Factsheet: Rules of calculus

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Summary

A list of common rules in calculus.

Please note: clickable links lead to study guides where the rule is introduced.

Rules of differentiation

Limit definition of the derivative: If f(x) is a continuous function, then (if it exists) the derivative f'(x) is defined by

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sum/difference and constant rule: If f(x) and g(x) are differentiable functions, then

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x)\pm g(x)) = f'(x)\pm g'(x) \quad \text{ and } \quad \frac{\mathrm{d}}{\mathrm{d}x}(cf(x)) = c\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) = cf'(x)$$

Product rule: If f(x) = u(x)v(x),

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} \left(u(x)v(x) \right) = u(x)v'(x) + u'(x)v(x)$$

Quotient rule: If f(x) = u(x)/v(x) and $v(x) \neq 0$, then

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{u(x)}{v(x)} \right) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$$

Chain rule: If f(x) = f(u(x)), then

$$f'(x) = \frac{\mathrm{d}f}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x} = f'(u(x)) \cdot u'(x)$$

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where f'(u(x)) is the derivative of f(u) with respect to u.

Implicit differentiation: If f(x,y) = 0 defines a function g(y) implicitly, then

$$\frac{\mathrm{d}}{\mathrm{d}x}(g(y)) = \frac{\mathrm{d}g}{\mathrm{d}y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = g'(y) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}$$

where g'(y) is the derivative of g(y) with respect to y.

Rules of integration

Sum/difference and constant rules: If f, g are functions and k is any number:

$$\int f(x) \pm g(x) \, \mathrm{d}x = \int f(x) \, \mathrm{d}x \pm \int g(x) \, \mathrm{d}x \quad \text{ and } \quad \int k f(x) \, \mathrm{d}x = k \int f(x) \, \mathrm{d}x$$

Limit manipulation: If f is a function and a, b are real numbers, then:

• for c such that a < c < b, then:

$$\int_a^b f(x) \, \mathrm{d}x = \int_a^c f(x) \, \mathrm{d}x + \int_c^b f(x) \, \mathrm{d}x$$

• if $a \leq b$, then:

$$\int_a^b f(x) \, \mathrm{d}x = -\int_b^a f(x) \, \mathrm{d}x$$

Integration by substitution: For an indefinite integral,

$$\int f(u(x)) \cdot u'(x) \, \mathrm{d}x = \int f(u) \, \mathrm{d}u$$

and for a definite integral

$$\int_a^b f(u(x)) \cdot u'(x) \, \mathrm{d}x = \int_{u(a)}^{u(b)} f(u) \, \mathrm{d}u$$

Integration by parts: For functions u, v of x:

$$\int uv'\mathrm{d}x = uv - \int vu'\,\mathrm{d}x$$

Integration of derivative over function: For a function f,

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C.$$

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