Answers: Rearranging equations involving trigonometry and logarithms

Ellie Gurini

Summary

This is an answer set relating to the questions based on Guide, Introduction to rearranging equations involving trigonometry and logarithms.

These are the answers to Questions: Introduction to rearranging equations using trigonometry and logarithms

Please attempt the questions before reading these answers!

Q1

Solve the trigonometric equations in radians.

1.1 For $sin(x) = \frac{\sqrt{2}}{2}$, x is equal to $\frac{\pi}{2}$ or 1.57.

1.2 For $cos(2x + 1) = \frac{1}{2}$, x is equal to $\frac{\pi-3}{6}$ or 0.0234.

1.3 For $tan(5x - 1) = \frac{\sqrt{2}}{2}$, x is equal to 0.323.

1.4 For $cos(x^2 + 4x + 3) = 1$, x is equal to -1 or -3. To do this, you use that $cos^{-1}(1) = 0$ and so you need to solve the quadratic equation $x^2 + 4x + 3 = 0$.

Q2

Rewrite $\cot$ and $\csc$ in terms of $\sin$, $\cos$, and $\tan$

$1 + \frac{1}{\tan^2(x)} = \frac{1}{\sin^2(x)}$

$1 + \frac{\cos^2(x)}{\sin^2(x)} = \frac{1}{\sin^2(x)}$

Then, multiply both sides of the equation by $\sin^2(x)$

$\sin^2(x) + \cos^2(x) = 1.$

Q3

Rewriting $5\cos(x)+9\sin(x)$ gives $\sqrt{106}\sin(x+0.507)$. Setting this equal to 10 and solving gives $x = 0.823$. If you have a slightly different answer, this may be due to rounding at different points in the process.

Q4

4.1 $a = 6$, $b = 36$, $c = 2$.

4.2 $a = 3$, $b = 2187$, $c = 2187$.

4.3 $a = e$, $b = y$, $c = x$.

4.4 $a = 2$, $b = 9$, $c = 3.17....$

4.5 $a = 2$, $b = 4$, $c = 2$.

Q5

5.1 The solution to $6\log_3(x) + \log_3(5) = 9$ is $x = \sqrt[6]{\frac{3^9}{5}}$, or approximately 3.97.

5.2 The solution to $\log_2(16x) = 6$ is $x = 4$.

5.3 If $e^{\ln(3x)} = y$, then $y = 3x$.

Q6

Firstly, substitute y into the first equation. This gives $2^{\log_2(x)} = 4x - 7$. Via example 7, you can see that this means $x = 4x - 7$. Rearranging this gives $x = \frac{7}{3}$ or approximately 2.33. Plugging this into the second equation gives $y = \log_2(\frac{7}{3})$ or approximately 1.22.

Q7

7.1 If $e^{-x} + 3e^x = 12$, then multiply everything by e^x and define y such that $e^x = y$. This makes $1 + 3y^2 = 12y$ and solving this gives $y = \frac{6\pm\sqrt{33}}{3}$. Then, $\ln(y) = x = 1.36$ or -2.46.

7.2 Using the same method detailed above $y = \frac{9\pm\sqrt{65}}{8}$ and $x = 0.757$ or -2.144.

Q8

8.1 If $\log_{16}(x) = \log_2(y)$, then $y = x^{\frac{1}{4}}$.

8.2 If $\log_3(x) = \log_{27}(y)$, then $y = x^3$.

8.3 If $\log_9(x) + \log_3(2x) = 6$, then $\log_9(x) = \log_3(x^{\frac{1}{2}})$. Substituting gives $\log_3(2x^{\frac{3}{2}}) = 6$, thus $3^6 = 2x^{\frac{3}{2}}$. This means that $x = 51.0$.