Questions: Introduction to partial differentiation

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Summary

A selection of questions for the study guide on the introduction to partial differentiation.

Before attempting these questions, it is highly recommended that you read Guide: Introduction to partial differentiation.

Q1

Find all possible first-order partial derivatives for each function f.

1.1.
$$f(x,y) = x^2y + y^3$$

1.2.
$$f(x,y) = 3x^3 - 2y^4 + xy$$

1.3.
$$f(x,y) = y\sin(2x) + 3$$

1.4.
$$f(x,y) = e^{xy} + 2x^2y^3$$

1.5.
$$f(x,y) = \ln(x) + x \ln(y) + 3x$$

1.6.
$$f(x,y) = \frac{y}{x} - \frac{x}{y}$$

1.7.
$$f(x,y) = x \exp(y^2)$$

1.8.
$$f(x,y) = \sqrt{x^2 + y^2}$$

1.9.
$$f(x,y) = (3x + 2y)^4$$

1.10.
$$f(x,y) = y\sin(xy)$$

1.11.
$$f(x,y) = \sin(x^2 + y^2)$$

1.12.
$$f(x,y) = \ln(1 + x^2y^2)$$

1.13.
$$f(x, y, z) = x^2 y \sin(z)$$

1.14.
$$f(x, y, z) = (x + y)(y + z)(z + x)$$

1.15.
$$f(x, y, z) = \frac{xyz}{x + y + z}$$

Q2

A function f(x,y) is called harmonic if it satisfies the equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

Show that each of these functions is harmonic by calculating the pure second-order partial derivatives and checking that their sum is zero.

- 2.1. $f(x,y) = x^2 y^2$
- 2.2. f(x,y) = xy
- 2.3. $f(x,y) = x^3 3xy^2$
- 2.4. $f(x,y) = \cos(x)\sinh(y)$
- 2.5. $f(x,y) = e^x \sin(y)$
- 2.6. $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$
- 2.7. $f(x,y) = \ln(x^2 + y^2)$

Q3

For each function f(x,y), calculate the mixed second-order partial derivatives and confirm that they satisfy the equation

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

- 3.1. $f(x,y) = x^2y + xy^2$
- 3.2. $f(x,y) = 2x^2 \cos(y)$
- 3.3. $f(x,y) = (x+y)^5$
- $3.4. \quad f(x,y) = \frac{x}{1+y}$
- 3.5. $f(x,y) = \sqrt{x^2 + y^2}$
- 3.6. $f(x,y) = x^2 \sin(y) + y^2 \cos(x)$
- 3.7. $f(x,y) = \tan^{-1}(xy)$

After attempting the questions above, please click this link to find the answers.

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