

# Factsheet: Rules of calculus

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## Summary

A list of common rules in calculus.

*Please note: clickable links lead to study guides where the rule is introduced.*

## Rules of differentiation

**Limit definition of the derivative:** If  $f(x)$  is a continuous function, then (if it exists) the derivative  $f'(x)$  is defined by

$$\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

**Sum/difference and constant rule:** If  $f(x)$  and  $g(x)$  are differentiable functions, then

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x) \quad \text{and} \quad \frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x)) = cf'(x)$$

**Product rule:** If  $f(x) = u(x)v(x)$ ,

$$f'(x) = \frac{d}{dx}(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

**Quotient rule:** If  $f(x) = u(x)/v(x)$  and  $v(x) \neq 0$ , then

$$f'(x) = \frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$$

**Chain rule:** If  $f(x) = f(u(x))$ , then

$$f'(x) = \frac{df}{du} \cdot \frac{du}{dx} = f'(u(x)) \cdot u'(x)$$

where  $f'(u(x))$  is the derivative of  $f(u)$  with respect to  $u$ .

**Implicit differentiation:** If  $f(x, y) = 0$  defines a function  $g(y)$  implicitly, then

$$\frac{d}{dx}(g(y)) = \frac{dg}{dy} \cdot \frac{dy}{dx} = g'(y) \cdot \frac{dy}{dx}$$

where  $g'(y)$  is the derivative of  $g(y)$  with respect to  $y$ .

## Rules of integration

**Sum/difference and constant rules:** If  $f, g$  are functions and  $k$  is any number:

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx \quad \text{and} \quad \int kf(x) dx = k \int f(x) dx$$

**Limit manipulation:** If  $f$  is a function and  $a, b$  are real numbers, then:

- for  $c$  such that  $a < c < b$ , then:

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

- if  $a \leq b$ , then:

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

**Integration by substitution:** For an indefinite integral,

$$\int f(u(x)) \cdot u'(x) dx = \int f(u) du$$

and for a definite integral

$$\int_a^b f(u(x)) \cdot u'(x) dx = \int_{u(a)}^{u(b)} f(u) du$$

**Integration by parts:** For functions  $u, v$  of  $x$ :

$$\int uv' dx = uv - \int vu' dx$$

**Integration of derivative over function:** For a function  $f$ ,

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C.$$

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## Version history

v1.0: created in 08/25 by tdhc.

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