

Answers: Trigonometric identities (degrees)

Dzhemma Ruseva

Summary

A selection of questions on trigonometric identities, using degrees to measure angles.

These are the answers to [Questions: Trigonometric identities \(degrees\)](#).

Please attempt the questions before reading these answers!

Q1

- 1.1. $2(6 \sin^2(\theta)) + 3(4 \cos^2(\theta)) = 12(\sin^2(\theta) + \cos^2(\theta)) = 12$
- 1.2. $10(7 \sin^2(\theta)) + 14(5 \cos^2(\theta)) = 70$
- 1.3. $5 \left(\frac{6}{\csc^2(\theta)} \right) + 15 \left(\frac{2}{\sec^2(\theta)} \right) = 30$
- 1.4. $(\cos^2(\theta) - \sin^2(\theta))^2 + 4 \sin^2(\theta) \cos^2(\theta) = \cos^2(2\theta) + \sin^2(2\theta) = 1$
- 1.5. $2 \sin(30) \cos(15) + 2 \cos(30) \sin(15) = 2 \sin(30 + 15) = 2 \sin(45) = \sqrt{2}$
- 1.6. $3 \cos(45) \cos(15) - 3 \sin(45) \sin(15) = 3 \cos(60) = \frac{3}{2}$
- 1.7. $\sin(150) + \sin(30) = 2 \sin\left(\frac{180}{2}\right) \cos\left(\frac{120}{2}\right) = 2 \sin(90) \cos(60) = 1$
- 1.8. $\cos(150) + \sin(30) = 2 \cos(90) \cos(60) = 0$

Q2

- 2.1. $\tan(\theta) \cos(-\theta) = \frac{\sin(\theta)}{\cos(\theta)} \cdot \cos(\theta) = \sin(\theta)$
- 2.2. $\tan(-\theta) \csc(-\theta) \sec(-\theta) = \left(-\frac{\sin(\theta)}{\cos(\theta)}\right) \left(\frac{1}{-\sin(\theta)}\right) \left(\frac{1}{\cos(\theta)}\right) = \left(\frac{1}{\cos^2(\theta)}\right) = \sec^2(\theta)$
- 2.3. $\tan^2(\theta) + \sin^2(\theta) + \cos^2(\theta) = \tan^2(\theta) + 1 = \sec^2(\theta)$
- 2.4. $\frac{2 \sin(\theta)}{\cos(\theta)(1 - \tan^2(\theta))} = \tan(2\theta)$

$$2.5. \frac{\sin(7\theta) + \sin(3\theta)}{\cos(7\theta) - \cos(3\theta)} = \frac{2\sin(5\theta)\cos(2\theta)}{-2\sin(5\theta)\sin(2\theta)} = -\cot(\theta)$$

$$2.6. \frac{\sin(5\theta) - \sin(\theta)}{\cos(5\theta) + \cos(\theta)} = \tan(2\theta)$$

Q3

$$3.1. \cos(210) = \frac{\sqrt{3}}{2}$$

3.2. Here $\sin(135) = \frac{1}{\sqrt{2}}$, and $\sin(225) = -\frac{1}{\sqrt{2}}$.

3.3. $\cos(130) = -0.766$ to three decimal places.

Q4

$$4.1. \sin(15) = \sin(45)\cos(30) - \cos(45)\sin(30) = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$4.2. \cos(15) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$4.3. \tan(15) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$4.4. \sin(75) = \sin(45)\cos(30) + \cos(45)\sin(30) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$4.5. \cos(75) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$4.6. \tan(75) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Version history and licensing

v1.0: initial version created 08/23 by Dzhemma Ruseva as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc, and split into versions for both degrees and radians.

This work is licensed under CC BY-NC-SA 4.0.