Questions: Introduction to partial differentiation

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Summary

A selection of questions for the study guide on the introduction to partial differentiation.

Before attempting these questions, it is highly recommended that you read Guide: Introduction to partial differentiation.

Q1

Find all possible first-order partial derivatives for each function f.

1.1. $f(x, y) = x^2y + y^3$

1.2. $f(x, y) = 3x^3 - 2y^4 + xy$

1.3. $f(x, y) = y\sin(2x) + 3$

1.4. $f(x, y) = e^{xy} + 2x^2y^3$

1.5. $f(x, y) = \ln(x) + x\ln(y) + 3x$

1.6. $f(x, y) = \frac{y}{x} - \frac{x}{y}$

1.7. $f(x, y) = x\exp(y^2)$

1.8. $f(x, y) = \sqrt{x^2 + y^2}$

1.9. $f(x, y) = (3x + 2y)^4$

1.10. $f(x, y) = y\sin(xy)$

1.11. $f(x, y) = \sin(x^2 + y^2)$

1.12. $f(x, y) = \ln(1 + x^2y^2)$

1.13. $f(x, y, z) = x^2y\sin(z)$

1.14. $f(x, y, z) = (x + y)(y + z)(z + x)$

1.15. $f(x, y, z) = \frac{xyz}{x + y + z}$

Q2

A function $f(x, y)$ is called harmonic if it satisfies the equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

Show that each of these functions is harmonic by calculating the pure second-order partial derivatives and checking that their sum is zero.

2.1. $f(x, y) = x^2 - y^2$

2.2. $f(x, y) = xy$

2.3. $f(x, y) = x^3 - 3xy^2$

2.4. $f(x, y) = \cos(x)\sinh(y)$

2.5. $f(x, y) = e^x \sin(y)$

2.6. $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$

2.7. $f(x, y) = \ln(x^2 + y^2)$

Q3

For each function $f(x, y)$, calculate the mixed second-order partial derivatives and confirm that they satisfy the equation

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

3.1. $f(x, y) = x^2 y + xy^2$

3.2. $f(x, y) = 2x^2 \cos(y)$

3.3. $f(x, y) = (x + y)^5$

3.4. $f(x, y) = \frac{x}{1 + y}$

3.5. $f(x, y) = \sqrt{x^2 + y^2}$

3.6. $f(x, y) = x^2 \sin(y) + y^2 \cos(x)$

3.7. $f(x, y) = \tan^{-1}(xy)$

After attempting the questions above, please click this link to find the answers.

Version history and licensing

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