# Answers: Expected value, variance, standard deviation

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**Summary**

Answers to questions relating to the guide on expected value, variance, and standard deviation.

*These are the answers to Questions: PMFs, PDFs, and CDFs.*

**Please attempt the questions before reading these answers!**

## Q1

For each of the following valid random variables with associated probability mass function, work out the expected value and variance.

**1.1.**

Here, $\mathbb{E}(X) = 2.8$ and $\mathbb{V}(X) = 0.76$.

**1.2.**

Here, $\mathbb{E}(X) = 2.4$ and $\mathbb{V}(X) = 2.04$.

**1.3.**

This is a Bernoulli trial, with probability of success $p = 0.7$. From Example 2 of Guide: Expected value, variance, standard deviation, you can say that $\mathbb{E}(X) = p = 0.3$. The variance can be worked out to be

$$\mathbb{V}(X) = 0.3 - (0.3)^2 = 0.21 = (0.3)(0.7) = p(1-p).$$

**1.4.**

Here, $\mathbb{E}(X) = 3$ and $\mathbb{V}(X) = 1$.

## Q2

For each of the following valid random variables with associated probability density function, work out the expected value and variance.

### 2.1.

This is the continuous uniform distribution with $a = 0$ and $b = 2$. From Example 2 of Guide: Expected value, variance, standard deviation, you can say that $\mathbb{E}(X) = (a + b)/2 = (0 + 2)/2 = 1$ and

$$\mathbb{V}(X) = \frac{1}{12}(b - a)^2 = \frac{1}{12}(2 - 0)^2 = \frac{4}{12} = \frac{1}{3}.$$

### 2.2.

Here, $\mathbb{E}(X) = \frac{2}{3}$ and $\mathbb{V}(X) = \frac{1}{6}$.

## Q3

You know that the expected value and variance of rolling one fair die are $\mathbb{E}(X) = \frac{7}{2}$ and $\mathbb{V}(X) = \frac{35}{12}$.

Because the roll of each die is an independent event, the random variable $Y$ of rolling seven dice is the same as

$$Y = X + X + X + X + X + X + X$$

which is seven lots of $X$.

You can use the properties of expected values and variance to get

$$\mathbb{E}(Y) = \mathbb{E}(X + X + X + X + X + X + X)$$
$$= 7 \cdot \frac{7}{2} = \frac{49}{2} = 24.5$$

and

$$\mathbb{V}(Y) = \mathbb{V}(X + X + X + X + X + X + X)$$
$$= 7 \cdot \frac{35}{12} = \frac{245}{12}$$

## Q4

When doing this question, you need to find the two integrals

$$\mathbb{E}(X) = \int_0^\infty \lambda x e^{-\lambda x}\,\mathrm{d}x \quad \text{and} \quad \mathbb{V}(X) = \int_0^\infty \lambda x^2 e^{-\lambda x}\,\mathrm{d}x.$$

You will need to use integration by parts; you should always differentiate the power of $x$ to reduce the size of the power. Use the given result to evaluate the integral when $x \to \infty$; the antiderivative should always be $0$ in this case.

## Version history and licensing

v1.0: initial version created 08/25 by tdhc.