Answers: Completing the square

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Summary

Answers to questions relating to the guide on completing the square.

These are the answers to Questions: Completing the square.

Please attempt the questions before reading these answers!

Q1

1.1. Here, $x^2 - 2x + 15 = (x-1)^2 + 14$, so in this question $p = -1$ and $q = 14$.

1.2. Here, $y^2 - 6y + 8 = (y-3)^2 - 1$, so in this question $p = -3$ and $q = -1$.

1.3. Here, $x^2 + 8x + 20 = (x+4)^2 + 4$, so in this question $p = 4$ and $q = 4$.

1.4. Here, $m^2 - 26m + 25 = (m-13)^2 - 144$, so in this question $p = -13$ and $q = -144$.

1.5. Here, $n^2 + 6n + 50 = (m+3)^2 + 41$, so in this question $p = 3$ and $q = 41$.

1.6. Here, $x^2 + 2x + 144 = (x+1)^2 + 143$, so in this question $p = 1$ and $q = 143$.

1.7. Here, $h^2 - 3h - 3 = \left(h - \frac{3}{2}\right)^2 + \frac{3}{4}$, so in this question $p = -3/2$ and $q = 3/4$.

1.8. Here, $x^2 + x - 3 = \left(x + \frac{1}{2}\right)^2 - \frac{13}{4}$, so in this question $p = 1/2$ and $q = -13/4$.

1.9. Here, $x^2 - 13x + 43 = \left(x - \frac{13}{2}\right)^2 + \frac{3}{4}$, so in this question $p = -13/2$ and $q = 3/4$.

1.10. Here, $y^2 - 8y + 16 = (y-4)^2$, so in this question $p = -4$ and $q = 0$.

1.11. Here, $x^2 + 13x + 9 = \left(x + \frac{13}{2}\right)^2 - \frac{133}{4}$, so in this question $p = 13/2$ and $q = -133/4$.

1.12. Here, $m^2 + 3m + 33 = \left(m + \frac{3}{2}\right)^2 - \frac{143}{4}$, so in this question $p = 3/2$ and $q = -143/4$.

Q2

2.1. Here, $2x^2 - 12x + 14 = 2(x-3)^2 - 4$, so in this question $a = 2$, $p = -3$ and $q = -4$.

2.2. Here, $5y^2 - 10y + 4 = 5(x-1)^2 - 1$, so in this question $a = 5$, $p = -1$ and $q = -1$.

2.3. Here, $4x^2 + 32x + 68 = 4(x + 4)^2 + 4$, so in this question $a = p = q = 4$. (Or, if you prefer, $(2x + 8)^2 + 4$.)

2.4. Here, $2m^2 + 2m + 2 = 2\left(m + \frac{1}{2}\right)^2 + \frac{3}{2}$, so in this question $a = 2$, $p = 1/2$ and $q = 3/2$.

2.5. Here, $3x^2 - 2x + 5 = 3\left(x - \frac{1}{3}\right)^2 + \frac{14}{3}$, so in this question $a = 3$, $p = -1/3$ and $q = 14/3$.

2.6. Here, $4x^2 - 4x + 1 = 4\left(x - \frac{1}{2}\right)^2$, so in this question $a = 4$, $p = -1/2$ and $q = 0$. (Or, if you prefer, $(2x - 1)^2$.)

2.7. Here, $2h^2 - 3h + 1 = 2\left(h - \frac{3}{4}\right)^2 - \frac{1}{8}$, so in this question $a = 2$, $p = -3/4$ and $q = -1/8$.

2.8. Here, $3x^2 + 5x + 2 = 3\left(x + \frac{5}{6}\right)^2 - \frac{3}{36}$, so in this question $a = 3$, $p = 5/6$ and $q = -3/36$.

Q3

Using your working from Q1 and Q2, solve the following quadratic equations.

3.1. You worked out in 1.2 that $y^2 - 6y + 8 = (y - 3)^2 - 1$. Rearranging $(y - 3)^2 - 1 = 0$ for y gives $y = 3 \pm 1$, so $y = 2$ or $y = 4$.

3.2. You worked out in 1.4 that $m^2 - 26m + 25 = (m - 13)^2 - 144$. Rearranging $(y - 3)^2 - 144 = 0$ for y gives $y = 13 \pm 12$, so $y = 1$ or $y = 25$.

3.3. You worked out in 1.3 that $x^2+8x+20 = (x+4)^2+4$. Using the fact that $(\pm 2i)^2 = -4$ (see [Guide: Introduction to complex numbers]), rearranging $(x + 4)^2 + 4 = 0$ for y gives $y = -4 \pm 2i$, so $y = -4 - 2i$ or $y = -4 + 2i$.

3.4. You worked out in 2.6 that $4x^2 - 4x + 1 = 4\left(x - \frac{1}{2}\right)^2$. Rearranging $4\left(x - \frac{1}{2}\right)^2 = 0$ for x gives $x = \frac{1}{2}$ (twice, see Guide: Introduction to quadratic equations).

3.5. You worked out in 2.3 that $4x^2 + 32x + 68 = 4(x + 4)^2 + 4$. Using the fact that $(\pm i)^2 = -1$ (see [Guide: Introduction to complex numbers]), rearranging $4(x + 4)^2 + 4 = 0$ for x gives $x = -4 \pm i$, so $x = -4 - i$ or $x = -4 + i$.

3.6. You worked out in 2.8 that $3x^2+5x+2 = 3\left(x + \frac{5}{6}\right)^2 - \frac{3}{36}$. Rearranging $3\left(x + \frac{5}{6}\right)^2 - \frac{3}{36} = 0$ for x gives $y = -\frac{5}{6} \pm \frac{1}{6}$, so $y = -1$ or $y = -2/3$.

Version history and licensing

v1.0: initial version created 09/24 by tdhc.