

# Answers: Multivariate chain rule

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## Summary

Answers to questions relating to the guide on the multivariate chain rule.

These are the answers to [Questions: Multivariate chain rule](#).

**Please attempt the questions before reading these answers!**

## Answers

### Q1

$$1.1. \quad \frac{dz}{dt} = 2e^{2t} \sin(t)(\cos(t) + \sin(t)).$$

$$1.2. \quad \frac{dz}{dt} = \frac{3}{t} - \tan(t).$$

$$1.3. \quad \frac{dz}{dt} = \frac{3}{2}\sqrt{t} + 6t(t^2 + 1)^2.$$

$$1.4. \quad \frac{dz}{dt} = \exp(t \ln(t + 1)) \left( \ln(t + 1) + \frac{t}{t + 1} \right) = (t + 1)^t \left( \ln(t + 1) + \frac{t}{t + 1} \right).$$

$$1.5. \quad \frac{dz}{dt} = 2t \cos(t) \sec^2(t^2) - \sin(t) \tan(t^2).$$

$$1.6. \quad \frac{dz}{dt} = 15 \cos(t)(2t - 1 + 25 \sin^2(t)) + 8t - 4 + 30 \sin(t).$$

$$1.7. \quad \frac{dz}{dt} = \frac{2t}{t - 2} - \frac{t^2 + 1}{(t - 2)^2} = \frac{t^2 - 4t - 1}{(t - 2)^2}.$$

$$1.8. \quad \frac{dz}{dt} = 0.$$

$$1.9. \quad \frac{dz}{dt} = t^2 e^t (t^4 + 6t^3 + e^t (2t + 3)).$$

$$1.10. \quad \frac{dz}{dt} = \frac{2}{t} + te^{-t}(2 - t).$$

$$1.11. \quad \frac{dz}{dt} = 4t(2 \ln(t) + 1).$$

$$1.12. \quad \frac{dz}{dt} = 3(t^3 + 1)(2t^2 \sin(3t) + (t^3 + 1) \cos(3t)).$$

$$1.13. \quad \frac{dz}{dt} = e^{\cosh(t)} \sinh(t) + e^{\sinh(t)} \cosh(t).$$

$$1.14. \quad \frac{dz}{dt} = \frac{1}{t^2 + 1}.$$

$$1.15. \quad \frac{dz}{dt} = \frac{\exp(\sqrt{t})}{t+2} + \frac{\ln(t+2)\exp(\sqrt{t})}{2\sqrt{t}}.$$

## Q2

$$2.1. \quad \frac{\partial z}{\partial s} = 2(s+t)(2s^2 + st - t^2) \text{ and } \frac{\partial z}{\partial t} = 2(s+t)(s^2 - st - 2t^2).$$

$$2.2. \quad \frac{\partial z}{\partial s} = 1 \text{ and } \frac{\partial z}{\partial t} = \frac{\cos(t) - \sin(t)}{\cos(t) + \sin(t)}.$$

$$2.3. \quad \frac{\partial z}{\partial s} = 3t(s^2t^2 - 2s - t) \text{ and } \frac{\partial z}{\partial t} = 3s(s^2t^2 - s - 2t).$$

$$2.4. \quad \frac{\partial z}{\partial s} = 2st \exp(s^2) \text{ and } \frac{\partial z}{\partial t} = \exp(s^2).$$

$$2.5. \quad \frac{\partial z}{\partial s} = \sin(st) + t(s - t^2) \cos(st) \text{ and } \frac{\partial z}{\partial t} = -2t \sin(st) + s(s - t^2) \cos(st).$$

$$2.6. \quad \frac{\partial z}{\partial s} = 2 \sin(s) \cos(s) \cos(2t) \text{ and } \frac{\partial z}{\partial t} = 2 \sin(t) \cos(t) \cos(2s).$$

$$2.7. \quad \frac{\partial z}{\partial s} = 4s + 2t \text{ and } \frac{\partial z}{\partial t} = 2s.$$

$$2.8. \quad \frac{\partial z}{\partial s} = \frac{1}{s+t} - \frac{1}{s} \text{ and } \frac{\partial z}{\partial t} = \frac{1}{s+t} - \frac{1}{t}.$$

$$2.9. \quad \frac{\partial z}{\partial s} = (2s+1) \sec^2(s^2 + s + t^2 - t) \text{ and } \frac{\partial z}{\partial t} = (2t-1) \sec^2(s^2 + s + t^2 - t).$$

$$2.10. \quad \frac{\partial z}{\partial s} = -\frac{2t}{s^2 + t^2} \text{ and } \frac{\partial z}{\partial t} = \frac{2s}{s^2 + t^2}.$$

## Q3

$$3.1. \quad \frac{\partial w}{\partial s} = 2s(t^2 + 2) \text{ and } \frac{\partial w}{\partial t} = 2t(s^2 + 2).$$

$$3.2. \quad \frac{\partial w}{\partial s} = t(2s + t + u) \text{ and } \frac{\partial w}{\partial t} = s(s + 2t + u) + 1 \text{ and } \frac{\partial w}{\partial u} = st + 1.$$

$$3.3. \quad \frac{\partial w}{\partial s} = 2st^2 \cos(s^2t^2) - \sin(s+t) \text{ and } \frac{\partial w}{\partial t} = 2s^2t \cos(s^2t^2) - \sin(s+t).$$

$$3.4. \quad \frac{\partial w}{\partial s} = 4s + 4u \text{ and } \frac{\partial w}{\partial t} = 4t \text{ and } \frac{\partial w}{\partial u} = 4s + 4u.$$

## **Version history and licensing**

v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

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