Overview: History of numbers

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Summary

An overview of the history of numbers and the decimal number system.

What are numbers

Numbers are used to count, measure, order, and perform calculations. In this overview you will see a brief history of numbers, how the commonly used number system came about and how it works, as well as a few important different types of number.

Representations

Numbers have a place even in very early history. Bones and other artifacts carved with lines that many believed to be tally marks provide evidence of the very earliest humans making use of counting, perhaps for counting quantities of things like crops and livestock, or for tracking passing time (Barrow-Green, Gray and Wilson, 2019, pp.6-8).

Numbers can be represented in many ways. Tally marks are one way of representing numbers, they can be useful for some purposes, but as you have more and more marks it will become easier to lose track and miscalculate.

You are perhaps familiar with roman numerals as another way of representing numbers. Roman numerals are surprisingly efficient for some calculations, but the lack of a place value system can complicate things.

Lots of different cultures evolved different ways of representing numbers based on different number systems. The kind you most commonly use today are Hindu–Arabic numerals, using the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$. The common number symbols you use today originate in the middle east, but 0, and the place value system originate in India, from the Brahmi numeral system (Barrow-Green, Gray and Wilson, 2019, p.201).

Place value

Place value systems assign value to a digit based on its position within a number. Having a place value system can make writing numbers and performing calculations with them more efficient. Number systems using place value have roots in ancient civilization, but the present decimal system was perfected by Indian mathematicians before spreading to the rest of the world (Barrow-Green, Gray and Wilson, 2019, pp.200-201). Zero plays an important role in the number system as it allows us to denote the position of a digit in the number.

> **Example 1**
>
> Place value allows you to distinguish the different values of the digits. For example you can express 3582.5 in the following way:
> $3582.5 = 3 \cdot 1000 + 5 \cdot 100 + 8 \cdot 10 + 2 \cdot 1 + 5 \cdot 0.1$

Bases

The base of a number system is the number of unique digits within the number system. The decimal system, a base-10 system, uses digits $0 - 9$. Ancient Egyptians where the first to use a base-10 system, around 3000BCE (Ritter, J. (2000), pp.116-127). The most common theory is that the base-10 system came about because humans have ten fingers, making it a natural way to count and represent numbers.

You might be familiar with the binary system, a base-2 system using only 0 and 1. Binary code is often used in computing. While computers primarily use binary, humans often use hexadecimal (base-16) as a shorthand for representing binary data, each hexadecimal digit corresponds to four binary digits.

Many of the earliest number systems, like those used by the Sumerians and Babylonians, favored sexagesimal systems, (base-60) (Barrow-Green, Gray and Wilson, 2019, p.24). Remnants of this still exist in how you measure time, angles and geographic coordinates. At first glance, 60 may seem an odd choice, but 60 is a highly composite number, it has factors $1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$, making it easy to divide with.

The Mayans used a vigesimal (base-20) system (Aveni, 2011, p.189), as did the Basque region. Remnants of this exist in the french language, 80 is quatre-vingts, meaning four-twentys. (Barrow-Green, Gray and Wilson, 2019, p.8)

You're able to change bases using the following algorithm.

> Algorithm for changing bases.
>
> Let a and $b > 1$ be integers. To find the representation of a in base b:
> **Step 1**: Divide a by b: $a = qb + r$
> **Step 2**: Set r to be the last digit of the expansion
> **Step 3**: Rename: $a := q$

Step 4: Repeat the above steps until $a = 0$

Now you can see this algorithm used.

Example 2

Lets convert 152 from the usual base 10, to hexadecimal (base-16):
Let $a = 152$ and $b = 16$:
Divide a by b: $152 = 9 \cdot 16 + 8$
Set 8 to be the last digit of the expansion
Rename: $a := 9$
Divide a by b: $9 = 0 \cdot 16 + 9$
Set 9 to be the second to last digit of the expansion
Rename: $a := 0$
STOP
So, 152 is 98 in base-16.

Catagories of numbers

Cardinal and ordinal numbers

Cardinal numbers are used to count, or represent a quantity of something.

Definition a cardinal number

Cardinal numbers are numbers such as:

$$1, 2, 3, 4...$$

Ordinal numbers, on the other hand, represent the position or order within a list.

Definition a ordinal number

Ordinal numbers are numbers such as:

$$1st, 2nd, 3rd, 4th...$$

Positive and negative numbers

Positive numbers are numbers such as:

$$1, 2, 3...$$

Negative numbers are numbers such as:

$$-1, -2, -3...$$

> **Definition a positive and negative numbers**
>
> A positive number is a number greater than 0, conversely a negative number is a number less than 0.

Negative numbers where first introduced in Chinese mathematics around 200BCE, as a way of representing debts and deficits (Barrow-Green, Gray and Wilson, 2019, pp.214-220). Indian mathematics began incorporating negatives as well, Brahmagupta laid out sets of rules for working with negative numbers in the 7th century (Barrow-Green, Gray and Wilson, 2019, pp.206-207), but it took until the 19th century for negative numbers to be fully accepted in western mathematics (Mumford, 2010, p.114).

Algebraic and transendental numbers

Algebraic vs transcendental is another important way of distinguishing numbers. Before defining algebraic and transcendental numbers, you need to define a polynomial.

> **Definition of a polynomial**
>
> A **polynomial** is an expression of the form:
>
> $$a_n x^n + a_{n-1} x^{n-1} + ...a_1 x + a_0$$
>
> The a_n here are the coefficients. The solutions to the equation, $a_n x^n + a_{n-1} x^{n-1} + ...a_1 x + a_0 = 0$, are the **roots** of the polynomial.

You can now define an algebraic number.

> **Definition of an algebraic number**
>
> An algebraic number is a number that is a root to some non-zero polynomial with rational coefficients.

> ℹ Definition of a transendental number
>
> A transcendental number is a number that's not algebraic.

It's difficult **AM I ALLOWED TO SAY DIFFICULT HERE?** to prove that a number is transcendental. It was first assumed that all numbers where algebraic, but in 1844 Joseph Liouville proved that his number, the Liouville constant $= 0.110001000000000001...$, was transcendental. Other mathematicians followed suit and proved that both π and e are also transcendental. Fascinatingly, there are actually more transcendental number than algebraic numbers.

Bibliography

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Further reading

For more reading on types of number, please go to **Overview: Number sets.**

Version history

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