

# Factsheet: Lognormal distribution

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## Summary

A factsheet for the lognormal distribution.

Lognormal( $\mu = 3.00, \sigma = 0.50$ )

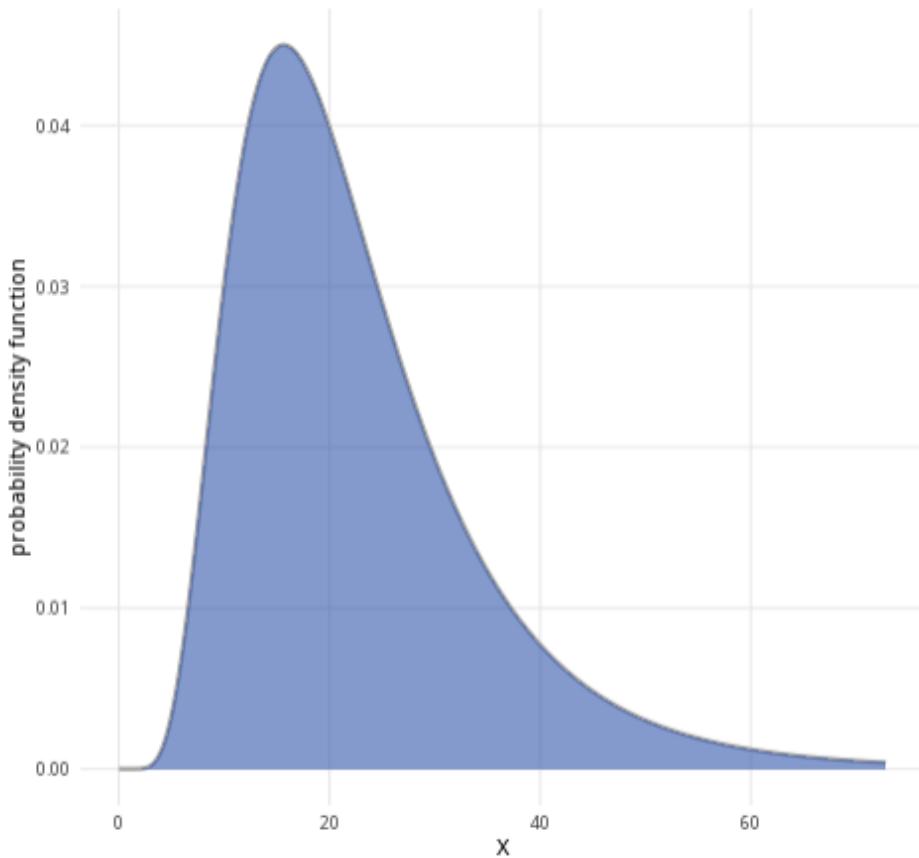


Figure 1: An example of the lognormal distribution with  $\mu = 3$  and  $\sigma = 0.5$ .

**Where to use:** The lognormal distribution is used to model continuous random variables with values that are both real and non-negative, wherein the logarithms of these variables follow a normal distribution. That is to say, if the random variable  $X$  is lognormally distributed, then the random variable  $Y = \ln(X)$  is normally distributed (where  $\ln$  is the natural logarithm).

**Notation:**  $X \sim \text{Lognormal}(\mu, \sigma^2)$

**Parameters:** As with the normal distribution, two numbers  $\mu$  and  $\sigma^2$  where:

- $\mu$  is the expected value of the normally distributed random variable  $Y = \ln(X)$ ,
- $\sigma^2$  is the variance of the normally distributed random variable  $Y = \ln(X)$ .

Quantity	Value	Notes
<b>Mean</b>	$\mathbb{E}(X) = \exp(\mu + \frac{\sigma^2}{2})$	$\exp(y) = e^y$
<b>Variance</b>	$\mathbb{V}(X) = [\exp(\sigma^2) - 1] \exp(2\mu + \sigma^2)$	$\exp(y) = e^y$
<b>PDF</b>	$\mathbb{P}(X = x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(x) - \mu)^2}{2\sigma^2}\right)$	$\exp(y) = e^y$
<b>CDF</b>	$\mathbb{P}(X \leq x) = \frac{1}{2} \left[ 1 + \operatorname{erf}\left(\frac{\ln(x) - \mu}{\sigma\sqrt{2}}\right) \right]$	$\operatorname{erf}(x)$ is the error function of $x$

**Example:** The logarithms of Cantor's Confectionery's stock prices follow a normal distribution. The mean of the stock prices' natural logarithms is 8.01, whereas the variance of the stock prices' natural logarithms is 3. This can be expressed as  $X \sim \text{Lognormal}(8.01, 3)$ , meaning the logarithm of the location parameter is 8.01 and the logarithm of scale parameter is 3.

## Further reading

This interactive element appears in Overview: Probability distributions. Please click this link to go to the guide.

## Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

- v1.1: moved to factsheet form and populated with material from Overview: Probability distributions by tdhc.

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