Answers: Trigonometric identities (degrees)

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Summary

A selection of questions on trigonometric identities, using degrees to measure angles.

These are the answers to Questions: Trigonometric identities (degrees).

Please attempt the questions before reading these answers!

Q1

1.1.
$$2(6\sin^2(\theta)) + 3(4\cos^2(\theta)) = 12(\sin^2(\theta) + \cos^2(\theta)) = 12$$

1.2.
$$10(7\sin^2(\theta)) + 14(5\cos^2(\theta)) = 70$$

1.3.
$$5\left(\frac{6}{\csc^2(\theta)}\right) + 15\left(\frac{2}{\sec^2(\theta)}\right) = 30$$

1.4.
$$(\cos^2(\theta) - \sin^2(\theta))^2 + 4\sin^2(\theta)\cos^2(\theta) = \cos^2(2\theta) + \sin^2(2\theta) = 1$$

1.5.
$$2\sin(30)\cos(15) + 2\cos(30)\sin(15) = 2\sin(30+15) = 2\sin(45) = \sqrt{2}$$

1.6.
$$3\cos(45)\cos(15) - 3\sin(45)\sin(15) = 3\cos(60) = \frac{3}{2}$$

1.7.
$$\sin(150) + \sin(30) = 2\sin\left(\frac{180}{2}\right)\cos\left(\frac{120}{2}\right) = 2\sin(90)\cos(60) = 1$$

1.8.
$$\cos(150) + \sin(30) = 2\cos(90)\cos(60) = 0$$

Q2

2.1.
$$\tan(\theta)\cos(-\theta) = \frac{\sin(\theta)}{\cos(\theta)} \cdot \cos(\theta) = \sin(\theta)$$

$$2.2 \quad \tan(-\theta)\csc(-\theta)\sec(-\theta) = \left(-\frac{\sin(\theta)}{\cos(\theta)}\right) \left(\frac{1}{-\sin(\theta)}\right) \left(\frac{1}{\cos(\theta)}\right) = \left(\frac{1}{\cos^2(\theta)}\right) = \sec^2(\theta)$$

2.3.
$$\tan^2(\theta) + \sin^2(\theta) + \cos^2(\theta) = \tan^2(\theta) + 1 = \sec^2(\theta)$$

2.4.
$$\frac{2\sin(\theta)}{\cos(\theta)(1-\tan^2(\theta))} = \tan(2\theta)$$

2.5.
$$\frac{\sin(7\theta) + \sin(3\theta)}{\cos(7\theta) - \cos(3\theta)} = \frac{2\sin(5\theta)\cos(2\theta)}{-2\sin(5\theta)\sin(2\theta)} = -\cot(\theta)$$

2.6.
$$\frac{\sin(5\theta) - \sin(\theta)}{\cos(5\theta) + \cos(\theta)} = \tan(2\theta)$$

Q3

3.1.
$$\cos(210) = \frac{\sqrt{3}}{2}$$

3.2. Here
$$\sin(135) = \frac{1}{\sqrt{2}}$$
, and $\sin(225) = -\frac{1}{\sqrt{2}}$.

3.3. cos(130) = -0.766 to three decimal places.

Q4

4.1.
$$\sin(15) = \sin(45)\cos(30) - \cos(45)\sin(30) = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

4.2.
$$\cos(15) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

4.3.
$$\tan(15) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

4.4.
$$\sin(75) = \sin(45)\cos(30) + \cos(45)\sin(30) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

4.5.
$$\cos(75) = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

4.6.
$$\tan(75) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Version history and licensing

v1.0: initial version created 08/23 by Dzhemma Ruseva as part of a University of St Andrews STEP project.

• v1.1: edited 05/24 by tdhc, and split into versions for both degrees and radians.

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