2D Conic Sections Fact Sheet

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Summary

This fact sheet provides an overview of 2D conic sections, including general equations, discriminants, types of conics, standard forms, key features, and labeled graphs.

General Quadratic Equation for Conics

A conic section can be represented algebraically by the **general quadratic equation**:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Discriminant and Conic Type

The discriminant is:

$$\Delta = B^2 - 4AC$$

Use this to classify the conic:

Discriminant	Conic Type	Notes
$\Delta < 0$	Ellipse	Circle if $A = C$ and $B = 0$
$\Delta = 0$	Parabola	
$\Delta > 0$	Hyperbola	

Conic Sections

Circle

Standard form:

$$(x-h)^2 + (y-k)^2 = r^2$$

Key features:

- Centre: (h, k)
- Radius: r

Ellipse

Standard form:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad (a > b)$$

Key features:

- Centre: (h, k)
- Semi-major axis: a
- Semi-minor axis: b
- Foci: $(h \pm c, k)$ for horizontal ellipse, $(h, k \pm c)$ for vertical, where $c = \sqrt{a^2 - b^2}$

Parabola

Standard forms:

- Vertical: $(x-h)^2 = 4p(y-k)$
- Horizontal: $(y-k)^2 = 4p(x-h)$

Key features:

- Vertex: (h, k)
- Focus: $(h, k+p)$ or $(h+p, k)$
- Directrix: $y = k - p$ or $x = h - p$
- Axis of symmetry: passes through vertex and focus

Hyperbola

Standard forms:

- Horizontal: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
- Vertical: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Key features:

- Centre: (h, k)
- Vertices: $(h \pm a, k)$ or $(h, k \pm a)$
- Foci: $(h \pm c, k)$ or $(h, k \pm c)$, $c = \sqrt{a^2 + b^2}$
- Asymptotes: $y - k = \pm\frac{b}{a}(x - h)$ (horizontal), $y - k = \pm\frac{a}{b}(x - h)$ (vertical)

Quick Reference Table

Conic	Standard Form	Key Features
Circle	$(x-h)^2 + (y-k)^2 = r^2$	Centre (h, k), radius r
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	Centre, axes, foci
Parabola	$(x-h)^2 = 4p(y-k)$	Vertex, focus, directrix
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	Centre, vertices, foci, asymptotes

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References

- Interactive Desmos graphs for visualization
- Geometry textbooks on conic sections

Version history

v1.0: initial version created 11/25 by Abigail Carpenter as part of a University of St Andrews VIP project.