Questions: The scalar product

Ritwik Anand

Summary

A selection of questions for the study guide on the scalar product

Before attempting these questions, it is highly recommended that you read Guide: The scalar product, as well as Guide: Introduction to quadratic equations.

Q1

Find the scalar product of $\mathbf{a}$ and $\mathbf{b}$.

1.1. $\mathbf{a} = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

1.2. $\mathbf{a} = \begin{bmatrix} 10 \\ -7 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ -5 \\ 13 \end{bmatrix}$

1.3. $\mathbf{a} = \begin{bmatrix} -44 \\ -12 \\ 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 61 \\ -25 \\ 93 \end{bmatrix}$

1.4. $\mathbf{a} = \begin{bmatrix} 54 \\ 38 \\ 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 32 \\ -55 \\ 13 \end{bmatrix}$

1.5. $\mathbf{a} = 2\mathbf{i} + 7\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$

1.6. $\mathbf{a} = -3\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 12\mathbf{j} + 9\mathbf{k}$

1.7. $\mathbf{a} = 17\mathbf{j} + 23\mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} - 23\mathbf{j} - 8\mathbf{k}$

1.8. $\mathbf{a} = \mathbf{i}$ and $\mathbf{b} = \mathbf{j}$.

What can you say about the result of 1.8.? Can you deduce similar conclusions for the scalar product of different combinations of the vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$?

Q2

Using the geometric definition of the scalar products, find the smallest angle θ in between $\mathbf{a}$ and $\mathbf{b}$ in degrees. If your answer is not a whole number, give your answer to an accuracy of one decimal place.

2.1. $\mathbf{a} = \begin{bmatrix} -5 \\ 2 \\ -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ -2 \\ 11 \end{bmatrix}$

2.2. $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

2.3. $\mathbf{a} = \begin{bmatrix} -8 \\ 1 \\ -4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ -5 \\ 7 \end{bmatrix}$

2.4. $\mathbf{a} = \begin{bmatrix} 1.2 \\ -1.4 \\ -3.1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -5.4 \\ 9.7 \\ -7.5 \end{bmatrix}$

2.5. $\mathbf{a} = \begin{bmatrix} 45 \\ 65 \\ 54 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -19 \\ -58 \\ 71 \end{bmatrix}$

2.6. $\mathbf{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

2.7. $\mathbf{a} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ -5 \\ 6 \end{bmatrix}$

2.8. $\mathbf{a} = \begin{bmatrix} -17 \\ 3 \\ 8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 12 \\ -19 \\ -16 \end{bmatrix}$

Q3

Find the value(s) of λ for which $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

3.1. $\mathbf{a} = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ \lambda \\ -2 \end{bmatrix}$

3.2. $\mathbf{a} = \begin{bmatrix} 0 \\ 1 \\ \lambda \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

3.3. $\mathbf{a} = \begin{bmatrix} 9 \\ -2 \\ 11 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} \lambda \\ -\lambda \\ 3 \end{bmatrix}$

3.4. $\mathbf{a} = \begin{bmatrix} \lambda \\ 6 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} \lambda \\ \lambda \\ 8 \end{bmatrix}$

3.5. $\mathbf{a} = \begin{bmatrix} -2\lambda^2 \\ 4 \\ 14 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 2\lambda \\ 1 \end{bmatrix}$

3.6. $\mathbf{a} = \begin{bmatrix} -5 \\ 9 \\ 2\lambda \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} \lambda \\ -2 \\ \lambda \end{bmatrix}$

3.7. $\mathbf{a} = \begin{bmatrix} -7 \\ 4 \\ 2\lambda \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2\lambda \\ 1 \\ 6\lambda \end{bmatrix}$

3.8. $\mathbf{a} = \begin{bmatrix} -25 \\ -\lambda^2 \\ -2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3\lambda \\ -11 \\ 7 \end{bmatrix}$

After attempting the questions above, please click this link to find the answers.

Version history and licensing

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- v1.1: edited 05/24 by tdhc.

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