Proof: properties of sigma notation

Tom Coleman

Summary

An explanation as to why some of the properties of sigma notation are true.

Before reading this proof sheet, it is recommended that you read Guide: Introduction to sigma notation and [Guide: Proof by induction].

Proof of properties of sigma notation

Distributivity

i Distributivity

Hello

i Proof of distributivity

You can see this is true by writing the entire sum out, like this:

$$\begin{array}{rcl} \sum_{i=k}^{n} Ca_{i} & = & Ca_{k} + Ca_{k+1} + Ca_{k+2} + \ldots + Ca_{n} \\ & = & C(a_{k} + a_{k+1} + a_{k+2} + \ldots + a_{n}) \\ & = & C\sum_{i=k}^{n} a_{k} \end{array}$$

Combining and decomposing sums

i Combining and decomposing sums

Hello

i Proof of combining and decomposing sums

Similar to the distributive property, you can show this is true by writing the entire sum out:

$$\begin{array}{lll} \sum_{i=k}^n a_i + \sum_{i=k}^n b_i & = & (a_k + a_{k+1} + \ldots + a_n) + (b_k + b_{k+1} + \ldots + b_n) \\ & = & (a_k + b_k) + (a_{k+1} + b_{k+1}) + \ldots + (a_n + b_n) \\ & = & \sum_{i=k}^n (a_k + b_k). \end{array}$$

In a similar way, you can show that $\sum_{i=k}^n a_i - \sum_{i=k}^n b_i = \sum_{i=k}^n (a_i - b_i)$ is also true.

Further reading

Guide: Using the quadratic formula

Questions: Using the quadratic formula

Version history

v1.0: created in 04/24 by tdhc.