Answers: Expected value, variance, standard deviation

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Summary

Answers to questions relating to the guide on expected value, variance, and standard deviation.

These are the answers to Questions: PMFs, PDFs, and CDFs.

Please attempt the questions before reading these answers!

Q1

For each of the following valid random variables with associated probability mass function, work out the expected value and variance.

1.1.

Here, $\mathbb{E}(X) = 2.8$ and $\mathbb{V}(X) = 0.76$.

1.2.

Here, $\mathbb{E}(X) = 2.4$ and $\mathbb{V}(X) = 2.04$.

1.3.

This is a Bernoulli trial, with probability of success $p = 0.7$. From Example 2 of Guide: Expected value, variance, standard deviation, you can say that $\mathbb{E}(X) = p = 0.3$. The variance can be worked out to be

$$\mathbb{V}(X) = 0.3 - (0.3)^2 = 0.21 = (0.3)(0.7) = p(1-p).$$

1.4.

Here, $\mathbb{E}(X) = 3$ and $\mathbb{V}(X) = 1$.

Q2

For each of the following valid random variables with associated probability density function, work out the expected value andvariance.

2.1.

This is the continuous uniform distribution with $a = 0$ and $b = 2$. From Example 2 of Guide: Expected value, variance, standard deviation, you can say that $\mathbb{E}(X) = (a+b)/2 = (0+2)/2 = 1$ and

$$\mathbb{V}(X) = \frac{1}{12}(b-a)^2 = \frac{1}{12}(2-0)^2 = \frac{4}{12} = \frac{1}{3}.$$

2.2.

Here, $\mathbb{E}(X) = \frac{2}{3}$ and $\mathbb{V}(X) = \frac{1}{6}$.

Q3

You know that the expected value and variance of rolling one fair die are $\mathbb{E}(X) = \frac{7}{2}$ and $\mathbb{V}(X) = \frac{35}{12}$.

Because the roll of each die is an independent event, the random variable Y of rolling seven dice is the same as

$$Y = X + X + X + X + X + X + X$$

which is seven lots of X.

You can use the properties of expected values and variance to get

$$\begin{aligned}\mathbb{E}(Y) &= \mathbb{E}(X + X + X + X + X + X + X) \\ &= 7 \cdot \frac{7}{2} = \frac{49}{2} = 24.5\end{aligned}$$

and

$$\begin{aligned}\mathbb{V}(Y) &= \mathbb{V}(X + X + X + X + X + X + X) \\ &= 7 \cdot \frac{35}{12} = \frac{245}{12}\end{aligned}$$

Q4

When doing this question, you need to find the two integrals

$$\mathbb{E}(X) = \int_0^\infty \lambda x e^{-\lambda x}\,\mathrm{d}x \quad \text{and} \quad \mathbb{V}(X) = \int_0^\infty \lambda x^2 e^{-\lambda x}\,\mathrm{d}x.$$

You will need to use integration by parts; you should always differentiate the power of x to reduce the size of the power. Use the given result to evaluate the integral when $x \to \infty$; the antiderivative should always be 0 in this case.

Version history and licensing

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