

Answers: Trigonometric identities (radians)

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Summary

A selection of questions on trigonometric identities, using radians to measure angles.

These are the answers to [Questions: Trigonometric identities \(radians\)](#).

Please attempt the questions before reading these answers!

Q1

- 1.1. $2(6 \sin^2(\theta)) + 3(4 \cos^2(\theta)) = 12(\sin^2(\theta) + \cos^2(\theta)) = 12$
- 1.2. $10(7 \sin^2(\theta)) + 14(5 \cos^2(\theta)) = 70$
- 1.3. $5 \left(\frac{6}{\csc^2(\theta)} \right) + 15 \left(\frac{2}{\sec^2(\theta)} \right) = 30$
- 1.4. $(\cos^2(\theta) - \sin^2(\theta))^2 + 4 \sin^2(\theta) \cos^2(\theta) = \cos^2(2\theta) + \sin^2(2\theta) = 1$
- 1.5. $2 \sin(\pi/6) \cos(\pi/12) + 2 \cos(\pi/6) \sin(\pi/12) = 2 \sin(\pi/6 + \pi/12) = 2 \sin(\pi/4) = \sqrt{2}$
- 1.6. $3 \cos(\pi/4) \cos(\pi/12) - 3 \sin(\pi/4) \sin(\pi/12) = 3 \cos(\pi/3) = \frac{3}{2}$
- 1.7. $\sin(5\pi/6) + \sin(\pi/6) = 2 \sin\left(\frac{180}{2}\right) \cos\left(\frac{120}{2}\right) = 2 \sin(\pi/2) \cos(\pi/3) = 1$
- 1.8. $\cos(5\pi/6) + \sin(\pi/6) = 2 \cos(\pi/2) \cos(\pi/3) = 0$

Q2

- 2.1. $\tan(\theta) \cos(-\theta) = \frac{\sin(\theta)}{\cos(\theta)} \cdot \cos(\theta) = \sin(\theta)$
- 2.2. $\tan(-\theta) \csc(-\theta) \sec(-\theta) = \left(-\frac{\sin(\theta)}{\cos(\theta)}\right) \left(\frac{1}{-\sin(\theta)}\right) \left(\frac{1}{\cos(\theta)}\right) = \left(\frac{1}{\cos^2(\theta)}\right) = \sec^2(\theta)$
- 2.3. $\tan^2(\theta) + \sin^2(\theta) + \cos^2(\theta) = \tan^2(\theta) + 1 = \sec^2(\theta)$

$$2.4. \frac{2 \sin(\theta)}{\cos(\theta)(1 - \tan^2(\theta))} = \tan(2\theta)$$

$$2.5. \frac{\sin(7\theta) + \sin(3\theta)}{\cos(7\theta) - \cos(3\theta)} = \frac{2 \sin(5\theta) \cos(2\theta)}{-2 \sin(5\theta) \sin(2\theta)} = -\cot(\theta)$$

$$2.6. \frac{\sin(5\theta) - \sin(\theta)}{\cos(5\theta) + \cos(\theta)} = \tan(2\theta)$$

Q3

3.1. $\cos(5\pi/6) = \frac{\sqrt{3}}{2}$

3.2. Here $\sin(3\pi/4) = \frac{1}{\sqrt{2}}$, and $\sin(5\pi/4) = -\frac{1}{\sqrt{2}}$.

3.3. $\cos(13\pi/18) = -0.766$ to three decimal places.

Q4

4.1. $\sin(\pi/12) = \sin(\pi/4) \cos(\pi/6) - \cos(\pi/4) \sin(\pi/6) = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

4.2. $\cos(\pi/12) = \frac{\sqrt{3}+1}{2\sqrt{2}}$

4.3. $\tan(\pi/12) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

4.4. $\sin(5\pi/12) = \sin(\pi/4) \cos(\pi/6) + \cos(\pi/4) \sin(\pi/6) = \frac{\sqrt{3}+1}{2\sqrt{2}}$

4.5. $\cos(5\pi/12) = \frac{\sqrt{3}-1}{2\sqrt{2}}$

4.6. $\tan(5\pi/12) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Version history and licensing

v1.0: initial version created 08/23 by Dzhemma Ruseva as part of a University of St Andrews STEP project.

- v1.1: edited 05/24 by tdhc, and split into versions for both degrees and radians.

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