Answers: Integration by substitution

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Summary

Answers to questions relating to the guide on integration by substitution.

These are the answers to Questions: Integration by substitution.

Please attempt the questions before reading these answers!

Answers

Q1

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int (ax+b)^n \, \mathrm{d}x = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

1.1. $\frac{1}{8}(2x+5)^4 + C$ using the substitution $u = 2x + 5$ or the chain rule with $a = 2$.

1.2. $-\frac{1}{24}(3-4x)^6 + C$ using the substitution $u = 3 - 4x$ or the chain rule with $a = -4$.

1.3. $\frac{2}{5}\left(\frac{x}{2} - 1\right)^5 + C$ using the substitution $u = \frac{x}{2} - 1$ or the chain rule with $a = \frac{1}{2}$.

1.4. $-\frac{1}{10}(5x-2)^{-2} + C$ using the substitution $u = 5x - 2$ or the chain rule with $a = 5$.

1.5. $\frac{1}{3}(4-3x)^{-1} + C$ using the substitution $u = 4 - 3x$ or the chain rule with $a = -3$.

1.6. $-\frac{1}{4}(2x+7)^{-2} + C$ using the substitution $u = 2x + 7$ or the chain rule with $a = 2$.

1.7. $-\frac{5}{3}\left(\frac{x}{5} + 3\right)^{-3} + C$ using the substitution $u = \frac{x}{5} + 3$ or the chain rule with $a = \frac{1}{5}$.

1.8. $-(1-2x)^{1/2} + C$ using the substitution $u = 1 - 2x$ or the chain rule with $a = -2$.

1.9. $-\frac{2}{3}(3x+4)^{-1/2} + C$ using the substitution $u = 3x + 4$ or the chain rule with $a = 3$.

1.10. $-\frac{1}{2}(5-6x)^{1/3} + C$ using the substitution $u = 5 - 6x$ or the chain rule with $a = -6$.

Q2

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int \sin(ax+b)\,\mathrm{d}x = -\frac{1}{a}\cos(ax+b) + C$$
$$\int \cos(ax+b)\,\mathrm{d}x = \frac{1}{a}\sin(ax+b) + C$$

2.1. $\sin(x) + C$ using the substitution $u = x$ or the chain rule with $a = 1$.

2.2. $-\frac{1}{2}\cos(2x) + C$ using the substitution $u = 2x$ or the chain rule with $a = 2$.

2.3. $\frac{5}{6}\sin(x) + C$ using the substitution $u = x$ or the chain rule with $a = 1$.

2.4. $\frac{1}{3}\sin(3x) + C$ using the substitution $u = 3x$ or the chain rule with $a = 3$.

2.5. $-3\cos\left(\frac{x}{3}\right) + C$ using the substitution $u = \frac{x}{3}$ or the chain rule with $a = \frac{1}{3}$.

2.6. $\frac{4}{15}\sin\left(3x - \frac{\pi}{4}\right) + C$ using the substitution $u = 3x - \frac{\pi}{4}$ or the chain rule with $a = 3$.

2.7. $-\frac{9}{4}\cos\left(\frac{\pi}{3} - \frac{4x}{9}\right) + C$ using the substitution $u = \frac{\pi}{3} - \frac{4x}{9}$ or the chain rule with $a = -\frac{4}{9}$.

2.8. $-\frac{1}{6}\sin\left(3x + \frac{\pi}{2}\right) + C$ using the substitution $u = 3x + \frac{\pi}{2}$ or the chain rule with $a = 3$.

2.9. $-16\cos\left(\frac{x}{4} - \frac{\pi}{2}\right) + C$ using the substitution $u = \frac{x}{4} - \frac{\pi}{2}$ or the chain rule with $a = \frac{1}{4}$.

2.10. $-\frac{3}{25}\sin\left(\frac{\pi}{6} - 5x\right) + C$ using the substitution $u = \frac{\pi}{6} - 5x$ or the chain rule with $a = -5$.

Q3

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int \mathrm{e}^{ax+b}\,\mathrm{d}x = \frac{1}{a}\mathrm{e}^{ax+b} + C$$
$$\int \frac{1}{ax+b}\,\mathrm{d}x = \frac{1}{a}\ln|ax+b| + C$$

3.1. $\frac{5}{2}\mathrm{e}^{2x+1} + C$ using the substitution $u = 2x+1$ or the chain rule with $a = 2$.

3.2. $-\frac{7}{3}\mathrm{e}^{-3x+4} + C$ using the substitution $u = -3x+4$ or the chain rule with $a = -3$.

3.3. $\frac{1}{3}\mathrm{e}^{-3(x-2)} + C$ using the substitution $u = -3(x-2)$ or the chain rule with $a = -3$.

3.4. $6\exp\left(\frac{x}{3} - 5\right) + C$ using the substitution $u = \frac{x}{3} - 5$ or the chain rule with $a = \frac{1}{3}$.

3.5. $2\ln|3x-7| + C$ using the substitution $u = 3x-7$ or the chain rule with $a = 3$.

3.6. $-2\ln|5-2x| + C$ using the substitution $u = 5-2x$ or the chain rule with $a = -2$.

3.7. $\frac{3}{2}\ln|2x+5| + C$ using the substitution $u = 2x+5$ or the chain rule with $a = 2$.

3.8. $-\frac{3}{5}\ln|5(x-2)+1| + C$ using the substitution $u = 5(x-2)+1$ or the chain rule with $a = 5$.

Q4

4.1. $\frac{1}{5}(3x^2+2)^5 + C$ using the substitution $u = 3x^2+2$.

4.2. $\frac{1}{4}(5x-7)^4 + C$ using the substitution $u = 5x-7$.

4.3. $\exp(4x^2-1) + C$ using the substitution $u = 4x^2-1$.

4.4. $-\frac{1}{x^2+x+5} + C$ using the substitution $u = x^2+x+5$.

4.5. $\sin(3x^2+2) + C$ using the substitution $u = 3x^2+2$.

4.6. $\exp(x^2+3x) + C$ using the substitution $u = x^2+3x$.

4.7. $-\frac{1}{\sqrt{x^2+1}} + C$ using the substitution $u = x^2+1$.

4.8. $\frac{1}{10}\ln|2\mathrm{e}^{5x}+3| + C$ using the substitution $u = 2\mathrm{e}^{5x}+3$.

4.9. $-\cos(4-2x^2) + C$ using the substitution $u = 4-2x^2$.

4.10. $-\frac{1}{x^3+1} + C$ using the substitution $u = x^3+1$.

Version history and licensing

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.