Factsheet: Beta distribution

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Summary

A factsheet about the beta distribution.

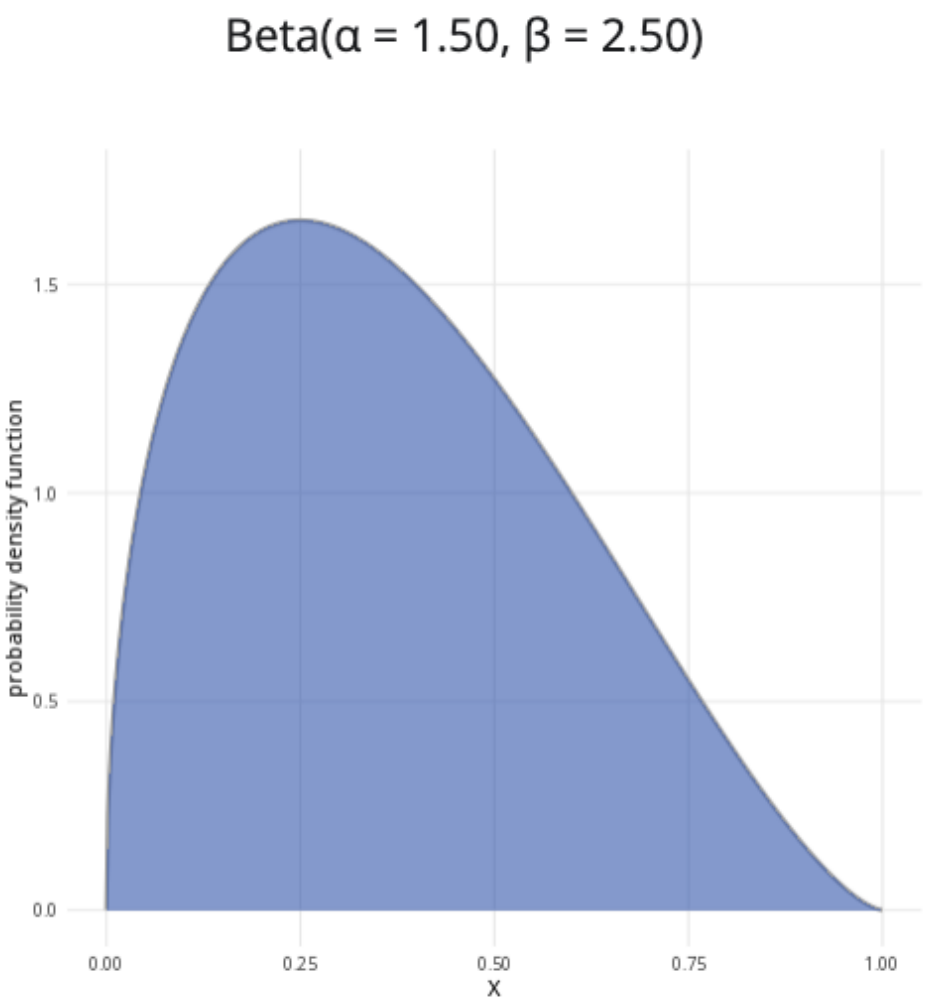

Figure 1: An example of the beta distribution with $\alpha = 1.5$ and $\beta = 2.5$.

Where to use: The beta distribution is used to model the distribution of *probabilities* or proportions. Hence, the random variable $0 \leq X \leq 1$.

Notation: $X \sim \text{Beta}(\alpha, \beta)$

Parameters: Two positive real numbers α, β, which are shape parameters. These can be specified as follows in terms of n and k where n is the number of Bernoulli trials and k is the number of successes:

- $\alpha = k + 1$
- $\beta = n - k + 1$

Quantity	Value	Notes
Mean	$\mathbb{E}(X) = \dfrac{\alpha}{\alpha + \beta}$	
Variance	$\mathbb{V}(X) = \dfrac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$	
PDF	$\mathbb{P}(X = x) = \dfrac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha, \beta)}$	$\mathrm{B}(x, y)$ is the beta function
CDF	$\mathbb{P}(X \leq x) = I_x(\alpha, \beta)$	$I_x(a, b)$ is the regularized incomplete beta function

Example: Cantor's Confectionery is visited by 10 customers, and 6 of them purchase something from the store. Taking the buying customers as successes and the total visiting customers as number of trials, there would be 6 successes, allowing you to find the following parameters:

- $\alpha = 6 + 1 = 7$
- $\beta = 10 - 6 + 1 = 5$

Then the distribution of the probabilities of a customer purchasing from Cantor's Confectionery can be expressed as $X \sim \mathrm{Beta}(7, 5)$, meaning the first shape parameter is 7 and the second shape parameter is 5.

Further reading

This interactive element appears in Overview: Probability distributions. Please click this link to go to the guide.

Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

- v1.1: moved to factsheet form and populated with material from Overview: Probability distributions by tdhc.