Answers: Rearranging equations involving trigonometry and logarithms

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Summary

This is an answer set relating to the questions based on Guide, Introduction to rearranging equations involving trigonometry and logarithms.

These are the answers to Questions: Introduction to rearranging equations using trigonometry and logarithms

Please attempt the questions before reading these answers!

Q1

Solve the trigonometric equations in radians.

1.1 For $sin(x) = \frac{\sqrt{2}}{2}$, x is equal to $\frac{\pi}{2}$ or 1.57.

1.2 For $cos(2x+1) = \frac{1}{2}$, x is equal to $\frac{\pi-3}{6}$ or 0.0234.

1.3 For $tan(5x-1) = \frac{\sqrt{2}}{2}$, x is equal to 0.323.

1.4 For $cos(x^2+4x+3) = 1$, x is equal to -1 or -3. To do this, you use that $cos^{-1}(1) = 0$ and so you need to solve the quadratic equation $x^2+4x+3 = 0$.

Q2

Rewrite $\cot$ and $\csc$ in terms of $\sin$, $\cos$, and $\tan$

$1 + \frac{1}{\tan^2(x)} = \frac{1}{\sin^2(x)}$

$1 + \frac{\cos^2(x)}{\sin^2(x)} = \frac{1}{\sin^2(x)}$

Then, multiply both sides of the equation by $\sin^2(x)$

$\sin^2(x) + \cos^2(x) = 1.$

Q3

Rewriting $5\cos(x) + 9\sin(x)$ gives $\sqrt{106}\sin(x+0.507)$. Setting this equal to 10 and solving gives $x = 0.823$. If you have a slightly different answer, this may be due to rounding at different points in the process.

Q4

4.1 $a = 6$, $b = 36$, $c = 2$.

4.2 $a = 3$, $b = 2187$, $c = 2187$.

4.3 $a = e$, $b = y$, $c = x$.

4.4 $a = 2$, $b = 9$, $c = 3.17....$

4.5 $a = 2$, $b = 4$, $c = 2$.

Q5

5.1 The solution to $6\log_3(x) + \log_3(5) = 9$ is $x = \sqrt[6]{\frac{3^9}{5}}$, or approximately 3.97.

5.2 The solution to $\log_2(16x) = 6$ is $x = 4$.

5.3 If $e^{\ln(3x)} = y$, then $y = 3x$.

Q6

Firstly, substitute y into the first equation. This gives $2^{\log_2(x)} = 4x - 7$. Via example 7, you can see that this means $x = 4x - 7$. Rearranging this gives $x = \frac{7}{3}$ or approximately 2.33. Plugging this into the second equation gives $y = \log_2(\frac{7}{3})$ or approximately 1.22.

Q7

7.1 If $e^{-x} + 3e^x = 12$, then multiply everything by e^x and define y such that $e^x = y$. This makes $1 + 3y^2 = 12y$ and solving this gives $y = \frac{6 \pm \sqrt{33}}{3}$. Then, $\ln(y) = x = 1.36$ or -2.46.

7.2 Using the same method detailed above $y = \frac{9 \pm \sqrt{65}}{8}$ and $x = 0.757$ or -2.144.

Q8

8.1 If $\log_{16}(x) = \log_2(y)$, then $y = x^{\frac{1}{4}}$.

8.2 If $\log_3(x) = \log_{27}(y)$, then $y = x^3$.

8.3 If $\log_9(x) + \log_3(2x) = 6$, then $\log_9(x) = \log_3(x^{\frac{1}{2}})$. Substituting gives $\log_3(2x^{\frac{3}{2}}) = 6$, thus $3^6 = 2x^{\frac{3}{2}}$. This means that $x = 51.0$.