

Answers: Vector addition and scalar multiplication

Renee Knapp, Kin Wang Pang

Summary

Answers to questions relating to the guide on vector addition and scalar multiplication.

These are the answers to [Questions: Addition and scalar multiplication](#).

Please attempt the questions before reading these answers!

Q1

1.1. For the **i** component, $4 + 8 = 12$. For the **j** component, $5 + 2 = 7$. For the **k** component, $7 + 4 = 11$. So the answer is $\mathbf{a} + \mathbf{b} = 12\mathbf{i} + 7\mathbf{j} + 11\mathbf{k}$.

1.2. $\mathbf{a} + \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$.

1.3. $\mathbf{a} - \mathbf{b} = 2\mathbf{i} - 11\mathbf{j} + 14\mathbf{k}$.

1.4. You can solve this by doing addition componentwise. **i** component: $4 - (3 + 11) = -10$, **j** component: $12 - (-3 - 4) = 19$, **k** component: $-7 - (-2 + 9) = -14$. So the answer is $-10\mathbf{i} + 19\mathbf{j} - 14\mathbf{k}$.

Q2

$$2.1. \mathbf{a} + \mathbf{b} = \begin{bmatrix} 4x \\ 7y \\ 0 \end{bmatrix}$$

$$2.2. \mathbf{a} - \mathbf{b} = \begin{bmatrix} 7 \\ 3y - 2x \\ -z \end{bmatrix}$$

$$2.3. \mathbf{a} + \mathbf{b} - \mathbf{c} = \mathbf{0} \text{ or } \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

2.4. **a.**

Q3

3.1. $3\mathbf{u} = (3)5\mathbf{j} + (3)6\mathbf{k} = 15\mathbf{j} + 18\mathbf{k}$.

3.2. $-6\mathbf{v} = \begin{bmatrix} 0 \\ 18 \\ -42 \end{bmatrix}$.

3.3. $4\mathbf{v} - 3\mathbf{u} = \begin{bmatrix} 0 \\ -27 \\ 10 \end{bmatrix}$

3.4. $-2\mathbf{w} - (4\mathbf{u} - 2\mathbf{v}) = \begin{bmatrix} -4 \\ -32 \\ -2 \end{bmatrix}$

Q4

4.1. By the laws of vector addition, $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB}$, where \overrightarrow{OA} and \overrightarrow{OB} are the respective coordinates of A and B written in vector form. You can find \overrightarrow{AB} by solving

the above equation. $\overrightarrow{AB} = \begin{bmatrix} -2 - 3 \\ 5 - 4 \\ 7 - 5 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \\ 2 \end{bmatrix}$

4.2. $\overrightarrow{AB} = \begin{bmatrix} 4 \\ 6 \\ 0 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} -2 \\ -4 \\ -5 \end{bmatrix}$. $\overrightarrow{AB} - \overrightarrow{AC} = \begin{bmatrix} 6 \\ 10 \\ 5 \end{bmatrix}$. You can also calculate this by noticing

$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$. Then $\overrightarrow{CB} = \begin{bmatrix} 6 - 0 \\ 11 - 1 \\ 7 - 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 5 \end{bmatrix}$ as required.

4.3. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$. $\begin{bmatrix} 1 \\ 5 \\ 9 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ -2 \end{bmatrix}$. Solving this gives $A = (-5, -2, 11)$.

4.4. Let λ and μ be scalars. $\lambda\mathbf{a} + \mu\mathbf{b} = 13\mathbf{i} - 9\mathbf{j}$. This gives you the simultaneous equations

$$2\lambda + 3\mu = 13 \quad (\mathbf{i} \text{ component})$$

$$3\lambda - 5\mu = -9 \quad (\mathbf{j} \text{ component})$$

Solving this gives $\mu = 3$, $\lambda = 2$, which gives the answer $2\mathbf{a} + 3\mathbf{b}$.

4.5. $2 \begin{bmatrix} 2 \\ 5 \\ z \end{bmatrix} + 3 \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$. Solving this gives $x = 3$, $y = 1$ and $z = -6$.

4.6. As they are parallel $\mathbf{a} = \lambda\mathbf{b}$ for some real scalar λ . This gives the simultaneous equations

$$x - 7 = -2\lambda \quad (\mathbf{i} \text{ component})$$

$$5x + 1 = 8\lambda \quad (\mathbf{k} \text{ component})$$

Eliminating λ and solving gives $x = 3$.

Version history and licensing

v1.0: initial version created 08/23 by Renee Knapp, Kin Wang Pang as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc.

This work is licensed under [CC BY-NC-SA 4.0](https://creativecommons.org/licenses/by-nc-sa/4.0/).