

Answers: Rationalizing the denominator

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Summary

Answers to questions relating to the guide on rationalizing the denominator.

These are the answers to [Questions: Rationalizing the denominator](#).

Please attempt the questions before reading these answers!

Q1

$$1.1. \quad \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

$$1.2. \quad \frac{7}{2\sqrt{5}} = \frac{7\sqrt{5}}{10}$$

$$1.3. \quad \frac{11}{4\sqrt{7}} = \frac{11\sqrt{7}}{28}$$

$$1.4. \quad \frac{8}{5\sqrt{6}} = \frac{4\sqrt{6}}{15}$$

$$1.5. \quad \frac{3\sqrt{2}}{\sqrt{5}} = \frac{3\sqrt{10}}{5}$$

$$1.6. \quad \frac{9}{\sqrt{10}} = \frac{9\sqrt{10}}{10}$$

$$1.7. \quad \frac{\sqrt{7}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$$

$$1.8. \quad \frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{3}}{3}$$

$$1.9. \quad \frac{12}{\sqrt{11}} = \frac{12\sqrt{11}}{11}$$

$$1.10. \quad \frac{\sqrt{8}}{\sqrt{2}} = 2$$

$$1.11. \quad \frac{15}{3\sqrt{7}} = \frac{5\sqrt{7}}{7}$$

$$1.12. \quad \frac{6\sqrt{3}}{\sqrt{10}} = \frac{3\sqrt{30}}{5}$$

$$\begin{aligned}
1.13. \quad & \frac{\sqrt{18}}{\sqrt{9}} = \sqrt{2} \\
1.14. \quad & \frac{2\sqrt{5}}{\sqrt{12}} = \frac{\sqrt{30}}{3} \\
1.15. \quad & \frac{4}{\sqrt{2}} = 2\sqrt{2} \\
1.16. \quad & \frac{10}{5\sqrt{13}} = \frac{2\sqrt{13}}{13}
\end{aligned}$$

Q2

$$\begin{aligned}
2.1. \quad & \frac{5}{2 + \sqrt{3}} = 10 - 5\sqrt{3} \\
2.2. \quad & \frac{7}{4 - \sqrt{2}} = \frac{4 + \sqrt{2}}{2} \\
2.3. \quad & \frac{3}{\sqrt{5} + 1} = \frac{3\sqrt{5} - 3}{4} \\
2.4. \quad & \frac{\sqrt{7}}{\sqrt{3} - 1} = \frac{\sqrt{21} + \sqrt{7}}{2} \\
2.5. \quad & \frac{2 + \sqrt{5}}{1 - \sqrt{2}} = -2 - 2\sqrt{2} - \sqrt{5} - \sqrt{10} \\
2.6. \quad & \frac{3\sqrt{2} + 5}{4 + \sqrt{6}} = \frac{12\sqrt{2} - 6\sqrt{3} + 20 - 5\sqrt{6}}{10} \\
2.7. \quad & \frac{8}{3 - \sqrt{7}} = 12 + 4\sqrt{7} \\
2.8. \quad & \frac{6}{2 + \sqrt{5}} = -12 + 6\sqrt{5} \\
2.9. \quad & \frac{\sqrt{10}}{\sqrt{2} + 3} = \frac{3\sqrt{10} - 2\sqrt{5}}{7} \\
2.10. \quad & \frac{2\sqrt{3} + 5}{\sqrt{7} - 1} = \frac{2\sqrt{21} + 5\sqrt{7} + 2\sqrt{3} + 5}{6} \\
2.11. \quad & \frac{\sqrt{6} - \sqrt{2}}{2 + \sqrt{5}} = -2\sqrt{6} + 2\sqrt{5} + 2\sqrt{2} - \sqrt{10} \\
2.12. \quad & \frac{4 + \sqrt{3}}{5 - \sqrt{7}} = \frac{4\sqrt{7} + 5\sqrt{3} + \sqrt{21} + 20}{18} \\
2.13. \quad & \frac{2}{4 - \sqrt{11}} = \frac{8 + 2\sqrt{11}}{5}
\end{aligned}$$

$$2.14. \quad \frac{\sqrt{8} + \sqrt{3}}{\sqrt{7} - 2} = \frac{2\sqrt{14} + 4\sqrt{2} + \sqrt{21} + 2\sqrt{3}}{3}$$

Q3

3.1. To prove this equation, rationalize the denominator of the left hand side of the equation.

Since the denominator contains two square roots you can multiply the numerator and denominator by $-2\sqrt{3} + \sqrt{5}$ or by $2\sqrt{3} - \sqrt{5}$ to rationalize the denominator.

If you multiply the numerator and denominator by $2\sqrt{3} - \sqrt{5}$ you get:

$$\frac{\sqrt{11}}{2\sqrt{3} + \sqrt{5}} \cdot \frac{2\sqrt{3} - \sqrt{5}}{2\sqrt{3} - \sqrt{5}} = \frac{\sqrt{11}(2\sqrt{3} - \sqrt{5})}{(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5})}$$

Expanding the brackets in both the numerator and the denominator gives you:

$$\frac{\sqrt{11}(2\sqrt{3} - \sqrt{5})}{(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5})} = \frac{2\sqrt{33} - \sqrt{55}}{(2\sqrt{3})^2 - 2\sqrt{15} + 2\sqrt{15} - (\sqrt{5})^2}$$

Simplifying the denominator then gives you:

$$\frac{2\sqrt{33} - \sqrt{55}}{(2\sqrt{3})^2 - 2\sqrt{15} + 2\sqrt{15} - (\sqrt{5})^2} = \frac{2\sqrt{33} - \sqrt{55}}{4(3) - 5}$$

Simplifying further gives you the final answer and the right hand side of the equation you are proving:

$$\frac{2\sqrt{33} - \sqrt{55}}{4(3) - 5} = \frac{2\sqrt{33} - \sqrt{55}}{7}$$

If you instead multiply the numerator and denominator by $-2\sqrt{3} + \sqrt{5}$ you get:

$$\frac{\sqrt{11}}{2\sqrt{3} + \sqrt{5}} \cdot \frac{-2\sqrt{3} + \sqrt{5}}{-2\sqrt{3} + \sqrt{5}} = \frac{\sqrt{11}(-2\sqrt{3} + \sqrt{5})}{(2\sqrt{3} + \sqrt{5})(-2\sqrt{3} + \sqrt{5})}$$

Expanding the brackets in both the numerator and the denominator gives you:

$$\frac{\sqrt{11}(-2\sqrt{3} + \sqrt{5})}{(2\sqrt{3} + \sqrt{5})(-2\sqrt{3} + \sqrt{5})} = \frac{\sqrt{11}(-2\sqrt{3} + \sqrt{5})}{-(2\sqrt{3})^2 + 2\sqrt{15} - 2\sqrt{15} + (\sqrt{5})^2}$$

Simplifying the denominator gives you:

$$\frac{\sqrt{11}(-2\sqrt{3} + \sqrt{5})}{-(2\sqrt{3})^2 + 2\sqrt{15} - 2\sqrt{15} + (\sqrt{5})^2} = \frac{-2\sqrt{33} + \sqrt{55}}{5 - 4(3)}$$

Further simplifying the denominator then gives you:

$$\frac{-2\sqrt{33} + \sqrt{55}}{5 - 4(3)} = \frac{-2\sqrt{33} + \sqrt{55}}{-7}$$

To get a positive denominator, multiplying both the numerator and the denominator by -1 gives you the right hand side of the equation you are proving:

$$\frac{-2\sqrt{33} + \sqrt{55}}{-7} = \frac{2\sqrt{33} - \sqrt{55}}{7}$$

$$3.2. \quad \frac{5 - \sqrt{2}}{\sqrt{10} - \sqrt{3}} = \frac{5\sqrt{10} + 5\sqrt{3} - 2\sqrt{5} - \sqrt{6}}{7}$$

Version history and licensing

v1.0: initial version created 12/24 by Maximilian Volmar.

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