

# Answers: PMFs, PDFs, and CDFs

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## Summary

Answers to questions relating to the guide on PMFs, PDFs, and CDFs.

*These are the answers to [Questions: PMFs, PDFs, and CDFs](#).*

**Please attempt the questions before reading these answers!**

## Q1

### 1.1.

The given PMF is valid because:

**Non-negativity:** All  $P(X = x) \geq 0$

**Honesty:** The sum of all probabilities equals 1:

$$\sum_{x=1}^4 p(x) = \sum_{x=1}^4 P(X = x) = \frac{1}{10} + \frac{1}{5} + \frac{1}{2} + \frac{1}{5} = 1$$

$$P(X = 4) = \frac{1}{5}.$$

### 1.2.

The given PMF is valid because:

**Non-negativity:** All  $P(X = x) \geq 0$

**Honesty:** The sum of all probabilities equals 1:

$$\sum_{x=1}^4 p(x) = \sum_{x=1}^4 P(X = x) = 0.25 + 0.35 + 0.05 + 0.2 + 0.1 = 1$$

$$P(X = 3 \text{ or } X = 4) = 0.05 + 0.2 = 0.25$$

### 1.3.

The completed PMF table for the biased coin toss is:

$x$	Heads	Tails
$P(X = x)$	0.3	0.7

This is a valid PMF because:

**Non-negativity:** Both  $P(X = x) \geq 0$

**Honesty:** The sum of both probabilities equal 1:

$$\sum_x p(x) = \sum_x P(X = x) = 0.3 + 0.7 = 1$$

#### 1.4.  $\{-\}$

This is not a valid PMF since it fails the honesty condition:

**Honesty:** The sum of the given probabilities does not equal 1:

$$\sum_{x=1}^7 p(x) = \sum_{x=1}^7 P(X = x) = 0.1 + 0.05 + 0.05 + 0.3 + 0.25 + 0.75 + 0.35 = 1.85 \neq 1$$

1.5.

(a)  $P(\text{Blue}) = \frac{3}{10} = 0.3$

(b) The PMF for the given scenario is:

$x$	Red	Blue	Green
$P(X = x)$	0.5	0.3	0.2

This is a valid PMF because:

**Non-negativity:** All  $P(X = x) \geq 0$

**Honesty:** The sum of all three probabilities equals to 1:

$$\sum_x p(x) = \sum_x P(X = x) = 0.5 + 0.3 + 0.2 = 1$$

1.6.

(a) For the given PMF to be valid, you must have  $p = \frac{1}{10}$ .

(b) For  $p = \frac{1}{10}$ , then  $P(X = 3) = \frac{3}{10}$ .

## Q2

### 2.1.

This is a valid PDF because:

**Non-negativity:**  $f(x) \geq 0$  for all values of  $x$ .

**Honesty:**  $\int_{-\infty}^{\infty} f(x) dx = \int_0^2 \frac{1}{2} dx = \left[ \frac{x}{2} \right]_0^2 = 1$

$$P(1 \leq x \leq 2) = \int_1^2 \frac{1}{2} dx = \left[ \frac{x}{2} \right]_1^2 = \frac{1}{2}$$

### 2.2.

This is a valid PDF because:

**Non-negativity:**  $f(x) \geq 0$  for all values of  $x$

**Honesty:**  $\int_{-\infty}^{\infty} f(x) dx = \int_0^1 \frac{x}{2} dx = \left[ \frac{x^2}{2} \right]_0^1 = 1$

$$(a) P(0.5 \leq X \leq 1) = \int_{0.5}^1 2x dx = [x^2]_{0.5}^1 = 1^2 - (0.5)^2 = 1 - 0.25 = 0.75$$

$$(b) P(0.25 \leq X \leq 0.75) = \int_{0.25}^{0.75} 2x dx = [x^2]_{0.25}^{0.75} = (0.75)^2 - (0.25)^2 = 0.5625 - 0.0625 = 0.5$$

### 2.3.

This is a valid PDF because:

**Non-negativity:**  $f(x) \geq 0$  for all values of  $x$

**Honesty:**  $\int_{-\infty}^{\infty} f(x) dx = \int_3^7 \frac{1}{4} dx = \left[ \frac{x}{4} \right]_3^7 = 1$

$$P(3 \leq X \leq 6) = \int_3^6 \frac{1}{4} dx = \left[ \frac{x}{4} \right]_3^6 = \frac{6}{4} - \frac{3}{4} = \frac{3}{4}$$

### 2.4.

This is not a valid PDF since it does not meet the honesty condition:

**Honesty:**  $\int_{-\infty}^{\infty} f(x) dx = \int_1^4 \frac{1}{9} dx + \int_5^7 \frac{1}{4} dx \neq 1$

Calculating the individual integrals:

- $\int_1^4 \frac{1}{9} dx = \frac{1}{9} [x]_1^4 = \frac{1}{3}$
- $\int_5^7 \frac{1}{4} dx = \frac{1}{4} [x]_5^7 = \frac{1}{2}$

And adding them together:

$$\int_{-\infty}^{\infty} f(x) dx = \frac{1}{3} + \frac{1}{2} = \frac{5}{6} \neq 1$$

## 2.5.

(a) For the given PDF to be valid, you must have  $k = 3$ .

(b)  $P(0.2 \leq X \leq 0.3) = \int_{0.2}^{0.3} 3x^2 dx = 3 \left[ \frac{x^3}{3} \right]_{0.2}^{0.3} = [x^3]_{0.2}^{0.3} = 0.019$

## 2.6.

This is a valid PDF because:

**Non-negativity:**  $f(x) \geq 0$  for all values of  $x$

**Honesty:**  $\int_{-\infty}^{\infty} f(x) dx = \int_0^{0.5} 4x dx + \int_{0.5}^{0.75} (4 - 4x) dx + \int_{0.75}^1 0.5 dx$

Calculating the individual integrals:

- $\int_0^{0.5} 4x dx = [2x^2]_0^{0.5} = 0.5$
- $\int_{0.5}^{0.75} (4 - 4x) dx = [4x - 2x^2]_{0.5}^{0.75} = 0.375$
- $\int_{0.75}^1 0.5 dx = [0.5x]_{0.75}^1 = 0.125$

and adding them together gives  $0.5 + 0.375 + 0.125 = 1$ .

## Q3

### 3.1.

(a)  $F(3) = P(X \leq 3) = 0.1 + 0.3 + 0.5 = 0.9$

(b)  $P(X > 2) = 1 - P(X \leq 2) = 1 - (0.1 + 0.3 + 0.5) = 1 - 0.9 = 0.1$

### 3.2.

(a) The CDF for values 0.5, 1, and 2:

$$\blacksquare F(0.5) = \int_0^{0.5} \frac{1}{2} dx = \left[ \frac{x}{2} \right]_0^{0.5} = \frac{0.5}{2} = 0.25$$

$$\blacksquare F(1) = \int_0^1 \frac{1}{2} dx = \left[ \frac{x}{2} \right]_0^1 = \frac{1}{2} = 0.5$$

$$\blacksquare F(2) = \int_0^2 \frac{1}{2} dx = \left[ \frac{x}{2} \right]_0^2 = \frac{2}{2} = 1$$

(b)  $F(3) = 1$  (since the CDF for any  $x \geq 2$  is 1.)

### 3.3.

(a) The CDF at points 4, 5, and 6:

$$\blacksquare F(4) = \int_3^4 \frac{1}{4} dx = \left[ \frac{x}{4} \right]_3^4 = \frac{4}{4} - \frac{3}{4} = \frac{1}{4}$$

$$\blacksquare F(5) = \int_3^5 \frac{1}{4} dx = \left[ \frac{x}{4} \right]_3^5 = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\blacksquare F(6) = \int_3^6 \frac{1}{4} dx = \left[ \frac{x}{4} \right]_3^6 = \frac{6}{4} - \frac{3}{4} = \frac{3}{4}$$

(b)  $P(X > 5) = 1 - F(5) = 1 - \frac{1}{2} = \frac{1}{2}$ .

### 3.4.

This is not a valid CDF because the CDF should be non-decreasing as  $x$  increases.

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## Version history and licensing

v1.0: initial version created 12/24 by Sophie Chowgule as part of a University of St Andrews VIP project.

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