# Answers: Introduction to partial differentiation

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#### Summary

Answers to questions relating to the guide on introduction to partial differentiation.

*These are the answers to Questions: Introduction to partial differentiation.*

**Please attempt the questions before reading these answers!**

## Answers

### Q1

1.1. $\dfrac{\partial f}{\partial x} = 2xy$ and $\dfrac{\partial f}{\partial y} = x^2 + 3y^2$.

1.2. $\dfrac{\partial f}{\partial x} = 9x^2 + y$ and $\dfrac{\partial f}{\partial y} = x - 8y^3$.

1.3. $\dfrac{\partial f}{\partial x} = 2y\cos(2x)$ and $\dfrac{\partial f}{\partial y} = \sin(2x)$.

1.4. $\dfrac{\partial f}{\partial x} = y\mathrm{e}^{xy} + 4xy^3$ and $\dfrac{\partial f}{\partial y} = x\mathrm{e}^{xy} + 6x^2y^2$.

1.5. $\dfrac{\partial f}{\partial x} = \dfrac{1}{x} + \ln(y) + 3$ and $\dfrac{\partial f}{\partial y} = \dfrac{x}{y}$.

1.6. $\dfrac{\partial f}{\partial x} = -\dfrac{y}{x^2}$ and $\dfrac{\partial f}{\partial y} = \dfrac{1}{x} + \dfrac{x}{y^2}$.

1.7. $\dfrac{\partial f}{\partial x} = \exp(y^2)$ and $\dfrac{\partial f}{\partial y} = 2xy\exp(y^2)$.

1.8. $\dfrac{\partial f}{\partial x} = \dfrac{x}{\sqrt{x^2+y^2}}$ and $\dfrac{\partial f}{\partial y} = \dfrac{y}{\sqrt{x^2+y^2}}$.

1.9. $\dfrac{\partial f}{\partial x} = 12(3x+2y)^3$ and $\dfrac{\partial f}{\partial y} = 8(3x+2y)^3$.

1.10. $\dfrac{\partial f}{\partial x} = y^2\cos(xy)$ and $\dfrac{\partial f}{\partial y} = x\cos(xy) - x^2y\sin(xy)$.

1.11. $\dfrac{\partial f}{\partial x} = 2x\ln(xy) + 2y$ and $\dfrac{\partial f}{\partial y} = \dfrac{x}{y} + 2x\ln(xy)$.

1.12. $\dfrac{\partial f}{\partial x} = \dfrac{1}{2\sqrt{x}}\tan(y) + 5\ln(x^2)\cos(2y) + \dfrac{10}{x}\cos(2y)$ and $\dfrac{\partial f}{\partial y} = \sqrt{x}\sec^2(y) + 5x$.

1.13. $\dfrac{\partial f}{\partial x} = 2xy\sin(z)$ and $\dfrac{\partial f}{\partial y} = x^2\sin(z)$ and $\dfrac{\partial f}{\partial z} = x^2y\cos(z)$.

1.14. $\dfrac{\partial f}{\partial x} = 2y(z+x)$ and $\dfrac{\partial f}{\partial y} = 2x(z+y)$ and $\dfrac{\partial f}{\partial z} = 2y(x+z)$.

1.15. $\dfrac{\partial f}{\partial x} = \dfrac{yz(x+y+z)-xyz}{(x+y+z)^2}$ and $\dfrac{\partial f}{\partial y} = \dfrac{xz(x+y+z)-xyz}{(x+y+z)^2}$ and $\dfrac{\partial f}{\partial z} = \dfrac{xy(x+y+z)-xyz}{(x+y+z)^2}$.

**Q2**

2.1. $\dfrac{\partial^2 f}{\partial x^2} = 2$ and $\dfrac{\partial^2 f}{\partial y^2} = -2$ so $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$.

2.2. $\dfrac{\partial^2 f}{\partial x^2} = 0$ and $\dfrac{\partial^2 f}{\partial y^2} = 0$ so $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$.

2.3. $\dfrac{\partial^2 f}{\partial x^2} = 6x$ and $\dfrac{\partial^2 f}{\partial y^2} = -6x$ so $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$.

2.4. $\dfrac{\partial^2 f}{\partial x^2} = -\cos(x)\sinh(y)$ and $\dfrac{\partial^2 f}{\partial y^2} = \cos(x)\sinh(y)$ so $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$.

2.5. $\dfrac{\partial^2 f}{\partial x^2} = \mathrm{e}^x\sin(y)$ and $\dfrac{\partial^2 f}{\partial y^2} = -\mathrm{e}^x\sin(y)$ so $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$.

**Q3**

3.1. $\dfrac{\partial^2 f}{\partial x\,\partial y} = 2x+2y$ and $\dfrac{\partial^2 f}{\partial y\,\partial x} = 2x+2y$ so $\dfrac{\partial^2 f}{\partial x\,\partial y} = \dfrac{\partial^2 f}{\partial y\,\partial x}$.

3.2. $\dfrac{\partial^2 f}{\partial x\,\partial y} = 0$ and $\dfrac{\partial^2 f}{\partial y\,\partial x} = 0$ so $\dfrac{\partial^2 f}{\partial x\,\partial y} = \dfrac{\partial^2 f}{\partial y\,\partial x}$.

3.3. $\dfrac{\partial^2 f}{\partial x\,\partial y} = 20(x+y)^3$ and $\dfrac{\partial^2 f}{\partial y\,\partial x} = 20(x+y)^3$ so $\dfrac{\partial^2 f}{\partial x\,\partial y} = \dfrac{\partial^2 f}{\partial y\,\partial x}$.

3.4. $\dfrac{\partial^2 f}{\partial x\,\partial y} = 0$ and $\dfrac{\partial^2 f}{\partial y\,\partial x} = 0$ so $\dfrac{\partial^2 f}{\partial x\,\partial y} = \dfrac{\partial^2 f}{\partial y\,\partial x}$.

3.5. $\dfrac{\partial^2 f}{\partial x\,\partial y} = \dfrac{y}{(x^2+y^2)^{3/2}}$ and $\dfrac{\partial^2 f}{\partial y\,\partial x} = \dfrac{y}{(x^2+y^2)^{3/2}}$ so $\dfrac{\partial^2 f}{\partial x\,\partial y} = \dfrac{\partial^2 f}{\partial y\,\partial x}$.

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