Determinants

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Summary

A selection of questions for the study guide on determinants.

Before attempting these questions, it is recommended that you read Guide: Determinants.

Q1

Compute the determinant of each matrix.

1.1. $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$

1.2. $A = \begin{bmatrix} -1 & 2 \\ 6 & 3 \end{bmatrix}$

1.3. $A = \begin{bmatrix} 7 & 0 \\ -3 & 5 \end{bmatrix}$

1.4. $A = \begin{bmatrix} 4 & -2 \\ 1 & 8 \end{bmatrix}$

1.5. $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$

Q2

Compute the determinant using Sarrus' Rule.

2.1. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

2.2. $A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 0 & 5 & 2 \end{bmatrix}$

2.3. $A = \begin{bmatrix} -1 & 3 & 2 \\ 4 & 0 & 1 \\ 2 & -2 & 5 \end{bmatrix}$

2.4. $A = \begin{bmatrix} 5 & 2 & 1 \\ 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

2.5. $A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 0 & 3 \\ 5 & -1 & 2 \end{bmatrix}$

Q3

Determinants of triangular and diagonal matrices.

3.1. $A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 6 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

3.2. $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3.3. $A = \begin{bmatrix} 4 & 0 & 0 \\ -3 & 2 & 0 \\ 1 & 5 & -6 \end{bmatrix}$

3.4. $A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

3.5. $A = \begin{bmatrix} 2 & 5 & 7 \\ 0 & 4 & 6 \\ 0 & 0 & 8 \end{bmatrix}$

Q4

Compute the determinant using cofactor expansion.

4.1. $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 0 \\ 5 & 0 & 6 \end{bmatrix}$

4.2. $A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 4 \\ 5 & 0 & 6 \end{bmatrix}$

4.3. $A = \begin{bmatrix} 7 & 0 & 0 \\ 1 & -2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

4.4. $A = \begin{bmatrix} 0 & 0 & 5 \\ 2 & 3 & 1 \\ 4 & -1 & 2 \end{bmatrix}$

4.5. $A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 3 & 4 & 0 & 0 \\ 5 & 0 & 6 & 0 \\ 0 & 7 & 1 & 8 \end{bmatrix}$

Q5

Determinant properties.

5.1. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$, compute $\det(3A)$.

5.2. If $\det(B) = -6$, compute $\det(2B)$.

5.3. If one row of a 3×3 matrix is multiplied by 5, how does the determinant change?

5.4. A matrix C has two identical rows. What is $\det(C)$?

5.5. Swapping two rows of a matrix results in a determinant of 18. What was the determinant before the swap?

5.6. A matrix D has $\det(D) = -4$. Compute $\det(D^{-1})$.

5.7. If $\det(E) = 9$, compute $\det(E^T)$.

5.8. Let $G = \begin{bmatrix} 2A & 0 \\ 0 & 3I_2 \end{bmatrix}$ with $\det(A) = 5$. Compute $\det(G)$.

5.9. If one row is replaced by 3 times that row plus another row, how does the determinant change?

5.10. If $\det(K) = 0$, what does this imply about the rows or columns of K?

5.11. A 4×4 matrix M has $\det(M) = 8$. Compute $\det(-M)$.

5.12. If $\det(N) = 7$ for a 3×3 matrix, compute $\det(N^2)$.

5.13. A matrix has determinant 10. After swapping two rows and multiplying a row by 3, what is the new determinant?

5.14. If $P = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ with $\det(A) = 2$, $\det(B) = -3$, compute $\det(P)$.

5.15. Does adding a multiple of one row to another row change the determinant? Explain.

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