Answers: The scalar product

Ritwik Anand

Summary

Answers to questions relating to the guide on the scalar product.

These are the answers to Questions: The scalar product.

Please attempt the questions before reading these answers!

Q1

1.1. For $\mathbf{a} = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$, the scalar product is $\mathbf{a} \cdot \mathbf{b} = 26$.

1.2. For $\mathbf{a} = \begin{bmatrix} 10 \\ -7 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ -5 \\ 13 \end{bmatrix}$, the scalar product is $\mathbf{a} \cdot \mathbf{b} = 117$.

1.3. For $\mathbf{a} = \begin{bmatrix} -44 \\ -12 \\ 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 61 \\ -25 \\ 93 \end{bmatrix}$, the scalar product is $\mathbf{a} \cdot \mathbf{b} = -2237$.

1.4. For $\mathbf{a} = \begin{bmatrix} 54 \\ 38 \\ 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 32 \\ -55 \\ 13 \end{bmatrix}$, the scalar product is $\mathbf{a} \cdot \mathbf{b} = -362$.

1.5. For $\mathbf{a} = 2\mathbf{i} + 7\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$, the scalar product is $\mathbf{a} \cdot \mathbf{b} = 48$.

1.6. For $\mathbf{a} = -3\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 12\mathbf{j} + 9\mathbf{k}$, the scalar product is $\mathbf{a} \cdot \mathbf{b} = -195$.

1.7. For $\mathbf{a} = 17\mathbf{j} + 23\mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} - 23\mathbf{j} - 8\mathbf{k}$, the scalar product is $\mathbf{a} \cdot \mathbf{b} = -575$.

1.8. For $\mathbf{a} = \mathbf{i}$ and $\mathbf{b} = \mathbf{j}$, the scalar product is $\mathbf{a} \cdot \mathbf{b} = 0$.

As the scalar product of $\mathbf{a} = \mathbf{i}$ and $\mathbf{b} = \mathbf{j}$ is 0, they are perpendicular to each other. This is true for any combination of any *distinct* pair of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$. However, since any vector is parallel to itself, it follows that $\mathbf{i} \cdot \mathbf{i} = |\mathbf{i}||\mathbf{i}| = |1||1| = 1$; similar results hold for $\mathbf{j} \cdot \mathbf{j}$ and $\mathbf{k} \cdot \mathbf{k}$.

Q2

2.1. For $\mathbf{a} = \begin{bmatrix} -5 \\ 2 \\ -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ -2 \\ 11 \end{bmatrix}$, the angle θ is $132.2°$.

2.2. For $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, the angle θ is $70.5°$.

2.3. For $\mathbf{a} = \begin{bmatrix} -8 \\ 1 \\ -4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ -5 \\ 7 \end{bmatrix}$, the angle θ is $108.7°$.

2.4. For $\mathbf{a} = \begin{bmatrix} 1.2 \\ -1.4 \\ -3.1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -5.4 \\ 9.7 \\ -7.5 \end{bmatrix}$, the angle θ is $86.2°$.

2.5. For $\mathbf{a} = \begin{bmatrix} 45 \\ 65 \\ 54 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -19 \\ -58 \\ 71 \end{bmatrix}$, the angle θ is $95.1°$.

2.6. For $\mathbf{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, the angle θ is $90°$.

2.7. For $\mathbf{a} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ -5 \\ 6 \end{bmatrix}$, the angle θ is $43.0°$.

2.8. For $\mathbf{a} = \begin{bmatrix} -17 \\ 3 \\ 8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 12 \\ -19 \\ -16 \end{bmatrix}$, the angle θ is $137.8°$.

Q3

3.1. For $\mathbf{a} = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ \lambda \\ -2 \end{bmatrix}$ to be perpendicular, then $\lambda = 3$.

3.2. For $\mathbf{a} = \begin{bmatrix} 0 \\ 1 \\ \lambda \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ to be perpendicular, then $\lambda = -\frac{2}{3}$.

3.3. For $\mathbf{a} = \begin{bmatrix} 9 \\ -2 \\ 11 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} \lambda \\ -\lambda \\ 3 \end{bmatrix}$ to be perpendicular, then $\lambda = -3$.

3.4. For $\mathbf{a} = \begin{bmatrix} \lambda \\ 6 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} \lambda \\ \lambda \\ 8 \end{bmatrix}$ to be perpendicular, then $\lambda = -2$ or $\lambda = -4$.

3.5. For $\mathbf{a} = \begin{bmatrix} -2\lambda^2 \\ 4 \\ 14 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 2\lambda \\ 1 \end{bmatrix}$ to be perpendicular, then $\lambda = \frac{7}{3}$ or $\lambda = -1$.

3.6. For $\mathbf{a} = \begin{bmatrix} -5 \\ 9 \\ 2\lambda \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} \lambda \\ -2 \\ \lambda \end{bmatrix}$ to be perpendicular, then $\lambda = \frac{9}{2}$ or $\lambda = -2$.

3.7. For $\mathbf{a} = \begin{bmatrix} -7 \\ 4 \\ 2\lambda \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2\lambda \\ 1 \\ 6\lambda \end{bmatrix}$ to be perpendicular, then $\lambda = \frac{2}{3}$ or $\lambda = \frac{1}{2}$.

3.8. For $\mathbf{a} = \begin{bmatrix} -25 \\ -1\lambda^2 \\ -2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3\lambda \\ -11 \\ 7 \end{bmatrix}$ to be perpendicular, then $\lambda = 7$ or $\lambda = -\frac{2}{11}$.

Version history and licensing

v1.0: initial version created 08/23 by Ritwik Anand as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc.