Questions: Multivariate implicit differentiation

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Summary

A selection of questions for the study guide on multivariate implicit differentiation.

Before attempting these questions, it is highly recommended that you read Guide: Multivariate implicit differentiation.

Q1

Let $z = f(x, y) = 0$ and define y as an implicit function of x.

For each function, use the multivariate implicit differentiation rule to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

1.1. $x^2 + y^2 - 25 = 0$

1.2. $x^3 y + y^3 - 7 = 0$

1.3. $x^2 - \frac{3y+2}{y-1} = 0$

1.4. $\sin(xy) + x = y$

1.5. $xe^y + y^2 = 4$

1.6. $x^2 y - 3xy^2 + 5 = 0$

1.7. $\ln(x) + \ln(y) = 1$

1.8. $\tan^{-1}\left(\frac{y}{x}\right) - x^2 = 0$

1.9. $y^3 + \cos(xy) = x$

1.10. $x \sin(y) + y \cos(x) = 0$

1.11. $x^2 + 2xy + y^2 - 1 = 0$

1.12. $e^{xy} + x - y = 0$

1.13. $x^3 + y^3 - 3xy - 7 = 0$

1.14. $\sqrt{x} + y^2 - 3 = 0$

1.15. $\frac{x+y}{x-y} - \ln(x) = 0$

Q2

Let $w = f(x, y, z) = 0$ and define z as an implicit function of x and y.

For each function, use the multivariate implicit differentiation rule to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

2.1. $4x^2 + 3y^2z^2 - 5x^2y + 2z^3 - 7 = 0$

2.2. $x^2 + y^2 + z^2 = 1$

2.3. $xyz = 1$

2.4. $e^{xz} + y - z = 0$

2.5. $\sin(xz) + \cos(yz) - 2 = 0$

2.6. $\ln(x) + \ln(y) + \ln(z) - 1 = 0$

2.7. $x^3 + y^3 + z^3 - 3xyz = 0$

2.8. $x^2z + y^2z + \sqrt{z} - 4 = 0$

2.9. $e^x + y^2z - \tan^{-1}(z) = 0$

2.10. $\ln(xz) + xy - z = 0$

2.11. $xe^{yz} + ye^{xz} - 5 = 0$

2.12. $\sin(x)\cos(z) + yz^2 - 1 = 0$

2.13. $x^2 + ye^z + z = 0$

2.14. $\dfrac{x+y}{z} + \ln(z) - 3 = 0$

2.15. $\sqrt{xyz} + x - y - z = 0$

After attempting the questions above, please click this link to find the answers.

Version history and licensing

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.