(A + BG) = -1 \pm j \Rightarrow G = \begin{bmatrix} -2 & -2 \end{bmatrix}$
  - $\sigma(A - KC) = -2 \pm j \Rightarrow K = \begin{bmatrix} -1 & 5 \end{bmatrix}^T$

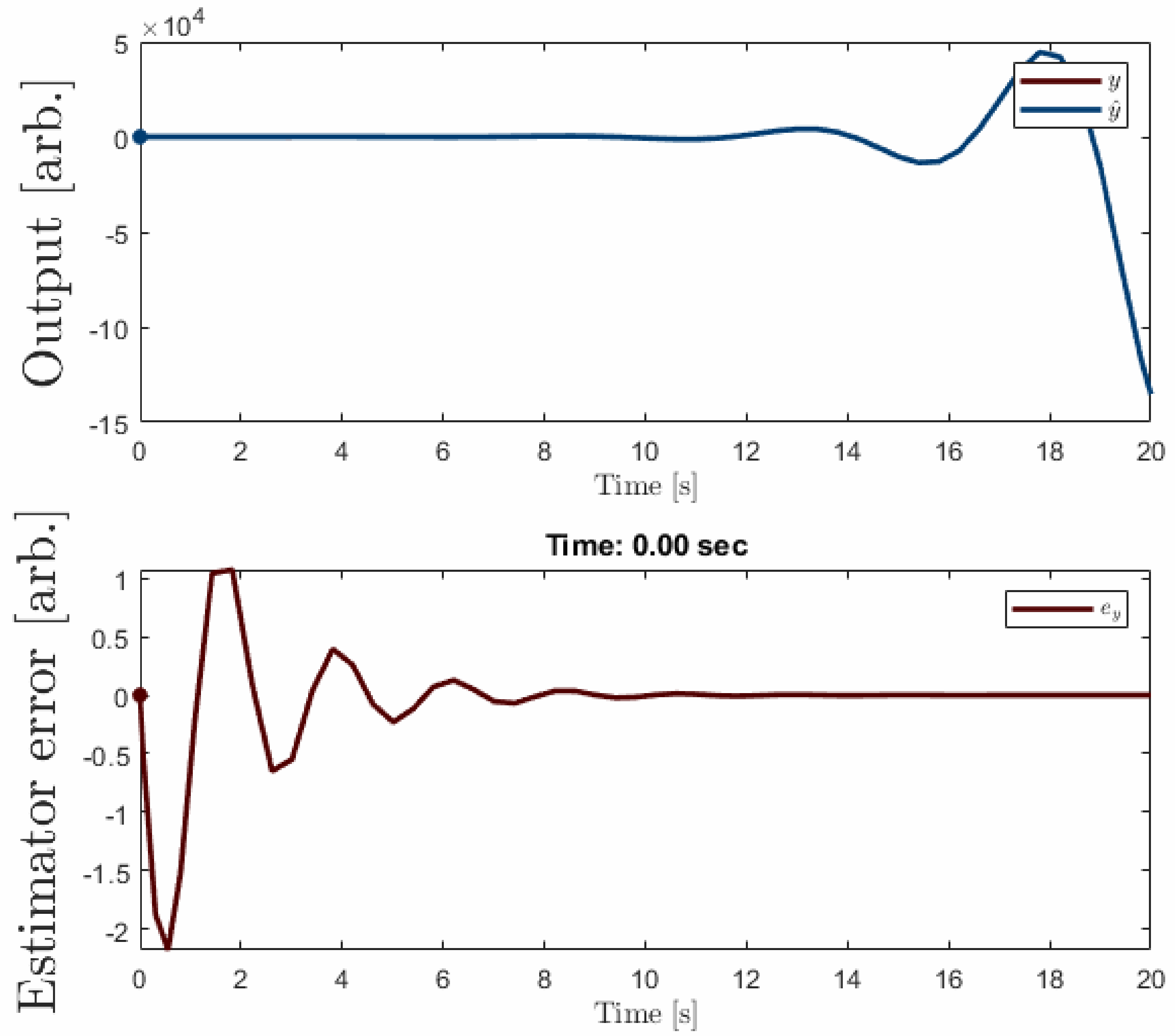


Separation principle controller is stable.

Augmentation example

Recovering $\mathcal{P}_A$

- But **suppose A became $\tilde{A}$:**
  - $\dot{x} = \tilde{A}x + Bu,\ y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [\,1 \quad 1\,]$
  - Min. phase with $Z(A, B, C) = -1$
- Separation principle controller:
  - $u = G\hat{x}$
  - $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
  - $\hat{y} = C\hat{x}$
- With set gains:
  - $\sigma(\tilde{A} + BG) = 0.5 \pm 1.3j \Rightarrow G = [\,-2 \quad -2\,]$
  - $\sigma(\tilde{A} - KC) = -0.5 \pm 2.7j \Rightarrow K = [\,-1 \quad 5\,]^T$

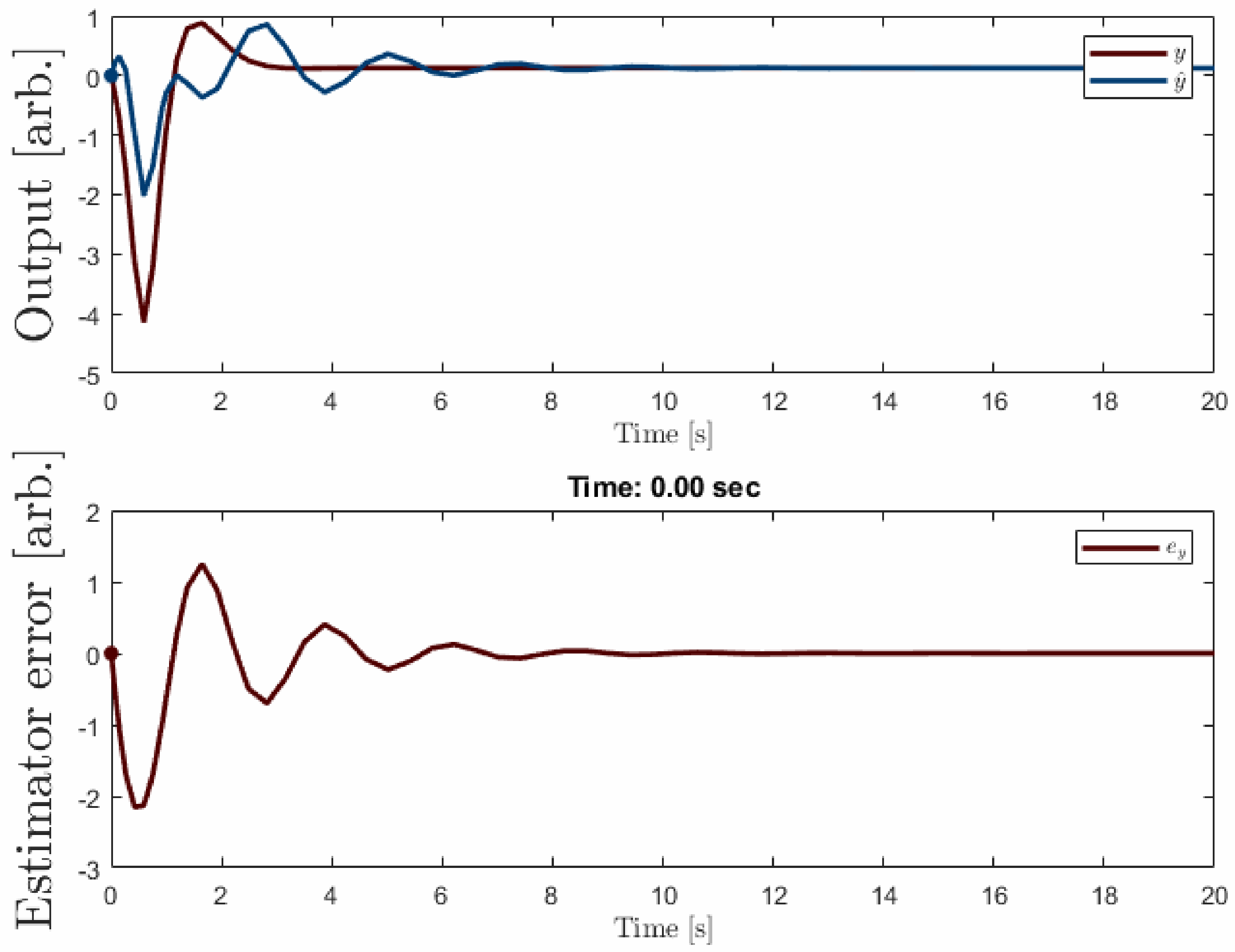


Perturbed separation principle controller is not stable.

Augmentation example

Recovering $\mathcal{P}_A$

- But suppose A became $\tilde{A}$ and I have augmented the system with an adaptive outer loop:
  - $\dot{x} = \tilde{A}x + Bu,\ y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 1]$
  - Min. phase with $Z(A, B, C) = -1$
- Adaptive separation principle controller:
  - $u = G\hat{x} + Ly$
  - $\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y})$
  - $\hat{y} = C\hat{x}$
- With same set gains and adaptive law:
  - $\dot{L} = -yy^T\sigma,\ \sigma > 0$

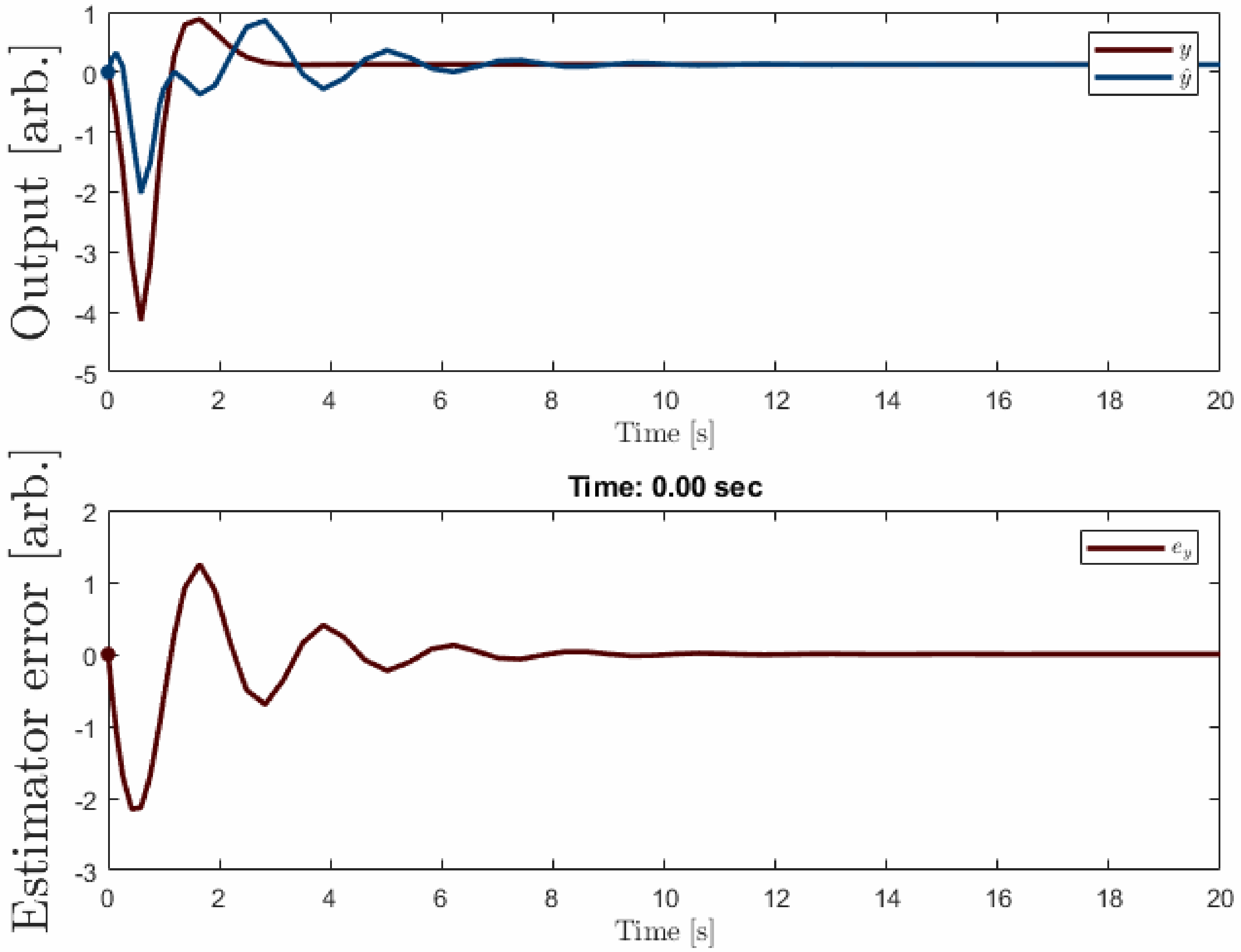


Adaptive separation principle controller is stable.

Augmentation example

Recovering $\mathcal{P}_A$

- Does this happen with gain scheduled controllers?
- We treated a significant constant perturbation adaptively
- Remark: Adaptive controllers are especially good at handling significant, slower disturbances
  - Robust controllers are especially good at small, fast disturbances
  - $\therefore$ **we should generally consider the adaptive augmentation of robust controllers.**



Adaptive separation principle controller is stable.

Adaptive Control is not Complicated

Adaptive Control is not Complicated

- Given:
  - $\begin{cases} \dot{x} & = Ax + Bu \\ y & = Cx \end{cases}$
  - (A, B, C) ctrb/obsv (i.e. minimal description of $P(s) = C(sI - A)^{-1}B$)
- Recall Kimura-Davison sufficient conditions:
  - $M \equiv \text{rank } B = \text{rank } C = M$ (square)
  - (A, B, C) ctrb/obsv
  - $M \geq \frac{N+1}{2};\ N = \dim x$
    - $\exists G_* \ni \sigma(A + BG_*C)$ that assigns pole locations arbitrarily

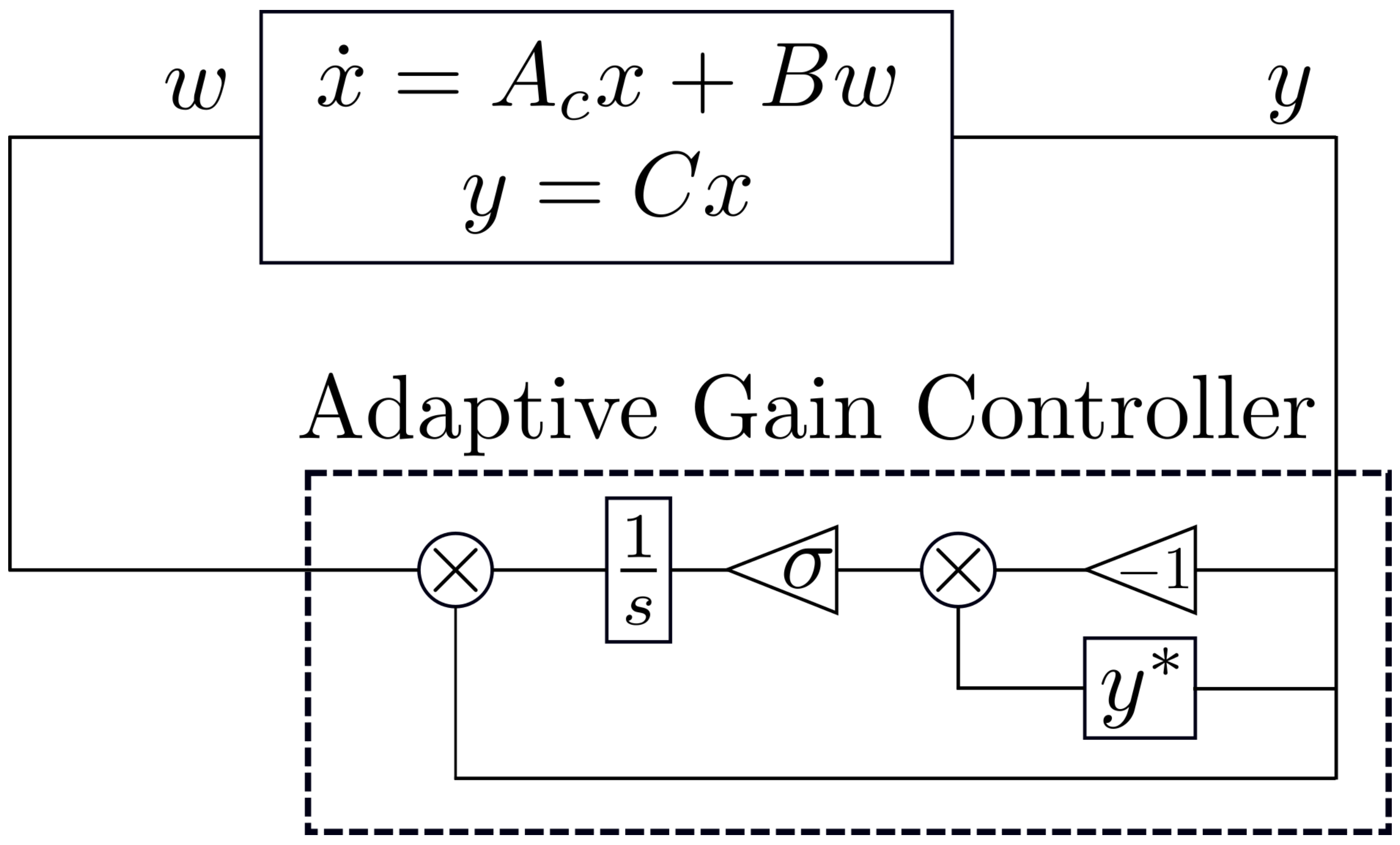
Adaptive Control is not Complicated

- Sufficient conditions for arb. pole placement but we must **know** (A, B, C) in detail to find G_*!
- This can be onerous, but if G_* exists, the system is called output feedback stabilizable
- Ex:
  - $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & \varepsilon \end{bmatrix}$
  - With $G_* = -g$, $A + BG_*C = \begin{bmatrix} 0 & 1 \\ -g & -g\varepsilon \end{bmatrix}$
  - $\det(\lambda I - A_c) = \lambda^2 + g\varepsilon\lambda + g$
  - $\therefore$ output feedback stabilizable when $\varepsilon > 0$ only!
    - Note: $\exists P > 0 \ni A_c^T P + PA_c = -Q > 0$

Adaptive Control is not Complicated

Adaptive Regulator using Output Feedback Only

- Plant: $\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases}$ (square)
- Regulator: $\begin{cases} u &= Gy \\ \dot{G} &= -yy^T\sigma \end{cases}$
- Let $G \equiv G_* + \Delta G$. Closed loop system is:
  - $\begin{cases} \dot{x} &= \underbrace{(A + BG_*C)}_{A_c}x + B\underbrace{\Delta G y}_{w} \\ y &= Cx \\ \Delta\dot{G} &= \dot{G} = -yy^T\sigma,\ \sigma > 0 \end{cases}$

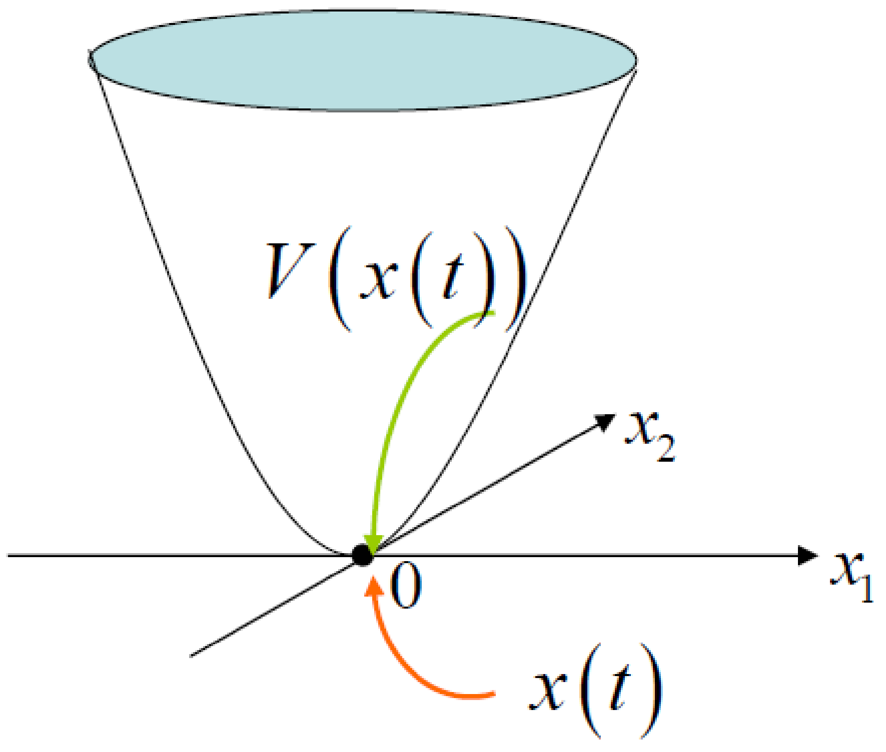


Adaptive regulator architecture.

Adaptive Control is not Complicated

Lyapunov Stability Argument

- If a scalar function $V(x,t)$ satisfies
  - function is lower bounded
  - Time derivative $\dot{V}(x,t)$ is negative semidefinite: $\sigma\big(\dot{V}(x,t)\big) \leq 0$
  - Time derivative $\dot{V}(x,t)$ is uniformly continuous in t: derivative is bounded
- Then $\lim_{t\to\infty} \dot{V}(x,t) = 0$
- and we have a theoretical stability guarantee.

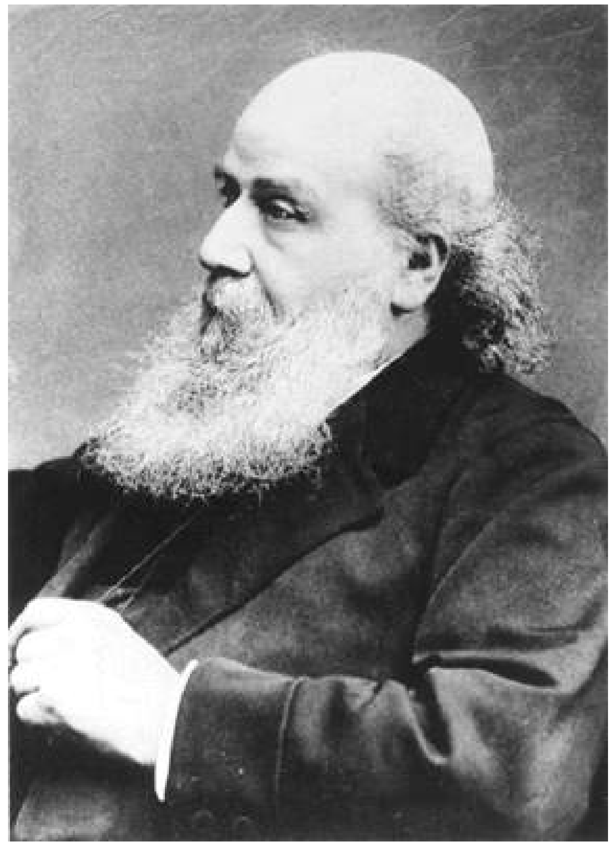


Example Lyapunov candidate function

Adaptive Control is not Complicated

Lyapunov Stability Argument

- Here, P from $A_c^T P + PA_c = -Q$ yields a quadratic, lower bounded function
  - $\frac{\lambda_{\min}(P)}{2}||x||^2 \leq V_1(x) \equiv \frac{1}{2}x^* Px \leq \frac{\lambda_{\max}(P)}{2}||x||^2$
  - which meets our first requirement.
- Notice
  - $$\begin{aligned} \dot{V}_1(x) \equiv \triangle V_1 \dot{x} &= x^* P[A_c x + Bw] \\ &= x^* PA_c x + x^* PBw \\ &\leq -\frac{1}{2}x^* Qx + x^* C^* w \\ &\leq -1/2\lambda_{\min}(Q)||x||^2 + (y, w) \end{aligned}$$
  - which may or may not be negative semidefinite, but is bound.



James Joseph Sylvester

Adaptive Control is not Complicated

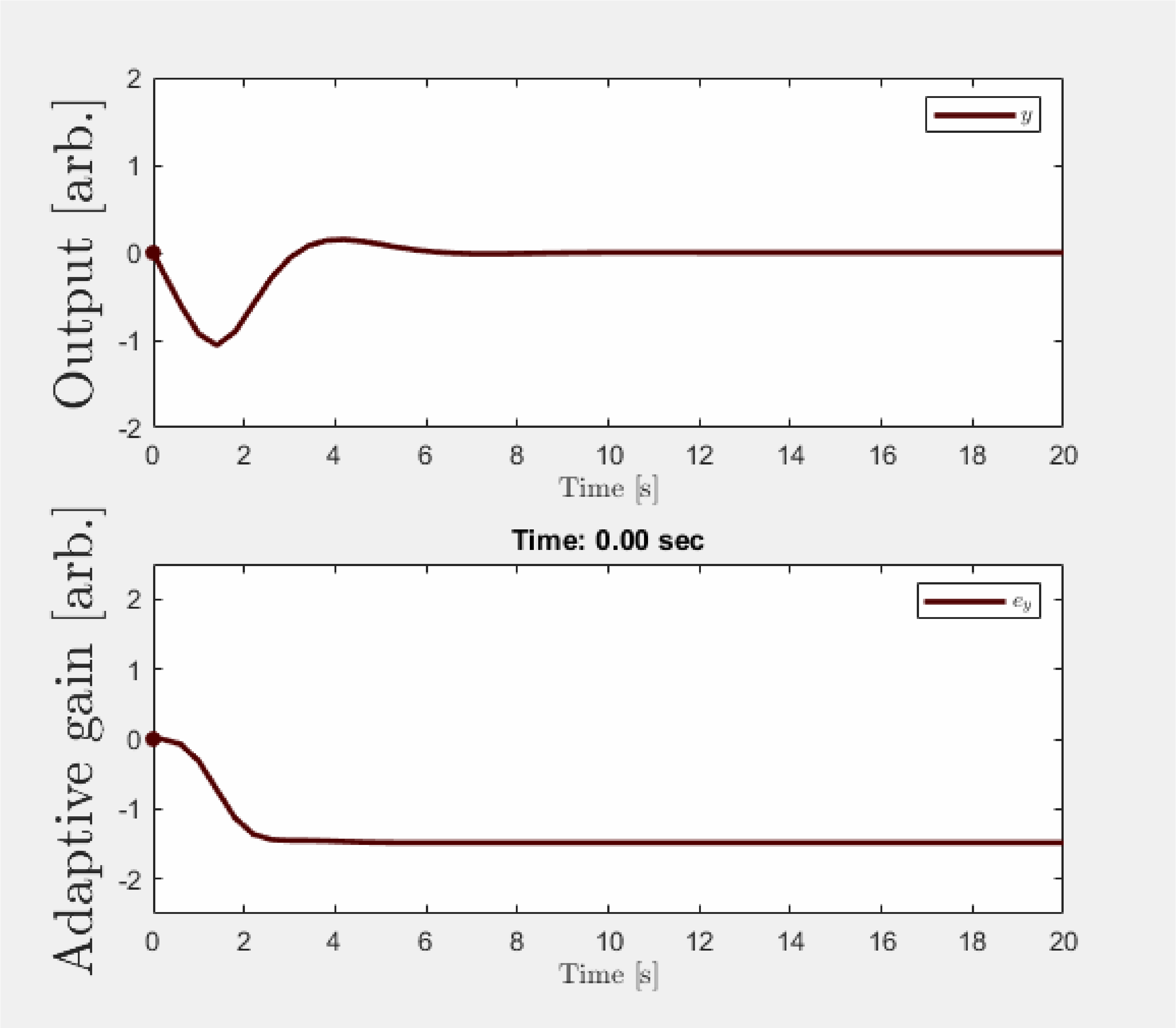
Lyapunov Stability Argument

- $\dot{V}_1(x) \leq -1/2\lambda_{\min}(Q)||x||^2 + (y, w)$
  - which may or may not be negative semidefinite, but is bound.
- However, we have not checked the stability of the adaptive gain G
  - Consider $V_2(\Delta G) \equiv \frac{1}{2}\mathrm{tr}(\Delta G \sigma^{-1} \Delta G^*)$
  - $$\begin{aligned} \dot{V}_2 &= \mathrm{tr}(\Delta \dot{G} \sigma^{-1} \Delta G^*) \\ &= \mathrm{tr}(-yy^* \sigma \sigma^{-1} \Delta G^*) \\ &= -\mathrm{tr}(y\underbrace{y^* \Delta G^*}_{w^*}) = -\mathrm{tr}(w^* y) \text{ scalar!} \\ &= -(y, w) \end{aligned}$$
- Which "conveniently" yields:
  - $$\begin{aligned} \dot{V}(x, \Delta G, t) &= \dot{V}_1(x, t) + \dot{V}_2(\Delta G, t) \\ &\leq -1/2\lambda_{\min}(Q)||x||^2 + (y, w) - (y, w) \\ &\leq -1/2\lambda_{\min}(Q)||x||^2 \end{aligned}$$
- Since x, G are now bound, composite system is bound. V is negative semidefinite. Therefore, by Lyapunov, $x \Rightarrow 0$.

Augmentation example

Double integrator example

- Returning to our double integrator example:
  - $\dot{x} = \tilde{A}x + Bu,\ y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
- Adaptive regulator:
  - $u = Gy$
- With adaptive law:
  - $\dot{G} = -yy^T\sigma,\ \sigma > 0$



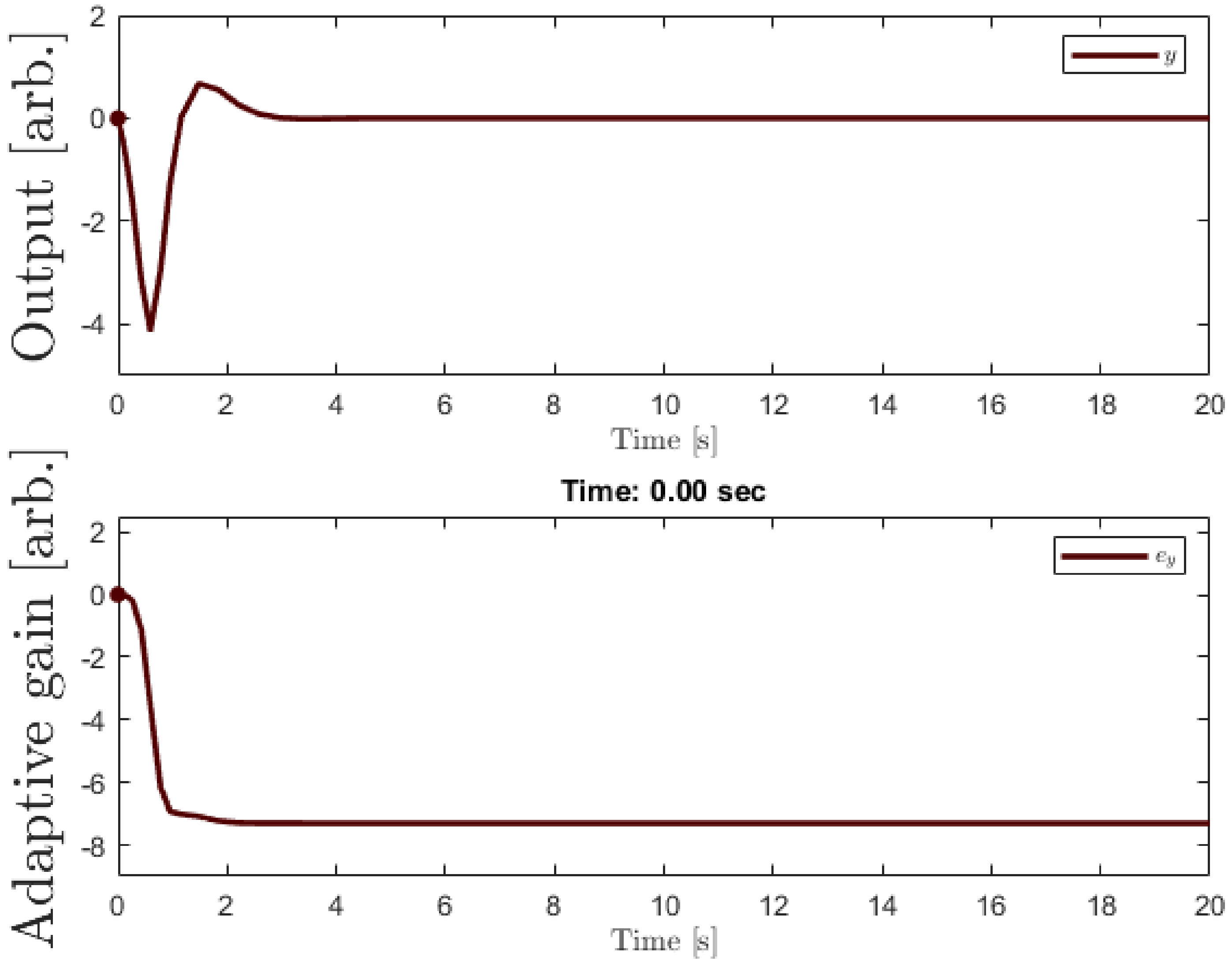
Adaptive controller is stable.

Augmentation example

Double integrator example

- Returning to our double integrator example:
  - $\dot{x} = \tilde{A}x + Bu,\ y = Cx$
  - $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
- Adaptive regulator:
  - $u = Gy$
- With adaptive law:
  - $\dot{G} = -yy^T\sigma,\ \sigma > 0$

Achieve exponential stability with exactly the same controller!



Same controller is stable for a different plant.

Adaptive Unknown Input Estimators

Adaptive Unknown Input Estimators

Estimator overview

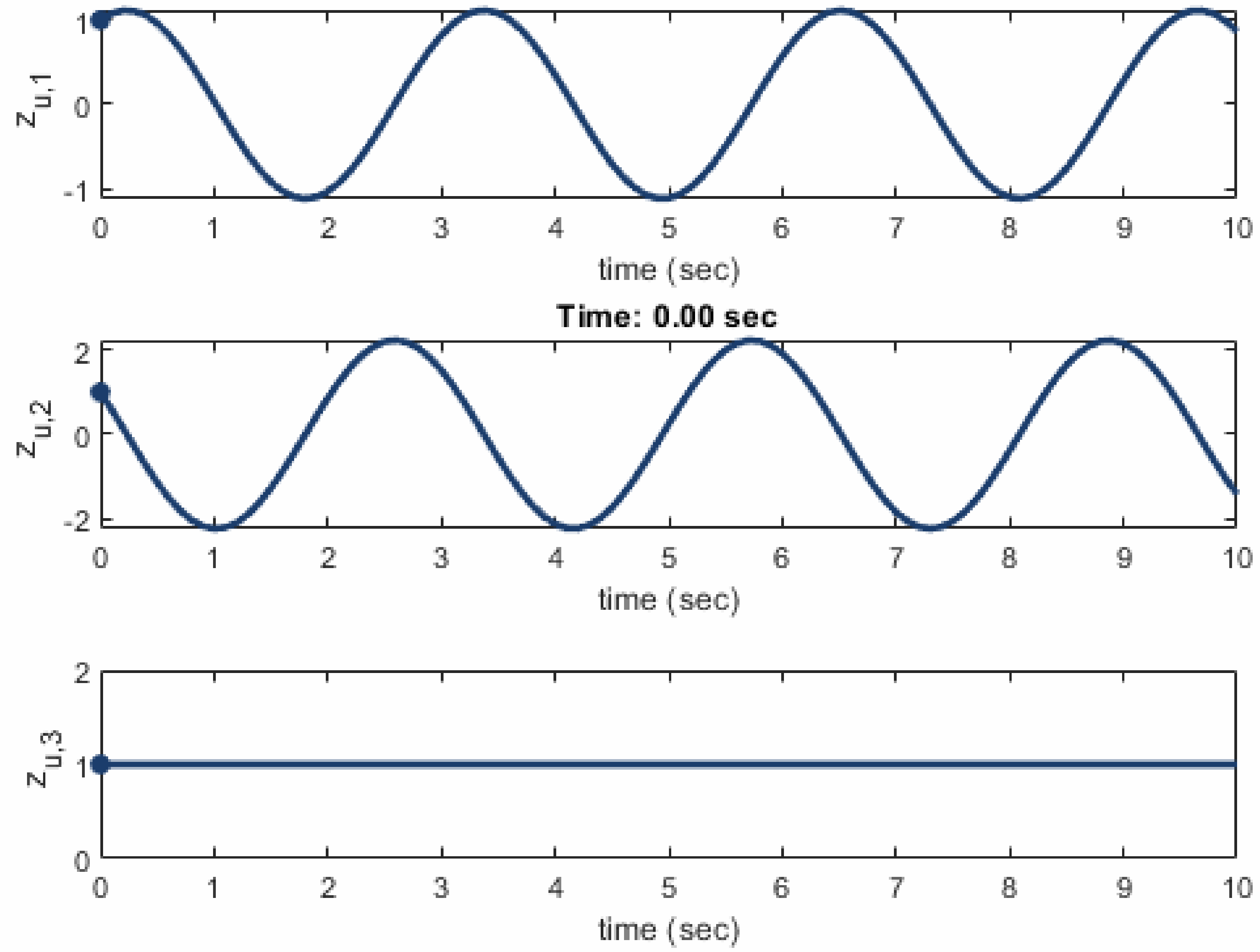
- Three significant uncertainties
  - Input u is unknown, external, deterministic
  - State matrix A may have uncertainty
  - Known, Lipschitz nonlinear internal dynamics $g(x)$
- **Can we synthesize $\boldsymbol{u}$ and correct $\boldsymbol{A}$?**

$$\dot{x} = Ax + g(x) + Bu$$
$$y = Cx$$

Adaptive Unknown Input Estimators

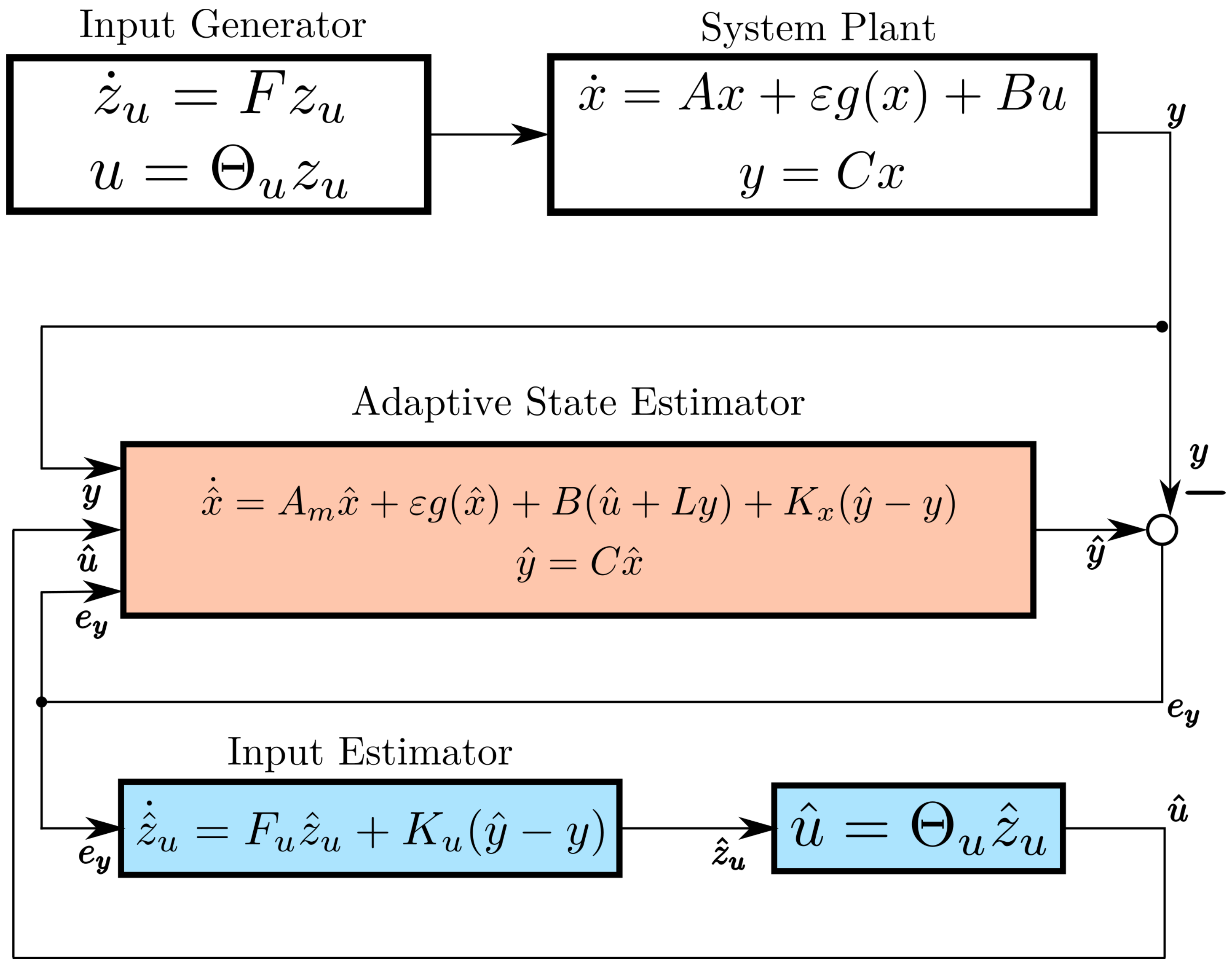
Modeling unknown inputs

- Approximate input space $\mathbb{U}$
  - $\hat{u} = \sum_{i=1}^{N} c_i f_i(t)$
- Persistent Inputs
  - $\dot{z}_u = F_u z_u$
  - $\hat{u} = \Theta_u z_u$
  - $F_u = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$



Adaptive Unknown Input Estimators

Architecture and estimator error



Recover A with adaptive scheme

$$A \equiv A_m + BL_* C$$

$$\dot{L} = -e_y y^* \gamma_e;\ \gamma_e > 0$$

Error dynamics

$$\dot{e} = (\bar{A} + \bar{K}\bar{C})e + \bar{B}\Delta L y + \varepsilon \Delta g$$

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_z \end{bmatrix} = \underbrace{\begin{bmatrix} A_m + K_x C & B\Theta_u \\ K_u C & F \end{bmatrix}}_{\bar{A}_c} \begin{bmatrix} e_x \\ e_z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w + \varepsilon \begin{bmatrix} g(\hat{x}) - g(x) \\ 0 \end{bmatrix}$$

Adaptive Unknown Input Estimators

Architecture and estimator error

- ASD plant dynamics
  - $\bar{A}_c^* \bar{P} + \bar{P}\bar{A}_c = -\bar{Q}$
  - $\bar{P}\bar{B} = \bar{C}^*$
- A Hurwitz
- Bounded L_*

- Error in state and input converges to zero
  - $V(e, \Delta L) = \frac{1}{2} e^* \bar{P} e + \frac{1}{2} \mathrm{tr}(\Delta L \gamma_e^{-1} \Delta L^*)$
  - $\dot{V}(e, \Delta L) \leq -\Big(\underbrace{\tfrac{1}{2}\lambda_{\min}(\bar{Q}) - \varepsilon\mu\lambda_{\max}(\bar{P})}_{\bar{\alpha}>0}\Big)||e||^2$

$$0 < \varepsilon < \frac{\lambda_{\min}(\bar{Q})}{2\mu\lambda_{\max}(\bar{P})} \iff \bar{\alpha} > 0.$$

Illustrative example

$$\begin{aligned}\dot{x} &= A_m x + \varepsilon g(x) + Bu \\ &= \begin{bmatrix} -4 & 1 & 2 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} x + \sin(x) + Bu \\ y &= Cx\end{aligned}$$

$$\begin{aligned}\dot{x} &= Ax + \varepsilon g(x) + Bu \\ &= \begin{bmatrix} -2.86 & 1 & 4.7 \\ 1.8 & -1 & 6.7 \\ -9 & 1 & -17.2 \end{bmatrix} x + \sin(x) + Bu \\ y &= Cx\end{aligned}$$

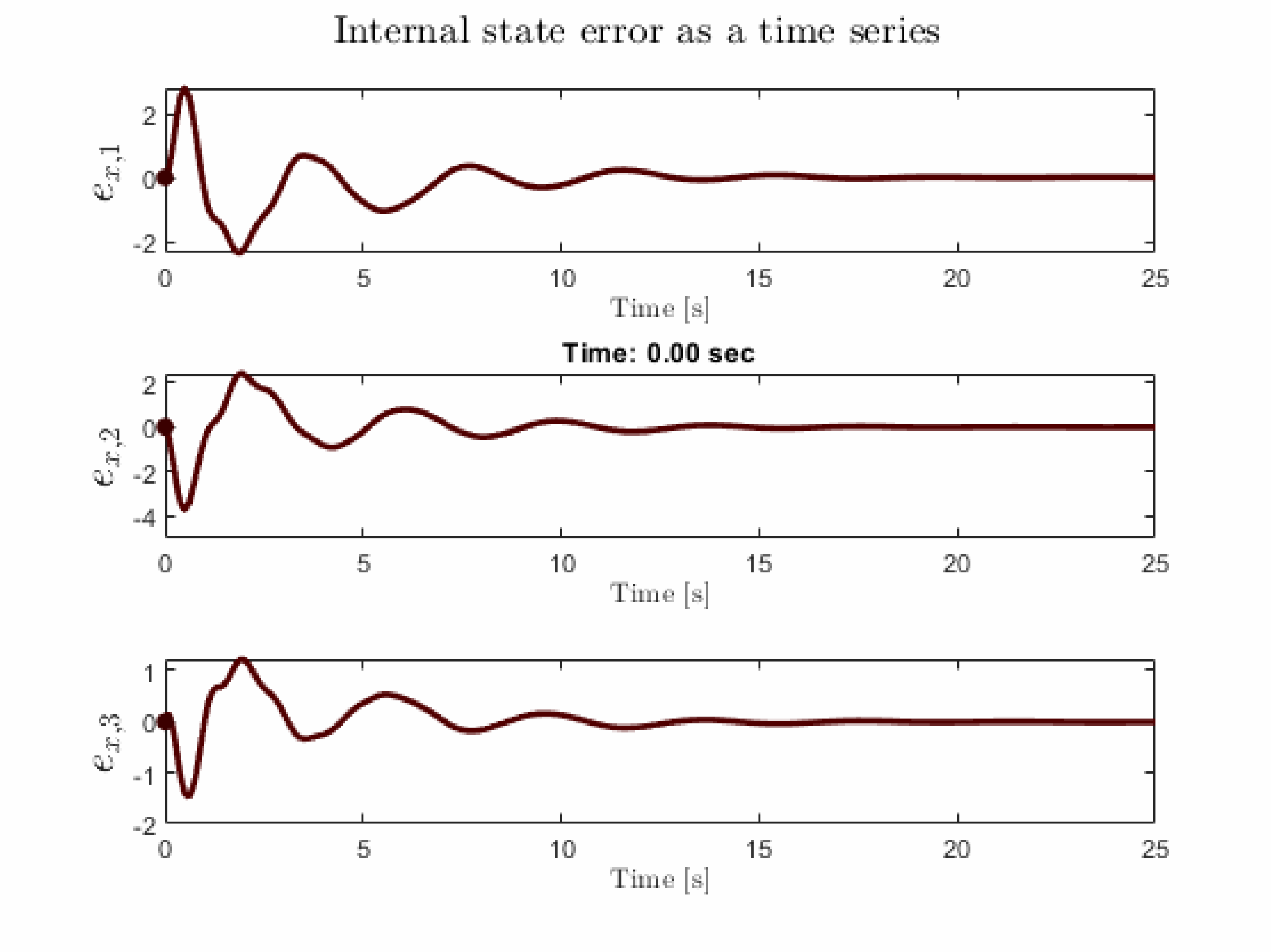
$$L* = \begin{bmatrix} -8 & 1 \\ 2 & -7 \end{bmatrix}$$

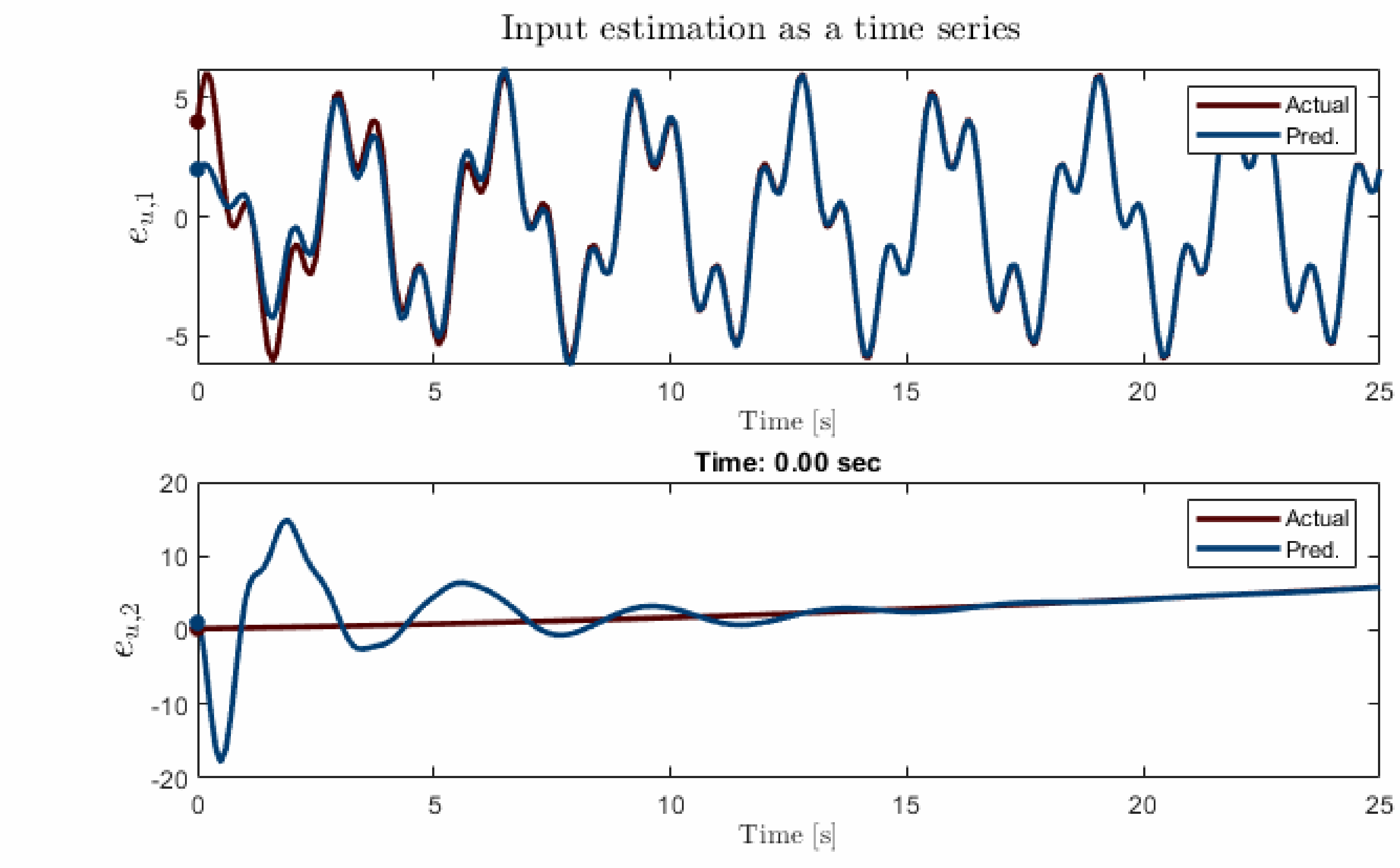
$$u_1(t) = c_{11}\sin(2t) + c_{12}\cos(2t) + c_{13}\sin(7t) + c_{14}\cos(7t)$$

$$u_2(t) = c_{11} + c_{22}t + c_{23}t^2 + c_{24}t^3$$

Illustrative example

Both the state error and the input error converge simultaneously

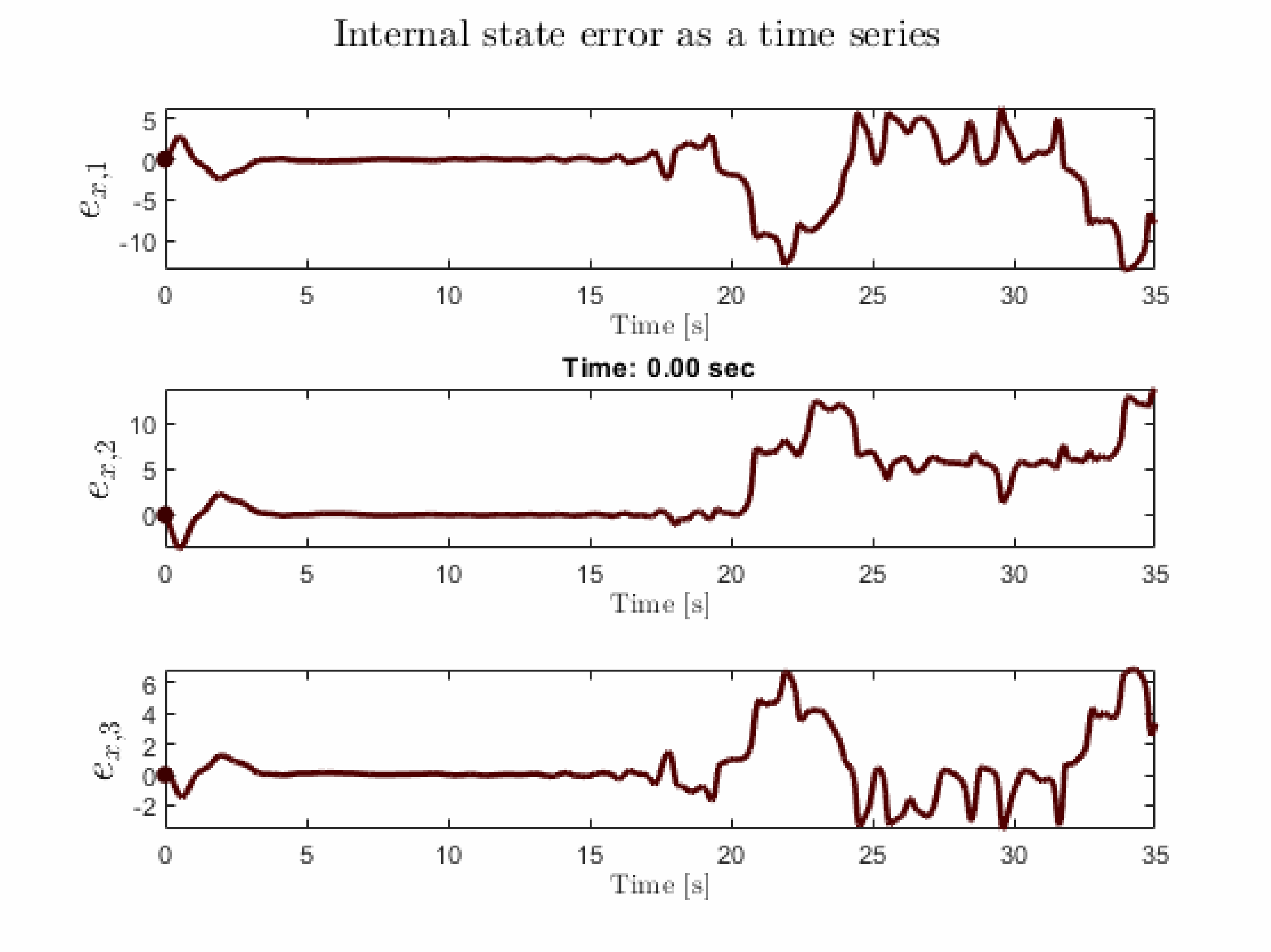


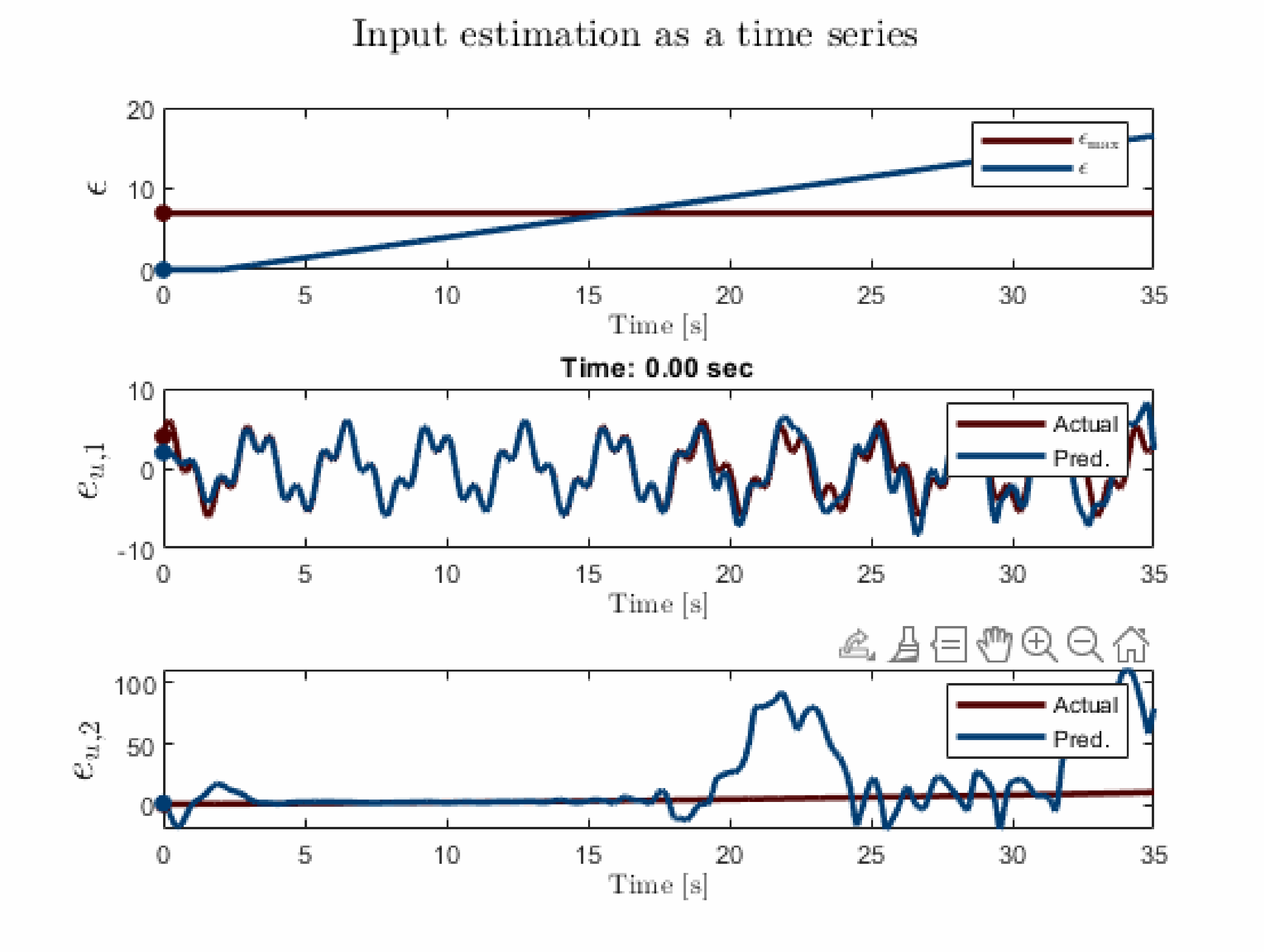


Illustrative example

provided ϵ is not too great

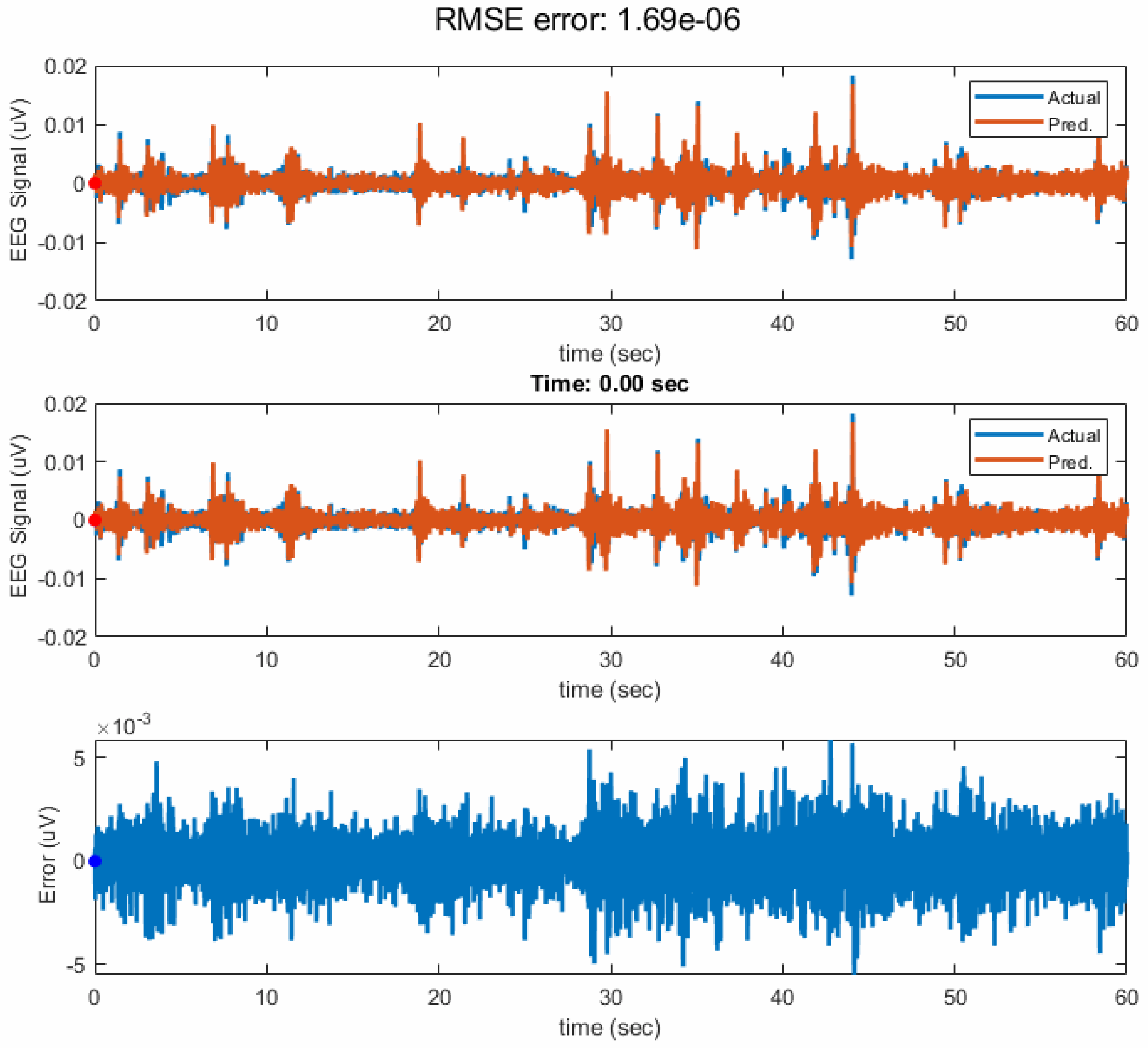
$$0 < \epsilon < \frac{\lambda_{\min}(\bar{Q})}{2\mu\lambda_{\max}(\bar{P})}$$



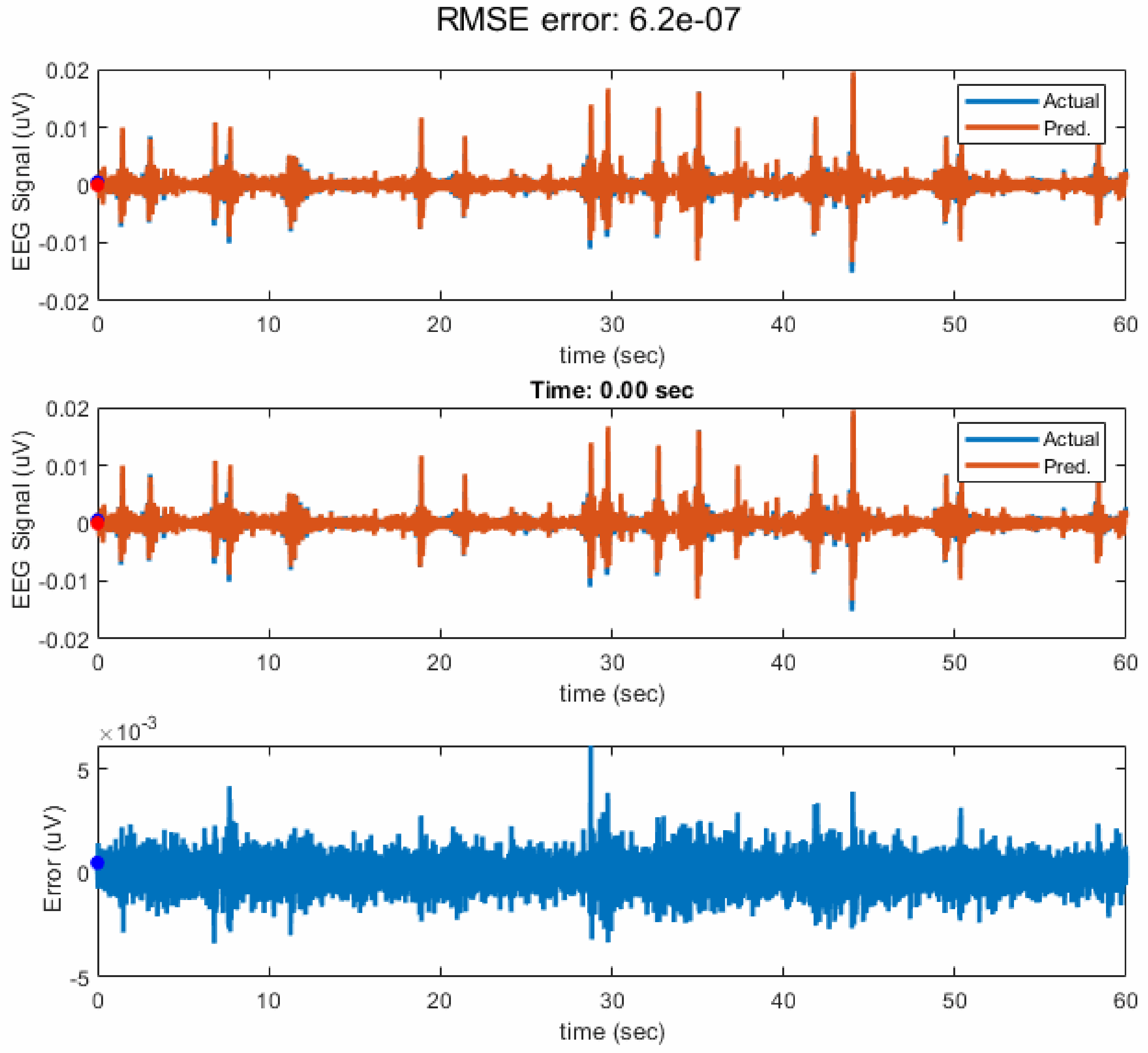


Application: Biomarker dynamics

Kalman filtering

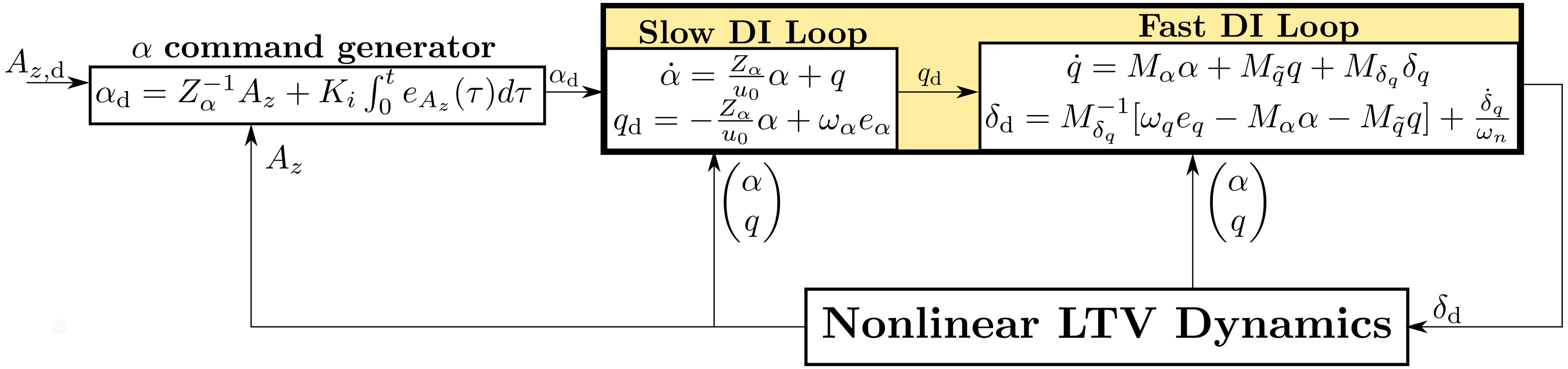


aUIO



Application: Dynamic inversion for High Speed Projectile

Adaptive DI scheme



Most sensitive to error in outer loop coefficients: $\dot{Z}_\alpha^{-1} = e_{A_z} A_z \sigma$

Open problems:

- Methods to certify flight critical systems not readily available
  - Existing validation methods are analogous but not immediate.
  - Stability margins? Validation of closed loop performance?

We lived in a sloppy world,

but we were precise, very precise.

- Carrying the Fire