Answers: PMFs, PDFs, and CDFs

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Summary

Answers to questions relating to the guide on PMFs, PDFs, and CDFs.

These are the answers to [Questions: PMFs, PDFs, and CDFs.]

Please attempt the questions before reading these answers!

Q1

1.1

The given PMF is valid because:

Non-negativity: All $P(X = x) \geq 0$

Honesty: The sum of all probabilities equals 1:

$$\sum_{x=1}^{4} p(x) = \sum_{x=1}^{4} P(X = x) = \frac{1}{10} + \frac{1}{5} + \frac{1}{2} + \frac{1}{5} = 1$$

$P(X = 4) = \frac{1}{5}$.

1.2

The given PMF is valid because:

Non-negativity: All $P(X = x) \geq 0$

Honesty: The sum of all probabilities equals 1:

$$\sum_{x=1}^{4} p(x) = \sum_{x=1}^{4} P(X = x) = 0.25 + 0.35 + 0.05 + 0.2 + 0.1 = 1$$

$P(X = 3 \text{ or } X = 4) = 0.05 + 0.2 = 0.25$

1.3

The completed PMF table for the biased coin toss is:

$$p(x)$$

x	Heads	Tails
$P(X=x)$	0.3	0.7

This is a valid PMF because:

Non-negativity: Both $P(X=x) \geq 0$

Honesty: The sum of both probabilities equal 1:

$$\sum_x p(x) = \sum_x P(X=x) = 0.3 + 0.7 = 1$$

1.4

This is not a valid PMF since it fails the honesty condition:

Honesty: The sum of the given probabilities does not equal 1:

$$\sum_{x=1}^{7} p(x) = \sum_{x=1}^{7} P(X=x) = 0.1 + 0.05 + 0.05 + 0.3 + 0.25 + 0.75 + 0.35 = 1.85 \neq 1$$

1.5

a. $P(\text{Blue}) = \dfrac{3}{10} = 0.3$
b. The PMF for the given scenario is:

$p(x)$

x	Red	Blue	Green
$P(X=x)$	0.5	0.3	0.2

This is a valid PMF because:

Non-negativity: All $P(X=x) \geq 0$

Honesty: The sum of all three probabilities equals to 1:

$$\sum_x p(x) = \sum_x P(X=x) = 0.5 + 0.3 + 0.2 = 1$$

1.6

a. For the given PMF to be valid, you must have $p = \dfrac{1}{10}$

b. For $p = \dfrac{1}{10}$, $P(X = 3) = \dfrac{3}{10}$

Q2

2.1

This is a valid PDF because:

Non-negativity: $f(x) \geq 0$ for all values of x.

Honesty: $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \int_0^2 \frac{1}{2}\,dx = \left[\frac{x}{2}\right]_0^2 = 1$

$\displaystyle P(1 \leq x \leq 2) = \int_1^2 \frac{1}{2}\,dx = \left[\frac{x}{2}\right]_1^2 = \frac{1}{2}$

2.2

This is a valid PDF because:

Non-negativity: $f(x) \geq 0$ for all values of x

Honesty: $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 \frac{x}{2}\,dx = \left[\,x^2\,\right]_0^1 = 1$

a. $\displaystyle P(0.5 \leq X \leq 1) = \int_{0.5}^{1} 2x\,dx = [x^2]_{0.5}^{1} = 1^2 - (0.5)^2 = 1 - 0.25 = 0.75$

b. $\displaystyle P(0.25 \leq X \leq 0.75) = \int_{0.25}^{0.75} 2x\,dx = [x^2]_{0.25}^{0.75} = (0.75)^2 - (0.25)^2 = 0.5625 - 0.0625 = 0.5$

2.3

This is a valid PDF because:

Non-negativity: $f(x) \geq 0$ for all values of x

Honesty: $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \int_3^7 \frac{1}{4}\,dx = \left[\frac{x}{4}\right]_3^7 = 1$

$\displaystyle P(3 \leq X \leq 6) = \int_3^6 \frac{1}{4}\,dx = \left[\frac{x}{4}\right]_3^6 = \frac{6}{4} - \frac{3}{4} = \frac{3}{4}$

2.4

This is not a valid PDF since it does not meet the honesty condition:

Honesty: $\int_{-\infty}^{\infty} f(x)\,dx = \int_{1}^{4} \frac{1}{9}\,dx + \int_{5}^{7} \frac{1}{4}\,dx \neq 1$

Calculating the individual integrals:

- $\int_{1}^{4} \frac{1}{9}\,dx = \frac{1}{9}\,[x]_{1}^{4} = \frac{1}{3}$
- $\int_{5}^{7} \frac{1}{4}\,dx = \frac{1}{4}\,[x]_{5}^{7} = \frac{1}{2}$

And adding them together:

$\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{3} + \frac{1}{2} = \frac{5}{6} \neq 1$

2.5

a. For the given PDF to be valid, you must have $k = 3$

b. $P(0.2 \leq X \leq 0.3) = \int_{0.2}^{0.3} 3x^2\,dx = 3\left[\frac{x^3}{3}\right]_{0.2}^{0.3} = \left[x^3\right]_{0.2}^{0.3} = 0.019$

2.6

This is a valid PDF because:

Non-negativity: $f(x) \geq 0$ for all values of x

Honesty: $\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{0.5} 4x\,dx + \int_{0.5}^{0.75} (4-4x)\,dx + \int_{0.75}^{1} 0.5\,dx$

Calculating the individual integrals:

- $\int_{0}^{0.5} 4x\,dx = \left[2x^2\right]_{0}^{0.5} = 0.5$
- $\int_{0.5}^{0.75} (4-4x)\,dx = \left[4x - 2x^2\right]_{0.5}^{0.75} = 0.375$
- $\int_{0.75}^{1} 0.5\,dx = \left[0.5x\right]_{0.75}^{1} = 0.125$

And adding them together gives:

$0.5 + 0.375 + 0.125 = 1$

Q3

3.1

a. $F(3) = P(X \leq 3) = 0.1 + 0.3 + 0.5 = 0.9$

b. $P(X > 2) = 1 - P(X \leq 2) = 1 - (0.1 + 0.3 + 0.5) = 1 - 0.9 = 0.1$

3.2

a. The CDF for values 0.5, 1, and 2:

- $F(0.5) = \int_0^{0.5} \frac{1}{2}\, dx = \left[\frac{x}{2}\right]_0^{0.5} = \frac{0.5}{2} = 0.25$
- $F(1) = \int_0^{1} \frac{1}{2}\, dx = \left[\frac{x}{2}\right]_0^{1} = \frac{1}{2} = 0.5$
- $F(2) = \int_0^{2} \frac{1}{2}\, dx = \left[\frac{x}{2}\right]_0^{2} = \frac{2}{2} = 1$

b. $F(3) = 1$ (since the CDF for any $x \geq 2$ is 1)

3.3

a. The CDF at points 4, 5, and 6:

- $F(4) = \int_3^{4} \frac{1}{4}\, dx = \left[\frac{x}{4}\right]_3^{4} = \frac{4}{4} - \frac{3}{4} = \frac{1}{4}$
- $F(5) = \int_3^{5} \frac{1}{4}\, dx = \left[\frac{x}{4}\right]_3^{5} = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$
- $F(6) = \int_3^{6} \frac{1}{4}\, dx = \left[\frac{x}{4}\right]_3^{6} = \frac{6}{4} - \frac{3}{4} = \frac{3}{4}$

b. $P(X > 5) = 1 - F(5) = 1 - \frac{1}{2} = \frac{1}{2}$

3.4

a. This is not a valid CDF because the CDF should be non-decreasing as x increases.

Version history and licensing

v1.0: initial version created 12/24 by Sophie Chowgule