Trigonometric Identities

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Summary

Trigonometric identities are equations expressed in terms of trigonometric functions that hold true for all values of the variable defined. It is important to familiarize yourself with the standard trigonometric identities as they come in handy later whether that be for deriving further trigonometric identities or solving integrals involving trigonometric functions.

Before reading this guide, it is recommended that you read (Guide: Trigonometry) first.

What is a trigonometric identity?

Think of **trigonometric identities** as rules or shortcuts that help us solve certain types of problems involving triangles, angles, and cycles more easily.

For example, if you're trying to find out information about a right triangle – maybe you know the length of one side and the size of one angle, and you want to find out the length of another side – trigonometric identities can help you do that.

You've probably heard of the sine ($\sin$), cosine ($\cos$), and tangent ($\tan$) functions. These functions are a way of relating the angles and sides of a triangle.

Trigonometric Identities are used in numerous fields of study, such as physics, engineering, astronomy, architecture, and even geography, so the knowledge you gather here could potentially be used in further studies.

Definition of Trigonometric Identity

A **trigonometric identity** is a mathematical equation that is always true for any values of the variables where both sides of the equation are defined, and that involves the trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant.
These identities can be derived from the definitions of these trigonometric functions and the Pythagorean theorem, and they hold for all real numbers (and complex numbers, if you choose to expand your understanding to include complex analysis).
In more technical terms, these identities are equalities that involve trigonometric functions and are true for every single value of the occurring variables. It's important to understand that trigonometric identities are not equations to solve for variables, but rather, they are tools to simplify trigonometric expressions or to solve other trigonometric equations.

Fundamental Trigonometric Identities

There are several fundamental types of trigonometric identities. Each type provides a different way of relating the various trigonometric functions to one another. Here are the basic categories:

1. **Pythagorean identities**: These are derived from the Pythagorean theorem. The basic one is

   - $\sin^2(\theta) + \cos^2(\theta) = 1.$

   If you divide the entire equation by $\sin^2(\theta)$ or $\cos^2(\theta)$, you get the other two identities.

   - $1 + \cot^2(\theta) = \csc^2(\theta)$
   - $\tan^2(\theta) + 1 = \sec^2(\theta)$

Proof

You know $\sin(\theta) = \frac{Opposite}{Hypotenuse}$ and $\cos(\theta) = \frac{Adjacent}{Hypotenuse}$ (Hence referred to as O, H and A).
$O = (H)(\sin(\theta))$ and $A = (H)(\cos(\theta))$
From Pythagoras' Theorem, you also know:
$(O)^2 + (A)^2 = (H)^2$
Replacing O and A, you get
$(H)(\sin(\theta))^2 + (H)(\cos(\theta))^2 = (H)^2$
Divide everything by H^2
$\frac{(H)(\sin(\theta))^2}{(H)^2} + \frac{(H)(\cos(\theta))^2}{(H)^2} = \frac{(H)^2}{(H)^2}$
Therefore $\sin^2(\theta) + \cos^2(\theta) = 1$
You can substitute $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$ and $\csc(\theta)$ in a similar manner to arrive to a similar conclusion for the remaining identities.

Example 1.1

$6 + 12sin^2(\theta) + 12cos^2(\theta)$
Solution:
$= 6 + 12(\sin^2(\theta) + \cos^2(\theta))$
$= 6 + 12(1)$
$= 18$

2. **Reciprocal identities**: These express csc, sec, and cot in terms of sin, cos, and tan.

   - $\csc(\theta) = \frac{1}{\sin(\theta)}$
   - $\sec(\theta) = \frac{1}{\cos(\theta)}$
   - $\cot(\theta) = \frac{1}{\tan(\theta)}$