Answers: Law of Total Probability and Bayes' Theorem

Sophie Chowgule

Summary

Answers to questions relating to the guide on the Law of Total Probability and Bayes' Theorem.

These are the answers to [Questions: Law of Total Probability and Bayes' Theorem].

Please attempt the questions before reading these answers.

Q1

1.1.

You know:

- $P(\text{Ward A}) = 0.4$
- $P(\text{Recover} \mid \text{Ward A}) = 0.8$
- $P(\text{Ward B}) = 0.6$
- $P(\text{Recover} \mid \text{Ward B}) = 0.6$

Using the law of total probability:

$$P(\text{Recover}) = (\frac{4}{10})(\frac{8}{10}) + (\frac{6}{10})(\frac{6}{10}) = 0.32 + 0.36 = 0.68$$

So the probability that a randomly chosen patient recovers is 0.68.

1.2.

You know:

- $P(\text{Veg}) = 0.5$, $P(\text{Finish} \mid \text{Veg}) = 0.9$

- $P(\text{Chicken}) = 0.3$, $P(\text{Finish} \mid \text{Chicken}) = 0.7$
- $P(\text{Fish}) = 0.2$, $P(\text{Finish} \mid \text{Fish}) = 0.8$

Using the law of total probability:

$$P(\text{Finish}) = (0.5)(0.9) + (0.3)(0.7) + (0.2)(0.8) = 0.45 + 0.21 + 0.16 = 0.82$$

So the probability that a randomly chosen student finishes their lunch is 0.82.

1.3.

You know:

- $P(F_1) = 0.2$, $P(\text{Defective} \mid F_1) = 0.05$

- $P(F_2) = 0.3$, $P(\text{Defective} \mid F_2) = 0.02$

- $P(F_3) = 0.5$, $P(\text{Defective} \mid F_3) = 0.01$

Using the law of total probability:

$$P(\text{Defective}) = (0.2)(0.05) + (0.3)(0.02) + (0.5)(0.01) = 0.01 + 0.006 + 0.005 = 0.021$$

So the probability that a randomly selected product is defective is 0.021.

1.4.

You know:

- $P(\text{Home}) = 0.5$, $P(\text{Complete} \mid \text{Home}) = 0.7$

- $P(\text{Library}) = 0.3$, $P(\text{Complete} \mid \text{Library}) = 0.9$

- $P(\text{Café}) = 0.2$, $P(\text{Complete} \mid \text{Café}) = 0.6$

Using the law of total probability:

$$P(\text{Complete}) = (0.5)(0.7) + (0.3)(0.9) + (0.2)(0.6) = 0.35 + 0.27 + 0.12 = 0.74$$

So the probability that the student completes their homework is 0.74.

Q2

2.1.

You know:

- $P(D) = 0.02$
- $P(\text{Pos} \mid D) = 0.95$
- $P(\text{Pos} \mid \neg D) = 0.1$ (where $\neg D$ means the person does not have the disease)
- $P(\neg D) = 0.98$

Using the law of total probability:

$$P(\text{Pos}) = (0.02)(0.95) + (0.98)(0.1) = 0.019 + 0.098 = 0.117$$

Now applying Bayes' theorem:

$$P(D \mid \text{Pos}) = \frac{(0.95)(0.02)}{0.117} \approx 0.162$$

So the probability that the person has the disease, given that they test positive, is approximately 0.162.

2.2.

You know:

- $P(\text{Rain}) = 0.4$
- $P(\text{Dry}) = 0.6$
- $P(F \mid \text{Rain}) = 0.8$
- $P(F \mid \text{Dry}) = 0.1$

Using the law of total probability:

$$P(F) = (0.4)(0.8) + (0.6)(0.1) = 0.32 + 0.06 = 0.38$$

Then applying Bayes' theorem gives:

$$P(\text{Rain} \mid F) = \frac{(0.8)(0.4)}{0.38} \approx 0.842$$

So the probability that it actually rains, given that the forecast predicts rain, is approximately 0.842.

2.3.

You know:

- $P(A) = 0.7$
- $P(B) = 0.3$
- $P(F \mid A) = 0.02$
- $P(F \mid B) = 0.05$

Using the law of total probability:

$$P(F) = (0.7)(0.02) + (0.3)(0.05) = 0.014 + 0.015 = 0.029$$

Then applying Bayes' theorem gives:

$$P(B \mid F) = \frac{(0.05)(0.3)}{0.029} \approx 0.517$$

So the probability that the item came from Machine B, given that it is faulty, is approximately 0.517.

2.4.

You know:

- $P(\text{Red}) = 0.4$

- $P(\text{Blue}) = 0.6$
- $P(W \mid \text{Red}) = 0.3$
- $P(W \mid \text{Blue}) = 0.7$

Using the law of total probability:

$$P(W) = (0.4)(0.3) + (0.6)(0.7) = 0.12 + 0.42 = 0.54$$

Then applying Bayes' theorem gives:

$$P(\text{Red} \mid W) = \frac{(0.3)(0.4)}{0.54} \approx 0.222$$

So the probability that the sweet is red, given that it has a wrapper, is approximately 0.222.

Version history and licensing

v1.0: initial version created 05/25 by Sophie Chowgule as part of a University of St Andrews VIP project.

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