

Answers: Introduction to complex numbers

Tom Coleman

Summary

Answers to questions relating to the guide on introduction to complex numbers.

These are the answers to [Questions: Introduction to complex numbers](#).

Please attempt the questions before reading these answers!

Q1

Using complex numbers, find solutions to the following equations.

- 1.1. Here, $x = i$ and $x = -i$ are the two solutions.
- 1.2. Here, $x = 3i$ and $x = -3i$ are the two solutions.
- 1.3. Here, $x = 12i$ and $x = -12i$ are the two solutions.
- 1.4. Here, $x = 1$ and $x = -1$ are the two solutions. (Real numbers are complex numbers too!)

Q2

For each of the complex numbers below, give their real and imaginary parts. (In this question, a, b are real numbers.)

- 2.1. The real part of z_1 is $\text{Re}(z_1) = 2$ and the imaginary part of z_1 is $\text{Im}(z_1) = 3$.
- 2.2. The real part of z_2 is $\text{Re}(z_2) = -23$ and the imaginary part of z_2 is $\text{Im}(z_2) = 32$.
- 2.3. The real part of z_3 is $\text{Re}(z_3) = 3$ and the imaginary part of z_3 is $\text{Im}(z_3) = -3$.
- 2.4. The real part of z_4 is $\text{Re}(z_4) = 0$ and the imaginary part of z_4 is $\text{Im}(z_4) = 3$.
- 2.5. The real part of z_5 is $\text{Re}(z_5) = -3$ and the imaginary part of z_5 is $\text{Im}(z_5) = -2$.
- 2.6. The real part of z_6 is $\text{Re}(z_6) = a$ and the imaginary part of z_6 is $\text{Im}(z_6) = 2b$.
- 2.7. The real part of z_7 is $\text{Re}(z_7) = 2$ and the imaginary part of z_7 is $\text{Im}(z_7) = 0$.

- 2.8. The real part of z_8 is $\operatorname{Re}(z_8) = 3/2$ and the imaginary part of z_8 is $\operatorname{Im}(z_8) = 2/3$.
- 2.9. The real part of z_9 is $\operatorname{Re}(z_9) = 22$ and the imaginary part of z_9 is $\operatorname{Im}(z_9) = -33$.
- 2.10. The real part of z_{10} is $\operatorname{Re}(z_{10}) = 333$ and the imaginary part of z_{10} is $\operatorname{Im}(z_{10}) = 22$.
- 2.11. The real part of z_{11} is $\operatorname{Re}(z_{11}) = -2$ and the imaginary part of z_{11} is $\operatorname{Im}(z_{11}) = 2$.
- 2.12. The real part of z_{12} is $\operatorname{Re}(z_{12}) = -2$ and the imaginary part of z_{11} is $\operatorname{Im}(z_{11}) = -3$.

Q3

The complex conjugate of $z_1 = 2 + 3i$ is $\bar{z}_1 = 2 - 3i$.

The complex conjugate of $z_2 = -23 + 32i$ is $\bar{z}_2 = -23 - 32i$.

The complex conjugate of $z_3 = 3 - 3i$ is $\bar{z}_3 = 3 + 3i$.

The complex conjugate of $z_4 = 3i$ is $\bar{z}_4 = -3i$.

The complex conjugate of $z_5 = -3 - 2i$ is $\bar{z}_5 = -3 + 2i$.

The complex conjugate of $z_6 = a + 2bi$ is $\bar{z}_6 = a - 2bi$.

The complex conjugate of $z_7 = 2$ is $\bar{z}_7 = 2$.

The complex conjugate of $z_8 = 3/2 + 2i/3$ is $\bar{z}_8 = 3/2 - 2i/3$.

The complex conjugate of $z_9 = 22 - 33i$ is $\bar{z}_9 = 22 + 33i$.

The complex conjugate of $z_{10} = 333 + 22i$ is $\bar{z}_{10} = 333 - 22i$.

The complex conjugate of $z_{11} = 2i - 2$ is $\bar{z}_{11} = -2i - 2$.

The complex conjugate of $z_{12} = -3i - 2$ is $\bar{z}_{12} = 3i - 2$.

Q4

See Figure 1 for the Argand diagram. You can notice that the complex conjugates of the complex numbers can be obtained by reflecting the point in the real axis.

Version history and licensing

v1.0: initial version created 10/24 by tdhc.

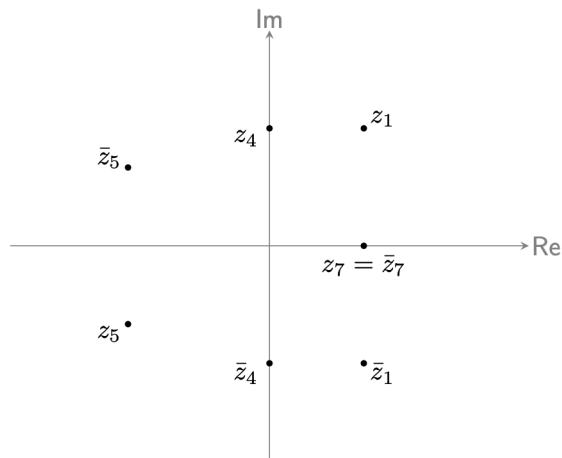


Figure 1: An Argand diagram with the seven complex numbers $z_1, \bar{z}_1, z_4, \bar{z}_4, z_5, \bar{z}_5$, in Example 5.

This work is licensed under CC BY-NC-SA 4.0.