

# Questions: Introduction to partial differentiation

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## Summary

A selection of questions for the study guide on the introduction to partial differentiation.

*Before attempting these questions, it is highly recommended that you read [Guide: Introduction to partial differentiation](#).*

## Q1

Find all possible first-order partial derivatives for each function  $f$ .

1.1.  $f(x, y) = x^2y + y^3$

1.2.  $f(x, y) = 3x^3 - 2y^4 + xy$

1.3.  $f(x, y) = y \sin(2x) + 3$

1.4.  $f(x, y) = e^{xy} + 2x^2y^3$

1.5.  $f(x, y) = \ln(x) + x \ln(y) + 3x$

1.6.  $f(x, y) = \frac{y}{x} - \frac{x}{y}$

1.7.  $f(x, y) = x \exp(y^2)$

1.8.  $f(x, y) = \sqrt{x^2 + y^2}$

1.9.  $f(x, y) = (3x + 2y)^4$

1.10.  $f(x, y) = y \sin(xy)$

1.11.  $f(x, y) = \sin(x^2 + y^2)$

1.12.  $f(x, y) = \ln(1 + x^2y^2)$

1.13.  $f(x, y, z) = x^2y \sin(z)$

1.14.  $f(x, y, z) = (x + y)(y + z)(z + x)$

1.15.  $f(x, y, z) = \frac{xyz}{x + y + z}$

## Q2

A function  $f(x, y)$  is called harmonic if it satisfies the equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

Show that each of these functions is harmonic by calculating the pure second-order partial derivatives and checking that their sum is zero.

2.1.  $f(x, y) = x^2 - y^2$

2.2.  $f(x, y) = xy$

2.3.  $f(x, y) = x^3 - 3xy^2$

2.4.  $f(x, y) = \cos(x) \sinh(y)$

2.5.  $f(x, y) = e^x \sin(y)$

2.6.  $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$

2.7.  $f(x, y) = \ln(x^2 + y^2)$

## Q3

For each function  $f(x, y)$ , calculate the mixed second-order partial derivatives and confirm that they satisfy the equation

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

3.1.  $f(x, y) = x^2y + xy^2$

3.2.  $f(x, y) = 2x^2 \cos(y)$

3.3.  $f(x, y) = (x + y)^5$

3.4.  $f(x, y) = \frac{x}{1 + y}$

3.5.  $f(x, y) = \sqrt{x^2 + y^2}$

3.6.  $f(x, y) = x^2 \sin(y) + y^2 \cos(x)$

3.7.  $f(x, y) = \tan^{-1}(xy)$

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After attempting the questions above, please click [this link](#) to find the answers.

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## **Version history and licensing**

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

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