Answers: Laws of indices

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Summary

Answers to questions relating to using laws of indices.

These are the answers to Questions: Laws of indices.

Please attempt the questions before reading these answers!

Q1

1.1. $3^4 = 81$

1.2. $125^{\frac{2}{3}} = 5^2 = 25$

1.3. $32^{\frac{2}{5}} = 2^2 = 4$

1.4. $729^{\frac{-2}{3}} = 9^{-2} = \frac{1}{81}$

1.5. $4^3 \cdot 2^5 = 2^6 \cdot 2^5 = 2^{11} = 2048$

1.6. $2^2 \cdot 3^2 = (2 \cdot 3)^2 = 6^2$

1.7. $8^5 \cdot 6^5 = (8 \cdot 6)^5 = 48^5$

1.8. $12^6 \cdot 3^6 = (12 \cdot 3)^6 = 36^6 = 2176782336$

1.9. $\dfrac{9^2}{27^2} = 3^{-2} = \dfrac{1}{9}$

1.10. $(5^2)^2 = 5^4 = 625$

1.11. $(35^0)^9 = 1$

1.12. $(35^9)^0 = 1$

1.13. $(729^9)^{\frac{1}{9}} = 729^{\frac{9}{9}} = 729$

1.14. $7^{-3} = \dfrac{1}{7^3} = \dfrac{1}{343}$

1.15. $\left(\frac{4^5}{2^5}\right) = \left(\frac{4}{2}\right)^5 = 2^5$

1.16. $\left(\frac{2^{-2}}{13^{-2}}\right) = \left(\frac{2}{13}\right)^{-2} = \left(\frac{13}{2}\right)^2$

1.17. $64^{\frac{4}{3}} = 4^4 = 256$

1.18. $\left(\dfrac{4^3 \cdot 3^3}{6^3}\right) = \left(\dfrac{4 \cdot 3}{6}\right)^3 = \left(\dfrac{12}{6}\right)^3 = 2^3 = 8$

1.19. $\left(\dfrac{4^2 \cdot 8^2}{2^2}\right) \cdot \left(\dfrac{1}{2}\right)^2 = \left(\dfrac{4 \cdot 8}{2}\right)^2 \cdot \left(\dfrac{1}{2}\right)^2 = \left(\dfrac{4 \cdot 8 \cdot 1}{2 \cdot 2}\right)^2 = \left(\dfrac{32}{4}\right)^2 = 8^2 = 64$

1.20. $\dfrac{\left[\left(\frac{-2}{3}\right)^{-3} \cdot \left(\frac{-3}{5}\right)^{-3}\right]}{\left(\frac{2}{3}\right)^{-3}} = \dfrac{\left(\frac{6}{15}\right)^{-3}}{\left(\frac{2}{3}\right)^{-3}} = \dfrac{\left(\frac{15}{6}\right)^3}{\left(\frac{3}{2}\right)^3} = \left(\dfrac{15 \cdot 2}{6 \cdot 3}\right)^3 = \left(\dfrac{5}{3}\right)^3 = \dfrac{125}{27}$

1.21. $\dfrac{\left(\frac{1}{2}\right)^4 \left(\frac{3}{5}\right)^4}{\left(\frac{8}{3}\right)^4} = \dfrac{\left(\frac{5}{6}\right)^4}{\left(\frac{8}{3}\right)^4} = \left(\dfrac{15}{48}\right)^4 = \left(\dfrac{5}{16}\right)^4 = \dfrac{625}{65536}$

1.22. $\left(\dfrac{2}{3}\right)^{14} \cdot \left(\dfrac{9}{12}\right)^{14} = \left(\left(\dfrac{2}{3}\right) \cdot \left(\dfrac{9}{12}\right)\right)^{14} = \left(\dfrac{18}{36}\right)^{14} = \left(\dfrac{1}{2}\right)^{14} = \dfrac{1}{16384}$

Q2

2.1. $(b^7)^4 = b^{28}$

2.2. $y^{13} \cdot y^5 = y^{18}$

2.3. $a^2 \cdot b^2 = (ab)^2$

2.4. $\dfrac{x^{13}}{x^5} = x^8$

2.5. $(3y^{-2})^5 = (3)^5 \cdot (y^{-2})^5 = 243y^{-10}$

2.6. $(a)^{-4} \cdot (b)^{-4} = (ab)^{-4} = \frac{1}{(ab)^4}$

2.7. $(7z^{-5})^3 = (7)^3 \cdot (z^{-5})^3 = 343z^{-15}$

2.8. $\left(\frac{8x^5}{4x^{-5}}\right) = 2x^{(5+5)} = 2x^{10}$

2.9. $((x^2)^3 \cdot x^5) = x^6 \cdot x^5 = x^{11}$

2.10. $\dfrac{2a^{-4}}{3a^{-2}} = \left(\dfrac{2}{3}\right) \cdot (a^{-4+2}) = \dfrac{2}{3a^2}$

2.11. $\dfrac{x^5}{y^5} = \left(\dfrac{x}{y}\right)^5$

2.12. $\dfrac{2y^3}{2y^5} = y^{-2}$

2.13. $\left(\dfrac{2}{a}\right)^4 \cdot \left(\dfrac{a}{12}\right)^3 = \dfrac{2^4 \cdot a^3}{a^4 \cdot 12^3} = \dfrac{16}{1728a} = \dfrac{1}{108a}$

2.14. $\dfrac{25t^{-4}}{60t^5} = \dfrac{5}{12t^9}$

2.15. $\left(\dfrac{a}{b}\right)^{-4} \cdot \left(\dfrac{c}{d}\right)^4 \cdot \left(\dfrac{e}{f}\right)^4 = \left(\dfrac{bce}{adf}\right)^4$

2.16. $\dfrac{5^{x+1} \cdot 6^{x+1}}{3^{x+1}} = \left(\dfrac{5 \cdot 6}{3}\right)^{x+1} = 10^{x+1}$

2.17. $a^{\frac{1}{2}} \cdot b^{-\frac{1}{2}} = \left(\dfrac{a}{b}\right)^{\frac{1}{2}} = \sqrt{\dfrac{a}{b}}$

2.18. $\left(\frac{a}{b}\right)^{n} \cdot \left(\frac{c}{d}\right)^{-n} = \left(\left(\frac{a}{b}\right) \cdot \left(\frac{d}{c}\right)\right)^{n} = \left(\frac{ad}{bc}\right)^{n}$

Version history and licensing

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