

# Factsheet: Hyperbolic identities

Tom Coleman

## Summary

A list of hyperbolic trig identities.

*These are common definitions and identities for hyperbolic functions. For derivatives and antiderivatives, please see [Factsheet: List of derivatives](#) and [Factsheet: List of integrals](#) respectively.*

## Definitions of hyperbolic functions

For all real numbers  $x$ :

$$\begin{aligned}\cosh(x) &= \frac{e^x + e^{-x}}{2} \\ \sinh(x) &= \frac{e^x - e^{-x}}{2} \\ \tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \\ \coth(x) &= \frac{1}{\tanh(x)} = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \\ \operatorname{sech}(x) &= \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \\ \operatorname{csch}(x) &= \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}\end{aligned}$$

## Hyperbolic identities

### Pythagorean formulas

For all real numbers  $x$ :

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$

$$\coth^2(x) - 1 = \operatorname{csch}^2(x)$$

## Sum and difference formulas

For all real numbers  $x, y$ :

$$\cosh(x + y) = \cosh(x) \cosh(y) + \sinh(x) \sinh(y)$$

$$\cosh(x - y) = \cosh(x) \cosh(y) - \sinh(x) \sinh(y)$$

$$\sinh(x + y) = \sinh(x) \cosh(y) + \cosh(x) \sinh(y)$$

$$\sinh(x - y) = \sinh(x) \cosh(y) - \cosh(x) \sinh(y)$$

$$\tanh(x + y) = \frac{\tanh(x) + \tanh(y)}{1 + \tanh(x) \tanh(y)}$$

$$\tanh(x - y) = \frac{\tanh(x) - \tanh(y)}{1 - \tanh(x) \tanh(y)}$$

## Double angle formulas

For all real numbers  $x$ :

$$\cosh(2x) = \cosh^2(x) + \sinh^2(x)$$

$$\sinh(2x) = 2 \sinh(x) \cosh(x)$$

$$\tanh(2x) = \frac{2 \tanh(x)}{1 + \tanh^2(x)}$$

## Definitions of inverse hyperbolic functions

---

function	logarithmic definition	validity
$\sinh^{-1}(x)$	$\ln\left(x + \sqrt{x^2 + 1}\right)$	
$\cosh^{-1}(x)$	$\ln\left(x + \sqrt{x^2 - 1}\right)$	$x \geq 1$
$\tanh^{-1}(x)$	$\frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$	$ x  < 1$
$\coth^{-1}(x)$	$\frac{1}{2} \ln\left(\frac{x+1}{x-1}\right)$	$ x  > 1$
$\operatorname{sech}^{-1}(x)$	$\ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)$	$0 < x \leq 1$

function	logarithmic definition	validity
$\text{csch}^{-1}(x)$	$\ln \left( \frac{1}{x} + \sqrt{\frac{1}{x^2} + 1} \right)$	$x \neq 0$

## Version history

v1.0: created in 08/25 by tdhc.

This work is licensed under CC BY-NC-SA 4.0.