Answers: Introduction to probability

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Summary

Answers to questions relating to the guide on introduction to probability.

These are the answers to Questions: Introduction to probability.

Please attempt the questions before reading these answers!

Q1

(Note: accept all simplified fractions, or probabilities represented in decimal and percentage forms)

1.1.

(a)

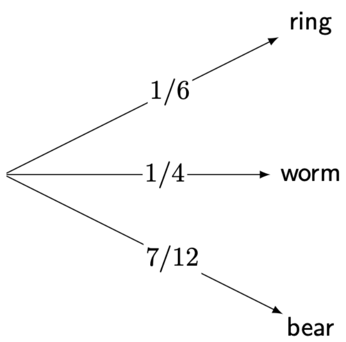

Figure 1: Answer to Q1.1(a)

(b)

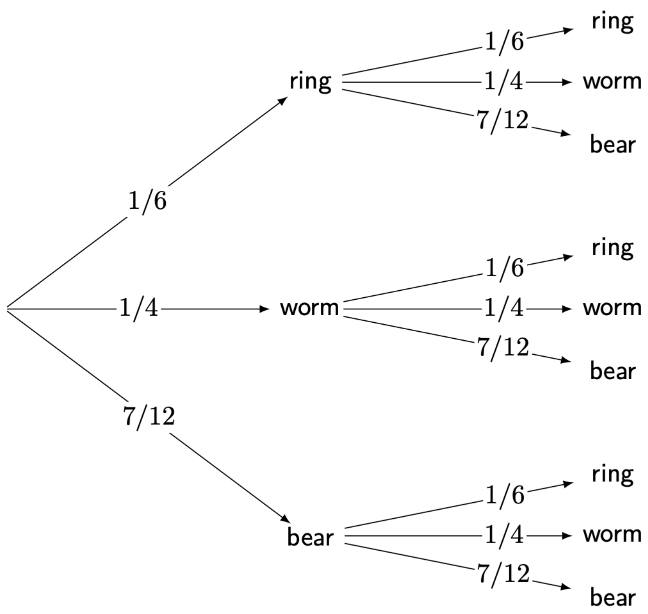

Figure 2: Answer to Q1.1(b)

(c)

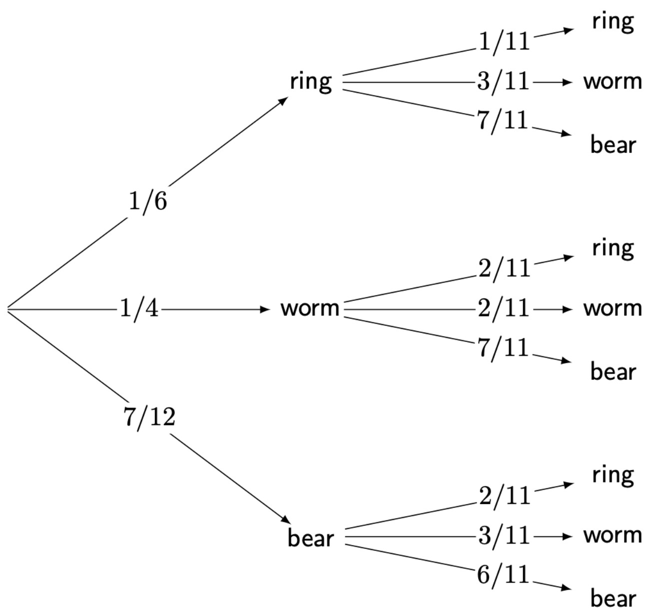

Figure 3: Answer to Q1.1(c)

1.2.

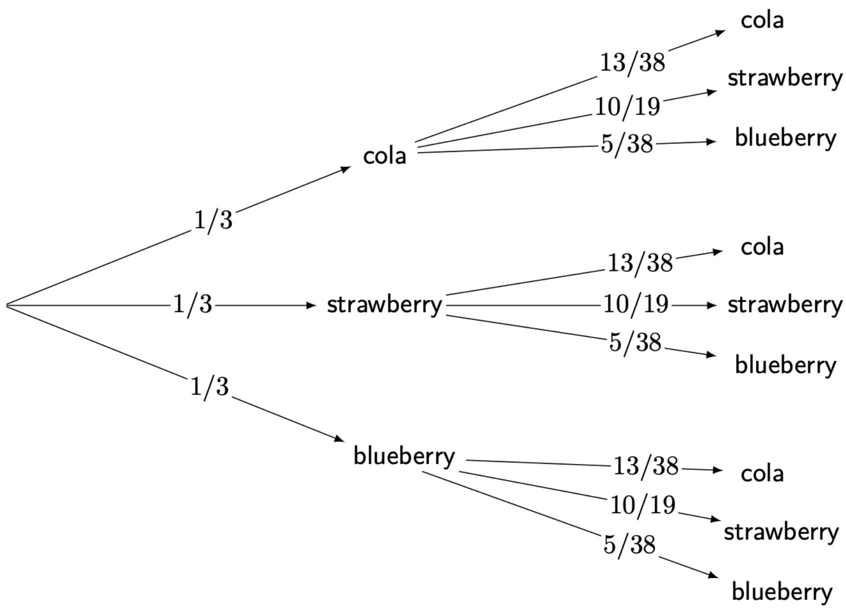

Figure 4: Answer to Q1.2

1.3.

- The events in Q1.1.(b) are independent.
- The events in Q1.1.(c) are dependent.
- The events in Q1.2. are independent.

Q2

(Note: accept all simplified fractions, or probabilities represented in decimal and percentage forms)

2.1.

(a) $\mathbb{P}(\text{gummy bear}) = \frac{7}{12}$. Therefore, apply the complement rule to calculate the complement of $\mathbb{P}(\text{gummy bear})$, so $\mathbb{P}(\text{gummy bear}') = 1 - \frac{7}{12} = \frac{5}{12}$.

(b) The probability of drawing a gummy ring the first time is $\frac{2}{12}$, and the probability of drawing a gummy ring the second time is also $\frac{2}{12}$. Therefore, $\mathbb{P}(\text{gummy ring and gummy ring}) = (\frac{2}{12})(\frac{2}{12}) = \frac{4}{144} = \frac{1}{36}$.

(c) The probability of drawing a gummy bear the first time is $\frac{7}{12}$, and the probability of drawing a gummy worm the second time is $\frac{3}{11}$. Therefore, $\mathbb{P}(\text{gummy bear then gummy worm}) = (\frac{7}{12})(\frac{3}{11}) = \frac{21}{132} = \frac{7}{44}$.

2.2. The probability of drawing a cola flavored jelly bean the first time is $\frac{10}{30}$, and the probability of drawing a strawberry flavored jelly bean the second time is $\frac{20}{38}$. Therefore, $\mathbb{P}(\text{soda and strawberry}) = (\frac{10}{30})(\frac{20}{38}) = \frac{200}{1140} = \frac{10}{57}$.

Q3

3.1. This is an example of experimental probability.

3.2. The total number of spins is 60, and it lands on white 17 times. Therefore, $\mathbb{P}(\text{white}) = \frac{17}{60}$, so by the complement rule, $\mathbb{P}(\text{white}') = 1 - \frac{17}{60} = \frac{43}{60}$.

3.3. The spinner is unbiased (meaning the probability is uniform), so there are four possible colors that the spinner is equally likely to land on. Calculating the theoretical probability, $\mathbb{P}(\text{red}) = \frac{1}{4}$.

3.4. As you spin the spinner more times, the experimental probabilities of each color will get closer to their theoretical probabilities.

Q4

4.1. The sample space is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, which contains 20 possible outcomes. The subset of the sample space with a value above 12 is $\{13, 14, 15, 16, 17, 18, 19, 20\}$, which contains 8 out of the 20 possible outcomes. Therefore, $\mathbb{P}(\text{success}) = \frac{8}{20} = \frac{2}{5}$.

4.2. The possible outcomes of the 5-sided dice roll are $\{1, 2, 3, 4, 5\}$. Adding 3 to each number, the sample space becomes $\{(1+3), (2+3), (3+3), (4+3), (5+3)\} = \{4, 5, 6, 7, 8\}$. The sample space contains 5 possible outcomes, and the subset of the sample space that contains values 5 and above is $\{5, 6, 7, 8\}$, which contains 4 out of 5 possible outcomes. Therefore, $\mathbb{P}(x \geq 5) = \frac{4}{5}$ where x points of damage are dealt to the dragon.

4.3. Here is the sample space represented as a table:

	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)

	1	2	3	4
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

For convenience purposes, all outcomes that do not contain a 4 are then marked with an asterisk (although any way of marking or counting these outcomes are acceptable):

	1	2	3	4
1	(1,1)*	(1,2)*	(1,3)*	(1,4)
2	(2,1)*	(2,2)*	(2,3)*	(2,4)
3	(3,1)*	(3,2)*	(3,3)*	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

There are 16 total possible outcomes in the sample space, and there are 9 outcomes that do not contain a 4. Therefore, $\mathbb{P}(\text{failure}) = \frac{9}{16}$.

4.4. Here is the sample space represented as a table:

	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)
5	(5,1)	(5,2)	(5,3)	(5,4)
6	(6,1)	(6,2)	(6,3)	(6,4)
7	(7,1)	(7,2)	(7,3)	(7,4)
8	(8,1)	(8,2)	(8,3)	(8,4)
9	(9,1)	(9,2)	(9,3)	(9,4)

The results of the two dice rolls can then be added together:

	1	2	3	4
1	2	3	4	5

	1	2	3	4
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10
7	8	9	10	11
8	9	10	11	12
9	10	11	12	13

For convenience purposes, all outcomes with skill levels that are greater than 9 are then marked with an asterisk (although any way of marking or counting these outcomes are acceptable):

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10*
7	8	9	10*	11*
8	9	10*	11*	12*
9	10*	11*	12*	13*

There are 36 total possible outcomes in the sample space, and there are 10 outcomes where the skill level exceeds 9 points. Therefore, $\mathbb{P}(x > 9) = \frac{10}{36} = \frac{5}{18}$ where x is the number of skill level points.

Version history and licensing

v1.0: initial version created 04/25 by Michelle Arnetta as part of a University of St Andrews VIP project.