

# Questions: Expected value, variance, standard deviation

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## Summary

A selection of questions to test your understanding of expected values, variance, and standard deviation.

*Before attempting these questions it is highly recommended that you read [Guide: Expected value, variance, standard deviation](#).*

## Q1

For each of the following valid random variables with associated probability mass function, work out the expected value and variance.

### 1.1.

Let  $X$  be the random variable representing the result of rolling a biased four sided-die. The PMF of  $X$  is given by:

| $x$        | 1              | 2             | 3             | 4             |
|------------|----------------|---------------|---------------|---------------|
| $P(X = x)$ | $\frac{1}{10}$ | $\frac{1}{5}$ | $\frac{1}{2}$ | $\frac{1}{5}$ |

### 1.2.

A discrete random variable  $X$  has five possible outcomes (1, 2, 3, 4, or 5), and the PMF is given by:

| $x$        | 1    | 2    | 3    | 4   | 5   |
|------------|------|------|------|-----|-----|
| $P(X = x)$ | 0.25 | 0.35 | 0.05 | 0.2 | 0.1 |

**1.3.**

A coin is tossed, where the probability of tails is 70 and heads is 30. Let  $X$  represent the result of the coin toss. Complete the table below:

| $x$        | Heads | Tails |
|------------|-------|-------|
| $P(X = x)$ | 0.7   | 0.3   |

**1.4.**

The PMF for a random variable  $X$  is given as:

| $x$        | 1    | 2    | 3    | 4    |
|------------|------|------|------|------|
| $P(X = x)$ | 1/10 | 2/10 | 3/10 | 4/10 |

**Q2**

For each of the following valid random variables with associated probability density function, work out the expected value and variance.

**2.1.**

Let  $X$  be a continuous random variable on the interval  $[0, 2]$  with the PDF:

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

**2.2.**

Let  $X$  be a continuous random variable with the PDF:

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

**Q3**

Give the expected value and variance for rolling seven fair 6-sided dice. You may assume that each roll is independent of every other roll.

## Q4

This question refers to the exponential distribution for a continuous random variable. You can find more information about this and [Factsheet: Exponential distribution](#).

The PDF of the exponential distribution is  $\mathbb{P}(X = x) = \lambda e^{-\lambda x}$ . Using integration by parts (see [Guide: Integration by parts]) and the fact that

$$\lim_{x \rightarrow \infty} x^n e^{-\lambda x} = 0$$

for any natural number  $n$  and real  $\lambda > 0$ , show that

(a) the mean  $\mu$  of the exponential distribution is  $\frac{1}{\lambda}$ ;

(b) the variance  $\sigma^2$  of the exponential distribution is  $\frac{1}{\lambda^2}$ .

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[After attempting the questions above, please click this link to find the answers.](#)

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## Version history and licensing

v1.0: initial version created 08/25 by tdhc.

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