

Proof: Trigonometric identities

Shanelle Advani, Krish Chaudhary, Tom Coleman, Dzhermma Ruseva

Summary

Explanations as to why certain trigonometric identities are true.

Before reading this proof sheet, it is recommended that you read [Guide: Trigonometric identities \(degrees\)](#) or [Guide: Trigonometric identities \(radians\)](#).

Proof of Pythagorean identities

Remember from [Guide: Trigonometric identities \(degrees\)](#) or [Guide: Trigonometric identities \(radians\)](#) that the **Pythagorean identities** are:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

i Proof of $\sin^2(\theta) + \cos^2(\theta) = 1$

You know from [Guide: Trigonometry \(degrees\)](#) or [Guide: Trigonometry \(radians\)](#) that

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \quad \text{and} \quad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}.$$

You can shorten these to O for opposite, A for adjacent and H for hypotenuse. Rearranging gives $A = H\cos(\theta)$ and $O = H\sin(\theta)$.

From Pythagoras' Theorem, you also know that $A^2 + O^2 = H^2$.

Replacing A and O with the expressions above, you get

$$(H\cos(\theta))^2 + (H\sin(\theta))^2 = H^2$$

Using the laws of indices (see [Guide: Laws of indices](#)), and using the standard notation $(\cos(\theta))^2 = \cos^2(\theta)$ and $(\sin(\theta))^2 = \sin^2(\theta)$ you can write

$$H^2 \cos^2(\theta) + H^2 \sin^2(\theta) = H^2$$

Divide everything by the non-zero H^2 to get:

$$\frac{H^2 \cos^2(\theta)}{H^2} + \frac{H^2 \sin^2(\theta)}{H^2} = \frac{H^2}{H^2}$$

Therefore $\cos^2(\theta) + \sin^2(\theta) = 1$.

Proof of sum identities

Further reading

[Guide: Trigonometric identities \(degrees\)](#)

[Questions: Trigonometric identities \(degrees\)](#)

Version history

v1.0: created in 04/24 by tdhc.