

# Answers: Introduction to complex numbers

Tom Coleman

## Summary

Answers to questions relating to the guide on introduction to complex numbers.

*These are the answers to [Questions: Introduction to complex numbers](#).*

**Please attempt the questions before reading these answers!**

## Q1

Using complex numbers, find solutions to the following equations.

- 1.1. Here,  $x = i$  and  $x = -i$  are the two solutions.
- 1.2. Here,  $x = 3i$  and  $x = -3i$  are the two solutions.
- 1.3. Here,  $x = 12i$  and  $x = -12i$  are the two solutions.
- 1.4. Here,  $x = 1$  and  $x = -1$  are the two solutions. (Real numbers are complex numbers too!)

## Q2

For each of the complex numbers below, give their real and imaginary parts. (In this question,  $a, b$  are real numbers.)

- 2.1. The real part of  $z_1$  is  $\operatorname{Re}(z_1) = 2$  and the imaginary part of  $z_1$  is  $\operatorname{Im}(z_1) = 3$ .
- 2.2. The real part of  $z_2$  is  $\operatorname{Re}(z_2) = -23$  and the imaginary part of  $z_2$  is  $\operatorname{Im}(z_2) = 32$ .
- 2.3. The real part of  $z_3$  is  $\operatorname{Re}(z_3) = 3$  and the imaginary part of  $z_3$  is  $\operatorname{Im}(z_3) = -3$ .
- 2.4. The real part of  $z_4$  is  $\operatorname{Re}(z_4) = 0$  and the imaginary part of  $z_4$  is  $\operatorname{Im}(z_4) = 3$ .
- 2.5. The real part of  $z_5$  is  $\operatorname{Re}(z_5) = -3$  and the imaginary part of  $z_5$  is  $\operatorname{Im}(z_5) = -2$ .
- 2.6. The real part of  $z_6$  is  $\operatorname{Re}(z_6) = a$  and the imaginary part of  $z_6$  is  $\operatorname{Im}(z_6) = 2b$ .
- 2.7. The real part of  $z_7$  is  $\operatorname{Re}(z_7) = 2$  and the imaginary part of  $z_7$  is  $\operatorname{Im}(z_7) = 0$ .

- 2.8. The real part of  $z_8$  is  $\operatorname{Re}(z_8) = 3/2$  and the imaginary part of  $z_8$  is  $\operatorname{Im}(z_8) = 2/3$ .
- 2.9. The real part of  $z_9$  is  $\operatorname{Re}(z_9) = 22$  and the imaginary part of  $z_9$  is  $\operatorname{Im}(z_9) = -33$ .
- 2.10. The real part of  $z_{10}$  is  $\operatorname{Re}(z_{10}) = 333$  and the imaginary part of  $z_{10}$  is  $\operatorname{Im}(z_{10}) = 22$ .
- 2.11. The real part of  $z_{11}$  is  $\operatorname{Re}(z_{11}) = -2$  and the imaginary part of  $z_{11}$  is  $\operatorname{Im}(z_{11}) = 2$ .
- 2.12. The real part of  $z_{12}$  is  $\operatorname{Re}(z_{12}) = -2$  and the imaginary part of  $z_{11}$  is  $\operatorname{Im}(z_{11}) = -3$ .

### Q3

The complex conjugate of  $z_1 = 2 + 3i$  is  $\bar{z}_1 = 2 - 3i$ .

The complex conjugate of  $z_2 = -23 + 32i$  is  $\bar{z}_2 = -23 - 32i$ .

The complex conjugate of  $z_3 = 3 - 3i$  is  $\bar{z}_3 = 3 + 3i$ .

The complex conjugate of  $z_4 = 3i$  is  $\bar{z}_4 = -3i$ .

The complex conjugate of  $z_5 = -3 - 2i$  is  $\bar{z}_5 = -3 + 2i$ .

The complex conjugate of  $z_6 = a + 2bi$  is  $\bar{z}_6 = a - 2bi$ .

The complex conjugate of  $z_7 = 2$  is  $\bar{z}_7 = 2$ .

The complex conjugate of  $z_8 = 3/2 + 2i/3$  is  $\bar{z}_8 = 3/2 - 2i/3$ .

The complex conjugate of  $z_9 = 22 - 33i$  is  $\bar{z}_9 = 22 + 33i$ .

The complex conjugate of  $z_{10} = 333 + 22i$  is  $\bar{z}_{10} = 333 - 22i$ .

The complex conjugate of  $z_{11} = 2i - 2$  is  $\bar{z}_{11} = -2i - 2$ .

The complex conjugate of  $z_{12} = -3i - 2$  is  $\bar{z}_{12} = 3i - 2$ .

### Q4

See Figure 1 for the Argand diagram. You can notice that the complex conjugates of the complex numbers can be obtained by reflecting the point in the real axis.



### Version history and licensing

v1.0: initial version created 10/24 by tdhc.



Figure 1: An Argand diagram with the seven complex numbers  $z_1, \bar{z}_1, z_4, \bar{z}_4$ , in Example 5.

This work is licensed under CC BY-NC-SA 4.0.