Questions: Integration by substitution

Donald Campbell

Summary

A selection of questions for the study guide on integration by substitution.

Before attempting these questions, it is highly recommended that you read [Guide: Integration by substitution].

Q1

Using an appropriate substitution or the chain rule for integration, integrate the following expressions with respect to \boldsymbol{x} .

1.1.
$$\int (2x+5)^3 dx$$

1.2.
$$\int (3-4x)^5 dx$$

1.3.
$$\int \left(\frac{x}{2} - 1\right)^4 dx$$

1.4.
$$\int (5x-2)^{-3} dx$$

1.5.
$$\int (4-3x)^{-2} \, \mathrm{d}x$$

1.6.
$$\int (2x+7)^{-3} \, \mathrm{d}x$$

$$1.7. \qquad \int \left(\frac{x}{5} + 3\right)^{-4} \, \mathrm{d}x$$

1.8.
$$\int (1-2x)^{-1/2} \, \mathrm{d}x$$

1.9.
$$\int (3x+4)^{-3/2} \, \mathrm{d}x$$

1.10.
$$\int (5-6x)^{-2/3} \, \mathrm{d}x$$

Q2

Using an appropriate substitution or the chain rule for integration, integrate the following trigonometric functions with respect to x.

2.1.
$$\int \cos(x) \, \mathrm{d}x$$

2.2.
$$\int \sin(2x) \, \mathrm{d}x$$

$$2.3. \qquad \int \frac{5}{6} \cos(x) \, \mathrm{d}x$$

2.4.
$$\int \cos(3x) \, \mathrm{d}x$$

2.5.
$$\int \sin\left(\frac{x}{3}\right) dx$$

2.6.
$$\int \frac{4}{5} \cos \left(3x - \frac{\pi}{4}\right) dx$$

$$2.7. \qquad \int \sin\left(\frac{\pi}{3} - \frac{4x}{9}\right) \, \mathrm{d}x$$

2.8.
$$\int -\frac{1}{2}\cos\left(3x + \frac{\pi}{2}\right) \,\mathrm{d}x$$

2.9.
$$\int 4\sin\left(\frac{x}{4} - \frac{\pi}{2}\right) \,\mathrm{d}x$$

2.10.
$$\int \frac{3}{5} \cos \left(\frac{\pi}{6} - 5x\right) dx$$

Q3

Using an appropriate substitution or the chain rule for integration, integrate the following exponential and reciprocal linear functions with respect to x.

$$3.1. \qquad \int 5e^{2x+1} \, \mathrm{d}x$$

3.2.
$$\int 7e^{-3x+4} dx$$

3.3.
$$\int -e^{-3(x-2)} dx$$

3.4.
$$\int 2\exp\left(\frac{x}{3} - 5\right) \mathrm{d}x$$

$$3.5. \qquad \int \frac{6}{3x - 7} \, \mathrm{d}x$$

$$3.6. \qquad \int \frac{4}{5 - 2x} \, \mathrm{d}x$$

$$3.7. \qquad \int \frac{3}{2x+5} \, \mathrm{d}x$$

3.8.
$$\int -\frac{3}{5(x-2)+1} \, \mathrm{d}x$$

Q4

Using an appropriate substitution, integrate the following functions with respect to x. Express your answers in terms of x only.

4.1.
$$\int 6x(3x^2+2)^4 \, \mathrm{d}x$$

4.2.
$$\int 5(5x-7)^3 \, \mathrm{d}x$$

4.3.
$$\int 8x \exp(4x^2 - 1) dx$$

4.4.
$$\int \frac{2x+1}{(x^2+x+5)^2} \, \mathrm{d}x$$

$$4.5. \qquad \int 6x \cos(3x^2 + 2) \, \mathrm{d}x$$

4.6.
$$\int (2x+3) \exp(x^2 + 3x) \, \mathrm{d}x$$

4.7.
$$\int \frac{x}{(x^2+1)^{3/2}} \, \mathrm{d}x$$

4.8.
$$\int \frac{e^{5x}}{2e^{5x} + 3} dx$$

4.9.
$$\int -4x \sin(4-2x^2) dx$$

4.10.
$$\int \frac{3x^2}{(x^3+1)^2} \, \mathrm{d}x$$

After attempting the questions above, please click this link to find the answers.

Version history and licensing

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

This work is licensed under CC BY-NC-SA 4.0.