

# Answers: Completing the square

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## Summary

Answers to questions relating to the guide on completing the square.

*These are the answers to [Questions: Completing the square](#).*

**Please attempt the questions before reading these answers!**

## Q1

- 1.1. Here,  $x^2 - 2x + 15 = (x - 1)^2 + 14$ , so in this question  $p = -1$  and  $q = 14$ .
- 1.2. Here,  $y^2 - 6y + 8 = (y - 3)^2 - 1$ , so in this question  $p = -3$  and  $q = -1$ .
- 1.3. Here,  $x^2 + 8x + 20 = (x + 4)^2 + 4$ , so in this question  $p = 4$  and  $q = 4$ .
- 1.4. Here,  $m^2 - 26m + 25 = (m - 13)^2 - 144$ , so in this question  $p = -13$  and  $q = -144$ .
- 1.5. Here,  $n^2 + 6n + 50 = (n + 3)^2 + 41$ , so in this question  $p = 3$  and  $q = 41$ .
- 1.6. Here,  $x^2 + 2x + 144 = (x + 1)^2 + 143$ , so in this question  $p = 1$  and  $q = 143$ .
- 1.7. Here,  $h^2 - 3h - 3 = \left(h - \frac{3}{2}\right)^2 + \frac{3}{4}$ , so in this question  $p = -3/2$  and  $q = 3/4$ .
- 1.8. Here,  $x^2 + x - 3 = \left(x + \frac{1}{2}\right)^2 - \frac{13}{4}$ , so in this question  $p = 1/2$  and  $q = -13/4$ .
- 1.9. Here,  $x^2 - 13x + 43 = \left(x - \frac{13}{2}\right)^2 + \frac{3}{4}$ , so in this question  $p = -13/2$  and  $q = 3/4$ .
- 1.10. Here,  $y^2 - 8y + 16 = (y - 4)^2$ , so in this question  $p = -4$  and  $q = 0$ .
- 1.11. Here,  $x^2 + 13x + 9 = \left(x + \frac{13}{2}\right)^2 - \frac{133}{4}$ , so in this question  $p = 13/2$  and  $q = -133/4$ .
- 1.12. Here,  $m^2 + 3m + 33 = \left(m + \frac{3}{2}\right)^2 - \frac{143}{4}$ , so in this question  $p = 3/2$  and  $q = -143/4$ .

## Q2

- 2.1. Here,  $2x^2 - 12x + 14 = 2(x - 3)^2 - 4$ , so in this question  $a = 2$ ,  $p = -3$  and  $q = -4$ .
- 2.2. Here,  $5y^2 - 10y + 4 = 5(y - 1)^2 - 1$ , so in this question  $a = 5$ ,  $p = -1$  and  $q = -1$ .

2.3. Here,  $4x^2 + 32x + 68 = 4(x + 4)^2 + 4$ , so in this question  $a = p = q = 4$ . (Or, if you prefer,  $(2x + 8)^2 + 4$ .)

2.4. Here,  $2m^2 + 2m + 2 = 2\left(m + \frac{1}{2}\right)^2 + \frac{3}{2}$ , so in this question  $a = 2$ ,  $p = 1/2$  and  $q = 3/2$ .

2.5. Here,  $3x^2 - 2x + 5 = 3\left(x - \frac{1}{3}\right)^2 + \frac{14}{3}$ , so in this question  $a = 3$ ,  $p = -1/3$  and  $q = 14/3$ .

2.6. Here,  $4x^2 - 4x + 1 = 4\left(x - \frac{1}{2}\right)^2$ , so in this question  $a = 4$ ,  $p = -1/2$  and  $q = 0$ . (Or, if you prefer,  $(2x - 1)^2$ .)

2.7. Here,  $2h^2 - 3h + 1 = 2\left(h - \frac{3}{4}\right)^2 - \frac{1}{8}$ , so in this question  $a = 2$ ,  $p = -3/4$  and  $q = -1/8$ .

2.8. Here,  $3x^2 + 5x + 2 = 3\left(x + \frac{5}{6}\right)^2 - \frac{3}{36}$ , so in this question  $a = 3$ ,  $p = 5/6$  and  $q = -3/36$ .

### Q3

Using your working from Q1 and Q2, solve the following quadratic equations.

3.1. You worked out in 1.2 that  $y^2 - 6y + 8 = (y - 3)^2 - 1$ . Rearranging  $(y - 3)^2 - 1 = 0$  for  $y$  gives  $y = 3 \pm 1$ , so  $y = 2$  or  $y = 4$ .

3.2. You worked out in 1.4 that  $m^2 - 26m + 25 = (m - 13)^2 - 144$ . Rearranging  $(y - 3)^2 - 144 = 0$  for  $y$  gives  $y = 13 \pm 12$ , so  $y = 1$  or  $y = 25$ .

3.3. You worked out in 1.3 that  $x^2 + 8x + 20 = (x + 4)^2 + 4$ . Using the fact that  $(\pm 2i)^2 = -4$  (see [Guide: Introduction to complex numbers]), rearranging  $(x + 4)^2 + 4 = 0$  for  $y$  gives  $y = -4 \pm 2i$ , so  $y = -4 - 2i$  or  $y = -4 + 2i$ .

3.4. You worked out in 2.6 that  $4x^2 - 4x + 1 = 4\left(x - \frac{1}{2}\right)^2$ . Rearranging  $4\left(x - \frac{1}{2}\right)^2 = 0$  for  $x$  gives  $x = \frac{1}{2}$  (twice, see [Guide: Introduction to quadratic equations](#)).

3.5. You worked out in 2.3 that  $4x^2 + 32x + 68 = 4(x + 4)^2 + 4$ . Using the fact that  $(\pm i)^2 = -1$  (see [Guide: Introduction to complex numbers]), rearranging  $4(x + 4)^2 + 4 = 0$  for  $x$  gives  $x = -4 \pm i$ , so  $x = -4 - i$  or  $x = -4 + i$ .

3.6. You worked out in 2.8 that  $3x^2 + 5x + 2 = 3\left(x + \frac{5}{6}\right)^2 - \frac{3}{36}$ . Rearranging  $3\left(x + \frac{5}{6}\right)^2 - \frac{3}{36} = 0$  for  $x$  gives  $y = -\frac{5}{6} \pm \frac{1}{6}$ , so  $y = -1$  or  $y = -2/3$ .



## **Version history and licensing**

v1.0: initial version created 09/24 by tdhc.

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