Answers: Introduction to quadratic equations

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Summary

Answers to questions relating to the guide on introduction to quadratic equations.

These are the answers to Questions: Introduction to quadratic equations.

Please attempt the questions before reading these answers!

Q1

For each of the quadratic equations below, identify the variable and the coefficients a, b, c.

1.1. For $x^2 - 7x + 6 = 0$, the variable is x, and the coefficients are $a = 1, b = -7, c = 6$.

1.2. For $y^2 + 14y + 49 = 0$, the variable is y, and the coefficients are $a = 1, b = 14, c = 49$.

1.3. For $h^2 - h - 56 = 0$, the variable is h, and the coefficients are $a = 1, b = -1, c = -56$.

1.4. For $7y^4 - y^2 = 0$, the variable is y^2, and the coefficients are $a = 7, b = -1, c = 0$.

1.5. For $5n^2 - 14n + 100 = 0$, the variable is n, and the coefficients are $a = 5, b = -14, c = 100$.

1.6. For $A^2 - 144 = 0$, the variable is A, and the coefficients are $a = 1, b = 0, c = -144$.

1.7. For $25M^2 = 0$, the variable is M, and the coefficients are $a = 25, b = 0, c = 0$.

1.8. For $e^{2x} - 4e^x + 4 = 0$, the variable is e^x, and the coefficients are $a = 1, b = -4, c = 4$.

1.9. For $-9s^4 + 3s^2 - 1 = 0$, the variable is s^2, and the coefficients are $a = -9, b = 3, c = -1$.

1.10. For $2e^{6x} + e^{3x} + 1 = 0$, the variable is e^{3x}, and the coefficients are $a = 2, b = 1, c = 1$.

1.11. For $\cos^2(x) + 4\cos(x) - 4 = 0$, the variable is $\cos(x)$, and the coefficients are $a = 1, b = 4, c = -4$.

1.12. For $8x^8 - 4x^4 - 1 = 0$, the variable is x^4, and the coefficients are $a = 8, b = -4, c = -1$.

Q2

2.1. The discriminant of the equation $x^2 - 7x + 6 = 0$ is $D = 25$, and therefore the equation has two distinct real roots.

2.2. The discriminant of the equation $y^2 + 14y + 49 = 0$ is $D = 0$, and therefore the equation has one distinct real root.

2.3. The discriminant of the equation $h^2 - h - 56 = 0$ is $D = 217$, and therefore the equation has two distinct real roots.

2.4. The discriminant of the equation $7y^4 - y^2 = 0$ is $D = 1$, and therefore the equation has two distinct real roots.

2.5. The discriminant of the equation $5n^2 - 14n + 100 = 0$ is $D = -1804$, and therefore the equation has no real roots (two distinct complex roots).

2.6. The discriminant of the equation $A^2 - 144 = 0$ is $D = 576$, and therefore the equation has two distinct real roots.

2.7. The discriminant of the equation $25M^2 = 0$ is $D = 0$, and therefore the equation has one distinct real root.

2.8. The discriminant of the equation $e^{2x} - 4e^x + 4 = 0$ is $D = 0$, and therefore the equation has one distinct real root r in e^x. Whether or not it has a real root in x depends on whether or not r is positive. If r is positive, there is exactly one real root $x = \ln(r)$; if r is negative, then there are no real roots.

2.9. The discriminant of the equation $-9s^4 + 3s^2 - 1 = 0$ is $D = -27$, and therefore the equation has no real roots. This is true even with s^2 as the variable, as if s^2 is complex then s must also be complex.

2.10. The discriminant of the equation $2e^{6x} + e^{3x} + 1 = 0$ is $D = -7$, and therefore the equation has no real roots. This is true even with e^{3x} as the variable, as if e^{3x} is complex then x must also be complex.

2.11. The discriminant of the equation $\cos^2(x) + 4\cos(x) - 4 = 0$ is $D = 32$, and therefore the equation has two distinct real roots r_1 and r_2 in $\cos(x)$. Whether or not it has a real root in x depends on whether or not either of the roots is between -1 and 1. If both r_1 and r_2 are outside this range, then there are no real roots. If one of r_1 or r_2 is between -1 and 1, then there are infinitely many solutions.

2.12. The discriminant of the equation $8x^8 - 4x^4 - 1 = 0$ is $D = 48$, and therefore the equation has two distinct real roots r_1 and r_2 in x^4. The amount of real roots depend on the signs of r_1 and r_2.

- If r_1 and r_2 are both positive, then there are four real roots in x. This is because

$x^2 = \pm\sqrt{r_1}$ or $x^2 = \pm\sqrt{r_2}$; square rooting the positive terms gives the roots in x as $\pm\sqrt{(\sqrt{r_1})} = \pm\sqrt[4]{r_1}$ and $\pm\sqrt{(\sqrt{r_2})} = \pm\sqrt[4]{r_2}$. Any other roots must be complex, since you are taking square roots of the negative numbers $-\sqrt{r_1}$ and $-\sqrt{r_2}$.

- If exactly one of r_1 and r_2 is positive (say r_i), then there are two real roots in x given by $\pm\sqrt[4]{r_i}$. All other roots are complex.
- If both r_1 and r_2 are negative, then then there are no real roots in x.

Q3

3.1. Rearranging gives $x^2 + x - 1 = 0$. The discriminant of this is $D = 5$, and therefore the equation has two distinct real roots.

3.2. Rearranging gives $y^2 + 10 = 0$. The discriminant of this is $D = -40$, and therefore the equation has no real roots (two distinct complex roots).

3.3. Rearranging gives $4m^2 + 4m + 1 = 0$. The discriminant of this is $D = 0$, and therefore the equation has one distinct real root.

3.4. Rearranging gives $t^4 + 1 = 0$. The discriminant of this is $D = -4$, and therefore the equation has no real roots. This is true even with t^2 as the variable, as if t^2 is complex then t must also be complex.

3.5. Rearranging gives $5x^2 - 11x - 1 = 0$. The discriminant of this is $D = 101$, and therefore the equation has two distinct real roots.

3.6. Rearranging gives $e^{2x} - 2e^x + 1 = 0$. The discriminant of this is $D = 0$, and therefore the equation has one distinct real root r in e^x. Whether or not it has a real root in x depends on whether or not r is positive. If r is positive, there is exactly one real root $x = \ln(r)$; if r is negative, then there are no real roots.

Version history and licensing

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