Proof: the quadratic formula

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Summary

An explanation as to why the quadratic formula is true.

*Before reading this proof sheet, it is recommended that you read* [*Guide: Completing the square*](../studyguides/completingthesquare.qmd)*.*

# Proof of the quadratic formula

Remember from [Guide: Using the quadratic formula](../studyguides/quadraticformula.qmd) that the **quadratic formula** is used to find roots of any quadratic equation:

|  |
| --- |
| The quadratic formula |
| Let be a quadratic equation (where ). The roots to this quadratic equation are given by  where one of the roots is given by the term and the other given by the term . |

In order to prove that these really are the solutions to the quadratic, you can **complete the square** on using the fact that . See [Guide: Completing the square](../studyguides/completingthesquare.qmd) for why this works.

## Proof of the quadratic formula

First of all, as you can divide through by to get

Taking the term over to the other side gives

Completing the square (see [Guide: Completing the square](../studyguides/completingthesquare.qmd)) gives

You can rearrange to get

Now the result is starting to come together. Taking square roots of both sides (not forgetting that it could be positive or negative) gives

and rearranging gives

as required.

# Further reading

[Guide: Using the quadratic formula](../studyguides/quadraticformula.qmd)

[Questions: Using the quadratic formula](../questions/qs-quadraticformula.qmd)

## Version history and licensing

v1.0: created in 04/24 by tdhc.

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