Answers: Multivariate implicit differentiation

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Summary

Answers to questions relating to the guide on multivariate implicit differentiation.

These are the answers to [Questions: Multivariate implicit differentiation](#).

Please attempt the questions before reading these answers!

Answers

Q1

1.1. Implicit

1.2. Explicit

1.3. Explicit

1.4. Implicit

1.5. Explicit

1.6. Implicit

1.7. Explicit

1.8. Implicit

1.9. Explicit

1.10. Implicit

Q2

2.1. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{x}{y}$

2.2. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{3x^2y}{x^3 + 3y^2}$

2.3. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2}{5}x(y-1)^2$

2.4. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{y\cos(xy)+1}{x\cos(xy)-1}$

2.5. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{\mathrm{e}^y}{x\mathrm{e}^y+2y}$

2.6. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2xy-3y^2}{x^2-6xy}$

2.7. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{y}{x}$

2.8. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{x} + 2(x^2+y^2)$

2.9. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y\sin(xy)+1}{3y^2-x\sin(xy)}$

2.10. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{\sin(y)-y\sin(x)}{x\cos(y)+\cos(x)}$

2.11. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -1$

2.12. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{y\mathrm{e}^{xy}+1}{x\mathrm{e}^{xy}-1}$

2.13. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{x^2-y}{y^2-x}$

2.14. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{4y\sqrt{x}}$

2.15. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{x} + \dfrac{(x-y)^2}{2x^2}$

Q3

3.1. $\dfrac{\partial z}{\partial x} = -\dfrac{x}{z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{y}{z}$.

3.2. $\dfrac{\partial z}{\partial x} = -\dfrac{4x-5xy}{3y^2z+3z^2}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{6yz^2-5x^2}{6y^2z+6z^2}$.

3.3. $\dfrac{\partial z}{\partial x} = -\dfrac{z}{x}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{z}{y}$.

3.4. $\dfrac{\partial z}{\partial x} = -\dfrac{z\mathrm{e}^{xz}}{x\mathrm{e}^{xz}-1}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{1}{x\mathrm{e}^{xz}-1}$.

3.5. $\dfrac{\partial z}{\partial x} = -\dfrac{z\cos(xz)}{x\cos(xz)-y\sin(yz)}$ and $\dfrac{\partial z}{\partial y} = \dfrac{z\sin(yz)}{x\cos(xz)-y\sin(yz)}$.

3.6. $\dfrac{\partial z}{\partial x} = -\dfrac{z}{x}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{z}{y}$.

3.7. $\dfrac{\partial z}{\partial x} = -\dfrac{x^2-yz}{z^2-xy}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{y^2-xz}{z^2-xy}$.

3.8. $\dfrac{\partial z}{\partial x} = -\dfrac{4xz^{3/2}}{2z^{1/2}(x^2+y^2)+1}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{4yz^{3/2}}{2z^{1/2}(x^2+y^2)+1}$.

3.9. $\dfrac{\partial z}{\partial x} = -\dfrac{(1+z^2)\mathrm{e}^x}{y^2(1+z^2)-1}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{2yz(1+z^2)}{y^2(1+z^2)-1}$.

3.10. $\dfrac{\partial z}{\partial x} = -\dfrac{z}{x(1-z)} - \dfrac{yz}{1-z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{xz}{1-z}$.

3.11. $\dfrac{\partial z}{\partial x} = -\dfrac{\mathrm{e}^{yz}+yz\mathrm{e}^{xz}}{xy\mathrm{e}^{yz}+yx\mathrm{e}^{xz}}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{xz\mathrm{e}^{yz}+\mathrm{e}^{xz}}{xy\mathrm{e}^{yz}+yx\mathrm{e}^{xz}}$.

3.12. $\dfrac{\partial z}{\partial x} = \dfrac{\cos(x)\cos(z)}{\sin(x)\sin(z)-2yz}$ and $\dfrac{\partial z}{\partial y} = \dfrac{z^2}{\sin(x)\sin(z)-2yz}$.

3.13. $\dfrac{\partial z}{\partial x} = -\dfrac{2x}{y\mathrm{e}^z+1}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{\mathrm{e}^z}{y\mathrm{e}^z+1}$.

3.14. $\dfrac{\partial z}{\partial x} = -\dfrac{z^2}{z-x-y}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{z^2}{z-x-y}$.

3.15. $\dfrac{\partial z}{\partial x} = -\dfrac{yz+2\sqrt{xyz}}{xy-2\sqrt{xyz}}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{xz-2\sqrt{xyz}}{xy-2\sqrt{xyz}}$.

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