Questions: Integration by substitution

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Summary

A selection of questions for the study guide on integration by substitution.

Before attempting these questions, it is highly recommended that you read [Guide: Integration by substitution].

Q1

Using an appropriate substitution or the chain rule for integration, integrate the following expressions with respect to x.

1.1. $\displaystyle\int (2x+5)^3 \,\mathrm{d}x$

1.2. $\displaystyle\int (3-4x)^5 \,\mathrm{d}x$

1.3. $\displaystyle\int \left(\frac{x}{2}-1\right)^4 \,\mathrm{d}x$

1.4. $\displaystyle\int (5x-2)^{-3} \,\mathrm{d}x$

1.5. $\displaystyle\int (4-3x)^{-2} \,\mathrm{d}x$

1.6. $\displaystyle\int (2x+7)^{-3} \,\mathrm{d}x$

1.7. $\displaystyle\int \left(\frac{x}{5}+3\right)^{-4} \,\mathrm{d}x$

1.8. $\displaystyle\int (1-2x)^{-1/2} \,\mathrm{d}x$

1.9. $\displaystyle\int (3x+4)^{-3/2} \,\mathrm{d}x$

1.10. $\displaystyle\int (5-6x)^{-2/3} \,\mathrm{d}x$

Q2

Using an appropriate substitution or the chain rule for integration, integrate the following trigonometric functions with respect to x.

2.1. $\int \cos(x)\,\mathrm{d}x$

2.2. $\int \sin(2x)\,\mathrm{d}x$

2.3. $\int \frac{5}{6}\cos(x)\,\mathrm{d}x$

2.4. $\int \cos(3x)\,\mathrm{d}x$

2.5. $\int \sin\left(\frac{x}{3}\right)\,\mathrm{d}x$

2.6. $\int \frac{4}{5}\cos\left(3x-\frac{\pi}{4}\right)\,\mathrm{d}x$

2.7. $\int \sin\left(\frac{\pi}{3}-\frac{4x}{9}\right)\,\mathrm{d}x$

2.8. $\int -\frac{1}{2}\cos\left(3x+\frac{\pi}{2}\right)\,\mathrm{d}x$

2.9. $\int 4\sin\left(\frac{x}{4}-\frac{\pi}{2}\right)\,\mathrm{d}x$

2.10. $\int \frac{3}{5}\cos\left(\frac{\pi}{6}-5x\right)\,\mathrm{d}x$

Q3

Using an appropriate substitution or the chain rule for integration, integrate the following exponential and reciprocal linear functions with respect to x.

3.1. $\int 5\mathrm{e}^{2x+1}\,\mathrm{d}x$

3.2. $\int 7\mathrm{e}^{-3x+4}\,\mathrm{d}x$

3.3. $\int -\mathrm{e}^{-3(x-2)}\,\mathrm{d}x$

3.4. $\int 2\exp\left(\frac{x}{3}-5\right)\mathrm{d}x$

3.5. $\int \frac{6}{3x-7}\,\mathrm{d}x$

3.6. $\int \frac{4}{5-2x}\,\mathrm{d}x$

3.7. $\int \frac{3}{2x+5}\,\mathrm{d}x$

3.8. $\int -\frac{3}{5(x-2)+1}\,\mathrm{d}x$

Q4

Using an appropriate substitution, integrate the following functions with respect to x. Express your answers in terms of x only.

4.1. $\displaystyle\int 6x(3x^2+2)^4\,\mathrm{d}x$

4.2. $\displaystyle\int 5(5x-7)^3\,\mathrm{d}x$

4.3. $\displaystyle\int 8x\exp(4x^2-1)\,\mathrm{d}x$

4.4. $\displaystyle\int \frac{2x+1}{(x^2+x+5)^2}\,\mathrm{d}x$

4.5. $\displaystyle\int 6x\cos(3x^2+2)\,\mathrm{d}x$

4.6. $\displaystyle\int (2x+3)\exp(x^2+3x)\,\mathrm{d}x$

4.7. $\displaystyle\int \frac{x}{(x^2+1)^{3/2}}\,\mathrm{d}x$

4.8. $\displaystyle\int \frac{\mathrm{e}^{5x}}{2\mathrm{e}^{5x}+3}\,\mathrm{d}x$

4.9. $\displaystyle\int -4x\sin(4-2x^2)\,\mathrm{d}x$

4.10. $\displaystyle\int \frac{3x^2}{(x^3+1)^2}\,\mathrm{d}x$

After attempting the questions above, please click this link to find the answers.

Version history and licensing

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