

# Factsheet: Gamma distribution

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## Summary

A factsheet for the gamma distribution.

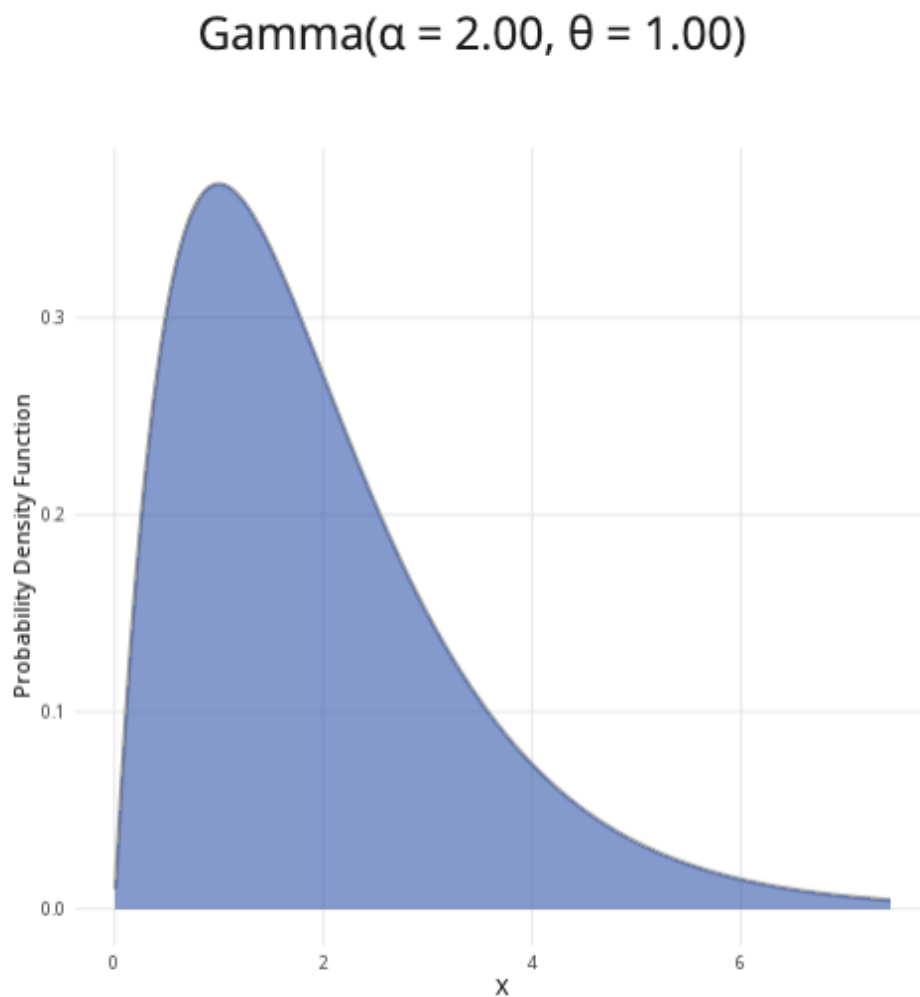


Figure 1: An example of the gamma distribution with  $\alpha = 2$  and  $\theta = 1$ .

**Where to use:** The gamma distribution generalizes the exponential distribution, allowing for greater or lesser variance. It is used to model positive continuous random variables that have skewed distributions.

**Notation:**  $X \sim \text{Gamma}(\alpha, \theta)$  or  $X \sim \text{Gam}(\alpha, \theta)$

**Parameters:** Two real numbers  $\alpha$  and  $\theta$ , which are related to the mean  $\mu$  and variance  $\sigma^2$ :

- $\alpha = \frac{\mu^2}{\sigma^2}$  (shape parameter)
- $\theta = \frac{\sigma^2}{\mu}$  (scale parameter)

Quantity	Value	Notes
<b>Mean</b>	$\mathbb{E}(X) = \alpha\theta$	
<b>Variance</b>	$\mathbb{V}(X) = \alpha\theta^2$	
<b>PDF</b>	$\mathbb{P}(X = x) = \frac{x^{\alpha-1} \exp(-\frac{x}{\theta})}{\Gamma(\alpha)\theta^\alpha}$	$\Gamma(x)$ the gamma function of $x$
<b>CDF</b>	$\mathbb{P}(X \leq x) = \frac{\text{Gam}(\alpha, \frac{x}{\theta})}{\Gamma(\alpha)}$	$\text{Gam}(\alpha, \theta)$ is the PDF of the gamma distribution

**Example:** You collect historical data on the time to failure of a machine from Cantor's Confectionery. The mean is 83 days and the variance is 50.3. You can then use this to estimate the shape and scale parameters of the gamma distribution:

- $\alpha = \frac{83^2}{50.3} = 136.958250497 \approx 137$
- $\theta = \frac{50.3}{83} = 0.60602409638 \approx 0.61$

The distribution can be expressed as  $X \sim \text{Gam}(137, 0.61)$ , where the shape parameter is 137 and the scale parameter is 0.61.

## Further reading

This interactive element appears in [Overview: Probability distributions](#). Please click this link to go to the guide.

## Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

- v1.1: moved to factsheet form and populated with material from [Overview: Probability distributions](#) by tdhc.

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