

# Answers: Integration by substitution

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## Summary

Answers to questions relating to the guide on integration by substitution.

These are the answers to [Questions: Integration by substitution](#).

**Please attempt the questions before reading these answers!**

## Answers

### Q1

In these questions, you can either use an appropriate  $u = ax + b$  substitution or use the chain rule for integration

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C$$

- 1.1.  $\frac{1}{8}(2x + 5)^4 + C$  using the substitution  $u = 2x + 5$  or the chain rule with  $a = 2$ .
- 1.2.  $-\frac{1}{24}(3 - 4x)^6 + C$  using the substitution  $u = 3 - 4x$  or the chain rule with  $a = -4$ .
- 1.3.  $\frac{2}{5}\left(\frac{x}{2} - 1\right)^5 + C$  using the substitution  $u = \frac{x}{2} - 1$  or the chain rule with  $a = \frac{1}{2}$ .
- 1.4.  $-\frac{1}{10}(5x - 2)^{-2} + C$  using the substitution  $u = 5x - 2$  or the chain rule with  $a = 5$ .
- 1.5.  $\frac{1}{3}(4 - 3x)^{-1} + C$  using the substitution  $u = 4 - 3x$  or the chain rule with  $a = -3$ .
- 1.6.  $-\frac{1}{4}(2x + 7)^{-2} + C$  using the substitution  $u = 2x + 7$  or the chain rule with  $a = 2$ .
- 1.7.  $-\frac{5}{3}\left(\frac{x}{5} + 3\right)^{-3} + C$  using the substitution  $u = \frac{x}{5} + 3$  or the chain rule with  $a = \frac{1}{5}$ .
- 1.8.  $-(1 - 2x)^{1/2} + C$  using the substitution  $u = 1 - 2x$  or the chain rule with  $a = -2$ .
- 1.9.  $-\frac{2}{3}(3x + 4)^{-1/2} + C$  using the substitution  $u = 3x + 4$  or the chain rule with  $a = 3$ .
- 1.10.  $-\frac{1}{2}(5 - 6x)^{1/3} + C$  using the substitution  $u = 5 - 6x$  or the chain rule with  $a = -6$ .

## Q2

In these questions, you can either use an appropriate  $u = ax + b$  substitution or use the chain rule for integration

$$\int \sin(ax + b) \, dx = -\frac{1}{a} \cos(ax + b) + C$$
$$\int \cos(ax + b) \, dx = \frac{1}{a} \sin(ax + b) + C$$

- 2.1.  $\sin(x) + C$  using the substitution  $u = x$  or the chain rule with  $a = 1$ .
- 2.2.  $-\frac{1}{2} \cos(2x) + C$  using the substitution  $u = 2x$  or the chain rule with  $a = 2$ .
- 2.3.  $\frac{5}{6} \sin(x) + C$  using the substitution  $u = x$  or the chain rule with  $a = 1$ .
- 2.4.  $\frac{1}{3} \sin(3x) + C$  using the substitution  $u = 3x$  or the chain rule with  $a = 3$ .
- 2.5.  $-3 \cos\left(\frac{x}{3}\right) + C$  using the substitution  $u = \frac{x}{3}$  or the chain rule with  $a = \frac{1}{3}$ .
- 2.6.  $\frac{4}{15} \sin\left(3x - \frac{\pi}{4}\right) + C$  using the substitution  $u = 3x - \frac{\pi}{4}$  or the chain rule with  $a = 3$ .
- 2.7.  $-\frac{9}{4} \cos\left(\frac{\pi}{3} - \frac{4x}{9}\right) + C$  using the substitution  $u = \frac{\pi}{3} - \frac{4x}{9}$  or the chain rule with  $a = -\frac{4}{9}$ .
- 2.8.  $-\frac{1}{6} \sin\left(3x + \frac{\pi}{2}\right) + C$  using the substitution  $u = 3x + \frac{\pi}{2}$  or the chain rule with  $a = 3$ .
- 2.9.  $-16 \cos\left(\frac{x}{4} - \frac{\pi}{2}\right) + C$  using the substitution  $u = \frac{x}{4} - \frac{\pi}{2}$  or the chain rule with  $a = \frac{1}{4}$ .
- 2.10.  $-\frac{3}{25} \sin\left(\frac{\pi}{6} - 5x\right) + C$  using the substitution  $u = \frac{\pi}{6} - 5x$  or the chain rule with  $a = -5$ .

## Q3

In these questions, you can either use an appropriate  $u = ax + b$  substitution or use the chain rule for integration

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + C$$

- 3.1.  $\frac{5}{2}e^{2x+1} + C$  using the substitution  $u = 2x + 1$  or the chain rule with  $a = 2$ .
- 3.2.  $-\frac{7}{3}e^{-3x+4} + C$  using the substitution  $u = -3x + 4$  or the chain rule with  $a = -3$ .
- 3.3.  $\frac{1}{3}e^{-3(x-2)} + C$  using the substitution  $u = -3(x-2)$  or the chain rule with  $a = -3$ .
- 3.4.  $6 \exp\left(\frac{x}{3} - 5\right) + C$  using the substitution  $u = \frac{x}{3} - 5$  or the chain rule with  $a = \frac{1}{3}$ .
- 3.5.  $2 \ln |3x - 7| + C$  using the substitution  $u = 3x - 7$  or the chain rule with  $a = 3$ .
- 3.6.  $-2 \ln |5 - 2x| + C$  using the substitution  $u = 5 - 2x$  or the chain rule with  $a = -2$ .
- 3.7.  $\frac{3}{2} \ln |2x + 5| + C$  using the substitution  $u = 2x + 5$  or the chain rule with  $a = 2$ .
- 3.8.  $-\frac{3}{5} \ln |5(x-2) + 1| + C$  using the substitution  $u = 5(x-2) + 1$  or the chain rule with  $a = 5$ .

#### Q4

- 4.1.  $\frac{1}{5}(3x^2 + 2)^5 + C$  using the substitution  $u = 3x^2 + 2$ .
- 4.2.  $\frac{1}{4}(5x - 7)^4 + C$  using the substitution  $u = 5x - 7$ .
- 4.3.  $\exp(4x^2 - 1) + C$  using the substitution  $u = 4x^2 - 1$ .
- 4.4.  $-\frac{1}{x^2 + x + 5} + C$  using the substitution  $u = x^2 + x + 5$ .
- 4.5.  $\sin(3x^2 + 2) + C$  using the substitution  $u = 3x^2 + 2$ .
- 4.6.  $\exp(x^2 + 3x) + C$  using the substitution  $u = x^2 + 3x$ .
- 4.7.  $-\frac{1}{\sqrt{x^2 + 1}} + C$  using the substitution  $u = x^2 + 1$ .
- 4.8.  $\frac{1}{10} \ln |2e^{5x} + 3| + C$  using the substitution  $u = 2e^{5x} + 3$ .
- 4.9.  $-\cos(4 - 2x^2) + C$  using the substitution  $u = 4 - 2x^2$ .
- 4.10.  $-\frac{1}{x^3 + 1} + C$  using the substitution  $u = x^3 + 1$ .

## **Version history and licensing**

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

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