Answers: Introduction to complex numbers

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Summary

Answers to questions relating to the guide on introduction to complex numbers.

These are the answers to Questions: Introduction to complex numbers.

Please attempt the questions before reading these answers!

Q1

Using complex numbers, find solutions to the following equations.

1.1. Here, $x = i$ and $x = -i$ are the two solutions.

1.2. Here, $x = 3i$ and $x = -3i$ are the two solutions.

1.3. Here, $x = 12i$ and $x = -12i$ are the two solutions.

1.4. Here, $x = 1$ and $x = -1$ are the two solutions. (Real numbers are complex numbers too!)

Q2

For each of the complex numbers below, give their real and imaginary parts. (In this question, a, b are real numbers.)

2.1. The real part of z_1 is $\text{Re}(z_1) = 2$ and the imaginary part of z_1 is $\text{Im}(z_1) = 3$.

2.2. The real part of z_2 is $\text{Re}(z_2) = -23$ and the imaginary part of z_2 is $\text{Im}(z_2) = 32$.

2.3. The real part of z_3 is $\text{Re}(z_3) = 3$ and the imaginary part of z_3 is $\text{Im}(z_3) = -3$.

2.4. The real part of z_4 is $\text{Re}(z_4) = 0$ and the imaginary part of z_4 is $\text{Im}(z_4) = 3$.

2.5. The real part of z_5 is $\text{Re}(z_5) = -3$ and the imaginary part of z_5 is $\text{Im}(z_5) = -2$.

2.6. The real part of z_6 is $\text{Re}(z_6) = a$ and the imaginary part of z_6 is $\text{Im}(z_6) = 2b$.

2.7. The real part of z_7 is $\text{Re}(z_7) = 2$ and the imaginary part of z_7 is $\text{Im}(z_7) = 0$.

2.8. The real part of z_8 is $\text{Re}(z_8) = 3/2$ and the imaginary part of z_8 is $\text{Im}(z_8) = 2/3$.

2.9. The real part of z_9 is $\text{Re}(z_9) = 22$ and the imaginary part of z_9 is $\text{Im}(z_9) = -33$.

2.10. The real part of z_{10} is $\text{Re}(z_{10}) = 333$ and the imaginary part of z_{10} is $\text{Im}(z_{10}) = 22$.

2.11. The real part of z_{11} is $\text{Re}(z_{11}) = -2$ and the imaginary part of z_{11} is $\text{Im}(z_{11}) = 2$.

2.12. The real part of z_{12} is $\text{Re}(z_{12}) = -2$ and the imaginary part of z_{11} is $\text{Im}(z_{11}) = -3$.

Q3

The complex conjugate of $z_1 = 2 + 3i$ is $\bar{z}_1 = 2 - 3i$.

The complex conjugate of $z_2 = -23 + 32i$ is $\bar{z}_2 = -23 - 32i$.

The complex conjugate of $z_3 = 3 - 3i$ is $\bar{z}_3 = 3 + 3i$.

The complex conjugate of $z_4 = 3i$ is $\bar{z}_4 = -3i$.

The complex conjugate of $z_5 = -3 - 2i$ is $\bar{z}_5 = -3 + 2i$.

The complex conjugate of $z_6 = a + 2bi$ is $\bar{z}_6 = a - 2bi$.

The complex conjugate of $z_7 = 2$ is $\bar{z}_7 = 2$.

The complex conjugate of $z_8 = 3/2 + 2i/3$ is $\bar{z}_8 = 3/2 - 2i/3$.

The complex conjugate of $z_9 = 22 - 33i$ is $\bar{z}_9 = 22 + 33i$.

The complex conjugate of $z_{10} = 333 + 22i$ is $\bar{z}_{10} = 333 + 22i$.

The complex conjugate of $z_{11} = 2i - 2$ is $\bar{z}_{11} = -2i - 2$.

The complex conjugate of $z_{12} = -3i - 2$ is $\bar{z}_{12} = 3i - 2$.

Q4

See Figure 1 for the Argand diagram. You can notice that the complex conjugates of the complex numbers can be obtained by reflecting the point in the real axis.

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v1.0: initial version created 10/24 by tdhc.

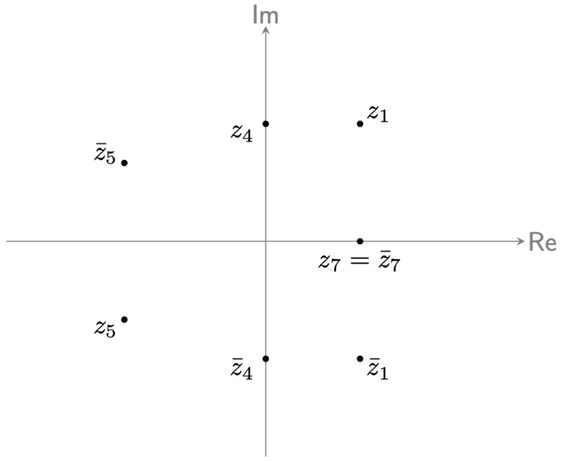

Figure 1: An Argand diagram with the seven complex numbers $z_1, \bar{z}_1, z_4, \bar{z}_4,$ in Example 5.