

# Determinants

Abigail Carpenter

## Summary

A selection of questions for the study guide on determinants.

*Before attempting these questions, it is recommended that you read [Guide: Determinants](#).*

## Q1

Compute the determinant of each matrix.

$$1.1. A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

$$1.2. A = \begin{bmatrix} -1 & 2 \\ 6 & 3 \end{bmatrix}$$

$$1.3. A = \begin{bmatrix} 7 & 0 \\ -3 & 5 \end{bmatrix}$$

$$1.4. A = \begin{bmatrix} 4 & -2 \\ 1 & 8 \end{bmatrix}$$

$$1.5. A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$$

## Q2

Compute the determinant using Sarrus' Rule.

$$2.1. A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$2.2. A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 0 & 5 & 2 \end{bmatrix}$$

$$2.3. A = \begin{bmatrix} -1 & 3 & 2 \\ 4 & 0 & 1 \\ 2 & -2 & 5 \end{bmatrix}$$

$$2.4. A = \begin{bmatrix} 5 & 2 & 1 \\ 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

$$2.5. A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 0 & 3 \\ 5 & -1 & 2 \end{bmatrix}$$

### Q3

Determinants of triangular and diagonal matrices.

$$3.1. A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 6 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

$$3.2. A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$3.3. A = \begin{bmatrix} 4 & 0 & 0 \\ -3 & 2 & 0 \\ 1 & 5 & -6 \end{bmatrix}$$

$$3.4. A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$3.5. A = \begin{bmatrix} 2 & 5 & 7 \\ 0 & 4 & 6 \\ 0 & 0 & 8 \end{bmatrix}$$

## Q4

Compute the determinant using cofactor expansion.

$$4.1. A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 0 \\ 5 & 0 & 6 \end{bmatrix}$$

$$4.2. A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 4 \\ 5 & 0 & 6 \end{bmatrix}$$

$$4.3. A = \begin{bmatrix} 7 & 0 & 0 \\ 1 & -2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$4.4. A = \begin{bmatrix} 0 & 0 & 5 \\ 2 & 3 & 1 \\ 4 & -1 & 2 \end{bmatrix}$$

$$4.5. A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 3 & 4 & 0 & 0 \\ 5 & 0 & 6 & 0 \\ 0 & 7 & 1 & 8 \end{bmatrix}$$

## Q5

Determinant properties.

$$5.1. \text{ If } A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}, \text{ compute } \det(3A).$$

$$5.2. \text{ If } \det(B) = -6, \text{ compute } \det(2B).$$

5.3. If one row of a  $3 \times 3$  matrix is multiplied by 5, how does the determinant change?

5.4. A matrix  $C$  has two identical rows. What is  $\det(C)$ ?

5.5. Swapping two rows of a matrix results in a determinant of 18. What was the determinant before the swap?

5.6. A matrix  $D$  has  $\det(D) = -4$ . Compute  $\det(D^{-1})$ .

5.7. If  $\det(E) = 9$ , compute  $\det(E^T)$ .

5.8. Let  $G = \begin{bmatrix} 2A & 0 \\ 0 & 3I_2 \end{bmatrix}$  with  $\det(A) = 5$ . Compute  $\det(G)$ .

5.9. If one row is replaced by 3 times that row plus another row, how does the determinant change?

5.10. If  $\det(K) = 0$ , what does this imply about the rows or columns of  $K$ ?

5.11. A  $4 \times 4$  matrix  $M$  has  $\det(M) = 8$ . Compute  $\det(-M)$ .

5.12. If  $\det(N) = 7$  for a  $3 \times 3$  matrix, compute  $\det(N^2)$ .

5.13. A matrix has determinant 10. After swapping two rows and multiplying a row by 3, what is the new determinant?

5.14. If  $P = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$  with  $\det(A) = 2$ ,  $\det(B) = -3$ , compute  $\det(P)$ .

5.15. Does adding a multiple of one row to another row change the determinant? Explain.

---

[After attempting the questions above, please click this link to find the answers.](#)

---

## Version history and licensing

v1.0: initial version created 12/25 by Abigail Carpenter as part of a University of St Andrews VIP project.

[This work is licensed under CC BY-NC-SA 4.0.](#)