Factsheet: Lognormal distribution

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Summary

A factsheet for the lognormal distribution.

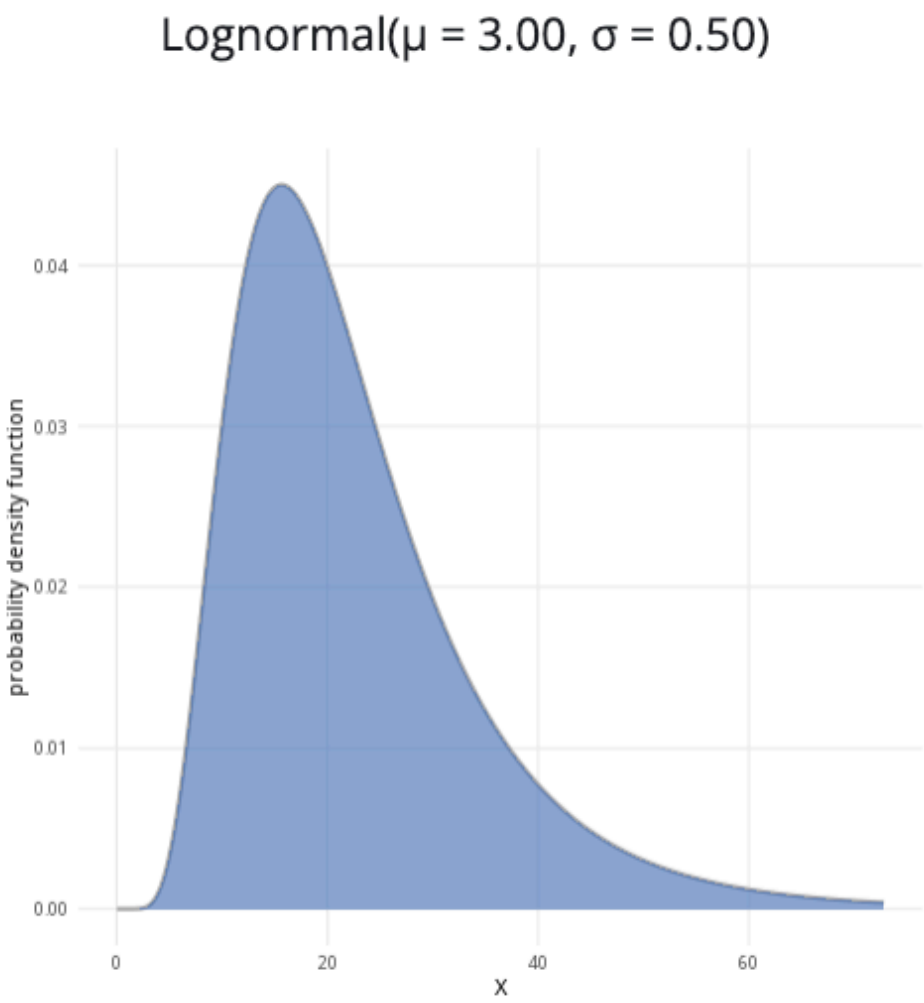

Figure 1: An example of the lognormal distribution with $\mu = 3$ and $\sigma = 0.5$.

Where to use: The lognormal distribution is used to model continuous random variables with values that are both real and non-negative, wherein the logarithms of these variables follow a normal distribution. That is to say, if the random variable X is lognormally distributed, then the random variable $Y = \ln(X)$ is normally distributed (where $\ln$ is the natural logarithm).

Notation: $X \sim \text{Lognormal}(\mu, \sigma^2)$

Parameters: As with the normal distribution, two numbers μ and σ^2 where:

- μ is the expected value of the normally distributed random variable $Y = \ln(X)$,
- σ^2 is the variance of the normally distributed random variable $Y = \ln(X)$.

Quantity	Value	Notes
Mean	$\mathbb{E}(X) = \exp(\mu + \frac{\sigma^2}{2})$	$\exp(y) = e^y$
Variance	$\mathbb{V}(X) = [\exp(\sigma^2) - 1]\exp(2\mu + \sigma^2)$	$\exp(y) = e^y$
PDF	$\mathbb{P}(X = x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(x) - \mu)^2}{2\sigma^2}\right)$	$\exp(y) = e^y$
CDF	$\mathbb{P}(X \leq x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\ln(x) - \mu}{\sigma\sqrt{2}}\right)\right]$	$\operatorname{erf}(x)$ is the error function of x

Example: The logarithms of Cantor's Confectionery's stock prices follow a normal distribution. The mean of the stock prices' natural logarithms is 8.01, whereas the variance of the stock prices' natural logarithms is 3. This can be expressed as $X \sim \text{Lognormal}(8.01, 3)$, meaning the logarithm of the location parameter is 8.01 and the logarithm of scale parameter is 3.

Further reading

This interactive element appears in Overview: Probability distributions. Please click this link to go to the guide.

Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

- v1.1: moved to factsheet form and populated with material from Overview: Probability distributions by tdhc.