Questions: PMFs, PDFs, and CDFs

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Summary

A selection of questions to test your understanding of probability mass functions (PMFs), probability density functions (PDFs), and cumulative distribution functions (CDFs).

*Before attempting these questions it is highly recommended that you read* [*Guide: PMFs, PDFs, and CDFs*](../studyguides/pmfspdfscdfs.qmd)*.*

## Q1

For each of the scenarios below, determine if the given distribution is a valid PMF and answer the following questions.

#### 1.1.

Let be the random variable representing the result of rolling a biased four sided-die. The PMF of is given by:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |

What is ?

#### 1.2.

A discrete random variable has five possible outcomes ( or ), and the PMF is given by:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |

What is the probability of or ?

#### 1.3.

A coin is tossed, where the probability of tails is and heads is . Let represent the result of the coin toss. Complete the table below:

|  | Heads | Tails |
| --- | --- | --- |
|  |  |  |

#### 1.4.

A discrete random variable has the possible outcomes or , with the following PMF:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |

Is this a valid PMF? Justify your answer either way.

#### 1.5.

A bag contains red, blue, and green sweets from a sweet shop. Let represent the color of a randomly picked sweet:

1. What is the probability of picking a blue sweet?
2. Construct the PMF for this scenario by completing the table:

|  | Red | Blue | Green |
| --- | --- | --- | --- |
|  |  |  |  |

#### 1.6.

The PMF for a random variable is given as:

|  | 1 | 2 | 3 | 4 |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |

1. For what value of is this a valid PMF?
2. For this value of , what is ?

## Q2

For each of the scenarios below, determine if the given distribution is a valid PDF and answer the following questions.

#### 2.1.

Let be a continuous random variable on the interval with the PDF:

What is the probability that lies between 1 and 2?

#### 2.2.

Let be a continuous random variable with the PDF:

1. What is the probability that lies between and ?
2. What is ?

#### 2.3.

Let be a continuous random variable uniformly distributed between and . The PDF is:

What is the probability that lies between and ?

#### 2.4.

The PDF of a random variable is given by:

Is this a valid PDF? Justify your answer either way.

#### 2.5.

Consider the PDF:

1. For what value of is this a valid PDF?
2. For this value of , what is ?

#### 2.6.

The PDF of is given by:

Is this a valid PDF? Justify your answer either way.

## Q3

For each of the scenarios below, answer the following questions.

#### 3.1.

In a scenario involving a discrete random variable, the following CDF is given:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |

1. What is ?
2. What is ?

#### 3.2.

For the random variable uniformly distributed on as seen in Q2.2:

1. Calculate the CDF at values , , and .
2. What is ?

#### 3.3.

For the PDF given in Q2.3:

1. Calculate the CDF at points , , and .
2. What is ?

#### 3.4.

The CDF of for a scenario is given by:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |

Is this a valid CDF? Justify your answer either way.

[After attempting the questions above, please click this link to find the answers.](../answers/as-pmfspdfscdfs.qmd)

## Version history and licensing

v1.0: initial version created 12/24 by Sophie Chowgule as part of a University of St Andrews VIP project.

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