Converting Units

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Summary

Units are essential when describing everything around us. Sometimes different people will use different units when talking about the same thing, so converting between units is very important. Understanding strategies of converting between unit systems, area, volume, and time is an important skill in science.

Motivation for the Importance of Units and Converting Units

Imagine your friend is going to the market, and you want them to buy 5 apples. Instead, you say, "buy me five fruits." Later that day, your friend returns with 5 oranges, which is not what you wanted. The reason you didn't get apples is that you didn't specify the type of fruit you wanted. In other words, you must specify the **unit** of the item you're referring to. In this case, the unit of the item you wanted is an apple.

Now imagine you're baking a cake from a recipe that calls for 8 ounces of butter. Unfortunately the butter you have is measured in grams, so you do not know how much butter you need to put into your cake. To solve this problem, you must know how many grams are in 8 ounces.This is the process of **converting** one unit to another.

> **Definition of a unit**
>
> A unit expresses any quantity which you are trying to measure.

Units allow you to communicate measurements clearly. Let's say that a distance is 5 metres long is more precise than saying it is "far away". If two quantities are measured in the same units, they are more intuitive to compare. Some examples of units which you have probably seen are metres to measure length, seconds to measure time, and kilograms to measure mass.

Unit Systems

Depending on where you are in the world, you may use different units as someone else to measure the same thing. A very good example of this is height. Suppose that there is a man called Bob from France who is 183 cm tall. If Bob traveled to the United States and told someone his height, they may be confused as they do not usually measure height in centimetres. However if Bob says that he is 6 feet tall they will probably understand, as in the United States height is normally measured in feet and inches.

Today, the two most common unit systems you have are the **metric system** and the **imperial system**.

> **Metric System**
>
> Includes units such as metres for length, kilograms for mass, and litres for volume.

> **Imperial System**
>
> Includes units such as feet for length, pounds for weight, and gallons for volume.

> **Warning**
>
> No matter what units you decide to use, make sure you keep them the same for the problem you are working on. In 1998, the science and technology organisation, NASA, launched the Mars Climate Orbiter to study the planet Mars. After it was launched, the orbiter approached Mars too close and was destroyed by the atmosphere. A later investigation found that the error was due to two groups of people using two different unit systems. Making a mistake with units can be disastrous!

Converting Between Unit Systems

To convert between unit systems, you need to know the relationship between the two units.

A strategy you can use when converting units is;

- Step 1: Identify the quantity you want to find
- Step 2: Determine how much of one unit is equal to another
- Step 3: Multiply or divide to scale the quantity to the value you want

Example 1

Let us look at the cake example from earlier. The recipe of the cake requires 8 ounces of butter, but you want to know how much that is in grams.
First, you must identify that you need to convert 8 ounces to some amount of grams.
Second, you need to know the the relationship between ounces and grams. An approximation is given here;

$$1 \text{ ounces} \approx 28.35 \text{ grams}$$

Lastly, you need to find out how many grams are in 8 ounces. You know the amount of grams in 1 ounce, so you can multiply that amount by 8 to get the number of grams in 8 ounces.

$$8 \text{ ounces} \approx 8 \cdot 28.35 \text{ grams}$$

$$8 \text{ oz} = 8 \cdot 28.35 \text{ g}$$

$8\ ounces = 226.20\ grams$
So you need 226.20 grams of butter in our cake

Example 2

A builder is assembling a cabinet which needs to be 40 centimetres in width. However, they only have a tape measure which has inches on it. What does the width of the cabinet need to be in inches?
First, you want to convert 40 cm to some amount of in.
Second, you will establish the relationship between inches and centimetres. An approximation is given below.
2.5 cm ≈ 1 in.
Lastly, you need to find how many inches are in 40 cm. To get from 2.5 cm to 40 cm, you can multiply by 16.
$2.5 \cdot 16$ cm $\approx 1 \cdot 16$ in
40 cm ≈ 1616 in

Unit Prefixes

Unit Prefixes

Unit prefixes are used to scale a unit to larger or smaller values. They are represented by a letter in front of the unit.

The metric system uses unit prefixes to scale a unit to larger or smaller values. It makes very large and very small units more natural to read. For example, I can say that my weight is $60,000$ grams. This is a large number and is not intuitive to understand its size. More commonly weight is measured in kilograms, so my weight can be written as 60 kilograms. This number is more useful if you want to write my weight down on a medical form as you don't need to write many of the 0 digits.

Other units which are not in the metric system may use unit prefixes. Examples include parsecs (which measures distances in space) or acres (which measure area of land).

Below are some common prefixes of base ten.

Prefix	Symbol	Multiplier	Meaning
giga-	G	10^9	one billion
mega-	M	10^6	one million
kilo-	k	10^3	one thousand
hecto-	h	10^2	one hundred
deca-	da	10^1	ten
(base unit)	-	10^0	one
deci-	d	10^{-1}	one tenth
centi-	c	10^{-2}	one hundredth
milli-	m	10^{-3}	one thousandth
micro-	μ	10^{-6}	one millionth
nano-	n	10^{-9}	one billionth

Tip: Unit prefix conversion

Converting to a **smaller unit** you should **multiply** by an appropriate power of 10.
Converting to a **larger unit** you should **divide** by an appropriate power of 10.

Example 3

A scientist is carrying out a study in the Amazon rainforest about a rare type of lizard. They measure the length of several lizards and find their average length to be $1.35\ \mathrm{cm}$. What is the average length of the lizards in both metres and millimetres?

You are going to use the same strategy as you did when converting between unit systems.

To find the length in metres, you are going to establish a relationship between centimetres and metres. Our base unit is going to be metres, because is does not have a prefix in front of it. From our table, the **centi-** prefix represents the 10^{-2} multiplier. This is the same thing as saying 1 centimetre is equal to 1 metre divided by 100. Writing this as an equation gives;

$1\ \mathrm{cm} = 1 \cdot 10^{-2}\ \mathrm{m} = \frac{1}{100}\ \mathrm{m}$

You want to know how many metres are in $1.35\ \mathrm{cm}$. So you can multiply the relationship you found by 1.35.

$1.35\ \mathrm{cm} = 1.35 \cdot \frac{1}{100}\ \mathrm{m}$

$1.35\ \mathrm{cm} = 0.0135\ \mathrm{m}$

Now, you will do the same for millimetres. You know that the multipliers centi and milli are 10^{-2} and 10^{-3} respectively. This means that the centi- prefix is ten times larger than the milli- prefix. So you can make an equation linking them both together.

$1\ \mathrm{cm} = 10\ \mathrm{mm}$

Now multiply the equation by 1.35.

$1.35\ \mathrm{cm} = 13.5\ \mathrm{mm}$

So the length of the lizard can be written as $0.0135\ \mathrm{m}$ or $13.5\ \mathrm{mm}$.

Example 4

A meteorologist is working in a coastal town. They find the air pressure to be $1013\ \mathrm{hPa}$, where Pa is a unit for pressure called Pascals. Convert this air pressure to MPa.

You know that the multipliers hecto- and Mega- are 10^2 and 10^6 respectively. This means that the mega- prefix is ten thousand times larger than the hecto- prefix.

$10,000\ \mathrm{hPa} = 1\ \mathrm{MPa}$

Divide both sides by $10,000$.

$1\ \mathrm{hPa} = \frac{1}{10{,}000}\ MPa$

Multiply both sides by 1013.

$1013\ \mathrm{hPa} = \frac{1013}{10{,}000}\ \mathrm{MPa}$

$1013\ \mathrm{hPa} = 0.1013\ \mathrm{MPa}$

So the air pressure is $0.1013\ \mathrm{MPa}$.

Converting Units of Area and Volume

you have seen examples of converting one unit of length to another. To do this you multiplied or divided by some scale factor. When converting area and volume to different units, you need to be careful because the scale factors are squared (for area) or cubed (for volume).

Definition of area

Area is the measure of the amount of space taken up by a two-dimensional surface.

Definition of volume

Volume is the measure of the amount of space taken up by a three-dimensional object.

If you have a square whose sides are length $1\ \mathrm{m}$, then its area will be $1\ \mathrm{m}^2$. To convert its area into cm^2, you first must convert the length of the sides to centimetres. The length of the sides are then equivalent to $100\ cm$, which means the area is $10,000\ \mathrm{cm}^2$. Our scale factor when converting the length is 10^2. When looking at the area, $1\ \mathrm{m}^2$ is equal to $10,000\ \mathrm{cm}^2$. This means our scale factor for area is 10^4, which can be written as $(10^2)^2$. you find that the area scale factor is the square of our length scale factor.

Doing a similar exercise with a cube, you can find that the volume scale is the cube of the length scale factor.

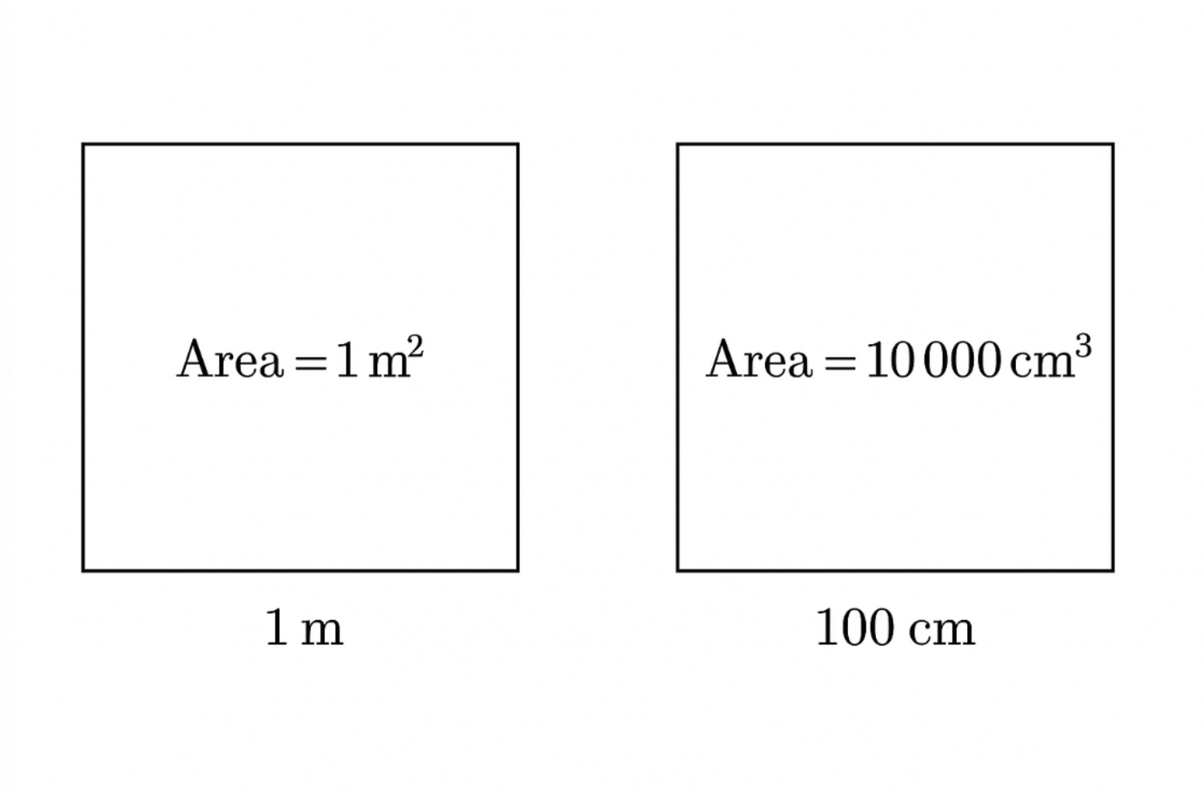

Figure 1: The same square mesured whose sides are measuerd in metres (left) and centimetres (right)

Example 5

You are a farmer who owns $800,000\ \mathrm{m}^2$ of land. Convert this area to km^2.

If you were converting length from metres to kilometres you would divide by 10^3. However because you want the area, so you must square the scale factor you use. So our area scale factor is $(10^3)^2$.

$1\ m^2 = \frac{1}{(10^3)^2}\ \mathrm{km}^2 = 10^{-6}\ km^2 = \frac{1}{1{,}000{,}000}\ \mathrm{km}^2$

$1\ m^2 = \frac{1}{1{,}000{,}000}\ \mathrm{km}^2$

Now multiply the equation by $800,000$.

$800,000\ \mathrm{m}^2 = \frac{800{,}000}{1{,}000{,}000}\ \mathrm{km}^2$

$800,000\ \mathrm{m}^2 = 0.8\ \mathrm{km}^2$

So the area of land can be written as $0.8\ \mathrm{km}^2$

Example 6

The volume of water in a pool is $75\ m^3$. Find how much water there is in litres.
Firstly, it is worth noting that $1\ \mathrm{l} = 1\ \mathrm{dm}^3$. So converting the area to cubic decimetres is the same as converting it to litres.
If you were converting length from metres to decimetres you would multiply by 10^1. However because you want the volume, so you must cube the scale factor you use. So our volume scale factor is $(10^1)^3$.
$1\ \mathrm{m}^3 = (10^1)^3\ \mathrm{dm}^3 = 10^3\ \mathrm{dm}^3 = 1000\ \mathrm{dm}^3$
$1\ \mathrm{m}^3 = 1000\ \mathrm{dm}^3$
Now multiply the equation by 75.
$75 \cdot 1\ \mathrm{mm}^3 = 75 \cdot 1000\ \mathrm{dm}^3$
$75\ \mathrm{m}^3 = 75,000\ \mathrm{dm}^3$
So the volume of water can be written as $75,000\ \mathrm{l}$.

Conversions for Time

Unlike the metric system, units of time do not completely use base ten prefixes. Some time conversions are seen below.

Unit	Symbol	Conversion
seconds	s	-
minutes	min	60 seconds
hours	h	60 minutes
days	d	24 hours
week	wk	7 days
(typical) year	yr	365 days

Sometimes you can use unit prefixes when describing time. This is usually done with time periods much smaller than one second.

Unit	Symbol	Conversion (s)
milliseconds	ms	0.001
nanosecond	ns	0.000000001

Example 7

How many many hours are in 3.5 days?
Writing the relationship between days and hours;
$1 \text{ d} = 24 \text{ h}$
Now multiply the equation by 3.5.
$1 \cdot \text{ d} = 3.5 \cdot 24 \text{ h}$
$3.5 \text{ d} = 84 \text{ h}$
Hence there are 84 hours in 3.5 days.

Example 8

The Silk Road was a network of ancient trade routes which spanned most of Asia. It can take a person 2 years to travel from China to Rome using the road. Calculate how long the journey will be in seconds.
We are not given a direct relationship between years and seconds. Instead, you can break down this question into multiple conversions.
First, youwill start by converting 2 years into days.
$2 \text{ yr} = 2 \cdot 365\text{d} = 730 \text{ d}$
Then you will convert this to hours.
$730 \text{ d} = 24 \cdot 730 \text{ h} = 17520 \text{ h}$
Then to minutes.
$17520 \text{ h} = 60 \cdot 17520 \text{ min} = 1,051,200 \text{ min}$
And finally to seconds.
$1,051,200 \text{ min} = 60 \cdot 1,051,200 \text{ s} = 63,072,000 \text{ s}$
Therefore it will take $63,072,000$ seconds to walk from China to Rome.
By methodically converting years to smaller units of time, you can find time into seconds without memorising the number of seconds in a year!

Compound Units

Definition of a compound unit.

Compound units are measures which combine two or more other units. Examples include m/s for speed, kg/m^3 for density, or N/m^2 for pressure.

When converting compound units, you may need to carry out more than one conversion. you will follow the same strategy you have been using so far.

Example 9

The top speed of a race car is $360\ \mathrm{km/h}$. Convert this speed to $\mathrm{m/s}$.
The speed of the car is made up of two components. The distance in km is on the numerator of the fraction. The time in s is on the denominator. you must convert both of these quantities to find a scale factor, and write them into a fraction.

$1 \cdot \frac{\mathrm{km}}{\mathrm{h}} = \frac{1000}{60 \cdot 60} \cdot \frac{\mathrm{m}}{\mathrm{s}} = \frac{1000}{3600} \cdot \frac{\mathrm{m}}{\mathrm{s}}$

$1 \cdot \frac{\mathrm{km}}{\mathrm{h}} = \frac{1000}{3600} \cdot \frac{\mathrm{m}}{\mathrm{s}}$

Now that you have a scale factor, you can multiply the equation by 360.

$360 \cdot \frac{\mathrm{km}}{\mathrm{h}} = 360 \cdot \frac{1000}{3600} \cdot \frac{\mathrm{m}}{\mathrm{s}}$

$360\ \mathrm{km/h} = 100\ \mathrm{m/s}$

Therefore, the speed of the race car is $100\ \mathrm{m/s}$.

Quick check problems

1. Given that $1\ km = 1.6\ mi$, convert 5 kilometers into miles.
2. Given that $1\ gal = 3.8\ l$, convert 0.5 a gallon to litres.
3. If the length of a screw is $44.5\ mm$, find its length in centimetres.
4. The mains electricity in the United Kingdom has a voltage of $0.23\ kV$. What is the voltage in volts?
5. If shape has an area of $3\ m^2$, find its area in cm^2.
6. If a shape has volume of $500\ mm^3$, find its volume on cm^3.
7. If a decade is 10 years, how many hours are in a decade.
8. If an airplane is flying with a speed of $250\ m/s$, find its speed in km/h.

Further reading

For more questions on the subject, please go to Questions: Completing the square.

Version history

v1.0: initial version created 09/24 by tdhc.