Answers: Rationalizing the denominator

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Summary

Answers to questions relating to the guide on rationalizing the denominator.

These are the answers to Questions: Rationalizing the denominator.

Please attempt the questions before reading these answers!

Q1

1.1. $\dfrac{5}{\sqrt{3}} = \dfrac{5\sqrt{3}}{3}$

1.2. $\dfrac{7}{2\sqrt{5}} = \dfrac{7\sqrt{5}}{10}$

1.3. $\dfrac{11}{4\sqrt{7}} = \dfrac{11\sqrt{7}}{28}$

1.4. $\dfrac{8}{5\sqrt{6}} = \dfrac{4\sqrt{6}}{15}$

1.5. $\dfrac{3\sqrt{2}}{\sqrt{5}} = \dfrac{3\sqrt{10}}{5}$

1.6. $\dfrac{9}{\sqrt{10}} = \dfrac{9\sqrt{10}}{10}$

1.7. $\dfrac{\sqrt{7}}{\sqrt{3}} = \dfrac{\sqrt{21}}{3}$

1.8. $\dfrac{\sqrt{2}}{\sqrt{6}} = \dfrac{\sqrt{3}}{3}$

1.9. $\dfrac{12}{\sqrt{11}} = \dfrac{12\sqrt{11}}{11}$

1.10. $\dfrac{\sqrt{8}}{\sqrt{2}} = 2$

1.11. $\dfrac{15}{3\sqrt{7}} = \dfrac{5\sqrt{7}}{7}$

1.12. $\dfrac{6\sqrt{3}}{\sqrt{10}} = \dfrac{3\sqrt{30}}{5}$

1.13. $\dfrac{\sqrt{18}}{\sqrt{9}} = \sqrt{2}$

1.14. $\dfrac{2\sqrt{5}}{\sqrt{12}} = \dfrac{\sqrt{30}}{3}$

1.15. $\dfrac{4}{\sqrt{2}} = 2\sqrt{2}$

1.16. $\dfrac{10}{5\sqrt{13}} = \dfrac{2\sqrt{13}}{13}$

Q2

2.1. $\dfrac{5}{2+\sqrt{3}} = 10 - 5\sqrt{3}$

2.2. $\dfrac{7}{4-\sqrt{2}} = \dfrac{4+\sqrt{2}}{2}$

2.3. $\dfrac{3}{\sqrt{5}+1} = \dfrac{3\sqrt{5}-3}{4}$

2.4. $\dfrac{\sqrt{7}}{\sqrt{3}-1} = \dfrac{\sqrt{21}+\sqrt{7}}{2}$

2.5. $\dfrac{2+\sqrt{5}}{1-\sqrt{2}} = -2 - 2\sqrt{2} - \sqrt{5} - \sqrt{10}$

2.6. $\dfrac{3\sqrt{2}+5}{4+\sqrt{6}} = \dfrac{12\sqrt{2} - 6\sqrt{3} + 20 - 5\sqrt{6}}{10}$

2.7. $\dfrac{8}{3-\sqrt{7}} = 12 + 4\sqrt{7}$

2.8. $\dfrac{6}{2+\sqrt{5}} = -12 + 6\sqrt{5}$

2.9. $\dfrac{\sqrt{10}}{\sqrt{2}+3} = \dfrac{3\sqrt{10} - 2\sqrt{5}}{7}$

2.10. $\dfrac{2\sqrt{3}+5}{\sqrt{7}-1} = \dfrac{2\sqrt{21} + 5\sqrt{7} + 2\sqrt{3} + 5}{6}$

2.11. $\dfrac{\sqrt{6}-\sqrt{2}}{2+\sqrt{5}} = -2\sqrt{6} + 2\sqrt{5} + 2\sqrt{2} - \sqrt{10}$

2.12. $\dfrac{4+\sqrt{3}}{5-\sqrt{7}} = \dfrac{4\sqrt{7} + 5\sqrt{3} + \sqrt{21} + 20}{18}$

2.13. $\dfrac{2}{4-\sqrt{11}} = \dfrac{8+2\sqrt{11}}{5}$

2.14. $\dfrac{\sqrt{8}+\sqrt{3}}{\sqrt{7}-2} = \dfrac{2\sqrt{14}+4\sqrt{2}+\sqrt{21}+2\sqrt{3}}{3}$

Q3

3.1. To prove this equation, rationalize the denominator of the left hand side of the equation.

Since the denominator contains two square roots you can multiply the numerator and denominator by $-2\sqrt{3}+\sqrt{5}$ or by $2\sqrt{3}-\sqrt{5}$ to rationalize the denominator.

If you multiply the numerator and denominator by $2\sqrt{3}-\sqrt{5}$ you get:

$$\frac{\sqrt{11}}{2\sqrt{3}+\sqrt{5}} \cdot \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}-\sqrt{5}} = \frac{\sqrt{11}(2\sqrt{3}-\sqrt{5})}{(2\sqrt{3}+\sqrt{5})(2\sqrt{3}-\sqrt{5})}$$

Expanding the brackets in both the numerator and the denominator gives you:

$$\frac{\sqrt{11}(2\sqrt{3}-\sqrt{5})}{(2\sqrt{3}+\sqrt{5})(2\sqrt{3}-\sqrt{5})} = \frac{2\sqrt{33}-\sqrt{55}}{(2\sqrt{3})^2-2\sqrt{15}+2\sqrt{15}-(\sqrt{5})^2}$$

Simplifying the denominator then gives you:

$$\frac{2\sqrt{33}-\sqrt{55}}{(2\sqrt{3})^2-2\sqrt{15}+2\sqrt{15}-(\sqrt{5})^2} = \frac{2\sqrt{33}-\sqrt{55}}{4(3)-5}$$

Simplifying further gives you the final answer and the right hand side of the equation you are proving:

$$\frac{2\sqrt{33}-\sqrt{55}}{4(3)-5} = \frac{2\sqrt{33}-\sqrt{55}}{7}$$

If you instead multiply the numerator and denominator by $-2\sqrt{3}+\sqrt{5}$ you get:

$$\frac{\sqrt{11}}{2\sqrt{3}+\sqrt{5}} \cdot \frac{-2\sqrt{3}+\sqrt{5}}{-2\sqrt{3}+\sqrt{5}} = \frac{\sqrt{11}(-2\sqrt{3}+\sqrt{5})}{(2\sqrt{3}+\sqrt{5})(-2\sqrt{3}+\sqrt{5})}$$

Expanding the brackets in both the numerator and the denominator gives you:

$$\frac{\sqrt{11}(-2\sqrt{3}+\sqrt{5})}{(2\sqrt{3}+\sqrt{5})(-2\sqrt{3}+\sqrt{5})} = \frac{\sqrt{11}(-2\sqrt{3}+\sqrt{5})}{-(2\sqrt{3})^2+2\sqrt{15}-2\sqrt{15}+(\sqrt{5})^2}$$

Simplifying the denominator gives you:

$$\frac{\sqrt{11}(-2\sqrt{3}+\sqrt{5})}{-(2\sqrt{3})^2+2\sqrt{15}-2\sqrt{15}+(\sqrt{5})^2}=\frac{-2\sqrt{33}+\sqrt{55}}{5-4(3)}$$

Further simplifying the denominator then gives you:

$$\frac{-2\sqrt{33}+\sqrt{55}}{5-4(3)}=\frac{-2\sqrt{33}+\sqrt{55}}{-7}$$

To get a positive denominator, multiplying both the numerator and the denominator by -1 gives you the right hand side of the equation you are proving:

$$\frac{-2\sqrt{33}+\sqrt{55}}{-7}=\frac{2\sqrt{33}-\sqrt{55}}{7}$$

3.2. $\dfrac{5-\sqrt{2}}{\sqrt{10}-\sqrt{3}}=\dfrac{5\sqrt{10}+5\sqrt{3}-2\sqrt{5}-\sqrt{6}}{7}$

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Version history and licensing

v1.0: initial version created 12/24 by Maximilian Volmar.