

# Answers: Expected value, variance, standard deviation

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## Summary

Answers to questions relating to the guide on expected value, variance, and standard deviation.

*These are the answers to [Questions: PMFs, PDFs, and CDFs](#).*

**Please attempt the questions before reading these answers!**

## Q1

For each of the following valid random variables with associated probability mass function, work out the expected value and variance.

### 1.1.

Here,  $\mathbb{E}(X) = 2.8$  and  $\mathbb{V}(X) = 0.76$ .

### 1.2.

Here,  $\mathbb{E}(X) = 2.4$  and  $\mathbb{V}(X) = 2.04$ .

### 1.3.

This is a Bernoulli trial, with probability of success  $p = 0.7$ . From Example 2 of [Guide: Expected value, variance, standard deviation](#), you can say that  $\mathbb{E}(X) = p = 0.7$ . The variance can be worked out to be

$$\mathbb{V}(X) = 0.7 - (0.7)^2 = 0.21 = (0.7)(0.3) = p(1 - p).$$

### 1.4.

Here,  $\mathbb{E}(X) = 3$  and  $\mathbb{V}(X) = 1$ .

## Q2

For each of the following valid random variables with associated probability density function, work out the expected value and variance.

### 2.1.

This is the continuous uniform distribution with  $a = 0$  and  $b = 2$ . From Example 2 of [Guide: Expected value, variance, standard deviation](#), you can say that  $\mathbb{E}(X) = (a + b)/2 = (0 + 2)/2 = 1$  and

$$\mathbb{V}(X) = \frac{1}{12}(b - a)^2 = \frac{1}{12}(2 - 0)^2 = \frac{4}{12} = \frac{1}{3}.$$

### 2.2.

Here,  $\mathbb{E}(X) = \frac{2}{3}$  and  $\mathbb{V}(X) = \frac{1}{6}$ .

## Q3

You know that the expected value and variance of rolling one fair die are  $\mathbb{E}(X) = \frac{7}{2}$  and  $\mathbb{V}(X) = \frac{35}{12}$ .

Because the roll of each die is an independent event, the random variable  $Y$  of rolling seven dice is the same as

$$Y = X + X + X + X + X + X + X$$

which is seven lots of  $X$ .

You can use the properties of expected values and variance to get

$$\begin{aligned}\mathbb{E}(Y) &= \mathbb{E}(X + X + X + X + X + X + X) \\ &= 7 \cdot \frac{7}{2} = \frac{49}{2} = 24.5\end{aligned}$$

and

$$\begin{aligned}\mathbb{V}(Y) &= \mathbb{V}(X + X + X + X + X + X + X) \\ &= 7 \cdot \frac{35}{12} = \frac{245}{12}\end{aligned}$$

## Q4

When doing this question, you need to find the two integrals

$$\mathbb{E}(X) = \int_0^{\infty} \lambda x e^{-\lambda x} dx \quad \text{and} \quad \mathbb{V}(X) = \int_0^{\infty} \lambda x^2 e^{-\lambda x} dx.$$

You will need to use integration by parts; you should always differentiate the power of  $x$  to reduce the size of the power. Use the given result to evaluate the integral when  $x \rightarrow \infty$ ; the antiderivative should always be 0 in this case.

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## Version history and licensing

v1.0: initial version created 08/25 by tdhc.

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