

# Questions: Multivariate chain rule

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## Summary

A selection of questions for the study guide on the multivariate chain rule.

*Before attempting these questions, it is highly recommended that you read [Guide: Multivariate chain rule](#).*

## Q1

Let  $z = f(x, y)$  be a function where both  $x$  and  $y$  depend on an independent variable  $t$ .

For each of the following, use the multivariate chain rule to find  $\frac{dz}{dt}$ , expressing your answer in terms of  $t$  only.

- 1.1.  $z = x^2y$  where  $x = \sin(t)$  and  $y = e^{2t}$ .
- 1.2.  $z = \ln(xy)$  where  $x = t^3$  and  $y = \cos(t)$ .
- 1.3.  $z = x^3 + y^3$  where  $x = \sqrt{t}$  and  $y = t^2 + 1$ .
- 1.4.  $z = e^{xy}$  where  $x = t$  and  $y = \ln(t + 1)$ .
- 1.5.  $z = x \tan(y)$  where  $x = \cos(t)$  and  $y = t^2$ .
- 1.6.  $z = x^2 + 3xy + y^3$  where  $x = 2t - 1$  and  $y = 5 \sin(t)$ .
- 1.7.  $z = \frac{x}{y}$  where  $x = t^2 + 1$  and  $y = t - 2$ .
- 1.8.  $z = \sqrt{x^2 + y^2}$  where  $x = \cos(t)$  and  $y = \sin(t)$ .
- 1.9.  $z = xy^2 + yx^2$  where  $x = e^t$  and  $y = t^3$ .
- 1.10.  $z = \ln(x) + xy$  where  $x = t^2$  and  $y = e^{-t}$ .
- 1.11.  $z = x^2y$  where  $x = 2t$  and  $y = \ln(t)$ .
- 1.12.  $z = x^2 \sin(y)$  where  $x = t^3 + 1$  and  $y = 3t$ .
- 1.13.  $z = e^x + e^y$  where  $x = \cosh(t)$  and  $y = \sinh(t)$ .
- 1.14.  $z = \tan^{-1}\left(\frac{y}{x}\right)$  where  $x = t$  and  $y = t^2$ .
- 1.15.  $z = xe^y$  where  $x = \ln(t + 2)$  and  $y = \sqrt{t}$ .

## Q2

Let  $z = f(x, y)$  be a function where both  $x$  and  $y$  depend on two independent variables  $s$  and  $t$ .

For each of the following, use the multivariate chain rule to find  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$ , expressing your answers in terms of  $s$  and  $t$  only.

2.1.  $z = x^2y$  where  $x = s + t$  and  $y = s^2 - t^2$ .

2.2.  $z = \ln(x + y)$  where  $x = e^s \cos(t)$  and  $y = e^s \sin(t)$ .

2.3.  $z = x^3 - 3xy$  where  $x = st$  and  $y = s + t$ .

2.4.  $z = e^{x+y}$  where  $x = s^2$  and  $y = \ln(t)$ .

2.5.  $z = x \sin(y)$  where  $x = s - t^2$  and  $y = st$ .

2.6.  $z = x^2 + y^2$  where  $x = \cos(s) \sin(t)$  and  $y = \sin(s) \cos(t)$ .

2.7.  $z = xy + x^2$  where  $x = s + t$  and  $y = s - t$ .

2.8.  $z = \ln(x) - \ln(y)$  where  $x = s + t$  and  $y = st$ .

2.9.  $z = \tan(x + y)$  where  $x = s^2 - t$  and  $y = s + t^2$ .

2.10.  $z = \tan^{-1}\left(\frac{y}{x}\right)$  where  $x = s^2 - t^2$  and  $y = 2st$ .

## Q3

Let  $w = f(x_1, \dots, x_n)$  be a function that depends on variables  $x_1, \dots, x_n$ , where each  $x_i$  is itself a function of  $t_1, \dots, t_m$ .

For each of the following, use a tree diagram to write the multivariate chain rule, and find the resulting partial derivatives.

3.1.  $w = x^2 + y^2 + z^2$  where 
$$\begin{cases} x = s + t \\ y = s - t \\ z = st \end{cases}$$

3.2.  $w = xy + z$  where 
$$\begin{cases} x = s + t + u \\ y = st \\ z = u + t \end{cases}$$

3.3.  $w = \sin(xy) + \cos(z)$  where 
$$\begin{cases} x = s^2 \\ y = t^2 \\ z = s + t \end{cases}$$

3.4.  $w = x^2 + y^2$  where  $\begin{cases} x = s + t + u \\ y = s - t + u \end{cases}$

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[After attempting the questions above, please click this link to find the answers.](#)

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