

Answers: Integration by substitution

Donald Campbell

Summary

Answers to questions relating to the guide on integration by substitution.

These are the answers to [Questions: Integration by substitution](#).

Please attempt the questions before reading these answers!

Answers

Q1

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C$$

- 1.1. $\frac{1}{8}(2x + 5)^4 + C$ using the substitution $u = 2x + 5$ or the chain rule with $a = 2$.
- 1.2. $-\frac{1}{24}(3 - 4x)^6 + C$ using the substitution $u = 3 - 4x$ or the chain rule with $a = -4$.
- 1.3. $\frac{2}{5} \left(\frac{x}{2} - 1\right)^5 + C$ using the substitution $u = \frac{x}{2} - 1$ or the chain rule with $a = \frac{1}{2}$.
- 1.4. $-\frac{1}{10}(5x - 2)^{-2} + C$ using the substitution $u = 5x - 2$ or the chain rule with $a = 5$.
- 1.5. $\frac{1}{3}(4 - 3x)^{-1} + C$ using the substitution $u = 4 - 3x$ or the chain rule with $a = -3$.
- 1.6. $-\frac{1}{4}(2x + 7)^{-2} + C$ using the substitution $u = 2x + 7$ or the chain rule with $a = 2$.
- 1.7. $-\frac{5}{3} \left(\frac{x}{5} + 3\right)^{-3} + C$ using the substitution $u = \frac{x}{5} + 3$ or the chain rule with $a = \frac{1}{5}$.
- 1.8. $-(1 - 2x)^{1/2} + C$ using the substitution $u = 1 - 2x$ or the chain rule with $a = -2$.
- 1.9. $-\frac{2}{3}(3x + 4)^{-1/2} + C$ using the substitution $u = 3x + 4$ or the chain rule with $a = 3$.
- 1.10. $-\frac{1}{2}(5 - 6x)^{1/3} + C$ using the substitution $u = 5 - 6x$ or the chain rule with $a = -6$.

Q2

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + C$$
$$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + C$$

- 2.1. $\sin(x) + C$ using the substitution $u = x$ or the chain rule with $a = 1$.
- 2.2. $-\frac{1}{2} \cos(2x) + C$ using the substitution $u = 2x$ or the chain rule with $a = 2$.
- 2.3. $\frac{5}{6} \sin(x) + C$ using the substitution $u = x$ or the chain rule with $a = 1$.
- 2.4. $\frac{1}{3} \sin(3x) + C$ using the substitution $u = 3x$ or the chain rule with $a = 3$.
- 2.5. $-3 \cos\left(\frac{x}{3}\right) + C$ using the substitution $u = \frac{x}{3}$ or the chain rule with $a = \frac{1}{3}$.
- 2.6. $\frac{4}{15} \sin\left(3x - \frac{\pi}{4}\right) + C$ using the substitution $u = 3x - \frac{\pi}{4}$ or the chain rule with $a = 3$.
- 2.7. $-\frac{9}{4} \cos\left(\frac{\pi}{3} - \frac{4x}{9}\right) + C$ using the substitution $u = \frac{\pi}{3} - \frac{4x}{9}$ or the chain rule with $a = -\frac{4}{9}$.
- 2.8. $-\frac{1}{6} \sin\left(3x + \frac{\pi}{2}\right) + C$ using the substitution $u = 3x + \frac{\pi}{2}$ or the chain rule with $a = 3$.
- 2.9. $-16 \cos\left(\frac{x}{4} - \frac{\pi}{2}\right) + C$ using the substitution $u = \frac{x}{4} - \frac{\pi}{2}$ or the chain rule with $a = \frac{1}{4}$.
- 2.10. $-\frac{3}{25} \sin\left(\frac{\pi}{6} - 5x\right) + C$ using the substitution $u = \frac{\pi}{6} - 5x$ or the chain rule with $a = -5$.

Q3

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + C$$

- 3.1. $\frac{5}{2}e^{2x+1} + C$ using the substitution $u = 2x + 1$ or the chain rule with $a = 2$.
- 3.2. $-\frac{7}{3}e^{-3x+4} + C$ using the substitution $u = -3x + 4$ or the chain rule with $a = -3$.
- 3.3. $\frac{1}{3}e^{-3(x-2)} + C$ using the substitution $u = -3(x-2)$ or the chain rule with $a = -3$.
- 3.4. $6 \exp\left(\frac{x}{3} - 5\right) + C$ using the substitution $u = \frac{x}{3} - 5$ or the chain rule with $a = \frac{1}{3}$.
- 3.5. $2 \ln |3x - 7| + C$ using the substitution $u = 3x - 7$ or the chain rule with $a = 3$.
- 3.6. $-2 \ln |5 - 2x| + C$ using the substitution $u = 5 - 2x$ or the chain rule with $a = -2$.
- 3.7. $\frac{3}{2} \ln |2x + 5| + C$ using the substitution $u = 2x + 5$ or the chain rule with $a = 2$.
- 3.8. $-\frac{3}{5} \ln |5(x - 2) + 1| + C$ using the substitution $u = 5(x - 2) + 1$ or the chain rule with $a = 5$.

Q4

- 4.1. $\frac{1}{5}(3x^2 + 2)^5 + C$ using the substitution $u = 3x^2 + 2$.
- 4.2. $\frac{1}{4}(5x - 7)^4 + C$ using the substitution $u = 5x - 7$.
- 4.3. $\exp(4x^2 - 1) + C$ using the substitution $u = 4x^2 - 1$.
- 4.4. $-\frac{1}{x^2 + x + 5} + C$ using the substitution $u = x^2 + x + 5$.
- 4.5. $\sin(3x^2 + 2) + C$ using the substitution $u = 3x^2 + 2$.
- 4.6. $\exp(x^2 + 3x) + C$ using the substitution $u = x^2 + 3x$.
- 4.7. $-\frac{1}{\sqrt{x^2 + 1}} + C$ using the substitution $u = x^2 + 1$.
- 4.8. $\frac{1}{10} \ln |2e^{5x} + 3| + C$ using the substitution $u = 2e^{5x} + 3$.
- 4.9. $-\cos(4 - 2x^2) + C$ using the substitution $u = 4 - 2x^2$.
- 4.10. $-\frac{1}{x^3 + 1} + C$ using the substitution $u = x^3 + 1$.

Version history and licensing

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

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