Factsheet: Trigonometric identities (radians)

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Summary

A list of trigonometric identities with angles measured in radians.

The main study guide for this factsheet is Guide: Trigonometric identities (radians). If you would like to know more about these, please read the guide.

This factsheet measures angles in radians. For the associated factsheet measuring angles in degrees, please go to Factsheet: Trigonometric identities (degrees).

Trigonometric identities

Periodicity and parity

For all angles θ and for all whole numbers $k \in \mathbb{Z}$:

$$\begin{aligned}
\cos(-\theta) &= \cos(\theta) \\
\sin(-\theta) &= -\sin(\theta) \\
\tan(-\theta) &= -\tan(\theta) \\
\cos(\theta + 2k\pi) &= \cos(\theta) \\
\sin(\theta + 2k\pi) &= \sin(\theta) \\
\tan(\theta + k\pi) &= \tan(\theta)
\end{aligned}$$

Pythagorean formulas

For all angles θ

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

Sum and difference formulas

For all angles α, β:

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$$

Double angle formulas

For all angles θ:

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

Shift formulas

For all angles θ:

$$\cos\left(\theta+\frac{\pi}{2}\right) = -\sin(\theta)$$

$$\cos\left(\theta-\frac{\pi}{2}\right) = \sin(\theta)$$

$$\sin\left(\theta+\frac{\pi}{2}\right) = \cos(\theta)$$

$$\sin\left(\theta-\frac{\pi}{2}\right) = -\cos(\theta)$$

$$\cos\left(\theta\pm\pi\right) = -\cos(\theta)$$

$$\sin\left(\theta\pm\pi\right) = -\sin(\theta)$$

$$\sin\left(\pi-\theta\right) = \sin(\theta)$$

$$\cos\left(\pi-\theta\right) = -\cos(\theta)$$

Sine and cosine rules

For a triangle with corners A, B, C, angles α, β, γ respectively at those corners, and sides

a, b, c opposite their respective corners, the **sine rule** is

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

and the **cosine rule** is

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha).$$

Common values of trigonometric functions

Angle θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Version history

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