

Answers: Introduction to partial differentiation

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Summary

Answers to questions relating to the guide on introduction to partial differentiation.

These are the answers to [Questions: Introduction to partial differentiation](#).

Please attempt the questions before reading these answers!

Answers

Q1

$$1.1. \quad \frac{\partial f}{\partial x} = 2xy \quad \text{and} \quad \frac{\partial f}{\partial y} = x^2 + 3y^2.$$

$$1.2. \quad \frac{\partial f}{\partial x} = 9x^2 + y \quad \text{and} \quad \frac{\partial f}{\partial y} = x - 8y^3.$$

$$1.3. \quad \frac{\partial f}{\partial x} = 2y \cos(2x) \quad \text{and} \quad \frac{\partial f}{\partial y} = \sin(2x).$$

$$1.4. \quad \frac{\partial f}{\partial x} = ye^{xy} + 4xy^3 \quad \text{and} \quad \frac{\partial f}{\partial y} = xe^{xy} + 6x^2y^2.$$

$$1.5. \quad \frac{\partial f}{\partial x} = \frac{1}{x} + \ln(y) + 3 \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{x}{y}.$$

$$1.6. \quad \frac{\partial f}{\partial x} = -\frac{y}{x^2} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{1}{x} + \frac{x}{y^2}.$$

$$1.7. \quad \frac{\partial f}{\partial x} = \exp(y^2) \quad \text{and} \quad \frac{\partial f}{\partial y} = 2xy \exp(y^2).$$

$$1.8. \quad \frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}}.$$

$$1.9. \quad \frac{\partial f}{\partial x} = 12(3x + 2y)^3 \quad \text{and} \quad \frac{\partial f}{\partial y} = 8(3x + 2y)^3.$$

$$1.10. \quad \frac{\partial f}{\partial x} = y^2 \cos(xy) \quad \text{and} \quad \frac{\partial f}{\partial y} = x \cos(xy) - x^2y \sin(xy).$$

$$1.11. \quad \frac{\partial f}{\partial x} = 2x \ln(xy) + 2y \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{x}{y} + 2x \ln(xy).$$

- 1.12. $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x}} \tan(y) + 5 \ln(x^2) \cos(2y) + \frac{10}{x} \cos(2y)$ and $\frac{\partial f}{\partial y} = \sqrt{x} \sec^2(y) + 5x$.
- 1.13. $\frac{\partial f}{\partial x} = 2xy \sin(z)$ and $\frac{\partial f}{\partial y} = x^2 \sin(z)$ and $\frac{\partial f}{\partial z} = x^2 y \cos(z)$.
- 1.14. $\frac{\partial f}{\partial x} = 2y(z+x)$ and $\frac{\partial f}{\partial y} = 2x(z+y)$ and $\frac{\partial f}{\partial z} = 2y(x+z)$.
- 1.15. $\frac{\partial f}{\partial x} = \frac{yz(x+y+z) - xyz}{(x+y+z)^2}$ and $\frac{\partial f}{\partial y} = \frac{xz(x+y+z) - xyz}{(x+y+z)^2}$ and $\frac{\partial f}{\partial z} = \frac{xy(x+y+z) - xyz}{(x+y+z)^2}$.

Q2

- 2.1. $\frac{\partial^2 f}{\partial x^2} = 2$ and $\frac{\partial^2 f}{\partial y^2} = -2$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.2. $\frac{\partial^2 f}{\partial x^2} = 0$ and $\frac{\partial^2 f}{\partial y^2} = 0$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.3. $\frac{\partial^2 f}{\partial x^2} = 6x$ and $\frac{\partial^2 f}{\partial y^2} = -6x$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.4. $\frac{\partial^2 f}{\partial x^2} = -\cos(x) \sinh(y)$ and $\frac{\partial^2 f}{\partial y^2} = \cos(x) \sinh(y)$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.5. $\frac{\partial^2 f}{\partial x^2} = e^x \sin(y)$ and $\frac{\partial^2 f}{\partial y^2} = -e^x \sin(y)$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

Q3

- 3.1. $\frac{\partial^2 f}{\partial x \partial y} = 2x + 2y$ and $\frac{\partial^2 f}{\partial y \partial x} = 2x + 2y$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.2. $\frac{\partial^2 f}{\partial x \partial y} = 0$ and $\frac{\partial^2 f}{\partial y \partial x} = 0$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.3. $\frac{\partial^2 f}{\partial x \partial y} = 20(x+y)^3$ and $\frac{\partial^2 f}{\partial y \partial x} = 20(x+y)^3$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.4. $\frac{\partial^2 f}{\partial x \partial y} = 0$ and $\frac{\partial^2 f}{\partial y \partial x} = 0$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.5. $\frac{\partial^2 f}{\partial x \partial y} = \frac{y}{(x^2 + y^2)^{3/2}}$ and $\frac{\partial^2 f}{\partial y \partial x} = \frac{y}{(x^2 + y^2)^{3/2}}$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

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