Proof: Trigonometric identities

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Summary

Explanations as to why certain trigonometric identities are true.

Before reading this proof sheet, it is recommended that you read Guide: Trigonometric identities (degrees) or Guide: Trigonometric identities (radians).

Proof of Pythagorean identities

Remember from Guide: Trigonometric identities (degrees) or Guide: Trigonometric identities (radians) that the **Pythagorean identities** are:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

Proof of $\sin^2(\theta) + \cos^2(\theta) = 1$

You know from Guide: Trigonometry (degrees) or Guide: Trigonometry (radians) that

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \quad \text{and} \quad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}.$$

You can shorten these to O for opposite, A for adjacent and H for hypotenuse. Rearranging gives $A = H\cos(\theta)$ and $O = H\sin(\theta)$.
From Pythagoras' Theorem, you also know that $A^2 + O^2 = H^2$.
Replacing A and O with the expressions above, you get

$$(H\cos(\theta))^2 + (H\sin(\theta))^2 = H^2$$

Using the laws of indices (see Guide: Laws of indices), and using the standard notation $(\cos(\theta))^2 = \cos^2(\theta)$ and $(\sin(\theta))^2 = \sin^2(\theta)$ you can write

$$H^2\cos^2(\theta) + H^2\sin^2(\theta) = H^2$$

Divide everything by the non-zero H^2 to get:

$$\frac{H^2\cos^2(\theta)}{H^2} + \frac{H^2\sin^2(\theta)}{H^2} = \frac{H^2}{H^2}$$

Therefore $\cos^2(\theta) + \sin^2(\theta) = 1$.

Proof of sum identities

Further reading

Guide: Trigonometric identities (degrees)

Questions: Trigonometric identities (degrees)

Version history

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