Answers: Conditional probability

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Summary

Answers to questions relating to the guide on conditional probability.

These are the answers to Questions: Conditional probability.

Please attempt the questions before reading these answers.

Q1

1.1.

- $\mathbb{P}(A) = \dfrac{13}{52}$ (hearts)
- $\mathbb{P}(B) = \dfrac{26}{52}$ (red cards)
- $\mathbb{P}(A \cap B) = \dfrac{13}{52}$ (red hearts)

Using the definition of conditional probability:

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{13/52}{26/52} = \frac{1}{2}$$

So the probability that the card is a heart, given that it is red, is $1/2$.

1.2.

You are given:

- $\mathbb{P}(\text{Piano} \mid \text{Left-handed}) = 0.25$

So the probability that a randomly chosen student plays the piano, given that they are left-handed, is 0.25.

1.3.

- $\mathbb{P}(A \cap B) = 0.15$
- $\mathbb{P}(B) = 0.30$

Using the definition of conditional probability:

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{0.15}{0.30} = 0.5$$

So the probability that an employee takes Spanish, given that they take French, is 0.5.

1.4.

- $\mathbb{P}(A \cap B) = 0.25$
- $\mathbb{P}(B) = 0.40$

Using the definition of conditional probability:

$$\mathbb{P}(A \mid B) = \frac{0.25}{0.40} = \frac{5}{8} = 0.625$$

So the probability that the student is sixteen, given they bring a packed lunch, is 0.625.

Q2

2.1.

- $\mathbb{P}(\text{first green}) = \frac{3}{5}$
- $\mathbb{P}(\text{second green} \mid \text{first green}) = \frac{2}{4}$

Using the multiplication rule:

$$\mathbb{P}(\text{both green}) = \left(\frac{3}{5}\right)\left(\frac{2}{4}\right) = \frac{6}{20} = 0.3$$

So the probability that both sweets are green is 0.3.

2.2.

- $\mathbb{P}(\text{first pass}) = 0.9$
- $\mathbb{P}(\text{second pass} \mid \text{first pass}) = 0.95$

Using the multiplication rule:

$$\mathbb{P}(\text{both pass}) = (0.9)(0.95) = 0.855$$

So the probability that a box of Bayes Biscuits passes both inspections is 0.855.

2.3.

These are independent events, so

- $\mathbb{P}(\text{heads}) = 0.5$
- $\mathbb{P}(\text{roll a 6}) = \frac{1}{6}$

Using the multiplication rule:

$$\mathbb{P}(\text{heads and 6}) = (0.5)\left(\frac{1}{6}\right) = \frac{1}{12}$$

So the probability of getting heads and rolling a 6 is $\frac{1}{12}$.

2.4.

- $\mathbb{P}(\text{likes tea}) = 0.7$
- $\mathbb{P}(\text{likes coffee} \mid \text{likes tea}) = 0.6$

Using the multiplication rule:

$$\mathbb{P}(\text{likes both}) = (0.7)(0.6) = 0.42$$

So the probability that a random person likes both tea and coffee is 0.42.

Q3

3.1.

Given: $\mathbb{P}(A) = 0.4$, $\mathbb{P}(B) = 0.5$, $\mathbb{P}(A \cap B) = 0.2$

Check:

$$\mathbb{P}(A)\mathbb{P}(B) = (0.4)(0.5) = 0.2$$

Since $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$, the events are **independent**.

3.2.

Given: $\mathbb{P}(A) = 0.3$, $\mathbb{P}(A \mid B) = 0.3$

Since $\mathbb{P}(A \mid B) = \mathbb{P}(A)$, events are **independent**.

3.3.

Given: $\mathbb{P}(A) = 0.5$, $\mathbb{P}(B) = 0.4$, $\mathbb{P}(A \cap B) = 0.1$

Check:

$$\mathbb{P}(A)\mathbb{P}(B) = (0.5)(0.4) = 0.2 \neq 0.1 = \mathbb{P}(A \cap B)$$

Since $\mathbb{P}(A \cap B) \neq \mathbb{P}(A)\mathbb{P}(B)$, the events are **dependent**.

3.4.

Given: $\mathbb{P}(A) = 0.6$, $\mathbb{P}(A \mid B) = 0.2$

Since $\mathbb{P}(A \mid B) \neq \mathbb{P}(A)$, the events are **dependent**.

Version history and licensing

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