Answers: Multivariate chain rule

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Summary

Answers to questions relating to the guide on the multivariate chain rule.

These are the answers to Questions: Multivariate chain rule.

Please attempt the questions before reading these answers!

Answers

Q1

1.1. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = 2\mathrm{e}^{2t}\sin(t)\left(\cos(t)+\sin(t)\right)$

1.2. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{3}{t} - \tan(t)$

1.3. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{3}{2}t^{1/2} + 6t(t^2+1)^2$

1.4. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \exp(t\ln(t+1))\left(\ln(t+1)+\dfrac{t}{t+1}\right) = (t+1)^t\left(\ln(t+1)+\dfrac{t}{t+1}\right)$

1.5. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = -\sin(t)\tan(t^2) + 2t\cos(t)\sec^2(t^2)$

1.6. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = 8t - 4 + 30\sin(t) + 5\cos(t)\left(6t-3+75\sin^2(t)\right)$

1.7. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{2t}{t-2} - \dfrac{t^2+1}{(t-2)^2} = \dfrac{t^2-4t-1}{(t-2)^2}$

1.8. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = 0$

1.9. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = t^2\mathrm{e}^t\left(t^4+6t^3+\mathrm{e}^t(2t+3)\right)$

1.10. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{2}{t} + t\mathrm{e}^{-t}(2-t)$

1.11. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = 4t\left(2\ln(t)+1\right)$

1.12. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = (t^3+1)\left(6t^2\sin(3t)+3(t^3+1)\cos(3t)\right)$

1.13. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \exp\left(\cosh(t)\right)\sinh(t) + \exp\left(\sinh(t)\right)\cosh(t)$

1.14. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{1}{1+t^2}$

1.15. $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{\exp(\sqrt{t})}{t+2} + \dfrac{\ln(t+2)\exp(\sqrt{t})}{2\sqrt{t}}$

Q2

2.1. $\dfrac{\partial z}{\partial s} = 2(s+t)(2s^2+st-t^2)$ and $\dfrac{\partial z}{\partial t} = 2(s+t)(s^2-2t^2-st)$.

2.2. $\dfrac{\partial z}{\partial s} = 1$ and $\dfrac{\partial z}{\partial t} = \dfrac{\cos(t)-\sin(t)}{\cos(t)+\sin(t)}$.

2.3. $\dfrac{\partial z}{\partial s} = 3t(s^2t^2-2s-t)$ and $\dfrac{\partial z}{\partial t} = 3s(s^2t^2-s-2t)$.

2.4. $\dfrac{\partial z}{\partial s} = 2st\exp(s^2)$ and $\dfrac{\partial z}{\partial t} = \exp(s^2)$.

2.5. $\dfrac{\partial z}{\partial s} = \sin(st)+t(s-t^2)\cos(st)$ and $\dfrac{\partial z}{\partial t} = -2t\sin(st)+s(s-t^2)\cos(st)$.

2.6. $\dfrac{\partial z}{\partial s} = 2\sin(s)\cos(s)\cos(2t)$ and $\dfrac{\partial z}{\partial t} = 2\sin(t)\cos(t)\cos(2s)$.

2.7. $\dfrac{\partial z}{\partial s} = 2(2s+t)$ and $\dfrac{\partial z}{\partial t} = 2s$.

2.8. $\dfrac{\partial z}{\partial s} = \dfrac{1}{s+t} - \dfrac{1}{s}$ and $\dfrac{\partial z}{\partial t} = \dfrac{1}{s+t} - \dfrac{1}{t}$.

2.9. $\dfrac{\partial z}{\partial s} = (2s+1)\sec^2(s^2+s+t^2-t)$ and $\dfrac{\partial z}{\partial t} = (2t-1)\sec^2(s^2+s+t^2-t)$.

2.10. $\dfrac{\partial z}{\partial s} = -\dfrac{2t}{s^2+t^2}$ and $\dfrac{\partial z}{\partial t} = \dfrac{2s}{s^2+t^2}$.

Q3

3.1. $\dfrac{\partial w}{\partial s} = 2s(2+t^2)$ and $\dfrac{\partial w}{\partial t} = 2t(2+s^2)$.

3.2. $\dfrac{\partial w}{\partial s} = t(2s+t+u)$ and $\dfrac{\partial w}{\partial t} = s(s+2t+u)+1$ and $\dfrac{\partial w}{\partial u} = st+1$.

3.3. $\dfrac{\partial w}{\partial s} = 2st^2\cos(s^2t^2)-\sin(s+t)$ and $\dfrac{\partial w}{\partial t} = 2s^2t\cos(s^2t^2)-\sin(s+t)$.

3.4. $\dfrac{\partial w}{\partial s} = 4(s+u)$ and $\dfrac{\partial w}{\partial t} = 4t$ and $\dfrac{\partial w}{\partial u} = 4(s+u)$.

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