

# Questions: The scalar product

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## Summary

A selection of questions for the study guide on the scalar product

Before attempting these questions, it is highly recommended that you read [Guide: The scalar product](#), as well as [Guide: Introduction to quadratic equations](#).

## Q1

Find the scalar product of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$1.1. \mathbf{a} = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$1.2. \mathbf{a} = \begin{bmatrix} 10 \\ -7 \\ 4 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 3 \\ -5 \\ 13 \end{bmatrix}$$

$$1.3. \mathbf{a} = \begin{bmatrix} -44 \\ -12 \\ 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 61 \\ -25 \\ 93 \end{bmatrix}$$

$$1.4. \mathbf{a} = \begin{bmatrix} 54 \\ 38 \\ 0 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 32 \\ -55 \\ 13 \end{bmatrix}$$

$$1.5. \mathbf{a} = 2\mathbf{i} + 7\mathbf{j} + \mathbf{k} \text{ and } \mathbf{b} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$$

$$1.6. \mathbf{a} = -3\mathbf{i} + 10\mathbf{j} - 8\mathbf{k} \text{ and } \mathbf{b} = \mathbf{i} - 12\mathbf{j} + 9\mathbf{k}$$

$$1.7. \mathbf{a} = 17\mathbf{j} + 23\mathbf{k} \text{ and } \mathbf{b} = 6\mathbf{i} - 23\mathbf{j} - 8\mathbf{k}$$

$$1.8. \mathbf{a} = \mathbf{i} \text{ and } \mathbf{b} = \mathbf{j}.$$

What can you say about the result of 1.8.? Can you deduce similar conclusions for the scalar product of different combinations of the vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ ?

## Q2

Using the geometric definition of the scalar products, find the smallest angle  $\theta$  in between  $\mathbf{a}$  and  $\mathbf{b}$  in degrees. If your answer is not a whole number, give your answer to an accuracy of one decimal place.

$$2.1. \mathbf{a} = \begin{bmatrix} -5 \\ 2 \\ -3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ -2 \\ 11 \end{bmatrix}$$

$$2.2. \mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$2.3. \mathbf{a} = \begin{bmatrix} -8 \\ 1 \\ -4 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -1 \\ -5 \\ 7 \end{bmatrix}$$

$$2.4. \mathbf{a} = \begin{bmatrix} 1.2 \\ -1.4 \\ -3.1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -5.4 \\ 9.7 \\ -7.5 \end{bmatrix}$$

$$2.5. \mathbf{a} = \begin{bmatrix} 45 \\ 65 \\ 54 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -19 \\ -58 \\ 71 \end{bmatrix}$$

$$2.6. \mathbf{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$2.7. \mathbf{a} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 4 \\ -5 \\ 6 \end{bmatrix}$$

$$2.8. \mathbf{a} = \begin{bmatrix} -17 \\ 3 \\ 8 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 12 \\ -19 \\ -16 \end{bmatrix}$$

### Q3

Find the value(s) of  $\lambda$  for which  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular.

$$3.1. \mathbf{a} = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ \lambda \\ -2 \end{bmatrix}$$

$$3.2. \mathbf{a} = \begin{bmatrix} 0 \\ 1 \\ \lambda \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$3.3. \mathbf{a} = \begin{bmatrix} 9 \\ -2 \\ 11 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} \lambda \\ -\lambda \\ 3 \end{bmatrix}$$

$$3.4. \mathbf{a} = \begin{bmatrix} \lambda \\ 6 \\ 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} \lambda \\ \lambda \\ 8 \end{bmatrix}$$

$$3.5. \mathbf{a} = \begin{bmatrix} -2\lambda^2 \\ 4 \\ 14 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 3 \\ 2\lambda \\ 1 \end{bmatrix}$$

$$3.6. \mathbf{a} = \begin{bmatrix} -5 \\ 9 \\ 2\lambda \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} \lambda \\ -2 \\ \lambda \end{bmatrix}$$

$$3.7. \mathbf{a} = \begin{bmatrix} -7 \\ 4 \\ 2\lambda \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2\lambda \\ 1 \\ 6\lambda \end{bmatrix}$$

$$3.8. \mathbf{a} = \begin{bmatrix} -25 \\ -\lambda^2 \\ -2 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 3\lambda \\ -11 \\ 7 \end{bmatrix}$$

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After attempting the questions above, please click [this link](#) to find the answers.

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## **Version history and licensing**

v1.0: initial version created 08/23 by Ritwik Anand as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc.

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