

# Answers: Logarithms

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## Summary

Answers to questions relating to the study guide on logarithms.

*These are answers to: [Questions: Logarithms](#).*

**Please attempt the questions before reading these answers!**

Throughout this answer sheet, the natural logarithm  $\log_e(x)$  is written as  $\ln(x)$ .

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## Q1

- 1.1.  $\log_7(x) = 1$  rearranged gives  $7^1 = x$  so  $x = 7$ .
- 1.2.  $\log_8(x) = 3$  rearranged gives  $8^3 = x$  so  $x = 512$ .
- 1.3.  $\log_{12}(x) = 0$  rearranged gives  $12^0 = x$  so  $x = 1$ .
- 1.4.  $\log_{10}(100) = x$  rearranged gives  $10^x = 100$  so  $x = 2$ .
- 1.5.  $\log_2(64) = x$  rearranged gives  $2^x = 64$  so  $x = 6$ .
- 1.6.  $\log_4(2) = x$  rearranged gives  $4^x = 2$  so  $x = \frac{1}{2}$ .
- 1.7.  $\log_3(27) = x$  rearranged gives  $3^x = 27$  so  $x = 3$ .
- 1.8.  $\log_{10}(1) = x$  rearranged gives  $10^x = 1$  so  $x = 0$ .
- 1.9.  $\log_x(16) = 4$  rearranged gives  $x^4 = 16$  so  $x = \sqrt[4]{16} = 2$ .
- 1.10.  $\log_x(49) = 2$  rearranged gives  $x^2 = 49$  so  $x = \sqrt{49} = 7$ .
- 1.11.  $\log_x(13) = 4$  rearranged gives  $x^4 = 13$  so  $x = \sqrt[4]{13}$ .
- 1.12.  $\log_{2x}(12) = -1$  rearranged gives  $(2x)^{-1} = 12$  so  $x = \frac{1}{24}$ .

## Q2

The product rule:  $\log_a(M \cdot N) = \log_a(M) + \log_a(N)$

The quotient rule:  $\log_a \left( \frac{M}{N} \right) = \log_a(M) - \log_a(N)$

The power rule:  $\log_a(M^k) = k \cdot \log_a(M)$

The zero rule:  $\log_a(1) = 0$

The identity rule:  $\log_a(a) = 1$

2.1. The solution to  $\log_3\left(\frac{1}{27}\right) = x$  is  $x = -1/3$ .

2.2. The solution to  $4\log_4(2) = x$  is  $x = 2$ .

2.3. The solution to  $\log_5(10) + \log_5\left(\frac{5}{2}\right) = x$  is  $x = 2$ .

2.4. The solution to  $3\log_7(a^{1/3}) - \frac{1}{2}\log_7(a^2) = x$  is  $x = 0$ .

2.5. The solution to  $\log_x(YZ) = M$  is  $x = \sqrt[M]{YZ}$ .

2.6. The solution to  $\log_a(y) - \log_a(x) = 11$  is  $x = ya^{-11}$ .

### Q3

3.1.  $\log_3(25)$  is equal to  $\frac{2}{\log_5(3)}$ .

3.2.  $\log_8(3)$  is equal to  $\frac{4\log_{16}(3)}{3}$ .

3.3.  $\log_e(10)$  is equal to  $\frac{1}{\log_{1000}(e^3)}$ .

3.4.  $\ln(27)$  is equal to  $\frac{3}{\log_3(e)}$ .

3.5.  $\log_4(8x)$  is equal to  $\frac{3}{2} + \log_2(\sqrt{2})$ .

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### Version history and licensing

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- v1.1: edited 05/24 by tdhc.

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