

# Answers: Solving equations involving logarithms

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## Summary

Answers to questions relating to solving equations involving logarithms.

*These are the answers to [Questions: Solving equations involving logarithms](#)*

**Please attempt the questions before reading these answers!**

Throughout this answer sheet, the natural logarithm  $\log_e(x)$  is written as  $\ln(x)$ .

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## Q1

1.1. Here,  $y = x^{1/4}$  and  $x = y^4$ .

1.2. Here,  $y = x^3$  and  $x = y^{1/3}$ .

1.3. Here,  $x = \sqrt{y^{1/2}x^3}$ ,  $y = x^4/z^6$ ,  $z = \sqrt[3]{x^2y^{-1/2}}$ .

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## Q2

2.1. The solution to  $6\log_3(x) + \log_3(5) = 9$  is  $x = \sqrt[6]{\frac{3^9}{5}}$ .

2.2. The solution to  $\log_2(16x) = 6$  is  $x = 4$ .

2.3. The solution to  $\log_{12} e^{2t} = 4$  is  $t = 2\ln(12) = \ln(144)$ .

2.4. The solution to  $\log_9(x) + \log_3(3x) = 6$  is  $x = 3^{10/3}$ .

2.5. The solution to  $4\ln\sqrt{x} - \ln(1-2x) = 0$  is  $x = -1 + \sqrt{2}$ .

2.6. The solution to  $\ln(x+1) - \ln(x) = e$  is  $x = \frac{1}{e^e - 1}$ .

2.7. There are no solutions to  $\log_{10}(2y+10) = \log_{10}(y-2)$ .

2.8. The solutions to  $\log_3\sqrt{x} - \log_9\sqrt{4x-3} = 0$  are  $x = 1$  and  $x = 3$ .

2.9. The solutions to  $\log_3(2 - 3x) = \log_9(6x^2 - 19x + 2)$  are  $x = -1/3$  and  $x = -2$ .

2.10. The solutions to  $\log_3(x) - 2\log_x(3) = 1$  are  $x = 9$  and  $x = 1/3$ .

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### Q3

The solutions are  $x = 15$  and  $y = 1/2$ .

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### Version history and licensing

v1.0: initial version created 08/23 by Ellie Gurini as part of a University of St Andrews STEP project, and updated 10/25 by tdhc.

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