

# Answers: Multivariate implicit differentiation

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## Summary

Answers to questions relating to the guide on multivariate implicit differentiation.

*These are the answers to [Questions: Multivariate implicit differentiation](#).*

**Please attempt the questions before reading these answers!**

## Answers

### Q1

$$1.1. \quad \frac{dy}{dx} = -\frac{x}{y}.$$

$$1.2. \quad \frac{dy}{dx} = -\frac{3x^2y}{x^3 + 3y^2}.$$

$$1.3. \quad \frac{dy}{dx} = -\frac{2}{5}x(y-1)^2.$$

$$1.4. \quad \frac{dy}{dx} = -\frac{y \cos(xy) + 1}{x \cos(xy) - 1}.$$

$$1.5. \quad \frac{dy}{dx} = -\frac{e^y}{xe^y + 2y}.$$

$$1.6. \quad \frac{dy}{dx} = -\frac{2xy - 3y^2}{x^2 - 6xy}.$$

$$1.7. \quad \frac{dy}{dx} = -\frac{y}{x}.$$

$$1.8. \quad \frac{dy}{dx} = \frac{y}{x} + 2(x^2 + y^2).$$

$$1.9. \quad \frac{dy}{dx} = \frac{y \sin(xy) + 1}{3y^2 - x \sin(xy)}.$$

$$1.10. \quad \frac{dy}{dx} = \frac{y \sin(x) - \sin(y)}{x \cos(y) + \cos(x)}.$$

$$1.11. \quad \frac{dy}{dx} = -1.$$

$$1.12. \quad \frac{dy}{dx} = -\frac{ye^{xy} + 1}{xe^{xy} - 1}.$$

$$1.13. \quad \frac{dy}{dx} = -\frac{x^2 - y}{y^2 - x}.$$

$$1.14. \quad \frac{dy}{dx} = -\frac{1}{4\sqrt{xy}}.$$

$$1.15. \quad \frac{dy}{dx} = \frac{x^2 + y^2}{2x^2}.$$

## Q2

$$2.1. \quad \frac{\partial z}{\partial x} = \frac{5xy - 4x}{3y^2z + 3z^2} \text{ and } \frac{\partial z}{\partial y} = \frac{5x^2 - 6yz^2}{6y^2z + 6z^2}.$$

$$2.2. \quad \frac{\partial z}{\partial x} = -\frac{x}{z} \text{ and } \frac{\partial z}{\partial y} = -\frac{y}{z}.$$

$$2.3. \quad \frac{\partial z}{\partial x} = -\frac{z}{x} \text{ and } \frac{\partial z}{\partial y} = -\frac{z}{y}.$$

$$2.4. \quad \frac{\partial z}{\partial x} = -\frac{ze^{xz}}{xe^{xz} - 1} \text{ and } \frac{\partial z}{\partial y} = -\frac{1}{xe^{xz} - 1}.$$

$$2.5. \quad \frac{\partial z}{\partial x} = -\frac{z \cos(xz)}{x \cos(xz) - y \sin(yz)} \text{ and } \frac{\partial z}{\partial y} = \frac{z \sin(yz)}{x \cos(xz) - y \sin(yz)}.$$

$$2.6. \quad \frac{\partial z}{\partial x} = -\frac{z}{x} \text{ and } \frac{\partial z}{\partial y} = -\frac{z}{y}.$$

$$2.7. \quad \frac{\partial z}{\partial x} = \frac{x^2 - yz}{xy - z^2} \text{ and } \frac{\partial z}{\partial y} = \frac{y^2 - xz}{xy - z^2}.$$

$$2.8. \quad \frac{\partial z}{\partial x} = -\frac{4xz^{3/2}}{2z^{1/2}(x^2 + y^2) + 1} \text{ and } \frac{\partial z}{\partial y} = -\frac{4yz^{3/2}}{2z^{1/2}(x^2 + y^2) + 1}.$$

$$2.9. \quad \frac{\partial z}{\partial x} = -\frac{(z^2 + 1)e^x}{y^2(z^2 + 1) - 1} \text{ and } \frac{\partial z}{\partial y} = -\frac{2yz(z^2 + 1)}{y^2(z^2 + 1) - 1}.$$

$$2.10. \quad \frac{\partial z}{\partial x} = \frac{z(xy + 1)}{x(z - 1)} \text{ and } \frac{\partial z}{\partial y} = \frac{xz}{z - 1}.$$

$$2.11. \quad \frac{\partial z}{\partial x} = -\frac{yze^{xz} + e^{yz}}{xy(e^{xz} + e^{yz})} \text{ and } \frac{\partial z}{\partial y} = -\frac{xze^{yz} + e^{xz}}{xy(e^{xz} + e^{yz})}.$$

$$2.12. \quad \frac{\partial z}{\partial x} = \frac{\cos(x) \cos(z)}{\sin(x) \sin(z) - 2yz} \text{ and } \frac{\partial z}{\partial y} = \frac{z^2}{\sin(x) \sin(z) - 2yz}.$$

$$2.13. \quad \frac{\partial z}{\partial x} = -\frac{2x}{ye^z + 1} \text{ and } \frac{\partial z}{\partial y} = -\frac{e^z}{ye^z + 1}.$$

$$2.14. \quad \frac{\partial z}{\partial x} = \frac{z}{x + y - z} \text{ and } \frac{\partial z}{\partial y} = -\frac{z}{x + y - z}.$$

2.15.  $\frac{\partial z}{\partial x} = \frac{z(2x + \sqrt{xyz})}{x(2z - \sqrt{xyz})}$  and  $\frac{\partial z}{\partial y} = -\frac{z(2y - \sqrt{xyz})}{y(2z - \sqrt{xyz})}$ .

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v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

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