

Answers: The product rule

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Summary

Answers to questions relating to the guide on the product rule.

These are the answers to [Questions: The product rule](#).

Please attempt the questions before reading these answers!

1.1. $\frac{d}{dx}(xe^x) = e^x + xe^x.$

1.2. $\frac{d}{dx}(x^2e^{2x}) = 2xe^{2x} + 2x^2e^{2x}.$

1.3. As $\tan(x) = \sin(x)/\cos(x)$, the function becomes $5x^3 \sin(x)$ and so

$$\frac{d}{dx}(5x^3 \tan(x) \cos(x)) = \frac{d}{dx}(5x^3 \sin(x)) = 15x^2 \sin(x) + 5x^3 \cos(x).$$

1.4. $\frac{d}{dx}(x \ln(x)) = \ln(x) + 1.$

1.5. $\frac{d}{dx}((x^3 + x^2 - 5)(x + 1)) = (3x^2 + 2x)(x + 1) + (x^3 + x^2 - 5).$

1.6. $\frac{d}{dx}((13x^2 + 5x + 2)(x^3 + 2)) = (26x + 5)(x^3 + 2) + 3x^2(13x^2 + 5x + 2).$

1.7. Take the x inside the first bracket so the function becomes $(5x^3 + 3x^2 + 2x)(x^2 + x + 1)$.

Then

$$\begin{aligned}\frac{d}{dx} (x(5x^2 + 3x + 2)(x^2 + x + 1)) &= \frac{d}{dx} ((5x^3 + 3x^2 + 2x)(x^2 + x + 1)) \\ &= (10x^2 + 6x + 2)(x^2 + x + 1) + (5x^3 + 3x^2 + 2x)(2x + 1).\end{aligned}$$

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$$1.8. \quad \frac{d}{dx} ((10x^2 + 21) \cos(x)) = 20x \cos(x) - (10x^2 + 21) \sin(x).$$

1.9. Using the definitions of $\cosh(2x)$ and $\sinh(3x)$:

$$\begin{aligned}\frac{d}{dx} (\cosh(2x) \sinh(3x)) &= \frac{d}{dx} \left(\left(\frac{e^{2x} + e^{-2x}}{2} \right) \left(\frac{e^{3x} - e^{-3x}}{2} \right) \right) \\ &= \left(\frac{e^{2x} + e^{-2x}}{2} \right) \left(\frac{3e^{3x} + 3e^{-3x}}{2} \right) + \left(\frac{2e^{2x} - 2e^{-2x}}{2} \right) \left(\frac{e^{3x} - e^{-3x}}{2} \right) \\ &= 3 \cosh(2x) \cosh(3x) + 2 \sinh(2x) \sinh(3x)\end{aligned}$$

$$1.10. \quad \frac{d}{dx} ((x^2 + 3) \ln(x)) = 2x \ln(x) + \frac{x^2 + 3}{x}.$$

$$1.11. \quad \frac{d}{dx} (\sin(x) \sqrt{x}) = \cos(x) \sqrt{x} + \frac{\sin(x)}{2\sqrt{x}}.$$

1.12. Since $\cosh(x) = \frac{e^x + e^{-x}}{2}$, it follows that

$$\begin{aligned}\frac{d}{dx} (\cosh(x) \ln(x)) &= \frac{d}{dx} \left(\left(\frac{e^x + e^{-x}}{2} \right) \ln(x) \right) \\ &= \left(\frac{e^x - e^{-x}}{2} \right) \ln(x) + \left(\frac{e^x + e^{-x}}{2} \right) \frac{1}{x} \\ &= \sinh(x) \ln(x) + \frac{\cosh(x)}{x}\end{aligned}$$

$$\text{since } \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

1.13. Factorize to get $x^2(\sqrt{x} + \cos(x))$, then

$$\begin{aligned}\frac{d}{dx}(x^2\sqrt{x} + x^2\cos(x)) &= \frac{d}{dx}(x^2(\sqrt{x} + \cos(x))) \\ &= 2x(\sqrt{x} + \cos(x)) + x^2\left(\frac{1}{2\sqrt{x}} - \sin(x)\right).\end{aligned}$$

1.14. $\frac{d}{dx}(e^{-5x}(x^3 + 5)) = 3x^2e^{-5x} - 5e^{-5x}(x^3 + 5).$

1.15. $\frac{d}{dx}\left(\frac{2}{5}\sinh(x) + \frac{2}{13}\cosh(x)\right) = \frac{2}{5}\cosh(x) + \frac{2}{13}\sinh(x).$

1.16. Using the product rule twice here:

$$\frac{d}{dx}(\ln(x)\ln(3x)\ln(100x)) = \frac{\ln(3x)\ln(100x) + \ln(x)\ln(100x) + \ln(x)\ln(3x)}{x}.$$

1.17. $\frac{d}{dx}((x^2 + 5x + 2)\sin(x)) = (2x + 5)\sin(x) + (x^2 + 5x + 2)\cos(x).$

1.18. $\frac{d}{dx}(-\ln(x)\ln(3x)) = -\frac{1}{x}(\ln(3x) + \ln(x)).$

1.19. Using the product rule twice:

$$\begin{aligned}\frac{d}{dx}((x^5 + 3)(x^2 + 3x)(x^7 + x^4)) \\ = 5x^4(x^2 + 3x)(x^7 + x^4) + (x^5 + 3)(2x + 3)(x^7 + x^4) + (x^5 + 3)(x^2 + 3x)(7x^6 + 4x^3).\end{aligned}$$

1.20. $\frac{d}{dx}((\sin(x) + 3x)e^{-x}) = (\cos(x) + 3)e^{-x} - (\sin(x) + 3x)e^{-x}.$

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