Answers: Vector addition and scalar multiplication

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Summary

Answers to questions relating to the guide on vector addition and scalar multiplication.

These are the answers to Questions: Addition and scalar multiplication.

Please attempt the questions before reading these answers!

Q1

1.1. For the $\mathbf{i}$ component, $4+8=12$.For the $\mathbf{j}$ component, $5+2=7$.For the $\mathbf{k}$ component, $7+4=11$. So the answer is $\mathbf{a}+\mathbf{b}=12\mathbf{i}+7\mathbf{j}+11\mathbf{k}$.

1.2. $\mathbf{a}+\mathbf{b}=2\mathbf{i}+3\mathbf{j}+9\mathbf{k}$.

1.3. $\mathbf{a}-\mathbf{b}=2\mathbf{i}-11\mathbf{j}+14\mathbf{k}$.

1.4. You can solve this by doing addition componentwise. $\mathbf{i}$ component: $4-(3+11)=-10$, $\mathbf{j}$ component: $12-(-3-4)=19$, $\mathbf{k}$ component: $-7-(-2+9)=-14$. So the answer is $-10\mathbf{i}+19\mathbf{j}-14\mathbf{k}$.

Q2

2.1. $\mathbf{a}+\mathbf{b}=\begin{bmatrix}4x\\7y\\0\end{bmatrix}$

2.2. $\mathbf{a}-\mathbf{b}=\begin{bmatrix}7\\3y-2x\\-z\end{bmatrix}$

2.3. $\mathbf{a}+\mathbf{b}-\mathbf{c}=\mathbf{0}$ or $\begin{bmatrix}0\\0\\0\end{bmatrix}$.

2.4. $\mathbf{a}$.

Q3

3.1. $3\mathbf{u} = (3)5\mathbf{j} + (3)6\mathbf{k} = 15\mathbf{j} + 18\mathbf{k}$.

3.2. $-6\mathbf{v} = \begin{bmatrix} 0 \\ 18 \\ -42 \end{bmatrix}$.

3.3. $4\mathbf{v} - 3\mathbf{u} = \begin{bmatrix} 0 \\ -27 \\ 10 \end{bmatrix}$

3.4. $-2\mathbf{w} - (4\mathbf{u} - 2\mathbf{v}) = \begin{bmatrix} -4 \\ -32 \\ -2 \end{bmatrix}$

Q4

4.1. By the laws of vector addition, $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB}$, where $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the respective coordinates of A and B written in vector form. You can find $\overrightarrow{AB}$ by solving the above equation. $\overrightarrow{AB} = \begin{bmatrix} -2-3 \\ 5-4 \\ 7-5 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \\ 2 \end{bmatrix}$

4.2.$\overrightarrow{AB} = \begin{bmatrix} 4 \\ 6 \\ 0 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} -2 \\ -4 \\ -5 \end{bmatrix}$. $\overrightarrow{AB} - \overrightarrow{AC} = \begin{bmatrix} 6 \\ 10 \\ 5 \end{bmatrix}$. You can also calculate this by noticing $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$. Then $\overrightarrow{CB} = \begin{bmatrix} 6-0 \\ 11-1 \\ 7-2 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 5 \end{bmatrix}$ as required.

4.3. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$. $\begin{bmatrix} 1 \\ 5 \\ 9 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ -2 \end{bmatrix}$. Solving this gives $A = (-5, -2, 11)$.

4.4. Let λ and μ be scalars. $\lambda\mathbf{a} + \mu\mathbf{b} = 13\mathbf{i} - 9\mathbf{j}$. This gives you the simultaneous equations

$$2\lambda + 3\mu = 13 \qquad (\mathbf{i} \text{ component})$$
$$3\lambda - 5\mu = -9 \qquad (\mathbf{j} \text{ component})$$

Solving this gives $\mu = 3$, $\lambda = 2$, which gives the answer $2\mathbf{a} + 3\mathbf{b}$.

4.5. $2\begin{bmatrix} 2 \\ 5 \\ z \end{bmatrix} + 3\begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$. Solving this gives $x = 3$, $y = 1$ and $z = -6$.

4.6. As they are parallel $\mathbf{a} = \lambda\mathbf{b}$ for some real scalar λ. This gives the simultaneous equations

$$x - 7 = -2\lambda \qquad (\mathbf{i}\text{ component})$$
$$5x + 1 = 8\lambda \qquad (\mathbf{k}\text{ component})$$

Eliminating λ and solving gives $x = 3$.

Version history and licensing

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