Questions: Multivariate implicit differentiation

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Summary

A selection of questions for the study guide on multivariate implicit differentiation.

Before attempting these questions, it is highly recommended that you read Guide: Multivariate implicit differentiation.

Q1

Decide whether the following equations represent explicit or implicit functions.

1.1.
$$x^3 + y^3 - 6xy = 0$$

1.2.
$$y = 3x^2 + 2x - 5$$

1.3.
$$y = \frac{4x}{x-1}$$

1.4.
$$x^2 + y^2 - 9 = 0$$

1.5.
$$y = \sqrt{x^2 + 4}$$

1.6.
$$x^2 + y^2 - 25 = 0$$

1.7.
$$x + y = 4$$

1.8.
$$x^2 + y^2 + 2xy = 4$$

1.9.
$$x^2 - 3y = 7$$

$$1.10. \quad x^3 + 2xy + y^2 = 10$$

Q2

Use the multivariate implicit differentiation rule to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for each equation of the form w=f(x,y)=0, where y is defined implicitly as a function of x.

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$$2.1. \quad x^2 + y^2 - 25 = 0$$

2.2.
$$x^3y + y^3 - 7 = 0$$

2.3.
$$x^2 - \frac{3y+2}{y-1} = 0$$

2.4.
$$\sin(xy) + x = y$$

2.5.
$$xe^y + y^2 = 4$$

$$2.6. \quad x^2y - 3xy^2 + 5 = 0$$

2.7.
$$ln(x) + ln(y) = 1$$

$$2.8. \quad \tan^{-1}\left(\frac{y}{x}\right) - x^2 = 0$$

$$2.9. \quad y^3 + \cos(xy) = x$$

2.10.
$$x \sin(y) + y \cos(x) = 0$$

$$2.11. \quad x^2 + 2xy + y^2 - 1 = 0$$

2.12.
$$e^{xy} + x - y = 0$$

$$2.13. \quad x^3 + y^3 - 3xy - 7 = 0$$

2.14.
$$\sqrt{x} + y^2 - 3 = 0$$

2.15.
$$\frac{x+y}{x-y} - \ln(x) = 0$$

Q3

Use the multivariate implicit differentiation rule to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for each equation of the form w=f(x,y,z)=0, where z is defined implicitly as a function of x and y.

$$3.1. \quad x^2 + y^2 + z^2 = 1$$

3.2.
$$4x^2 + 3y^2z^2 - 5x^2y + 2z^3 - 7 = 0$$

$$3.3. \quad xyz = 1$$

3.4.
$$e^{xz} + y - z = 0$$

3.5.
$$\sin(xz) + \cos(yz) - 2 = 0$$

3.6.
$$\ln(x) + \ln(y) + \ln(z) - 1 = 0$$

$$3.7. \quad x^3 + y^3 + z^3 - 3xyz = 0$$

3.8.
$$x^2z + y^2z + \sqrt{z} - 4 = 0$$

3.9.
$$e^x + y^2z - \tan^{-1}(z) = 0$$

3.10.
$$\ln(xz) + xy - z = 0$$

3.11.
$$xe^{yz} + ye^{xz} - 5 = 0$$

3.12.
$$\sin(x)\cos(z) + yz^2 - 1 = 0$$

3.13.
$$x^2 + ye^z + z = 0$$

3.14.
$$\frac{x+y}{z} + \ln(z) - 3 = 0$$

$$3.15. \quad \sqrt{xyz} + x - y - z = 0$$

After attempting the questions above, please click this link to find the answers.

Version history and licensing

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

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