Answers: Trigonometric identities (radians)

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Summary

A selection of questions on trigonometric identities, using radians to measure angles.

These are the answers to Questions: Trigonometric identities (radians).

Please attempt the questions before reading these answers!

Q1

1.1. $2(6\sin^2(\theta)) + 3(4\cos^2(\theta)) = 12(\sin^2(\theta) + \cos^2(\theta)) = 12$

1.2. $10(7\sin^2(\theta)) + 14(5\cos^2(\theta)) = 70$

1.3. $5\left(\dfrac{6}{\csc^2(\theta)}\right) + 15\left(\dfrac{2}{\sec^2(\theta)}\right) = 30$

1.4. $(\cos^2(\theta) - \sin^2(\theta))^2 + 4\sin^2(\theta)\cos^2(\theta) = \cos^2(2\theta) + \sin^2(2\theta) = 1$

1.5. $2\sin(\pi/6)\cos(\pi/12) + 2\cos(\pi/6)\sin(\pi/12) = 2\sin(\pi/6 + \pi/12) = 2\sin(\pi/4) = \sqrt{2}$

1.6. $3\cos(\pi/4)\cos(\pi/12) - 3\sin(\pi/4)\sin(\pi/12) = 3\cos(\pi/3) = \dfrac{3}{2}$

1.7. $\sin(5\pi/6) + \sin(\pi/6) = 2\sin\left(\dfrac{180}{2}\right)\cos\left(\dfrac{120}{2}\right) = 2\sin(\pi/2)\cos(\pi/3) = 1$

1.8. $\cos(5\pi/6) + \sin(\pi/6) = 2\cos(\pi/2)\cos(\pi/3) = 0$

Q2

2.1. $\tan(\theta)\cos(-\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} \cdot \cos(\theta) = \sin(\theta)$

2.2 $\tan(-\theta)\csc(-\theta)\sec(-\theta) = \left(-\dfrac{\sin(\theta)}{\cos(\theta)}\right)\left(\dfrac{1}{-\sin(\theta)}\right)\left(\dfrac{1}{\cos(\theta)}\right) = \left(\dfrac{1}{\cos^2(\theta)}\right) = \sec^2(\theta)$

2.3. $\tan^2(\theta) + \sin^2(\theta) + \cos^2(\theta) = \tan^2(\theta) + 1 = \sec^2(\theta)$

2.4. $\dfrac{2\sin(\theta)}{\cos(\theta)(1-\tan^2(\theta))} = \tan(2\theta)$

2.5. $\dfrac{\sin(7\theta)+\sin(3\theta)}{\cos(7\theta)-\cos(3\theta)} = \dfrac{2\sin(5\theta)\cos(2\theta)}{-2\sin(5\theta)\sin(2\theta)} = -\cot(\theta)$

2.6. $\dfrac{\sin(5\theta)-\sin(\theta)}{\cos(5\theta)+\cos(\theta)} = \tan(2\theta)$

Q3

3.1. $\cos(5\pi/6) = \dfrac{\sqrt{3}}{2}$

3.2. Here $\sin(3\pi/4) = \dfrac{1}{\sqrt{2}}$, and $\sin(5\pi/4) = -\dfrac{1}{\sqrt{2}}$.

3.3. $\cos(13\pi/18) = -0.766$ to three decimal places.

Q4

4.1. $\sin(\pi/12) = \sin(\pi/4)\cos(\pi/6) - \cos(\pi/4)\sin(\pi/6) = \dfrac{\sqrt{3}}{2\sqrt{2}} - \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

4.2. $\cos(\pi/12) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

4.3. $\tan(\pi/12) = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

4.4. $\sin(5\pi/12) = \sin(\pi/4)\cos(\pi/6) + \cos(\pi/4)\sin(\pi/6) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

4.5. $\cos(5\pi/12) = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

4.6. $\tan(5\pi/12) = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

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