Answers: Law of total probability and Bayes’ theorem

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Summary

Answers to questions relating to the guide on the law of total probability and Bayes’ theorem.

*These are the answers to* [*Questions: Law of total probability and Bayes’ theorem*](../questions/qs-bayestheorem.qmd)*.*

**Please attempt the questions before reading these answers.**

## Q1

#### 1.1.

You know:



Using the law of total probability:

So the probability that a randomly chosen patient recovers is .

#### 1.2.

You know:

* ,
* ,
* ,

Using the law of total probability:

So the probability that a randomly chosen student finishes their lunch is .

#### 1.3.

You know:

* ,
* ,
* ,

Using the law of total probability:

So the probability that a randomly selected product is defective is .

#### 1.4.

You know:

* ,
* ,
* ,

Using the law of total probability:

So the probability that the student completes their homework is .

## Q2

#### 2.1.

You know:

* (where means the person does not have the disease)

Using the law of total probability:

Now applying Bayes’ theorem:

So the probability that the person has the disease, given that they test positive, is approximately . Not a very good test!

#### 2.2.

You know:



Using the law of total probability:

Then applying Bayes’ theorem gives:

So the probability that it actually rains in St Andrews, given that the forecast predicts rain, is approximately .

#### 2.3.

You know:



Using the law of total probability:

Then applying Bayes’ theorem gives:

So the probability that the broken biscuit came from Machine B, given that it is broken, is approximately .

#### 2.4.

You know:



Using the law of total probability:

Then applying Bayes’ theorem gives:

So the probability that the sweet is red, given that it has a wrapper, is approximately .

## Version history and licensing

v1.0: initial version created 05/25 by Sophie Chowgule as part of a University of St Andrews VIP project.

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