Answers: Determinants

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Summary

Answers to questions relating to the guide on determinants.

These are the answers to Questions: Determinants.

Please attempt the questions before reading these answers!

Q1

1.1. $\det(A) = 3 \cdot 4 - 5 \cdot 2 = 2$

1.2. $\det(A) = (-1) \cdot 3 - 2 \cdot 6 = -3 - 12 = -15$

1.3. $\det(A) = 7 \cdot 5 - 0 \cdot (-3) = 35$

1.4. $\det(A) = 4 \cdot 8 - (-2) \cdot 1 = 32 + 2 = 34$

1.5. $\det(A) = 0 \cdot (-5) - 3 \cdot 2 = -6$

Q2

2.1. $\det(A) = 1(4 \cdot 6 - 5 \cdot 0) - 2(0 \cdot 6 - 5 \cdot 0) + 3(0 \cdot 0 - 4 \cdot 0) = 24$

2.2. $\det(A) = 2(-1 \cdot 2 - 4 \cdot 5) - 1(3 \cdot 2 - 4 \cdot 0) + 0(...) = 2(-2 - 20) - 1(6 - 0) = -44 - 6 = -50$

2.3. $\det(A) = -1(0 \cdot 5 - 1 \cdot (-2)) - 3(4 \cdot 5 - 1 \cdot 2) + 2(4 \cdot (-2) - 0 \cdot 2) = -1(0 + 2) - 3(20 - 2) + 2(-8 - 0) = -2 - 54 - 16 = -72$

2.4. $\det(A) = 5(-3 \cdot 2 - 4 \cdot 0) - 2(0 \cdot 2 - 4 \cdot 1) + 1(0 \cdot 0 - (-3) \cdot 1) = 5(-6 - 0) - 2(0 - 4) + 1(0 + 3) = -30 + 8 + 3 = -19$

2.5. $\det(A) = 2(0 \cdot 2 - 3 \cdot (-1)) - 4(1 \cdot 2 - 3 \cdot 5) + 1(1 \cdot (-1) - 0 \cdot 5) = 2(0 + 3) - 4(2 - 15) + 1(-1 - 0) = 6 - 4(-13) - 1 = 6 + 52 - 1 = 57$

Q3

3.1. Triangular: $\det(A) = 3 \cdot 6 \cdot 5 = 90$

3.2. Diagonal: $\det(A) = (-2)(7)(1) = -14$

3.3. Triangular (upper part): $\det(A) = 4 \cdot 2 \cdot (-6) = -48$

3.4. Diagonal: $\det(A) = 9 \cdot (-1) \cdot (-3) = 27$

3.5. Triangular (upper part): $\det(A) = 2 \cdot 4 \cdot 8 = 64$

Q4

4.1. $\det(A) = -16$

4.2. $\det(A) = 25$

4.3. $\det(A) = -189$

4.4. $\det(A) = -70$

4.5. $\det(A) = 128$

Q5

5.1. $\det(3A) = 90$

5.2. $\det(2B) = 2^3 \cdot (-6) = 8 \cdot (-6) = -48$

5.3. Determinant is multiplied by 5.

5.4. $\det(C) = 0$

5.5. Determinant before swap $= -18$

5.6. $\det(D^{-1}) = 1/\det(D) = -\frac{1}{4}$

5.7. $\det(E^T) = \det(E) = 9$

5.8. $\det(G) = \det(2A) \cdot \det(3I_2) = 2^2 \cdot 5 \cdot 3^2 = 4 \cdot 5 \cdot 9 = 180$

5.9. Determinant does not change.

5.10. Rows or columns are linearly dependent.

5.11. $\det(-M) = (-1)^4 \det(M) = 1 \cdot 8 = 8$

5.12. $\det(N^2) = (\det(N))^2 = 7^2 = 49$

5.13. Swap changes sign, multiply row by 3: $10 \to -10 \cdot 3 = -30$

5.14. $\det(P) = \det(A) \cdot \det(B) = 2 \cdot (-3) = -6$

5.15. Determinant does not change.

Version history and licensing

v1.0: initial version created 12/25 by Abigail Carpenter as part of a University of St Andrews VIP project.