Proof: The square root of 2 is irrational

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Summary

Proof by contradiction of the irrationality of

Before reading this proof sheet, it is recommended that you read **Overview: Number sets.**

# Proof

You might remember from **Overview: Number sets** that an irrational number is a number that cannot be represented as a fraction of integers, here you can prove that is irrational. This particular proof dates back to the ancient Greeks and and relies on a method of proof called proof by contradiction. In a proof by contradiction you begin by assuming that what you’re tying to prove is false, then you show that from that assumption you can derive a contradiction, so your assumption must have been false.

Let’s prove that is irrational by contradiction.

Suppose is rational. Then it can be expressed as a fraction:

where and are integers with no common factors other than 1, meaning the fraction is in its simplest form, and .

Then you can square both sides:

Then multiply both sides by :

This implies that is even (since it is divisible by 2). Since the square of an odd number is odd, must be even. Let for some integer .

You can then substitute into the equation:

Dividing both sides by 2:

This shows that is also even, which means must be even.

Since both and are even, they share a common factor of 2, contradicting your assumption that and have no common factors other than 1.

This contradiction implies that your initial assumption was false, cannot be written as a fraction of integers, and so, is irrational.

## Version history

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