

Answers: Introduction to partial differentiation

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Summary

Answers to questions relating to the guide on the introduction to partial differentiation.

These are the answers to [Questions: Introduction to partial differentiation](#).

Please attempt the questions before reading these answers!

Q1

$$1.1. \quad \frac{\partial f}{\partial x} = 2xy \text{ and } \frac{\partial f}{\partial y} = x^2 + 3y^2.$$

$$1.2. \quad \frac{\partial f}{\partial x} = 9x^2 + y \text{ and } \frac{\partial f}{\partial y} = x - 8y^3.$$

$$1.3. \quad \frac{\partial f}{\partial x} = 2y \cos(2x) \text{ and } \frac{\partial f}{\partial y} = \sin(2x).$$

$$1.4. \quad \frac{\partial f}{\partial x} = ye^{xy} + 4xy^3 \text{ and } \frac{\partial f}{\partial y} = xe^{xy} + 6x^2y^2.$$

$$1.5. \quad \frac{\partial f}{\partial x} = \frac{1}{x} + \ln(y) + 3 \text{ and } \frac{\partial f}{\partial y} = \frac{x}{y}.$$

$$1.6. \quad \frac{\partial f}{\partial x} = -\frac{y}{x^2} - \frac{1}{y} \text{ and } \frac{\partial f}{\partial y} = \frac{1}{x} + \frac{x}{y^2}.$$

$$1.7. \quad \frac{\partial f}{\partial x} = \exp(y^2) \text{ and } \frac{\partial f}{\partial y} = 2xy \exp(y^2).$$

$$1.8. \quad \frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} \text{ and } \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}}.$$

$$1.9. \quad \frac{\partial f}{\partial x} = 12(3x + 2y)^3 \text{ and } \frac{\partial f}{\partial y} = 8(3x + 2y)^3.$$

$$1.10. \quad \frac{\partial f}{\partial x} = y^2 \cos(xy) \text{ and } \frac{\partial f}{\partial y} = x \cos(xy) - x^2y \sin(xy).$$

$$1.11. \quad \frac{\partial f}{\partial x} = 2x \cos(x^2 + y^2) \text{ and } \frac{\partial f}{\partial y} = 2y \cos(x^2 + y^2).$$

$$1.12. \quad \frac{\partial f}{\partial x} = \frac{2xy^2}{1 + x^2y^2} \text{ and } \frac{\partial f}{\partial y} = \frac{2x^2y}{1 + x^2y^2}.$$

- 1.13. $\frac{\partial f}{\partial x} = 2xy \sin(z)$ and $\frac{\partial f}{\partial y} = x^2 \sin(z)$ and $\frac{\partial f}{\partial z} = x^2 y \cos(z)$.
- 1.14. $\frac{\partial f}{\partial x} = (y+z)(2x+y+z)$ and $\frac{\partial f}{\partial y} = (x+z)(x+2y+z)$ and $\frac{\partial f}{\partial z} = (x+y)(x+y+2z)$.
- 1.15. $\frac{\partial f}{\partial x} = \frac{yz(y+z)}{(x+y+z)^2}$ and $\frac{\partial f}{\partial y} = \frac{xz(x+z)}{(x+y+z)^2}$ and $\frac{\partial f}{\partial z} = \frac{xy(x+y)}{(x+y+z)^2}$.

Q2

- 2.1. $\frac{\partial^2 f}{\partial x^2} = 2$ and $\frac{\partial^2 f}{\partial y^2} = -2$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.2. $\frac{\partial^2 f}{\partial x^2} = 0$ and $\frac{\partial^2 f}{\partial y^2} = 0$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.3. $\frac{\partial^2 f}{\partial x^2} = 6x$ and $\frac{\partial^2 f}{\partial y^2} = -6x$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.4. $\frac{\partial^2 f}{\partial x^2} = -\cos(x) \sinh(y)$ and $\frac{\partial^2 f}{\partial y^2} = \cos(x) \sinh(y)$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.5. $\frac{\partial^2 f}{\partial x^2} = e^x \sin(y)$ and $\frac{\partial^2 f}{\partial y^2} = -e^x \sin(y)$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.6. $\frac{\partial^2 f}{\partial x^2} = \frac{2xy}{(x^2 + y^2)^2}$ and $\frac{\partial^2 f}{\partial y^2} = -\frac{2xy}{(x^2 + y^2)^2}$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- 2.7. $\frac{\partial^2 f}{\partial x^2} = \frac{2(y^2 - x^2)}{(x^2 + y^2)^2}$ and $\frac{\partial^2 f}{\partial y^2} = \frac{2(x^2 - y^2)}{(x^2 + y^2)^2}$ so $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

Q3

- 3.1. $\frac{\partial^2 f}{\partial x \partial y} = 2x + 2y$ and $\frac{\partial^2 f}{\partial y \partial x} = 2x + 2y$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.2. $\frac{\partial^2 f}{\partial x \partial y} = -4x \sin(y)$ and $\frac{\partial^2 f}{\partial y \partial x} = -4x \sin(y)$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.3. $\frac{\partial^2 f}{\partial x \partial y} = 20(x+y)^3$ and $\frac{\partial^2 f}{\partial y \partial x} = 20(x+y)^3$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.4. $\frac{\partial^2 f}{\partial x \partial y} = -\frac{1}{(y+1)^2}$ and $\frac{\partial^2 f}{\partial y \partial x} = -\frac{1}{(y+1)^2}$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.5. $\frac{\partial^2 f}{\partial x \partial y} = -\frac{xy}{(x^2 + y^2)^{3/2}}$ and $\frac{\partial^2 f}{\partial y \partial x} = -\frac{xy}{(x^2 + y^2)^{3/2}}$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.6. $\frac{\partial^2 f}{\partial x \partial y} = 2x \cos(y) - 2y \sin(x)$ and $\frac{\partial^2 f}{\partial y \partial x} = 2x \cos(y) - 2y \sin(x)$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- 3.7. $\frac{\partial^2 f}{\partial x \partial y} = \frac{1 - (xy)^2}{(1 + (xy)^2)^2}$ and $\frac{\partial^2 f}{\partial y \partial x} = \frac{1 - (xy)^2}{(1 + (xy)^2)^2}$ so $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.



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