Answers: Integration by substitution

Donald Campbell

Summary

Answers to questions relating to the guide on integration by substitution.

These are the answers to Questions: Integration by substitution.

Please attempt the questions before reading these answers!

Answers

Q1

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int (ax + b)^n \, \mathrm{d}x = \frac{(ax + b)^{n+1}}{a(n + 1)} + C$$

1.1. $\dfrac{1}{8}(2x + 5)^4 + C$ using the substitution $u = 2x + 5$ or the chain rule with $a = 2$.

1.2. $-\dfrac{1}{24}(3 - 4x)^6 + C$ using the substitution $u = 3 - 4x$ or the chain rule with $a = -4$.

1.3. $\dfrac{2}{5}\left(\dfrac{x}{2} - 1\right)^5 + C$ using the substitution $u = \dfrac{x}{2} - 1$ or the chain rule with $a = \dfrac{1}{2}$.

1.4. $-\dfrac{1}{10}(5x - 2)^{-2} + C$ using the substitution $u = 5x - 2$ or the chain rule with $a = 5$.

1.5. $\dfrac{1}{3}(4 - 3x)^{-1} + C$ using the substitution $u = 4 - 3x$ or the chain rule with $a = -3$.

1.6. $-\dfrac{1}{4}(2x + 7)^{-2} + C$ using the substitution $u = 2x + 7$ or the chain rule with $a = 2$.

1.7. $-\dfrac{5}{3}\left(\dfrac{x}{5} + 3\right)^{-3} + C$ using the substitution $u = \dfrac{x}{5} + 3$ or the chain rule with $a = \dfrac{1}{5}$.

1.8. $-(1 - 2x)^{1/2} + C$ using the substitution $u = 1 - 2x$ or the chain rule with $a = -2$.

1.9. $-\dfrac{2}{3}(3x + 4)^{-1/2} + C$ using the substitution $u = 3x + 4$ or the chain rule with $a = 3$.

1.10. $-\dfrac{1}{2}(5 - 6x)^{1/3} + C$ using the substitution $u = 5 - 6x$ or the chain rule with $a = -6$.

Q2

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int \sin(ax + b) \, \mathrm{d}x = -\frac{1}{a}\cos(ax + b) + C$$

$$\int \cos(ax + b) \, \mathrm{d}x = \frac{1}{a}\sin(ax + b) + C$$

2.1. $\sin(x) + C$ using the substitution $u = x$ or the chain rule with $a = 1$.

2.2. $-\dfrac{1}{2}\cos(2x) + C$ using the substitution $u = 2x$ or the chain rule with $a = 2$.

2.3. $\dfrac{5}{6}\sin(x) + C$ using the substitution $u = x$ or the chain rule with $a = 1$.

2.4. $\dfrac{1}{3}\sin(3x) + C$ using the substitution $u = 3x$ or the chain rule with $a = 3$.

2.5. $-3\cos\left(\dfrac{x}{3}\right) + C$ using the substitution $u = \dfrac{x}{3}$ or the chain rule with $a = \dfrac{1}{3}$.

2.6. $\dfrac{4}{15}\sin\left(3x - \dfrac{\pi}{4}\right) + C$ using the substitution $u = 3x - \dfrac{\pi}{4}$ or the chain rule with $a = 3$.

2.7. $-\dfrac{9}{4}\cos\left(\dfrac{\pi}{3} - \dfrac{4x}{9}\right) + C$ using the substitution $u = \dfrac{\pi}{3} - \dfrac{4x}{9}$ or the chain rule with $a = -\dfrac{4}{9}$.

2.8. $-\dfrac{1}{6}\sin\left(3x + \dfrac{\pi}{2}\right) + C$ using the substitution $u = 3x + \dfrac{\pi}{2}$ or the chain rule with $a = 3$.

2.9. $-16\cos\left(\dfrac{x}{4} - \dfrac{\pi}{2}\right) + C$ using the substitution $u = \dfrac{x}{4} - \dfrac{\pi}{2}$ or the chain rule with $a = \dfrac{1}{4}$.

2.10. $-\dfrac{3}{25}\sin\left(\dfrac{\pi}{6} - 5x\right) + C$ using the substitution $u = \dfrac{\pi}{6} - 5x$ or the chain rule with $a = -5$.

Q3

In these questions, you can either use an appropriate $u = ax + b$ substitution or use the chain rule for integration

$$\int e^{ax+b}\, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{1}{ax+b}\, dx = \frac{1}{a}\ln|ax+b| + C$$

3.1. $\dfrac{5}{2}e^{2x+1} + C$ using the substitution $u = 2x + 1$ or the chain rule with $a = 2$.

3.2. $-\dfrac{7}{3}e^{-3x+4} + C$ using the substitution $u = -3x + 4$ or the chain rule with $a = -3$.

3.3. $\dfrac{1}{3}e^{-3(x-2)} + C$ using the substitution $u = -3(x-2)$ or the chain rule with $a = -3$.

3.4. $6\exp\left(\dfrac{x}{3} - 5\right) + C$ using the substitution $u = \dfrac{x}{3} - 5$ or the chain rule with $a = \dfrac{1}{3}$.

3.5. $2\ln|3x - 7| + C$ using the substitution $u = 3x - 7$ or the chain rule with $a = 3$.

3.6. $-2\ln|5 - 2x| + C$ using the substitution $u = 5 - 2x$ or the chain rule with $a = -2$.

3.7. $\dfrac{3}{2}\ln|2x + 5| + C$ using the substitution $u = 2x + 5$ or the chain rule with $a = 2$.

3.8. $-\dfrac{3}{5}\ln|5(x-2) + 1| + C$ using the substitution $u = 5(x-2) + 1$ or the chain rule with $a = 5$.

Q4

4.1. $\dfrac{1}{5}(3x^2 + 2)^5 + C$ using the substitution $u = 3x^2 + 2$.

4.2. $\dfrac{1}{4}(5x - 7)^4 + C$ using the substitution $u = 5x - 7$.

4.3. $\exp(4x^2 - 1) + C$ using the substitution $u = 4x^2 - 1$.

4.4. $-\dfrac{1}{x^2 + x + 5} + C$ using the substitution $u = x^2 + x + 5$.

4.5. $\sin(3x^2 + 2) + C$ using the substitution $u = 3x^2 + 2$.

4.6. $\exp(x^2 + 3x) + C$ using the substitution $u = x^2 + 3x$.

4.7. $-\dfrac{1}{\sqrt{x^2 + 1}} + C$ using the substitution $u = x^2 + 1$.

4.8. $\dfrac{1}{10}\ln|2e^{5x} + 3| + C$ using the substitution $u = 2e^{5x} + 3$.

4.9. $-\cos(4 - 2x^2) + C$ using the substitution $u = 4 - 2x^2$.

4.10. $-\dfrac{1}{x^3 + 1} + C$ using the substitution $u = x^3 + 1$.

Version history and licensing

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.