

# 2D Conic Sections Fact Sheet

Abigail Carpenter

## Summary

This fact sheet provides an overview of 2D conic sections, including general equations, discriminants, types of conics, standard forms, key features, and labeled graphs.

## General Quadratic Equation for Conics

A conic section can be represented algebraically by the **general quadratic equation**:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

## Discriminant and Conic Type

The discriminant is:

$$\Delta = B^2 - 4AC$$

Use this to classify the conic:

Discriminant	Conic Type	Notes
$\Delta < 0$	Ellipse	Circle if $A = C$ and $B = 0$
$\Delta = 0$	Parabola	
$\Delta > 0$	Hyperbola	

## Conic Sections

### Circle

Standard form:

$$(x - h)^2 + (y - k)^2 = r^2$$

**Key features:**

- Centre:  $(h, k)$
  - Radius:  $r$
- 

## Ellipse

**Standard form:**

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \quad (a > b)$$

**Key features:**

- Centre:  $(h, k)$
  - Semi-major axis:  $a$
  - Semi-minor axis:  $b$
  - Foci:  $(h \pm c, k)$  for horizontal ellipse,  $(h, k \pm c)$  for vertical, where  $c = \sqrt{a^2 - b^2}$
- 

## Parabola

**Standard forms:**

- Vertical:  $(x - h)^2 = 4p(y - k)$
- Horizontal:  $(y - k)^2 = 4p(x - h)$

**Key features:**

- Vertex:  $(h, k)$
  - Focus:  $(h, k + p)$  or  $(h + p, k)$
  - Directrix:  $y = k - p$  or  $x = h - p$
  - Axis of symmetry: passes through vertex and focus
-

# Hyperbola

## Standard forms:

- Horizontal:  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
- Vertical:  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

## Key features:

- Centre:  $(h, k)$
  - Vertices:  $(h \pm a, k)$  or  $(h, k \pm a)$
  - Foci:  $(h \pm c, k)$  or  $(h, k \pm c)$ ,  $c = \sqrt{a^2 + b^2}$
  - Asymptotes:  $y - k = \pm \frac{b}{a}(x - h)$  (horizontal),  $y - k = \pm \frac{a}{b}(x - h)$  (vertical)
- 

## Quick Reference Table

Conic	Standard Form	Key Features
Circle	$(x - h)^2 + (y - k)^2 = r^2$	Centre $(h, k)$ , radius $r$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	Centre, axes, foci
Parabola	$(x - h)^2 = 4p(y - k)$	Vertex, focus, directrix
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	Centre, vertices, foci, asymptotes

---

## References

- Interactive Desmos graphs for visualization
- Geometry textbooks on conic sections

## Version history

v1.0: initial version created 11/25 by Abigail Carpenter as part of a University of St Andrews VIP project.

[This work is licensed under CC BY-NC-SA 4.0.](#)