Questions: Expected value, variance, standard deviation

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Summary

A selection of questions to test your understanding of expected values, variance, and standard deviation.

Before attempting these questions it is highly recommended that you read Guide: Expected value, variance, standard deviation.

Q1

For each of the following valid random variables with associated probability mass function, work out the expected value and variance.

1.1.

Let X be the random variable representing the result of rolling a biased four sided-die. The PMF of X is given by:

x	1	2	3	4
$P(X = x)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$

1.2.

A discrete random variable X has five possible outcomes $(1, 2, 3, 4,$ or $5)$, and the PMF is given by:

x	1	2	3	4	5
$P(X = x)$	0.25	0.35	0.05	0.2	0.1

1.3.

A coin is tossed, where the probability of tails is 70 and heads is 30. Let X represent the result of the coin toss. Complete the table below:

x	Heads	Tails
$P(X = x)$	0.7	0.3

1.4.

The PMF for a random variable X is given as:

x	1	2	3	4
$P(X = x)$	$1/10$	$2/10$	$3/10$	$4/10$

Q2

For each of the following valid random variables with associated probability density function, work out the expected value and variance.

2.1.

Let X be a continuous random variable on the interval $[0, 2]$ with the PDF:

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

2.2.

Let X be a continuous random variable with the PDF:

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Q3

Give the expected value and variance for rolling seven fair 6-sided dice. You may assume that each roll is independent of every other roll.

Q4

This question refers to the exponential distribution for a continuous random variable. You can find more information about this and Factsheet: Exponential distribution.

The PDF of the exponential distribution is $\mathbb{P}(X = x) = \lambda e^{-\lambda x}$. Using integration by parts (see [Guide: Integration by parts]) and the fact that

$$\lim_{x \to \infty} x^n e^{-\lambda x} = 0$$

for any natural number n and real $\lambda > 0$, show that

(a) the mean μ of the exponential distribution is $\dfrac{1}{\lambda}$;

(b) the variance σ^2 of the exponential distribution is $\dfrac{1}{\lambda^2}$.

After attempting the questions above, please click this link to find the answers.

Version history and licensing

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