

# Factsheet: Trigonometric identities (radians)

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## Summary

A list of trigonometric identities with angles measured in radians.

*The main study guide for this factsheet is [Guide: Trigonometric identities \(radians\)](#). If you would like to know more about these, please read the guide.*

*This factsheet measures angles in radians. For the associated factsheet measuring angles in degrees, please go to [Factsheet: Trigonometric identities \(degrees\)](#).*

## Trigonometric identities

### Periodicity and parity

For all angles  $\theta$  and for all whole numbers  $k \in \mathbb{Z}$ :

$$\cos(-\theta) = \cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\cos(\theta + 2k\pi) = \cos(\theta)$$

$$\sin(\theta + 2k\pi) = \sin(\theta)$$

$$\tan(\theta + k\pi) = \tan(\theta)$$

### Pythagorean formulas

For all angles  $\theta$

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

### Sum and difference formulas

For all angles  $\alpha, \beta$ :

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$$

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha) \tan(\beta)}$$

### Double angle formulas

For all angles  $\theta$ :

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}$$

### Shift formulas

For all angles  $\theta$ :

$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin(\theta)$$

$$\cos\left(\theta - \frac{\pi}{2}\right) = \sin(\theta)$$

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos(\theta)$$

$$\sin\left(\theta - \frac{\pi}{2}\right) = -\cos(\theta)$$

$$\cos(\theta \pm \pi) = -\cos(\theta)$$

$$\sin(\theta \pm \pi) = -\sin(\theta)$$

$$\sin(\pi - \theta) = \sin(\theta)$$

$$\cos(\pi - \theta) = -\cos(\theta)$$

### Sine and cosine rules

For a triangle with corners  $A, B, C$ , angles  $\alpha, \beta, \gamma$  respectively at those corners, and sides

$a, b, c$  opposite their respective corners, the **sine rule** is

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

and the **cosine rule** is

$$a^2 = b^2 + c^2 - 2bc \cos(\alpha).$$

### Common values of trigonometric functions

Angle $\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	un- def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

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### Version history

v1.0: created in 08/25 by tdhc.

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