Introduction to integration

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Summary

The idea of integration...

Before reading this guide, it is recommended that you read [Guide: Properties of functions], Guide: Laws of indices, Guide: Logarithms, and [Guide: Tangents].

What is integration

This guide will look at the idea of differentiation; where it comes from, how it can be used, and how you can apply its techniques to functions that you may be familiar with.

Example 4

Determine the behaviour of the function $f(x) = 2\ln(3x) - x$ when $x = 1$.
Here, you will first need to differentiate the function $f(x)$ to find $f'(x)$. Then, you will need to evaluate the derivative $f'(x)$ when $x = 1$ to see how the function behaves.
Using your rules of differentiation as you found above, you can say that the derivative of $2\ln(3x)$ is $2/x$, and the derivative of x is 1. Therefore, the derivative of the function $f(x) = 2\ln(3x) - x$ is

$$f'(x) = \frac{2}{x} - 1.$$

You can evaluate the derivative $f'(x)$ at $x = 1$ to get

$$f'(1) = \frac{2}{1} - 1 = 2 - 1 = 1$$

and so the derivative is positive at $x = 1$. This implies that the function $f(x) = 2\ln(3x) - x$ is increasing at the point $x = 1$.
It also means that the gradient of the tangent to the function $f(x)$ at the point $(1, 2\ln(3) - 1)$ is 1. You can see this in the figure below.

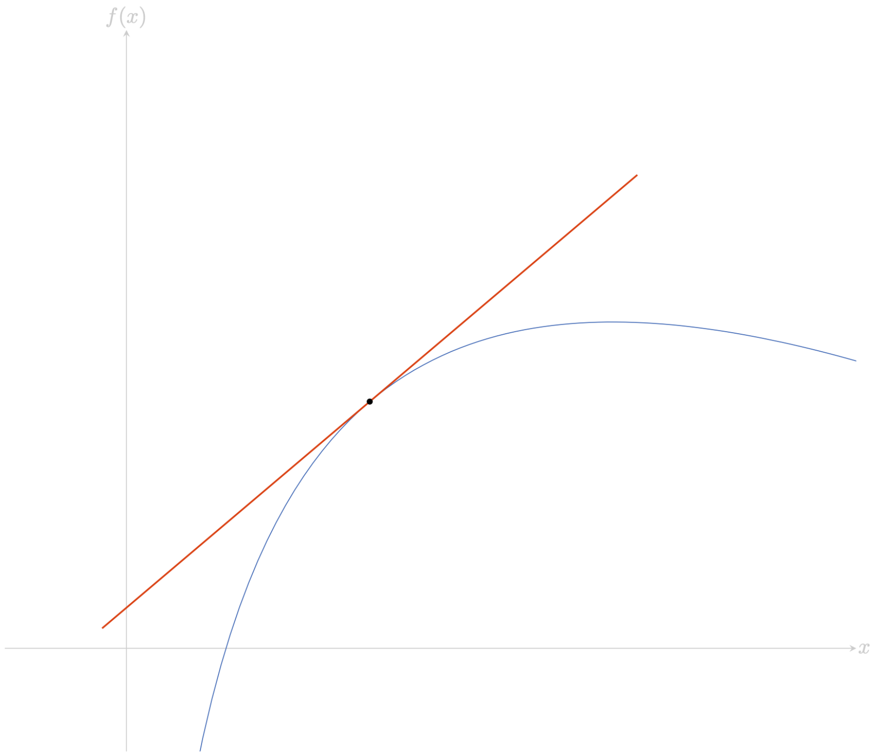

Figure 1: The graph of $f(x) = 2\ln(3x) - x$, with the tangent to the graph at $(1, 2\ln(3) - 1)$ illustrated, demonstrating that the function is increasing at $x = 1$.

Summary

Here's a table of derivatives that you should remember going into any further reading on differentiation. Here, a, b, c, n are any real numbers.

Function $f(x)$	Derivative $f'(x)$	Notes
$f(x) = c$	$f'(x) = 0$	
$f(x) = ax + b$	$f'(x) = a$	
$f(x) = ax^n$	$f'(x) = anx^{n-1}$	$n \neq 0$
$f(x) = ae^{bx}$	$f'(x) = abe^{bx}$	
$f(x) = a\sin(bx)$	$f'(x) = ab\cos(bx)$	
$f(x) = a\cos(bx)$	$f'(x) = -ab\sin(bx)$	
$f(x) = a\ln(bx)$	$f'(x) = \frac{a}{x}$	

Quick check problems

1. Answer the following questions true or false:

(a) The derivative of a function at $x = a$ is equal to the gradient of the tangent to $f(x)$ at $x = a$.

(b) If $f'(a) < 0$, then the function is increasing at $x = a$.

(c) If $f(x) = c$, then the derivative $f'(x) = c - 1$.

(d) The derivative of $f(x) = \cos(x)$ is $f'(x) = -\sin(x)$.

(e) The derivative of $f(x) = \frac{1}{x}$ is $f'(x) = \ln(x)$.

(f) The power of x in the derivative of $f(x) = \frac{1}{\sqrt{x}}$ is $-3/2$.

2. Differentiate the following functions with respect to x.

(a) $f(x) = 3x^7 - 14x$

(b) $f(x) = -4\cos(3x)$

(c) $f(x) = -15\sin(x) + e^{8x}$

Further reading

For more questions on the subject, please go to Questions: Introduction to differentiation and the derivative.

For more about techniques of differentiation, please see [Guide: The product rule], [Guide: The quotient rule], and [Guide: The chain rule].

For more about where the derivatives in the above table come from, please see Proof sheet: Derivatives of functions from first principles and [Proof sheet: Derivatives of other common functions]. For more about why the rules of differentiation are true, please see [Proof sheet: Rules of differentiation].

Version history

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