Overview: Number sets

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Summary

An overview of what numbers and sets are, and some key number sets you can use.

What are numbers

Numbers are used to count, measure, order, and perform calculations. In this overview you will see some common sets of numbers, what defines them, and how they all fit together.

What are sets

In mathematics a set is a collection of things. In this overview the sets you'll look at will be sets of numbers. The elements of a set are usually called members. Curly brackets are commonly used for set notation. Sets can be empty, they can contain a finite number of elements, or an infinite number of elements. The sets you'll see discussed in detail in this guide are all infinity large sets.

Example 1

This is how you might write a set containing the first 8 letters in the alphabet. You can call this set $'Alpha8'$.

$$Alpha8 = \{a, b, c, d, e, f, g, h\}$$

There are some symbols you can use to describe containment within sets.

Definition of element of

If you want to say an element is a member of a set, you can use $\in$.
$a \in A$ Means that a is a member of A. Similarly, if you want to say an element is not a member of a set, you can use $\notin$. $b \notin A$ Means that b is not a member of A.

Example 2

Using the set containing the first 8 letters in the alphabet from above:

$$Alpha8 = \{a, b, c, d, e, f, g, h\}$$

Since a is in $Alpha8$, you can write $a \in Alpha8$.
Since z is not in $Alpha8$, you can write $z \notin Alpha8$.

Definition of subset

If you want to say all the members of one set are contained in another, you can use $\subseteq$. $B \subseteq A$ Means that all the members of B are also members of A. Here you would say B is a **subset** of A. Similarly you can use $\nsubseteq$ to say something isn't a subset. $D \nsubseteq C$ Means that all the members of D aren't also members of C.

Example 3

You could say the set containing the first 4 letters of the alphabet is a subset of $Alpha8$.
$\{a, b, c, d\} \subseteq Alpha8$.
But the set containing $\{g, h, i, j\}$ isn't a subset of $Alpha8$, since i and j aren't members of $Alpha8$.
So $\{g, h, i, j\} \nsubseteq Alpha8$.

Types of number sets

Now you can look at some different sets of numbers.

Natural numbers

The natural numbers are counting numbers. Important sets often have standard notation, the symbol commonly used for the natural numbers is $\mathbb{N}$.

Definition of the natural numbers

$\mathbb{N} = \{0, 1, 2, 3, 4, 5...\}$

Opinions vary on whether or not zero should be considered a natural number or not. For this overview you can assume zero is a natural number.

number line diagram

Integers

The natural numbers allow you to count positive values, but what about measuring debts or deficits, for those you have the integers. The integers are the natural numbers as well as the negatives of each natural number. The symbol commonly used for the integers is $\mathbb{Z}$. The 'Z' coming from the German word, Zahlen, meaning numbers.

> **Definition of the integers**
>
> $\mathbb{Z} = \{... - 3, -2, -1, 0, 1, 2, 3...\}$

number line diagram

Since the integers contain the natural numbers you can say $\mathbb{N} \subseteq \mathbb{Z}$.

Rational numbers

The natural numbers and the integers let you count in whole numbers, but what if you need to split a whole number. Imagine you have a cake you want to split between three people, for this you need the rationals. The rational numbers are all numbers that can be described as fractions of integers. The symbol used for the rational numbers is $\mathbb{Q}$, 'Q' coming from quotient.

> **Definition of the rational numbers**
>
> $\mathbb{Q} = \{a/b : a, b \in \mathbb{Z} \quad and \quad b \neq 0\}$

It's **very important** here that you don't allow b to be 0, as you can't divide by 0.

> **Example 4**
>
> Here are some examples of rational numbers, $-3/7 \in \mathbb{Q}, \quad -0.5 \in \mathbb{Q}, \quad 2 \in \mathbb{Q}$.

The rational numbers contain the integers, and, by extension, the natural numbers.

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$.

number line diagram

Irrational numbers

The numbers you have so far are sufficient for counting positively and negatively, and dividing, but what about measuring. How would you measure the diagonal of a square? For this you need the irrationals.

The irrational numbers are all numbers that can't be expressed as a fractions for integers. A group of ancient Greek mathematicians, the Pythagoreans, believed all numbers where rational. Legend has it that when a member of their order, Hippasus of Metapontum, proved that $\sqrt{2}$ is not rational, his brothers left him to drown in the Mediterranean as punishment.

Hippasus was, off course, correct. $\sqrt{2}$ Cannot be described as a fraction of integers, you can read a proof of this here, **Proof: The square root of 2 is irrational.** In fact there are lots more irrational numbers. There are actually more irrational numbers than rational.

> **Example 5**
>
> Here are some more examples of irrational numbers: π, $-\sqrt{3}$, e.

number line diagram

Real numbers

Together, the rational numbers and the irrational numbers describe what you call the real numbers. The symbol used for the real numbers is $\mathbb{R}$.

The real numbers then, necessarily contain all of the above number types: the natural numbers, the integers, the rationals, and the irrationals.

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$.

number line diagram

The name, real numbers, is attributed to René Descartes, and serves to distinguish them from another set of numbers you can read about below.

Complex numbers

The numbers you have so far will let you count, measure, order, and perform some calculations. But how would you solve a problem like $x^2 + 1 = 0$? For more on this type of equation please read **Guide: Introduction to quadratic equations.**

The complex numbers came about while 16th century Italian mathematician, Girolamo Cardano, wrestled with solutions to cubic equations, that is, equations of the form $ax^3 + bx^2 +$

$cx + d$. He ran into the problem of having to square root a negative number. He defined the $\sqrt{-1}$ to be i, the imaginary number, it's name evidencing some of the mocking this faced at the time.

Complex numbers are numbers of the form $a + bi$, where a and b are real numbers. You can think of complex numbers as having a real part and an imaginary part, though either or both of these can be zero. The symbol used for the complex numbers is $\mathbb{C}$.

Definition of the complex numbers

$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$

Example 6

The following are examples of complex numbers: $1 + i \in \mathbb{C}, \quad 5 - 7i \in \mathbb{C}, \quad 4/5 \in \mathbb{C}, \quad \sqrt{3}i \in \mathbb{C}, \quad 0 \in \mathbb{C}, \quad -17/3 + 7/8i \in \mathbb{C}.$

The complex numbers contain all of the above number sets. $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$.

You can read more about complex numbers at **Guide: Introduction to complex numbers.**

insert here argan diagram, plots of examples?

then have here an demonstration of the nesting of these sets

Further reading

For more on this topic, please go to **Guide: Introduction to complex numbers** and **Guide: Arithmetic on complex numbers.**

Version history

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