Questions: Law of total probability and Bayes' theorem

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Summary

A selection of questions to test your understanding of the law of total probability and Bayes' theorem.

Before attempting these questions it is highly recommended that you read [Guide: Law of total probability and Bayes' theorem]

Q1

Use the law of total probability to answer the following.

1.1.

In a hospital:

- 40% of patients are treated in Ward A
- 60% in Ward B
- The probability of recovery is 80% in Ward A
- The probability of recovery is 60% in Ward B

Let R be the event that a patient recovers. What is $P(R)$?

1.2.

A school has three types of lunches:

- 50% of students choose vegetarian

- 30% choose chicken
- 20% choose fish

The probability that a student finishes their lunch is:

- 90% for vegetarian
- 70% for chicken
- 80% for fish

What is the probability that a randomly chosen student finishes their lunch?

1.3.

A product is manufactured in three factories:

- 20% from Factory 1 (with a defect rate 5%)
- 30% from Factory 2 (with a defect rate 2%)
- 50% from Factory 3 (with a defect rate 1%)

What is the probability that a randomly chosen product is defective?

1.4.

A student can study in three locations:

- At home (50% of the time)
- In the library (30%)
- In a café (20%)

The probability they complete their homework is:

- 70% at home

- 90% in the library
- 60% in the café

What is the probability that a randomly selected student completes their homework?

Q2

Use Bayes' theorem to answer the following.

2.1.

A test for a disease is:

- 95% accurate for infected individuals (true positive)
- 90% accurate for uninfected individuals (true negative)
- 2% of the population has the disease

Let D be the event that a person has the disease and T the event they test positive. What is $P(D \mid T)$?

2.2.

In a certain region:

- 60% of days are dry
- 40% are rainy

A forecast predicts rain:

- 80% of the time on rainy days
- 10% of the time on dry days

If the forecast predicts rain, what is the probability that it will actually rain?

2.3.

In a factory:

- 70% of items are made by Machine A
- 30% by Machine B

The probability of a faulty item is:

- 2% from Machine A
- 5% from Machine B

If an item is found to be faulty, what is the probability it came from Machine B?

2.4.

A bag contains:

- 40% red sweets
- 60% blue sweets

A red sweet has a 30% chance of having a wrapper and a blue sweet has a 70% chance of having a wrapper. If a sweet is picked at random and has a wrapper, what is the probability it is red?

[After attempting the questions above, please click this link to find the answers.]

Version history and licensing

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