Factsheet: Rules of calculus

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Summary

A list of common rules in calculus.

Please note: clickable links lead to study guides where the rule is introduced.

Rules of differentiation

Limit definition of the derivative: If $f(x)$ is a continuous function, then (if it exists) the derivative $f'(x)$ is defined by

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sum/difference and constant rule: If $f(x)$ and $g(x)$ are differentiable functions, then

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x) \pm g(x)) = f'(x) \pm g'(x) \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}x}(cf(x)) = c\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) = cf'(x)$$

Product rule: If $f(x) = u(x)v(x)$,

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(u(x)v(x)\right) = u(x)v'(x) + u'(x)v(x)$$

Quotient rule: If $f(x) = u(x)/v(x)$ and $v(x) \neq 0$, then

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u(x)}{v(x)}\right) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$$

Chain rule: If $f(x) = f(u(x))$, then

$$f'(x) = \frac{\mathrm{d}f}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x} = f'(u(x)) \cdot u'(x)$$

where $f'(u(x))$ is the derivative of $f(u)$ with respect to u.

Implicit differentiation: If $f(x, y) = 0$ defines a function $g(y)$ implicitly, then

$$\frac{\mathrm{d}}{\mathrm{d}x}(g(y)) = \frac{\mathrm{d}g}{\mathrm{d}y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = g'(y) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}$$

where $g'(y)$ is the derivative of $g(y)$ with respect to y.

Rules of integration

Sum/difference and constant rules: If f, g are functions and k is any number:

$$\int f(x) \pm g(x)\,\mathrm{d}x = \int f(x)\,\mathrm{d}x \pm \int g(x)\,\mathrm{d}x \quad \text{and} \quad \int kf(x)\,\mathrm{d}x = k\int f(x)\,\mathrm{d}x$$

Limit manipulation: If f is a function and a, b are real numbers, then:

- for c such that $a < c < b$, then:

$$\int_a^b f(x)\,\mathrm{d}x = \int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x$$

- if $a \leq b$, then:

$$\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x$$

Integration by substitution: For an indefinite integral,

$$\int f(u(x)) \cdot u'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$$

and for a definite integral

$$\int_a^b f(u(x)) \cdot u'(x)\,\mathrm{d}x = \int_{u(a)}^{u(b)} f(u)\,\mathrm{d}u$$

Integration by parts: For functions u, v of x:

$$\int uv' \mathrm{d}x = uv - \int vu' \, \mathrm{d}x$$

Integration of derivative over function: For a function f,

$$\int \frac{f'(x)}{f(x)} \mathrm{d}x = \ln |f(x)| + C.$$

Version history

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