Factsheet: Chi-squared distribution

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Summary

A factsheet for the χ^2 distribution.

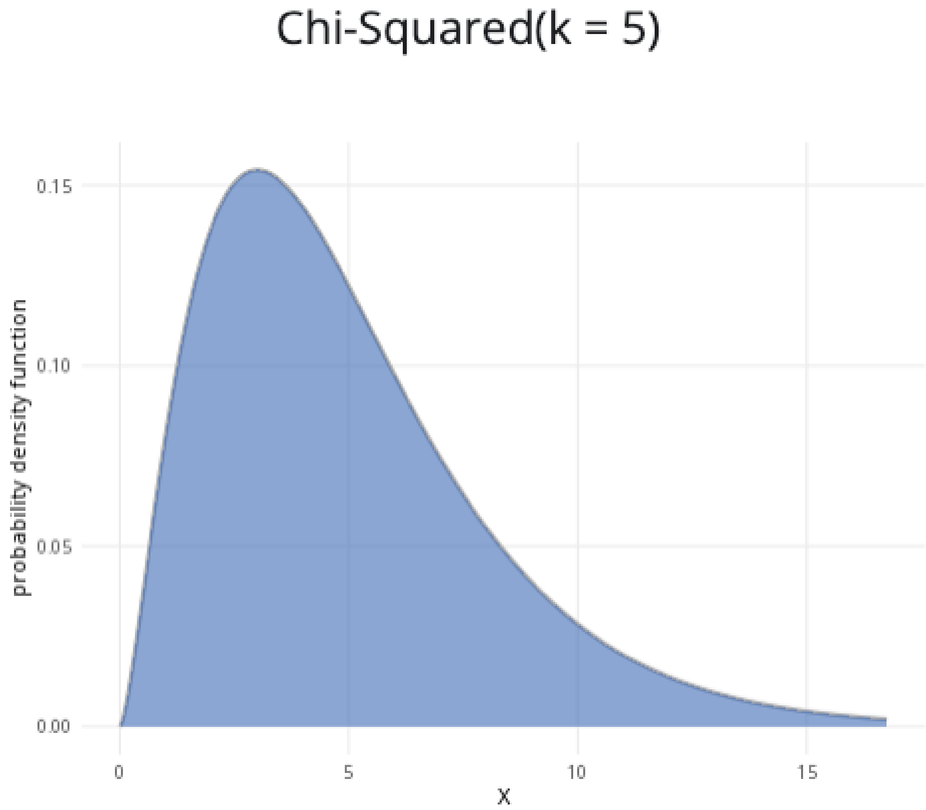

Figure 1: An example of the chi-squared distribution with $k = 5$.

Where to use: The χ^2 distribution is used for hypothesis testing, such as for goodness of fit tests and tests for independence. (See Guide: Introduction to hypothesis testing for more.) It is a special case of the gamma distribution, as $\chi^2(k) = \mathrm{Gam}(\frac{k}{2}, 2)$.

Notation: $X \sim \chi^2(k)$

Parameter: The integer k is the number of degrees of freedom in the sample.

Quantity	Value	Notes
Mean	$\mathbb{E}(X) = k$	

Quantity	Value	Notes
Variance	$\mathbb{V}(X) = 2k$	
PDF	$\mathbb{P}(X = x) = \dfrac{x^{\frac{k}{2}-1}\exp\left(-\frac{x}{2}\right)}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)}$	$\Gamma(x)$ is the gamma function
CDF	$\mathbb{P}(X \leq x) = \dfrac{1}{\Gamma\left(\frac{k}{2}\right)}\mathrm{Gam}\left(\frac{k}{2}, \frac{x}{2}\right)$	$\Gamma(x)$ is the gamma function, $\mathrm{Gam}(\alpha, \theta)$ is the PDF of the gamma distribution

Examples:

- **Goodness of fit example:** You have a six-sided die with six possible outcomes: $1, 2, 3, 4, 5$, and 6. You calculate the expected frequencies of each outcome. Then you roll the die many times and record the observed frequencies of each outcome. Since there are 6 categories,

  $$\text{degrees of freedom} = \text{number of categories} - 1 = 6 - 1 = 5$$

  This can be expressed as $X \sim \chi^2(5)$, meaning the degrees of freedom is 5.

- **Test for independence example:** You are investigating whether there is a correlation between two variables: candy colour and flavour. You have 5 categories of colours and 3 categories of flavours. Calculating the degrees of freedom can be done with the formula:

  $$(\text{categories of colours} - 1)(\text{categories of flavours} - 1) = (5-1)(3-1) = (4)(2) = 8.$$

  You can model $X \sim \chi^2(8)$, meaning that there are 8 degrees of freedom.

Further reading

This interactive element appears in Overview: Probability distributions. Please click this link to go to the guide.

Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

- v1.1: moved to factsheet form and populated with material from Overview: Probability distributions by tdhc.