

# Answers: Vector addition and scalar multiplication

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## Summary

Answers to questions relating to the guide on vector addition and scalar multiplication.

*These are the answers to [Questions: Addition and scalar multiplication](#).*

**Please attempt the questions before reading these answers!**

## Q1

1.1. For the **i** component,  $4 + 8 = 12$ . For the **j** component,  $5 + 2 = 7$ . For the **k** component,  $7 + 4 = 11$ . So the answer is  $\mathbf{a} + \mathbf{b} = 12\mathbf{i} + 7\mathbf{j} + 11\mathbf{k}$ .

1.2.  $\mathbf{a} + \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$ .

1.3.  $\mathbf{a} - \mathbf{b} = 2\mathbf{i} - 11\mathbf{j} + 14\mathbf{k}$ .

1.4. You can solve this by doing addition componentwise. **i** component:  $4 - (3 + 11) = -10$ , **j** component:  $12 - (-3 - 4) = 19$ , **k** component:  $-7 - (-2 + 9) = -14$ . So the answer is  $-10\mathbf{i} + 19\mathbf{j} - 14\mathbf{k}$ .

## Q2

$$2.1. \mathbf{a} + \mathbf{b} = \begin{bmatrix} 4x \\ 7y \\ 0 \end{bmatrix}$$

$$2.2. \mathbf{a} - \mathbf{b} = \begin{bmatrix} 7 \\ 3y - 2x \\ -z \end{bmatrix}$$

$$2.3. \mathbf{a} + \mathbf{b} - \mathbf{c} = \mathbf{0} \text{ or } \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

2.4. **a.**

### Q3

3.1.  $3\mathbf{u} = (3)5\mathbf{j} + (3)6\mathbf{k} = 15\mathbf{j} + 18\mathbf{k}$ .

3.2.  $-6\mathbf{v} = \begin{bmatrix} 0 \\ 18 \\ -42 \end{bmatrix}$ .

3.3.  $4\mathbf{v} - 3\mathbf{u} = \begin{bmatrix} 0 \\ -27 \\ 10 \end{bmatrix}$

3.4.  $-2\mathbf{w} - (4\mathbf{u} - 2\mathbf{v}) = \begin{bmatrix} -4 \\ -32 \\ -2 \end{bmatrix}$

### Q4

4.1. By the laws of vector addition,  $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB}$ , where  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$  are the respective coordinates of  $A$  and  $B$  written in vector form. You can find  $\overrightarrow{AB}$  by solving

the above equation.  $\overrightarrow{AB} = \begin{bmatrix} -2 - 3 \\ 5 - 4 \\ 7 - 5 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \\ 2 \end{bmatrix}$

4.2.  $\overrightarrow{AB} = \begin{bmatrix} 4 \\ 6 \\ 0 \end{bmatrix}$ ,  $\overrightarrow{AC} = \begin{bmatrix} -2 \\ -4 \\ -5 \end{bmatrix}$ .  $\overrightarrow{AB} - \overrightarrow{AC} = \begin{bmatrix} 6 \\ 10 \\ 5 \end{bmatrix}$ . You can also calculate this by noticing

$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$ . Then  $\overrightarrow{CB} = \begin{bmatrix} 6 - 0 \\ 11 - 1 \\ 7 - 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 5 \end{bmatrix}$  as required.

4.3.  $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ .  $\begin{bmatrix} 1 \\ 5 \\ 9 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ -2 \end{bmatrix}$ . Solving this gives  $A = (-5, -2, 11)$ .

4.4. Let  $\lambda$  and  $\mu$  be scalars.  $\lambda\mathbf{a} + \mu\mathbf{b} = 13\mathbf{i} - 9\mathbf{j}$ . This gives you the simultaneous equations

$$2\lambda + 3\mu = 13 \quad (\mathbf{i} \text{ component})$$

$$3\lambda - 5\mu = -9 \quad (\mathbf{j} \text{ component})$$

Solving this gives  $\mu = 3$ ,  $\lambda = 2$ , which gives the answer  $2\mathbf{a} + 3\mathbf{b}$ .

4.5.  $2 \begin{bmatrix} 2 \\ 5 \\ z \end{bmatrix} + 3 \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ . Solving this gives  $x = 3$ ,  $y = 1$  and  $z = -6$ .

4.6. As they are parallel  $\mathbf{a} = \lambda\mathbf{b}$  for some real scalar  $\lambda$ . This gives the simultaneous equations

$$x - 7 = -2\lambda \quad (\mathbf{i} \text{ component})$$

$$5x + 1 = 8\lambda \quad (\mathbf{k} \text{ component})$$

Eliminating  $\lambda$  and solving gives  $x = 3$ .

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## Version history and licensing

v1.0: initial version created 08/23 by Renee Knapp, Kin Wang Pang as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc.

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