Answers: Logarithms

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Summary

Answers to questions relating to the study guide on logarithms.

These are answers to: Questions: Logarithms.

Please attempt the questions before reading these answers!

Throughout this answer sheet, the natural logarithm $\log_e(x)$ is written as $\ln(x)$.

Q1

1.1. $\log_7(x) = 1$ rearranged gives $7^1 = x$ so $x = 7$.

1.2. $\log_8(x) = 3$ rearranged gives $8^3 = x$ so $x = 512$.

1.3. $\log_{12}(x) = 0$ rearranged gives $12^0 = x$ so $x = 1$.

1.4. $\log_{10}(100) = x$ rearranged gives $10^x = 100$ so $x = 2$.

1.5. $\log_2(64) = x$ rearranged gives $2^x = 64$ so $x = 6$.

1.6. $\log_4(2) = x$ rearranged gives $4^x = 2$ so $x = \frac{1}{2}$.

1.7. $\log_3(27) = x$ rearranged gives $3^x = 27$ so $x = 3$.

1.8. $\log_{10}(1) = x$ rearranged gives $10^x = 1$ so $x = 0$.

1.9. $\log_x(16) = 4$ rearranged gives $x^4 = 16$ so $x = \sqrt[4]{16} = 2$.

1.10. $\log_x(49) = 2$ rearranged gives $x^2 = 49$ so $x = \sqrt{49} = 7$.

1.11. $\log_x(13) = 4$ rearranged gives $x^4 = 13$ so $x = \sqrt[4]{13}$.

1.12. $\log_{2x}(12) = -1$ rearranged gives $(2x)^{-1} = 12$ so $x = \frac{1}{24}$.

Q2

The product rule: $\log_a(M \cdot N) = \log_a(M) + \log_a(N)$

The quotient rule: $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$

The power rule: $\log_a(M^k) = k \cdot \log_a(M)$

The zero rule: $\log_a(1) = 0$

The identity rule: $\log_a(a) = 1$

2.1. The solution to $\log_3(\frac{1}{27}) = x$ is $x = -1/3$.

2.2. The solution to $4\log_4(2) = x$ is $x = 2$.

2.3. The solution to $\log_5(10) + \log_5\left(\frac{5}{2}\right) = x$ is $x = 2$.

2.4. The solution to $3\log_7\left(a^{1/3}\right) - \frac{1}{2}\log_7(a^2) = x$ is $x = 0$.

2.5. The solution to $\log_x(YZ) = M$ is $x = \sqrt[M]{YZ}$.

2.6. The solution to $\log_a(y) - \log_a(x) = 11$ is $x = ya^{-11}$.

Q3

3.1. $\log_3(25)$ is equal to $\dfrac{2}{\log_5(3)}$.

3.2. $\log_8(3)$ is equal to $\dfrac{4\log_{16}(3)}{3}$.

3.3. $\log_e(10)$ is equal to $\dfrac{1}{\log_{1000}(e^3)}$.

3.4. $\ln(27)$ is equal to $\dfrac{3}{\log_3(e)}$.

3.5. $\log_4(8x)$ is equal to $\dfrac{3}{2} + \log_2\left(\sqrt{2}\right)$.

Version history and licensing

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- v1.1: edited 05/24 by tdhc.