Answers: Multivariate implicit differentiation

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Summary

Answers to questions relating to the guide on multivariate implicit differentiation.

These are the answers to Questions: Multivariate implicit differentiation.

Please attempt the questions before reading these answers!

Answers

Q1

1.1. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{x}{y}$.

1.2. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{3x^2y}{x^3+3y^2}$.

1.3. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2}{5}x(y-1)^2$.

1.4. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{y\cos(xy)+1}{x\cos(xy)-1}$.

1.5. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{e^y}{xe^y+2y}$.

1.6. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2xy-3y^2}{x^2-6xy}$.

1.7. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{y}{x}$.

1.8. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{x} + 2(x^2+y^2)$.

1.9. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y\sin(xy)+1}{3y^2-x\sin(xy)}$.

1.10. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y\sin(x)-\sin(y)}{x\cos(y)+\cos(x)}$.

1.11. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -1$.

1.12. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{ye^{xy}+1}{xe^{xy}-1}$.

1.13. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{x^2-y}{y^2-x}$.

1.14. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{4\sqrt{x}y}$.

1.15. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x^2+y^2}{2x^2}$.

Q2

2.1. $\dfrac{\partial z}{\partial x} = \dfrac{5xy-4x}{3y^2z+3z^2}$ and $\dfrac{\partial z}{\partial y} = \dfrac{5x^2-6yz^2}{6y^2z+6z^2}$.

2.2. $\dfrac{\partial z}{\partial x} = -\dfrac{x}{z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{y}{z}$.

2.3. $\dfrac{\partial z}{\partial x} = -\dfrac{z}{x}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{z}{y}$.

2.4. $\dfrac{\partial z}{\partial x} = -\dfrac{ze^{xz}}{xe^{xz}-1}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{1}{xe^{xz}-1}$.

2.5. $\dfrac{\partial z}{\partial x} = -\dfrac{z\cos(xz)}{x\cos(xz)-y\sin(yz)}$ and $\dfrac{\partial z}{\partial y} = \dfrac{z\sin(yz)}{x\cos(xz)-y\sin(yz)}$.

2.6. $\dfrac{\partial z}{\partial x} = -\dfrac{z}{x}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{z}{y}$.

2.7. $\dfrac{\partial z}{\partial x} = \dfrac{x^2-yz}{xy-z^2}$ and $\dfrac{\partial z}{\partial y} = \dfrac{y^2-xz}{xy-z^2}$.

2.8. $\dfrac{\partial z}{\partial x} = -\dfrac{4xz^{3/2}}{2z^{1/2}(x^2+y^2)+1}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{4yz^{3/2}}{2z^{1/2}(x^2+y^2)+1}$.

2.9. $\dfrac{\partial z}{\partial x} = -\dfrac{(z^2+1)e^x}{y^2(z^2+1)-1}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{2yz(z^2+1)}{y^2(z^2+1)-1}$.

2.10. $\dfrac{\partial z}{\partial x} = \dfrac{z(xy+1)}{x(z-1)}$ and $\dfrac{\partial z}{\partial y} = \dfrac{xz}{z-1}$.

2.11. $\dfrac{\partial z}{\partial x} = -\dfrac{yze^{xz}+e^{yz}}{xy(e^{xz}+e^{yz})}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{xze^{yz}+e^{xz}}{xy(e^{xz}+e^{yz})}$.

2.12. $\dfrac{\partial z}{\partial x} = \dfrac{\cos(x)\cos(z)}{\sin(x)\sin(z)-2yz}$ and $\dfrac{\partial z}{\partial y} = \dfrac{z^2}{\sin(x)\sin(z)-2yz}$.

2.13. $\dfrac{\partial z}{\partial x} = -\dfrac{2x}{ye^z+1}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{e^z}{ye^z+1}$.

2.14. $\dfrac{\partial z}{\partial x} = \dfrac{z}{x+y-z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{z}{x+y-z}$.

2.15. $\dfrac{\partial z}{\partial x} = \dfrac{z(2x + \sqrt{xyz})}{x(2z - \sqrt{xyz})}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{z(2y - \sqrt{xyz})}{y(2z - \sqrt{xyz})}$.

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